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The Value of Constraints on Discretionary Government Policy

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Abstract

Societies often rely on simple monetary and fiscal rules to restrict the size and behavior of governments. I study the merit of these constraints in a dynamic stochastic model in which fiscal and monetary policies are jointly determined. For all types of shocks considered, the best rule is a limit on the primary deficit. Most welfare gains arise from constraining government behavior during normal times, which to a large extent is sufficient to discipline policy in adverse times. Monetary policy rules are not generally desirable as they severely hinder distortion-smoothing. Debt ceilings are generally benign, but always dominated by deficit rules. For an economy calibrated to the postwar U.S., the optimal rule is a primary surplus of roughly half a percent of output.

Keywords: time-consistency, discretion, government debt, inflation, deficit, fiscal constraints, inflation targeting, institutional design, political frictions.

JEL classification: E52, E58, E61, E62.

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1 Introduction

A perennial debate in the design of political institutions is the trade-off between commitment and flexibility, also commonly referred to as rules versus discretion. At the heart of the issue is a time-consistency problem, that is, the temptation to revise ex ante optimal policy plans.

Allowing policymakers to exercise too much discretion raises the potential for bad policy outcomes, such as, high inflation, large debt accumulation or excessive capital taxation.[1] Unfortunately, forcing policymakers to implement benevolent rules is not straighforward. Ex ante optimal policy plans are oftentimes complicated objects that cannot be easily legislated and require a great deal of foreknowledge of all possible future states of the world. There is virtue in simplicity when binding the behavior of future policymakers; simple, straightforward rules are easy to write down and make non-compliance easy to verify.

Political considerations tend to exacerbate time-inconsistency problems. Policymakers may, for example, be short-sighted due to political turnover, have a desire for "empire-building" or be subjected to clientelism. Thus, even in situations where a benevolent planner would not face strong temptations to revise ex ante optimal plans, there is still a role for constraining the behavior of political actors, which in the end, are the ones putting policy into effect.

Societies have tried to resolve the issues raised above by designing institutions that constrain government policy. There are several illustrative examples of this practice. First, the adoption of economic convergence criteria by prospective members of the European Economic and Monetary Union (the "Eurozone") allowed some countries to impose discipline on their governments by targeting polices more in line with those of strong performing economies.[2] Second, many countries, such as Australia, Canada, New Zealand, Sweden and the U.K., have adopted inflation targets. Although the specific implementation varies somewhat across countries, there is widespread agreement that inflation targets have been successful in keeping inflation low and stable.[3] Third, the U.S. has several formal constraints on fiscal policy. The debt ceiling legislation forces the executive to seek Congressional approval when increasing debt beyond the pre-established limit. In addition, most states are subjected to balanced-budget rules and there have been repeated proposals to impose one at the Federal level. Fourth, perhaps more applicable to developing countries, currency substitution is a simple and effective way to adopt the monetary policy of a more disciplined country.[4] At the moment, there are several countries exclusively using foreign currency; e.g., Ecuador, El Salvador and Panama all use the U.S. dollar.

In practice, however, institutional constraints on government policy may not work as intended. Although membership to the Eurozone was granted conditional on meeting explicit convergence criteria, the reality was that many countries did not meet them (Greece being a notable example as it met none of the criteria upon entry). Recently, around 2014–2015, even core countries such as France were not satisfying European Union deficit targets. In the U.K., inflation was allowed to grow above its target band as a response to the deep recession and elevated unemployment levels that followed the 2007-08 financial crisis. In the U.S., the debt ceiling has arguably done very little to curtail the recent growth of public debt, which has

[1] See Strotz (1956), Kydland and Prescott (1977), Barro and Gordon (1983), Benhabib and Rustichini (1997), Albanesi et al. (2003), Martin (2010), among many others.

[2] These constraints were very effective in terms of inflation, interest rates and deficits. See Martin and Waller (2012).

[3] See Mishkin (1999) and Svensson (1999) for analyses of the international experience with inflation targeting and its comparison to other, less formally institutionalized, monetary policy regimes.

[4] A currency board, such as the one adopted by Argentina (1991-2002), Hong-Kong (since 1983) and Bulgaria (since 1997), is a weaker version of this type of constraint. There are also examples of countries allowing the legal circulation of both domestic and foreign currencies.

reached levels not seen since the end of World War II.

There is a natural tension between the desirability of constraining government behavior in normal and abnormal times. As wise as it may be to discipline policymakers, severe adverse shocks may require some degree of flexibility, in particular, the relaxation or outright abandonment of pre-existing rules. For example, the U.S. government arguably responded in a discretionary manner during the American Civil War and the two World Wars, but it would likely have been detrimental to limit its capacity to issue debt during these episodes.[5] More recently, some countries in the Eurozone have questioned the benefits of delegating monetary policy to a supranational entity that does not internalize regional concerns and pondered the desirability of abandoning the monetary union. In all these cases it is hard to separate the value of flexibility from the gains of political expediency.

In this paper, I provide a systematic study of institutional constraints on government policy. I take the view that governments are naturally discretionary and not fully benevolent, and study the effects of the types of policy constraints that we see implemented in the real world, as enumerated above, i.e., inflation targets, interest rate rules, limits on deficits and debt ceilings. The purpose is to understand the effectiveness and welfare properties of these constraints.

I consider economies subjected to aggregate fluctuations, such as shocks to aggregate demand, public expenditure, productivity and liquidity. The analysis in this paper is guided by several pertinent questions. First, how would a discretionary government behave in such an environment? Second, would placing constraints on the policy response improve welfare? If so, which constraints are more effective? And what are the optimal levels of such constraints? Third, would it be desirable to suspend rules during adverse times or is it better to impose constraints in all states of the world? Fourth, are mistakes costly? That is, what is the welfare cost of not hitting the correct value for a policy constraint? Fifth, how do these results depend on the likelihood, duration and magnitude of shocks?

To provide answers to the questions posed above, I extend the model of fiscal and monetary policy of Martin (2011, 2013). The environment is a monetary economy based on Lagos and Wright (2005), with the addition of a government that uses distortionary taxes, money and nominal bonds to finance the provision of a valued public good.[6] The government is not fully benevolent, preferring higher public expenditure than private agents, and lacks the ability to commit to policy choices beyond the current period. Under full discretion, government policy is determined by the interaction of three main forces: distortion-smoothing, a time-consistency problem and political frictions. The incentive to smooth distortions intertemporally follows the classic arguments in Barro (1979) and Lucas and Stokey (1983). Time-consistency problems arise from the interaction between debt and monetary policy, as analyzed in Martin (2009, 2011, 2013): how much debt the government inherits affects its monetary policy since inflation reduces the real value of nominal liabilities; in turn, the anticipated response of future monetary policy affects the current demand for money and bonds, and thereby how the government today internalizes policy trade-offs. The political friction, creates an upward bias in public expenditure and inflation.

In an economy without uncertainty where the government is non-benevolent, the optimal values for policy constraints are very close to the policies implemented in steady state by a benevolent government (except for the case of debt). For an economy calibrated to the postwar U.S., the best constraint is to impose a minimum primary surplus of 0.8% of output, which yields a welfare gain equivalent to 0.7% of private consumption. Note that the economy without uncertainty is constrained-efficient at the steady state. That is, endowing the government with

[5]See Barro (1979), Ohanian (1997), Aiyagari et al. (2002) and Martin (2012).

[6]Most of the analysis and lessons here would carry over to economies with a cash-in-advance constraint or money-in-the-utility function, although at the cost of lower analytical tractability.

commitment power at the steady state would not affect equilibrium policy. Thus, all the welfare gains come from correcting the political frictions stemming from the non-benevolent nature of the government.

When allowing for aggregate fluctuations several lessons arise. First, imposing a small primary surplus, of about half a percent of output, is always the best policy. Second, inflation targets have small (and sometimes detrimental) welfare effects relative to full discretion. Third, the optimal values for fiscal policy constraints are similar for stochastic and non-stochastic economies. Fourth, most welfare gains come from imposing constraints in normal times. In addition, except for public expenditure shocks, the welfare loss from suspending constraints during bad or abnormal times is minimal. Fifth, mistakes can sometimes be costly. Specifically, picking the wrong inflation target may lead to large welfare losses.

The classical approach in the literature has been to compare the outcomes under full commitment and full discretion. Here, instead, I focus on comparing full discretion with constrained discretionary policy. Related work on fiscal policy constraints includes Brennan and Buchanan (1977), Engineer (1990), Bohn and Inman (1996), Athey et al. (2005), Bassetto and Sargent (2006), Chari and Kehoe (2007), Niepelt (2007), Azzimonti et al. (2016), Halac and Yared (2014) and Hatchondo et al. (2017). Related work on inflation targeting includes Mishkin (1999), Svensson (1999) and Martin (2015).

2 Model

2.1 Environment

Consider an economy populated by a continuum of infinitely-lived agents, which discount the future by factor $\beta \in (0,1)$. Each period, two competitive markets open in sequence, for expositional convenience labeled *day* and *night*. All goods produced in the economy are perishable and cannot be stored from one subperiod to the next.

At the beginning of each period, agents receive an idiosyncratic shock that determines their role in the day market. With probability $\eta \in (0,1)$ an agent wants to consume but cannot produce the day-good x, while with probability $1-\eta$ an agent can produce but does not want consume. A *consumer* derives utility $u(x)$, where u is twice continuously differentiable, satisfies Inada conditions and $u_{xx} < 0 < u_x$. A *producer* incurs in utility cost $\phi > 0$ per unit produced.

Agents are anonymous and lack commitment. Thus, credit arrangements are not feasible and some medium of exchange is necessary for day trade to occur.[7] Exchange media in this economy takes the form of government-issued liabilities: cash and one-period nominal bonds. Cash is universally recognized and can be used in all transactions. Following Kiyotaki and Moore (2002), assume that agents may pledge a fraction $\theta \in [0,1)$ of their government bond holdings to finance day market expenditures.

At night, all agents can produce and consume the night-good, c. The production technology is assumed to be linear in labor, such that n hours worked produce ζn units of output, where $\zeta > 0$. Assuming perfect competition in factor markets, the wage rate is equal to productivity ζ. Utility at night is given by $\gamma U(c) - \alpha n$, where U is twice continuously differentiable, $U_{cc} < 0 < U_c$, $\gamma > 0$ and $\alpha > 0$.

There is a government that supplies a valued public good g at night. Agents derive utility from the public good according to $v(g)$, where v is twice continuously differentiable, satisfies Inada conditions and $v_{gg} < 0 < v_g$. To finance its expenditure, the government may use

[7]See Kocherlakota (1998), Wallace (2001), Shi (2006) and Williamson and Wright (2010), among others.

proportional labor taxes τ, print fiat money at rate μ and issue one-period nominal bonds, which are redeemable in fiat money. Government policy choices for the period are announced at the beginning of each day, before agents' idiosyncratic shocks are realized. The government only actively participates in the night market, i.e., taxes are levied on hours worked at night and open-market operations are conducted in the night market. The public good is transformed one-to-one from the night-good.

Let $s \equiv \{\gamma, \zeta, \theta, \omega\}$ denote the exogenous aggregate state of the economy, which is revealed to all agents at the beginning of each period. The economy is thus subject to a variety of aggregate shocks: demand (γ), productivity (ζ), liquidity (θ) and government type (ω)—the role played by this last parameter will be explained below. The set of all possible realizations for the stochastic state is S. Let $E[s'|s]$ be the expected value of future state s' given current state s.

All nominal variables—except for bond prices—are normalized by the aggregate money stock. Thus, today's aggregate money supply is equal to 1 and tomorrow's is $1+\mu$. The government budget constraint can be written as

$$p_c(\tau\zeta n - g) + (1+\mu)(1+qB') - (1+B) = 0, \tag{1}$$

where B is the current aggregate bond-money ratio, p_c is the—normalized—market price of the night-good c, and q is the price of a bond that earns one unit of fiat money in the following night market. "Primes" denote variables evaluated in the following period. Thus, B' is tomorrow's aggregate bond-money ratio. In equilibrium, prices and policy variables depend on the aggregate state (B, s); this dependence is omitted from the notation to simplify exposition.

2.2 Problem of the agent

Let $V(m, b, B, s)$ be the value of entering the day market with (normalized) money balances m and bond balances b, when the aggregate state of the economy is (B, s). Upon entering the night market, the composition of an agent's nominal portfolio (money and bonds) is irrelevant, since bonds are redeemed in fiat money at par. Thus, let $W(z, B, s)$ be the value of entering the night market with total (normalized) nominal balances z.

In the day market, consumers and producers exchange money and bonds for goods at (normalized) price p_x. Let x be the individual quantity consumed and κ the individual quantity produced; these quantities are generally different in equilibrium, unless there is an equal measure of consumers and producers. A consumer with starting balances (m, b) has total liquidity $m + \theta b$ to purchase day output. The problem of a consumer is

$$V^c(m, b, B, s) = \max_x \; u(x) + W(m + b - p_x x, B, s)$$

subject to: $p_x x \leq m + \theta b$. The problem of a producer is

$$V^p(m, b, B, s) = \max_\kappa \; -\phi\kappa + W(m + b + p_x\kappa, B, s).$$

The *ex ante* value of an agent at the start of the period satisfies

$$V(m, b, B, s) \equiv \eta V^c(m, b, B, s) + (1-\eta)V^p(m, b, B, s).$$

The problem of an agent at night, arriving with net nominal balances z is

$$W(z, B, s) = \max_{c,n,m',b'} \; \gamma U(c) - \alpha n + v(g) + \beta E[V(m', b', B', s')|s]$$

subject to: $p_c c + (1+\mu)(m' + qb') = p_c(1-\tau)\zeta n + z$.

2.3 Monetary equilibrium

The resource constraints in the day and night equate total consumption to total production in each subperiod. The resource constraint in the day is $\eta x = (1-\eta)\kappa$. Given the assumptions on preferences, individual consumption at night is the same for all agents, whereas individual labor depends on whether an agent was a consumer or a producer in the day. Hence, the resource constraint at night is given by $c + g = \zeta[\eta n^c + (1-\eta)n^p]$, where n^c and n^p denote night-labor by agents that were consumers or producers in the day, respectively. As shown in Lagos and Wright (2005), the preference specification also implies that all agents make the same portfolio choice. Market clearing at night implies $m' = 1$ and $b' = B'$.

The literature on government optimal policy with distortionary instruments typically adopts what is known as the *primal approach*, which consists of using the first-order conditions of the agent's problem to substitute prices and policy instruments for allocations in the government budget constraint. Following this approach, the problem of a government with limited commitment can be written in terms of choosing debt and allocations. After some work (see Appendix A), we get the following conditions characterizing prices (p_x, p_c, q) and policy instruments (μ, τ) in a monetary equilibrium:

$$p_x = \frac{(1+\theta B)}{x} \tag{2}$$

$$p_c = \frac{\gamma U_c(1+\theta B)}{\phi x} \tag{3}$$

$$q = \frac{E[\frac{x'(\eta\theta' u'_x + (1-\eta\theta')\phi)}{1+\theta' B'}|s]}{E[\frac{x'(\eta u'_x + (1-\eta)\phi)}{1+\theta' B'}|s]}, \tag{4}$$

$$\mu = \frac{\beta(1+\theta B)}{\phi x} E\left[\frac{x'(\eta u'_x + (1-\eta)\phi)}{(1+\theta' B')}\Big| s\right] - 1 \tag{5}$$

$$\tau = 1 - \frac{\alpha}{\zeta\gamma U_c} \tag{6}$$

Condition (5) states that, for a given expected future day-good allocation (which in equilibrium is a function of debt choice, B' and the exogenous state s'), a higher money growth rate μ implies lower day-good consumption x. In other words, given current debt policy and future monetary policy, the allocation of the day-good is a function of *current* monetary policy. Thus, we can interchangeably refer to variations in the day-good allocation and variations in current monetary policy. Similarly, from (6) a higher tax rate τ is equivalent to lower night consumption c.

Using these conditions, we can write the government budget constraint (1) in a monetary equilibrium as

$$(\gamma U_c - \alpha/\zeta)\, c - (\alpha/\zeta) g - \frac{\phi x(1+B)}{1+\theta B} + \beta E\left[\frac{\phi x'(1+B')}{1+\theta' B'}\Big| s\right] + \beta\eta E[x'(u'_x - \phi)|s] = 0 \tag{7}$$

for all $s \in S$. Condition (7) is also known as an implementability constraint, as it restricts the set of allocations that a government can implement in a monetary equilibrium.

3 Discretionary Policy

3.1 Problem of the government

Assume the government can commit to policy announcements for the current period, but cannot commit to policies implemented in future periods. That is, at the beginning of the period, the current government chooses $\{B', \mu, \tau, g\}$—equivalently, implements $\{B', x, c, g\}$—taking as given expected future policy. Policies implemented by the government in the future affect its current budget constraint. This is reflected by the presence of the future allocation x' in the government budget (implementability) constraint (7), due to the fact that future monetary policy affects the current demand for money and bonds. Due to limited commitment, the current government cannot directly control future policy, even though it can affect future policy through its choice of debt. Future allocations depend on the policy expected to be implemented by the government, which in turn, depends on the level of debt it inherits and the exogenous aggregate state of the economy. Let $x' = \mathcal{X}(B', s')$ be the policy that the current government anticipates will be implemented by future governments. The function $\mathcal{X}$ is an equilibrium object, but the current government takes it as given.

From the day resource constraint, we can write production in equilibrium as a function of consumption: $\kappa = \eta x/(1-\eta)$. Thus, an agent's expected flow utility in the day is equal to $\eta[u(x) - x]$. Night output is equal to the consumption of private and public goods and so, we can use the night resource constraint to write expected night labor as $(c+g)/\zeta$. The *ex ante* period utility of an agent can be thus written in terms of the bundle (x, c, g) and the aggregate state of the economy s. Let $\mathcal{U}(x, c, g, s) \equiv \eta[u(x) - \phi x] + \gamma U(c) - \alpha(c+g)/\zeta + v(g)$.

As described in the introduction, the analysis presumes the government is not benevolent. Following Martin (2015), suppose the government values the utility of its subjects, but may value public expenditure differently: its flow utility is given by $\mathcal{U}(x, c, g, s) + \mathcal{R}(g, \omega)$, where $\mathcal{R}$ is increasing in public expenditure, g and decreasing in the level of government benevolence, $\omega > 0$. Let $R(g, 1) = 0$, so that $\omega = 1$ indicates the government is benevolent. When $\omega \in (0, 1)$, which is the focus here, the government prefers larger public expenditure than private agents. This expenditure bias may arise from a variety of sources: a desire for empire-building, the spoils of patronage and clientelism, the existence of a self-serving public bureaucracy or the support of the sovereign's lifestyle. Critical to the analysis below is that private agents would prefer the government to spend less, but cannot directly control nor limit this choice.

Let $\Gamma \equiv [\underline{B}, \overline{B}]$] be the set of possible debt levels, where $\underline{B} < \overline{B}$ are such that they do not constrain government choices. Taking as given future government policy $\{\mathcal{B}, \mathcal{X}, \mathcal{C}, \mathcal{G}\}$ the problem of the current government can be written as

$$\max_{B', x, c, g} \; \mathcal{U}(x, c, g, s) + \mathcal{R}(g, \omega) \; + \beta E[\mathcal{V}(B', s')|s]$$

subject to (7) and given

$$\mathcal{V}(B', s') \equiv \mathcal{U}(\mathcal{X}(B', s'), \mathcal{C}(B', s'), \mathcal{G}(B', s'), s') + \mathcal{R}(\mathcal{G}(B', s'), \omega') + \beta E[\mathcal{V}(\mathcal{B}(B', s'), s'')|s'].$$

We can now define an equilibrium in this economy.

Definition 1 *A Markov-Perfect Monetary Equilibrium (MPME) is a set of functions* $\{\mathcal{B}, \mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{V}\}$: $\Gamma \times S \to \Gamma \times \mathbb{R}^3_+ \times \mathbb{R}$, *such that for all* $B \in \Gamma$ *and all* $s \in S$*:*

$$\{\mathcal{B}(B, s), \mathcal{X}(B, s), \mathcal{C}(B, s), \mathcal{G}(B, s)\} = \underset{B', x, c, g}{\operatorname{argmax}} \; \mathcal{U}(x, c, g, s) + \mathcal{R}(g, \omega) \; + \beta E[\mathcal{V}(B', s')|s]$$

subject to

$$(\gamma U_c - \alpha/\zeta)\, c - (\alpha/\zeta) g - \frac{\phi x(1+B)}{1+\theta B} + \beta E\left[\frac{\phi x'(1+B')}{1+\theta' B'}\Big| s\right] + \beta\eta E[x'(u'_x - \phi)|s] = 0$$

where $x' \equiv \mathcal{X}(B', s')$ *and where*

$$\mathcal{V}(B, s) \equiv \mathcal{U}(\mathcal{X}(B, s), \mathcal{C}(B, s), \mathcal{G}(B, s), s) + \mathcal{R}(\mathcal{G}(B, s), \omega) + \beta E[\mathcal{V}(\mathcal{B}(B, s), s')|s].$$

With Lagrange multiplier λ associated with the government budget constraint and multiplier function $\Lambda(B,s)$ associated with future policy $\{\mathcal{B},\mathcal{X},\mathcal{C},\mathcal{G}\}$, the first-order conditions of the government's problem imply:

$$
\begin{aligned}
E\left[\frac{\phi x'(1-\theta')(\lambda-\Lambda')}{(1+\theta' B')^2}\Big| s\right]+\lambda E\left[\mathcal{X}'_B\left\{\eta(u'_x+u'_{xx}x'-\phi)+\frac{\phi(1+B')}{1+\theta' B'}\right\}\Big| s\right] &= 0 \quad (8)\\
\eta(u_x-\phi)-\frac{\lambda(1+B)}{1+\theta B} &= 0 \quad (9)\\
\gamma U_c-\alpha/\zeta+\lambda\left\{\gamma U_c-\alpha/\zeta+\gamma U_{cc}c)\right\} &= 0 \quad (10)\\
v_g-\alpha/\zeta+\mathcal{R}_g-\lambda(\alpha/\zeta) &= 0 \quad (11)
\end{aligned}
$$

for all $B\in\Gamma$ and all $s\in S$. See Martin (2011) for an extended analysis of these conditions in the non-stochastic case. A *differentiable* MPME is a set of differentiable (a.e.) functions $\{\mathcal{B},\mathcal{X},\mathcal{C},\mathcal{G},\Lambda\}$ that solve (7)–(11) for all (B,s).

As shown in Martin (2011, 2015) the non-stochastic version of this economy features the property that the steady state of the Markov-perfect equilibrium is constrained-efficient. Thus, endowing the government with commitment at the steady state would not affect the allocation. The result is summarized in the following proposition.

Proposition 1 *Assume $S=\{s^*\}$ and initial debt equal to $B^*\equiv\mathcal{B}(B^*,s^*)$. Then, a government with commitment and a government without commitment will both implement the allocation $\{x^*,c^*,g^*\}$ and choose debt level B^* in every period.*

Proof. After a substitution of variables $1+\hat{B}\equiv(1+B)/(1+\theta B)$, the proof proceeds exactly as in Martin (2015). ■

In the absence of aggregate fluctuations, private agents cannot be made better-off at the steady state, by endowing the government with more commitment power. The only long-run inefficiency in this economy stems from the political friction (i.e., the misalignment in preferences between agents and government). Outside the steady state or in the presence of aggregate fluctuations, government policy will exhibit inefficiencies due to both a time-consistency problem and the political friction.

4 Constrained Government Policy

Though private agents cannot dictate the government how much to spend, they may be able to regulate other components of the budget. In order to place constraints on government policy we first need to define some relevant macroeconomic variables: GDP, actual and expected inflation, primary and total deficits, the nominal interest rate and debt over GDP.

4.1 Accounting

Let us start by computing nominal GDP. Day and night output are equal to ηx_t and c_t+g_t, respectively. Then, nominal output is defined as $Y_t\equiv p_{x,t}\eta x_t+p_{c,t}(c_t+g_t)$, which using (2) and (3) implies

$$Y_t=\frac{(1+\theta_t B_t)[\eta\phi x_t+\gamma_t U_{c,t}(c_t+g_t)]}{\phi x_t}. \tag{12}$$

Nominal GDP, like other nominal variables, is normalized by the aggregate money stock.

Let $\varsigma_{x,t}$ and $\varsigma_{c,t}$ be the day-good and night-good expenditure shares, respectively. Thus, $\varsigma_{x,t}\equiv p_{x,t}\eta x_t/Y_t=(1+1/\Upsilon_t)^{-1}$ and $\varsigma_{c,t}\equiv p_{c,t}(c_t+g_t)/Y_t=(1+\Upsilon_t)^{-1}$, where $\Upsilon_t\equiv\eta\phi x_t[\gamma_t U_{c,t}(c_t+$

$g_t)]^{-1}$. Fixing expenditure shares to those corresponding to the non-stochastic steady state (B^*, x^*, c^*, g^*), yields ς_x^* and ς_c^*. The price level can then be defined as: $P_t \equiv \varsigma_x^* p_{x,t} + \varsigma_c^* p_{c,t}$. Using (2) and (3) we obtain

$$P_t = \frac{(1+\theta_t B_t)(\varsigma_x^*\phi + \varsigma_c^*\gamma_t U_{c,t})}{\phi x_t}. \tag{13}$$

Thus, we can define inflation as $\pi_t \equiv P_t(1+\mu_{t-1})/P_{t-1} - 1$ and expected inflation as $\pi_{t+1}^e \equiv E_t[P_{t+1}](1+\mu_t)/P_t - 1$. Using (5) and (13) we get

$$\pi_{t+1}^e = \beta E_t\left[\frac{(1+\theta_{t+1}B_{t+1})(\varsigma_x^*\phi + \varsigma_c^*\gamma_{t+1}U_{c,t+1})}{\phi x_{t+1}(\varsigma_x^*\phi + \varsigma_c^*\gamma_t U_{c,t})}\right] E_t\left[\frac{x_{t+1}(\eta u_{x,t+1} + (1-\eta)\phi)}{(1+\theta_{t+1}B_{t+1})}\right] - 1. \tag{14}$$

The nominal interest rate is defined as $i_t \equiv q_t^{-1} - 1$. Hence, from (4):

$$i_t = \frac{E_t[\frac{x_{t+1}(\eta u_{x,t+1}+(1-\eta)\phi)}{1+\theta_{t+1}B_{t+1}}]}{E_t[\frac{x_{t+1}(\eta\theta_{t+1}u_{x,t+1}+(1-\eta\theta_{t+1})\phi)}{1+\theta_{t+1}B_{t+1}}]} - 1. \tag{15}$$

The primary deficit is the difference between government expenditure before interest payments and tax revenue. The primary deficit over GDP is defined as: $d_t \equiv p_{c,t}(g_t - \tau_t\zeta_t n_t)/Y_t$. Using (3), (6) and (12) we obtain

$$d_t = \frac{(\alpha/\zeta_t)(c_t + g_t) - \gamma_t U_{c,t}c_t}{\eta\phi x_t + \gamma_t U_{c,t}(c_t + g_t)}. \tag{16}$$

The total fiscal deficit includes the primary deficit plus interest payments on the debt. The deficit over GDP is defined as: $D_t \equiv d_t + \frac{(1+\mu_t)(1-q_t)B_{t+1}}{Y_t}$. Using (5), (4), (12) and (16) we get

$$D_t = \frac{(\alpha/\zeta_t)(c_t + g_t) - \gamma_t U_{c,t}c_t + \beta\eta B_{t+1}E_t[\frac{(1-\theta_{t+1})x_{t+1}(u_{x,t+1}-\phi)}{(1+\theta_{t+1}B_{t+1})}]}{\eta\phi x_t + \gamma_t U_{c,t}(c_t + g_t)}. \tag{17}$$

Debt is measured at the end of the period, as in the data. Thus, debt-over-GDP is defined as

$$\frac{(1+\mu_t)B_{t+1}}{Y_t} = \frac{\beta B_{t+1}}{\eta\phi x_t + \gamma_t U_{c,t}(c_t + g_t)} E_t\left[\frac{x_{t+1}(\eta u_{x,t+1} + (1-\eta)\phi)}{(1+\theta_{t+1}B_{t+1})}\right] \tag{18}$$

4.2 Policy constraints

The constraints on government actions studied in this paper can be categorized in three groups, depending on which policy variable they target: monetary policy, deficit and debt.

Consider first constraints on monetary policy. An inflation target restricts a government to implement policy so that expected inflation is within a given interval, that is, $\pi_{t+1}^e \in [\underline{\pi}, \bar{\pi}]$, where π_{t+1}^e is defined in (14). Similarly, an interest rate rule restricts policy to be consistent with the nominal interest rate fluctuating within a given interval, that is, $i_t \in [\underline{i}, \bar{i}]$, where i_t is defined in (15). For the purpose of the exercises in this paper, I will focus on strict rules: an inflation target, $\underline{\pi} = \bar{\pi}$ and an interest rate peg, $\underline{i} = \bar{i}$.[8]

A Taylor rule, as first proposed by Taylor (1993), can be thought of another type of constraint on monetary policy. There has been a recent push in the U.S. to legislate such a rule, so it is of

[8]Allowing for small intervals around a monetary target did not seem to have any measurable impact on the results. Ceilings or floors proved to be worse than strict rules and hence omitted from the exercises presented here.

interest to consider its effect in the context of this paper. Consider the following forward-looking variant of the Taylor rule:

$$1 + i_t = (1 + r^*)(1 + \pi^e_{t+1})^{\varphi}(1 + \pi^T)^{-\varphi} \tag{19}$$

where i_t is the nominal interest rate given by (15), π^e_{t+1} is the expected inflation rate given by (14), π^T is the desired target for inflation, r^* is the real interest rate in a non-stochastic steady state (standing-in here for the "natural real rate") and $\varphi > 0$. By the Fisher equation, $1 + i_t \equiv (1 + \pi^e_{t+1})(1 + r^e_t)$. Using (14) and (15) and imposing a steady state, we obtain

$$r^* = \frac{\phi}{\beta[\eta\theta^* u^*_x + (1 - \eta\theta^*)\phi]} - 1. \tag{20}$$

Note that if government bonds are illiquid, $\theta^* = 0$, then $r^* = \beta^{-1} - 1$. If monetary policy follows a Taylor rule, then the government is constrained to implement expected inflation and the nominal interest rate consistent with (19). If we take the parameter φ as given (say, at the standard value of 1.5 used in the literature), then the choice of the optimal Taylor-rule constraint involves a choice of π^T.

Constraints on deficits and debt are inequalities relative to an upper bound. Consider ceilings on the primary deficit $\bar{d}$ and the total deficit $\bar{D}$, both in terms of GDP, which take the form $d_t \leq \bar{d}$, $D_t \leq \bar{D}$, respectively, where d_t and D_t are as defined in (16) and (17). There are two types of debt constraints: an upper limit on debt over GDP and a ceiling on the nominal value of outstanding debt. That is, constraints of the form: $(1 + \mu_t)B_{t+1}/Y_t \leq \bar{b}$ and $B_{t+1} \leq \bar{B}$. The former is akin to the Maastricht convergence criteria on debt, while the latter is similar to the debt ceiling imposed by the U.S. Congress on the federal government. Note that even though B is the bond-money ratio, the latter constraint should be interpreted as a limit on the nominal stock of debt.

Importantly, different types of policy constraints differ in how they restrict government actions. Compare, for example, an inflation target with a primary deficit ceiling: as we can see from (14) and (16), both constraints depend on $\{x_t, c_t, g_t, s_t\}$, but the inflation target also depends on the choice of debt, B_{t+1} and the expected realization of future allocations, $\{x_{t+1}, c_{t+1}\}$. Thus, the inflation target is intrinsically dynamic while the primary deficit ceiling is static. The debt ceiling, on the other hand, simply restricts the maximum amount of debt that can be accumulated, regardless of allocations. Note that none of the policy constraints depend directly on the inherited level of debt, B_t. However, they all interact with inherited debt through the budget constraint, (7).

Constraints can be imposed on all exogenous states of the world or on select ones. For example, it may be undesirable to restrict government behavior when output is low (say, during a deep recession). Alternatively, this may be precisely the time when government behavior *ought* to be restricted. I will consider all these possible cases in the analysis below.

Let us now include the policy constraints in the recursive formulation of the government's problem. Let $i = \{1, \ldots, 7\}$ index the type of constraint, where $i = 1$ corresponds to an inflation target, $i = 2$ to an interest rate peg, $i = 3$ to a Taylor rule, $i = 4$ to a primary deficit ceiling, $i = 5$ to a total deficit ceiling, $i = 6$ to a debt over GDP ceiling and $i = 7$ to a nominal debt limit. The indicator function $\mathcal{I}^i(s)$ states whether a constraint of type i is in effect in state s. Constraints on policy can be written then as

$$\mathcal{I}^i(s)\Omega^i(B', x, c, g, s; \mathcal{X}, \mathcal{C}) \geq 0 \quad (= 0 \text{ if } i = \{1, 2, 3\}) \tag{21}$$

The function Ω^i corresponds to each of the constraints described above. In general, constraints depend on the current policy choice (B', x, c, g), the current state s and expected future policy choices $\mathcal{X}(B', s')$ and $\mathcal{C}(B', s')$. Some constraints depend trivially on some of the arguments. For example, an interest rate target ($i = 2$) only depends non-trivially on B', s and $\mathcal{X}(B', s')$, while a nominal debt ceiling only depends non-trivially on B'. The problem of the government and the definition of a MPME can be written similarly to the unconstrained case, but with the addition of (21) for all $B \in \Gamma$ and $s \in S$.

4.3 Are primary deficit constraints special?

As anticipated in the introduction, the quantitative exercises conducted below will all show that constraining the primary deficit yields superior welfare to private agents than all other types of constrains. This begs the question: is there anything qualitatively intrinsic to primary deficit constraints? The answer is yes and is related to the static nature of this constraint, as described above.

Primary deficit constraints are the only one in the set of policy constraints considered here that do not depend on B', either directly or indirectly through future polices $\mathcal{X}$ or $\mathcal{C}$. Why is this important? Lack of dependence on B' implies that the Generalized Euler Equation (8) is left functionally intact after adding constraint (21) to the government's problem. This is not the case for any of the other policy constraints. Condition (8) determines how the government is trading off distortion smoothing—the first term in (8)—with the time-consistency problem—the second term in (8). If (21) depends non-trivially on B', then the trade-off in (8) will be upset. In other words, the imposition of a policy constraint, though beneficial as a disciplining device, will hinder the government's ability to smooth distortions intertemporally, which carries an important welfare cost to agents.

A ceiling on the primary deficit will only affect equations (9)–(11). The added term in each condition is the Lagrange multiplier associated with constraint (21) times the marginal change of the constraint with respect to the relevant variable (x, c, or g, respectively). Hence, this type of constraint will alter the way the government views the static trade-offs between monetary policy, taxation and expenditure. The key to an effective constraint is to make the government internalize the cost of excessive spending by making the static distortions more costly, as captured by the additional terms in (9)–(11). Constraints on the the primary deficit achieve this without affecting the dynamic policy trade-offs in (8).

4.4 Alternative implementations

As described above, constraints on government policy are imposed in terms of observable policy variables (inflation, deficit, etc). Alternatively, one could restrict government actions by placing constraint on allocations. For example, in an economy without aggregate shocks, a specific inflation rate could be achieved by imposing a particular day-good allocation x. Suppose we restrict the government to conduct policy such that it implements $x = x^T$. Since now $\mathcal{X}_B = 0$, the system (7)–(11) implies $B = B'$ (since now $\lambda = \Lambda'$—see Martin, 2015 for derivation details) and hence, from (14) we get $1 + \pi^e_{t+1} = \beta(\eta u^T_x/\phi + 1 - \eta) \equiv 1 + \pi^T$. In other words, there is a one-to-one mapping between x^T and π^T. A constraint $x = x^T$ *and* the assumption that future governments also set $x' = x^T$ is a very different object than constraint (14), which in the non-stochastic case takes the form: $1+\pi_{t+1} = \frac{\beta(\varsigma^*_x\phi+\varsigma^*_c U_{c,t+1})(\eta u_{x,t+1}/\phi+1-\eta)}{\varsigma^*_x\phi+\varsigma^*_c U_{c,t}}$. This constraint depends on the current choice for c_t, but also on x_{t+1} and c_{t+1} which the current government cannot directly control. Since in equilibrium, $x_{t+1} = \mathcal{X}(B_{t+1})$ and $c_{t+1} = \mathcal{C}(B_{t+1})$, the way a government facing constraint (14) can comply is by appropriately choosing B_{t+1} and c_t (i.e., debt and taxes), rather than directly through the implementation of a day-good allocation x_t.

Note that both of the approaches described above would successfully implement the same inflation target. The sources of the different implementation are the limited commitment friction and the presence of debt, which is an endogenous state variable. The current government is best-responding to future policy. In turn, future policy may itself be restricted, due to a policy constraint; but the shape of the policy function is an equilibrium outcome and not a constraint on the current government.

The above example has important implications. In general, similar policy outcomes could in principle be achieved by placing restrictions on allocations. This approach could even be desirable from a welfare perspective.[9] Consistent with the objective of evaluating real-world constraints, and to keep the paper focused, I here take the stand that policy constraints take the form of constraints on observed policy variables instead of allocations. The fact that we can write these problems in terms of allocations rather than policy variables is solely for analytical convenience.

5 Optimal constraints in non-stochastic economies

I first explore the optimality of policy constraints in the absence of shocks.

5.1 Calibration

Consider the following functional forms: $u(x) = \frac{x^{1-\sigma}-1}{1-\sigma}$; $U(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$; $v(g) = \ln g$; and $\mathcal{R}(g,\omega) = (\omega^{-1} - 1)g$. The parameter $\omega > 0$ determines the degree of benevolence of the government, where $\omega = 1$ means the government is fully benevolent.

The non-stochastic version of the economy is calibrated to the post-war, pre-Great Recession U.S., 1955–2008. Government in the model corresponds to the federal government and period length is set to a fiscal year. The variables targeted in the calibration are: debt over GDP, inflation, nominal interest rate, outlays (not including interest payments) over GDP and revenues over GDP. All variables are taken from the Congressional Budget Office. Government debt is defined as debt held by the public, excluding holdings by the Federal Reserve system.

Calibrating the extent of political frictions is more challenging. In principle, one would like to have an estimate of the socially optimal level of government expenditure. Such an estimate is of course hard, if not impossible, to come by. Instead, I use an indirect approach by assuming that a benevolent government would set the long-run inflation rate at 2% annual, which corresponds to the explicit target adopted by the Federal Reserve since 2012 (and implicitly before then) and by most inflation-targeting central banks around the world. Thus, the set of calibrated parameters need to hit two economies simultaneously: one targeting the actual U.S. economy and another one which shares all the same parameter values, except for $\omega = 1$, and that implements 2% inflation in steady state. For robustness, I also study how the results change when we vary the degree of government benevolence.

Tables 1 and 2 present the benchmark parameterization and target statistics, respectively. As we can see, expenditure over GDP in the benevolent economy is about 3 percentage points lower than in the calibrated economy: 14.8% of GDP vs 18.0%. The size of the benevolent government would thus be similar to the actual U.S. federal government around the mid-1950s.

[9] One can show numerically that this is indeed the case for an inflation target.

Table 1: Benchmark Calibration

α	β	σ	η	ϕ	θ	ω
8.9790	0.9452	3.7009	0.3776	3.7617	0.3747	0.3400

Normalized parameters: $\gamma = \zeta = 1$.

Table 2: Non-stochastic steady state statistics

Variable	Statistic	Benchmark	Benevolent	Big Government	Low Inflation
Debt over GDP	$\frac{B(1+\mu)}{Y}$	0.325	0.319	0.330	0.325
Inflation rate	π	0.036	0.020	0.052	0.020
Nominal interest rate	i	0.058	0.048	0.068	0.040
Revenue over GDP	$\frac{p_c \tau n}{Y}$	0.180	0.152	0.206	0.180
Expenditure over GDP	$\frac{p_c g}{Y}$	0.180	0.148	0.210	0.180

Note: "benevolent" refers to an economy with $\omega = 1$*; "big government" assumes* $\omega = 0.235$*; "low inflation" recalibrates to target* 2% *annual inflation—see Table 5 for parameter values.*

5.2 Optimal policy constraints

Table 3 presents the optimal values of each policy constraint for the case of the non-stochastic economy, as calibrated in the previous section. The values are compared to the steady state statistics of the calibrated and benevolent economies. Recall that the steady state is constrained-efficient, so all the welfare gains come from how well policy constraints "starve the beast", i.e., mitigate the government's expenditure bias.[10]

Table 3: Optimal constraints in non-stochastic economy—benchmark

Constraint	Formula	Optimal Constraint	Welfare Gain
Inflation target	$\pi_{t+1} = \bar{\pi}$	0.040	0.0%
Interest rate peg	$i_t = \bar{i}$	0.055	0.2%
Taylor rule	$(1+i_t) = (1+r^*)(\frac{1+\pi_{t+1}}{1+\pi^T})^{\varphi}$	0.051	0.1%
Primary deficit	$\frac{p_{c,t}(g_t - \tau_t n_t)}{Y_t} \leq \bar{d}$	−0.006	0.9%
Deficit	$\frac{p_{c,t}(g_t - \tau_t n_t) + (1+\mu_t)(1-q_t)B_{t+1}}{Y_t} \leq \bar{D}$	0.005	0.6%
Debt over GDP	$\frac{(1+\mu_t)B_{t+1}}{Y_t} \leq \bar{b}$	0.274	0.2%
Debt ceiling	$B_{t+1} \leq \bar{B}$	0.238	0.3%

Note: The "debt ceiling" $\bar{B}$ *is reported as a fraction of* Y^*.

Each type of constraint is imposed on all levels of debt. The optimal value for a constraint is picked by evaluating welfare at the steady state of the non-stochastic fully discretionary economy and includes the full transition to the new steady state. Welfare is expressed in terms of equivalent compensation, measured in units of night-good consumption. Formally, welfare is

[10] Although the phrase "starving the beast" may commonly be interpreted as limiting spending by cutting taxes, I use it here in a more general sense to include any mechanism that constraints government actions to curb spending.

measured as the proportion Δ that solves

$$\eta[u(x^*) - \phi x^*] + U(c^*(1+\Delta)) + v(g^*) - \alpha(c^* + g^*) + \beta V(B^*) = \bar{V}(B^*)$$

where $\{b^*, x^*, c^*, g^*\}$ is the fully discretionary steady state, with associated agent's value function $V(B)$, and $\bar{V}(B)$ corresponds to the agent's value function in a Markov-perfect equilibrium associated with a particular policy constraint. Given the assumptions on functional forms, the equivalent compensation has a closed-form solution: $\Delta = \{(1-\sigma)[\bar{V}(B^*) - V(B^*)]\}^{1/(1-\sigma)} - 1$.

As Table 3 shows, all types of constraints are effective in increasing agents' welfare. Gains range from a maximum of 0.9% for the case of a primary deficit ceiling to a minimum of essentially zero for the case of an inflation target. Deficit rules are the most effective. Debt ceilings appear less desirable than deficit rules, a characteristic that will recur throughout the exercises conducted below.

5.3 Big government

Consider now the case of an economy with an even less benevolent government. Table 2 shows the steady state statistics of an economy with $\omega = 0.235$. In this case, public expenditure over GDP is 21%, i.e., 3 percentage points higher than the calibrated economy and 6 percentage higher than the benevolent economy. As a result, inflation, deficits and debt are all higher. Table 4 presents the optimal constraint for this economy.

Table 4: Optimal constraints in non-stochastic economy—alternative calibrations

Constraint	Big Government		Low Inflation	
	Optimal Constraint	Welfare Gain	Optimal Constraint	Welfare Gain
Inflation target	0.060	0.1%	0.024	0.0%
Interest rate peg	0.060	1.0%	0.037	0.4%
Taylor rule	0.088	0.7%	0.032	0.2%
Primary deficit	−0.013	5.9%	−0.006	0.7%
Deficit	−0.007	3.7%	0.002	0.5%
Debt over GDP	0.260	0.6%	0.274	0.5%
Debt ceiling	0.215	1.2%	0.244	0.5%

Note: The "debt ceiling" $\bar{B}$ is reported as a fraction of Y^*.

When compared to the benchmark results in Table 3, the optimal constraints are all stricter when facing a less benevolent (i.e., bigger) government. Welfare gains for all types of constraints increase by about an order of magnitude. Notably, the welfare ranking of constraints remains the same; the best prescription is still to run a primary surplus, about 1.3% of GDP in this case.

5.4 Low inflation

Arguably, inflation is the one policy variable that in the last two decades looks significantly different from the postwar average.[11] In order to account for this and perhaps deliver recommendations more applicable to the current economy, I consider an alternative calibration that

[11] Debt is currently also far from the postwar average, but did not look significantly different right before the most recent recession.

delivers a steady state inflation of 2% annual. Parameters are presented in Table 5 while the steady state statistics are presented in Table 2, together with the previous economies. The degree of government benevolence, ω is set so that with the "low inflation" parameterization, the benevolent government (the one with $\omega = 1$) has the same expenditure over GDP as in the benchmark economy. As we can see in Table 2, inflation and the nominal interest rate are lower, consistent with the new targets, but fiscal variables are the same as in the benchmark economy.

Table 5: Low Inflation Calibration

α	β	σ	η	ϕ	θ	ω
7.4042	0.9615	5.2009	0.2432	4.8780	0.3289	0.3900

Normalized parameters: $\gamma = \zeta = 1$.

The optimal constraints for the low inflation economy are presented in the last two columns of Table 4. The lessons from the benchmark economy apply to the low inflation economy: the best constraint is to impose a primary surplus (also 0.6% of GDP in this case); deficit and debt constraints are superior to monetary policy constraints; and the best monetary policy constraint is an interest rate peg. One difference from the benchmark case is that the welfare gains from imposing fiscal constraints seem more compressed in the low inflation economy, although the primary deficit ceiling still delivers significantly higher gains than the alternatives.

6 Optimal constraints in stochastic economies

6.1 Numerical approach

The exogenous state of the economy is given by the values of parameters $\{\gamma, \zeta, \theta, \omega\}$. To keep the analysis as transparent as possible and draw useful lessons, I consider economies with one type of shock at a time. Each economy has three exogenous states, $S = \{s_1, s_2, s_3\}$. Let ϖ_{ij} be the probability of going from state s_i today to state s_j tomorrow. I will interpret s_2 as "normal" times, similar to where the economy lies in the non-stochastic version of the economy. The state s_1 corresponds to "bad" times and s_3 ("good" times) is included for symmetry and so that the stochastic economy fluctuates around the calibrated non-stochastic steady state. The label "bad" refers to states of the world that feature what are generally deemed undesirable macroeconomic outcomes: low aggregate demand, high public expenditure, low average productivity and low real interest rate.

The transition matrix is characterized by two values ϖ and ϖ^* such that $\varpi_{1,1} = \varpi_{33} = \varpi$, $\varpi_{1,2} = \varpi_{3,1} = 1-\varpi$, $\varpi_{13} = \varpi_{3,1} = 0$, $\varpi_{22} = \varpi^*$ and $\varpi_{2,1} = \varpi_{2,3} = (1-\varpi^*)/2$. In other words, ϖ^* is the probability of remaining in the normal state of the world, with an equal chance of transitioning to bad times (s_1) or good times (s_3). During bad (good) times there is a chance $1-\varpi$ of transitioning back to normal times and it is not possible to immediately transition to the good (bad) state.

For the numerical simulations, I will assume $\varpi^* = 0.98$ and $\varpi = 0.90$. That is, normal times last on average 50 years and bad (good) times have an expected duration of 10 years. I will also consider more frequent abnormal times, to verify the robustness of results. For each economy, the corresponding parameter in states s_1 and s_3 is a multiple of the parameter in state s_2, which is equal to the calibrated parameter from Table 1. The parameterization is shown on Table 6.

Table 6: Stochastic economy parameterization

Economy	s_1	s_2	s_3
Demand shock	$\gamma(1-\varrho_\gamma)$	γ	$\gamma(1+\varrho_\gamma)$
Productivity shock	$\zeta(1-\varrho_\zeta)$	ζ	$\zeta(1+\varrho_\zeta)$
Liquidity shock	$\theta(1+\varrho_\theta)$	θ	$\theta(1-\varrho_\theta)$
Expenditure shock	$\omega(1-\varrho_\omega)$	ω	$\omega(1+\varrho_\omega)$

$\varrho_\gamma = 0.40$	$\varrho_\zeta = 0.15$	$\varrho_\theta = 0.20$	$\varrho_\omega = 0.30$

For each type of shock and each type of constraint, I evaluate four scenarios: (i) constraints apply to all states of the world; (ii) constraints are suspended in the bad state s_1, and so only imposed in states s_2 and s_3; (iii) constraints are only imposed during normal times, i.e., state s_2; and (iv) constraints are suspended in the good state s_3, and so only imposed in states s_1 and s_2. For each case, the optimal constraints are calculated.

In all cases, welfare is evaluated as the equivalent compensation, in terms of night consumption, at the initial state (B^*, s_2), relative to the full discretionary outcome. *Welfare gains over full discretion* are defined as the difference between welfare in a particular stochastic constrained case and the fully discretionary stochastic equilibrium.

6.2 Optimal policy constraints for demand shocks

As a benchmark case, consider an economy subjected to fluctuations in aggregate demand, i.e., with shocks to γ. I will study this case exhaustively and afterwards, verify that the main results obtained for demand shocks also apply to other types of shocks.

Table 7 summarizes the welfare effects of imposing constraints on policy in an economy facing demand shocks. The four right-most columns show the welfare effects of imposing, respectively, policy constraints: (i) always; (ii) in normal and good times (suspended in bad times); (iii) in normal times only; and (iv) in bad and normal times (suspended in good times). The best case is shown in bold. For each type of policy constraint, the column labeled "optimal value" shows the value that corresponds to the best case (the best values for the remaining cases are omitted to simplify exposition but can be seen on Figure 1).

Table 7: Welfare gains over full discretion

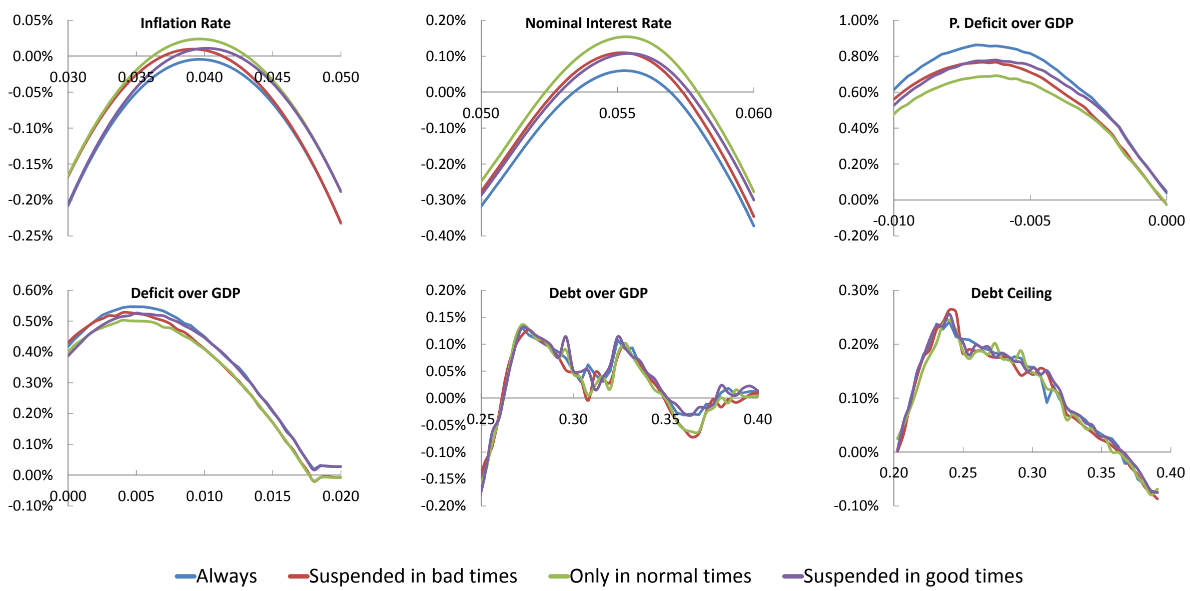
Constraint	Optimal Value	Always	Suspended in bad times	Only in normal times	Suspended in good times
Inflation target	0.040	0.0%	0.0%	**0.0%**	0.0%
Interest rate peg	0.055	0.1%	0.1%	**0.2%**	0.1%
Taylor rule	0.052	0.1%	0.1%	**0.1%**	0.1%
P. Deficit over GDP	−0.006	**0.9%**	0.8%	0.7%	0.8%
Deficit over GDP	0.005	**0.5%**	0.5%	0.5%	0.5%
Debt over GPD	0.272	0.1%	0.1%	**0.1%**	0.1%
Debt ceiling (over GDP)	0.240	**0.3%**	0.2%	0.2%	0.2%

Note: The "debt ceiling" is reported as a fraction of GDP. For each type of policy constraint, "optimal value" corresponds to the best scenario (bolded).

There are several important observations coming out of Table 7. First, placing an upper limit on the primary deficit over GDP improves welfare the most. The optimal value is to have a small primary surplus of about half a percent of output. Notably, this is the same result we obtained in the non-stochastic case. Second, for all types of constraints, most of the welfare gains come from imposing constraints in normal times. Third, suspending constraints during abnormal (both bad and good) implies only small differences in welfare, for all types.

Figure 1 expands on the results summarized in Table 7. For each case, the figure plots the welfare gains associated with imposing a particular policy constraint at specific times. One property that pops up immediately is that, for each type of constraint, the optimal value is similar whether we allow the constraint to be sometimes suspended or not. As mentioned above, the welfare changes of temporarily suspending constraint is minor relative to the overall welfare gains of imposing them in the first place. Both these results are significant for implementation, as there may be other reasons (say, political) for wanting to suspend constraint on government actions at certain times. Note, however, that these conclusions rely on the fact that constraints are to be reimposed when normal times come back.

Figure 1: Welfare gains over full discretion



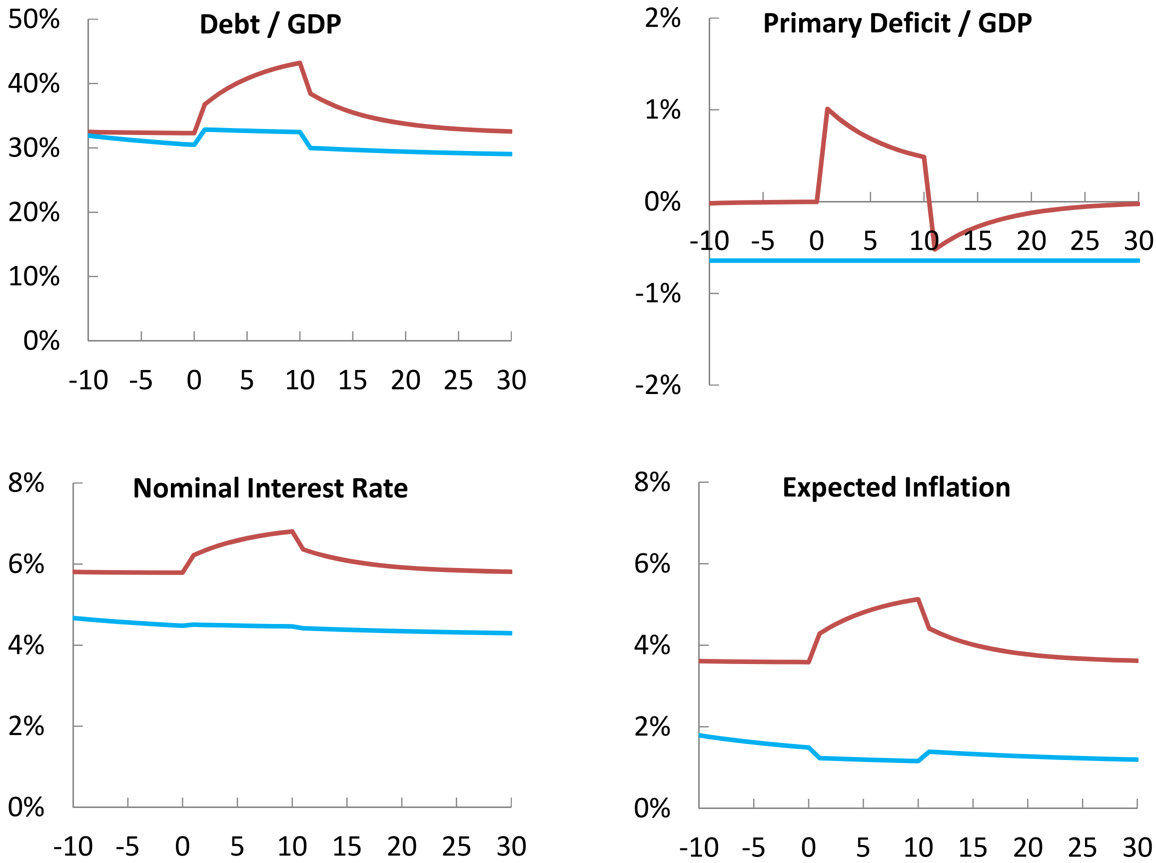
Is it costly to set the wrong value for a constraint? As Figure 1 illustrates, the answer depends on the type of constraint. Constraints on the primary and total deficits are benign for a significant range around the optimum. For example, small primary surpluses are always beneficial, so getting the exact value for the constraint right is not critical, which is an added benefit as it reduces the costs of improper implementation. In contrast, monetary policy targets can quickly turn a gain into a substantial loss. Coupled with the fact that the welfare gains of the optimal value are fairly small to begin with, this implies that monetary targets are not desirable constraints.

Debt constraints are good as long as they are not too tight, as they interact with the ability of the government to smooth distortions. A limit on debt over GDP, as the one imposed on Eurozone countries, has the peculiar property of two local maxima. Note however, that the stricter limit yields higher welfare. The welfare gains of a debt ceiling, as the one implemented nominally in the U.S., are single peaked, but note that they rapidly convert into losses when it is set too high. The reason for this result is that too-high debt ceilings do not provide the benefits of lower expenditure, but still hinder tax-smoothing.

6.3 Policy dynamics

Figure 2 compares the policy response to a negative demand shock under full discretion vs the optimal primary deficit ceiling, as indicated by the results in Table 7. The charts start off each economy at the non-stochastic steady state level of debt, 10 periods before the negative shock hits. This shock lowers real GDP by about 10% for 10 years (which is the calibrated duration of the bad state). The economy then returns to normal.

Figure 2: Full discretion vs optimal primary deficit ceiling



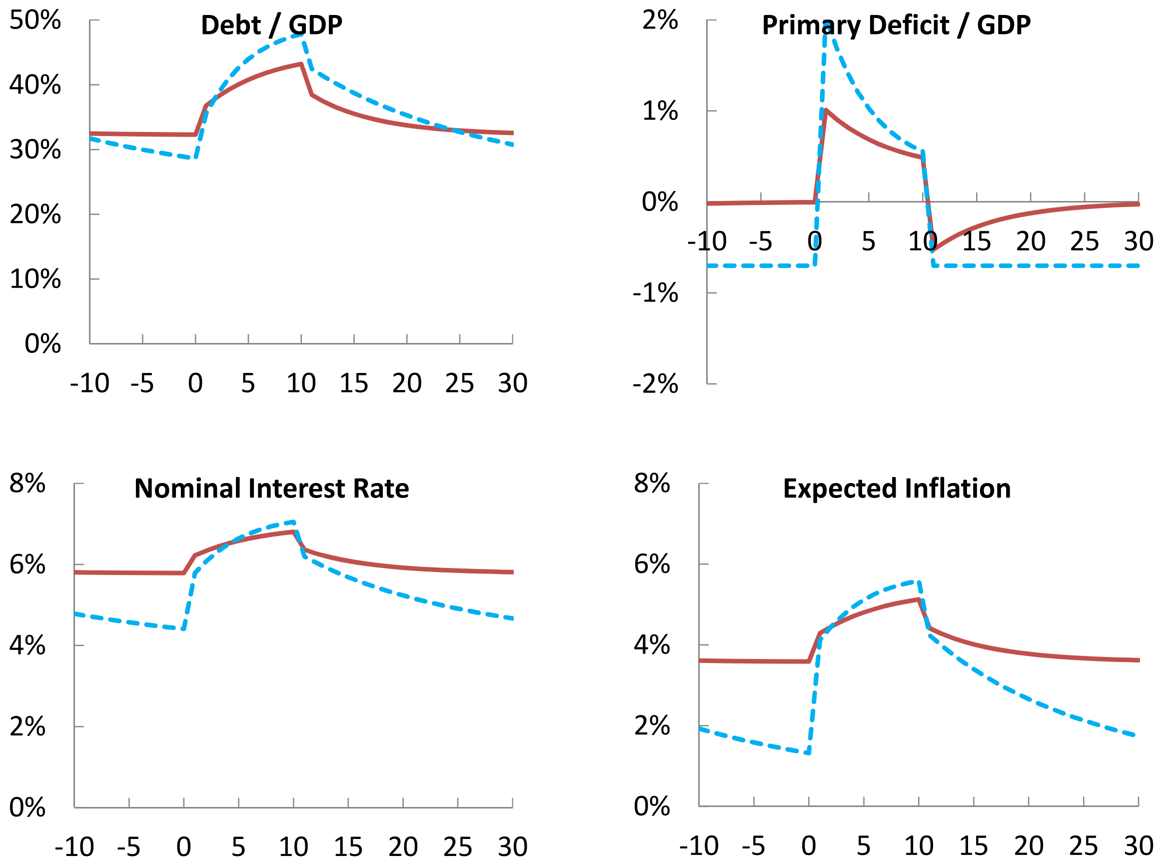
Note: full discretion (red solid line) and primary deficit ceiling (light blue line).

The constrained policy displays a significantly more muted response to the adverse shock. The better welfare performance of the optimal primary deficit constraint comes from the lower inflation distortion it allows. In effect, by implementing a primary surplus, inflation can be lower, both in normal and adverse times. In the appendix, Figure 5 shows the implied dynamics in allocations. Lower inflation under the deficit constraint is associated with higher day-good consumption. Notably, under the primary deficit constraint day-good consumption does not react significantly once the adverse shock hits. The optimal policy constraint also implies lower public expenditure. They drop significantly more during bad times, which explains why not suspending the constraint leads to larger welfare gains.

Figure 3 considers the case when we allow for the primary deficit ceiling to be suspended in abnormal times. As shown in Table 7, most of the welfare gains from a primary deficit ceiling came from imposing it during normal times. The constrained policy response looks now qualitatively more similar to the fully discretionary policy. There are two important differences.

First, during normal times, the requirement of a primary surplus induces a lower inflation than under discretion, which mitigated the social losses due to political frictions. Second, when the economy returns to normal, both debt and inflation transition gradually back to their (long-run) normal levels. I.e., even though the government is constrained to run a surplus, it is still able to adequately smooth distortions over time, which is always desirable. This is why an inflation target imposed during normal times only (of say 2% annual) does not work as effectively; although inflation is typically lower, once the economy returns back to normal, inflation needs to adjust immediately, which is costly since it does not allow for sufficient distortion-smoothing.

Figure 3: Full discretion vs optimal primary deficit ceiling in only normal times



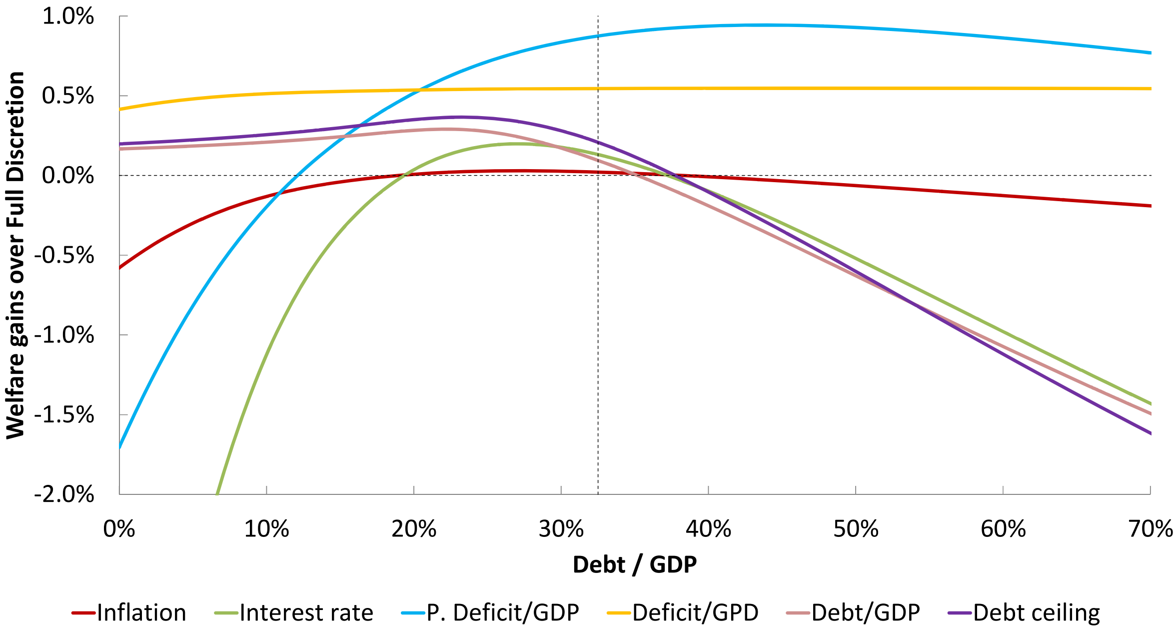
Note: full discretion (red solid line) and primary deficit ceiling in normal times only (light blue dashed line).

6.4 Welfare and the timing of reform

Another potential concern is the fact that constraints could be imposed at inappropriate times. For example, the calculations for optimal constraints rely on them being implemented around the non-stochastic steady state (which is trivially close to the stochastic steady state in normal times with full discretion). What happens when constraints are placed far from this state? In particular, how does the welfare derived from imposing the optimal values for each policy constraint depend on the level of debt at the moment of introduction? Figure 4 provides an answer to this question.

The optimal inflation target can lead to some important welfare losses when implemented far from the steady state. These losses increase dramatically when imposing the optimal interest

Figure 4: Welfare gains of optimal constraints as a function of initial debt level



rate target. In fact, there is only a small range of debt levels over which monetary policy constraints are welfare improving. The optimal primary deficit target typically leads to fairly consistent welfare gains, even when initial debt is fairly high. The exception is when initial debt is low, as the requirement of a primary deficit surplus severely limits the amount of debt accumulation and thus, mitigates distortion-smoothing. On the other hand, the optimal deficit ceiling offers consistent welfare gains for all levels of debt. The difference stems from the fact that at low levels of debt, the constrained government can now run a primary deficit, since the interest paid on debt is low. Hence, a deficit ceiling, as opposed to a primary deficit ceiling, might be a better idea for governments with low initial debt. Both debt constraints can lead to substantial welfare losses when initial debt is high. The reason for this is simple: the debt ceiling forces a sudden adjustment of debt, which goes against the desirability to smooth distortions.

Monetary targets (inflation and interest rates) have a minor upside and are instead potentially very costly when implemented far away from the non-stochastic steady state. Coupled with the findings in Figure 1, this suggests that monetary targets are generally not a good idea in economies with potentially large aggregate demand shocks. In contrast, as shown in Tables 3 and 4, they improve welfare significantly in non-stochastic economies (and by extension, probably also in economies subjected to very mild aggregate fluctuations).

6.5 Frequent abnormal times

Next we consider increasing the frequency of abnormal times or, equivalently, reducing the duration of normal times. Let $\varpi^* = \varpi = 0.9$; that is, all states have now a duration of 10 years. Table 8 present the results. As we can see, the results obtained for the benchmark case still apply. Even the optimal values for constraint are very close. The only significant difference is a slight decrease in welfare gains.

Table 8: Welfare gains over full discretion when abnormal times are frequent

Constraint	Optimal Value	Always	Suspended in bad times	Only in normal times	Suspended in good times
Inflation rate	0.040	−0.0%	0.0%	**0.0%**	0.0%
Nominal interest rate	0.055	−0.2%	0.0%	**0.1%**	0.0%
P. Deficit over GDP	−0.007	**0.8%**	0.5%	0.3%	0.5%
Deficit over GDP	0.005	**0.5%**	0.4%	0.3%	0.4%
Debt over GDP	0.272	0.1%	0.1%	**0.1%**	0.1%
Debt ceiling (over GPD)	0.240	**0.2%**	0.2%	0.2%	0.2%

Note: The "debt ceiling" is a constraint on the amount of debt, displayed here in terms of GDP. For each type of policy constraint, "optimal value" corresponds to the best scenario (bolded).

6.6 Productivity, liquidity and expenditure shocks

We now verify that the main results derived for aggregate demand shocks also apply to other types of shocks. Table 9 summarizes the welfare effects of imposing constraints on policy in economies facing productivity, liquidity and government expenditure shocks.

Although each case presents its own idiosyncracies, the similarities across economies are notable. For each type of shock the best prescription remains a small primary surplus, between 0.6% and 0.7% of GDP. It is always best not to suspend this constraint. Again, even if the constraint is imposed only during normal times, due to the distortion-smoothing motive, it is disciplining government behavior during abnormal times. Except for the case with expenditure shocks, the welfare loss of suspending a primary deficit constraint during abnormal times (good, bad or both) is very small. The same conclusions can be drawn about imposing the next best constraint, a ceiling on total deficit over GDP of around 0.4% to 0.5%. For economies with expenditure shocks, since the spending increase stems from the government becoming less benevolent, the gains from not suspending deficit constraints during bad times are significant.

Monetary policy constraints remain largely undesirable. There are many cases in which these constraints lead to welfare losses relative to full discretion. Ironically, their performance is especially poor in the presence of liquidity shocks. When they do improve over discretion, it is in the form of an interest rate target and the gains are small. Debt constraints fare better, especially a debt ceiling that is imposed at all times. Still the gains from debt constraints are dominated by those deficit constraints, even when they are not implemented optimally.

7 General Lessons and Conclusions

The exercises presented in this paper offer some important and novel lessons for the institutional design of government policy.

First, a small primary surplus is always the best policy and it is never optimal to suspend this constraint. For an economy calibrated to the U.S. and subject to a variety of shocks, the optimal primary surplus is a bit over half percent of output. A total deficit of about half a percent of output, imposed at all times, is the next best prescription. In both cases, most welfare gains come from imposing constraints in normal times, which also helps discipline government policy during abnormal times. The cost of implementing deficit constraints sub-optimally, e.g., picking the wrong ceiling or suspending constraints during abnormal times, carry small welfare costs. A notable exception is suspending deficit constraints when government expenditure temporarily

Table 9: Welfare gains over full discretion

Productivity shocks

Constraint	Optimal Value	Always	Suspended in bad times	Only in normal times	Suspended in good times
Inflation rate	0.040	−0.2%	−0.1%	**0.0**%	−0.1%
Nominal interest rate	0.055	0.0%	0.1%	**0.2**%	0.1%
P. Deficit over GDP	−0.007	**0.9**%	0.8%	0.7%	0.7%
Deficit over GDP	0.005	**0.6**%	0.6%	0.5%	0.5%
Debt over GPD	0.272	**0.2**%	0.2%	0.1%	0.1%
Debt ceiling (over GDP)	0.240	**0.3**%	0.3%	0.2%	0.2%

Liquidity shocks

Constraint	Optimal Value	Always	Suspended in bad times	Only in normal times	Suspended in good times
Inflation rate	0.040	0.0%	**0.0**%	0.0%	0.0%
Nominal interest rate	0.055	−0.3%	−0.1%	**0.1**%	−0.2%
P. Deficit over GDP	−0.006	**0.8**%	0.8%	0.7%	0.8%
Deficit over GDP	0.005	**0.6**%	0.5%	0.5%	0.5%
Debt over GPD	0.325	0.1%	**0.1**%	0.1%	0.1%
Debt ceiling (over GDP)	0.284	**0.3**%	0.3%	0.2%	0.3%

Government expenditure shocks

Constraint	Optimal Value	Always	Suspended in bad times	Only in normal times	Suspended in good times
Inflation rate	0.040	0.0%	0.0%	**0.0**%	0.0%
Nominal interest rate	0.056	0.2%	0.1%	0.1%	**0.2**%
P. Deficit over GDP	−0.007	**1.1**%	0.7%	0.6%	1.1%
Deficit over GDP	0.004	**0.7**%	0.5%	0.4%	**0.7**%
Debt over GPD	0.325	**0.2**%	0.2%	0.2%	**0.2**%
Debt ceiling (over GDP)	0.285	**0.4**%	0.3%	0.3%	0.3%

Note: The "debt ceiling" is a constraint on the amount of debt, displayed here in terms of GDP. For each type of policy constraint, "optimal value" corresponds to the best scenario (bolded).

increases (assuming agents do not value such an increase). Welfare gains from deficit constraints are still significant even when they are implemented far from steady state; the only exception being imposing a primary deficit ceiling with very low debt.

Second, monetary policy rules, i.e., inflation and interest rate targets, are not generally desirable constraints. Both in economies with and without aggregate fluctuations, monetary policy targets yield small welfare gains and even losses relative to full discretion. More problematic is the fact that slight mis-targeting or incorrect timing can lead to large welfare losses. The reason for these negative results is that monetary policy targets hinder the ability of governments to smooth distortions across states. In effect, inflation allows for less distortionary repayments of temporary debt increases.

Third, debt limits are generally effective, but are always dominated by deficit constraints, even when these latter constraints are not implemented at their best (e.g., because society allows them to be suspended in bad times). This suggest that the typical focus of government

Table 10: Welfare gains over full discretion with correlated demand and public expenditure shocks

Constraint	Optimal Value	Always	Suspend in bad	Only in normal	Suspend in good
Inflation rate	0.040	−0.1%	0.0%	**0.0**%	0.0%
Nominal interest rate	0.052	0.1%	0.3%	**0.4**%	0.3%
P. Deficit over GDP	−0.007	**0.8**%	0.7%	0.7%	0.7%
Deficit over GDP	0.004	0.5%	**0.5**%	0.5%	0.5%
Debt over GDP	0.272	**0.3**%	0.3%	0.3%	0.3%
Debt ceiling	0.240	0.1%	**0.2**%	0.1%	0.1%

Note: The "debt ceiling" is a constraint on the amount of debt, displayed here in terms of GDP. For each type of policy constraint, "optimal value" corresponds to the best scenario (bolded).

reformers on debt ceilings may be misplaced. It is always more desirable, and arguably easier in practice, to aim at constraining the deficit.

Fourth, and not least, the socially effective way to combat inefficiently high public expenditure, is not more pre-commitment to government actions, but rather commitment to rules that constraint government action. The difference is that the latter makes the government internalize the cost of socially suboptimal policy.

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A Derivation of monetary equilibrium conditions (2)–(6)

Here, we derive conditions (2)–(6) which characterize a monetary equilibrium. Let us start with the problem of an agent at night,

$$W(z,B,s) = \max_{c,n,m',b'} \gamma U(c) - \alpha n + v(g) + \beta E[V(m',b',B',s')|s]$$

subject to the budget constraint: $p_c c + (1+\mu)(m' + qb') = p_c(1-\tau)\zeta n + z$. Solving the budget constraint for n and replacing in the objective function, the first-order conditions imply:

$$\gamma U_c - \frac{\alpha}{(1-\tau)\zeta} = 0 \quad (22)$$

$$-\frac{\alpha(1+\mu)}{p_c(1-\tau)\zeta} + \beta E[V'_m|s] = 0 \quad (23)$$

$$-\frac{\alpha(1+\mu)q}{p_c(1-\tau)\zeta} + \beta E[V'_b|s] = 0 \quad (24)$$

The night-value function W is linear in z, $W_z = \frac{\alpha}{p_c(1-\tau)\zeta}$. Hence, $W(z,B,s) = W(0,B,s) + \frac{\alpha z}{p_c(1-\tau)\zeta}$, which we will use to rewrite the problem of the agent in the day. Accordingly, the problem of a consumer in the day can be rewritten as

$$V^c(m,b,B,s) = \max_x \ u(x) + W(0,B,s) + \frac{\alpha(m+b-p_x x)}{p_c(1-\tau)\zeta}$$

subject to the liquidity constraint $p_x x \leq m + \theta b$, with associated Lagrange multiplier ξ. The first-order condition is

$$u_x - \frac{\alpha p_x}{p_c(1-\tau)\zeta} - \xi p_x = 0 \quad (25)$$

Similarly, the problem of a producer can be rewritten as

$$V^p(m,b,B,s) = \max_\kappa \ -\phi\kappa + W(0,B,s) + \frac{\alpha(m+b+p_x\kappa)}{p_c(1-\tau)\zeta}$$

The first-order condition implies

$$-\phi + \frac{\alpha p_x}{p_c(1-\tau)\zeta} = 0 \quad (26)$$

Given $V(m,b,B,s) \equiv \eta V^c(m,b,B,s) + (1-\eta)V^p(m,b,B,s)$ and using (26) we obtain:

$$V_m = \phi/p_x + \eta\xi$$
$$V_b = \phi/p_x + \eta\theta\xi$$

Using these expressions, together with (26), we can rewrite (23) and (24) as

$$1+\mu = \frac{\beta p_x E[\phi/p'_x + \eta\xi'|s]}{\phi} \quad (27)$$

$$q = \frac{E[\phi/p'_x + \eta\theta'\xi'|s]}{E[\phi/p'_x + \eta\xi'|s]} \quad (28)$$

In equilibrium, we have $m' = 1$ and $b' = B'$. Furthermore, the day and night resource constraints imply $\kappa = \eta/(1-\eta)x$ and $n = c + g$, respectively. The liquidity constraint of consumers in the day holds with equality (wlog if it does not bind); thus,

$$p_x = \frac{1+\theta B}{x} \quad (29)$$

which gives us (2).

Next, notice that (22) can be rearranged to yield (6):

$$\tau = 1 - \frac{\alpha}{\zeta \gamma U_c} \tag{30}$$

Plugging (29) and (30) into (26) yields (3):

$$p_c = \frac{\gamma U_c (1 + \theta B)}{\phi x} \tag{31}$$

Given (29)–(31) we can solve for the Lagrange multiplier of the liquidity constraint:

$$\xi = \frac{(u_x - \phi) x}{(1 + \theta B)} \tag{32}$$

Hence, (27) and (28) imply, respectively, (5) and (4), i.e.,

$$q = \frac{E[\frac{x'(\eta\theta' u'_x + (1-\eta\theta')\phi)}{1+\theta' B'}|s]}{E[\frac{x'(\eta u'_x + (1-\eta)\phi)}{1+\theta' B'}|s]}, \tag{33}$$

$$\mu = \frac{\beta(1+\theta B)}{\phi x} E\left[\frac{x'(\eta u'_x + (1-\eta)\phi)}{(1+\theta' B')}\Big| s\right] - 1 \tag{34}$$

B Numerical algorithm

Economies without policy constraints are solved globally using a projection method with the following algorithm:

(i) Let $\Gamma = [\underline{B}, \bar{B}]$ be the debt state space. Define a grid of $N_\Gamma = 10$ points over Γ and set $N_S = 3$. Create the indexed functions $\mathcal{B}^i(B)$, $\mathcal{X}^i(B)$, $\mathcal{C}^i(B)$, and $\mathcal{G}^i(B)$, for $i = \{1, \ldots, N_S\}$, and set an initial guess.

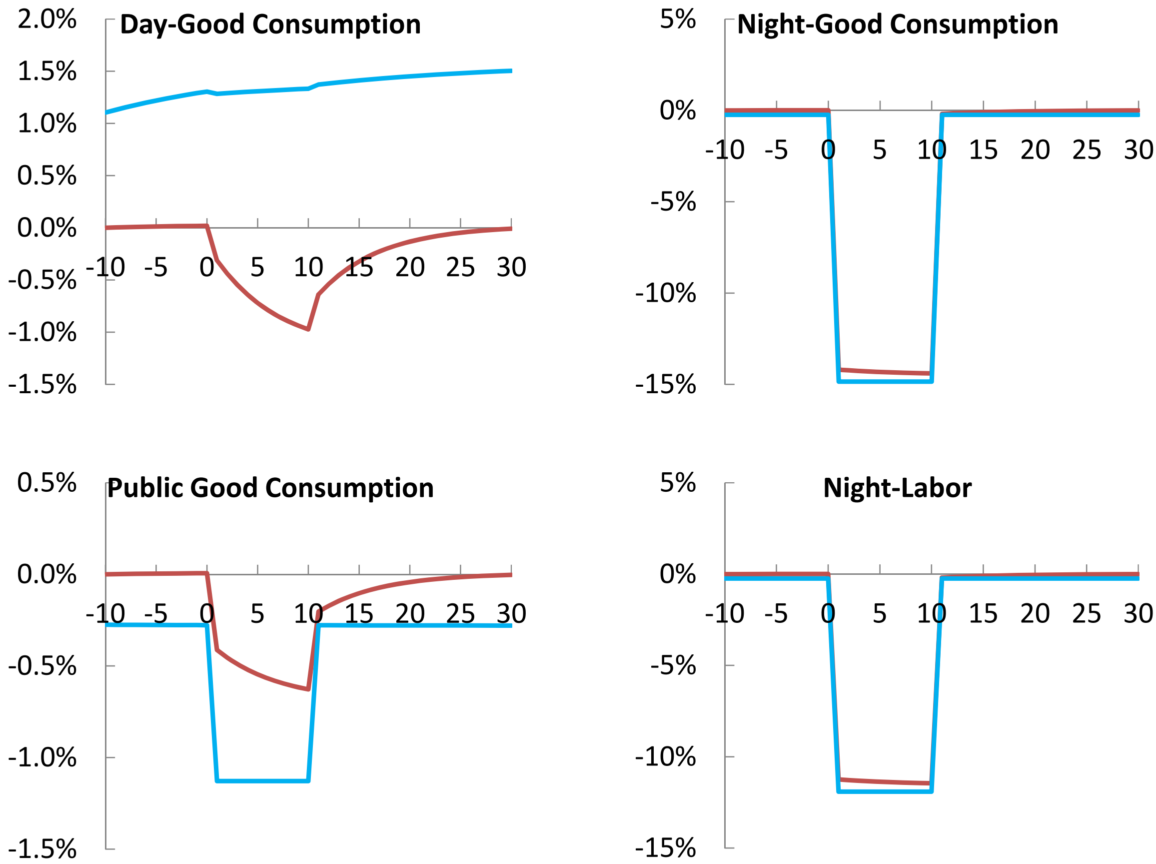
(ii) Construct the following system of equations: for every point in the debt and exogenous state grids, evaluate equations (7)—(11). Since (8) contains $\mathcal{X}^j(\mathcal{B}^i(B))$ (and its derivative) and $\mathcal{G}^j(\mathcal{B}^i(B))$, use cubic splines to interpolate between debt grid points and calculate the derivatives of policy functions.

(iii) Use a non-linear equations solver to solve the system in (ii). There are $N_\Gamma \times N_S \times 4 = 120$ equations. The unknowns are the values of the policy function at the grid points. In each step of the solver, the associated cubic splines need to be updated so that the interpolated evaluations of future choices are consistent with each new guess.

For economies that include constraints to policy in all or some states, I use value function iteration: solve the maximization problem of the government subject to the corresponding policy constraint, at every grid point. Update the policy and value functions and iterate until convergence is achieved.

C Additional Figures

Figure 5: Full discretion vs optimal primary deficit ceiling



Note: full discretion (red solid line) and primary deficit ceiling (light blue line). Each allocation is shown as deviations from non-stochastic steady state.