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Markets, Externalities, and the Dynamic Gains of Openness

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Abstract

Inflows of foreign knowledge are key for developing countries to catch up with the world technology frontier. In this paper, I construct a model to analyze the entry decisions of foreign firms that bring their know-how into a developing country, as well as the incentives of domestic firms to invest in their own know-how, given the exposure to foreign ideas and competition. The model embeds two diffusion mechanisms typically considered separately in the literature: *externalities* and *markets*. I find that their relative preponderance of markets vs. externalities substantially changes the dynamic implications of openness. Notably, openness allows developing countries to fully catch up *only* when market transactions dominate the diffusion of ideas. Externalities are never enough to catch up, and, in their presence, openness may even lead to losses in income and welfare. However, with a simple quantitative extension of the model, I argue that the dynamic gains of openness are large.

Keywords: Foreign Firms; Managerial know-how; Diffusion; Internalization; Compensating differentials.

JEL Codes: F23, F43, O19, O33, O34, O41.

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1 Introduction

Inflows of production-related knowledge have been widely regarded as the key for developing countries to catch up with the world technology frontier.[1] Besides static gains,[2] the premise that foreign ideas can be adopted by domestic firms has motivated many developing countries to open up not just to trade, but also to foreign direct investment (FDI.) The gains of catching up with foreign knowledge can be tantamount to a transition from rags to riches, as supported by the historical evidence on the diffusion of the Industrial Revolution to Europe, the U.S. and other Western countries, and, more recently, the successful adoption of Western technology by Japan, Korea, and China. Economists have long emphasized two mechanisms for the diffusion of knowledge: Externalities and markets.[3] In this paper, I build a model that incorporates both forms of diffusion and use it to examine the impact of *openness* for the output and welfare of a developing country.

I explicitly consider *(a)* the incentives of foreign firms to locate in a developing country –and bring their know-how with them– and *(b)* the incentives of domestic firms to invest in their own know-how. The model allows a clean, analytical characterization of the aggregate formation of knowledge inside a country, given the equilibrium exposure to ideas and competition from abroad, and the dynamic behavior of foreign ideas inflows in the country. Characterizing the equilibrium produces a number of simple but important implications for the literature on the cross-country diffusion of ideas.

First, contrary to the emphasis in the vast empirical literature and in recent theoretical models, externalities are neither necessary nor sufficient for the diffusion and adoption of foreign ideas. Indeed, I show that it is only when markets fully internalize the costs and benefits of knowledge transfers that countries end up catching up with the world leaders. Second, openness to foreign know-how necessarily requires destructive, reallocative and renewal forces for firms and occupations inside the country. These forces are only fully unleashed by knowledge transmitted by markets, and this mechanism is needed for the model to be consistent with salient empirical observations on the impact of FDI on developing countries. Third, using a simple quantitative extension of the model, I find that the dynamic gains of openness can be very large, an order of magnitude higher than the static gains. This holds whether knowledge formation is driven by markets or by externalities. Interestingly, the gains can be an increasing function of the preponderance of markets in knowledge formation, a proposition I support throughout the paper.

The basic model is as follows: Entrepreneurs set up firms, production teams in which the entrepreneur is the manager of a group of workers. In the quantitative model, firms are teams of the entrepreneur, middle-managers and workers. As in Lucas (1978), the knowledge of the entrepreneur determines the productivity of the team.[4] The environment is an OLG economy in which some of

[1] Among others, see Lucas (2009.)

[2] See for example, Antras, Garicano and Rossi-Hansberg (2006), Burstein and Monge-Naranjo (2009) and Eeckhout and Jovanovic (2012). The gains are even larger if productivities are not driven by skills but by non-rival factors, as in Ramondo (2013) and McGrattan and Prescott (2009).

[3] See Romer (1986), Klenow (1998), Lucas (2002), Klenow and Rodriguez-Clare (2005), Jones (2006) just to cite a few of the many papers emphasizing externalities. See Boyd and Prescott (1987a,b), Chari and Hopenhayn (1991), Jovanovic and Nyarko (1995) and Boldrin and Levine (2009) for models in which knowledge is diffused by markets.

[4] A long line in the literature links the productivity of firms to the quality of their management, e.g. Kaldor (1934), Lucas (1978), Oi (1983), Rosen (1982), Prescott and Visscher (1980), Garicano (2000) and Bloom and Van Reenen (2007).

the young build up knowledge to set, manage and profit from firms when old. Knowledge, the engine of growth in the economy, has a dual nature here. On one hand, it is a rival factor, the skills of an individual with a limited span-of-control on production activities. On the other hand, knowledge may also be a non-rival factor, the productive ideas that could be used by anybody in the country for building up their own skills. I consider "closed" and "open" countries. In a closed country, only national entrepreneurs can set up firms; in an open country, foreign firms are free to enter.[5] The entry of foreign firms not only impacts the set of entrepreneurial skills implemented in a country, but also the set of productive ideas available to its young generation.

The ideas upon which a young entrepreneur builds up his know-how come from two sources: *(i)* the specific know-how running the firm in which he is a worker and an apprentice; and *(ii)* the productive ideas implemented by the entire set of firms operating in the country. We assume that the first source is fully internalized by market transactions. The second source is a pure externality; it would be highly unrealistic to expect a market arrangement to internalize it. In this way, the model encompasses, as special cases, two common –but often conflicting– views of the accumulation and diffusion of knowledge. In one extreme, the young individual's own firm is the only source of ideas, as in Boyd and Prescott (1987a,b), Chari and Hopenhayn (1991), Jovanovic and Nyarko (1995) and similar to Boldrin and Levine (2009). In the other extreme, the productive ideas implemented by each firm are uniformly exposed to all the young in the country. Variants of such externalities have a dominant presence in the growth literature (e.g., Romer 1986, Klenow 1998, and Jones 2006) and trade and growth (e.g., Stokey 1991, Young 1991.) Notably, by including at least some degree of markets in the diffusion of knowledge the model can be consistent with two key empirical observations on the impact of FDI in developing countries. First, openness to FDI can push *pre-existing* domestic firms to *reduce* their productivity, as documented by Aitken and Harrison (1999), Xu (2000), and Alfaro et al. (2006). Second, entry of firms from developed countries can generate the emergence of new sectors of domestic firms in developing countries, as described by Rhee and Belot (1990).[6]

The preponderance of markets vis-à-vis externalities is governed by a parameter in the model. The knowledge externalities are determined by the set of skills operating in the country at large, and therefore, driven by the past investments of older entrepreneurs and the entry of foreign ones. The market (or internal) element for the transfer of knowledge is embedded in labor market relationships, apprenticeships that internalize the costs and returns of the learning opportunities that each firm offers to its workers relative to those offered by all other firms.

On the basis of available learning opportunities, each young entrepreneur builds up his own skills, foreseeing the set of skills, domestic and foreign, with which he will be competing against. The endogenous formation of skills leads to non-trivial dynamics even for a closed economy. I provide simple parameter conditions upon which the accumulation of entrepreneurial knowledge is an engine of sustained growth and the country exhibits a balanced growth path (BGP.) I also show that the convergence to the BGP exhibits an interesting form of collapsing heterogeneity.

Openness to foreign entrepreneurs impacts the accumulation of skills of the host country in three

[5]The emphasis on the cross-border reallocation of management conforms with the observation that multinational firms heavily rely on home expatriates –and home trained individuals– to manage their operations, especially in developing countries (see Chapters 5 and 6 of UNCTAD 1994).

[6]For support of internal diffusion at the industry level, see Keppler (2001, 2002, 2006) for the car industry and Agarwal et al. (2004), Filson and Franco (2006) and Franco (2005) for the rigid disk drive industry.

ways. First, foreign firms enhance the exposure to ideas of the domestic young working directly for them. Second, foreign firms may have positive externalities (spillovers) on the set of ideas circulating in the country, which benefits all the domestic, future entrepreneurs, including those working for domestic firms. While these two are positive effects, a third one is detrimental: Foreign entrepreneurs bid up the cost of labor in the country for all future periods, reducing the returns and, therefore, the incentives of domestic entrepreneurs to invest in know-how. I show that in the presence of externalities, openness to foreign firms can even slow down the formation of domestic knowledge and can even possibly reduce welfare.

Open economies may exhibit an interesting vintage structure for the population of domestic firms. With less-than-perfect internalization, the domestic entrepreneurs who build up their skills working for foreign firms do not fully catch up with their foreign counterparts. Moreover, the young workers working for them will lag further behind next period as entrepreneurs. Then, endogenously, each vintage will fall below the previous vintages in size and productivity. This equilibrium structure is similar to that of Chari and Hopenhayn (1991), but with two important additional aspects: First, the productivity level of each vintage is endogenous. Second, the set of productive ideas circulating in the country is also endogenous, determined by domestic investments and foreign entry.

The model implies large –potentially huge– dynamic gains from openness for countries far below the world knowledge frontier.[7] Openness would enable poor, unproductive countries to build up skills on the basis of the more advanced knowledge of developed countries. In present value terms, the enhanced exposure to ideas more than compensates the negative effect of a higher price for (unskilled) labor and the negative impact on the current cohort of domestic entrepreneurs. Interestingly, openness can lead to *leapfrogging* among developing countries if a complementarity between domestic and foreign sources of ideas is not too strong in a sense explained below. In this case, when two developing countries open, the initially poorer one may surpass and temporarily remain ahead while both countries eventually converge to the same position in the BGP.

The gains from openness are even stronger if occupation choices are introduced. As in Antras, Garicano and Rossi-Hansberg (2006), Burstein and Monge-Naranjo (2009) and more forcefully in Eeckhout and Jovanovic (2012), the static gains of openness are larger when individuals can choose between managerial and labor occupations. With endogenous skill formation, occupation choices can enhance the gains of openness even further. Not only can they change the structure of the BGPs toward more productive ones, but occupation choices can also redirect an open economy away from a laggard (interior) BGP and toward fully catching up with developed countries. Occupation choices can also accelerate the convergence. Introducing occupation choices leads to a different and starker form of leapfrogging: After openness, a very poor country may end up fully catching up with developed countries while a more advanced country would remain in an inferior BGP.

To gauge the order of magnitude of the dynamic gains of openness, I extend the basic model to allow for middle-managers. In the extended model, only middle-managers can accumulate know-how as they are the young individuals directly interacting with the top decision-makers of firms. The extension allows using standard values for the two key parameters needed for the quantitative evaluation of the dynamic gains of openness: the span-of-control of the firm and the share of entrepreneurs in the economy. With standard values of these two parameters, I assess the output and

[7]This is quite the opposite from the gains of openness to trade in Stokey (1991) and Young (1991).

welfare consequences for different preponderance of markets vis-à-vis externalities and for different initial conditions.

These simple exercises generate conclusive messages. First, the output and welfare gains from openness can be very substantial. Second, for most developing countries, the present value of the gains of openness would be in the same order of magnitude across different parameter specifications even if they lead to limiting dynamics in the diffusion of foreign knowledge. Third, the preponderance of markets vis-à-vis externalities is important for countries close to the world frontier. For them, if externalities are the main conduit for the diffusion of ideas, then openness can be detrimental to growth as the exposure to better foreign ideas is more than compensated by the lower incentives to innovate arising for higher foreign competition. Lastly, the welfare gains from openness can be larger when the diffusion of ideas is fully internalized (and hence foreigners are compensated for it) than when it is via externalities and host countries are hoping for a free lunch.

Related Literature. In the last few years, there has been renewed interest in models of the diffusion of ideas. Starting with the earlier work of Jovanovic and Rob (1989), a number of papers have modeled the exposure to ideas as arising from random meetings across individual units. In this vein, Alvarez et al. (2008), Luttmer (2012), Lucas and Moll (2014) and Perla and Tonetti (2014) characterize the limiting behavior of the firms' productivities that result from these random meetings. Within this model structure, Perla et al. (2015) and Sampson (2016) show that trade leads to better knowledge formation and productivity because of selection: After openness, the low productivity firms exit and no longer pollute the pool of learning opportunities inside a country. Buera and Oberfield (2016) generalize these frameworks by allowing additional randomness in the acquisition of knowledge. Besides trade, their model also allows knowledge diffusion from FDI. The model of Buera and Oberfield (2016) is designed to naturally blend with state-of-the-art static multicountry quantitative models of trade and FDI,[8] as they derive conditions under which the transition laws for the firms' productivities lead to the Fréchet distributions that make these settings tractable.

The search decisions in Perla et al. (2015), Sampson (2016) and Buera and Oberfield (2016) are one-sided, and the adoption of ideas is assumed costless both for the learner and for the teacher. Ultimately, the learning mechanisms are entirely driven by externalities. These papers abstract from the incentives for the individuals with superior knowledge to expose their ideas to those who can learn from them. That is, the market mechanism highlighted in this paper is either absent or severely restricted in those papers. To be sure, it would be extremely interesting to evaluate the implications of openness in models that incorporate both markets and externalities within the richer stochastic framework put forth by these authors.

Dasgupta (2012) considers a markets-only model of knowledge transfers, which are exogenous but random, generating sustained heterogeneity across workers and managers. The distribution of skills is continuous and the equilibrium exhibits positive-assortative matching. Interestingly, Dasgupta finds that the response to openness can be Pareto improving, as low-skilled domestic firms provide early training for the future workers of more advanced foreign firms. Sampson (2013) considers economies with complementarity between skills and pure country productivities (à la Burstein and Monge-Naranjo, 2009). Such complementarity implies that the most advanced firms remain in the most advanced countries, and therefore, knowledge diffusion will remain incomplete. Both papers abstract

[8] For example, Eaton and Kortum (2002) and Ramondo and Rodriguez-Clare (2013).

from the endogeneity of the investment in skills across firms with different learning opportunities, a key margin in the response to openness considered here.

To focus on the accumulation of entrepreneurial knowledge, the paper abstracts from many other aspects studied in the literature on multinational activity, such as the endogenous choice of organization (see the survey by Antras and Rossi-Hansberg 2009), the choice of technologies sent by multinational firms to their subsidiaries (e.g., Helpman 1984 and Keller and Yeaple 2013), and the interactions between technology diffusion, multinational activity and international trade in goods as referenced above. The paper also abstracts from cross-country education differences and hence, it is silent on the interaction between schooling and technology adoption (e.g., Stokey 2010) and between schooling and entrepreneurial know-how (e.g., Dasgupta 2012.) The paper ignores multinational firms' externalities to a country's non-rival productivity (i.e. the country-embedded productivity in Burstein and Monge-Naranjo 2009) and ignores any other cross-country spillovers (e.g., Klenow and Rodriguez-Clare 2005.) Finally, the paper abstracts from international flows of workers and physical capital (e.g. Klein and Ventura 2009, Gourinchas and Jeanne 2013 and Monge-Naranjo et al. 2018), and ignores frictions and distortions in the allocation of workers across firms inside a country (e.g., Buera and Shin, 2010, Cagetti and De Nardi, 2006, Guner et al. 2008, among many others.)

The rest of the paper proceeds as follows. Section 2 sets up the basic model. Section 3 characterizes the competitive equilibria of closed and open economies and analyzes the dynamic gains from openness. Section 4 introduces occupation choices and explores their role for the gains from openness. Section 5 lays out the extended model in which production involves entrepreneurial knowledge, mid-management and labor services. This section also reports the quantitative assessment of the dynamic gains of openness. An appendix contains analytical details omitted in the main body.

2 The Model

Consider a discrete time, infinite horizon OLG economy with a single consumption good and individuals that live for two periods. The utility of an individual born at time t who consumes c_t^t and c_{t+1}^t in periods t and $t+1$ is

$$U^t = c_t^t + \beta c_{t+1}^t,$$

where $0 < \beta < 1$.

The size (measure) of all generations is equal to one. All individuals have an endowment of one unit of time every period they are alive. The young supply their time endowment as labor; the old can become entrepreneurs, i.e. set up and control a firm, or can remain workers. The young foresee the value of their career options and decide whether and how much to invest in acquiring entrepreneurial know-how.

As in Lucas (1978), the consumption good is produced by firms, teams of one entrepreneur (or manager) and a group of workers. The entrepreneur is the residual claimant of the single firm he sets up and manages.[9] The (person-specific) skills or knowledge z of the entrepreneur determines

[9] This formulation of equilibrium is equivalent to one in which firms with constant returns to scale (and zero profits in equilibrium) are the ones hiring managerial services from the entrepreneurs. For a model that distinguishes between the economic functions of entrepreneurs and managers, see Holmes and Schmitz (1990).

the productivity of the firm under his control. With z units of entrepreneurial skills and n units of labor, a firm produces

$$y = zn^{\alpha},$$

units of the consumption good. The span-of-control parameter $\alpha \in (0, 1)$ is the degree of decreasing returns to the amount of labor n.[10]

The core of the analysis is in the accumulation and diffusion of entrepreneurial know-how. Therefore, unlike much of the existing span-of-control models, entrepreneurial skills are endogenously determined as the outcome of optimal investments, the maximization of a young person's foreseen returns of setting up and controlling a firm minus the cost of building up the required skills. Both the costs and the returns of entrepreneurial skills are determined in equilibrium, as I will explain. The costs are determined by the set of productive ideas a person is exposed to when young; the returns, by the skills of the other competing entrepreneurs when old.

The exposure to ideas of a young person, denoted $z^E \geq 0$, subsumes the contributions of two sources: (i) the productive know-how z of the entrepreneur that controls the firm where the young individual is a worker, and (ii) an average Z^O of the know-how of all the entrepreneurs actively operating inside the country at that time. Therefore, $z^E = F\left(z, Z^O\right)$, where $F : \mathbb{R}^2_+ \to \mathbb{R}_+$ is a positive and linearly homogeneous function that is increasing in both arguments. For most of the paper, I will use as a baseline case a Cobb-Douglas aggregator:

$$z^E = (z)^{\gamma} \left(Z^O\right)^{1-\gamma}, \tag{1}$$

where $0 \leq \gamma \leq 1$ is an *internalization* parameter that determines how much a person learns inside his own job relative to how much he learns from outside. The second section of Appendix A contains a discussion of the implications of using a more general CES aggregator.

The average Z^O summarizes the set of productive ideas outside each individual's firm. It is a national public good, i.e. a non-rival factor to which everyone in the country has free access.[11] It is determined as follows: Let μ_z be the (endogenous) probability measure that indicates the allocation of the country's total labor across the firms with different know-how levels. That is, for any Borel set $B \subset \mathbb{R}_+$, $\mu_z(B)$ indicates the share of the labor in control of entrepreneurs with know-how levels in B. Then, Z^O is a generalized (or Hölder) weighted mean of all the active firms:

$$Z^O = \left[\int_{\mathbb{R}_+} (z)^{\rho} \mu_z(dz)\right]^{\frac{1}{\rho}}, \tag{2}$$

where the parameter ρ can assume any value in the extended real numbers. This formulation encompasses many familiar ways of averaging the know-how levels of active firms in a country. If $\rho \to -\infty$, Z^O is the minimum value (Leontieff function); if $\rho \to \infty$, it is the maximum value. The arithmetic, geometric and harmonic means correspond to, respectively, $\rho = 1, 0, -1$.

[10] As Lucas (1978), I call these teams firms even if they can be seen equally as parts of a conglomerate of teams within the boundaries of the same firm. However, see Garicano (2000), Oi (1983) and Rosen (1982) for related issues.

[11] Notice the dual nature of entrepreneurial knowledge. On one hand, as in Boldrin and Levine 2009, knowledge equals skills, and as such, a rival factor that is tied to the time of the holder and cannot be used simultaneously in multiple tasks. On the other hand, as in Romer 1986, knowledge equals ideas, non-rival, partially non-excludable (if $\gamma > 0$) factors that could be used by any young future entrepreneur in the country, without crowding out the use by others.

Given the exposure to productive ideas z^E, the cost (in terms of current consumption) for a young individual to acquire a next-period level of skills $z' \geq 0$ is $z^E \phi\left(z'/z^E\right)$, where $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a non-negative, continuously differentiable and strictly convex function with $\lim_{x\rightarrow 0} \phi(x) = \phi'(x) = 0$ and $\lim_{x\rightarrow\infty} \phi(x) = \phi'(x) = \infty$. Then, the total and marginal costs of investing are strictly increasing and strictly convex in z' and strictly decreasing in z^E. It is convenient to focus on the special case

$$\phi\left(\frac{z'}{z^E}\right) = \frac{v_0}{1+v}\left(\frac{z'}{z^E}\right)^{1+v}, \tag{3}$$

where v_0, $v > 0$. The marginal cost of z' is $\phi'\left(z'/z^E\right) = v_0\left(z'/z^E\right)^v$, which depends only on the ratio z'/z^E, i.e., how far an individual accumulates skills relative to his exposure to ideas z^E. I shall keep using $\phi(\cdot)$ and $\phi'(\cdot)$ as shorthand in some of the formulas below.

The parameters ρ, v and γ are key for the diffusion of know-how. The *curvature* parameter v determines the impact of z^E on the costs of acquiring z'; it determines how quickly know-how will grow over time and the learning opportunity differences across firms. The *diffusion* parameter ρ determines how easily superior ideas diffuse inside a country. The higher the value of ρ, the higher the impact of superior ideas on the common pool Z^O. In the extreme, if $\rho = +\infty$, only the very best of all the ideas are considered in Z^O. In the opposite extreme, a value $\rho = -\infty$, implies that only the worst ideas are understood and can be used to build up skills.

Most importantly, by allowing that the exposure to ideas to be given by an aggregator $z^E = F\left(z, Z^O\right)$, the model encompasses two common –but conflicting– views of the accumulation and diffusion of knowledge. To see this it suffices to consider the parameter $0 \leq \gamma \leq 1$ in the baseline Cobb-Douglas case. On one hand, if $\gamma = 0$, then a common value $z^E = Z^O$ holds for everyone and externalities are the only engine of accumulation and diffusion. Such an assumption has a dominant presence in the economic growth literature (e.g., Romer 1986 and Lucas 2002), as well as on the impact of openness to trade (e.g., Stokey 1991) and to multinational firms (e.g., Findlay 1978.) On the other hand, if $\gamma = 1$, then the exposure to ideas –and hence, the ability to accumulate skills– is uniquely determined by one's own firm. This gives rise to a richer relationship between young and old entrepreneurs, one that fully internalizes the costs and benefits of accumulating skills. Such is the view in Boyd and Prescott (1987a,b), Chari and Hopenhayn (1991), Jovanovic and Nyarko (1995), and others. By allowing any $0 \leq \gamma \leq 1$, the model here combines the impact of externalities with labor markets that compensate for differences in the learning opportunities across firms with different knowledge levels.

I consider two types of economies. In a closed economy, only domestic entrepreneurs can set up firms and hire local labor. In an open economy, foreign entrepreneurs can enter the country and a free entry condition determines how much of the local labor is hired by foreign firms.

3 The Formation and Diffusion of Knowledge

In this section I construct the objects needed to define a competitive equilibrium and then characterize and compare the accumulation of knowledge in closed and open economies.

3.1 Equilibrium Preliminaries

I consider perfect foresight competitive equilibria. The discount factor β pins down the interest rate with which all future payoffs are discounted. The key component of the price system is a sequence of wage functions $\{w_t : \mathbb{R}_+ \rightarrow \mathbb{R}_+\}_{t=0}^{\infty}$. The wage $w_t(z)$ indicates the price that an entrepreneur with skills z must pay for a unit of labor at time t. The dependence of wages on the skills of the entrepreneur is driven by the learning opportunities offered by the firm under his control as explained below.

The economic decisions of an old person are whether to remain a worker or become an entrepreneur and, if an entrepreneur, how much labor to hire. As in Lucas (1978) this occupation choice is made on the basis of the individual's own skills z and the market wage; the difference is that the wage $w(z)$ may depend on his own z. Should he become an active entrepreneur, the net rents for such an old person would be

$$\begin{aligned} \pi[z, w(z)] &\equiv \max_{\{n\}} \{zn^{\alpha} - w(z)\, n\} \\ &= \theta z^{\frac{1}{1-\alpha}} [w(z)]^{\frac{-\alpha}{1-\alpha}}, \end{aligned} \tag{4}$$

where $\theta \equiv (1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha}} > 0$. Given $w(z)$, $\pi[\cdot, w(z)]$ is strictly increasing and convex; given z, $\pi[w(z), \cdot]$ is strictly decreasing in $w(z)$. Likewise, the optimal demand for labor would also be increasing in z and decreasing in $w(z)$:

$$n^*[z, w(z)] = \left[\frac{\alpha z}{w(z)}\right]^{\frac{1}{1-\alpha}}. \tag{5}$$

An old individual becomes an active entrepreneur if and only if the rents $\pi[z, w(z)]$ dominate the entire schedule of wages available to workers, $w(\cdot)$.

The economic decisions of a young person are first, selecting the firm for which to work, and second, conditional on that decision, whether and how much to invest in entrepreneurial skills. With respect to the latter, given the exposure to ideas z^E and the next period's cost of labor $w_{t+1}(\cdot)$, the optimal investment in entrepreneurial skills z' solves

$$V\left[z^E, w_{t+1}(\cdot)\right] \equiv \max_{z'} \left\{\beta\pi[z', w_{t+1}(z')] - z^E \phi\left(z'/z^E\right)\right\}. \tag{6}$$

The key determinant of the optimal z' are z^E and $w_{t+1}(\cdot)$. Differences in learning opportunities are fully perceived by all young individuals when choosing which firms to work for. As in Chari and Hopenhayn (1991), for simplicity, I assume that all young individuals are identical. Then, in equilibrium young workers must be indifferent to work for any of the active firms, except, perhaps, for those only employing old workers. For all the others, wages must compensate for differences in learning opportunities. Two of those active firms, with say know-how levels $z_0 < z_1$ pay wages that satisfy:

$$w_t(z_0) - w_t(z_1) = V\left[z_1^E, w_{t+1}(\cdot)\right] - V\left[z_0^E, w_{t+1}(\cdot)\right], \tag{7}$$

where $z_i^E \equiv F\left(z_i, Z^O\right)$ for $i = 0, 1$. Less skilled managers must pay higher wages as the right-hand-side of this equation is positive. It is important to keep in mind that the proper interpretation of (7) is as differences in the cost of effective units of labor across firms, which may not translate directly to observed differences in earnings in the data, because of potential heterogeneity of these units across workers.[12]

Under the conditions laid out and discussed below, optimal investments in skills are determined by the condition

$$\beta\left\{\pi_1\left[z',w_{t+1}\left(z'\right)\right]+\pi_2\left[z',w_{t+1}\left(z'\right)\right]\frac{\partial w_{t+1}\left(z'\right)}{\partial z'}\right\}=\phi'\left(\frac{z'}{z^E}\right), \tag{8}$$

where $\pi_1(\cdot)$ stands for the derivative of profits π with respect to the skills z of the manager, regardless of the wages he expects to face for his own workers. The term $\pi_2(\cdot)$ is the derivative with respect to the wages $w_{t+1}(z)$ that the future entrepreneur will have to pay for each worker, which depend on the skills z' as they determine the learning opportunities that the entrepreneur will offer.

For the future entrepreneur, the right-hand-side of this equation is simply the marginal cost of his investment in skills z'. Note that a better exposure to ideas (a higher z^E) naturally reduces the marginal (and total) costs of investment, leading to higher skill investment z'. The left-hand-side is the discounted marginal returns of the investment. The first term captures the direct gain in the profits of higher skills ($\pi_1>0$). The second captures the impact of the better learning opportunities to the future workers that are induced by a higher z'. This effect is positive when the learning opportunities are at least partially internalized in lower wages ($\frac{\partial w_{t+1}(z')}{\partial z'}<0$) and lower wages increase profits ($\pi_2<0$.)

Combining (4), (7) and (8), and using the envelope conditions (7), the following lemma that characterizes the optimal skill accumulation of a young entrepreneur for a generic aggregator of internal and external exposure to ideas, $z^E\equiv F\left(z,Z^O\right)$.

Lemma 1 *Denote by* $\xi_{z,z^E}\equiv\left(\partial z^E/\partial z\right)/\left(z^E/z\right)$*, the elasticity of a worker's exposure to ideas with respect to his employer's level of skills, and by* $\theta_{z',z^E}\equiv z^E\phi\left(\frac{z'}{z^E}\right)/w(z)$ *the share of income that a young worker invests in acquiring skills. Then, the optimal acquisition of skills* z' *of a future entrepreneur satisfies*

$$\beta\left[\frac{\alpha z'}{w_{t+1}\left(z'\right)}\right]^{\frac{\alpha}{1-\alpha}}\left[1+\alpha\ v\ \xi_{z',(z^E)'}\theta_{z'',(z^E)'}\right]=v_0\left(\frac{z'}{z^E}\right)^v,$$

where $\left(z^E\right)'$ *is the implied exposure to ideas that the entrepreneur will provide to his future workers and* z'' *is their acquired level of know-how.*

For the baseline case of a Cobb-Douglas aggregator, $z^E=(z)^\gamma\left(Z^O\right)^{1-\gamma}$, the implied elasticity is constant, $\xi_{z,\ z^E}=\gamma$, and the skill accumulation condition simplifies to

$$\beta\left(\frac{\alpha z'}{w'\left(z'\right)}\right)^{\frac{\alpha}{1-\alpha}}\left[1+\alpha v\gamma\theta_{z'',(z^E)'}\right]=v_0\left(\frac{z'}{z^E}\right)^v, \tag{9}$$

[12]For instance, consider an economy with heterogeneous entrepreneurs, heterogeneous workers and fixed costs of hiring each worker. More productive firms would want to hire more units of effective labor, and to minimize on the fixed costs, in equilibrium they would hire the workers endowed with the most effective units. Such a simple positive assortative matching could lead to higher earnings for workers in the more productive firms.

where $\theta_{z'',\,(z^E)'} = \frac{\frac{v_0}{(1+v)}(z^E)'\left[z''/(z^E)'\right]^{1+v}}{w'(z')}$. In what follows, the analysis will focus on this baseline case, deferring to Appendix A the analysis of a more general CES aggregator of internal and external ideas.

Workers may have the option of forsaking entrepreneurship and remaining workers when old. Beyond the standard, ex-post decisions, these occupation choices would have an interesting interaction in the individual decisions to accumulate skills. Indeed, as explained below, if a young worker's exposure to ideas z^E is too low, he might even opt for the corner solution of $z' = 0$ and then remain a worker during old age. For a country as a whole, occupation choices can completely reshape the equilibria, changing both the set of BGPs and/or the transition dynamics for both, open and closed economies. For clarity of exposition, for most of the text, I assume that all the old become entrepreneurs and hence all the young invest in skills according to (9.) Section 4 fully develops the economy with occupation choices, highlights how they interact with the skill accumulation of individuals and how they change the dynamics of closed and open economies.

3.2 Skill Formation in Closed Economies

Consider a closed economy. Since only the domestic old can supply the entrepreneurial skills and operate firms in the country, they are the sole source of entrepreneurial ideas for the domestic young generation. Let the probability measure ϕ_z^t describe the distribution of skills of the old generation in period t. Assume that ϕ_z^t has a strictly positive support $[z_L^t, z_H^t] \subset \mathbb{R}_+$ that is bounded from above. Given wages $w_t(z)$, the amount of labor $n_t^*(z)$ hired by an entrepreneur with skill level z is given by (5). In turn, the distribution of labor employed across skill levels is given by $\mu_z^t(B) = \left(\int_B n_t^*(z)\,\phi_z^t(dz)\right) / \int_{z_L^t}^{z_H^t} n_t^*(z)\,\phi_z^t(dz)$ for any Borel set $B \subset [z_L^t, z_H^t]$. With μ_z^t thus determined, the exposure to external ideas, Z_t^O, is determined by the expression (2), and the total exposure z_t^E of ideas of the young who work in firms with know-how z is $(z)^\gamma \left(Z_t^O\right)^{1-\gamma}$.

The equilibrium evolution of the skill distribution $\left\{\phi_z^t\right\}$ is driven by the investment decisions of all individual young entrepreneurs. These decisions are shaped by two opposite forces. A force towards equality, the common, economywide external exposure Z_t^O, drives all future entrepreneurs towards common skill levels. A force towards inequality, the differential, internalized exposure of ideas, can lead to widening gaps in their skill levels. The following lemma provides sufficient conditions upon the latter which dominates the former.

Lemma 2 *If it exists, an equilibrium wage function $w_t(z')$ must be non-increasing. Let the function $z'(z)$ indicate the optimal skills investments in skills z' for any internal z, given two external values Z^O, $\left(Z^O\right)'$. Then, if $v > \alpha/(1-\alpha)$,[13] the function $z'(\cdot)$ is strictly increasing. If also $\gamma > 1 - \alpha/[(1-\alpha)v]$, then $z'(z_1)/z'(z_0) > z_1/z_0$ for any $z_1 > z_0$.*

Therefore, when internalized learning accounts for a large share of the exposure to the ideas, i.e. $\gamma > 1 - \alpha/[(1-\alpha)v]$, the pre-existing differences in the exposure to ideas will lead to *widening* gaps in skill formation. Widening gaps in know-how also lead to widening gaps in firm sizes and shares of employment. Asymptotically, the economy will exhibit only a negligible mass of workers

[13] This assumption is needed for (6) to have a non-zero solution. Given $w(z)$, the function π is strictly convex in z with an elasticity of $1/(1-\alpha)$. Given z^E, the elasticity of ϕ w.r.t. z' is $1+v$; the condition $v > \alpha/(1-\alpha)$ ensures that the latter is 'more convex' which is needed for an interior solution.

outside very top end of the know-how distribution. Using this result I argue below that under these circumstances, a closed economy converges to a homogeneous population of entrepreneurs.

Regardless of the value of γ, if at any point in time the old generation of entrepreneurs is homogenous, then all subsequent generations will remain homogeneous forever. To see this, assume that all initial old individuals possess the same level of know-how $Z > 0$. Then, in equilibrium, all the young workers receive the same wage w and are exposed to the same level of ideas $z^E = Z$. They will invest the same amount in skills Z' and in the next period, their own workers will be exposed to the same $\left(z^E\right)' = Z'$, and receive the same wage w', and so on. In every period, all firms will hire the same units of labor, $n^* = 1$, and market-clearing wages and earnings for entrepreneurs are $w_t = \alpha Z_t$ and $\pi_t = (1-\alpha) Z_t$. Imposing these conditions, equation (9) implies that the growth in skills $G_t \equiv Z_{t+1}/Z_t$ must satisfy the difference equation

$$\beta \left[1 + \frac{\gamma v v_0}{(1+v)} \left(G_{t+1}\right)^{1+v}\right] = v_0 \left(G_t\right)^v . \tag{10}$$

An equilibrium balanced growth path (BGP) requires that entrepreneurial knowledge grows at a constant rate over time, i.e. $G_t = G > 0$ all t. For this to hold, G must be a root of the equation (10) when $G_{t+1} = G_t = G$. The following proposition establishes sufficient conditions for the existence and uniqueness of an equilibrium BGP, and for the economy to exhibit positive growth.

Proposition 1 *Assume that in addition of $v > 1/(1-\alpha)$, either: (i) $\gamma = 0$ and $\beta < [v_0 (1+v)]^{\frac{1}{1+v}}$, or (ii) $\gamma > 0$ and $\beta \leq (v_0/[\gamma^v (1+v)])^{\frac{1}{1+v}}$, then an equilibrium BGP with homogeneous skills exists and it is unique. Moreover, there is positive growth, $G > 1$, if $\beta > v_0 (1+v)/(1+v+vv_0\gamma)$.*

For a BGP to exist, the curvature parameter v must be high enough to guarantee the existence of a solution to the individual maximization program. The condition $\beta < [v_0 (1+v)]^{\frac{1}{1+v}}$ is needed to prevent negative consumption. When $\gamma = 0$, the value of skills is only on the direct impact of the future entrepreneur's profits and not in their ability to train his workers, and since the individual's profits accrue for only one period, there is no need to impose an upper bound on β for the individual problem to be well defined.[14] When $\gamma > 0$, the value of accumulating skills is not only for the direct impact on the next period profits, but also in the value of the skills in teaching the future entrepreneur's apprentices, which in turn would also teach their own apprentices, and so on. Hence, we also need an upper bound on β, otherwise, the left-hand-side could always lay above the right-hand-side, and skill accumulation would degenerate to $+\infty$.

Having established conditions for the existence and uniqueness of a closed economy BGP with homogeneous firms, I now establish two forms of stability. The first one is that under the same conditions for existence and uniqueness, we can rule out local, self-fulfilling (extrinsic) fluctuations in G_t. Second, under some parameter conditions, any pre-existent heterogeneity will eventually disappear and the economy must converge in the limit to a homogeneous firms BGP.

Proposition 2 *Assume that in addition of $v > 1/(1-\alpha)$, either: (i) $\gamma = 0$ and $\beta < [v_0 (1+v)]^{\frac{1}{1+v}}$, or (ii) $\gamma > 0$ and $\beta \leq (v_0/[\gamma^v (1+v)])^{\frac{1}{1+v}}$, then: (1) for a closed economy initially populated by homogeneous entrepreneurs, the only equilibrium is the unique BGP, i.e., other non-explosive fluctuations*

[14] When $\gamma = 0$, the gross growth rate is given by $G = (\beta/v_0)^{\frac{1}{v}}$. For the equilibrium present value of the aggregate endowment to be finite it is needed that $\beta G < 1$, i.e. $\beta < (v_0)^{\frac{1}{1+v}}$.

in G_t are ruled out; (2) if either $\gamma > 1 - \alpha / [(1-\alpha) v]$ or $\gamma = 0$, then any equilibrium starting with initial distribution with bounded support will asymptotically converge to a homogenous firms BGP.

The second part of the proposition is perhaps the most interesting one. Along the lines of Lemma 1, if $\gamma > 1 - \alpha / [(1-\alpha) v]$, the economy exhibits dispersion-induced homogeneity: the economy converges to a pool of homogeneous entrepreneurs precisely because the top end of the distribution reproduces at a faster pace than the lower end. Over time, the progeny of the initially most productive entrepreneur will progressively dominate the entire economy, and after a possible episode of expanding inequality, the economy would converge to a homogeneous pool of entrepreneurs.

Interestingly, under two polar opposite cases in the preponderance of markets (when γ is close to 1) vs. externalities (when $\gamma = 0$), a closed economy would converge to homogeneous entrepreneurs.[15] For the remainder of the analysis, I will consider only the homogeneous skills closed economy BGP and use it in two ways: *(a)* as the initial conditions for both the home and foreign countries at the time the home country opens up; and *(b)* as the benchmark to assess the gains from openness.

3.3 The Diffusion of Foreign Know-how in Open Economies

Consider now a country that freely allows old foreign entrepreneurs to set up firms and hire domestic labor.[16] I use, respectively, the names "home" and "foreign," and the indexes h and f, for the host country and for the rest of the world. I assume that initially both home and foreign are in the BGP as described in Proposition 1 and that at time $t = 0$, openness takes place unexpectedly and permanently. In addition, I assume that home is *less developed*, that is, at time $t = 0$, $Z_h < Z_f$. I also assume that home is *small,* i.e., it does not affect the equilibrium path of the foreign country.

The entry of foreign entrepreneurs impacts the accumulation of skills at home in three ways. First, foreign firms enhance the exposure to ideas of the domestic young working directly for them. Second, foreign firms have spillovers on the country's level of Z^O, which benefits all the local young, including those working for domestic firms. While these two effects are positive, a third one is detrimental: Foreign entrepreneurs bid up the cost of labor $w_{t+1}(\cdot)$ in the home country for all future periods t.

In each period, a free entry condition endogenously determines the mass of foreign know-how in the country. Foreign entrepreneurs enter until they are indifferent between operating at home or remaining in the foreign country. While in the foreign country, they earn $\pi_f = (1-\alpha) Z_f$ and have no mobility frictions, so their indifference between the home and the foreign locations can only happen when the (effective) cost of labor is the same in both countries.[17] Therefore, with openness, the domestic market-clearing wages for foreign firms in the home country, w_h^f, must be equal to the

[15] The economy may still converge to a homogeneous population for intermediate levels of γ, but providing sufficient conditions for those intermediate cases, for any initial distribution ϕ_z^0 is much more difficult.

[16] I will assume that individuals from the home country cannot move. This is without loss of generality for old entrepreneurs and workers, since, in equilibrium they will be indifferent between moving to a foreign country or remaining home. However, the possibility for domestic young potential entrepreneurs to move to foreign and grow up there would enhance their learning possibilities. The implications for home would hinge on whether there is reverse migration or not. No doubt those issues are important and deserving of separate treatment.

[17] The key is that the effective unit of labor is the same in the two countries. For clarity, we assume that both countries have the same ratio of effective-to-physical units of labor. In the language of Burstein and Monge-Naranjo (2009), I am abstracting from differences in country-embedded productivities. Adding those differences would add extra notation but no substance to the results in this paper.

foreign wage:

$$w_h^f = w_f = \alpha Z_f. \tag{11}$$

Facing the same effective wages, each foreign firm hires the same number of labor units, $n_f^* = 1$, as if they had remained in the foreign country.[18]

To set up the analysis of an open economy, it is convenient to consider the ratios $R \equiv z/Z_f$, i.e., the knowledge z of a domestic entrepreneur relative to the knowledge Z_f of the foreign entrepreneur. Likewise, $R^O \equiv Z^O/Z_f$ denotes the external exposure to ideas in the home country relative to that in the foreign country.[19] In a BGP, necessarily the know-how levels of both home and foreign firms grow at a rate $G > 1$ and the ratios R and R^O would be constant over time.

Openness can give rise to an interesting vintage structure for the population of domestic firms. To see this, Table 1 illustrates the diffusion of foreign know-how into a country that opens up at time $t = 0$. Denote by $j = 0, 1, 2, ...$ the different vintages of domestic firms as offspring of foreign entrepreneurs. That is, $j = 0$ indicates the foreign firm itself; $j = 1$ indicates the vintage of domestic entrepreneurs that directly worked for a foreign firm when young; $j = 2$ are the domestic entrepreneurs that worked for a foreign-trained domestic entrepreneur; $j + 1$ are those who were trained by a member of vintage j, etc. The ratio R_j^t indicates the skills, relative to skills in the foreign country, Z_f^t, of a vintage j of domestic firms at time t.

For every period $t \geq 0$, a mass of foreign firms enters, each one carrying skills Z_f^t, the same skills as those firms operating in the developed country. Then, since foreign firms are at the frontier, $R_0^t = 1$, and because in each period $w_h^f = w_f$, each of the foreign firms hires $n_f^t = 1$ domestic workers. Those domestic young individuals working in foreign firms or subsidiaries are exposed to ideas in the ratio $R_0^{E,t} = (1)^\gamma \left(R^{O,t}\right)^{1-\gamma} \leq 1$ and acquire know-how in the ratio R_1^{t+1}. In the subsequent period $t+1$, they will hire n_1^{t+1} young workers and expose them to ideas in the ratio $R_2^{E,t+1} = \left(R_1^{t+1}\right)^\gamma \left(R^{O,t+1}\right)^{1-\gamma}$; these n_1^{t+1} young workers will mature into entrepreneurs with relative skills R_2^{t+2}. Each of those will go on in period $t+2$ to spawn n_2^{t+2} entrepreneurs with skills R_2^{t+3} that will be active at $t+3$, and so on. This process is indicated by the diagonal arrows in Table 1. Clearly, with the passage of every period, an additional vintage of domestic firms is added.

Period\Vintage	0	1	2	3	$\cdots$	j	$\cdots$	∞
0	1 $\searrow$	-	-	-	$\cdots$	-	$\cdots$	-
1	1 $\searrow$	R_1^1 $\searrow$	-	-	$\cdots$	-	$\cdots$	-
2	1 $\searrow$	R_1^2 $\searrow$	R_2^2 $\searrow$	-	$\cdots$	-	$\cdots$	-
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	1 $\searrow$	R_1^t $\searrow$	R_2^t $\searrow$	R_3^t $\searrow$	$\cdots$	R_j^t $\searrow$	$\cdots$	-
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
∞	1 $\rightarrow$	R_1^∞ $\rightarrow$	R_2^∞ $\rightarrow$	R_3^∞ $\rightarrow$	$\cdots$	R_j^∞ $\rightarrow$	$\cdots$	R_∞^∞

Table 1: Relative skills R_j^t of the j-th vintage of a foreign entrepreneur at period $t \geq 0$.

[18] This equality is in terms of effective units of labor. Therefore, the model can accommodate cross-country differences in workers' earnings by introducing differences in effective units between workers of different countries. For instance, if workers in the home country have fewer units of effective labor per unit of physical labor, foreign firms will be larger than the domestic firms in the foreign country. Moreover, since $Z_f > Z_h$, foreign firms are also larger than domestic firms at home.

[19] Here $Z_f^O = Z_f$ because the foreign country is in a BGP.

Asymptotically there can be an infinite number of vintages of domestic firms that have, directly or indirectly, built up their relative skills with ideas that were at the world frontier.[20] In a steady or balanced growth path, all the ratios R_j^t –and the average $R^{O,t}$– would have converged to an array of constants $\left\{R_j^\infty\right\}_{j=0}^\infty$, and the absolute skills Z_j^t of all vintages grow over time at the gross rate G. As indicated by the horizontal arrows in the table, in the BGP, each vintage j exposes their workers to ideas in the constant ratio $R_j^E = (R_j)^\gamma \left(R^O\right)^{1-\gamma}$, these workers form skills in the ratio $R_{j+1} \leq R_j$.

For simplicity, and without the risk of confusion, I will ignore the superindex ∞ when discussing the BGP.

3.3.1 Balance Growth Paths

In a BGP of an open economy, a constant mass $m_f \geq 0$ of foreign firms enters the country and a constant distribution of skill levels R (the relative gaps with respect to the foreign productivity Z_f) characterizes the population of domestic firms. Obviously, the external exposure inside the home country relative to that in the foreign country, $R^O = Z^O/Z_f$, would also be constant along a BGP.

It is convenient to separate the more general case, $0 < \gamma < 1$, when both markets and externalities operate in the diffusion of know-how, from the two extreme cases, $\gamma = 0$ and $\gamma = 1$, in which only one of these is present.

Markets and Externalities: $0 < \gamma < 1$**.** When $0 < \gamma < 1$, it is easy to see that there are two BGPs. The first one is *full-convergence*, $1 = R^O = R_j$ all j, is a BGP. In such a BGP, the (net) entry of foreign firms is zero, $m_f = 0$, all domestic firms are homogeneous and identical to foreign firms in terms of skills, wages, and learning opportunities. If such BGP is reached, the equilibrium conditions are the same as in the closed-economy BGP.[21]

Another *interior* or *partial convergence* BGP exists in which the skills and ideas circulating in the home country are inferior to those in the developed foreign countries, i.e., $R^O = Z^O/Z_f < 1$, and $R_j \leq 1$ all j with the inequality strict for at least some $j < \infty$. To lay out the equilibrium conditions for this interior BGP, let w_j denote the wage that a domestic firm of vintage j must pay their workers. Because all young individuals are ex-ante identical and all have the option to work for any firm, whenever $R_j < 1$, the wage w_j must be higher than $w_f = \alpha Z_f$ to compensate for the inferior learning opportunities. Define $d_j \geq 0$ to be the relative compensating differential in terms of Z_f, i.e., $w_j = (\alpha + d_j) Z_f$. In a BGP the compensating differentials must be an array of non-negative constants $\{d_j\}_{j=0}^\infty$. By construction, $R_0 = 1$, $d_0 = 0$, $n_0 = 1$, and $R_0^E = \left(R^O\right)^{1-\gamma}$. Since the aggregate mass of labor is normalized to 1, and $n_f = 1$, then the mass of workers in the country working for foreign firms, m_0, must be equal to the mass of foreign firms in the country, m_f, that is $m_0 = m_f$.

Definition 1 *An equilibrium BGP of an open economy is a domestic-to-foreign ratio of external exposure of ideas* $0 < R^O \leq 1$*, as well as an array of non-negative employment shares, firm sizes, relative skills, compensating differentials and exposures to know-how* $\left\{m_j, n_j, R_j, d_j, R_j^E\right\}_{j=0}^\infty$ *such that,* $\forall j = 0, 1, 2, \ldots$ ***(a)*** *old entrepreneurs of vintage* j *maximize profits, hiring workers in the amount*

$$n_j = \left[\frac{\alpha}{\alpha + d_j} R_j\right]^{\frac{1}{1-\alpha}}; \tag{12}$$

[20] Once again, when the occupation choices are introduced in Section 4, the number of vintages may be finite.

[21] Of course, a third, trivial BGP when $R_j = R^O = 0$ also exists.

(b) *the employment shares for each vintage* $j = 0, 1, 2...$ *are*

$$m_j = m_0 \prod_{i=0}^{j} n_i; \tag{13}$$

(c) *and the entry of foreign firms* ($m_f = m_0$) *ensures that the country's labor market clears*

$$m_0 = \frac{1}{\sum_{k=0}^{\infty} \prod_{j=0}^{k} n_j}; \tag{14}$$

(d) *the external exposure of ideas in the home country is the weighted mean*

$$R^O = \left[\sum_{j=0}^{\infty} m_j \left(R_j\right)^{\rho} \right]^{\frac{1}{\rho}}; \tag{15}$$

(e) *the relative exposure to ideas in each firm of vintage* j *is*

$$R_j^E = \left(R_j\right)^{\gamma} \left(R^O\right)^{1-\gamma}. \tag{16}$$

(f) *Skill formation* R_j *in vintage* $j-1$ *workers are optimal, exposure* R_{j-1}^E *and future wages* $(\alpha + d_j)\, GZ_f$,

$$\beta \left(\frac{\alpha R_j}{\alpha + d_j} \right)^{\frac{\alpha}{1-\alpha}} \left[1 + \frac{\alpha R_j^E}{\alpha + d_j} \frac{\gamma v v_0}{1+v} \left(G \frac{R_{j+1}}{R_j^E} \right)^{1+v} \right] = v_0 \left(G \frac{R_j}{R_{j-1}^E} \right)^{v}; \tag{17}$$

(g) *and young individuals are indifferent to work in the across active firms* $j \geq 1$

$$d_j = \beta G \left[\pi\left(R_1, \alpha + d_1\right) - \pi\left(R_{j+1}, \alpha + d_{j+1}\right) \right] + R_j^E \phi \left(G \frac{R_{j+1}}{R_j^E} \right) - \phi \left(G \frac{R_1}{R_0^E} \right), \tag{18}$$

where the shorthands $\pi(\cdot, \cdot)$ *and* $\phi(\cdot)$ *are as defined above.*

Using the parameter values discussed in Section 5, Figure 1 illustrates the behavior of the different vintages of domestic firms in an interior BGP with $0 < \gamma < 1$. Because $R^O < 1$, all the young domestic entrepreneurs are less exposed to productive ideas than their foreign counterparts. Even those working in foreign firms are only exposed to the ratio $\left(R^O\right)^{1-\gamma} < 1$. For those working in older vintages, the exposure to ideas decays consistently, $R_j^E \geq R_{j+1}^E$. As shown in the upper-left panel, this results in a formation of skills that consistently decays with the age of the vintage. The declining exposure to ideas also explains that compensating differentials d_j must increase with the vintage j as shown in the upper-right panel of the figure. Both the declining skills R_j and the increasing wages $\alpha + d_j$ explain the decline in the labor hired by each firm n_j and in the share m_j of the total labor force hired by the vintage j (bottom-left panel). In this example, the decline is so rapid that shares m_j are negligible for $j \geq 5$.

The bottom-right panel of the figure reports the difference between the earnings (relative to Z_f) of someone who, having worked in vintage j when young, invests in skills and becomes an entrepreneur (i.e., $v_j \equiv V_j / Z_f$) as well as the income $\beta G (\alpha + d_j)$ value of remaining a worker in the same vintage

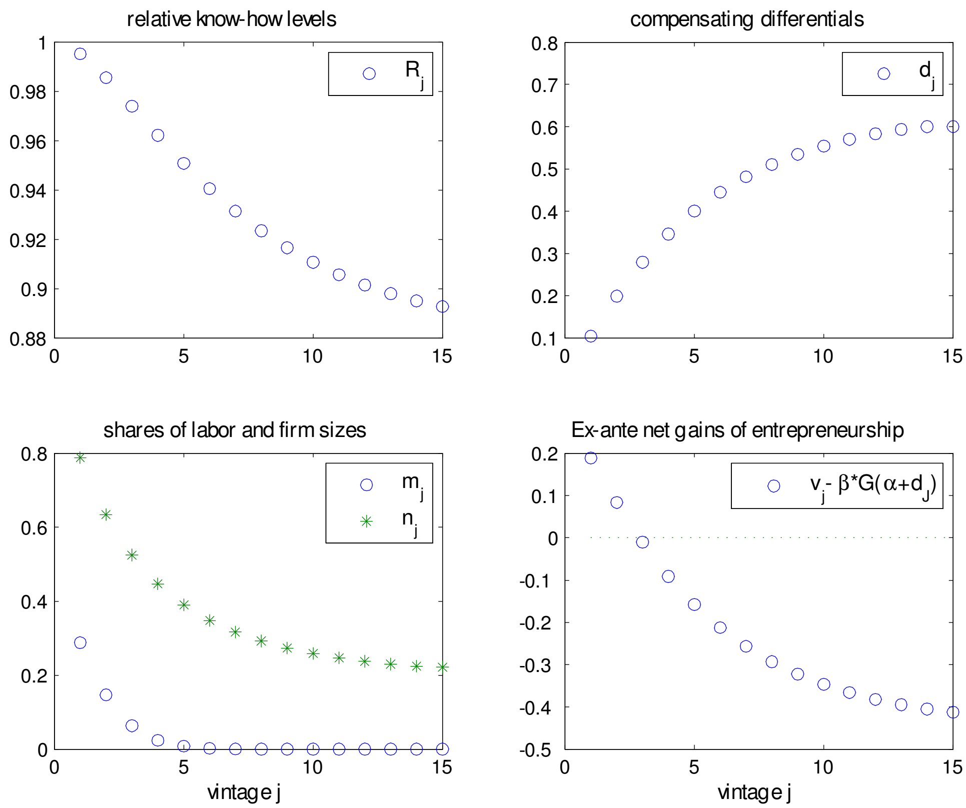

Figure 1: Interior BGP of an open economy (no occupation choices).

j next period (and thus setting skill formation to zero.) The fact that $v_j - \beta G(\alpha + d_j)$ becomes negative for higher j indicates that entrepreneurship choices will be binding for high enough vintages j, and some of the old would want to remain workers. We consider the model with those occupation choices in Section 4.

Markets Only: $\gamma = 1$**.** In this case the diffusion of ideas is entirely within the firm, $z^E = z$. The outside set of ideas R^O and parameter ρ are irrelevant. In particular, regardless of how productive the domestic firms in the home country are, workers in a foreign firm are exposed to exactly the same set of ideas as foreign workers in the foreign country. They will be will find it feasible and optimal to acquire exactly the same level of skills as their foreign peers, i.e., $R_0^{t+1} = 1$. In the next period, these **new domestic entrepreneurs** will expose their own workers to the same level of skills as the foreigners. This second generation of workers will also accumulate the same level of skills as young foreigners, i.e. $R_{j+1}^{t+1} = 1$, and so on. Because of this, in any period t, and all vintages $j \leq t-1$, we have $R_j^t = R_{j+1}^t = 1$. Then, to characterize the equilibrium, it suffices to keep track of (i) the mass of foreign firms m_f^t, (ii) the total mass of a sector of **new domestic firms** m_{new}^t (i.e., the cumulative mass of all the different generations j of foreign-trained domestic entrepreneurs) and (iii) the relative productivity, R_t^{old}, of the old or pre-existing domestic firms at the time $t = 0$ when the country opened up.

As time passes by, the cumulative entry $\sum_{\tau=0}^{t} m_f^\tau$ of foreign firms builds up the mass m_{new}^t of foreign-trained domestic entrepreneurs. The older sector of domestic firms is eventually pushed away of existence and the new domestic sector will eventually overtake the entire labor force of the country, i.e., $m^t_{\text{new}} \to 1$. In the BGP, these domestic entrepreneurs alone push the equilibrium domestic wage to the foreign level w_f, and foreign firms will no longer have gains of entering, i.e., $m_f = 0$.

Externalities Only: $\gamma = 0$**.** In this other extreme, the diffusion of foreign ideas is only through spillovers on Z^O. Because $z^E = Z^O$, domestic and foreign firms both offer the same learning opportunities to their workers and must pay the same wages. Contrary to the case of $\gamma = 1$, the productivity of domestic and foreign firms can exhibit persistent gaps, $R < 1$, if in equilibrium there are persistent differences in the set of ideas circulating in the home and in the foreign countries, $R^O < 1$. In such an interior BGP, the future inflow of competing foreign skills reduces the domestic return of investing in skills in the same proportion as the current inflow of foreign ideas reduces the marginal cost of investing. The interior BGP is unique and globally stable, implying that externalities only cannot push the country to the frontier. The only way that a country can catch up is if, somehow, it is already there, $R = R^O = 1$. As discussed in Section 4, this message is nuanced by the presence of occupation choices.

When $\gamma = 0$, the vintage structure is not needed, as $R_j = R$ and $d_j = 0$ for all $j \geq 1$. We can write the transition function for $R^t \equiv Z^t_h / Z^t_f$ in closed form. Given Z^t_h and $w^t = \alpha Z^t_f$, each domestic entrepreneur hires $n^t_h = (R^t)^{\frac{1}{1-\alpha}}$ units of labor. Since each foreign firm hires $n^t_f = 1$, the clearing of the domestic labor market requires $m^t_f = 1 - (R^t)^{\frac{1}{1-\alpha}}$. The shares of labor hired by domestic and foreign firms are, respectively, $1 - m^t_f$ and m^t_f, implying that the relative exposure to ideas for youth at home is

$$R^{E,t} = R^{O,t} = \left[1 + \left(R^t\right)^{\rho + \frac{1}{1-\alpha}} - \left(R^t\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\rho}}. \tag{19}$$

Openness always improves the domestic exposure to ideas, that is, $R^{E,t} > R^t$, because $R^{E,t}$ is an average of 1 and $R^t \leq 1$. However, $R^{E,t}$ might not be monotone increasing in R^t because of two countervailing forces. On the one hand, a higher R^t increases $R^{E,t}$ because domestic firms are a better source of ideas. On the other hand, a higher ratio R^t reduces the entry of foreign m^t_f and the country's exposure to foreign ideas. When $\rho \leq -1/(1-\alpha)$, the first effect dominates and $R^{E,t}$ is always increasing in R^t because of a strong complementarity between the domestic and foreign sources of ideas. However, if $\rho > -1/(1-\alpha)$, the negative effect dominates for low values of R^t and the relative exposure $R^{E,t}$ increases with a higher R^t.

Openness does not lead to full convergence; that is, even if $R^{E,t} > R^t$, the relative skills of domestic firms will remain below those of foreign firms, i.e., $R^{t+1} < 1$, because they build up their skills on the basis of interior ideas, $R^{E,t} < 1$, but have to pay the same wages $w_{t+1} = \alpha G Z^t_f$ as foreign firms. To see this, solve for the optimal accumulation of skills (17) for $\gamma = 0$ and use (19), which leads to

$$R^{t+1} = \left(R^{E,t}\right)^{\mu} = \left[1 + \left(R^t\right)^{\rho + \frac{1}{1-\alpha}} - \left(R^t\right)^{\frac{1}{1-\alpha}}\right]^{\frac{\mu}{\rho}}, \tag{20}$$

where $\mu \equiv v / [v - \alpha/(1-\alpha)] > 1$. Thus, R^{t+1}, the next period's relative level of skills of domestic entrepreneurs is a strictly convex function of the current period relative exposure $R^{E,t}$. An obvious fixed point in this mapping is when $R = R^E = 1$. But, as long as $-\infty < \rho < +\infty$, another unique and (stable) interior fixed point exists R^{int} in which $0 < R^{\text{int}} \leq R^{O,\text{int}} \leq R^{E,\text{int}} < 1$. This is (barely) shown by Figure 2, which is constructed with the parameter values discussed in Section 5. In one limit, when $\rho = -\infty$, the interior BGP converges to $R^{\text{int}} = 0$ and all firms in the country will be

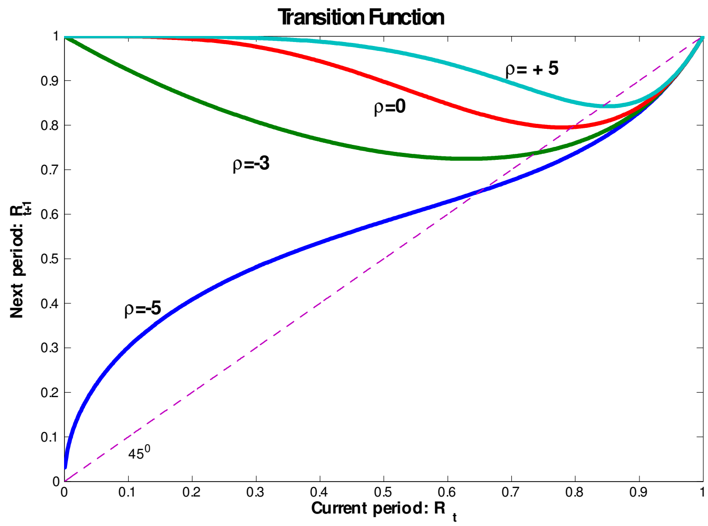

Figure 2: Transitions for an Open Economy, Externalities Only ($\gamma = 0$.)

controlled by foreign skills. In the other limit, when $\rho = +\infty$, the interior BGP collapses to $R = 1$, and after just one period of openness the home country will fully catch up with the foreign countries.

Appendix A contains the proof of the following result:

Proposition 3 (BGP open economy) *Assume that the parameter assumptions of Proposition* 1 *hold. Then:* ***(a)*** *Full convergence, i.e.,* $R^O = 1$ *is always a BGP equilibrium;* ***(b)*** *if either* $\gamma = 1$ *or* $\rho = \infty$, *then full convergence is the unique BGP and the country converges to it from any initial condition;* ***(c)*** *if* $\gamma = 0$ *and* $\rho < \infty$, *then there exists a unique interior equilibrium, i.e.,* $0 < R^{O,int} < 1$ *and the country converges to it from any initial condition* $R^0 = Z_h^0 / Z_f^0$.

The equivalent of part (c) for $0 < \gamma < 1$ is harder to prove analytically because of the multiplicity of dimensions and the potentially non-monotonicity of R^O as a function of $\{R_j\}$. However, it is straightforward to examine numerically. Indeed, with the parameter values used in Section 5, uniqueness and global stability of the interior BGP was routinely verified when $\gamma > \alpha / [v(1-\alpha)]$, as suggested by Lemma 1.

The analytical solution of the externalities only case, $\gamma = 0$, yields two interesting results about the gains from openness. The first result shows that after openness, countries that lag behind could surpass other developing countries that started ahead. Denote by R_i^t, the relative know-how of domestic firms at time t of a country that opened up at $t = 0$ with initial relative productivity R_i. Then:

Corollary 1 (Leapfrogging 1) *Assume that* $\gamma = 0$ *and* $\rho > -1/(1-\alpha)$. *Then there are two initial levels* $R_1 < R_2$ *such that* $R_1^t > R_2^t$ *for* $t \geq 1$.

When $\rho > -1/(1-\alpha)$, the relative exposure R^E is initially decreasing with respect to the relative productivity of domestic firms R. This is because, at low levels of R, the complementarity between domestic and foreign sources of ideas is not strong enough to outdo the higher entry of foreign ideas. Then, taking two small countries that under closeness lag behind the developed world, if both of them open, the country that is further behind will receive more entry, and its youth will be exposed to more ideas. Because of this, the domestic firms of that country will be managed with more knowledge in the next period. Interestingly, the form of the transition function implies that after a period, countries leave the decreasing region and remain in the increasing portion of the transition. Then, even if both countries eventually converge to the same R^{int}, during the entire transition, the country initially behind will stay ahead.

The second result is in terms of welfare and is relevant for the elusive quest for externalities in the literature. Aggregation in the case of $\gamma = 0$ is straightforward. In the closed economy, aggregate geographic and national output are equal and given by $Y^t_{\text{closed}} = Z^t_h$. Subtracting the costs of learning, aggregate consumption is $C^t_{\text{closed}} = Z^t_h [1 - \phi(G)]$. For an open economy, free entry of foreign firms implies that domestic (geographic) output will also be equal to $Y^{D,t}_{\text{open}} = Z^t_f$, and subtracting foreign profits, national income is $Y^{N,t}_{\text{open}} = Z^t_f[\alpha + (1-\alpha) R_t^{\frac{1}{1-\alpha}}]$. Obviously $Y^{N,t}_{\text{open}}$ is increasing in R_t (less of the output goes to foreign profits when the local entrepreneurs are more skillful). After some easy manipulations, it can be shown that national consumption is

$$C^t_{\text{open}} = Z^t_f[\alpha + (1-\alpha) R_t^{\frac{1}{1-\alpha}} - \phi(G) \left(R^E_t\right)^{\mu+(\mu-1)v}].$$

Consumption is a decreasing function of R^E_t because it increases the investment in skills of the young generation. Define the *steady-state* welfare gains of openness as $C^{\text{int}}_{\text{open}}/C^0_{\text{closed}} - 1$.[22] We can show that these gains are

$$\frac{C^{\text{int}}_{\text{open}}}{C^0_{\text{closed}}} = \frac{1}{R^0}\left[\frac{\alpha + [1-\alpha-\phi(G)] \left(R^{\text{int}}\right)^{\frac{1}{1-\alpha}}}{1-\phi(G)}\right],$$

which immediately lead to the following result:

Corollary 2 (Steady-state gains from openness) *The steady-state output and welfare gains of openness are strictly decreasing in R^0; if $R_L \equiv [\alpha + (1-\alpha-\phi(G))(R^{int})^{\frac{1}{1-\alpha}}]/[1-\phi(G)] < 1$, then countries with $R_L < R^0 < 1$ have negative welfare gains from openness.*

How can a country lose with openness when it brings superior knowledge from abroad? It is because the *future* inflow of foreign skills reduces the incentives of each individual in the current generation to build up skills. Collectively, this reduces the value of the public good Z^O, and it reduces the ability to form skills for everyone in the future generation.

To see this more clearly, consider the extreme case in which $z^E = Z_h$, i.e. the domestic young can only learn from the domestic old. This case is in the spirit of Stokey (1991) where openness changes the relative price of factors of production –in favor of labor, the factor that does not require investment– but does not allow for international diffusion of knowledge. In this case, the transition function boils down to $R_{t+1} = R^{\mu}_t$, and the stable BGP is $R^{\text{int}} = 0$. Then, whenever the initial R^0 lays below 1, openness leads the country to destroy the platform of ideas upon which each generation built up their skills. In the limit, the country ends up fully specialized in providing labor (to foreign

[22] In Section 5 I account for transitional dynamics when computing the gains of openness.

firms). Aggregate consumption and national income both would equal αZ_f^t. In this extreme case, the steady-state gains of openness are positive only if $R^0 < \alpha / [1 - \phi(G)]$.

4 Occupation Choices and the Diffusion of Know-how

Entrepreneurship choices have a prominent presence in the development literature (e.g., Banerjee and Newman 1993). Sorting individuals between managerial and labor occupations can enhance the static gains of openness as shown by Antras, Garicano and Rossi-Hansberg (2006), Burstein and Monge-Naranjo (2009) and more forcefully by Eeckhout and Jovanovic (2009). In this section I will argue that occupation choices can also determine whether –and how quickly– a developing country can catch up with the rest of the world. Specifically, I will show that occupation choices: (a) can change the form of the BGPs; (b) can push an open economy away from the interior BGP and toward fully catching up; and (c) can accelerate the convergence.

In the model, an old person carrying a skill level z would only become an active entrepreneur if his rents $\pi[z, w(z)]$ are above the maximum wage as a worker, i.e., only if

$$\pi[z, w_t(z)] \geq \sup_{\zeta \in \text{ support}(t)} w_t(\zeta), \tag{21}$$

where "*support*" refers to the entire set of entrepreneurial knowledge –domestic or foreign– active in the country.

The option of choosing an occupation when old can change the investment in skills for a young person. For a given exposure to ideas z^E, a young person would only invest in skills if:

$$V\left[z^E, w_{t+1}(\cdot)\right] \geq \beta \sup_{\zeta \in \text{ support } (t+1)} w_{t+1}(\zeta). \tag{22}$$

This lower bound in the career value of a job $V[\cdot,\cdot]$ can reduce the equilibrium gap between the wages paid by active entrepreneurs with different skills. Specifically, consider two entrepreneurs with skill levels $z_0 < z_1$. If the two of them fall below a certain threshold z_t^*, they will both pay the same wage; if the two fall above the threshold, the wage difference will be given by (7) of the previous section, reflecting the difference in the learning opportunities of the two jobs. Finally, if the two skill levels fall on different sides of the threshold, i.e., $z_0 < z_t^* < z_1$, the two wages paid satisfy:

$$w_t(z_1) = w_t(z_0) + \beta \sup_{\zeta \in \text{ support } (t+1)} w_{t+1}(\zeta) - V\left[z_1^E, w_{t+1}(\cdot)\right] < w_t(z_0) = w_t(z_t^*).$$

Obviously, $w_t(\cdot)$ is flat up to the threshold z_t^*, after which it becomes strictly decreasing.

Identical results as Lemma 1 and Proposition 1 of Section 3 hold even with a flat region of $w_t(\cdot)$. More interestingly, when $\gamma > 1 - \alpha / [(1-\alpha) v]$, convergence to the BGP with homogeneous entrepreneurs can be even faster as the lower tail of the skill distribution is being eliminated each period; the only ones to reproduce are the entrepreneurs on the higher end.

Before analyzing the impact on an open economy, notice that occupation choices can also change the BGP in a closed economy.

Lemma 2 (BGP closed economy, occupation choices) *Under the same parameter assumptions as in Proposition* 1*, there exists a unique BGP in a closed economy with occupation choices. If the G in the BGP without occupation choices satisfies* $\beta(1-2\alpha) > v_0 G^v / (1+v)$*, then it is also the BGP with occupation choices. If not, the unique BGP is described by a fraction* ω *of young individuals who invest in skills as well as an intergenerational growth rate* G *of skills, such that (i) young individuals are ex-ante indifferent between the two occupations:*

$$\beta\left[\frac{2-\omega}{\omega}\right]^{\alpha}\left[\frac{2(1-\alpha)-\omega}{2-\omega}\right] = \frac{v_0}{1+v}G^v;$$

and (ii) those who invest do so optimally, i.e. G *is the lower root of*

$$\beta\left[\frac{2-\omega}{\omega}\right]^{\alpha}\left[1+\frac{\gamma v v_0}{(1+v)}G^{1+v}\right] = v_0 G^v.$$

For the rest of the analysis, however, I will focus on the case where $\beta(1-2\alpha) > v_0 G^v / (1+v)$, and in the closed economy BGP all the young become entrepreneurs when old.[23]

4.1 Open Economies

It is convenient to keep separate the different cases for γ:

Markets Only: $\gamma = 1$**.** As before, full convergence is the only BGP for an open economy. Those who work for a foreign firm will mature next period into a domestic entrepreneur with exactly the same level of know-how as the contemporaneous foreign entrepreneurs. Over time, this new group of domestic entrepreneurs fully takes over the domestic labor of the country, and foreign firms will cease to enter.

Occupation choices can accelerate the convergence to $R = 1$. To illustrate this, consider a country initially endowed with domestic entrepreneurs with very low skills, $R < \alpha/(1-\alpha)$. These initial old entrepreneurs would rather supply their labor, and all the young will work for foreign firms. Because of this, the country will attain $R = 1$ the next period after openness. Notice that without occupation choices the convergence would only be asymptotic as the initial old would have to remain active and would reproduce over time.

But even if the old generation of domestic entrepreneurs is skilled enough to remain active, $R > \alpha/(1-\alpha)$, occupation choices can accelerate the convergence to the BGP. For instance, their own workers may not find it optimal to invest in skills due to their relative inferior exposure to knowledge. If so, in the next period they would be workers, and the economy will converge after two periods of openness. It is possible that convergence requires an integer n of generations of progeny of the pre-existing domestic entrepreneurs until the n-th one finds it optimal not to invest.

In any case, with occupation choices, convergence is in finite time, not just asymptotic.

Externalities Only: $\gamma = 0$**.** Absent occupation choices, the interior BGP was globally stable. With occupation choices, however, the interior BGP may no longer exist and an open economy necessarily converges to $R = 1$.

[23] The other case is a knife-edge for open economies. As becomes evident in this section, if $\beta(1-2\alpha) \leq v_0 G^v / (1+v)$, upon openness, the country will fully catch up with developed countries after one period (if $R < \alpha/(1-\alpha)$ as all the old domestic entrepreneurs choose to be workers) or after two periods (if $\alpha/(1-\alpha) < R < 1$ as the current young do not invest).

There are two possibilities. The first is driven by the occupation choices of the old: with relative skills R^{int} the old may be better off as workers, not as entrepreneurs. This happens when

$$R^{\text{int}} < [\alpha/(1-\alpha)]^{1-\alpha}, \tag{23}$$

because the domestic profits $\pi_h = \theta Z_h^{\frac{1}{1-\alpha}} [w_f]^{\frac{-\alpha}{1-\alpha}}$ fall short of the wages $w_f = \alpha Z_f$. The second possibility is that the young, being exposed to $R^{O,\text{int}} = \left[1 + \left(R^{\text{int}}\right)^{\rho + \frac{1}{1-\alpha}} - \left(R^{\text{int}}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\rho}}$, could be better off remaining workers and not investing. To see this, as a worker in the next period, the young foresees discounted earnings of $w^{t+1} = \beta G \alpha Z_f^t$; should he opt to be an entrepreneur his optimal acquisition of skills is $Z_h^{t+1} = Z_f^t G \left(R^{O,\text{int}}\right)^{\mu}$, and the net discounted payoff would be equal to $Z_f \left\{\beta (1-\alpha) G \left(R^{O,\text{int}}\right)^{\frac{\mu}{1-\alpha}} - R^{O,\text{int}} \phi \left[G \left(R^{O,\text{int}}\right)^{\mu-1}\right]\right\}$. After some basic simplifications, it can be shown that a young person would not become an entrepreneur when

$$R^{O,\text{int}} < \left[\frac{\beta G \alpha}{\beta (1-\alpha) G - \phi(G)}\right]^{\frac{1-\alpha}{\mu}}, \tag{24}$$

i.e., if his exposition to ideas is too low relative to that of the future foreign competition.

If either of these two conditions holds, after opening up, a country will always catch up with developed countries in a finite number of periods. This is because, regardless of initial conditions, R^t eventually gets near R^{int}, and at that point, after one or two periods, the country would jump to $R = 1$.

Yet, even if neither (23) nor (24) rule out the interior BGP, *depending on initial conditions*, occupation choices can lead an open country to catch up, i.e. $R = 1$. Indeed, if in any period R^t falls below the threshold $[\alpha/(1-\alpha)]^{1-\alpha}$, then all the old would opt to remain workers, clearing the way for the young to be exposed to only foreign ideas, i.e. $R^{O,t} = 1$, thus leading to $R^{t+1} = 1$ in the next period. Likewise, the occupation choice of the young could trigger the convergence: If $[\alpha/(1-\alpha)]^{1-\alpha} < R_t < R^{O,t} < (\beta G \alpha / [\beta (1-\alpha) G - \phi(G)])^{\frac{1-\alpha}{\mu}}$, the current old domestic entrepreneurs remain active but the young ones do not invest, $R^{t+1} = 0$. Therefore, all the entrepreneurial skills operating in the country at $t+1$ will be foreign, and hence $R^{O,t+1} = 1$. After such a destruction of local entrepreneurship, a new crop of domestic entrepreneurs at $t+2$ will invest at the rate $R^{t+2} = 1$, that is, will catch up with the world leaders, and all subsequent generations thereafter will remain there.

In any event, the model cleanly highlights the notion that the only way a developing country can catch up with the world leaders is through a period of destruction of the inferior know-how operating in the economy. There is no doubt that the model is extremely stark, but it also shows that these forces can be unleashed either by the full internalization of knowledge formation by markets or by the occupation choices of the young generations of domestic entrepreneurs.

Markets and Externalities: $0 < \gamma < 1$. Once occupation choices are present, it is possible that only a finite number $J < \infty$ of vintages of domestic firms can remain active. This was suggested by Figure 1, where young entrepreneurs in older vintages j would be better off remaining workers. Therefore, in equilibrium there is a last vintage, a finite value of J. All the young individuals working in last vintage, $j = J$, would not invest in skills because they foresee that they will return as workers in the vintage $j = J$ of period $t+1$ (which will be managed by the current young trainees in vintage $J-1$.)

Denote by d_J the compensating differential (in units of Z_f) received by workers of the last vintage J. Because entrepreneurs do not care who is providing the labor, both young and old receive wages equal to $w_J = (\alpha + d_J) Z_f$. For a young person to be indifferent between being a worker in both periods of life, the implied lifetime earnings $(\alpha + d_J)(1 + \beta G)$ must be equal to the lifetime earnings attained from any other option, in particular $j = 0$, the foreign firm. Equating the net-present values from these two options, the compensating differential d_J must be equal to

$$d_J = \frac{\beta G \left[\pi \left(R_1, \alpha + d_1\right) - \alpha\right] - R_0^E \phi \left(G \frac{R_1}{R_0^E}\right)}{1 + \beta G}. \tag{25}$$

In all the other vintages, $j = 0, ..., J - 1$, d_j behaves as in the previous section, with labor fully provided by young individuals who invest in skills and become entrepreneurs when old. Since there is no population growth, it has to be the case that half of the workers in the last vintage J are young and half old.

Therefore, an interior BGP equilibrium (i.e., $0 < R^O < 1$) of an open economy with occupation choices is defined as a $J \in \mathbb{N}$ (possibly ∞ if occupation choices do not bind) and an array $\left\{m_j, n_j, R_j, d_j, R_j^E\right\}_{j=0}^J$ that satisfies conditions (a)-(b) and (d)-(g) of the previous section, but with the following modifications: (i) the summations run from $j = 0$ to $j = J$; (ii) d_J is given by (25); and (iii) while condition (c) holds, the absolute mass of foreign firms is $m_f = m_0 (1 + n_J/2)$ since the total labor force includes $n_J/2 > 0$ old persons supplying labor for the last vintage of domestic firms.

If occupation choices not bind, a number $J < \infty$ of vintages is determined by the following conditions: (1) workers in vintage $J - 1$ find it optimal to invest at the rate R_J instead of remaining a worker:

$$\beta G \left[\pi \left(R_J, \alpha + d_J\right) - \left(\alpha + d_J\right)\right] - R_{J-1}^E \phi \left(G \frac{R_J}{R_{J-1}^E}\right) > 0;$$

and (2) workers in vintage R_J find it optimal to remain workers, given that, as entrepreneurs they would not spawn entrepreneurs, i.e.,

$$\max_R \left\{\beta G \left[\pi \left(R, \alpha + d_J\right) - \left(\alpha + d_J\right)\right] - R_J^E \phi \left(G \frac{R}{R_J^E}\right)\right\} < 0.$$

This vintage structure is similar to that in Chari and Hopenhayn (1991), except for two crucial additional aspects. First, the levels R_j of each vintage $j \geq 1$ are endogenous. Second, the level of the externality R^O is also determined endogenously.

Comparing economies with and without occupation choices, it is evident that occupation choices have two different positive effects on the formation of domestic know-how. First, a *selection effect*, as the older, less productive vintages are no longer active. Their labor must be re-allocated to the younger, more productive vintages. This reallocation pushes the economy to a higher ratio R^O of ideas. Second, there is an *investment effect*: the young who work in any of the active vintages $j = 0, ..., J - 1$ invest more because of the lower compensating differentials d_j that they would have to pay, given the better career options in vintages $j \geq J$.

Using the parameter values of Section 5, Figure 3 compares the interior BGPs of economies with occupation choices (dots) and without them (stars). As shown by the lower-right panel, only six

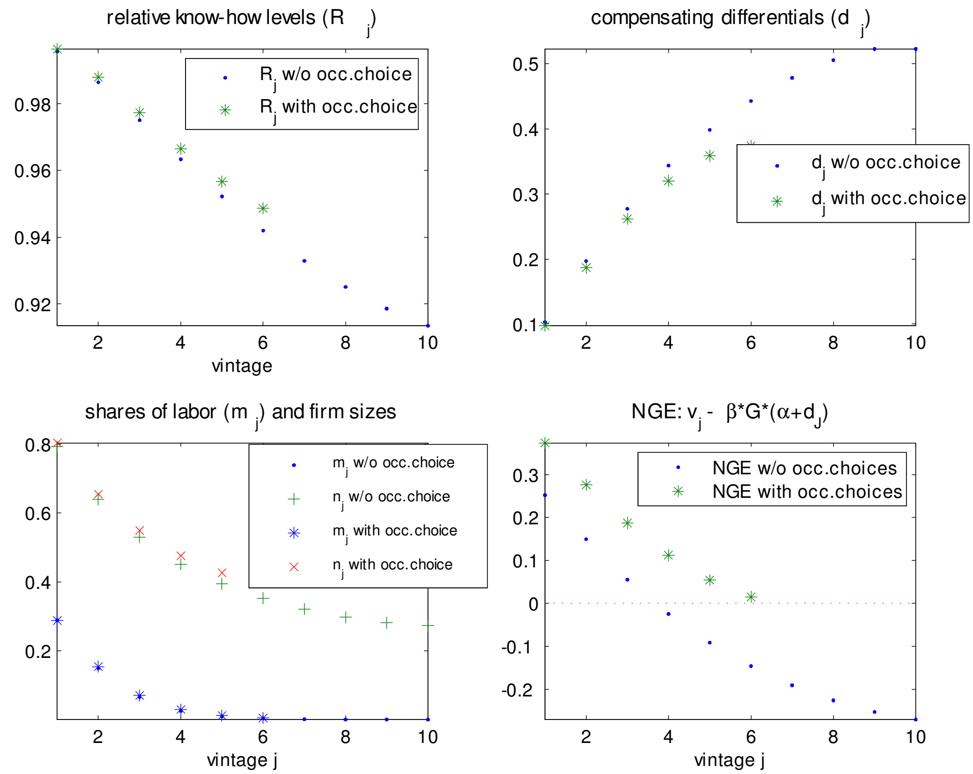

Figure 3: BGP open economy, occupation choices; only six vintages of domestic firms.

vintages of domestic firms would remain active. Comparing these six vintages with the respective ones in economies without occupation choices, we can see that the relative productivities R_j are higher (upper-left panel) and the compensating differentials d_j are lower (upper-right panel) in economies with occupation choices. Therefore, it is easy to explain why the labor units n_j and labor shares m_j are also higher in economies with occupation choice (lower-left panel). The lower-right panel reports the net present value gains of opting for entrepreneurship in the second period.

As in the case of $\gamma = 0$, the impact of occupation choices goes beyond changing the structure of interior BGPs. First, they can remove the interior BGP altogether, and regardless of initial conditions, openness would necessarily lead a country to fully catch up with developed countries. Second, even if the interior BGP persists, occupation choices can drastically change the dynamics of an open country because, depending on initial conditions, the country would move toward full convergence instead of the interior BGP.

Two simple corollaries summarize the impact of occupation choices on the diffusion of foreign entrepreneurial know-how in open economies.

Corollary 3 (Destruction for convergence) *Open countries only catch up with developed countries if pre-existent, inferior domestic skills stop being reproduced at some point.*

As before, an open country only catches up if, along the way, the pre-existent, less productive domestic sector is entirely replaced by a new domestic sector with the same productivity as foreign firms. Such destruction can be abrupt as described above when $\gamma = 0$ and there is only one vintage of domestic firms. The destruction can be more protracted when $\gamma > 0$ and multiple vintages of domestic firms arise along the transition.

Occupation choices can be the culprit of a different form of leapfrogging, one in which the more backward an open country is the more likely it is to catch up with the frontier.

Corollary 4 (Leapfrogging 2) *Assume the interior BGP exists (i.e.,* $R^{int} > [\alpha/(1-\alpha)]^{1-\alpha}$ *and* $R^{O,int} > \{\beta G\alpha/[\beta(1-\alpha)G-\phi(G)]\}^{\frac{1-\alpha}{\mu}}$*). Then, there are two initial levels* $R_1^0 < R_2^0$ *such that, upon openness at* $t=0$*,* $\lim_{t\to\infty} R_2^t = R^{int} < \lim_{t\to\infty} R_1^t = 1$*.*

Simply put, after openness the more backward countries may end up fully catching up with the world frontier while the relatively more advanced remain forever behind, stuck in the interior BGP $R^{\text{int}} < 1$. This form of leapfrogging is across BGPs, very different from the previous one which refereed to the transition dynamics to the same BGP.

5 Quantitative Analysis

This section provides a basic quantitative assessment of the aggregate gains that a developing country could attain from being open to foreign firms from more developed countries. To this end, I extend the basic model to connect more closely with some basic observed statistics on entrepreneurial activity and use parameter values that are standard in the literature. The extended model provides a more accurate depiction of the diffusion of entrepreneurial skills, as it is restricted to the subset of young workers in management who directly interact with the decision makers at the top of active firms.

After briefly setting up the extended model, I discuss the parameter values used, and then I illustrate the ensuing dynamics of a country after it opens up. Then, I assess the welfare gains for different initial conditions, as well as alternative values for the preponderance of markets vis-à-vis externalities.

5.1 A Model with Middle-Managers and Workers

Consumption goods are produced with entrepreneurial (top management) skills (z), mid-management services (n), and labor services (l):

$$y = zn^{\alpha}l^{\lambda}.$$

Each young cohort is composed of two groups: a fraction $0 < \omega < 1/2$ of potential managers and a fraction $1-\omega$ of perennial workers. Perennial workers provide labor services both periods, when young and old, and do not accumulate skills. On the contrary, potential managers in either period can be workers or middle-managers; when young they can invest in skills and become entrepreneurs (top-managers) when old. As in the basic model, their exposure to ideas z^E is determined by both, the skills z of the top-manager for whom the young person works and the average skill level Z^O implemented in the country, which follows a similar formula as in the basic model.[24]

[24] The weights μ_z in Z^O are the shares of the country's aggregate supply of middle-managers used by firms with the different levels z of skills. These shares are different from the shares of labor services because, as shown in the following, the relative price paid for mid-managers and workers depends on the skill levels z of the top manager.

Under the parameter values used and discussed below, along the closed economy BGP all the young potential managers become middle-managers, invest in skills and then become active entrepreneurs when old. Thus, assume that all the ω old entrepreneurs have skills Z_t. Then, if all of them are active top-entrepreneurs the aggregate supply of (top) managerial skills is ωZ_t. Likewise, if all the young potential middle-managers operate as such, the aggregate supply of middle-managerial services is simply ω. Finally, the two cohorts of perennial workers provide a total amount of labor equal to $2(1-\omega)$. In equilibrium, each production team is composed of one top manager, one middle manager and $\varrho \equiv 2(1-\omega)/\omega > 1$ workers. The incomes of workers (w_t), mid-managers (w_t^m) and entrepreneurs (π_t) are, respectively, $w_t = \lambda \varrho^{\lambda-1} Z_t$, $w_t^m = \alpha \varrho^{\lambda} Z_t$, and $\pi_t = (1-\alpha-\lambda)\varrho^{\lambda} Z_t$. It is straightforward to verify that if $\lambda/\varrho < \alpha < (1-\lambda)/2$ ex-post occupation choices are satisfied as an old person is better off being a top-manager than a middle-manager or a worker and, being a young manager is better than being a worker.

As detailed in Appendix B, along the closed-economy BGP, the gross growth rate of skills G is given by

$$\beta\left[\varrho^{\lambda} + \frac{\gamma v v_0}{1+v} G^{1+v}\right] = v_0 G^{v},$$

the same as expression (10) except for the term ϱ^{λ}. Since $\varrho^{\lambda} > 1$, top-managers have more workers under their control. Finally, for an equilibrium BGP it is required that potential managers opt to invest in skills instead of remaining mid-managers when old, i.e., $w_t^m + \beta w_{t+1}^m < w_t^m + \beta \pi_{t+1} - Z_t \phi(G)$. This inequality boils down to $\phi(G) < \beta(1-2\alpha-\lambda)\varrho^{\lambda} G$. Thus, under conditions very similar to the basic model, there exists a unique BGP for a closed economy.

The analysis of open economies is very similar to that in the basic model. In some of the formulas, for instance μ, the counterpart of the parameter α in the basic model is $\alpha + \lambda$ in the extended model. See the details in Appendix B.

5.2 Parameter Values

I discipline the value of some of the parameters using some basic considerations and broad observations. Table 2 presents the resulting parameter values. Each period of life is set to represent 15 years, a ballpark number for the years after starting to work, a mid– or assistant manager gets promoted to the top position of a firm. Using periods of 20 does not change the results. Anyway, the discount factor is set to $\beta = (1.04)^{15}$ so that the annual interest rate is equal to 4%, roughly the historical average for the U.S. I set $\omega = 0.1$ so that 10% of the population is in managerial occupations, 5% as top-managers and 5% as entrepreneurs. These values are in the in the low end of the numbers reported for related concepts, i.e. entrepreneurship (in Cagetti and De Nardi, 2006) or managerial occupations (in Eeckhout and Jovanovic 2010). Similarly, I set the span-of-control $\lambda + \alpha$ to 0.80, which is also in the low end of the values used in the literature (e.g. Buera and Shin 2010). Using low values for ω and $\lambda + \alpha$ leads to conservative estimates of the aggregate gains from openness. The individual values λ and α are of no particular interest as long as (i) the occupation choice conditions $\lambda/\varrho < \alpha < (1-\lambda)/2$ are satisfied, and (ii) the effect of ϱ^{λ} on the equilibrium G is undone with the calibration of the other parameters, which is precisely how I proceed.

	Parameter/Definition	Value	Criterion/Target
ω	fraction managers/entrepreneurs	0.1	% in managerial occupations
λ	output share, labor	0.725	see text
α	output share, mid-managers	0.075	see text
β	discount factor	$(1.04)^{-15}$	annual risk free-rate in the U.S.
v	curvature, cost of skills acquisition	25	existence of BGP for various ρ, γ
v_0	level, cost of skills acquisition	depends on (γ, ρ)	2% annual growth (BGP)
γ, ρ	internalization, external diffusion	see text	comparative statics

Table 2. Parameter values for the quantitative exercises.

As seen in Proposition 1, the existence of a BGP requires a high value for v, the curvature of the costs of skills $\phi(\cdot)$. I set $v = 25$, a value high enough to ensure the existence of a BGP for a wide range of the other parameters. While changing v affects the behavior of the economy, in the interest of space I shall focus on variations in γ and ρ, the internalization and external diffusion parameters. I experiment with different values of γ in $[0, 1]$ and values of ρ in $(-\infty, \infty)$ to examine the implications of alternative specifications of the diffusion of knowledge.

Finally, given all the other parameter values, I recalibrate the parameter v_0 so that the equilibrium gross growth rate is $G = (1.02)^{15}$, i.e., a 2% implied net annual growth rate of output.

5.3 The Inflow of Foreign Know-how after Openness

Figure 4 shows the response of a closed economy, previously in a BGP, when it permanently opens up at $t = 1$. In the interest of space, the illustrations consider only three parameter configurations: *(a)* a fully internalized diffusion case ($\gamma = 1$) where externalities are absent and the value of ρ is irrelevant; *(b)* a fully external case ($\gamma = 0$) with $\rho = 3.33$ to model a case in which externalities can have a strong impact on the accumulation of domestic know-how; and *(c)* an intermediate case ($\gamma = 0.75$, $\rho = 3.33$) where both externalities and internalized transfers are present. For all three cases, I assume that the initial domestic-to-foreign know-how ratio is $R = .7$, which is high enough so that occupation choices do not bind upon openness.

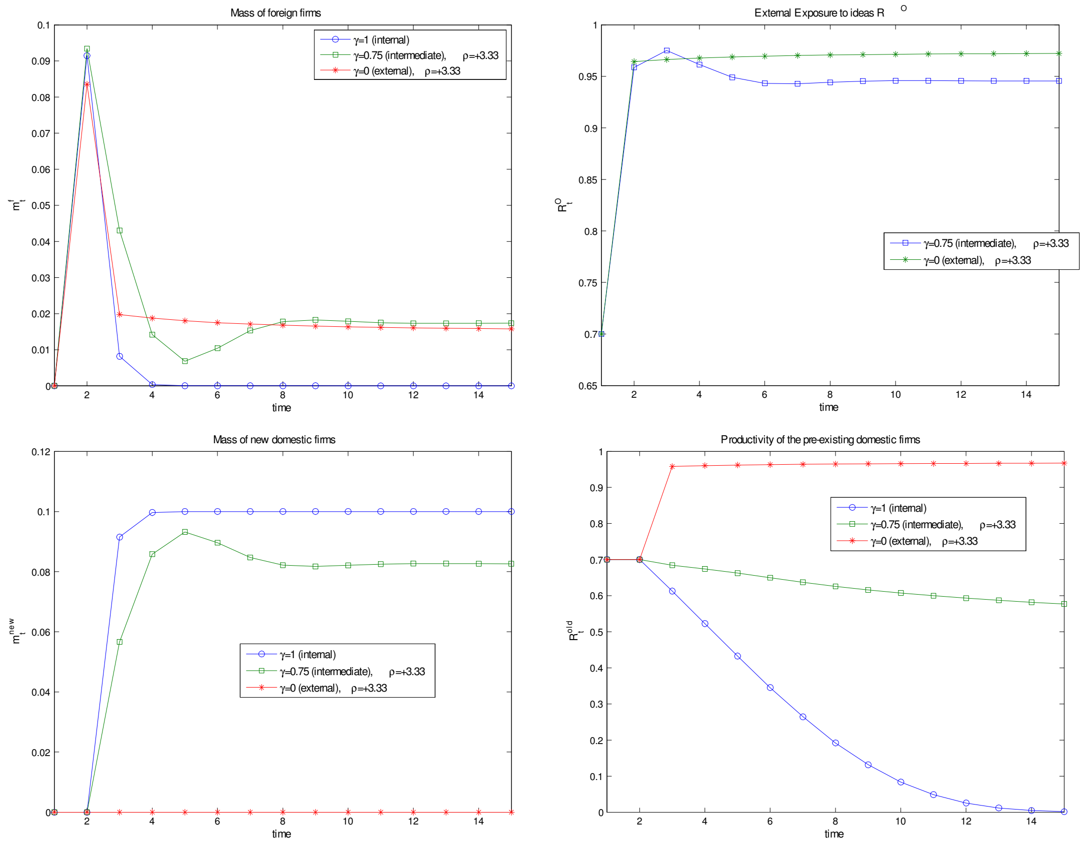

Figure 4: Foreign entry and diffusion of know-how after openness.

In the three cases, as shown by the upper-left panel of Figure 4, the initial lower costs of labor and mid-management services in the home country generate a burst of entry of foreign firms. Entry is higher for parameter cases (a) and (c) because the better career opportunities offered by foreign firms push further the costs of middle-managers for domestic firms. When $\gamma = 1$, these learning opportunities are so strong that the new sector of domestic firms (lower-left panel) quickly catch up with foreign firms and these practically stop entering the country just after three periods.[25] On the opposite extreme, when $\gamma = 0$,there is a sustained presence of foreign firms because the home country young entrepreneurs never reach the exposure to ideas as their foreign peers, i.e. R^O remains below 1 (upper-right panel).

The response in the intermediate case (c) captures aspects of both extreme cases (a) and (b). As in case (a), the entry of foreign firms generates a new sector of domestic firms (lower-left panel), but their know-how levels do not fully catch up with that of the foreign firms, as shown by Figure 3 (lower-right panel.) Indeed, as shown by the lower-left panel of Figure 4, the size or total mass of these new domestic firms is lower than in full internalization case (a). Because of this, as shown in the upper-left panel, foreign firms sustain their presence in the country. Interestingly, in the limiting BGP, the mass of foreign firms may be higher in an economy with partial internalization, i.e. case (c) with $\gamma > 0$, than in the pure externalities case (b) with $\gamma = 0$, because in the former economy foreign firms have an additional advantage of offering better career prospects to mid-managers than domestic firms.

[25] Keep in mind that each period represents a generation of entrepreneurs.

Perhaps the most interesting difference across the three economies is in the impact of openness on the productivity of pre-existent domestic firms. In the pure-externalities case (b), pre-existent domestic firms become more productive when their initial level is below the interior BGP ratio R^{int}. The opposite response, however, would be observed if initially $R > R^{\text{int}}$. Therefore, the pure externalities model appears completely at odds with the empirical results in Aitken and Harrison (1999), Xu (2000), and Alfaro et al. (2006) among others and this suggests that, if anything, the presence of foreign firms seems to have a negative impact on the productivity of domestic firms in developing countries.[26]

In light of the results in Figure 4, the empirical evidence can be interpreted as supporting the view that the diffusion of foreign-firm knowledge must involve, at least partially, some internalization, i.e., $\gamma > 0$. Most vividly, in case (a), when there is full internalization, i.e. $\gamma = 1$, pre-existing domestic respond to openness by substantially *reducing* their investments in know-how, leading to a rapid decline in the growth –and future levels– of their productivity. Albeit less strong, a similar response is observed for the intermediate case (c), $\gamma = 0.75$, because the negative impact of foreign competition is not compensated by the positive spillovers of foreign ideas.

Contrary to the implicit presumption in the empirical literature, externalities are neither sufficient nor necessary for openness to push the country forward. Indeed, some forms of internalized transfers of knowledge seem to explain the emergence of new production sectors in Bangladesh, Colombia, Indonesia and other countries described by Rhee and Belot (1990).[27] In any event, it important to distinguish the impact of openness on the country as a whole from that on the pre-existing domestic firms. Even if pre-existing domestic firms reduce their productivity, openness may be the culprit for countries to catch up with the world frontier, as they replaced their pre-existent firms (and their progeny) with a new sector of more productive domestic firms.

5.4 Welfare Gains from Openness

A central question in this paper is whether openness enhances the aggregate welfare of a developing country. In this section, I use alternative configurations of the model that vary on the preponderance of markets vs. externalities for the diffusion of ideas, and compute the welfare gains for a wide range of development levels, i.e. the country's initial ratio R. I define

$$\text{Welfare Gains of Openness} \equiv \frac{\sum_{t=0}^{\infty} \beta^t C_t^{\text{open}}}{\sum_{t=0}^{\infty} \beta^t C_t^{\text{closed}}} - 1,$$

[26] As discussed by Xu (2000), Alfaro et al. (2006) and Griffith et a. (2002), the evidence suggests positive spillovers on domestic firms, but only for developed countries. For developing countries, some authors (e.g. Javorcik [2004] and Kugler [2005]) have argued for the existence of inter-industry spillovers, specifically, from foreign firms to local suppliers. However, productivity gains are probably better seen as internalized transfers, not spillovers, since as Javorcik herself reports, foreign firms in her sample were directly involved, providing training, equipment, and know-how to the local suppliers.

[27] At the level of domestic industries, skill formation inside the firm seems to be a major mechanism for aggregate skill formation and dissemination, as indicated by the empirical evidence that links the characteristics and the outcomes of parent firms with their spin-offs. For the U.S. car industry, Keppler (2001, 2002, 2006) documents that the genesis of the most successful car makers can be traced to former employees of other car makers. Agarwal et al. (2004), Filson and Franco (2006), and Franco (2005) show the same for the rigid disk drive industry.

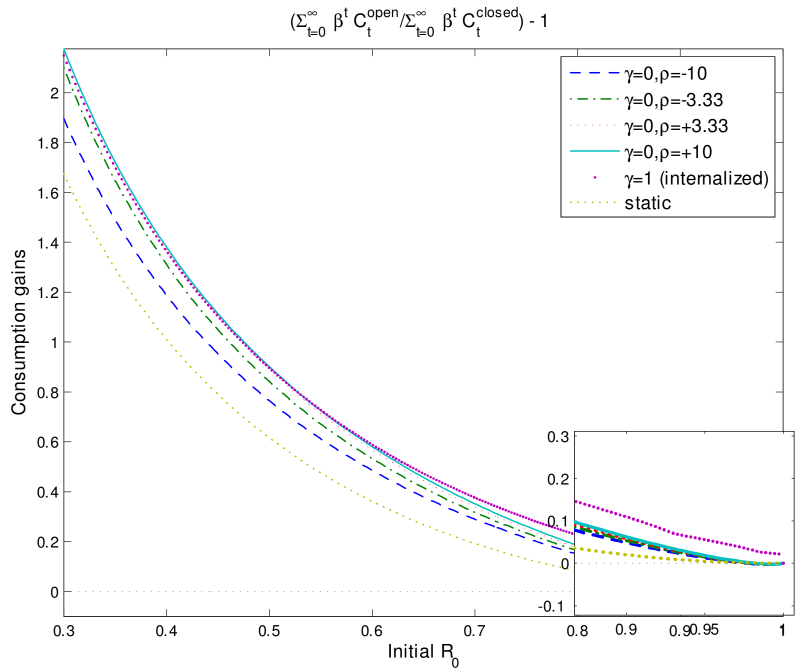

Figure 5: Welfare gains of openness under alternative initial conditions and parameters.

i.e., the net proportional gain in the present value of aggregate consumption that the country attains by opening up at time $t = 0$. In every period, aggregate consumption is equal to the country's geographic output (GDP), minus foreign profits (retribution to foreign know-how), minus the skill formation costs of all the country's young forming entrepreneurs. This definition of welfare gains, obviously, goes beyond the steady-state calculations of Section 2, as it fully accounts for the transitional dynamics. This measure is already in consumption units because of linear preferences,

Figure 5 displays the welfare gains for initial values of R in the rage between 0.3 and 1. The figure reports the gains from the pure externalities models $\gamma = 0$, for values of the external diffusion parameter ranging from the very low $\rho = -10$ to very high $\rho = 10$. It also reports the gains for the fully internalized formation $\gamma = 1$. As a benchmark, the figure also reports the static gains (dotted line).[28] The figure generates a number of important messages.

First, regardless of the parameter specification, the gains of openness are generally very high, especially for countries that lag far behind. For example, a country with only $R = 0.3$ could attain net gains above 200%, i.e., a threefold gain in present value consumption. A country with an initial ratio $R = 0.7$ has still substantial gains, in the order of 40%, across the different variations in the relative role of markets vs. externalities. These numbers are large, considering that they net out foreign profits and the costs of building up the skills.

Second, the dynamic gains of openness are substantially larger than the static gains. The response in the formation of domestic know-how accounts for a large fraction of the gains. This is true even in the extreme case of $\gamma = 0$ and $\rho = -10$, when the absorption of foreign ideas is persistently hindered

[28]Static gains coincide with an economy in which skills are exogenously kept constant at the initial R, there are zero costs of investments, and foreign entrepreneurs fully appropriate their marginal contribution to the country's output. Thus calculated, static gains coincide with the gains as computed by Burstein and Monge-Naranjo (2009) for the unilateral case when all countries have the same country-embedded productivity.

by the low level of domestic skills. Even then, the dynamic gains add about 20% of consumption relative to the static gains.

The third and most remarkable finding is that the gains can be larger when the diffusion of ideas takes places via markets as opposed to externalities. Indeed, the gains are larger for the externalities model only when R is low and ρ is high. Otherwise, the market mechanism would lead to larger gains. Recall that with full internalization, $\gamma = 1$, the country always gains with openness, no matter how far or close it is to the world frontier.

Fourth, very different parameter specifications lead to relatively similar global behavior in the gains across different levels of development, R. This finding is interesting, given the very different mechanisms of models of diffusion based on externalities vs. models of diffusion via markets. A final noteworthy finding is that the potential losses of openness in the case of externalities occur only in a small interval near the frontier, around 1, as shown by the magnified region of the graph.

6 Concluding Remarks

I construct a model where entrepreneurial skills in a country are built up every period on the basis of the productive knowledge implemented by domestic and foreign firms operating inside the country. The model encompasses as special cases two standard –but conflicting– models for the diffusion of knowledge: markets and externalities. The model also provides a simple framework that clearly distinguishes the impact of foreign knowledge on pre-existing and new domestic firms. Doing so, I argue that the dynamic gains from openness to foreign know-how necessarily require destructive, reallocative and renewal forces for domestic firms and occupations inside the country. These forces are only fully unleashed when knowledge is transmitted by markets. Interestingly, a market mechanism is needed for the model to be consistent with existent empirical estimates on the impact of FDI on developing countries.

A similar framework could be used to study a number of related issues. First, the paper has focused on a rather drastic policy choice, either being completely closed or completely open. An interesting elaboration would be to characterize the optimal taxes or subsidies for a developing country to set on foreign firms, considering their impact on the domestic formation of skills. The presence of externalities is likely to yield interesting trade-offs and time-consistency issues may constrain the optimal policy. Second, the analysis here has considered entrepreneurial skills as the single engine of growth. The accumulation of complementary technology, physical capital, schooling and other forms of human capital could all be important for the consequences of foreign entrepreneurial know-how in developing countries. Third, the analysis in the paper is vertical in the sense that it has focused on the impact of small developing countries. Adapting the recent stochastic multicountry settings for trade and FDI is the way to go for understanding also the gains of openness for developed countries. However, those settings should go beyond externalities and be extended to incorporate markets in the transmission of knowledge. Last but not least, the theory developed in this paper provides novel ways of looking at the increasingly available matched employer-employee data sets and for interpreting the dynamic outcomes of workers in domestic and foreign firms, including their entrepreneurial decisions.

A Analytical Aspects of the Model

A.1 Proofs

Proof of Lemma 1. To examine the accumulation of skills, derive expression (4) to obtain

$$\pi_1 = \left[\frac{\alpha z}{w(z)}\right]^{\frac{\alpha}{1-\alpha}}, \text{ and } \pi_2 = -\left[\frac{\alpha z}{w(z)}\right]^{\frac{1}{1-\alpha}}.$$

Next, from equilibrium condition (7), it follows that

$$\frac{\partial w_{t+1}(z')}{\partial z'} = -V_1\left[\left(z^E\right)', w_{t+2}(z'')\right]\frac{\partial\left(z^E\right)'}{\partial z'}.$$

The envelope condition of (6) implies that

$$V_1\left[\left(z^E\right)', w_{t+2}(z'')\right] = -\phi\left(\frac{z''}{(z^E)'}\right) + \left(\frac{z''}{(z^E)'}\right)\phi'\left(\frac{z''}{(z^E)'}\right) = -\frac{vv_0}{1+v}\left(\frac{z''}{(z^E)'}\right)^{1+v},$$

where $\left(z^E\right)'$ and (z'') are, respectively, the next period's exposure of ideas and the investments for the workers of the next period's entrepreneurs. The second equality results from the functional form of $\phi(\cdot)$.

Therefore, combining all of these expressions in (8), and using the functional form for $\phi(\cdot)$

$$\begin{aligned}
v_0\left(\frac{z'}{z^E}\right)^v &= \beta\left\{\left[\frac{\alpha z'}{w_{t+1}(z')}\right]^{\frac{\alpha}{1-\alpha}} + \left[\frac{\alpha z'}{w_{t+1}(z')}\right]^{\frac{1}{1-\alpha}}\frac{vv_0}{1+v}\left(\frac{z''}{(z^E)'}\right)^{1+v}\frac{\partial\left(z^E\right)'}{\partial z'}\right\} \\
&= \beta\left[\frac{\alpha z'}{w_{t+1}(z')}\right]^{\frac{\alpha}{1-\alpha}}\left\{1 + \frac{\alpha z'}{w_{t+1}(z')}\frac{vv_0}{1+v}\left(\frac{z''}{(z^E)'}\right)^{1+v}\frac{\partial\left(z^E\right)'}{\partial z'}\right\} \\
&= \beta\left[\frac{\alpha z'}{w_{t+1}(z')}\right]^{\frac{\alpha}{1-\alpha}}\left\{1 + \alpha v\left[\frac{\frac{v_0}{1+v}\left(\frac{z''}{(z^E)'}\right)^{1+v}\left(z^E\right)'}{w_{t+1}(z')}\right]\left[\frac{z'}{(z^E)'}\frac{\partial\left(z^E\right)'}{\partial z'}\right]\right\},
\end{aligned}$$

where the first equality follows from all the substitutions, the second simply takes the expression for $\pi_1(\cdot)$ as common factor and the third simply rearranges expressions after dividing and multiplying by $\left(z^E\right)'$. The expression in the text follows from using the definitions of $\xi_{z,\ z^E}$ and θ_{z',z^E}.■

Proof of Lemma 2 The value of V may be $+\infty$ as illustrated in the proof for Proposition 1 for the case of homogeneous firms. In such a case, the function $w(\cdot)$ is not well defined. However, if V is bounded, then the monotonicity of the function $w(\cdot)$ arises directly from the envelope condition on V. Next, the monotonicity of $z'(\cdot)$ arises from the fact that it is given by

$$\beta(\alpha z')^{\frac{\alpha}{1-\alpha}} g(z') = v_0\left(\frac{z'}{z^E}\right)^v,$$

where $g(z') \equiv w'(z')^{\frac{-\alpha}{1-\alpha}}\left[1 + \frac{\alpha\gamma vv_0\left(z''/\left(z^E\right)'\right)^{1+v}(z')^\gamma\left(Z^{O\prime}\right)^{1-\gamma}}{(1+v)w(z')}\right]$, which is non-decreasing in z'. For a maximization, the left-hand-side must cross the right-hand side from above, hence the need for the condition $v > \alpha/(1-\alpha)$. Thus, a z or higher z^E leads to a higher optimal value for z'. Finally, straightforward differentiation leads to

$$\frac{\partial\ln(z')}{\partial\ln(z^E)} = \frac{v}{\left[v - \frac{\alpha}{1-\alpha}\right] - \frac{\partial g(z')}{\partial z'}\frac{z'}{g(z')}} \geq \frac{v}{v - \frac{\alpha}{1-\alpha}} > 1,$$

where the first inequality holds because $\frac{\partial g(z')}{\partial z'}\frac{z'}{g(z')} \geq 0$ and the second because $v > \alpha/(1-\alpha)$. Since $z^E = z^\gamma\left(Z^O\right)^{1-\gamma}$, then,

$$\frac{\partial\ln(z')}{\partial\ln(z^E)} \geq \frac{\gamma v}{v - \frac{\alpha}{1-\alpha}},$$

which is greater than one if $\gamma > 1 - \frac{\alpha}{(1-\alpha)v}$, as stated.■

Proof of Proposition 1. First consider $\gamma = 0$. The unique solution is $G = (\beta/v_0)^{1/v}$. and the inequality $\beta < [v_0(1+v)]^{\frac{1}{1+v}}$ ensures that $\phi(G) < 1$ so the growth in skills is not so high so as to lead to negative aggregate consumption. Second, consider $\gamma > 0$. Define $L(G) \equiv \beta\left[1 + \frac{vv_0\gamma}{1+v}(G)^{1+v}\right]$, the marginal return to investing (the left-hand side of equation 10) and $R(G) \equiv v_0(G)^v$ the marginal cost of investing (the right-hand-side.) Notice that $L(0) > 0 = R(0)$ and that the curvatures of $L(\cdot)$ and $R(\cdot)$ are $1+v$ and v, respectively. Therefore, $L(\cdot)$ may lay above $R(\cdot)$ for all positive real numbers. In such a case, the optimal investment and the career value of a young person would both be degenerate, $G = +\infty$, $V = +\infty$. However, $L(\cdot)$ may lay below $R(\cdot)$ for some range; if so, because of its higher curvature, it would cross $R(\cdot)$ twice, first from above and then from below. From the intermediate value theorem, a sufficient condition for $L(\cdot)$ to cross $R(\cdot)$ is to find a point $G > 0$ for which $L(G) < R(G)$. It is straightforward to show that the two curves are parallel to each other, i.e. $\partial L/\partial G = \partial R/\partial G$, only at the point $\bar{G} = (\beta\gamma)^{-1}$, and obviously, $\partial L/\partial G < (>)\, \partial R/\partial G$ for $G < (>)\, \bar{G}$. It is easy to verify that $L(\bar{G}) < R(\bar{G})$ if $\beta < (v_0/[\gamma^v(1+v)])^{\frac{1}{1+v}}$. Uniqueness follows because the second root of (10) is not relevant, since it is a local minimum, i.e. the marginal costs would cut the marginal benefit from above. Not having a closed form for the root G implies that non-negativity of consumption has to be examined numerically. Finally, the inequality $\beta > v_0(1+v)/(1+v+vv_0\gamma)$ is equivalent to $L(1) > R(1)$ which ensures that the first crossing, if it exists, is higher than 1. Notice that the same condition applies when $\gamma = 0$.■

Proof of Proposition 2. We first prove the first part of the proposition. Solving equation (10) for G_{t+1} we get:

$$G_{t+1} = \left[\frac{1+v}{\gamma v v_0}\left(\frac{v_0}{\beta}(G_t)^v - 1\right)\right]^{\frac{1}{1+v}},$$

which is a strictly concave function of G_t that first crosses the 45^0 line from below, precisely at the valid BGP G. From any $\epsilon < 0$ and initial $G_0 \neq G^{BGP}$ such that $0 < \left|G_0 - G^{BGP}\right| < \epsilon$, the implied $\{G_{t+1}\}$ would diverge. Therefore, the only equilibrium possible is $G_t = G^{BGP}$ for all t. We now prove the second part of the proposition. First, consider $\gamma = 0$. In this case, $z^E = Z^O$ for all, regardless of the initial distribution. After one period, the economy converges to a uniform entrepreneurs BGP in which $z_{t+1} = (\beta/v_0)^{1/v} Z_t^O$. Second, consider $\gamma > 1 - \alpha/[(1-\alpha)v]$, and any initial non-degenerate distribution ϕ_z^0 with bounded support. Consider two skill levels, $z_0 < z_1$ and denote by z_i^t the skills levels of the t generation of a dynasty of entrepreneurs starting with z_i for $i = 0, 1$. From Lemma 1, $\lim_{t\to\infty} z_0^t/z_1^t = 0$. Given an initial mass M_i^0 of entrepreneurs of each type i, the mass M_i^t at time t equals $M_0^i \Pi_{\tau=0}^{t-1} n_i^\tau$, where n_i^τ is the labor employed at time τ by the dynasty i. Since $w_\tau(\cdot)$ is non-increasing then $n_0^\tau/n_1^\tau \leq (z_0^\tau/z_1^\tau)^{\frac{1}{1-\alpha}}$, which is a decreasing sequence that converges to 0. Then, $\lim_{t\to\infty} M_0^t/M_1^t = 0$. Since this holds for any pair $z_0 < z_1$, the only possibility is that the dynasty starting with the highest initial skill level takes over the entire population.■

Proof of Proposition 3. Part (a) is straightforward: since $R = 1$, the equilibrium conditions for the closed economy BGP apply to the open economy with $m_f = 0$ because $w_h = w_f$. Part (b) is also straightforward: assume that there is a BGP with $R_j \leq 1$ with the inequality strict for some $j < \infty$. Since the economy is open, then $w_h = w_f$ and $m_f > 0$. Otherwise, there would be an excess supply of labor because $n_j \leq 1$ with strict inequality for the vintages j s for which $R_j < 1$. Now, if $\gamma = 1$, and by definition $R_0 = 1$, then $R_0^E = 1$ and in equilibrium $R_1 = 1$. Similarly, if $\gamma < 1$ but $\rho = +\infty$, then $R^O = 1$ implying that $R_0^E = 1$, and then $R_1 = 1$. In either case, whenever $R_j = 1$, it is the case that $R_j^E = 1$ and $R_{j+1} = 1$. Therefore, by induction $R_j = 1$ for all $j = 1, 2, ...\infty$ and the result is established. For part (c) recall that the transition function is

$$R_{\text{next}} = F(R) \equiv \left[1 + (R)^{\rho + \frac{1}{1-\alpha}} - (R)^{\frac{1}{1-\alpha}}\right]^{\frac{\mu}{\rho}},$$

a twice continuously differentiable function. It is easy to verify that $F(1) = 1$ and that

$$F(0) = \begin{cases} 1 & \text{if } \rho > -1/(1-\alpha), \\ 2^{\frac{-\mu}{1-\alpha}} \in (0,1) & \text{if } \rho = -1/(1-\alpha), \\ 0 & \text{if } \rho < -1/(1-\alpha). \end{cases}$$

Moreover, the first derivative is

$$F'(R) = \left(\frac{\mu}{\rho}\right)\left[1 + (R)^{\rho+\frac{1}{1-\alpha}} - (R)^{\frac{1}{1-\alpha}}\right]^{\frac{\mu}{\rho}-1} \times \left[\left(\rho + \frac{1}{1-\alpha}\right)R^{\rho+\frac{\alpha}{1-\alpha}} - \left(\frac{1}{1-\alpha}\right)R^{\frac{\alpha}{1-\alpha}}\right],$$

which is obviously a continuous function, and moreover, regardless of the parameter values, $F'(1) = \mu > 1$. Then, there is an $\varepsilon_1 > 0$ (which may depend on the parameter values) such that $F(R) < R$ for all $R \in (1-\varepsilon_1, 1)$. Therefore, whenever $\rho \geq -1/(1-\alpha)$ the intermediate value theorem implies the existence of an interior BGP because there is an $R \in (0,1)$ such that $F(R) = R$. For the case of $-\infty < \rho < -1/(1-\alpha)$, using de l'Hôpital's rule, it is easy to verify that $\lim_{R\to 0} F'(R) = +\infty$. Then, there exists an $\varepsilon_0 > 0$ (which may depend on the parameter values) such that $F(R) > R$ for all $R \in (0, \varepsilon_0)$. Then, again from the intermediate value theorem the existence of the interior BGP is established for these sets of parameters.

Now, define $H(R) \equiv 1 + (R)^{\rho+\frac{1}{1-\alpha}} - (R)^{\frac{1}{1-\alpha}}$. Obviously $F(R) = H(R)^{\frac{\mu}{\rho}}$, $F'(R) = \left(\frac{\mu}{\rho}\right) H(R)^{\left(\frac{\mu}{\rho}-1\right)} H'(R)$ and

$$F''(R) = \left(\frac{\mu}{\rho}\right) H(R)^{\left(\frac{\mu}{\rho}-2\right)} \left\{H(R)H''(R) + \left(\frac{\mu}{\rho} - 1\right)[H'(R)]^2\right\},$$

where I have taken the term $\left(\frac{\mu}{\rho}\right) H(R)^{\left(\frac{\mu}{\rho}-2\right)}$ as a common factor and then simplified. This common term is positive for all $R > 0$ hence $sign\{F''(R)\} = sign\left\{H(R)H''(R) + \left(\frac{\mu}{\rho}-1\right)[H'(R)]^2\right\}$. Since $[H'(R)]^2$ is obviously positive, the term $\left(\frac{\mu}{\rho}-1\right)[H'(R)]^2$ is always positive, zero or negative, depending on whether $\frac{\mu}{\rho} \gtreqless 1$. The function $H(R)$ is always positive on $[0,1]$. Then, $F''(R)$ can only change signs if $H''(\cdot)$ changes signs. The unique inflection point of $H(\cdot)$–where the function changes from concave to convex or vice-versa– is $R^I = \left\{\alpha/\left[(1-\alpha)\left(\rho^2(1-\alpha)+\alpha\rho\right)\right]\right\}^{\frac{1}{\rho}}$, which may or may not fall in the interval $[0,1]$. In any event, the transition function $F(\cdot)$ has at most one inflection point, which is equal (larger than) [smaller than] R^I if $\frac{\mu}{\rho}=1$ ($\frac{\mu}{\rho}<1$) [$\frac{\mu}{\rho}>1$]. Because $\mu > 1$, if $F(\cdot)$ changes curvature it is necessarily from concave to convex and since $\lim_{R\to 1} F'(R) = \mu > 1$, then, $F(\cdot)$ crosses at most once the 45^0 line in the interval $[0,1]$.

Finally, to establish global stability of the interior BGP, recall that existence was established by showing $F(R) > R$ for a lower sub-interval of $[0,1]$ and $F(R) < R$ for a higher sub-interval. Because of this, the continuity of F and the fact that its crossing is unique, then $F(\cdot)$ must cross the 45^0 from above, which implies the global stability of the BGP.■

Proof of Lemma 3. If $\beta(1-2\alpha) \geq v_0 G^v/(1+v)$, then when all the initial old have the same skill level Z: (a) all of the old are better off being active entrepreneurs and (b) all young individuals would be better off investing and become entrepreneurs in the next period. Therefore, optimal occupation choices are satisfied, and, by assuming all the parameter conditions of Proposition 1, then the allocations are an equilibrium. Alternatively, if $\beta(1-2\alpha) < v_0 G^v/(1+v)$,if all the young invest, then the net present value $w - Z\phi(G) + \beta G\pi$ would be strictly below the option $w + \beta G w$ of not investing and remaining a worker next period. If only ω of the young invest, then, in the BGP, the aggregate supply of labor will be $2-\omega$ (all the mass 1 of young individuals plus the mass $1-\omega$ of the old who did not invest) and the supply of entrepreneurs will be ω. With homogeneous entrepreneurs with skills Z, workers' wages are equal to $w = \alpha[(2-\omega)/\omega]^{\alpha-1} Z$ and entrepreneurs' earnings are $\pi = (1-\alpha)[(2-\omega)/\omega]^{\alpha} Z$. By construction, the condition

$$\beta\left(\frac{2-\omega}{\omega}\right)^{\alpha}\left(\frac{2(1-\alpha)-\omega}{2-\omega}\right) = \frac{v_0}{1+v}G^v,$$

implies that the fraction ω makes young individuals to be indifferent between remaining worker in the next period (and not invest in skills) or investing at the rate G and becoming an entrepreneur; likewise, the first root of

$$\beta\left(\frac{2-\omega}{\omega}\right)^{\alpha}\left(1+\frac{\gamma v v_0}{(1+v)}G^{1+v}\right) = v_0 G^v.$$

implies that for those investing, the growth rate G is optimal, given than only ω of the current young are investing.■

A.2 Beyond the Cobb-Douglas Aggregator of Ideas

For the baseline model, I have assumed that the exposure to ideas of a young worker is a Cobb-Douglas function of the internal ideas z, those from the employer of the worker, and all the external ideas circulating in the country, Z^O, i.e.

$$z^E = (z)^{\gamma}\left(Z^O\right)^{1-\gamma},$$

where Z^O is a CES aggregator with curvature ρ. In this appendix, I consider the implications of the model when the aggregator of internal and external ideas is given by a more general CES function

$$z^E = \left[\gamma(z)^{\xi} + (1-\gamma)\left(Z^O\right)^{\xi}\right]^{\frac{1}{\xi}},$$

where $1/(1-\xi)$ is the elasticity of substitution. If $\xi = 0$, we are back to our Cobb-Douglas baseline case. If instead, $\xi \to -\infty$, then we get the Leontieff case, $z^E = \min\left\{z, Z^O\right\}$, and if $\xi \to +\infty$, then we get $z^E = \max\left\{z, Z^O\right\}$. For any $-\infty < \xi < +\infty$, we have that the elasticity of the exposure to ideas z^E with respect to internal ideas z is given by

$$\xi_{z,\ z^E} = \gamma\left(\frac{z}{z^E}\right)^{\xi}.$$

With it, the condition (9) for the young entrepreneur's accumulation of skills becomes

$$\beta\left(\frac{\alpha z'}{w'(z')}\right)^{\frac{\alpha}{1-\alpha}}\left[1+\alpha v\gamma\left(\frac{z'}{(z^E)'}\right)^{\xi}\theta_{z'',\ (z^E)'}\right] = v_0\left(\frac{z'}{z^E}\right)^{v}. \tag{26}$$

The evolution of the cross-section of skills in a closed economy: The equivalent analysis of Lemma 2 can become substantially more complex for a generic CES case, since $\theta_{z'',\ (z^E)'}$ is a function of the wage function $w''(\cdot)$. The extreme cases, $\xi = -\infty$ and $\xi = +\infty$, however can be described easily. First, consider the Leontieff case, $\xi = -\infty$, when

$$z^E = \begin{cases} z & \text{for } z < Z^O \\ Z^O & \text{for } z \geq Z^O. \end{cases}$$

For firms in the low-end of the distribution, the implied wages $w_t(z_t)$ must be decreasing in their productivity level z, up to the threshold $z = Z^O$, after which they are constant. In terms of skill investments, the workers of all firms with $z \geq Z^O$ will all invest in the same level $z' = \left(Z^O\right)'$, while all the others will invest below that level but increasingly in the knowledge or their own managers.

Second, consider the opposite case, when $\xi = \infty$, and

$$z^E = \begin{cases} Z^O & \text{for } z < Z^O \\ z & \text{for } z \geq Z^O. \end{cases}$$

Now, for firms in the low-end of the distribution, up to the threshold $z = Z^O$, the wages $w_t(z_t)$ must be constant. For firms managed with knowledge above that external level, wages must be decreasing. In terms of skill investments, the workers of all firms with know-how below Z^O will all invest the same $z' = \left(Z^O\right)'$, while those working with managers with $z > Z^O$ will invest above $\left(Z^O\right)'$, in direct relation with their manager's z.

Both cases are analytically tractable and both would lead a closed economy to converge to a BGP with homogeneous firms. In the first case, the lower end of the distribution would eventually catch up with the mass of firms in the upper end. In the second case, the forces of dispersion induced homogeneity of Lemma 2 are even stronger, and in the limit, the progeny of the initially most productive entrepreneur will overtake the entire economy.

Closed Economy BGPs: Interestingly, the parameter ξ does not affect the set of homogenous BGPs in closed economies. The BGPs are the same as in the baseline case ($\xi = 0$) because when $z' = Z' = \left(z^E\right)' = \left(Z^O\right)'$, we have $\xi_{z,\ z^E} = \gamma$.

Open Economies: For generic $-\infty < \xi < +\infty$, the differences in the BGPs in the open economy are only in R_j^E, the relative exposure to ideas of each vintage j and the determination in the skills levels R_j. For the first, instead of equation (16), the model with a CES aggregator requires

$$R_j^E = \left[\gamma \left(R_j\right)^\xi + (1-\gamma)\left(R^O\right)^\xi\right]^{\frac{1}{\xi}}.$$

For the second, instead of equation (17), the model with a CES requires

$$\beta\left(\frac{\alpha R_j}{\alpha + d_j}\right)^{\frac{\alpha}{1-\alpha}}\left[1 + \frac{\frac{v_0}{1+v}\left(G\frac{R_{j+1}}{R_j^E}\right)^{1+v} R_j^E}{\alpha + d_j}\alpha v \gamma\left(\frac{R_j}{R_j^E}\right)^\xi\right] = v_0\left(G\frac{R_j}{R_{j-1}^E}\right)^v.$$

All the other equations (12), (13), (14), (15), and (18) remain unchanged from the baseline Cobb-Douglas model. Obviously, the additional parameter ξ will have an impact on the number of vintages, and on the relative productivities R_j and relative exposure to ideas R_j^E of all active ones.

Finally, the extreme cases $\xi = -\infty$ and $\xi = \infty$ may have somewhat interesting contrasts with respect to the baseline case. However, these differences are ancillary to the general point that even with laissez-faire openness the home country has an interior BGP and never fully catches up with the world frontier. In both cases, and for the same two reasons discussed above, the vintage structure collapses to only one vintage of domestic firms.

First, consider $\xi = \infty$. For workers in a domestic firm with relative productivity $R < 1$ the implied exposure to ideas is $R^E = \max\left\{R, R^O\right\}$, and since $R^O = \left[m + (1-m)(R)^\rho\right]^{\frac{1}{\rho}}$ then,

$$R^E = \begin{cases} \left[m + (1-m)(R)^\rho\right]^{\frac{1}{\rho}} & \text{for } R < R^O \\ R & \text{for } R \geq R^O, \end{cases}$$

where $m \in [0,1]$ is the equilibrium entry. From this equation and the equilibrium accumulation of skills, it can be shown that eventually all domestic firms in the open economy would have the same level of relative know-how. Yet, this level of know-how will not converge to 1 (fully catch up) unless we also have $\rho = +\infty$ too. Obviously, the transition function has a discontinuity and exhibit additionally interesting dynamic behavior. But the main implication about the existence of stable interior is already captured by the baseline model with $\xi = 0$ used in the paper.

Second, consider $\xi = -\infty$. Then, $R^E = \min\left\{R, R^O\right\}$, and since $R^O = \left[m + (1-m)(R)^\rho\right]^{\frac{1}{\rho}}$ then,

$$R^E = \begin{cases} R & \text{for } R < R^O \\ \left[m + (1-m)(R)^\rho\right]^{\frac{1}{\rho}} & \text{for } R \geq R^O. \end{cases}$$

In this case, the transition also exhibits a discontinuity, but if it exists, there is only one interior equilibrium and it is stable. Interestingly, this interior equilibrium is not the extreme dependency theory or 'colonial' equilibrium where all domestic firms are wiped out and all domestic workers are perennial workers of foreign firms and do not acquire skills. That can only happen when also $\rho = -\infty$.

B The Model with Middle-Managers and Workers

Given a wage for the workers w, and a wage function $w_m(\cdot)$ for middle-managers, the earnings of an entrepreneur with skills z are

$$\pi[z; w_m(z), w] \equiv \max_{\{n,l\}} \left\{ z n^{\alpha} l^{\lambda} - w_m n - wl \right\} = (1-\alpha-\lambda) z^{\frac{1}{1-\alpha-\lambda}} \left(\frac{\alpha}{w_m(z,Z)} \right)^{\frac{\alpha}{1-\alpha-\lambda}} \left(\frac{\lambda}{w} \right)^{\frac{\lambda}{1-\alpha-\lambda}}.$$

Consider first a closed economy in which all entrepreneurs have skills $Z_h = Z > 0$. From the market-clearing conditions, $n = 1$ and $l = \varrho$, the wage for workers is $w = \lambda \varrho^{\lambda-1} Z$; for middle-managers the wage function $w_m(z, Z)$ must satisfy $w_m(Z, Z) = \alpha Z \varrho^{\lambda}$. In this case, the earnings of entrepreneurs are $\pi = \pi(Z, Z) = (1-\alpha-\lambda) Z \varrho^{\lambda}$. For any other skills level z, given Z, Z_{t+1} $w_{m,t+1}(z', Z_{t+1})$, and w_{t+1}, the entrepreneur would have to pay his middle-managers wages equal to

$$w_m(z, Z) = w_m(Z, Z) + V[Z, w_{m,t+1}(\cdot), w_{t+1}] - V\left[z^{\gamma} Z^{1-\gamma}, w_{m,t+1}(\cdot), w_{t+1}\right] \tag{27}$$

which compensate for better or worse learning possibilities. Here, similar to the basic model, the learning or career value $V(\cdot)$ of a mid-management job with exposure z^E of ideas is

$$V\left[z^E, w_{m,t+1}(\cdot), w_{t+1}\right] \equiv \max_{z'} \left\{ \beta \pi\left[z', w_{m,t+1}\left(z', Z_{t+1}\right), w_{t+1}\right] - z^E \phi\left(z'/z^E\right) \right\}. \tag{28}$$

If it is well defined, which requires that $v > (\alpha+\lambda)/(1-\alpha-\lambda)$, the optimal skill accumulation z' for a mid-manager who will go on to be an entrepreneur is given the FOC

$$\beta \left[\pi_1\left[z', w_{m,t+1}\left(z', Z_{t+1}\right), w_{t+1}\right] + \pi_2\left[z', w_{m,t+1}\left(z', Z_{t+1}\right), w_{t+1}\right] \frac{\partial w_{m,t+1}(z', Z_{t+1})}{\partial z'} \right] = \phi'\left(\frac{z'}{z^E}\right), \tag{29}$$

where $\pi_1(\cdot)$ and $\pi_2(\cdot)$ denote, respectively, the derivative of π with respect to the manager's skill z and with respect to the wage $w_{m,t+1}(z, Z)$ that he must pay his mid-managers. These derivatives are:

$$\pi_1(\cdot) = z^{\frac{\alpha+\lambda}{1-\alpha-\lambda}} \left(\frac{\alpha}{w_m(z,Z)} \right)^{\frac{\alpha}{1-\alpha-\lambda}} \left(\frac{\lambda}{w} \right)^{\frac{\lambda}{1-\alpha-\lambda}} \quad \text{and} \quad \pi_2(\cdot) = -z^{\frac{1}{1-\alpha-\lambda}} \left(\frac{\alpha}{w_m(z,Z)} \right)^{\frac{1--\lambda}{1-\alpha-\lambda}} \left(\frac{\lambda}{w} \right)^{\frac{\lambda}{1-\alpha-\lambda}}.$$

Finally, from (27), $\frac{\partial w_{t+1}(z')}{\partial z'} = -V_1\left[\left(z^E\right)', w_{t+2}(z'')\right] \frac{\partial \left(z^E\right)'}{\partial z'}$, and applying the envelope condition on V one period ahead:

$$V_1\left[\left(z^E\right)^{t+1}, w_{m,t+2}(\cdot), w_{t+2}\right] = -\frac{v v_0}{1+v} \left(\frac{z^{t+2}}{\left(z^E\right)^{t+1}} \right)^{1+v},$$

where I use simplifications derived from the functional form assumed for $\phi(\cdot)$.

BGP closed economies. Impose $z_t = z_t^E = Z_t$, and $G = Z_{t+1}/Z_t$ for all periods t. Plugging $w_{t+1} = \lambda \varrho^{\lambda-1} Z_{t+1}$ and $w_{m,t+1}(Z, Z) = \alpha \varrho^{\lambda} Z_{t+1},$ we get, that the derivatives boil down to $\pi_1 = \varrho^{\lambda}$ and $\pi_2 = -1$; we also get $V_1 = -\frac{v v_0}{1+v}(G)^{1+v}$, $\frac{\partial \left(z^E\right)'}{\partial z'} = \gamma$, and that $\phi'\left(\frac{z'}{z^E}\right) = v_0 G^v$. Using all of these in (29), delivers the expression in the text:

$$\beta \left(\varrho^{\lambda} + \frac{\gamma v v_0}{1+v} G^{1+v} \right) = v_0 G^v.$$

The remaining equilibrium conditions (ex-ante and ex-post optimality of occupation choices) follow exactly the same lines as discussed above for the basic model (i.e. $\lambda = 0$ and $\omega = 1$).

For this model is $\mu = v/\left[v - (\alpha+\lambda)/(1-\alpha-\lambda)\right]$ which is greater than one when $v > (\alpha+\lambda)/(1-\alpha-\lambda)$, as assumed throughout.

BGP open economies. Home being small and open, and foreign being in a BGP, then free entry implies that at home the wage of workers is equal to $w = \lambda \varrho^{\lambda-1} Z_f$, and that foreign firms pay domestic mid-managers wages equal to $w_{m,\ t+1}(Z_f) = \alpha \varrho^{\lambda} Z_f$. Then, a domestic firm with skills z must pay a wage

$$w_m(z, Z_f) = w_m(Z_f) + V\left[(Z_f)^{\gamma} \left(Z^O\right)^{1-\gamma}, w_{m,t+1}(\cdot), w_{t+1}\right] - V\left[z^{\gamma} \left(Z^O\right)^{1-\gamma}, w_{m,t+1}(\cdot), w_{t+1}\right], \tag{30}$$

where Z^O is the outside set of ideas circulating in the country, which must be determined as part of the equilibrium. Let $R = z/Z_f$ and denote by $d(R)$ the compensating differential, relative to Z_f, of an entrepreneur with skills $z = R Z_f$. That is, $w_m(z, Z_f) = \left[\alpha \varrho^{\lambda} + d(z/Z_f)\right] Z_f$, where to shorten the notation I have omitted indicating the dependence on Z^O. After simplifying, the derivatives π_1 and π_2 become

$$\pi_1 = \Phi R^{\frac{\alpha+\lambda}{1-\alpha-\lambda}} \left(\frac{\alpha}{\alpha \varrho^{\lambda} + d(R)} \right)^{\frac{\alpha}{1-\alpha-\lambda}} \quad \text{and } \pi_2 = -\Phi R^{\frac{1}{1-\alpha-\lambda}} \left(\frac{\alpha}{\alpha \varrho^{\lambda} + d(R)} \right)^{\frac{1-\lambda}{1-\alpha-\lambda}}. \tag{31}$$

where $\Phi \equiv [\varrho]^{\frac{\lambda(1-\lambda)}{1-\alpha-\lambda}}$. Next, using the fact that $Z_{f,\ t+2}/Z_{f,\ t+1} = G$, defining $R^E \equiv z^\gamma \left(Z^O\right)^{1-\gamma}/Z_f = R^\gamma \left(R^O\right)^{1-\gamma}$ –as I defined in the text for the basic model– then

$$V_1\left[\left(z^E\right)^{t+1}, w_{m,t+2}(\cdot), w_{t+2}\right] = -\frac{vv_0}{1+v}\left(G\frac{R_{t+2}}{R^E_{t+1}}\right)^{1+v}, \tag{32}$$

where R^E_{t+1} is the relative exposure to ideas that the entrepreneur will provide to his mid-managers in the next period and R_{t+2} is their relative accumulation of skills. It is also straightforward to compute,

$$\frac{\partial\left(z^E\right)^{t+1}}{\partial z^{t+1}} = \gamma\frac{R^E_{t+1}}{R_{t+1}} = \gamma\left(\frac{R^{O,t+1}}{R^{t+1}}\right)^{1-\gamma}. \tag{33}$$

Finally,

$$\phi'\left(\frac{z_{t+1}}{z^E_t}\right) = v_0\left[G\frac{R_{t+1}}{R^E_t}\right]^v. \tag{34}$$

Plugging conditions (31), (32), (33) and (34) into condition (29), implies that the optimal accumulation of skills R_{t+1} for a worker of a mid-manager of an entrepreneur with relative skills R_t

$$\beta\Phi R_{t+1}^{\frac{\alpha+\lambda}{1-\alpha-\lambda}}\left[\left(\frac{\alpha}{\alpha\varrho^\lambda + d\left(R_{t+1}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}} + \frac{\gamma vv_0 R^E_{t+1}}{1+v}\left(\frac{\alpha}{\alpha\varrho^\lambda + d\left(R_{t+1}\right)}\right)^{\frac{1-\lambda}{1-\alpha-\lambda}}\left(G\frac{R_{t+2}}{R^E_{t+1}}\right)^{1+v}\right] = v_0\left[G\frac{R_{t+1}}{R^E_t}\right]^v,$$

where I have grouped and simplified terms involving R_{t+1}. Because R^E_{t+1} obviously depends on R_{t+1}, it is instructive to re-write this condition as

$$\beta\Phi R_{t+1}^{\frac{\alpha+\lambda}{1-\alpha-\lambda}}\left[\left(\frac{\alpha}{\alpha\varrho^\lambda + d\left(R_{t+1}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}} + \frac{\gamma vv_0}{1+v}\left(\frac{\alpha}{\alpha\varrho^\lambda + d\left(R_{t+1}\right)}\right)^{\frac{1-\lambda}{1-\alpha-\lambda}}\frac{\left(GR_{t+2}\right)^{1+v}}{\left(R_{t+1}\right)^{\gamma v}\left(R^O_{t+1}\right)^{(1-\gamma)v}}\right] = v_0\left[G\frac{R_{t+1}}{R^E_t}\right]^v. \tag{35}$$

Notice that re-interpreting the index t not as a period but as a 'vintage' instead, this condition determines the time-invariant $\{R_j\}_{j=1}^\infty$ for an open economy BGP. Also, notice that forcing $\lambda = 0$, the condition boils down to the the equation in the text for basic model.

To compute the compensating differentials $d(R)$, observe that the earnings of an entrepreneur with skills z can be written as

$$\pi\left(z, Z_t\right) = Z_f\left[\Phi\left(1-\alpha-\lambda\right)R^{\frac{1}{1-\alpha-\lambda}}\left(\frac{\alpha}{\alpha\varrho^\lambda + d(R)}\right)^{\frac{\alpha}{1-\alpha-\lambda}}\right].$$

Let let R^1_{t+1} denote the relative skills of a domestic entrepreneur who was trained at time t in a foreign firm. Then, the career value for a mid-manager in a foreign firm is

$$V_{f,t} = Z_{f,t}\left[\beta G\Phi\left(1-\alpha-\lambda\right)\left(R^1_{t+1}\right)^{\frac{1}{1-\alpha-\lambda}}\left(\frac{\alpha}{\alpha\varrho^\lambda + d\left(R^1_{t+1}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}} - \left(R^O_t\right)^{1-\gamma}\frac{v_0}{1+v}\left(\frac{GR^1_{t+1}}{\left(R^O_t\right)^{1-\gamma}}\right)^{1+v}\right],$$

and his life-time utility is $\alpha\varrho^\lambda Z_{t+1} + V_{f,t}$. On the other hand, working for a domestic firm with relative skills R, a mid-manager attains

$$Z_{f,t}\left[\alpha\varrho^\lambda + d(R) + \beta G\Phi\left(1-\alpha-\lambda\right)\left(R^R_{t+1}\right)^{\frac{1}{1-\alpha-\lambda}}\left(\frac{\alpha}{\alpha\varrho^\lambda + d\left(R^R_{t+1}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}} - \frac{v_0}{1+v}\frac{\left(GR^R_{t+1}\right)^{1+v}}{\left(R^\gamma\left(R^O_t\right)^{1-\gamma}\right)^v}\right]$$

where R^R_{t+1} indicates the optimal skill accumulation –that solves (35)– when exposed to any level R. Equating this expression with the life-time utility attainable working for a foreign firm, then

$$\begin{aligned} d(R) &= \beta G\Phi\left(1-\alpha-\lambda\right)\left[\left(R^1_{t+1}\right)^{\frac{1}{1-\alpha-\lambda}}\left(\frac{\alpha}{\alpha\varrho^\lambda + d\left(R^1_{t+1}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}} - \left(R^R_{t+1}\right)^{\frac{1}{1-\alpha-\lambda}}\left(\frac{\alpha}{\alpha\varrho^\lambda + d\left(R^R_{t+1}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}}\right] \\ &\quad + \left(\frac{v_0}{1+v}\right)\frac{G^{1+v}}{\left(R^O_t\right)^{(1-\gamma)v}}\left[\frac{\left(R^R_{t+1}\right)^{1+v}}{R^{\gamma v}} - \left(R^1_{t+1}\right)^{1+v}\right]. \end{aligned}$$

Instead of solving for any arbitrary levels R, we can restrict this equation for the time-invariant vintage structure in a BGP for an open economy using j to index each vintage, i.e. $d_j \equiv d(R_j)$. Notice that this is a generalization of the expression in the text. First, it does not presume $\lambda = 0$; if it did, we recover the expression in the text for the basic model. Second, it allows the computation for time-varying $\left\{R_j^t,\ d_j^t,\ R^{O,t} : j \geq 1,\ t \geq 0\right\}$. Indeed, on the basis of these equations, I use a simple shooting algorithm on $R^{O,t}$ to compute for the transition dynamics of a closed economy starting in a BGP to the final steady state after openness.

Finally, the optimal amount of labor and mid-management units hired by a domestic entrepreneur with $0 \leq R \leq 1$, are equal to

$$l(R) = R^{\frac{1}{1-\alpha-\lambda}} \left(\frac{\alpha \varrho^{\frac{(1-\alpha)(1-\lambda)}{\alpha}}}{\alpha \varrho^{\lambda} + d(R)} \right)^{\frac{\alpha}{1-\alpha-\lambda}} \quad \text{and} \quad n(R) = R^{\frac{1}{1-\alpha-\lambda}} \left(\frac{\alpha \varrho^{\lambda}}{\alpha \varrho^{\lambda} + d(R)} \right)^{\frac{1-\lambda}{1-\alpha-\lambda}}.$$

Notice that the ratio $n(R)/l(R)$ is increasing in R when $d(R)$ is decreasing. The relative cost of mid-management services is lower for more productive as they offer better learning opportunities.

In a BGP of an open economy, the amount of mid-managers hire by entrepreneurs of vintage j (foreign firms are $j = 0$)

$$n_j = (R_j)^{\frac{1}{1-\alpha-\lambda}} \left(\frac{\alpha \varrho^{\lambda}}{\alpha \varrho^{\lambda} + d_j} \right)^{\frac{1-\lambda}{1-\alpha-\lambda}},$$

and the mass of domestic mid-managers in each vintage j is

$$\hat{m}_j = \hat{m}_0 \prod_{i=0}^{j-1} n_i.$$

To clear the domestic market for mid-managers, i.e. $\sum_{j=0}^{\infty} \hat{m}_j n_j = \omega$, the total mass entry of foreign firms must be $\hat{m}_0 = \omega / \left[\sum_{k=0}^{\infty} \prod_{j=0}^{k} n_j \right]$. Then, the shares (weights in R^O) are $m_j = \hat{m}_j / \omega$.

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