Liquidity Premiums on Government Debt and the Fiscal Theory of the Price Level

Authors	Aleksander Berentsen, and Christopher J. Waller
Working Paper Number	2017-008A
Creation Date	March 2017
Citable Link	https://doi.org/10.20955/wp.2017.008
Suggested Citation	Berentsen, A., Waller, C.J., 2017; Liquidity Premiums on Government Debt and the Fiscal Theory of the Price Level, Federal Reserve Bank of St. Louis Working Paper 2017-008. URL https://doi.org/10.20955/wp.2017.008

Published In	Journal of Economic Dynamics & Control
Publisher Link	https://doi.org/10.1016/j.jedc.2018.01.038

Liquidity Premiums on Government Debt and the Fiscal Theory of the Price Level*

Aleksander Berentsen
University of Basel and FRB St. Louis

Christopher Waller
FRB-St. Louis

March 2017

Abstract

We construct a dynamic general equilibrium model where agents use nominal government bonds as collateral in secured lending arrangements. If the collateral constraint binds, agents price in a liquidity premium on bonds that lowers the real rate on bonds. In equilibrium, the price level is determined according to the fiscal theory of the price level. However, the market value of government debt exceeds its fundamental value. We then examine the dynamic properties of the model and show that the market value of the government debt can fluctuate even though there are no changes to current or future taxes or spending. The price dynamics are driven solely by the liquidity premium on the debt.

JEL Codes: E31, E62
Keywords: Price level, Fiscal Policy, Liquidity

*The views in this paper are those of the authors and do not represent those of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the FOMC.

1 Introduction

The fiscal theory of the price is a controversial idea that states that fiscal policy, not monetary policy, pins down the aggregate price level. The argument for this follows from the intertemporal government budget constraint which states that the *real* value of the outstanding stock of government debt is pinned down by the discounted stream of real future surpluses.[1] Since the initial stock of *nominal* debt is given, for any fixed stream of surpluses, this theory argues that the price level must adjust to make the real value of the government debt satisfy the intertemporal government budget constraint. Furthermore, dynamic movements of the price level are driven by expected changes in future fiscal policy.

While a substantial debate has occurred over the last two decades regarding the validity of this theory, one assumption is common to both sides of the debate – the sole purpose of government debt is to reallocate taxes across time. However, government debt is commonly used as collateral for secured lending in financial markets. Consequently, its market value may include a liquidity premium that reflects its value for trading in addition to serving as a claim on the stream of future surpluses. This suggests that price level movements may be driven by changes in the liquidity value of government debt rather than changing expectations of fiscal policy.

Our objective in this paper is to construct a model where government debt serves as collateral in secured lending arrangements. We show that if the real value of the government debt is sufficiently high, there is no liquidity premium on the debt and the price level is pinned down in the usual way by the fiscal theory of the price level. However if the real value of government debt is sufficiently low, then collateral constraints bind and the price of government debt reflects a liquidity premium on the debt. In this case, the market value of government debt exceeds its "fundamental value" which is the discounted stream of future surpluses. When the collateral constraint binds, the price level is pinned down by a modified version of the fiscal theory of the price level.

Once the collateral value of government debt is accounted for, we obtain some interesting results that we believe are new to this literature. First, when the collateral constraint binds, any action that increases the real value of government debt relaxes

[1] Original proponents of this theory are Begg and Haque (1984), Auernheimer and Contreras (1990) Leeper (1991), Sims (1994), Woodford (1995) and Cochrane (1998). For a fulsome survey of the literature see Leeper and Leith (2015).

the collateral constraint and expands economic activity in the secured lending sector. For example, an increase in future taxes or a cut in government spending, raises the fundamental value of the government debt and thus the real value of the debt. This then leads to an expansion of economic activity via an increase in secured lending. As a result, surprisingly, raising taxes or cutting spending government spending are "stimulative".

Second, out of steady state, the dynamics of the price level are determined by changes in the liquidity premium on government debt. Thus, we can generate movements in the price level even though there are no expected changes in current or future fiscal policy. As a result, our model can help understand observed movements in the price level when there appears to be no change in perceived fiscal policy.

2 Environment

Time is discrete, the horizon is infinite and there is a measure one of infinitely-lived agents who consume perishable goods and discount only across periods at rate $\beta = 1/(1+r)$.[2] In each period agents engage in two sequential rounds of trade, denoted by "DM" and "CM". Markets differ in terms of economic activities and preferences. In the DM, a measure 1 of agents are always consumers and a measure 1 are always producers. We call the former agents consumers and the latter producers. Consumers get utility $u(q)$ from consuming $q > 0$ consumption, with $u'(q) > 0$, $u''(q) < 0$, $u'(0) = +\infty$, and $u'(\infty) = 0$. Producers incur a utility cost $c(q) \geq 0$ to produce q goods, with $c'(q) > 0$, $c''(q) \geq 0$ and $c'(0) = 0$. We assume that trade in the DM is pairwise and agents are matched randomly with probability σ.

In the CM all agents can consume and produce. Following Lagos and Wright (2005), in market two every agent of type j has quasilinear preferences $U(x) - h$, where the first term is utility from x consumption, and the second is disutility from h labor. We assume $U'(x) > 0$, $U''(x) \leq 0$, $U'(0) = +\infty$ and $U'(\infty) = 0$. For ease of notation we denote period $t + s$ values with a subscript $+s$ for all s.

[2] The basic framework builds on Lagos and Wright (2005).

3 Efficient Allocation

In order to define the efficient allocation, we consider a benevolent planner who treats agents identically and maximizes their lifetime utilities. The planner problem is given by

$$\underset{x,q}{Max}\ \frac{1}{1-\beta}\left\{U(x)-x+\sigma\left[u\left(q\right)-c\left(q\right)\right]\right\}$$

In the constrained-efficient allocation marginal consumption utility equals marginal production disutility in each market. Such allocation is therefore defined by $u'\left(q^{*}\right)=c'\left(q^{*}\right)$ in the DM and $U'(x)=1$ in the CM.

4 Decentralized Allocation

There is a government that issues nominal one-period discount bonds that specify a payout of one unit of an arbitrary numeraire. There is an initial nominal bond stock $B_0>0$. The government also collects lump-sum taxes T_t in the CM and buys G_t units of CM goods. Let $S_t\equiv T_t-G_t$ denote the government surplus in period t. The government's budget constraint is given by

$$\phi\rho B_{+1}=\phi B-S \tag{1}$$

where we assume all government spending is wasteful.

Government bonds are not portable across sub-periods. The key friction is limited commitment in the DM, which precludes private unsecured credit arrangements. However, secured credit arrangements are feasible. We assume, similar to Kiyotaki-Moore (1997), that government bonds can be pledged to finance the purchase of DM goods. DM consumers can credibly pledge to deliver a proportion θ of their CM bond holdings to pay for their purchase of DM goods.

Terms of trade in each round of trade are determined as follows. We assume that DM consumers make take-it-or-leave-it offers to producers while, in the CM, prices are determined in competitive Walrasian markets. Let ϕ denote the goods price for a unit of numeraire.

4.1 CM Trade

During the centralized market buyers also choose how much to consume and work but also how many bonds to carry into the next period. Pledged repayment of secured loans from the previous DM must also be settled. Let $j = c, p$ denote an agent's economic type in the DM. Furthermore, let $W(b^j, \ell^j)$ denote the value function for agent of type j coming into the CM with b^j units of government bonds and ℓ_j units outstanding secured loans where $\ell^j > 0$ means that the agent has borrowed. For consumers $\ell^c \geq 0$ and for producers $\ell^p \leq 0$. Let $V_j(b_{+1})$ denote the value function from entering the next period with b_{+1} units of bonds. Hence, CM problem for an agent is

$$W\left(b^j, \ell^j\right) = \max_{x,h,z_{+1}} \left\{U(x) - h + \beta V^j\left(b^j_{+1}\right)\right\} \tag{2}$$
$$s.t.\;\; x + \phi\rho b_{+1} = h + \phi\left(b^j - \ell^j\right) - \tau$$

where ρ is the numeraire price at time t for a bond maturing at $t+1$. The first-order conditions yield

$$U'(x) = 1 \tag{3}$$
$$\beta V^b_j\left(b^j_{+1}\right) \leq \phi\rho \quad (= 0 \;\; if \;\; b_{+1} > 0) \tag{4}$$

where $V^{jb}\left(b^j_{+1}\right)$ is the partial derivative of the value function $V^j\left(b^j_{+1}\right)$ with respect to b^j_{+1}. The envelope conditions satisfy

$$W^b\left(b^j, \ell^j\right) = \phi; \quad W^\ell\left(b, \ell^c\right) = -W^\ell\left(b, \ell^p\right) = -\phi,$$

where $W^b(b^j, \ell^j)$ is the partial derivative of the value function with respect to b.

4.2 DM Trade

The DM value functions are given by

$$\begin{aligned} V^p\left(b^p\right) &= \sigma\left[-c\left(q\right) + W\left(b^p, \ell^p\right)\right] + (1-\sigma)\, W\left(b^p, 0\right) \\ V^c\left(b^c\right) &= \sigma\left[u(q) + W\left(b^c, \ell^c\right)\right] + (1-\sigma)\, W\left(b^c, 0\right) \end{aligned}$$

When a DM consumer is paired with a DM seller, he makes an offer of $\ell^c = -\ell^p$

units of nominal bonds to be delivered in the next CM in return for q units of the DM good. This has to yield a payoff that is higher than simply walking away and entering the CM with $\ell^c = 0$. Thus, upon being matched the DM consumer's problem is

$$\max_{q,\ell^c} u(q) + W(b^c, \ell^c) - W(b^c, 0)$$
$$\text{s.t.} \quad \phi\ell^c \leq \phi\theta b_c \quad \text{and} \quad - c(q) + W(b^p, -\ell^c) - W(b^p, 0) \geq 0$$

Due to the quasi-linearity of CM utility we have $W(b^c, \ell^c) - W(b^c, 0) = -\phi\ell^c$ and $W(b^p, -\ell^c) - W(b^p, 0) = \phi\ell^p$. Thus, we can simplify the consumer problem as follows:

$$\max_{q} u(q) - \phi\ell^c$$
$$\text{s.t.} \quad \phi\ell^c \leq \theta\phi b \quad \text{and} \quad - c(q) + \phi\ell^c \geq 0$$

Under TIOLI the buyer's offer ensures that the last constraint binds with equality. Consequently, we have

$$\max_{q} u(q) - c(q) \quad \text{s.t.} \quad c(q) \leq \theta\phi b \tag{5}$$

Letting λ denote the Lagrangian multiplier on the constraint, the FOC yields

$$u(q) - (1 + \lambda) c'(q) = 0$$

If $\lambda = 0$, the consumer has more than enough bonds than he needs to pledge in order to obtain the first-best allocation $q = q^*$ and $c(q^*) < \theta\phi b$. If $\lambda > 0$, the consumer does not have enough bonds to obtain the first-best and so we have $q < q^*$ and $c(q) = \theta\phi b$.

Given these results and using the DM value functions for each type of DM agent we obtain

$$\begin{aligned} V^{pb}(b^p) &= \phi \\ V^{cb}(b^c) &= \sigma u'(q) \frac{\partial q}{\partial b^c} + \phi \left(1 - \sigma \frac{\partial \ell^c}{\partial b^c}\right). \end{aligned}$$

The producers have no payment use for the bonds and will simply carry them as a store of value. For consumers, if $\lambda = 0$ then $V_c'(b^c) = \phi$ and just like the producers, the marginal value of a bond is only for its store of value. However, if $\lambda > 0$ then

$\partial q/\partial b^c = \phi\ell^c/c'(q)$ and $\partial \ell^c/\partial b^c = \theta$ and we obtain

$$V^{cb}(b^c) = \sigma \frac{u'(q)}{c'(q)}\theta\phi + \phi(1-\sigma\theta).$$

Using (4) one period and updating the expressions above for producers we get

$$\beta\phi_{+1} \leq \phi\rho$$

while for consumers we get

$$\begin{aligned} \beta\phi_{+1} &\leq \phi\rho \;\; if \;\; \lambda = 0 \\ \beta\phi_{+1}[1+L(q_{+1})] &= \phi\rho \;\; if \;\; \lambda > 0. \end{aligned}$$

where

$$L(q_{+1}) \equiv \sigma\theta\left[\frac{u'(q_{+1})}{c'(q_{+1})} - 1\right] \geq 0 \tag{6}$$

is the liquidity premium on government debt. Note that if $\lambda > 0$ then $\beta\phi_{+1} < \phi\rho$ and so DM sellers will never acquire bonds since their real return is less that the time rate of discount.

5 Equilibrium

In equilibrium, when $\lambda = 0$ we have

$$\frac{\phi}{\phi_{+1}}\rho = \beta \text{ and } \phi\rho B_{+1} = \phi B - S$$

and when $\lambda > 0$

$$\begin{aligned} \frac{\phi}{\phi_{+1}}\rho &= \beta[1+L(q_{+1})] \\ \phi\rho B_{+1} &= \phi B - S \end{aligned} \tag{7}$$

From (5), the consumer's budget constraint in the DM satisfies $c(q) = \theta\phi b$. Since in any equilibrium, $b = B$, we obtain that for $\lambda = 0$, the real value of the government debt satisfies

$$\phi B > c(q^*)/\theta.$$

and for $\lambda > 0$ we have

$$\phi B = c\left(q\right)/\theta. \tag{8}$$

Assuming $\lambda_{+s} > 0$ for all s we can write the government's budget constraint (1) as follows:

$$\begin{aligned} \phi\rho B_{+1} &= \phi B - S \\ \frac{\phi}{\phi_{+1}}\rho &= \frac{c\left(q\right) - \theta S}{c\left(q_{+1}\right)} \end{aligned} \tag{9}$$

Combining (7) and (9) yields a dynamic equation in q

$$c\left(q_{+1}\right)\beta\left[1 + L\left(q_{+1}\right)\right] = c\left(q\right) - \theta S \tag{10}$$

Definition 1 *A monetary equilibrium is a sequence for* $\{q_t\}_{t=0}^{\infty}$ *satisfying (10) with* $q_t \in (0, q^*]$ *for all* t*. Assume that* S *is a constant for all* t*. A steady state equilibrium is a* $\bar{q}$ *that solves*

$$c\left(\bar{q}\right)\left\{1 - \beta\left[1 + L\left(\bar{q}\right)\right]\right\} = \theta S \tag{11}$$

Proposition 2 *If* $0 \leq \theta S \leq c\left(q^*\right)\left(1 - \beta\right)$*, a unique monetary equilibrium exists with* $\frac{d\bar{q}}{d\theta S} > 0$.

Proof of Proposition 2. Denote the left-hand side of (11) $\Psi(\bar{q})$. Then

$$\Psi'(\bar{q}) = c'\left(\bar{q}\right)\left\{1 - \beta\left[1 + L\left(\bar{q}\right)\right]\right\} - c\left(\bar{q}\right)\beta L'\left(\bar{q}\right) > 0$$

since

$$L'\left(\bar{q}\right) \equiv \sigma\theta\frac{u''\left(\bar{q}\right)c'\left(\bar{q}\right) - u'\left(\bar{q}\right)c''\left(\bar{q}\right)}{c'\left(\bar{q}\right)^2} < 0.$$

Use (11) to rewrite the derivative as follows:

$$\Psi'(\bar{q}) = c'\left(\bar{q}\right)\theta S/c\left(\bar{q}\right) - c\left(\bar{q}\right)\beta L'\left(\bar{q}\right) > 0$$

From (11), at $\bar{q} = q^*$, we have

$$\Psi(q^*) = c\left(q^*\right)\left(1 - \beta\right)$$

Thus, if $\theta S > c\left(q^*\right)\left(1 - \beta\right)$, $q = q^* \leq \phi B = \theta S$.

Next we need to show that $\lim_{\bar{q}->0} \Psi(\bar{q}) < \theta S$. For $\bar{q} \to 0$ we have

$$\lim_{\bar{q}->0} \Psi(\bar{q}) = \lim_{\bar{q}->0} c(\bar{q}) \{1 - \beta [1 + L(\bar{q})]\} = \lim_{\bar{q}->0} -\beta\sigma\theta \left[\frac{c(\bar{q}) u'(\bar{q})}{c'(\bar{q})}\right] \leq 0.$$

Hence, since $\Psi(q)$ is strictly increasing there exists a unique $\bar{q} \in (0, q^*]$ that solves (11).

Finally, note that

$$\frac{d\bar{q}}{d\theta S} = \frac{1}{\Psi'(\bar{q})} > 0.$$

■

Proposition 1 states under which conditions a steady state equilibrium exists. It is stated under the assumption that the government surplus is non negative. However, an equilibrium can exist for a negative surplus provided that the deficit is not too large. From (8), the steady state real value of debt satisfies,

$$\phi B = c(\bar{q}) / \theta \tag{12}$$

Since q is constant in a steady state equilibrium, the bond price must grow at the same rate at the growth rate of bonds. That is, (12) implies that

$$\frac{\phi}{\phi_{+1}} = \frac{B_{+1}}{B} = 1 + \pi$$

where π is the inflation rate which is equal to the growth rate of the government bonds.

An interesting aspect of Proposition 1 is that changes in fiscal policy that increase current or future primary surpluses *raise* the steady state value of q. In short, raising non-distortionary taxes or cutting government spending actually stimulate economic activity in the DM. The reason is that by raising the fundamental value of government debt, the real value of current debt increases which loosens the collateral constraints in the DM, which in turn lead to an increase in secured lending and consumption of goods. This is clearly a surprising result about the stimulative effects of fiscal policy. But this is a direct implication of the fiscal theory of the price level when government debt carries a liquidity premium.

6 Discussion

For the following discussion, we assume that $c(q) = q$. In this case,the steady state real value of debt satisfies $\phi = \bar{q}/\theta B$, which allows us to rewrite (11) as follows:

$$\overline{\phi B} = \frac{S}{1 - \beta\left[1 + \sigma\theta L\left(\theta\overline{\phi B}\right)\right]}$$

Let $\bar{\beta} \equiv \beta\left[1 + \sigma\theta L\left(\theta\overline{\phi B}\right)\right] \leq 1$ and $\bar{r} \equiv \left(1 - \bar{\beta}\right)/\bar{\beta}$. We can then rewrite this expression as follows:

$$\overline{\phi B} = \textstyle\sum_{s=0}^{\infty}\left(\frac{1}{1+\bar{r}}\right)^{s} S \tag{13}$$

There are several key results that result from expression (13). First, if the liquidity premium on government bonds is zero, then $\bar{r} = r$ and we get the standard expression for the intertemporal government budget constraint:

$$\widehat{\phi B} = \frac{S}{1-\beta} = \textstyle\sum_{s=0}^{\infty}\beta^{s}S = \sum_{s=0}^{\infty}\left(\frac{1}{1+r}\right)^{s} S. \tag{14}$$

As Cochrane (2005) shows, this is just a standard valuation equation where the current value of outstanding debt claims are equal to the PDV of the future "dividend" stream where the dividend is just the future fiscal surplus. For the remainder of this paper we will refer to the RHS of (14) as the ‘fundamental value’ of the government debt.

Second, when there is a liquidity premium, the real market value of the outstanding nominal bond stock *exceeds* its fundamental value; i.e., $\overline{\phi B} > \widehat{\phi B}$. In short, as in models of fiat money, there can be a "bubble" in the value of nominal government debt. When this happens it appears that the government is "violating" its intertemporal budget constraint since the market value of the debt exceeds its fundamental value. However, by construction, the government is satisfying its budget constraint in every period.

Third, the fiscal theory of the price level holds. From (13) we have

$$\frac{\overline{B}}{P}\left\{1 - \beta\left[1 + \sigma\theta L\left(\theta\frac{\overline{B}}{P}\right)\right]\right\} = S.$$

Since the RHS is fixed and B_0 is an initial condition, the numeraire price of goods must adjust to satisfy this expression. Furthermore, changes in the initial bond stock are neutral. From (12), if the initial nominal bond stock doubles, then the numeraire price of goods must also double. This is the same as money neutrality in models of fiat money.

Fourth, although the fiscal theory of the price level holds, there is no fiscal theory of inflation coming. Inflation is occurring from nominal bond growth but not from changes in the PDV of government surpluses. Out of steady state, price level dynamics are driven by (10) and these fluctuations occur even though there are no changes in the surplus S or the stock of bonds B. The inflation rate $P_{+1}/P = \phi/\phi_{+1} = 1 + \pi$ has no effect on the real allocation. This is because the inflation rate does not appear in the dynamic equation (10) nor in the steady state expression (11). All the inflation rate affects is the nominal price of government bonds via the consumer's budget constraint (12). As a result, inflation is "costless" in our framework. Changes in the inflation rate only affect the nominal interest rate. To see this, note that $\rho = 1/(1+i)$ and rewrite (7)as follows:

$$\frac{1+\pi}{1+i} = \beta \left[1 + \sigma\theta L(\bar{q})\right] \tag{15}$$

Since the RHS is fixed, any changes in π lead to a one-for-one increase in the nominal interest rate.

Fifth, the real interest rate $\bar{r} \equiv \left(1 - \bar{\beta}\right)/\bar{\beta}$ is lower than the time rate of discount since $\bar{\beta} \geq \beta$. Since government bonds have collateral value in trade, agents are willing to bid up the price of the bonds above their fundamental value. This in turn lowers the real interest rate earned on the bonds. As shown by Lagos (2010) having a liquidity premium on public debt helps to explain why the risk free rate appears to be too low.

Finally, when there is a liquidity premium on government bonds, the real value of the stock of government debt can have positive value even if $S_s = 0$ for all s. The intuition for this is straightforward. The term S_s is effectively the dividend payment on a government claim in a manner similar to a share of a Lucas tree. Because of its collateral value, the nominal claims are valued in equilibrium even if the dividend is zero. This is equivalent to a fiat money model in which money pays no interest, i.e., has no fundamental value. All we have shown here is that the same reasoning applies to nominal government bonds.

Our results, while supporting the fiscal theory literature, should also serve as a warning since we show that market value of the government debt can exceed its fundamental value. Thus, one cannot think of the value equation as a "solvency" condition as it is often claimed to be. This is only true when there is no liquidity or collateral premium associated with government debt.

7 Dynamics of Debt

In what follows we study the dynamics of the value of government debt. The evolution of the real debt is equivalent to the evolution of consumption q since $\phi B = c(q)/\theta$.[3] In what follows, we assume that $c(q) = q$, which simplifies the exposition without affecting the results in an important way. This allows us to rewrite (10) as follows

$$q = g(q_{+1}), \tag{16}$$

where $g(q_{+1}) = q_{+1}\beta[1 + L(q_{+1})] + \theta S$ with $L(q_{+1}) = \sigma\theta[u'(q_{t+1}) - 1]$.

Note that g is single-valued, whereas

$$h = g^{-1}(q_{+1}) \tag{17}$$

is typically not. A non-monetary steady state with $q_t = 0$ only exists for $\theta S \leq 0$. Assume that h is single valued and that $g'(0) > 0$, where

$$\begin{aligned} g'(q_{+1}) &= \beta[1 + L(q_{+1}) + q_{+1}L'(q_{+1})] \text{ and} & (18)\\ g''(q_{+1}) &= \beta[1 + 2L'(q_{+1}) + q_{+1}L''(q_{+1})]. & (19) \end{aligned}$$

Note that $g'(q_{+1}) < \beta[1 + L(q_{+1})]$ since $q_{+1}L'(q_{+1}) = \sigma\theta q_{+1}u''(q_{+1}) < 0$. Note also that $g'(q_{+1})$ depends only indirectly on the surplus S through the effect of S on q_{+1}.[4]

[3] The first paper that has studied the dynamics of the "New Monetarist" framework is Lagos and Wright (2003). Here, we closely follow their exposition.

[4] To begin with the analysis note that

$$g''(q_{+1}) = \beta[1 + 2L'(q_{+1}) + q_{+1}L''(q_{+1})].$$

7.1 Dynamics for $S = 0$

We first present the dynamics of q, respectively the dynamics of ϕB, when $S = 0$. In this case, the dynamics are well understood and has been studied for the first time by Lagos and Wright (2003). They consider fiat money, so the only difference to our model is that our medium of exchange are discounted government bonds that trade at price ρ.[5]

In steady state, from (11, $L(\bar{q}) = (1-\beta)\beta^{-1}$ and so (18) can be written as follows

$$g'(\bar{q}) = 1 + \beta \bar{q} L'(\bar{q}) < 1,$$

since $\bar{q}L'(\bar{q}) = \sigma\theta\bar{q}u''(\bar{q}) < 0$. The dynamic properties of the model around the steady state value $\bar{q}$ depend on the slope $g(\bar{q})$.

Assume that $g'(\bar{q}) > 0$. In this case, the steady state equilibrium is not stable. Furthermore, for $g'(0) > 1$ there exists a continuum of dynamic equilibria such that for all $q_0 \in (0, \bar{q})$ there is a path $\{q_t\}_{t=0}^{\infty}$ such that $q_t \to 0$.[6] If $q_0 > \bar{q}$, then the path $\{q_t\}_{t=0}^{\infty}$ is unbounded and hence not an equilibrium since it violates feasibility.

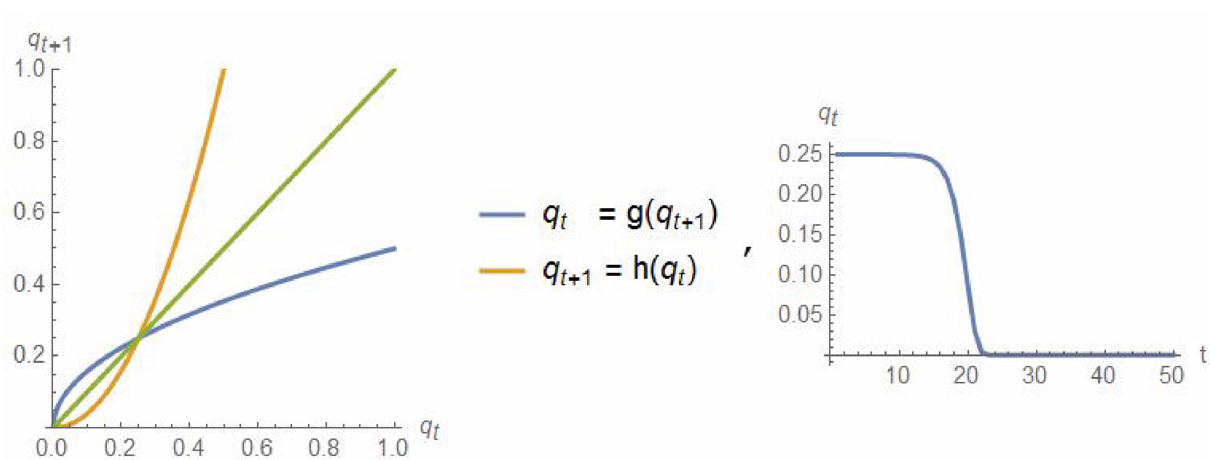

Figure 1. Inflationary pathes with a constant bond supply.

Assume a steady state exist, then if $0 > g'(\bar{q}) > -1$, the steady state equilibrium

[5] Other papers that study the dynamics of the Lagos and Wright framework are ....

[6] Note that, in contrast to Lagos and Wright, we do not have multiple steady state equilibria since we assume buyer-takes-all bargaining.

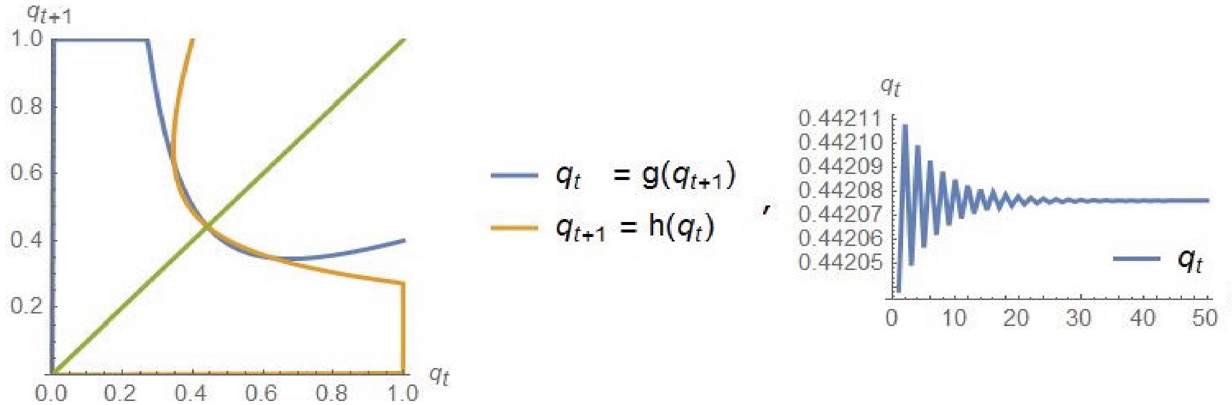

Figure 1: Figure 3: Stable equilibria with fluctuating price levels.

is unstable and for any initial value q_0 close to $\bar{q}$ the steady state is an "unstable" spiral.

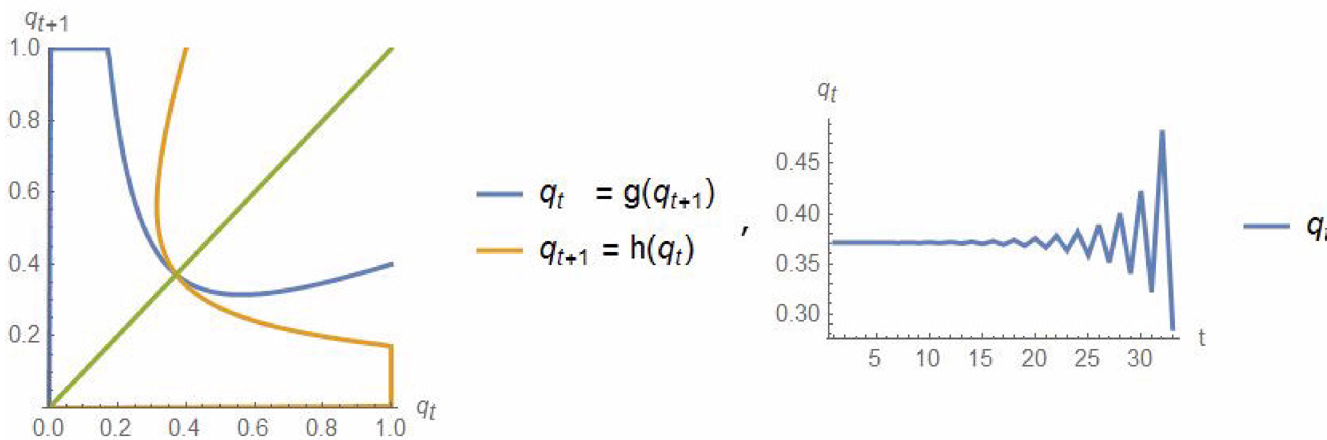

Figure 2. Unstable equilibria with fluctuating price level.

Assume a steady state exist, then if $g'(\bar{q}) < -1$, the steady state equilibrium is stable and for any initial value q_0 close to $\bar{q}$ the steady state is an "stable" spiral.

7.2 Dynamics for any S

In what follows we study the dynamics around the steady state for any values of S such that a steady state equilibrium exists. In steady state, $L(\bar{q}) = (1 - \theta S/\bar{q})\beta^{-1} - 1$ and so (18) can be written as follows

$$g'(\bar{q}) = \beta\left[1 + L(\bar{q}) + \bar{q}L'(\bar{q})\right] = 1 - \theta S/\bar{q} + \beta\bar{q}L'(\bar{q})$$

The dynamic properties of the model around the steady state value $\bar{q}$ depend again on the slope $g'(\bar{q})$. We have already described it above. What is of interest, here, is how these properties change as θS changes. For this purpose, consider the derivative of $g'(\bar{q})$ with respect to θS. We find

$$\frac{dg'(\bar{q})}{d\theta S} = \left[2\beta L'(\bar{q}) + \beta\bar{q}L''(\bar{q})\right]\frac{d\bar{q}}{d\theta S}$$

From $L(\bar{q}) = (1 - \theta S/\bar{q})\beta^{-1} - 1$, we have $L'(\bar{q}) = \theta S\beta^{-1}/\bar{q}^2$ and $L''(\bar{q}) = 2\bar{q}\theta S\beta^{-1}/\left(\bar{q}^2\right)^2$ and so we get

$$\frac{dg'(\bar{q})}{d\theta S} = 4\theta S/\bar{q}^2\frac{d\bar{q}}{d\theta S}$$

Thus the effect depends on the derivative $\frac{d\bar{q}}{d\theta S}$ which we have shown to be positive. The implication is that an increase in $d\theta S$ can transform a stable equilibrium into a unstable one.

A potential scenario would be as follows: Suppose a shock hits (earthquake, war, financial crisis etc.) and the government unexpectedly increases government spending by issuing more debt. Then future surpluses have to increase to satisfy the intertemporal budget constraint. An increase of future surpluses can move the economy from a locally stable steady state to a locally unstable steady state.

7.3 Examples

Example 1 *Assume $u(q) = A\log q$ and $c(q) = q$.*[7] *Then $q^* = A$ and (11) yields*

$$\bar{q} = \frac{\theta(S + A\beta\sigma)}{1 - \beta(1 - \sigma\theta)}$$

[7] Log utility does not satisfy our assumption that the utility function go through the origin. However, if we assume that utility is given by $u(q) = \frac{(q+z)^{1-\eta}}{1-\eta} - \frac{z^{1-\eta}}{1-\eta}$, then $u(q) \to \ln q$ as $\eta \to 1$ for z arbitrarily close to zero.

To ensure $\bar{q} \leq q^* = A$ requires $\theta S \leq c(q^*)(1-\beta) = A(1-\beta)$. This can be achieved by a large enough value of A or a sufficiently small value of θ. Given this condition holds we can write the real value of government debt as:

$$\overline{\phi B} = \frac{S + A\beta\sigma}{1-\beta(1-\sigma\theta)} = \sum_{s=0}^{\infty} [\beta(1-\sigma\theta)]^s (S + A\beta\sigma) = \sum_{s=0}^{\infty} \left(\frac{1-\sigma\theta}{1+r}\right)^s (S + A\beta\sigma) \tag{20}$$

The dynamic equation (17) reduces to

$$\begin{aligned} q &= q_{+1}\beta[1-\sigma\theta+\sigma\theta u'(q_{+1})] + \theta S \\ &= q_{+1}\beta(1-\sigma\theta) + A\beta\sigma\theta + \theta S \\ q_{+1} &= -\frac{A\beta\sigma\theta + \theta S}{\beta(1-\sigma\theta)} + \frac{1}{\beta(1-\sigma\theta)} q \end{aligned}$$

which is an unstable path in q. Thus, the steady state derived above is unstable.

Example 2 *Assume as in Lagos and Wright (2003) $u(q) = \frac{(q+b)^{1-\alpha} - b^{1-\alpha}}{1-\alpha}$, $c(q) = q$, and $S = 0$. Then, $u'(q) = (q+b)^{-\alpha}$ and the dynamic equation (16) reduces to*

$$q = q_{+1}\beta[1-\sigma\theta+\sigma\theta u'(q_{+1}+b)].$$

The steady state satisfies $1 = \beta[1-\sigma\theta+\sigma\theta u'(\bar{q}+b)]$; i.e.,

$$\bar{q} = \left(\frac{1-\beta+\beta\sigma\theta}{\beta\sigma\theta}\right)^{-1/\alpha} - b$$

The slope $g'(\bar{q})$ satisfies

$$\begin{aligned} g'(\bar{q}) &= \beta[1-\sigma\theta+\sigma\theta u'(\bar{q}+b)] + \bar{q}\beta\sigma\theta u''(\bar{q}+b) \\ &= 1 + \bar{q}\beta\sigma\theta u''(\bar{q}+b) \\ &= 1 - \alpha\beta\sigma\theta u'(\bar{q}+b) - b\beta\sigma\theta u''(\bar{q}+b) \\ &= 1 - a(1-\beta+\beta\sigma\theta) - b\beta\sigma\theta u''(\bar{q}+b) \end{aligned}$$

Assume b small. For $g'(\bar{q}) = 1 - a(1-\beta+\beta\sigma\theta) > 0$, the steady state equilibrium is unstable and for any initial value q_0 with $q_0 < \bar{q}$, q decreases monotonically with $q \to 0$. For $-1 < g'(\bar{q}) = 1 - a(1-\beta+\beta\sigma\theta) < 0$, the steady state equilibrium is unstable and for any initial value q_0 for close to $\bar{q}$ the steady state is an "unstable"spiral. For

$g'(\bar{q}) = 1 - a(1 - \beta + \beta\sigma\theta) < -1$, the steady state is a stable spiral since for any initial value q_0 close to $\bar{q}$ the steady state is approached where q cycles around $\bar{q}$.

Example 3 *Assume $u(q) = A \log q$ and $c(q) = q^2/2$. Then, $u'(q) = A/q$, $c'(q) = q$ and $L(q_{+1}) = \sigma\theta\left[\frac{A}{q_{+1}^2} - 1\right]$. the dynamic equation (10) reduces to*

$$\begin{aligned} q^2/2 &= \left(q_{+1}^2/2\right)\beta\left[1 + \sigma\theta\left(\frac{A}{q_{+1}^2} - 1\right)\right] + \theta S \\ &= \beta\left(q_{+1}^2/2\right)(1 - \sigma\theta) + \sigma\theta\beta A/2 + \theta S \\ q &= \left(\beta q_{+1}^2(1 - \sigma\theta) + \sigma\theta\beta A + 2\theta S\right)^{1/2} \end{aligned}$$

Note that

$$\frac{dq_{+1}}{dq} = \frac{1}{\beta(1 - \sigma\theta)} > 0$$

which implies that the steady state in unstable, where the steady state satisfies

$$\bar{q} = \left(\frac{\sigma\theta\beta A + 2\theta S}{1 - \beta(1 - \sigma\theta)}\right)^{1/2}$$

Example 4 *For $u(q) = (1 - \alpha)^{-1} q^{1-\alpha}$, $c(q) = q$ and $\sigma\theta = 1$ the dynamic equation reduces to $q = \beta q_{+1}^{1-\alpha} + \theta S$. Solving for q_{+1} yields*

$$q_{+1} = \left(\frac{q - \theta S}{\beta}\right)^{\frac{1}{1-\alpha}}$$

The steady state value of q satisfies

$$\bar{q}^{1-\alpha}\beta = \bar{q} - \theta S$$

The slope of the phase line defined by (17) satisfies

$$\frac{dq_{+1}}{dq} = \frac{1}{\beta(1 - \alpha)}\left(\frac{q - \theta S}{\beta}\right)^{\frac{\alpha}{1-\alpha}}$$

For $\alpha \in (0, 1)$, $\left.\frac{dq_{+1}}{dq}\right|_{q=\bar{q}} = (1 - \alpha)^{-1}\beta^{-1}\bar{q}^{\alpha} > 0$. This immediately implies that the steady is not stable. Furthermore, since $q - \theta S > 0$, it is monotonically increasing and convex in q in (q_t, q_{t-1}) space for $q_t < q^*$.
There are a continuum of equilibria leading to the non-monetary equilibrium: if $q_0 <$

$\bar{q}$, then t approaches 0 as t becomes arbitrarily large. For all these equilibria, q_t is decreasing and, holding B constant, the price level is increasing over time. If $q_0 > \bar{q}$, then the equilibrium then the sequence $\{q_t\}_{t=0}^{\infty}$ is unbounded and hence not an equilibrium.

8 Conclusion

We have constructed a dynamic general equilibrium model that generates the fiscal theory of the price level as an equilibrium outcome. Our main contribution is to show how a liquidity premium on government debt can make the market value of the government debt exceed its "fundamental value". Furthermore, we are able to show how price level dynamics can be the result of changes in the liquidity premium as opposed to changes in future fiscal policy. Finally, we obtain the counter-intuitive result that increases in taxes or cuts in government spending can be stimulative.

We have constructed a model with a minimal fiscal structure in the sense that taxes are lump-sum and government spending is constant. The main point is to focus on the liquidity value of government debt in addition to its intertemporal purpose of reallocating taxes across time. Interesting extensions would include studying optimal fiscal policy in the presence of distorting taxes (see Angletos, Collard and Dellas (2016) for an example of this work) or how the use of a Taylor rule for government debt affects the stability of the economy, particularly for dealing with unstable movements in liquidity premia. We leave this to future research.

References

[1] Angletos, G., F. Collard and H. Dellas (2016). "Public Debt as Private Liquidity: Optimal Policy", Manuscript, University of Bern.

[2] Auernheimer, L., and B. Contreras (1990). "Control of the Interest Rate with a Government Budget Constraint: Determinacy of the Price Level and Other Results", Manuscript, Texas A&M University.

[3] Begg, D. K. H., and B. Haque (1984). "A Nominal Interest Rate Rule and Price Level Indeterminacy Reconsidered", *Greek Economic Review*, 6 (1), 31–46.

[4] Cochrane, J. H. (1998). "A Frictionless View of U.S. Inflation", in NBER Macroeconomics Annual 1998, ed. by B. S. Bernanke, and J. J. Rotemberg, vol. 14, 323–384. MIT Press, Cambridge, MA.

[5] Cochrane, J. H. (2005). "Money as Stock", *Journal of Monetary Economics* 52 (3), 501-528.

[6] Kiyotaki, N. and J. Moore (1997). "Credit Cycles", *Journal of Political Economy* 105 (2), 211-248.

[7] Lagos, R. (2010). "Asset Prices and Liquidity in an Exchange Economy", *Journal of Monetary Economics,* 57 (8), 913-930

[8] Lagos, R. and Wright, R. (2003). "Dynamics, Cycles and Sunspot Equilibria in 'Genuinely Dynamic, Fundamentally Disaggregative' Models of Money", *Journal of Economic Theory* 109, 156-171.

[9] Lagos, R. and Wright, R. (2005). "A Unified Framework for Monetary Theory and Policy Analysis", *Journal of Political Economy* 113, 463-484.

[10] Leeper, E. M. (1991). "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies", *Journal of Monetary Economics*, 27 (1), 129–147.

[11] Leeper, E. and C. Leith (2015). "Inflation Through the Lens of the Fiscal Theory", prepared for the Handbook of Macroeconomics, volume 2 (John B. Taylor and Harald Uhlig, editors, Elsevier Press).

[12] Sims, C. (1994). "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy", *Economic Theory*, 4 (3), 381–399.

[13] Woodford, M. (1995): "Price-Level Determinacy Without Control of a Monetary Aggregate," Carnegie-Rochester Conference Series on Public Policy, 43, 1–46.