Openness and the Optimal Taxation of Foreign Know-How

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Openness and the Optimal Taxation of Foreign Know-How*

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Abstract

Developing countries frequently offer tax incentives and even subsidize the entry and operation of foreign firms. I examine the optimality of such policies in an economy where growth is driven by entrepreneurial know-how, a skill that is continuously updated on the basis of the productive ideas implemented in the country. Openness allows foreign ideas to disseminate inside a country and can foster the country’s domestic accumulation of know-how. With externalities, however, laissez-faire openness is suboptimal and can be growth- and even welfare-reducing. I examine the gains from openness under an optimal taxation program—the self-funding taxes on domestic and foreign firms that maximize the welfare of the recipient country, subject to the equilibrium behavior of national and foreign firms. Under optimal taxation, openness is always welfare enhancing and leads lagging countries to catch up with the world frontier. Yet, a country may want to subsidize the entry of foreign firms only if it can also subsidize the domestic accumulation of know-how. I also consider the optimal tax program under a number of restrictions that developing countries typically face. For instance, a country must not subsidize entry of foreign firms if doing so requires taxing the concurrent cohort of domestic firms. Similarly, an international agreement that requires equal taxation of domestic and foreign firms can be welfare reducing for a country close to the knowledge frontier.

Keywords: Pigou taxes; Ramsey program; Multinational firms; Gains from openness; Fiscal Constraints.

JEL Codes: H21, H25, O19, O31, O33, O34, O38.

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1 Introduction

Entrepreneurial know-how—the skills to combine technology and market opportunities to set up and manage firms—can be the limiting factor for a country’s aggregate productivity.\(^1\) Indeed, recent work suggests that developing countries can accrue significant output and consumption gains by opening up to the skills and productive ideas in foreign firms.\(^2\) However, does the presence of foreign know-how enhance or impair the country’s development of its own entrepreneurial skills over time? How far does openness lead a developing country to catch up with the world frontier? What is the optimal program of taxes for domestic and foreign firms? Should developing countries open up to foreign firms even if they face dire fiscal limitations? Should they sign international agreements that limit their taxation of foreign and domestic firms? This paper uses a simple general equilibrium growth model to answer these questions.

I consider an economy in which entrepreneurial know-how is the engine of growth. The model is an OLG extension of a standard Lucas (1978) span-of-control economy. Entrepreneurial know-how plays a dual role. On one side, as in Lucas, it determines the productivity of the firms managed by the old. On the other side, it provides the foundation for the young to build up their future skills. In a closed economy, only national entrepreneurs can set up firms, shape the aggregate productivity of the country and generate the ideas upon which the young can build up their know-how. In an open economy, foreign entrepreneurs can also enter the country, combine their know-how with local labor, and enhance aggregate productivity.\(^3\) Foreign firms, whose entry is endogenously determined by an indifference condition, make foreign ideas available to the domestic young for their acquisition of know-how. In this environment I characterize the taxation program that maximize the welfare of a developing country, subject to the equilibrium behavior of national and foreign firms and the taxation program of the rest of the world. Then, I characterize the gains of openness under the optimal taxation program and compare them with the more traditional gains from laissez-faire openness.

A key aspect for determining the optimal policies is the extent in which knowledge is excludable. In related work (Monge-Naranjo, 2011), I have considered a more general environment in which each young worker is exposed to ideas from two different sources: (i) the know-how of those


\(^2\) See Antras, Garicano, and Rossi-Hansberg (2006); Burstein and Monge-Naranjo (2009); and Eeckhout and Jovanovic (2010) The gains are even larger if these skills are nonrival factors, (i.e. can be used simultaneously in many locations.) See Ramondo (2008) and McGrattan and Prescott (2008).

\(^3\) The emphasis on the cross-border reallocation of management conforms with the observation that multinational firms heavily rely on home expatriates—home trained individuals—to manage their operations, especially in developing countries (see Chapters 5 and 6 of UNCTAD 1994). It also conforms with the emphasis of the literature on firm specific intangible assets (e.g. Barba-Navarretti 2004 and Markusen 2004).
running the firm for which he works; and (ii) an average of the knowledge embedded in all the firms operating in the country. In that paper, I characterize the equilibria that arises in economies with different relative weights of the two sources of ideas. In one extreme, if all knowledge is built up on the basis of source (i) – as in Boyd and Prescott (1987a,b), Chari and Hopenhayn (1991), and Boldrin and Levine (2009) – then ideas can be fully excludable, the costs and benefits of knowledge diffusion are fully internalized, and competitive allocations are efficient. In this case, laissez-faire openness is always welfare enhancing, is the optimal policy and always leads the country to catch up with the knowledge frontier. These properties no longer hold when the external source (ii) has positive weight. In such case, the optimal policies for the country are no longer trivial. For tractability, in this paper I consider the opposite extreme, where all the weight is in (ii). With externalities as the mechanism by which knowledge diffuses puts the analysis in this paper in line with a common assumption in the growth literature (e.g. Romer 1986, Klenow 1998 and Jones 2006), the impact of openness to trade on growth (e.g. Stokey 1991) and the impact of openness to multinational firms (e.g. Findlay 1978).

With knowledge externalities, not surprisingly, laissez-faire openness is not necessarily efficient, since the non-excludability of knowledge diffusion keeps market transactions from making sure that socially efficient knowledge transfers take place, both within and between countries. More interestingly, despite the apparent ‘free-lunch’ of knowledge spillovers, laissez-faire openness can be growth-reducing. Entry of foreign firms is determined by (marginal) profit equalization and fails to internalize the impact on the country’s exposure to learn. Indeed, laissez-faire (zero taxes) openness may fail to push developing countries to catch up with the rest of the world, even if individuals in those countries have the same preferences, policies and inherent capacity to learn, and no barriers to knowledge are present.\footnote{For a detailed analysis of these results and the role of occupation choices, see Monge-Naranjo (2011).}

In this paper, I examine the optimal (Pigou-Ramsey) dynamic taxation of both domestic and foreign firms, where the taxes are chosen so as to maximize the welfare of the recipient country, subject to the budget constraint of the government and the equilibrium behavior of national and foreign agents. I allow governments to impose proportional taxes rates on the net income (profits) of domestic and foreign firms operating in the country. Tax rates are uniform across individuals with the same nationality but can be different across nationalities. They can also vary over time. In the baseline case, I also allow taxes of the net income of the domestic workers (wages), which in this environment are effectively lump-sum taxes. The taxation program considered is in the context of a small economy which is initially less developed than the rest of the world. Taxes in the rest of the world play an important role shaping the optimal taxes in a country, and will be assumed to be constant over time. The baseline case assumes that the rest of the world also
sets taxes optimally. Most of the analysis also assumes full commitment on behalf of the home government. Later on, the paper explores suboptimal foreign taxation and the constraints imposed by a country’s temptation to expropriate foreign firms.

I use a primal approach for solving the optimal taxation program. Given a tax sequence, competitive allocations are fully pinned down by the time \( t = 0, 1, \ldots \) sequences of two numbers: a ratio \( R_t \) of the relative know-how of domestic firms relative to the know-how of foreign firms, and a fraction \( m_t \) of the domestic labor under the control of foreign firms. Then, optimal taxes on foreign and domestic firms are solved for with a simple Bellman equation that defines the optimal transition from \((R_t, m_t)\) to \((R_{t+1}, m_{t+1})\). Relative to a closed economy, the optimal program may allow for non-zero \( m_t \) and \( R_t \) may change over time even if the countries in the rest of the world tax their respective domestic firms optimally. Relative to laissez-faire openness, taxes (subsidies) on foreign firms can reduce (expand) the amount of entry of foreign know-how and the country’s overall exposure to productive ideas.

The prescription for optimal taxes on domestic know-how is as expected: A Pigouvian subsidy is needed to internalize the positive externality of productive ideas of active entrepreneurs on the know-how accumulation of future generations. More interestingly, the level of these Pigouvian subsidies depends on the country’s policies on openness, since the gap between the social and private returns to skills is larger in open economies. With respect to foreign firms, optimal taxes must balance the positive and negative impacts of openness in the net present value of resources for the country. There are two positive impacts: (a) static gains in the country’s national output, and (b) a reduction in the cost of accumulating domestic know-how. There is one negative impact, the negative impact of future entry and competition of foreign firms on the domestic return of accumulating know-how. The positives are associated with current entry and the negative with future entry, so the optimal taxation program must balance off the dynamic interplay between them.

The baseline case assumes that the rest of the world is the balanced-growth path (BGP) as defined by the optimal taxation program of a closed economy.\(^5\) As anticipated, the optimal tax program for an open economy dominates the best closed economy program (i.e. taxation programs restricted to \( m_t = 0 \) for all \( t \)); therefore, contrary to the case of laissez-faire, under optimal taxation, the gains from openness are always positive. Similarly, the optimal tax program also dominates laissez-faire openness. In general, while countries that are really low in know-how may find it optimal to collect some of the foreign profits via taxes, I find that countries that are close to the frontier find it optimal to subsidize entry of foreign firms, i.e. the optimal allocation

\(^5\)Since I am assuming that the country of interest is small, the relevant program for the rest of the world is the one associated to a closed economy.
exhibits shares $m_t$ higher than the corresponding ones under laissez-faire (zero taxes/subsidies). In those case, subsidies to foreign firms are also met with higher subsidies to domestic firms, and over time the ratio $R_t$ approaches 1. Therefore, contrary to laissez-faire openness, under optimal taxation, countries that are sufficiently developed end up catching up with the world frontier.

I consider two substantial departures from the baseline case. First, I consider cases in which the rest of the world follows a suboptimal taxation program. On one hand, if the rest of the world is under-accumulating know-how (subsidies below the Pigouvian rate), the optimal taxation program would lead an initially less-developed country to catch-up and eventually surpass the rest of the world. In such a case, the ratio $R_t$ would diverge to plus infinity as the country would consistently outgrew the rest of the world. On the other hand, if the rest of the world is over-accumulating know-how (subsidies above the Pigouvian rate), then catching up is not part of the optimal policy for the country. The country would converge to a BGP with the same growth rate as the rest of the world, but with know-how levels below the rest of the world ($R_\infty < 1$). The higher growth rate of the country would be sustained by positive entry of foreign firms ($m_\infty > 0$) overtime.

The second departure is in terms of limitations to the tax instruments that the country can use. Such limitations can arise, for example, from the inability of countries to tax workers. This would imply that any subsidies that are provided to foreign (domestic) firms must be financed with taxes on domestic (foreign) firms. The primal formulation of the problem could be readily adapted to add such a constraint for the case when that the budget constraint of the government must be balanced period by period. Relative to the unrestricted program, this constraint can substantially reduce the gains from openness. However, it is always the case that the gains from openness are positive.

Tax programs can be also limited by the country’s inability to tax foreign firms differently from domestic ones or to tax them at all. These restrictions could arise as part of international agreements, e.g. WTO or NAFTA. Consider first the restriction of zero taxes (or subsidies) to foreign firms. With such a restriction the gains from openness are reduced relative to the unrestricted program but are still positive relative to autarkic economies. Entry of foreign firms would be given by the laissez-faire condition and as such the planner cannot use taxes to internalize the transfer of know-how from foreign firms. However, subsidies on domestic firms can still be used in the open economy, and in general will be higher in response to the entry of foreign know-how. While the country might fail to catch up, in general domestic firms will attain a higher ratio $R_t$ as result of openness. Consider now a different restriction, equal taxation of domestic and foreign firms. Interestingly, in the context of this model, the equality of tax rates between foreign and domestic leads $m_{t+1}$ to be equal to the laissez-faire openness. The country can still choose a

\footnote{More generally, any other inelastic factor of production.}
different $R_{t+1}$ from that of laissez-faire, but in general the gains from openness can be severely reduced. Indeed, similarly to the laissez-faire case, the country may fail to catch up and the gains from openness can be negative if the country is initially close to the knowledge frontier.

As indicated above, the model of knowledge formation in this paper is based on Monge-Naranjo (2011). However, that paper and related work by Beaudry and Francois (2010), Dasgupta (2010) and Sampson (2011), restrict attention to comparing complete openness with complete closedness. The general trend in the literature is to use increasingly sophisticated models to study different aspects of the gains from openness, but still focus on simple open vs closed counterfactual policies. I depart from that trend in the literature and characterize the gains attainable under different policies, in particular under optimal Ramsey taxation. In doing so, I have abstracted from many aspects studied in the literature of multinational activity such as the endogenous choice of organization (see the recent survey by Antras and Rossi-Hansberg 2009 and references therein), and the choice of technologies that multinational firms send to their subsidiaries (e.g. Helpman 1984 and Keller and Yeaple 2010). The analysis also abstracts from international flows of labor (e.g., Rauch 1991; Klein and Ventura 2006) and of physical capital (e.g. Castro 2004, Gourinchas and Jeanne 2003). I have also abstracted from interactions between technology diffusion, multinational activity and international trade in goods (e.g. Grossman and Helpman 1991, Eaton and Kortum 2006, Rodriguez-Clare 2007, and Alvarez, Buera, and Lucas 2010). The paper also omits other forms of knowledge or human capital (e.g. Krishna and Chesnokova 2009) and their interaction with technology adoption (e.g. Stokey 2010), and does not consider specificity or appropriateness of technologies (e.g. Basu and Weil 1998). I have also abstracted from cross-country spillovers (e.g. Damsgaard and Krusell 2008 and Klenow and Rodriguez-Clare 2005) Finally, the paper assumes that there are no frictions or tax distortions in the allocation of workers across managers inside countries (e.g. Battaharya, Guner and Ventura, 2012, Buera and Shin, 2010, Cagetti and De Nardi, 2006, Gurer et al. 2008, Gennaioli and Caselli 2006, among others).

2 The Model

Consider a discrete time, infinite horizon OLG economy with a single consumption good. Individuals live for two periods. In each period, the population consists of equal sized cohorts (normalized to one) of young and old persons. A person born at time $t$ that consumes $c_t$ and $c_{t+1}$ in periods $t$ and $t + 1$, respectively, attains utility

$$U^t = c_t^\gamma + \beta c_{t+1}^\gamma,$$

where $0 < \beta < 1$. 
As in Lucas (1978), the consumption good is produced by ‘firms’, teams of one manager and a group of workers. Managers can also be seen as entrepreneurs since they will be the residual of firms. The (person-specific) skills or knowledge of an entrepreneur determines the productivity of the firm under his control. With \( z \geq 0 \) units of entrepreneurial skills and \( n \geq 0 \) units of labor, a firm produces

\[
y = zn^\alpha,
\]

units of the consumption good. The degree \( \alpha \in (0, 1) \) of decreasing returns to labor \( n \) is also the span-of-control parameter in this economy.

In each period of life, every person has an endowment of one unit of time. When young, that unit can only be supplied as labor; when old time can only be supplied as skilled entrepreneurial services, i.e. setting up and controlling a firm. The returns to entrepreneurship are foreseen by the young as they decide how much to invest in acquiring skills.

Accumulation of skills is made on the basis of the productive ideas to which youth is exposed to. Let \( z^E \geq 0 \) denote the exposure to ideas of an individual. It contains contributions from two sources: (i) the knowledge \( z \) of the particular entrepreneur for whom the youth works; and (ii) an average \( Z^O_t \) of the knowledge implemented by of all firms, domestic and foreign, that operate inside the country at the time. While \( z \) can vary across young persons, \( Z^O_t \) is the same for all of the agents in the country that are young in period \( t \). In particular, I will assume that

\[
z^E = (z)^{\gamma} (Z^O_t)^{1-\gamma},
\]

where \( 0 \leq \gamma \leq 1 \) will be called the internalization parameter because it determines how much a young person learns internally from his job, and, as explained below, the extent in which knowledge spillovers can be internalized with the contractual relationship between a manager and his workers. Notice that \( z^E \) is increasing and linearly homogeneous in the levels of both sources of knowledge. Moreover, notice that there are no “size” effects, i.e. the absolute number (mass) of firms does not impact the level \( z^E \).

The average \( Z^O_t \) is a national “public good”, i.e. a non-rival factor available to everyone in the country. It is determined as follows: Let \( \mu_t \) be the (endogenous, as explained below)
probability measure that indicates the allocation of the country’s total labor across firms with different knowledge levels. That is, for any Borel set \( B \subset \mathbb{R}_+ \), \( \mu_t(B) \) indicates the share of the labor in control of entrepreneurs with knowledge levels in \( B \). Then, \( Z_t^O \) is a generalized (or Hölder) weighted mean of all the active firms:

\[
Z_t^O = \left[ \int_{\mathbb{R}_+} (z)^\rho \mu_t(dz) \right]^{\frac{1}{\rho}},
\]

where the parameter \( \rho \) can assume any value in the extended real numbers. This formulation encompasses many familiar average formulas. The arithmetic, geometric and harmonic means correspond to, respectively, \( \rho = 1, 0, \) and \(-1\). If \( \rho \to -\infty \), \( Z_t^O \) is the lowest value in the support of \( \mu_t \), while if \( \rho \to \infty \), it is the highest value.

Given an exposure level to productive ideas \( z^E > 0 \), the cost (in terms of current consumption) for a young individual to acquire any level \( z' \geq 0 \) of skills for the next period is \( z^E \phi \left( \frac{z'}{z^E} \right) \), where \( \phi : \mathbb{R}_+ \to \mathbb{R}_+ \) is a non-negative, continuously differentiable and strictly convex function with \( \lim_{x \to 0} \phi(x) = \phi'(x) = 0 \) and \( \lim_{x \to \infty} \phi(x) = \phi'(x) = \infty \). Total and marginal costs of investing are strictly increasing and strictly convex in \( z' \) and strictly decreasing in \( z^E \). The marginal cost of \( z' \) is \( \phi' \left( \frac{z'}{z^E} \right) = v_0 \left( \frac{z'}{z^E} \right)^v \), which depends only on the ratio \( z'/z^E \), i.e. how far an individual’s future level of skills relative to the level of his exposure to ideas \( z^E \). It is convenient to focus on the functional form

\[
\phi \left( \frac{z'}{z^E} \right) = \frac{v_0}{1+v} \left( \frac{z'}{z^E} \right)^{1+v},
\]

where \( v_0, v > 0 \). I shall keep \( \phi(\cdot) \) and \( \phi'(\cdot) \) as shorthands in some of the formulas below.

The parameters \( \rho, v \) and \( \gamma \) are key for the formation and diffusion of knowledge. The curvature parameter \( v \) determines the impact of \( z^E \) on the costs of acquiring \( z' \); it determines whether and how quickly knowledge grows over time. The diffusion parameter \( \rho \) determines how easily superior ideas can contribute in the value of \( Z^O \) and, in particular, how foreign ideas may diffuse inside a country. The higher the value of \( \rho \), the higher the impact of superior ideas on the common pool \( Z^O \).

Most importantly, by allowing any value \( 0 \leq \gamma \leq 1 \), the model encompasses two common but conflicting views of the accumulation and diffusion of knowledge. On one hand, if \( \gamma = 0 \), then a common value \( z^E = Z^O \) holds for everyone and externalities are the only engine of accumulation and diffusion. Such assumption has a dominant presence in the literature on growth (e.g. Romer 1986 and Lucas 1988), the impact of openness to trade on growth (e.g. Stokey 1991) and the impact of openness to multinational firms (e.g. Findlay 1978). On the other hand, if \( \gamma = 1 \), then

and partially non-excludable factors that could be used by any young forming entrepreneur in the country without crowding out the use by others.
the exposure to ideas—and hence, the ability to accumulate skills—are uniquely determined by one’s own firm. This gives rise to a richer relationship between young and old entrepreneurs, one that fully internalize the costs and benefits of accumulating skills. Such is the view in Boyd and Prescott (1987a,b), Chari and Hopenhayn (1991), Jovanovic and Nyarko (1995), and others. By allowing any $0 \leq \gamma \leq 1$, the model here combines the impact of externalities with labor markets that compensate for differences in the learning opportunities across firms with different knowledge levels.

Finally, a government in the country collects taxes and (possibly) disburses subsidies. Following the Ramsey tradition, I assume that governments can only charge proportional taxes on the net-income of the different individuals. However, I allow these taxes to vary across types of individuals. In particular, let $\boldsymbol{\tau} = \{\tau_t^W, \tau_t^E, \tau_t^F\}_{t=0}^\infty$ denote, respectively, the tax rates on the net-earnings of domestic workers, domestic entrepreneurs and foreign entrepreneurs operating inside the country for each period $t \geq 0$. Tax rates can be negative (subsidies) but can never be above 1. For simplicity, I assume zero government expenditures but the analysis can be easily extended to economies in which the government spends a constant fraction of the country’s domestic output.

## 3 Competitive Equilibria

I consider perfect foresight competitive equilibria. The key component of the price system is a sequence of wages function $\mathbf{w} = \{w_t : \mathbb{R}_+ \rightarrow \mathbb{R}_+\}_{t=0}^\infty$, where $w_t(z)$ is the price that an entrepreneur with skills $z$ pays for a unit of labor at time $t$. The dependence of the price on the old manager’s skill $z$ is explained below. The discount factor $\beta$ pins down the interest rate. Equilibrium prices and allocations depend also on the tax policy $\boldsymbol{\tau}$, and a government budget constraint must be satisfied.

First I characterize the individually optimal decisions, given policies $\boldsymbol{\tau}$ and prices $\mathbf{w}$. Start with the decisions of an old entrepreneur who has already acquired a given level of skills $z$. Facing market wages $w_t(z)$, he attains pre-tax earnings $\pi[z, w_t(z)] \equiv \max_{\{n\}} \{zn^\alpha - w_t(z)n\}$. Net-of-taxes, his income is

$$
(1 - \tau_t^E) \pi[z_t, w_t(z)] = (1 - \tau_t^E) \left[ \alpha^{1 - \alpha} (1 - \alpha) \right] z^{\frac{1}{1 - \alpha}} [w_t(z)]^{\frac{1}{1 - \alpha}}. 
$$

Notice that because the tax $\tau_t^E$ is on his net-income it does not distort his optimal hiring of labor,

$$
n^* [z, w_t(z)] = \left[ \frac{\alpha z}{w_t(z)} \right]^{\frac{1}{1 - \alpha}},
$$

which is increasing and convex in $z$ and decreasing in $w_t(z)$. 

Given $w_t(z)$, $\pi[\cdot; w_t(z)]$ is strictly increasing and convex; given $z$, $\pi(z, \cdot)$ is strictly decreasing in $w_t(z)$. The total response of the functions $\pi$ and $n^*$ to variations in $z$ will be even more steeply positive and convex in $z$ because, as seen below, the equilibrium function $w_t(z)$ is non-increasing in $z$.

Next, consider the decision problem of a young person. He must select the firm for which to work, and how much to invest in entrepreneurial skills. With respect to the latter, given an exposure to ideas $z^E$ and the next period’s cost of labor $w_{t+1}(\cdot)$, the optimal investment in entrepreneurial skills $z'$ solves

$$V [z^E, w_{t+1} (\cdot),] \equiv \max_{z'} \left\{ (1 - \tau_{t+1}^E) \beta \pi [z', w_{t+1} (z')] - z^E \phi \left( \frac{z'}{z^E} \right) \right\}.$$  \hspace{1cm} (6)

Here, I am assuming that investments in skill formation $z^E \phi \left( \frac{z'}{z^E} \right)$ are not deductible from labor earning taxes.\(^{11}\) The key determinant of the optimal investment in skills $z'$ are the exposure to ideas $z^E$, the future cost of labor $w_{t+1} (\cdot)$ and the foreseen taxes $\tau_{t+1}^E$ on entrepreneurial labor. Under the conditions laid out in Proposition 1 below, optimal investments in skills are determined by the condition

$$\beta (1 - \tau_{t+1}^E) \left[ \pi_1 [z', w_{t+1} (z')] + \pi_2 [z', w_{t+1} (z')] \frac{\partial w_{t+1} (z')} {\partial z'} \right] = \phi' \left( \frac{z'}{z^E} \right),$$  \hspace{1cm} (7)

where $\pi_1 (\cdot)$ and $\pi_2 (\cdot)$ stand for, respectively, the first derivative of $\pi$ with respect to the the skill $z$ of the manager and the wage $w_{t+1} (\cdot)$ he will have to pay for labor.

Let $z_{t+1} = \zeta_t [z^E]$ denote the optimal accumulation of skills for each period $t$. It is increasing in $z^E$ as a better exposure of ideas reduces the marginal costs of investment, i.e. the RHS of equation (7). However, as discussed in detail in Monge-Naranjo (2011), if $z^E$ is too low, the optimal choice may be zero as those youth will remain workers when old. The function $\zeta_t (\cdot)$ is shaped by the wage function $w_{t+1} (\cdot)$. Higher future wages, i.e. higher levels for $w_{t+1} (\cdot)$, reduce the investment in skills because it reduces the marginal return to skills in production ($\pi_{12} > 0$). Moreover, the slope of $w_{t+1} (\cdot)$ also matters for investment in skills. Because $\pi_2 < 0$, the more skilled entrepreneurs will pay lower wages because their workers value the better learning opportunities. Finally, notice that $\zeta_t (\cdot)$ is directly affected by the tax rate $\tau_{t+1}^E$ on the returns to entrepreneurial knowledge.

When choosing which firms to work for, the young fully perceive the implied differences in learning opportunities across firms. For simplicity, as in Chari and Hopenhayn (1991) and others, all young individuals are assumed to be identical. In equilibrium, they must be indifferent to work for the different active firms. Then, the wages paid by firms with two different know-how levels

\(^{11}\)If investment costs were not tax-deductible, then $V [z^E, w_{t+1} (\cdot),] \equiv \max_{z'} \left\{ (1 - \tau_{t+1}^E) \beta \pi [z', w_{t+1} (z')] - (1 - \tau_{t+1}^W) z^E \phi \left( \frac{z'}{z^E} \right) \right\}$. With this alternative assumption, the precise formulas below will be changed but not the substance of the results.
\( z_0 < z_1 \) must compensate for differences in learning opportunities,

\[
(1 - \tau^W_{t+1}) [w_t(z_0) - w_t(z_1)] = V[z^E_1, w_{t+1}()] - V[z^E_0, w_{t+1}()] ,
\]

where the implied exposure to ideas in the two firms are \( z^E_0 = (z_0)^{0.5} (Z^O)^{0.5} < z^E_1 = (z_1)^{0.5} (Z^O)^{0.5} \).

Less skilled managers must pay higher wages, \( w_t(z_0) \geq w_t(z_1) \) as the higher skilled managers provide better learning opportunities, \( V[z^E_1, w_{t+1}()] \geq V[z^E_0, w_{t+1}()] \). \(^{12}\)

Let \( \lambda_t \) be a positive and fine measure that describes the managers operating in the country at time \( t \). In general, \( \lambda_t \) can be composed of a measure of domestic managers \( \lambda^H_t \) and a measure of foreign managers \( \lambda^F_t \). For reasons that will become apparent below, I assume that \( \lambda_t \) has a bounded support in the non-negative numbers. Given wages \( w_t(z) \), the amount of labor hired by an entrepreneur with skill level \( z \) is given by (5), and the distribution of labor employed across skill levels is given by

\[
\mu_t(B) = \frac{\int_{\mathbb{R}^+} n^* [z, w_t(z)] \lambda_t (dz)}{\int_{\mathbb{R}^+} n^* [z, w_t(z)] \lambda_t (dz)}, \quad \text{for any Borel set } B.
\]

In each period \( t \), the government collects (or pays if negative) taxes from domestic entrepreneurs, foreign entrepreneurs and domestic workers in the amounts. As of time \( t = 0 \), the government budget constraint, is given by

\[
\sum_{t=0}^{\infty} \beta^t \left[ \tau^E_t \int_{\mathbb{R}^+} \pi [z, w_t(z)] \lambda^H_t (dz) + \tau^F_t \int_{\mathbb{R}^+} \pi [z, w_t(z)] \lambda^F_t (dz) + \tau^W_t \int_{\mathbb{R}^+} w_t(z) \mu_t (dz) \right] \geq 0,
\]

because the government has zero expenditures.

Given a government policy \( \tau = \{\tau^E_t, \tau^W_t, \tau^F_t\} \), a competitive equilibrium is a price system \( \{w_t(\cdot)\}_{t=0}^{\infty} \), profit and labor hiring functions \( \pi [z, w_t(z)] \), \( n^* [z, w_t(z)] \), a pair of sequence of skill-acquisition functions \( \{V[z^E, w_{t+1}(\cdot)\}, \zeta_t(z^F)\}_{t=0}^{\infty} \), and sequences of aggregate exposure to ideas \( \{Z^0_t\}_{t=0}^{\infty} \), and non-negative measures of domestic and foreign firms \( \{\lambda^H_t, \lambda^F_t\}_{t=0}^{\infty} \), such that: (i) the government budget constraint (10) is satisfied for \( t = 0 \). For any \( t \geq 0 \); (ii) \( \pi [z, w_t(z)], n^* [z, w_t(z)] \) solve the profit maximization problem of the old; (iii) \( V[z^E, w_{t+1}(\cdot)\], \zeta_t(z^F) \) solve the optimal acquisition of skills for the young for any level \( z^E \geq 0 \), and \( w_t(\cdot) \) satisfies the indifference condition (8); (iv) the value \( Z^0_t \) is given by (2) for \( \mu_t(\cdot) \) defined by (9), given \( \lambda_t = \lambda^H_t + \lambda^F_t \); (iv) the distribution of skills for the domestic firms \( \{\lambda^H_t\}_{t=0}^{\infty} \) evolves according to \( \{\zeta_t\}_{t=0}^{\infty} \), i.e.

\(^{12}\)The proper interpretation of (8) is as differences in the cost of effective units of labor, which may not directly translate into differences into workers earnings differences when there is heterogeneity across workers too. For instance, an economy with heterogeneous entrepreneurs and heterogeneous workers and small fixed costs of hiring each worker. More productive firms would want to hire more units of effective labor, and to minimize on the fixed costs, in equilibrium they would hire the workers endowed with the most effective units. Such positive assortive matching could lead to higher earnings for workers in the more productive firms.
for any Borel set $A \subset \mathbb{R}_+$, \( \lambda_{t+1}^H(A) = \int_{\mathbb{R}_+} 1_A(\zeta_t(\gamma (Z_t^O)^{1-\gamma})) \mu_t(dz) \); and (v) an entry condition for foreign firms \( \{\lambda_t^F\}_{t=0}^{\infty} \) and (vi) market-clearing for the domestic labor market that pin down \( \{w_t(\cdot)\}_{t=0}^{\infty} \).

This definition is incomplete as it lacks the optimal entry decisions of foreign firms. In the next two sections, I complete the definition with alternative assumptions regarding whether the country is “closed” or “open” and on the behavior of the tax policy \( \tau \).

4 A Closed Economy

This section considers a closed economy, defined as the case when \( \lambda_t^F = 0 \) for all \( t \). For example, closed economies can be seen as a result of imposing confiscatory taxes, \( \tau_t^F = 1 \) all \( t \), on foreign firms. In a closed economy, domestic entrepreneurs are the only ones setting up firms, controlling using their knowledge and local labor to produce and are also the only source of ideas for future entrepreneurs.

4.1 Homogeneous Managers

Consider first the case when all of the have the same level of knowledge \( z = Z_0 > 0 \), i.e. \( \lambda_t^H = \delta Z_0 \), a Dirac distribution. Regardless of the value \( \rho \), the average \( Z_0^O \) is also equal to \( Z_0 \); likewise, regardless of the value of \( \gamma \), all young workers are exposed to the same level of ideas \( z^E = Z_0 \). Therefore, at time \( t = 0 \) all firms pay the same wage \( w_0 > 0 \) and hire the same units of labor, \( n_0^* = 1 \). Moreover, since all the young are exposed to the same level of ideas and foresee the same wage function \( w_1(\cdot) \) for \( t = 1 \), they invest the same amount in skills \( Z_1 > 0 \). Then \( \lambda_t^H = \delta Z_1 \). The same logic applies for any period \( t \) and the initial homogeneity will be preserved over all the generations. Thus, the entire dynamics of the country can be traced by a sequence \( \{Z_t\}_{t=0}^{\infty} \) of knowledge levels for each generation.

Under those circumstances, \( n_t^* = 1 \) for all firms. Workers wages and entrepreneurs’ profits are equal to \( w_t = \alpha Z_t \) and \( \pi_t = (1 - \alpha) Z_t \), respectively. Using these, and defining \( G_t \equiv Z_{t+1}/Z_t \) to be the gross growth rate of knowledge, the optimality condition (7) boils down to

\[
\beta \left( 1 - \tau_{t+1}^E \right) \left[ 1 + \frac{\gamma v w_0 \left( 1 - \tau_{t+1}^W \right)}{1 + v} (G_{t+1})^{1+v} \right] = v_0 (G_t)^v. \tag{11}
\]

Clearly, higher taxes \( \tau_{t+1}^E \) reduce the accumulation of knowledge \( G_t \) by the current young generation because they reduce their marginal net-of-taxes returns. More interestingly, the current accumulation \( G_t \) of knowledge is higher when future young generations are foreseen to accumulate more knowledge, i.e. when \( G_{t+1} \) is higher, because the returns to accumulate knowledge are not only in terms of producing goods but also in terms of producing skills for the future generations.
Restrict attention now to balance growth paths (BGP), equilibria in which entrepreneurial knowledge grows at a constant rate $G$. The value of $G$ must be a root of the implied equation (11) when $G_t = G_{t+1} = G$ and $\tau^E_{t+1} = \tau^E < 1$,

$$\beta (1 - \tau^E) \left[ 1 + \frac{\gamma v v_0 (1 - \tau^W)}{1 + v} (G)^{1 + v} \right] = v_0 (G)^v.$$  \hspace{1cm} (12)

The following results are proved in the appendix:

**Proposition 1** (Closed economy BGP) Consider a closed economy with constant tax rates $\tau^E \in \left(-\alpha/(1 - \alpha), 1\right)$ and $\tau^W = -\tau^E (1 - \alpha)/\alpha$. Then the following hold: (a) An equilibrium non-degenerate BGP exists if either (i) $\gamma = 0$ and $v > 1/(1 - \alpha)$ or (ii) $\gamma > 0$, and $(1 - \tau^E)^{(1/(1 - \alpha))} (\alpha + (1 - \alpha) v_0 (1 + v))^{1/(1 - \alpha)} < \beta < \frac{\alpha}{(1 - \tau^E)(\alpha(1 + (1 - \alpha))\tau^E)}$; (b) if an equilibrium BGP exists it is unique; (c) the economy exhibits sustained growth, i.e. $G > 1$ if either (i) $\gamma = 0$ and $\tau^E < 1 - v_0/\beta$, or (ii) $\gamma > 0$ and $[1 - \tau^E(\alpha + (1 - \alpha))] < \beta < \frac{\alpha}{(1 - \tau^E)(\alpha(1 + (1 - \alpha))\tau^E)}$; (d) under the conditions in (a), a closed economy initially populated by homogeneous entrepreneurs remains homogeneous and in the unique BGP; that is, other non-explosive fluctuations in $G_t$ are ruled out.

The curvature parameter $v$ must be high enough for a BGP to exist. Otherwise, it may be possible that the left-hand-side of (12) always lays above the right-hand-side; if so, the optimal accumulation would be degenerated to $+\infty$. Under conditions in part (a) there are two roots, but the higher one is ruled out because it corresponds to a local minimum. Being in the lower root also rules out self-fulfilling (extrinsic) fluctuations.

The condition $\tau^E > -\alpha/(1 - \alpha)$ arises from the budget constraint of the government (10) since the government cannot impose a tax $\tau^W \geq 1$ on workers to subsidize entrepreneurs. Other than that, the government could set any tax $\tau^E < 1$ because it effectively disposes of lump-sum taxation on workers.

### 4.2 Heterogeneous Managers

Consider now a non-degenerate but bounded support in the distribution of skills for the initial old generation. For any level $z$, and given $Z^O_{t+1}(\cdot)$, $w_{t+1}(\cdot)$ and $w_{t+2}(\cdot)$, the first order condition for the optimal acquisition of skills $z'$ is

$$\beta (1 - \tau^E_{t+1}) \left[ \frac{\alpha z'}{w_{t+1}(z')} \right]^{\gamma \alpha / \gamma} \left[ 1 + \frac{(1 - \tau^W_{t+1}) \alpha w_0 (z')^\gamma (Z^O_{t+1})^{1-\gamma}}{(1 + v) w_{t+1}(z')} \left[ (z')^\gamma (Z^O_{t+1})^{1-\gamma} \right]^{1 + v} \right] = v_0 \left( \frac{z'}{z^E} \right)^v,$$ \hspace{1cm} (13)

$^{13}$Boundedness is required. Otherwise only the limiting entrepreneur would hire the entire mass of young workers.
where \( z'' \) is the foreseen acquisition of skills of the young workers at period \( t+1 \) working for the currently forming entrepreneur. This expression is derived first using (8) to obtain,
\[
\frac{\partial w_{t+1}(z')}{\partial z'} = -\left(1 - \tau^W_{t+1}\right) V_1[(z^E)'] w_{t+2}(z'') \frac{\partial (z^E)'}{\partial z'},
\]
and then the envelope condition on (6) to get
\[
V_1[(z^E)'] w_{t+2}(z'') = -\frac{\psi_0}{1+v} \left[ \left( z'' \right)^\gamma \left( z_{t+1}^E \right)^{1-\gamma} \right].
\]
See Monge-Naranjo (2011) for further details.

The proof of the following limited but useful results are also in Monge-Naranjo (2011).

**Proposition 2** Assume constant taxes \( \tau^E_t = \tau^E > -\alpha/(1 - \alpha) \). If an equilibrium exist: (a) the wage function \( w_t(z) \) is non-increasing; (b) if \( v > \alpha/(1 - \alpha) \), the function \( z_{t+1} = \zeta_t(z) \) is strictly increasing. Additionally, (c) if \( \gamma > 1 - \alpha/[(1 - \alpha) v] \), then \( \zeta_t(z_1)/\zeta_t(z_0) > z_1/z_0 \) for any \( z_1 > z_0 \) in the support of \( \lambda_t \).

Albeit limited, this simple result has important implications for the limiting behavior of the skill distribution:

**Corollary 1** If either \( \gamma > 1 - \alpha/[(1 - \alpha) v] \) or \( \gamma = 0 \), then, any equilibrium starting with initial distribution with bounded support will asymptotically converge to a homogenous firms BGP.

Most obviously, if \( \gamma = 0 \), pre-existing heterogeneity disappears after one period. More interestingly, if \( \gamma > 1 - \alpha/[(1 - \alpha) v] \), i.e. one’s own manager is a leading source of ideas, then pre-existing differences in the exposure to ideas will lead to widening gaps in skill formation. In this case, the economy exhibits dispersion-induced homogeneity: It converges to a pool of homogeneous entrepreneurs because the top end of the distribution reproduces at a faster pace than the lower end; in the limit, all the remaining entrepreneurs would be the offsprings of the initially highest skilled entrepreneur(s).

### 4.3 Efficient Allocations

In this section I consider allocations that maximize the net-present value of the country’s aggregate consumption. I first discuss the social planner’s allocation of (young) labor \( n_t(z) \) across old managers and the investment decisions \( z_{t+1}(z) \) on each young worker at time \( t \). Then, I show how some of those allocations can be decentralized with the appropriate tax rates.

Given a initial distribution \( \lambda_0 \), a social planner would choose sequences \( \{n_t : \mathbb{R}_+ \rightarrow \mathbb{R}_+\}_{t=0}^\infty \) and \( \{z_{t+1} : \mathbb{R}_+ \rightarrow \mathbb{R}_+\}_{t=0}^\infty \), to maximize the value of
\[
S(\lambda_0) = \sum_{t=0}^\infty \beta^t \left\{ \int_{\mathbb{R}_+} \left[ z [n_t(z)]^\alpha - n_t(z) z^E_t(z) \phi \left( \frac{z_{t+1}(z)}{z^E_t(z)} \right) \right] \lambda_t(dz) \right\},
\]
subject to the adding up constraint
\[
\int_{\mathbb{R}_+} n_t(z) \lambda_t(dz) = 1,
\]

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and the law of motion for the distribution of skills

$$\lambda_{t+1} [A] = \int_{\mathbb{R}_+} 1_A \{ z_{t+1} (z) \} n_t (z) \lambda_t (dz)$$, for any Borel set \( A \subset \mathbb{R}_+ \).

Here \( z^E_t (z) \equiv z^\gamma [ \int x^n t_l (x) \lambda_t (dx) ]^{1 - \gamma} \) is the implied exposure to ideas of each worker, as pinned down by each person’s own manager \( z \), and the outside exposure to ideas as pinned down by \( \lambda_t \) and \( n_t \).

A social planner internalizes two aspects of labor allocations and learning decisions that are omitted in a laissez-faire competitive equilibrium. First, the investments \( z_{t+1} (z) \) also consider the impact on the exposure to ideas for all future young workers, not only those working for the individual entrepreneur. Second, the allocation of labor \( n_t (z) \) also consider the implied impact on the exposure to ideas of all current young workers. This effect is positive for high \( z \) and negative for low \( z \).

The internalization of these two forces magnifies the differences in the allocation of labor and in the learning investments across firms with different managerial skills levels. The proof for the following proposition is in the Appendix:

**Proposition 3** Let \( \{ n^{LF}_t (\cdot) \, , \, n^{LF}_{t+1} (\cdot) \}_{t=0}^{\infty} \) and \( \{ n^{SP}_t (\cdot) \, , \, n^{SP}_{t+1} (\cdot) \}_{t=0}^{\infty} \) denote, respectively, the labor allocation and knowledge formation for the laissez-faire and social planners allocations. If \( 0 \leq \gamma < 1 \) and \(-\infty < \rho < \infty\),

$$\frac{n^{LF}_t (z_1)}{n^{LF}_t (z_0)} < \frac{n^{SP}_t (z_1)}{n^{SP}_t (z_0)} \quad \text{and} \quad \frac{z^{LF}_t (z_1)}{z^{LF}_t (z_0)} < \frac{z^{SP}_t (z_1)}{z^{SP}_t (z_0)}$$

for any \( z_0 < z_1 \) and \( t \geq 0 \) for which the ratios are well defined.

Both of these forces implies that in any point in time, the efficient allocations leads to a more dispersion and therefore, to a faster convergence to homogeneity than in the laissez-faire allocation.

If (when) initially the old cohort of managers is homogeneous, the planning problem is fairly simple. Assume that all the current crop of old managers have the same expertise \( Z_t > 0 \). The planner must decide the units of labor to assign to each manager and the skills \( Z_{t+1} \) to invest in each of the young workers. Because learning is the same in all firms, the decreasing returns in production implies that all managers must command the same amount of labor, \( n_t \). Aggregating over firms, aggregate output of goods is \( Z_t \). It is evident also that it is optimal to invest the same knowledge \( Z_{t+1} \) in each of the future managers. The aggregate cost of learning formation is \( Z_t \phi (Z_{t+1}/Z_t) \).

In recursive form, the value function \( S (Z) \) for the planner is defined by the Bellman Equation (BE):

$$S (Z) = \max_{\{ Z' \geq 0 \}} \left\{ Z \left[ 1 - \phi \left( \frac{Z'}{Z} \right) \right] + \beta S (Z') \right\}.$$ (14)
Notice that the period return function \( Z [1 - \phi (Z' / Z)] \) is linearly homogeneous and jointly concave in \((Z, Z')\) and that the feasible set for \( Z' \) does not depend on \( Z \). These properties lead to the following result:

**Proposition 4** Assume that parameter conditions in Proposition 1 hold for \( \gamma = 1 \). Then, the unique solution to (14) has the form \( S(Z_n) = S_0 Z_n \) for \( 0 < S_0 < \infty \) given by

\[
S_0 = \max_{G \in [0, \infty]} \left\{ 1 - \frac{v_0 (G)}{1 + v} + \beta G S_0 \right\}
\]

Moreover, the value \( G \) that solves this maximization coincides with the laissez-faire \( G \) for \( \gamma = 1 \).

Let \( G^{SP} \) and \( G^{LF} \) denote the growth rate in the social planner’s and in the laissez-faire allocations, respectively. When \( \gamma < 1 \), \( G^{SP} > G^{LF} \), because the individual entrepreneur only captures the returns on his knowledge accumulation that accrued in his profits and not on the aggregate stock of ideas circulating for future generations. However, for the case of homogeneous managers, the implementation of the socially efficient accumulation of knowledge is fairly simple. It involves simple proportional Pigouvian taxes. The following result is straightforward to verify:

**Proposition 5** Assume \( 0 \leq \gamma < 1 \). If there is a tax rate \(-\alpha / (1 - \alpha) < \tau^E < 1\) such that

\[
\tau^E = 1 - \frac{v_0 (G^{SP})^v}{\beta \left[ 1 + (1 - \tau^W) \frac{G^{SP}}{1 + v} \right]}, \quad \text{and} \quad \tau^W = -\frac{\tau^E (1 - \alpha)}{\alpha} < 1, \quad (15)
\]

then the allocation of labor and formation of knowledge in a competitive equilibrium with constant taxes \((\tau^E, \tau^W)\) coincide with the socially efficient ones.

When \( \gamma < 1 \), a subsidy, i.e. \( \tau^E < 0 \), is required to induce young entrepreneurs to accumulate more knowledge and internalize the social benefits for subsequent generations. To finance these subsidies, a labor tax \( \tau^W > 0 \) is required.\(^{14}\)

## 5 Open Economies

Assume now that the home country allows foreign managers to set up firms and hire domestic labor.\(^{15}\) The entry of foreign skills and ideas can impact the domestic accumulation of knowledge.

\(^{14}\)Even if labor taxes are non-distortionary it may be possible that the efficient allocation cannot be implemented. The equation for \( \tau^E \) defines a quadratic expression, so it might be possible that two different tax rates pairs \((\tau^E, \tau^W)\) implement the efficient allocation.

\(^{15}\)I will assume that individuals from the home country cannot move. This is without loss of generality for workers and old entrepreneurs, since, in equilibrium they will be indifferent between moving to foreign or remaining in home. However, ruling out the possibility for domestic young potential entrepreneurs to move and “grow up” in the developed country is crucial. I will discuss further below the factual and analytical relevance of this assumption.
in two opposite ways. One one hand, foreign managers expose their ideas to local workers, directly to those under their control, and indirectly to all others via their impact on $Z_t^O$. On the other hand, the expectation of foreign entry increases the foreseeable future cost of labor and reduces the total and marginal return to investing in knowledge. The balance between the two forces is also determined by the taxation program chosen by the government.

For simplicity, I focus on the case where $\gamma = 0$, i.e. when all the exposure to ideas is external to the firm and all the young agents perceive the same set of ideas $Z_t^O$. Aside tractability, such a case also captures the literature’s emphasis on externalities.\textsuperscript{16} I assume that the home country is initially less developed than the rest of the world. Also, for clarity and concreteness, I make other ancillary assumptions: First, both home and foreign are initially populated by homogeneous managers. That is, at time $t = 0$, $Z^h_0 < Z^f_0$, where, as with all other variables, the super-indexes $h$ and $f$ stand for “home” and “foreign.” Second, the home country is “small,” i.e. its policies do not affect the aggregate dynamics of the foreign country. Third, the rest of the world is in a BGP with growth $G_f$, and foreign entrepreneurs face a constant tax (subsidy) rate $\tau^F_f$ if they remain in the foreign country. As a benchmark case I will use $\tau^F_f$ to be equal to the Pigouvian rate, i.e. $G_f = G^{sp}$; however, I will also comment on cases outside this benchmark.

I now define the relevant state variables for an open economy. Let $R_t \equiv Z^h_t/Z^f_t$, i.e. the level of know-how of domestic firms relative to the know-how of foreign firms. The ratio $R_t$ will be useful since the absolute levels of knowledge grow over time. Also, define $m_t$ to be the fraction of domestic labor under the control of foreign firms in any period $t$. The dynamics of the home country will be entirely determined by the behavior of these two variables. That is, the state of the economy is the pair $(m_t, R_t) \in [0, 1] \times R_+$.\textsuperscript{17}

For any arbitrary state $(m_t, R_t)$, I first characterize the determination of $(m_{t+1}, R_{t+1})$ and the value of other relevant variables. Then, I look at the behavior of the economy and the for the closed state $(m_0, R_0) = (0, R)$ to evaluate the gains from openness.

Given the state $(m_t, R_t)$, the average quality of the ideas surrounding the youth in the country is equal to

$$Z_t^O = \left[ (1 - m_t) (Z^h_t)^{\phi} + m_t (Z^f_t)^{\phi} \right]^{\frac{1}{\phi}},$$

an average of domestic and foreign ideas, with weights $1 - m_t$ and $m_t$, respectively. In relative terms, let $R_t^O$ be the ratio of ideas that the domestic young are exposed to, relative to their foreign peers, i.e.,

$$R_t^O \equiv \frac{Z_t^O}{Z_t^F}.$$

\textsuperscript{16}See Monge-Naranjo (2011) for the general case $0 \leq \gamma \leq 1$, but restricted to comparing laissez-faire openness ($\tau^F = 0$) with complete closedness ($\tau^F = 1$).
Obviously,
\[ R_t^O = \left[ (1 - m_t) (Z_t^h) + m_t (Z_t^f)^\beta \right]^{\frac{1}{\beta}}. \quad (16) \]

Inasmuch as \( Z_t^h < Z_t^f \) (i.e. \( R_t < 1 \)), the entry of foreign firms in period \( t \) (i.e. \( m_t > 0 \)), \( R_t^O > R_t \), as foreign ideas increase the average relative to \( R_t \). However, as long as \( m_t < 1 \), then \( R_t^O < 1 \), and the domestic youth are being exposed to inferior ideas than those available to the foreign young.

The previous formulas are in terms of \( m_t \) which is an equilibrium object. Specifically, foreign firms enter the country up to the point in which the last one is indifferent between moving or staying. In this simple model that holds when producing at home or abroad produce the same the after-tax profits. Given the taxes \( \tau_t^F \) in the foreign country and the tax \( \tau_t^F \) to be paid in the home country, and the domestic and foreign wages \( w_t^h \) and \( w_t^f \), the condition that determines entry of foreign firms is

\[ (1 - \tau_t^F) \pi(Z_t^f, w_t^f) \geq (1 - \tau_t^F) \pi(Z_t^h, w_t^h), \quad m_t \geq 0, \]

and at least one the inequalities holds with equality.\(^{17}\) Using the expressions for \( \pi(Z_t^f, w_t^h) \), and the domestic labor market clearing condition, the equilibrium share \( m_t \) is given by

\[
m_t = \begin{cases} 
0 & \text{if } R_t \geq \left( \frac{1 - \tau_t^F}{1 - \tau_t^F} \right)^{\frac{1-\alpha}{\alpha}} \\
1 - (R_t)^{\frac{1}{\alpha}} \left( \frac{1 - \tau_t^F}{1 - \tau_t^F} \right)^{\frac{1}{\alpha}} & \text{otherwise.}
\end{cases}
\]

Given that foreign wages are given by \( w_t^f = \alpha Z_t^f \), the indiﬁerence condition for foreign ﬁrms can only be satisﬁed if domestic wages are

\[
w_t^h = \begin{cases} 
\alpha Z_t^h & \text{if } R_t \geq \left( \frac{1 - \tau_t^F}{1 - \tau_t^F} \right)^{\frac{1-\alpha}{\alpha}} \\
\alpha Z_t^f \left( \frac{1 - \tau_t^F}{1 - \tau_t^F} \right)^{\frac{1-\alpha}{\alpha}} & \text{otherwise.}
\end{cases}
\]

As for the exposure to ideas \( R_t^O \), using the equilibrium \( m_t \), we get

\[
R_t^O = \begin{cases} 
R_t & \text{if } R_t \geq \left( \frac{1 - \tau_t^F}{1 - \tau_t^F} \right)^{\frac{1-\alpha}{\alpha}} \\
\left[ 1 + [(R_t)^\rho - 1] (R_t)^{\frac{1}{\beta}} \left( \frac{1 - \tau_t^F}{1 - \tau_t^F} \right)^{\frac{1}{\alpha}} \right]^{\frac{1}{\rho}} & \text{otherwise.}
\end{cases}
\]

In all these equations, the first branch indicates that foreign ﬁrms will not enter if home country taxes are too high relative to the foreign country taxes and/or the domestic competition, \( R_t \), is too high.

\(^{17}\)I am abstracting from mobility costs, quality of workers and differences in ‘country-embedded productivities’ (Burstein and Monge-Naranjo 2009). Adding those differences would add extra notation but no substance to the results in this paper.
The dynamic behavior of \( R_{t+1} \) is determined by the optimal accumulation of knowledge by domestic entrepreneurs. That is,

\[
Z^h_{t+1} = \arg \max_{z'} \left\{ \beta \left( 1 - \tau^E_{t+1} \right) \pi(z, w^h_t) - Z^O_t \phi \left( \frac{z'}{Z^O_t} \right) \right\}.
\]

The solution to this optimization problem depends on whether in the next period, foreign firms will enter the country or not. Using the functional form of \( \pi(\cdot, \cdot) \) and the equilibrium values for \( w^h_{t+1} \) when \( m_{t+1} > 0 \) and when \( m_{t+1} = 0 \), the dynamic behavior of the domestic knowledge \( Z^h_{t+1} \) is

\[
Z^h_{t+1} = \begin{cases}
\left[ \frac{\beta(1-\tau^E_{t+1})}{v_0} \right]^{1/v} R^O_t Z^f_t & \text{if } m_{t+1} = 0 ; \\
\left( \frac{\beta(1-\tau^E_{t+1})}{v_0} \right) \left( \frac{1-\tau^E_{t+1}}{1-\tau^f_{t+1}} \right)^{\theta/v} \frac{(R^O_t)^\theta}{(G^f_t)} Z^f_t & \text{if } m_{t+1} > 0 ;
\end{cases}
\]

where \( \theta \equiv v / [v - \alpha / (1 - \alpha)] > 1 \), because of the restriction imposed on \( v \) and \( \alpha \) for the concavity of the young entrepreneur’s problem.

In relative terms, using the fact that foreign growth is \( G^f_t = \left[ \frac{\beta \left( 1 - \tau^E_{t+1} \right) / v_0 }{1 - \tau^f_{t+1} / v_0} \right]^{1/v} \), the dynamics can be simplified to:

\[
R_{t+1} = \begin{cases}
\left[ \frac{1-\tau^E_{t+1}}{1-\tau^f_{t+1}} \right]^{\frac{1}{v}} R^O_t & \text{if } m_{t+1} = 0 ; \\
\left( \frac{1-\tau^E_{t+1}}{1-\tau^f_{t+1}} \right)^{\frac{1}{v}} (R^O_t)^\frac{\theta}{v} & \text{if } m_{t+1} > 0 ;
\end{cases}
\]

Notice that either description of the dynamics is incomplete, since it depends on the undetermined value of \( m_{t+1} \), which also has to be consistent with equilibrium conditions and the government budget constraint. This incompleteness arises because I have not yet specified the determination of the tax program \( \{ \tau_{t+1}^W, \tau_{t+1}^E, \tau_{t+1}^F \} \). In the reminder of this section and in the next, I consider the the cases of zero taxes (laissez-faire), optimal tax programs (Ramsey) and restricted optimal taxes, where at least one of the tax rates is exogenously restricted.

## 5.1 Laissez-Faire

First, consider the case in which the home country adheres to *laissez-faire*, where all taxes are zero. For simplicity, let us focus first on the case in which the foreign country also has zero taxes. Since foreign entrepreneurs can freely enter the home country, and since we are abstracting from mobility frictions or taxes, foreign entrepreneurs operating at home must earn \( \pi_f = (1 - \alpha) Z^f_t \), i.e. their profits at foreign. This can only happen under factor price equalization:

\[
w^h_t = w^f_t = \alpha Z^f_t ,
\]

i.e., the cost of labor (in efficiency units) is the same in both countries.\(^{18}\)

\(^{18}\)The key is that the cost of each efficiency unit and not physical unit of labor, which for simplicity are assumed to be the same. The model can easily accomodate cross-country differences in workers earnings by introducing differences in the ratio of effective-to-physical units across countries.
Facing the same effective wages, each foreign firm hires the same amount of labor units, \( n^f_t = 1 \), as if they had remained in the foreign country. When domestic wages are given by \( w^h_t = \alpha Z^f_t \), domestic firms with knowledge \( Z^h_t = R_t Z^h_t \) hire \( n^h_t = (R_t)^{1/\alpha} \) workers. Then, the labor market clearing condition is simply \( 1 = m_t + n^h_t \) and the foreign-controlled share of domestic labor is

\[
m_t = 1 - (R_t)^{1/\alpha},
\]

(18)

which is strictly decreasing in \( R_t \). If \( R_t = 1 \), domestic entrepreneurs are at par with the foreign ones and dissipate any saving in labor costs from moving across countries. On the contrary if \( R_t = 0 \), all the domestic labor force would be under foreign management.

Under laissez-faire, the domestic exposure to ideas has a closed-form solution. Plugging expression (18) in the formula (16), \( R^O_t \) is equal to:

\[
R^O_t = \left[ 1 + (R_t)^{\rho+1/\alpha} - (R_t)^{1/\alpha} \right]^{1/\rho}.
\]

(19)

Notice however that \( R^O_t \) may not be always increasing in \( R_t \). On the one hand, a higher \( R_t \) increases \( R^O_t \) because domestic firms are a better source of ideas. On the other hand, a higher ratio \( R_t \) reduces the entry of foreign firms and their productive ideas. Indeed, as shown in Figure 1, if \( \rho > -1/(1 - \alpha) \), the negative force dominates for values of \( R_t \) close to zero but the positive force dominates for higher values of \( R_t \). As shown in the figure, if \( \rho \leq -1/(1 - \alpha) \), the complementarity in the two source of ideas is so strong that \( R^O_t \) is globally increasing in \( R_t \).

The transition function under laissez-faire can also be solved in closed-form. The next period ratio \( R_{t+1} \) is simply given by

\[
R_{t+1} = (R^O_t)^{\theta} = \left[ 1 + (R_t)^{\rho+1/\alpha} - (R_t)^{1/\alpha} \right]^{\theta/\rho}.
\]

(20)

Since \( \theta > 1 \), the country’s relative knowledge for the next period \( R^O_t \) is strictly increasing and convex in the current relative exposure \( R^O_t \) to ideas. However, it is always the case that \( R_{t+1} < R^O_t \), i.e. domestic entrepreneurs, in relative terms, never fully match up with the level of ideas with which they are exposed to. The reason is that they foresee the next-period entry of (and competition from) foreign entrepreneurs who are being trained with better with overall better ideas \( Z^f_t \), i.e. \( 1 > R^O_t \).

Figure 1 is also very useful displaying the BGP’s of a laissez-faire open economy. Each BGP is a fixed point of the transition function. First, \( R = 1 \) is always a fixed point. Essentially, if the home country is initially at par with the rest of the world, it will remain so. The country does not have anything to learn from the world and foreign firms gain no cost advantage moving in, hence \( m_t = 0 \). Another interior fixed point, \( R^{\text{int}} < 1 \) always exists as long as \(-\infty < \rho < +\infty\). Moreover, the interior fixed point is unique and globally stable, as proved in Monge-Naranjo (2011),
Figure 1: **Transition Function: Laissez-Faire**

**Proposition 6** (BGPs open economy) $R = 1$ is an equilibrium BGP. If $-\infty < \rho < \infty$, then there exists a unique interior equilibrium $R^{\text{int}} \in (0, 1)$ and an open, laissez-faire country converges to it from any initial $R_0 \in (0, 1)$.

This simple result shows that, except for the leaders with $R = 1$, initial conditions do not matter for the long-run relative income of countries under a laissez-faire openness regime. However, this form of openness does not push developing countries all the way to catch up with the rest of the world. Even technologies, preferences and inherent capacity to learn are the same and barriers or distortions to the mobility of knowledge are absent, the presence of externalities implies that cross-country differences in the exposure to and creation of skills and ideas will be preserved over time.

Initial conditions matter, however, for the impact of openness on the relative income (transitional growth) and welfare of the country. On one hand, countries with a very low initial level of knowledge would experience dramatic gains from laissez-faire openness. On the other hand, countries that are initially close to the frontier, i.e. $R_0 \in (R^{\text{int}}, 1)$ would experience a slow-down and will end up lagging further behind their initial position.

Albeit more favorable, similar implications hold in terms of welfare. To see this, it is straightforward to show that because the marginal product of labor is equalized between the two countries, per-capita domestic or geographic output (GDP), $Y^{\text{D, open}}_t$, must be also equal to that of the foreign country, $Y^{\text{D, open}}_t = Z^f_t$. From that, the home country’s national income is only
\[ Y_t^{N, \text{open}} = Z_t^f [\alpha + (1 - \alpha) R_t^{1/\alpha}] \] once foreign firms collect their payout. Finally, subtracting the costs of knowledge formation, the country’s aggregate domestic consumption is \( C_t^{\text{open}} = Y_t^{N, \text{open}} - Z_t^O \phi \left( Z_{t+1}^h / Z_t^O \right) \), which, after some easy algebra, can be shown to be

\[ C_t^{\text{open}} = Z_t^f \left[ \alpha + (1 - \alpha) R_t^{1/\alpha} - \phi (G_f) \left( R_t^O \right)^{\theta + (\theta - 1) \nu} \right]. \]

Under the counter-factual of remaining closed, at time \( t \) aggregate consumption would have been \( C_t^{\text{closed}} = Z_t^h [1 - \phi (G_f)] \). As reported below, it is straightforward to compute transition paths and the implied net (present value) aggregate gains of openness, \( (\sum_{t=0}^{\infty} \beta^t C_t^{\text{open}}) / (\sum_{t=0}^{\infty} \beta^t C_t^{\text{closed}}) - 1 \).

A simpler calculation is the cross-BGP or steady state consumption gains between the interior BGP and the closed economy BGP with initial ratio \( R_0 \)

\[ \frac{C_t^{\text{open}}}{C_t^{\text{closed}}} = \frac{\alpha + [1 - \alpha - \phi (G_f)] \left( R_t^{\text{int}} \right)^{1/\alpha}}{R_0 [1 - \phi (G_f)]}, \]

an expression derived using the definition of \( \theta \) and then simplifying. The following result is immediate:

**Corollary 7** Let \( R_L \equiv [\alpha + (1 - \alpha - \phi (G)) (R_t^{\text{int}})^{1/\alpha}] / \left( [1 - \phi (G)] \right) \). Then laissez-faire openness lead to a (steady state) reduction in the aggregate consumption of countries with initial knowledge \( R_0 \in (R_L, 1) \).

How can a country lose domestic knowledge when it is exposed to superior knowledge from abroad? In this model, it is possible that the inflow of foreign ideas is more than compensated by a reduction in the domestic individual’s incentives to build up skills, which can result in an overall decline in sets of ideas \( \{Z_t^O\}_{t=0}^{\infty} \) in the country. Depending on initial conditions, this negative impact on profits can overcome the gains of openness on the wages of the young and national output can fall. For overall welfare to fall, however, it is needed that the net of these two effects is negative enough so that the potential gains from reductions in the cost of training \( Z_t^O \phi \left( Z_{t+1}^h / Z_t^O \right) \) is big enough. This model shows that that is certainly possible. See Monge-Naranjo (2011) for some quantitative evaluations of the gains from openness in an extended version of this model.

This section has examined the income and welfare gains from openness under the notion that openness narrowly understood as laissez-faire. That is the notion most commonly adopted in the literature. However, as argued in the ensuing section, this conceptual restriction is unwarranted.

### 5.2 A Ramsey Program

Following the Ramsey tradition, let us now assume that the objective of the home government is to maximize the average welfare of the country, but that the tools available are restricted to some
proportional taxes \( \{\tau^W_t, \tau^E_t, \tau^F_t\} \), taking as given the tax \( \tau^E_t \) and growth rate \( G_f \) in the foreign country. Specifically, I take maximizing \( \sum_{t=0}^{\infty} \beta^t C_t \) as the objective of the government, which, given the OLG structure, implicitly weights the welfare of different generations on the basis of the individuals’ discount factor \( \beta \).

To solve for the Ramsey program, I use the *primal approach*, i.e. instead of maximizing over the different taxes \( \{\tau^W_t, \tau^E_t, \tau^F_t\} \), I solve for a social planner’s problems in terms of allocations that correspond to competitive equilibria for feasible taxation programs, taking as given the initial level of knowledge of both countries \( Z^h_0 \) and \( Z^f_0 \) and the tax rate \( \tau^F_f \) and growth path \( G_f \) in the foreign country. In any period, the aggregate consumption equals the domestic wages and domestic profits, the taxes collected from (or the subsidies pay to) foreign profits minus the cost of acquiring skills, i.e.

\[
C_t = w^h_t + \pi^h_t + \tau^E_t \pi^f_t q^f_t - Z^h_t \phi \left( \frac{Z^h_t}{Z^E_t} \right),
\]

where \( q^f_t \) is the mass of foreign firms in the home country at time \( t \).

Ramsey allocations can be solved as follows: given the state \( R_t \), the planner chooses the mass \( m_t \) of labor controlled by foreign firms and the country’s next period knowledge level relative to foreign, \( R_{t+1} \). From (2??) a value of \( m_t > 0 \) implies

\[
\tau^F_t = 1 - \frac{(1 - \tau^F_f)}{(1 - m_t)^{\alpha}} (R_t)^{1 - \alpha} ;
\]

\( m_t = 0 \) is compatible with any \( \tau^F_t \geq \tau^F_f (R_t) \equiv 1 - (1 - \tau^F_f) (R_t)^{1 - \alpha} \). On the other hand, there is a highest attainable \( \bar{m} (R_t) < 1 \) which corresponds to the highest feasible subsidy \( -\tau^F_f (R_t) > 0 \) financed with taxes levied domestic workers and domestic firms.\(^\text{19}\) Thus, we can restrict attention to \( m_t \in \Gamma (R_t) = [0, \bar{m} (R_t)] \). In those cases:

\[
w^h_t = Z^f_t \frac{\alpha R_t}{(1 - m_t)^{1 - \alpha}},
\]

which, plugged into (4) implies

\[
\pi^h_t = (1 - \alpha) Z^f_t (1 - m_t)^{\alpha} R_t, \text{ and }
\]

\[
\pi^f_t = (1 - \alpha) Z^f_t (1 - m_t)^{\alpha} (R_t)^{1 - \alpha},
\]

and plugged into (5) implies \( n^h_t = 1 - m_t \) and \( n^f_t = (1 - m_t) / (R_t)^{1 - \alpha} \). Then, labor market-clearing requires a mass of foreign firms equal to \( q^f_t = (R_t)^{1 - \alpha} m_t / (1 - m_t) \), and the country’s total collection of taxes from (disbursement of subsidies to) foreign firms is

\[
\tau^F_t q^f_t \pi^f_t = (1 - \alpha) Z^f_t \frac{m_t}{1 - m_t} R_t \left[ (1 - m_t)^{\alpha} - (1 - \tau^F_f) (R_t)^{1 - \alpha} \right],
\]

\(^\text{19}\)With occupation choices, any \( m_t = 1 \) is attainable, a jump in the occupation choices as a response to taxes can imply that some \( m \in [0, 1] \) are not attainable.
which, given $R_t$ and $\tau_f^E$, is a non-montone (Laffer) curve; it is zero for $m_t = 0$ and $m_t = m_t^{LF} \equiv 1 - (1 - \tau_f^E)^{\frac{1}{v}} (R_t)^{\frac{1}{1-v}}$; positive inside these two points and negative for $m_t > m_t^{LF}$, where $m_t^{LF}$ indicates the level that would take place if the government imposes zero taxes at time $t$.

Finally, as a function of $m_t$, the relative exposure to ideas in the country is $R_t^E = [m_t + (1 - m_t) (R_t)^\gamma]^\frac{1}{\gamma}$ and the cost of acquiring a relative knowledge $R_{t+1}$ for the next generation is $Z_t^f R_t^E \phi \left( \frac{R_{t+1}}{R_t^E} G_f \right)$. Adding the elements in (21) and simplifying, we obtain $C_t = Z_t^f \varrho (R_t, m_t, R_{t+1})$ where

$$
\varrho (R_t, m_t, R_{t+1}) \equiv \frac{R_t \left[ (1 - m_t)^\alpha - (1 - \alpha) m_t (1 - \tau_f^E) (R_t)^{\frac{\alpha}{1-\alpha}} \right]}{1 - m_t} - \frac{v_0 (G_f)^{1+v}}{(1+v)} \frac{(R_{t+1})^{1+v}}{[m_t + (1 - m_t) (R_t)^\gamma]^\frac{1}{\gamma}}.
$$

As a fraction of $Z_t^f$, the first term indicates the amount of resources available to consume or to invest in the period. These resources are single-picked in $m_t$; initially they increase in $m_t$ but eventually they decrease and become negative as $m_t$ approaches 1. The second term is the cost of knowledge accumulation. It is always decreasing in $m_t$ and strictly increasing in $R_{t+1}$. There is an important complementarity between $m_t$ and $R_{t+1}$, as the marginal cost of investing in skills $R_{t+1}$ is decreasing in $m_t$.\(^{20}\) As a practical matter, this property will imply a complementarity between openness (and even subsidies) to foreign firms and government incentives for domestic investment in knowledge.

To solve for the Ramsey program, we can eliminate $Z_t^f$, and leave $R_t$ as the only state. The value function $\vartheta (R)$ associated with the Ramsey program solves the Bellman equation

$$
\vartheta (R) = \max_{m \in [0, \bar{m}(R)]} \left\{ \varrho (R, m, R') + \beta G_f \vartheta (R') \right\}.
$$

(22)

In the appendix, standard recursive methods are used to prove the following proposition.

**Proposition 8** Assume that the foreign government tax rate $\tau_f^E$ is such that $G_f < \beta^{-1}$. Then, there exists a unique $\vartheta (\cdot)$ that solves (22); $\vartheta (\cdot)$ is continuous and strictly increasing. Let $\{m_t^{Ramsey}, R_{Ramsey} \}_{t=0}^{t=\infty}$ denote the optimal Ramsey allocation. From any initial $R_0$: (a) a country would subsidize foreign firms, $m_t^{Ramsey} > m_t^{Laissez-faire} \equiv 1 - (R_t)^{\frac{1}{1-v}} (1 - \tau_f^E)^{\frac{1}{v}}$, if and only if $R_{t+1}^{Ramsey} > R_{t+1}^{Laissez-faire}$; moreover, (b) if $G_f \leq G^{SP}$, then, the home country converges, i.e. $\lim_{t \rightarrow \infty} R_{t+1}^{Ramsey} \geq 1$ and $\lim_{t \rightarrow \infty} m_t^{Ramsey} = 0$.

Figures 3 and 4 illustrate the solution of the Ramsey program under the assumption that the foreign country follows also the (closed economy) Ramsey policies, i.e. $\tau_f^E$ is given by (??) and $G_f = G^{SP}$. Figure 2 shows that for any $R_t$, the value of an open economy is always above the value of a closed economy (the straight line in the diagonal). The gains from openness are $^{20} \frac{\partial^2 \varrho(R_t, m_t, R_{t+1})}{\partial R_{t+1} \partial m_t} < 0$, when $R_t < 1$, i.e.the marginal cost of $R_{t+1}$ is reduced with a higher $m_t$. 

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Figure 2: Value Function of the Ramsey Program: Open and Closed Economies

Figure 3: Transition Function under the Ramsey Program.
always positive, except when $R = 1$, when the home country has nothing to learn from the rest of the world. Notice that the gains from openness are initially very steep: not only the learning opportunities are huge for laggard countries (low $R$) but they can also impose taxes from foreign firms that would still enter motivated by the low wages. For more advanced countries (high $R$), the learning opportunities may come at a fiscal cost. Furthermore Figure 3 shows that the transition function of an open country is always above the $45^0$ line except when $R = 1$; therefore, the open country always catches up.

Figure 4 shows the implied taxes $\{\tau^E_{t+1}, \tau^F_t\}$. From the figure, it is clear that more than subsidizing foreign knowledge (which could occur but only temporarily), the key of the optimal program is to incentivize the formation of domestic knowledge, given the enhanced learning opportunities from the exposure to foreign ideas.

Need to complete this section.  (1) Discussion of these results, e.g. for any $R<1$, Ramsey program implies $m > 0$ and $R' > R$. (2) add of extensions including: (i) restrictions in the tax program, e.g. $\tau^W_t = \tau^E_t$ or $\tau^F_t = \tau^E_t$; (ii) non-optimal foreign policies, $G_f \leq G^{SP}$. Sequel to this paper: large country issues, including the analysis of strategic interactions.

6 Restrictions on the Taxation Program

Up until now, I have characterized the optimal taxation programs under fairly unrestricted conditions. I have assumed that (a) the government is not bound by international agreements on the taxation of foreign firms; (b) the government can freely discriminate between domestic and
foreign firms; (c) the government can finance subsidies to foreign and/or domestic firms with taxes on workers (which in this environment are essentially lump-sum taxes); and (d) the government is able to commit to not to expropriate foreign firms once they have enter. In this section I examine the implications of relaxing each of these conditions on the optimal taxation program and on the gains from openness.

6.1 Zero Taxes/Subsidies on Foreign Firms

Bilateral and multilateral agreements (e.g. the WTO) may restrict the ability of governments to either impose taxes on or provide subsidies to foreign firms. In this spirit, consider the optimal taxation of domestic workers and firms under the constraint that in every point in time taxes on foreign firms are zero, i.e. $\tau_t^F = 0$. Given $R_t$, the restriction to zero taxes implies that $m_t$ equals the static laissez-faire or free entry (FE) level, i.e.

$$m_t = m_t^{LF} (R_t) \equiv \max \left\{ 0, 1 - (R_t) \frac{1}{1-\alpha} \left( 1 - \tau_t^F \right)^{\frac{1}{\alpha}} \right\}. \quad (23)$$

Under this policy, the country’s period payoff is entirely determined by the current state $R_t$ and the next period $R_{t+1}$. Define $\varrho_{LF} (R_t, R_{t+1}) = \varrho [R_t, m_t^{LF} (R_t), R_{t+1}]$. Then, a little algebra leads to

$$\varrho_{FE} (R_t, R_{t+1}) = \begin{cases} \frac{R_t - v_0 (G_f)^{1+v} (R_{t+1})^{1+v} [1-(1-\tau_f^E)^{\frac{1-\alpha}{\alpha}}]}{1-(1-\tau_f^E)^{\frac{1-\alpha}{\alpha}} (1-\tau_f^E)^{\frac{1}{\alpha}} (1+v) \left( 1+(1-\tau_f^E)^{\frac{1}{\alpha}} \right)^{\frac{1}{\alpha}} [1+(1-\tau_f^E)^{\frac{1}{\alpha}} (R_t)^{\frac{1}{\alpha}} [(R_t) - 1]^{\frac{1}{\alpha}}]} & \text{if } R_{t+1} \geq (1 - \tau_f^E)^{-\frac{(1-\alpha)}{\alpha}} \\ \frac{v_0 (G_f)^{1+v} (R_{t+1})^{1+v} (1-\tau_f^E)^{\frac{1}{\alpha}}}{(1+v) [1+(1-\tau_f^E)^{\frac{1}{\alpha}}]^{\frac{1}{\alpha}}} & \text{otherwise.} \end{cases}$$

Overtime, the government can still affect the path $m_t$ as the chosen domestic taxes influence the path of $\{R_t\}$. In recursive form, the primal of the optimal taxation of domestic firms under laissez-faire openness is given by the solution of

$$\tilde{\varrho}_{FE} (R) = \max_{R' \geq 0} \left\{ \varrho_{FE} (R, R') + \beta G_f \tilde{\varrho}_{FE} (R') \right\}.$$ 

Then, given the solution of the above program, for any initial $(m, R)$, the optimal taxation at the period before laissez-faire openness was imposed becomes

$$\varrho_{FE} (m, R) = \max_{R' \geq 0} \left\{ \varrho (R, m, R') + \beta G_f \tilde{\varrho}_{FE} (R') \right\}.$$ 

Results:

If $\tau_f^E = \tau_{SP}^E$, then the country will not catch up. If Precludes countries to catch up.

If $m_t = 0$, then $R_{t}^{Q} = R_{t} \implies$ closed economy. Then, $R_{t} = R_{0}^{Q}$ forever after.

Question: would it always converge to a point above $\hat{R} = (1 - \tau_{SP}^E)^{\frac{\alpha-1}{\alpha}} < 1$.

If instead, $\tau_f^E > \tau_{SP}^E$, then the country will catch up and surpass.
Even when \( m_t = 0 \), the home country would grow at rate \( G_{SP} > G_f \).

Gains from openness always positive, as laissez-faire openness does not impose restrictions on domestic subsidies. But restrictions can seriously reduce the gains from openness. **Graph**

### 6.2 Equal taxation of Domestic and Foreign Firms

An alternative restriction can be the inability of the government to differentiate between domestic or foreign firms for taxation purposes. One source of such restriction could be bilateral or multilateral agreements of the country. Another rationale for such restriction is that firms may be able to manipulate the legal incorporation of their operations to be of either nationality. In either case, I now consider the optimal taxation program subject to the constraint of equal taxation (ET) for any period \( t \) onwards, \( \tau^E_{t+1} = \tau^E_t \).

Forcing the tax program to equalize the tax rates on foreign and domestic profits sharply constrains the ability of the country to accumulate knowledge. Equalizing expressions \((tauE)\) and \((tauF)\) and leads to the following simple result:

**Lemma 9** Take foreign taxes \( \tau^F_f \) as given and assume that \( \tau^E_{t+1} = \tau^E_t \) is imposed in the tax program. If \( m_{t+1} > 0 \) then \( R_{t+1} = R^{LF}(m_t, R_t) \equiv [m_t + (1 - m_t) (R_t)^\theta]^2 \), i.e. the laissez-faire accumulation \( R_{t+1} = (R_t^2)^\theta \). Moreover, as long as \( R_t < 1 \), the optimal taxation program always select \( m_{t+1} > 0 \).

Interestingly, domestic accumulation of knowledge behaves similarly to the laissez-faire openness case. Given the predetermined values of \( m_t \) and \( R_t \), and as long as the tax rate is low enough so that \( m_{t+1} > 0 \). In this model, under uniform taxation, the tax (dis)incentives for domestic knowledge accumulation are exactly compensated by the aligned (dis)incentives provided to the entry of foreign knowledge.

Obviously, the allocations in this restricted optimal taxation program need not coincide with laissez-faire, as the program optimally reshapes the behavior of \( m_t \) over time, and with it, the behavior of \( R_t \). To define the taxation program in recursive primal form, define

\[
\varrho_{ET} (m_t, R_t) \equiv \varrho [m_t, R_t, R^{LF} (m_t, R_t)] = \frac{R_t \left[ (1 - m_t)^\alpha - (1 - \alpha) m_t (1 - \frac{\tau^F_f}{\tau^E_t}) (R_t)^{\frac{\alpha}{1 - \alpha}} \right]}{1 - m_t} - v_0 (G_f)^{1+v} m_t + (1 - v) m_t (1 - m_t) (R_t)^\theta
\]

The optimal taxation program subject to equal taxation of foreign and domestic firms is given by the solution to the BE

\[
\varrho_{ET} (m, R) = \max_{m' \geq 0} \left\{ \varrho_{ET} (m, R) + \beta G_f \varrho_{ET} \left[ m', R^{LF} (m, R) \right] \right\}.
\]

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The first order condition determining \( m \) is

\[
\frac{\partial \phi_{ET}(m, R)}{\partial m} + \beta G_f \frac{\partial \phi_{ET}(m', R')}{\partial R'} \frac{\partial R^F}{\partial m} = 0.
\]

6.3 No Taxation of Workers

The benchmark case analyzed in the previous sections and the constrained taxation programs studied in this one, all presume that the government can finance subsidies to domestic and or foreign firms by taxing the (young) workers in the country. Since current labor supply decisions of workers are inelastic, those taxes are effectively a source of lump-sum taxation. In practice, however, many countries face serious difficulties taxing its population.

In this subsection I consider the optimal taxation and the gains of openness when the government must balance its budget period by period and cannot resort to taxes on domestic labor. In such case, any subsidies provided to foreign (domestic) firms must necessarily entail a tax on domestic (foreign) firms. Specifically, when \( \tau^W_t = 0 \) and no borrowing is allowed, then the flow budget constraint of the government must be balanced, i.e.

\[
\tau^F_{t+1} q^f_{t+1} \pi^f_{t+1} + \tau^E_{t+1} \pi^h_{t+1} \geq 0.
\]

This constraint is equivalent to

\[
\left( \frac{R_{t+1}}{R^O_t} \right)^v \leq \left( \frac{1 - m_{t+1}}{1 - \tau^E_f} \right)^\alpha - m_{t+1} \left( \left( \frac{R_{t+1}}{R^O_t} \right)^{\alpha - \alpha} - \left( \frac{R_{t+1}}{R^O_t} \right)^v \right),
\]

which can be directly used to solve for the optimal taxation program in primal form. Then, the optimal taxation program with balanced tax revenues (BT) must solve

\[
\phi_{BT}(m, R) = \max_{\{m', R'\}} \{ \phi(R, m, R') + \beta G_f \phi_{BT}(m', R') \}
\]

s.t.:

\[
\left( \frac{R'}{R^O} \right)^v \leq \left( \frac{1 - m'}{1 - \tau^E_f} \right)^\alpha - m' \left( \left( \frac{R'}{R^O} \right)^{\alpha - \alpha} - \left( \frac{R'}{R^O} \right)^v \right).
\]

6.4 Expropriation Incentives

Now, consider the case in which the government cannot pre-commit not to expropriate the foreign firms after they have enter the country. That is, in any period \( t \), instead of collecting (or paying if negative) \( \tau^f_t q^f_t \pi^f_t \), the government could expropriate the entire amount of foreign profits \( q^f_t \pi^f_t \) and dispose of the country’s whole GDP

\[
q^f_{t+1} \pi^f_{t+1} + \pi^h_{t+1} + w^h_{t+1} = \frac{Z^f_{t+1} R_{t+1}}{(1 - m_{t+1})^{1-\alpha}}.
\]
Doing so, obviously, would trigger reputation and retaliation costs and possibly other punishments for the country, at least for some periods. For simplicity, assume that the penalty for expropriating is the permanent exclusion from the market of foreign firms. Therefore, the period after expropriating the foreign investors, the country would be a closed economy. Upon this even, by optimizing the domestic formation of knowledge, the country would attain a (normalized) value of 

\[ \phi_{\text{closed}}(R) = R^O \left[ \frac{1 - \phi(G_{SP})}{1 - \beta G_{SP}} \right] \]

Therefore, after some simple algebra, the value \( \tilde{\vartheta}(m, R) \) attainable for a country when it expropriates, given \((m, R)\), is

\[
\tilde{\vartheta}(m, R) = \frac{R}{(1 - m)^{1 - \alpha} - R^O \phi(G_{SP}) + \beta G_{SP} \phi_{\text{closed}}(R^O)} = \frac{R}{(1 - m)^{1 - \alpha} + [m (1 - R^0) + R^0]^\frac{1}{2} \left[ \frac{\beta G_{SP} - \phi(G_{SP})}{1 - \beta G_{SP}} \right]}
\]

In equilibrium, foreign firms can foresee the ex-post expropriation incentives of the country, and the entry of foreign knowledge \( m \) would be restricted to be below levels that trigger expropriation. The optimal taxation program must satisfy the restriction that in every period, the next period allocation \((m_{t+1}, R_{t+1})\) implies of value of not expropriating, \( \vartheta_{EI}(m_{t+1}, R_{t+1}) \) that is above the value of expropriation \( \tilde{\vartheta}(m_{t+1}, R_{t+1}) \). In recursive form, the optimal taxation problem restricted by expropriation incentives (EI) is solves

\[
\vartheta_{EI}(m, R) = \max_{\{m', R'\} \in [0,1] \times \mathbb{R}_+} \{ \vartheta(R, m, R') + \beta G_f \vartheta_{EI}(m', R') \}
\]

s.t.: \( \tilde{\vartheta}(m', R') \leq \vartheta_{EI}(m', R') \).

Contrary to the previous cases, this program does not define a contraction mapping. This is because the objective function \( \vartheta_{EI}(m', R') \) is also part of the definition of the the feasible set. However, it is immediate that the program defines a monotone operator. For the domain in which \( m, R \in [0,1]^2 \) the value \( \vartheta_{EI}(m, R) \) is bounded from above by the unconstrained program \( \vartheta(m, R) \) and from below by the function \( \tilde{\vartheta}(m, R) \). Then there exist a fixed point (see chapter 17 of Stockey, Lucas and Prescott [1989]). The relevant solution is the highest of such fixed points.

### 6.5 Occupation Choices

Entrepreneurship choices have a prominent presence in the development literature (e.g. Banerjee and Newman 1993). Sorting individuals between managerial and labor occupations can enhance the static gains of openness as shown by Antras, Garicano and Rossi-Hansberg (2006), Burstein and Monge-Naranjo (2009) and more forcefully by Eeckhout and Jovanovic (2009). In this section I will argue that occupation choices can also determine whether—and how quickly—a developing country can catch up with the rest of the world. Specifically, I will show that occupation choices:
(a) can change the form of the BGPs; (b) can push an open economy away from the interior BGP and instead to fully catch up; and (c) can accelerate the convergence.

In the model, an old person carrying a skill level \( z \) would only become an active entrepreneur if his rents \( \pi [z, w(z)] \) are above the maximum wage as a worker, i.e. only if

\[
(1 - \tau^E_t) \pi [z, w_t(z)] \geq (1 - \tau^W_t) \sup_{\zeta \in \text{support}(t)} w_t(\zeta),
\]

(24)

where ‘support’ refers to the entire set of entrepreneurial knowledge –domestic or foreign– active in the country.

The option of choosing occupation when old can change the investment in skills for a young person. For a given exposure to ideas \( z^E \), a young person would only invest in skills if:

\[
V[z^E, w_{t+1}(\cdot)] \geq \beta \sup_{\zeta \in \text{support}(t+1)} w_{t+1}(\zeta).
\]

(25)

This lower bound in the career value of a job \( V[\cdot, \cdot] \) can reduce the equilibrium gap between the wages paid by active entrepreneurs with different skills. Specifically, consider two entrepreneurs with skill levels \( z_0 < z_1 \). If the two of them fall below a certain threshold \( z_t^* \), they will both pay the same wage; if the two fall above the threshold, the wage difference will be given by (8) of the previous section, reflecting the difference in the learning opportunities of the two jobs. Finally, if the two skill levels fall on different sides of the threshold, i.e. \( z_0 < z_t^* < z_1 \), the two wages paid satisfy:

\[
w_t(z_1) = w_t(z_0) + \beta \sup_{\zeta \in \text{support}(t+1)} w_{t+1}(\zeta) - V[z_1^E, w_{t+1}(\cdot)] < w_t(z_0) = w_t(z_t^*).
\]

Obviously, \( w_t(\cdot) \) is flat up to the threshold \( z_t^* \), after which it becomes strictly decreasing.

From the point of view of the social planner, occupation choices offer additional instruments. In particular, regardless of their skills, older domestic managers can be allocated to supply labor. More interesting, foreseeing this future option, the planner can opt to not invest in skill formation for some or all of the domestic young workers, which could accelerate the adoption and catching up of foreign know-how.

7 Concluding Remarks

This paper deviated from the usual practice of comparing extreme openness vs. closedness, and instead characterize the output and welfare gains under a Ramsey program, where tax policies are set to maximize the welfare of recipient countries, subject to the equilibrium behavior of national and foreign agents. The paper argues that optimal taxation can change the gains from openness to foreign knowledge in a small developing country. Contrary to simple laissez-faire,
openness to foreign knowledge is always optimal when the country follows a Ramsey program. More interestingly, the paper shows that the optimal tax program always lead developing countries to catch-up with the productive knowledge in developed countries.

Ongoing work extends the analysis along a number of dimensions. A first extension is to solve numerically for the optimal policies for the general case \(0 < \gamma < 1\), allowing for the vintage structure described in Section 3. The second extension considers the optimal policies for home when foreign is not following the optimal Ramsey program. I find that if \(G_f > G^{SP}\) it is not optimal for home to catch up with foreign; instead, the country will be better off reaching a BGP in which the (excessive) growth of foreign knowledge pulls up the country via a positive presence of foreign firms. On the other hand, if \(G_f < G^{SP}\), the optimal policies for home country is to surpassing the foreign country; in this case, the ratio \(R\) will grow without bound at the rate \(G^{SP}/G_f > 1\). A third extension considers a two-country world in which the policies of home affect foreign and vice versa. In equilibrium, the tax program of one country must be the best response of the tax program of the other. Standard game theoretic constructs will be applied to this setting and the equilibrium outcomes will be contrasted with recent policies in the OECD and large emerging market countries.

A Proofs and Other Analytical Details

Proof of Proposition 1. Define \(L(G) = \beta (1 - \tau_E) \left[1 + \frac{\gamma v_0 (1 - \tau^W)}{1 + v} (G)^{1 + v}\right]\) and \(R(G) = v_0 (G)^v\). A gross growth rate \(G > 0\) is a BGP if \(L(G) = R(G)\) and that at that point \(R(\cdot)\) crosses \(L(\cdot)\). To prove both parts (a) and (b) first consider the case \(\gamma > 0\). Notice that both functions are continuous, that \(R(\cdot)\) is always increasing and that \(L(\cdot)\) is also increasing because \(\tau_E < 1\) and \(\tau^W < 1\). Since the curvature of \(L(\cdot)\) is \(1 + v\) and the curvature of \(R(\cdot)\) is only \(v\), then \(L(G) > R(G)\) as \(G \to \infty\). But also notice that \(L(0) = \beta (1 - \tau_E) > 0\) and \(R(0) = 0\). Therefore, either \(L(G) > R(G)\) for all \(G \geq 0\), or there exists a region \([G_L, G_H]\) such that \(L(G) < R(G)\) for the interior, \(G_L < G < G_H\), and \(L(G_L) = R(G_L)\) and \(L(G_H) = R(G_H)\) at the boundaries. For the latter to be the case, it is sufficient and necessary that at the parallel point \(G_p\), i.e. when \(R' (G_p) = L' (G_p)\), it is the case \(L(G_p) < R(G_p)\). Obviously, \(L' (G) = \beta (1 - \tau_E) \left[\gamma v_0 (1 - \tau^W) (G)^v\right]\) and \(R' (G) = v_0 (G)^v-1\), and solving for \(G_p\) we obtain

\[
G_p = \left[\beta \gamma (1 - \tau_E) \left(1 - \tau^W\right)\right]^{-1}.
\]

Then, the condition \(L(G_p) \leq R(G_p)\), is

\[
\beta (1 - \tau_E) \left[1 + \frac{\gamma v_0 (1 - \tau^W)}{1 + v} \left[\beta \gamma (1 - \tau_E) \left(1 - \tau^W\right)\right]^{-1-v}\right] \leq v_0 \left[\beta \gamma (1 - \tau_E) \left(1 - \tau^W\right)\right]^{-v},
\]

which can be simplified to

\[
\left[\beta (1 - \tau_E)\right]^{1+v} \leq \frac{v_0 \left[\gamma (1 - \tau^W)\right]^{-v}}{1 + v}.
\]

Finally, using the government budget constraint, \(\tau^W = -\frac{(1-\alpha)}{\alpha} \tau_E\), and the condition becomes

\[
\left[\beta (1 - \tau_E)\right]^{1+v} \leq \frac{v_0 \left[\frac{\gamma}{\alpha} (\alpha + (1-\alpha) \tau_E)\right]^{-v}}{1 + v},
\]

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as stated in the statement of the proposition. Under these conditions, the unique equilibrium $G_E$ is equal to the lower boundary $G_L = (0, G_p]$ because it is the only point in which $R(\cdot)$ crosses $L(\cdot)$ from above. If that condition does not hold, then there is no equilibrium. For the case when $\gamma = 0$, $L(\cdot) = \beta (1 - \tau^E)$ is constant and there exists a unique intersection $G_E = \left[\frac{\beta (1 - \tau^E)}{v_0}\right]^\frac{1}{\alpha} > 0$. In all these cases, the condition $v > 1/ (1 - \alpha)$ is required for the second order condition for a maximization to be satisfied. To prove (c) first consider $\gamma = 0$. In this case, $G_E > 1$ if $\frac{\beta (1 - \tau^E)}{v_0} > 1$, i.e. $\tau^E < 1 - \frac{v_0}{\beta}$ as stated in the text. Consider now the case of $\gamma > 0$. Conditions: (i) $L(1) > R(1)$, and (ii) $G_p > 1$ insure that $G_E > 1$. After some easy algebraic simplification, condition (i) becomes $\beta > \frac{v_0}{(1 - \tau^E)} \left[\frac{\alpha}{1 + \gamma v_0 (\alpha + (1 - \alpha)\tau^E)}\right]$; likewise, condition (ii) is simply $\beta < \frac{\alpha}{\gamma(1 - \tau^E)(\alpha + (1 - \alpha)\tau^E)}$. Combining these two inequalities lead to the condition in the text. The proof of (d) is exactly the same as in Monge-Naranjo (2011) for the case of $\tau^E = \tau^W = 0$.

References


