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# Persistence of Shocks and the Reallocation of Labor

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## Abstract

This paper proposes a theoretical and quantitative analysis of the reallocation of labor across firms in response to idiosyncratic shocks of different persistence. Creating and destroying jobs is costly and workers are paid a share of the value of the marginal worker. The model predicts that employment and labor costs react differently to transitory shocks and permanent shocks. Quantitative evaluation of the model on a panel of French firms shows the model's performance. Modest adjustment costs are needed to reproduce observed job reallocation and inaction rates. Removing adjustment costs leads to productivity gains of 1% at the steady state. These gains are 50% larger in a economy with only transitory shocks and an order of magnitude lower in an economy with only permanent shocks. Bargaining dampens the reallocation of labor across firms, leading to larger efficiency losses from adjustment costs.

JEL Codes: E24, J21, J23

Keywords: Firm Dynamics, Adjustment Costs, Misallocation, Persistence of Shocks

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# 1 Introduction

Data on individual firms reveal the importance of idiosyncratic shocks: Davis and Haltiwanger (1999) document the coexistence of both job creation and job destruction within narrowly defined industries and at all phases of the business cycles. There is also an enormous amount of heterogeneity in firm-level productivity: In the typical four-digit industry, the lowest decile producer is half as productive as the highest decile producer.<sup>1</sup> A recent growing literature, based on the seminal work of Hopenhayn (1992) and Hopenhayn and Rogerson (1993), and surveyed in Hopenhayn (2014), investigates whether the allocation of resources input within firms across countries can matter for aggregate outcomes. Hsieh and Klenow (2009) find much larger microeconomic gaps in productivity across firms in poor countries. They suggest impediments to the reallocation of resources from low to high-productivity firms can have important aggregate consequences. While there exists a body of work that analyzes the allocation of physical capital across firms within a country as well as the importance of financial markets,<sup>2</sup> much less is known about the impact of labor market regulations and, more generally, labor adjustment costs. These costs may be technological (e.g., reduced efficiency during the period of adjustment, or they may be institutional (e.g., employment protection legislation). The responsiveness of labor costs to shocks also alters a firm's incentives to adjust its workforce. For example, following a negative shock, a firm may not have to reduce its workforce if the cost of labor decreases sufficiently.

The persistence and variance of shocks are crucial parameters that determine the benefits of reallocating resources across firms.<sup>3</sup> Firms face both permanent shocks (such as a change in consumers' taste or a new software) and transitory shocks (such as a climate shock or a power outage) and may well adopt different strategies depending on the persistence of the shocks. While the distinction between transitory and permanent shock is very popular in the consumption literature,<sup>4</sup> the literature on firm dynamics typically assumes a univariate AR(1) process whose persistence is calibrated or estimated as a range of values that greatly varies across studies despite its first-order importance.

I propose a theoretical and quantitative analysis of the firm-level employment and wage responses to idiosyncratic shocks of different persistence. Specifically, I show that the combined assumptions of (1) decreasing returns to labor, (2) Nash bargaining with multiple workers as in Stole and Zwiebel (1996) and, (3) costly employment adjustment, imply that transitory shocks have a strong impact on wages and little effect on employment while permanent shocks have a strong effect on employment and little effect on wages.

The firm produces with decreasing returns to labor and is subject to transitory and permanent shocks to its profitability. The wage is negotiated every year, and workers are paid a share of

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<sup>1</sup>See Syverson (2011) for a survey.

<sup>2</sup>See, for instance, Quadriini (2000); Cagetti and De Nardi (2006); Buera et al. (2011) and Midrigan and Xu (2014).

<sup>3</sup>See Buera and Shin (2011, 2013); Gourio (2008); Moll (2014), and Midrigan and Xu (2014).

<sup>4</sup>See Meghir and Pistaferri (2011) for a survey.

the value of the marginal worker as in Stole and Zwiebel (1996). Finally, adjusting the level of employment is costly. The intuitive mechanism at work is as follows: If there is a shock that raises the marginal productivity of labor, and this shock is expected to last, then the firm pays the cost of hiring additional workers. Since the marginal product of labor is decreasing, this offsets the shock, so that the marginal worker is not much more valuable than before. Therefore, the wage does not increase much, but there is a substantial rise in employment. In the case of a transitory shock, it is not worthwhile to add more workers, because it will be costly to decrease employment after the shock expires. In this case, the marginal worker is more valuable in this period, so the wage rises. There is little change in employment in this case.

Then, I examine whether the model is able to quantitatively reproduce the patterns of employment and wage dynamics observed in firm-level data. In a preliminary step, I apply a simple econometric specification to a panel of French firms. I find that, at the firm level, transitory shocks have a strong effect on average wages, whereas permanent shocks have a very small effect on average wages. Then, I use the simulated method of moments and estimate the structural parameters – labor adjustment costs, workers bargaining power, and the sources of dispersion of the observed variables. In particular, I provide some evidence on the importance of adjustment costs and the strength of rent sharing. The estimation results indicate that the model is consistent with the data. Relatively modest adjustment costs (about two months of average annual wages per job created or destroyed) can reproduce the data well. This conflicts with the perceived rigidity of the French labor market.

I use the model to infer the cost of the regulation and its impact on labor productivity observed in the data, linking to the large body of work that followed Hsieh and Klenow (2009) and Restuccia and Rogerson (2008). These papers show that the dispersion in the productivity of factors inputs across firms (misallocation) has important consequences for aggregate output. I find that adjustment costs can explain one-fourth of the dispersion of labor productivity. Holding total employment constant, output increases by around 1% at the steady state if adjustment costs are removed. This number captures the misallocation of labor across firms. I find that the persistence of shocks is important for evaluating the impact of labor adjustment costs. Precisely, transitory shocks are responsible for more misallocation than permanent shocks, but the gains to reallocating labor are 50% larger in economies with only transitory shocks compared with the baseline economy. And, they are an order of magnitude lower in economies with only permanent shocks. With wage bargaining, labor costs tend to rise following positive shocks and they tend to fall following negative shocks. Hence, wage bargaining dampens the reallocation of labor across firms leading to larger efficiency losses from adjustment costs.

**Related Literature** This paper is related to a large literature on firm dynamics, labor market regulations, and misallocation.

A number of recent studies that followed Hsieh and Klenow (2009) and Restuccia and Roger-

son (2008) argue that factors of production are inefficiently allocated among firms (misallocation). Notably, Buera and Shin (2013), Moll (2014), and Midrigan and Xu (2014) quantitatively examine the impact of financing frictions on the dispersion of capital productivity. These authors show that the costs of misallocation strongly depend on the persistence of shocks. Yet, these papers neglect the distinction between less and more persistent variations in firm productivity, and it is customary to assume that firm productivity follows a stationary autoregressive process of order one.<sup>5</sup> This contrasts with the various dynamic models for individual workers that have been proposed in the earnings dynamics literature.<sup>6</sup>

Some empirical papers investigate the relationship between wages and profits but none consider this question simultaneously with that of employment flexibility. Georgiadis and Manning (2014) document a large amount of transitory volatility in firm-level average earnings and they attribute it to a mechanism in line with the theory below. Using matched employer-employee data from Italy, Guiso et al. (2005) find that wages are sensitive to permanent output shocks but not to transitory output shocks. They do not consider profitability shocks but output shocks, and they consider only the response of wages of workers who stay with the firm - not the response of employment. My paper studies the joint responses of wages and employment at the firm level while theirs focuses on the wages of workers who stay with the same firm. Also my paper is able to distinguish between profitability shocks and output shocks, while Guiso et al. (2005) consider only the reduced-form effect of output shocks that combine profitability shocks and the corresponding response of employment. The effects of a permanent shock are comparable in both papers. I find that the nonsignificant effect of transitory shocks on stayers' wages (Guiso et al., 2005) is compatible with a strong effect on firm-average wages (documented in this paper) if the elasticity of non-stayers wages is high enough. I return to this issue in more detail in Section 6.2.

The theoretical model builds on several contributions that incorporate firm size into search models. The wage setting mechanism used in this paper is borrowed from Stole and Zwiebel (1996) and Smith (1999). They generalize the Nash bargaining solution to a setting with downward-sloping labor demand. This has been used in a similar framework by Elsby and Michaels (2013).

Adjustment costs are used in many fields of economics to explain a wide range of facts.<sup>7</sup> In these models, the calibration procedure uses some measure of dismissal costs to assign values to the adjustment costs parameters.<sup>8</sup> However, this practice is not entirely satisfactory because adjustment costs have an implicit component, and there are many regulations that may not be well summarized

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<sup>5</sup>An important exception is Gourio (2008) who focuses on investment whereas I consider the joint response of wages and employment. Also, he obtains a closed-form solution by assuming quadratic adjustment costs and log-linearization of the model. Quadratic costs may be appropriate for approximating the behavior of very large firms, but they are at odds with observed patterns of factor adjustment at the plant level.

<sup>6</sup>See Meghir and Pistaferri (2011) for a survey.

<sup>7</sup>A non-exhaustive list includes the behavior of gross job flows and aggregate employment over the business cycle (Campbell and Fisher, 2000; Elsby and Michaels, 2013), and the impact of firing costs on productivity and employment (Bentolila and Bertola, 1990; Hopenhayn and Rogerson, 1993).

<sup>8</sup>Measures of dismissal costs are reported, for example, in Heckman et al. (2000).

by dismissal costs. Indeed, I find that relatively modest adjustment costs (about two months of wages) can reproduce the data well. One possible reason for the small estimated frictions is that firms have access to fixed-term contracts that allows them to separate from workers at relatively low cost. Recent other estimates of labor adjustment costs include Rota (2004), Trapeznikova (2014), and Aguirregabiria and Alonso-Borrego (2014).

**Organization of the Paper** The remainder of the paper proceeds as follows. Section 2 develops the theoretical framework and examines the impact of transitory and permanent shocks on wages and employment. Section 3 conducts a reduced-form empirical investigation. Section 4 covers the estimation method and presents the empirical results. Section 5 uses these estimates to conduct some policy experiments. Section 6 discusses the model’s assumptions.

## 2 A Model of Labor Demand

This section presents a model of the firm that combines both employment and wage decisions and distinguishes between permanent and transitory shocks to business conditions. The firm produces with decreasing returns to labor and is subject to transitory and permanent shocks to its profitability. The wage is negotiated every year and workers are paid a share of the value of the marginal worker, following Stole and Zwiebel (1996). There is a constant cost to creating or destroying jobs. The model shows that transitory and permanent shocks have different effects on wages and employment.

### 2.1 Framework

Time is discrete and runs forever. The risk-neutral firm produces a homogeneous good using labor  $n$ . Every period the firm faces two independent sources of profitability variations: (1) a transitory shock  $\epsilon$  that is serially uncorrelated and i.i.d. across firms and time and (2) a permanent shock  $\eta$  that is serially uncorrelated and i.i.d. across firms and time such that the permanent component of profitability  $A_t$  evolves over time  $t$  as  $A_{t+1} = A_t\eta_t$ . The revenue function is  $(\epsilon A)^{1-\alpha} n^\alpha$ , where  $0 < \alpha < 1$  reflects decreasing returns to labor and/or market power.<sup>9</sup> Combined with the wage function  $w(A, \epsilon, n)$ , this gives profits  $\pi$  as a function of  $A, \epsilon$ , and  $n$ :

$$\pi(A, \epsilon, n) = (\epsilon A)^{1-\alpha} n^\alpha - w(A, \epsilon, n)n. \quad (1)$$

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<sup>9</sup>I do not model explicitly physical capital but the model is consistent with two views: (1) It is perfectly flexible in which case it is proportional to  $A$  and  $\alpha$  cannot be interpreted as the elasticity of output with respect to employment, and (2) it is fixed in which case it appears as a fixed constant in  $A$ . Reality is likely to be somewhere in between.

There is a constant cost  $\bar{c}$  that is paid for every job destroyed and similarly a constant cost  $\underline{c}$  for every job created. The adjustment cost function  $C$  is a function of employment variations  $n_t - n_{t-1}$ :

$$C(n_t - n_{t-1}) = \underline{c}(n_t - n_{t-1})^+ - \bar{c}(n_t - n_{t-1})^-,$$

where  $x^+ = x$  if  $x$  is positive and zero otherwise and  $x^- = -x$  if  $x$  is negative and zero otherwise. I do not consider quadratic adjustment costs because they imply a smooth adjustment of employment to shocks, which is at odds with the data.<sup>10</sup> I consider linear but not fixed adjustment costs because it allows me to solve the wage function in closed form and considerably simplifies the numerical solution of the model that is complicated by the existence of both permanent and transitory shocks. Further, both fixed and linear adjustment costs imply that it may be optimal to maintain the same number of employees, and employment change tends to be concentrated in a single period.

The cost to create a job  $\underline{c}$  can readily be interpreted as a linear cost of posting a vacancy divided by the probability of filling a vacancy, which depends on aggregate conditions (see Elsby and Michaels, 2013). Since I focus on idiosyncratic shocks rather than business cycles, I abstract from this complication. Similarly, the cost of destroying jobs can be interpreted as a firing cost.

At the beginning of each period, the timing of events is as follows. Given  $(A, \epsilon, n)$ , the manager creates or destroys jobs  $d_t$ , which affects production in the current period. Then, the manager and workers bargain over current-period wages,  $w(A, \epsilon, n + d)$ . Finally, production takes place. The manager's problem is to choose a state-contingent sequence of employment to maximize the present discounted value of current and future profits. The parameter  $\beta \in (0, 1)$  represents the rate at which the manager discounts profits in future periods. Define the value function at period 0,  $V^*(A_0, \epsilon_0, n_{-1})$  as the present discounted value of current and future profits given initial productivity,  $A_0$ , lagged employment,  $n_{-1}$ , and initial transitory shock,  $\epsilon_0$ :

$$V^*(A_0, \epsilon_0, n_{-1}) = \sup_{\{n_t\}_{t \geq 0}} E \left\{ \sum_{t=0}^{\infty} \beta^t [\pi(A_t, \epsilon_t, n_t) - C(n_t - n_{t-1})] \mid A_0, \epsilon_0, n_{-1} \right\}. \quad (2)$$

Under standard conditions,  $V^*(A, \epsilon, n)$  is the unique solution to Bellman's equation:

$$V(A, \epsilon, n) = \max_d \{ \pi(A, \epsilon, n + d) - C(d) + \beta E[V(A\eta, \epsilon', n + d)] \}, \quad (3)$$

where  $E$  is an expectation over the joint distribution of  $\epsilon'$  and  $\eta$ . I assume the shocks are log-normally distributed with mean zero and variance  $\sigma_\epsilon^2$  and  $\sigma_\eta$ .

## 2.2 Employment Policy

Given the wage function (determined later on), and given  $(A, \epsilon, n)$ , the optimal choice  $d$ , of creating and destroying jobs must satisfy the first-order conditions:

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<sup>10</sup>A large literature documents that labor adjustment costs are non convex at the firm level (see Hamermesh, 1989; Cooper et al., 2007, and the references therein).

$$\pi_n(A, \epsilon, n + d) - \underline{c} + \beta E(V_n(A\eta, \epsilon', n + d)) \leq 0 \quad (4)$$

with equality if  $d > 0$ , and

$$\pi_n(A, \epsilon, n + d) + \bar{c} + \beta E(V_n(A\eta, \epsilon', n + d)) \leq 0 \quad (5)$$

with equality if  $d < 0$ .

The optimal choice for next-period employment  $n'(A, \epsilon, n)$ , given the state  $(A, \epsilon, n)$ , reads

$$n'(A, \epsilon, n) = \begin{cases} \bar{n}(A, \epsilon) & \text{if } n > \bar{n}(A, \epsilon) \\ n & \text{if } \underline{n}(A, \epsilon) < n < \bar{n}(A, \epsilon) \\ \underline{n}(A, \epsilon) & \text{if } n < \underline{n}(A, \epsilon) \end{cases} \quad (6)$$

where the target functions  $\bar{n}$  and  $\underline{n}$  are defined by,

$$\begin{aligned} \alpha(\epsilon A)^{1-\alpha} \bar{n}(A, \epsilon)^{\alpha-1} + \beta E(V_n(A\eta, \epsilon', \bar{n}(A, \epsilon))) &= w(A, \epsilon, \bar{n}(A, \epsilon)) + w_n(A, \epsilon, \bar{n}(A, \epsilon))\bar{n}(A, \epsilon) - \bar{c} \\ \alpha(\epsilon A)^{1-\alpha} \underline{n}(A, \epsilon)^{\alpha-1} + \beta E(V_n(A\eta, \epsilon', \underline{n}(A, \epsilon))) &= w(A, \epsilon, \underline{n}(A, \epsilon)) + w_n(A, \epsilon, \underline{n}(A, \epsilon))\underline{n}(A, \epsilon) + \underline{c}. \end{aligned}$$

Firms' hiring and firing decisions are based on the expected present value comparisons between labor's marginal revenue product, and wage plus turnover costs. There is also an additional term  $w_n$  that captures the impact of employment  $n$  on the wage rate  $w$ .

Optimality requires the firm to create and destroy jobs as needed to keep the marginal value of labor in the closed interval  $[-\underline{c}, \bar{c}]$ . Figure 1 plots optimal employment given shocks. The diagonal line is the 45 degree line. If employment at the beginning of the period is below the threshold  $\underline{n}(A, \epsilon)$  for job creation, the firm creates jobs to reach the target  $\underline{n}(A, \epsilon)$  independently of employment at the beginning of the period. Similarly, if employment at the beginning of the period is above the threshold for job destruction  $\bar{n}(A, \epsilon)$ , the firms destroys jobs to reach the target  $\bar{n}(A, \epsilon)$  independently of employment at the beginning of the period. If employment at the beginning of the period is between the two thresholds  $\underline{n}(A, \epsilon)$  and  $\bar{n}(A, \epsilon)$ , the firm stays inactive. This pattern will be shown to be consistent with the observed patterns of employment dynamics in Section 4.4.3.

### 2.3 Bargaining and Wages

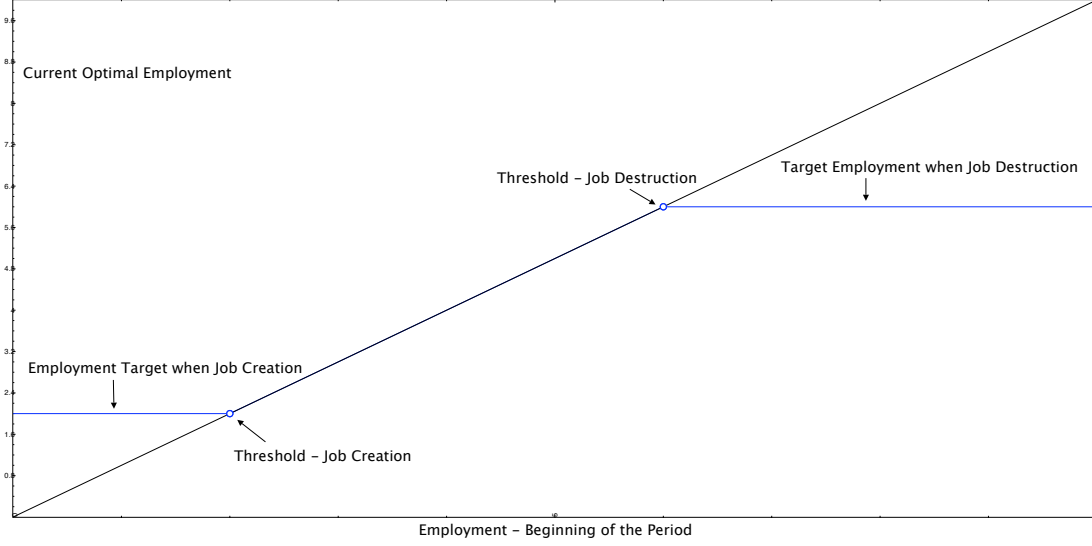
I adopt the wage bargaining solution of Stole and Zwiebel (1996) and Smith (1999) which generalizes the Nash solution to a setting with downward-sloping labor demand.<sup>11</sup> The firm cannot commit to long term contracts and costless renegotiation takes place every period. Workers are homogeneous and the current wage rate is the outcome of a sequence of bilateral negotiations between the firm

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<sup>11</sup>This approach has been used recently in a similar context by Acemoglu and Hawkins (2014) and Elsby and Michaels (2013). Brügemann et al. (2015) provides a micro-foundation of this wage equation based on the Rolodex Game.



Figure 1: Optimal decision rule conditional on shocks and past employment



and its employees, where each employee is regarded as the marginal worker. Wages are then the outcome of a Nash bargaining game over the marginal surplus. Workers and the firm each receive a given fraction,  $\gamma$  and  $1 - \gamma$ , of the marginal surplus.

A job-seeker threat point is the value achieved during the prospective employment period by disclaiming the current job opportunity and continuing to search, that is, the unemployment value  $U$ . Wages are set after employment has been determined.<sup>12</sup> Thus, hiring costs are sunk when wages are set. The firm's threat point is the value achieved by destroying a job, that is, the cost of destroying a job  $-\bar{c}$ . The solution takes the form,

$$(1 - \gamma) [W(A, \epsilon, n) - U] = \gamma [V_n(A, \epsilon, n) + \bar{c}], \quad (7)$$

where  $W(A, \epsilon, n)$  is the value of being employed in a firm with current state  $(A, \epsilon, n)$ . While employed, a worker receives a flow payoff equal to the bargained wage  $w(A, \epsilon, n)$ . A worker loses her job with some probability  $s$  next period,<sup>13</sup>

$$W(A, \epsilon, n) = w(A, \epsilon, n) + \beta E [sU + (1 - s)W(A\eta, \epsilon', n')],$$

where the expectation is taken over the distribution of shocks.

An unemployed worker receives flow payoff  $b$ , and she finds a job with probability  $f$ :

$$U = b + E [(1 - f)U + fW(A, \epsilon, n)].$$

<sup>12</sup>I implicitly assume there is no probationary period.

<sup>13</sup>It is unnecessary to characterize  $s$  because the value of working in a firm that is downsizing is equal to the value of unemployment.

In Appendix B.1, I show that the solution to Equation (7) is,

$$w(A, \epsilon, n) = (1 - \gamma) b + \bar{c} \gamma (1 - \beta (1 - f)) + \underline{c} \beta \gamma f + \frac{\gamma \alpha}{1 - \gamma(1 - \alpha)} (\epsilon A)^{1-\alpha} n^{\alpha-1}. \quad (8)$$

Wages are an affine function of labor productivity. Since  $\alpha < 1$ , employment and wages are inversely related conditional on the TFP level.

## 2.4 The Differential Impact of Transitory and Permanent Shocks

In Equation 8, permanent and transitory shocks enter symmetrically. Yet, they have a different impact on wages because of the behavior of employment. In this section, I explore the mechanisms underlining the differential impact of transitory and permanent shocks on wages.

A transitory shock increases only today's profits while a permanent shock increases both today's profits and future profits. This can be seen formally by examining the first-order conditions for a firm creating jobs:

$$\alpha (\epsilon \eta A)^{1-\alpha} (n + d)^{\alpha-1} - w(\eta A, \epsilon, n + d) - w_n(\eta A, \epsilon, n + d) (n + d) + \beta E(V_n(A\eta, \epsilon', n + d)) = \underline{c}. \quad (9)$$

The transitory shock  $\epsilon$  appears only on the derivative of the one-period profit function. Conversely, the permanent level of profitability  $A$  appears in the derivative of both the one-period profit function and the value function.

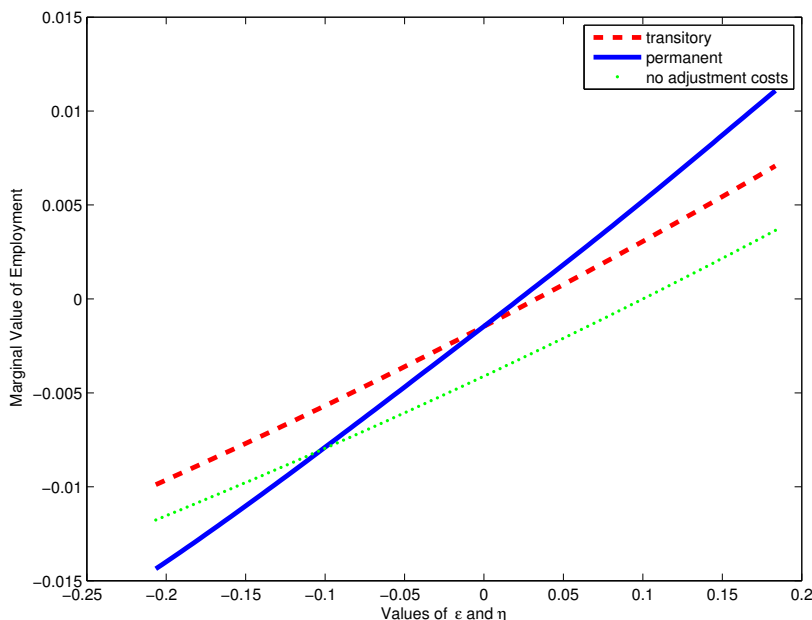
Figure 2 plots the left-hand side of Equation 9 for different values of the transitory shock  $\epsilon$  and the permanent shock  $\eta$ . It holds fixed employment  $n + d$  and the past level of profitability  $A$ . I use Section 4.3 to set parameter values. The left-hand side of Equation 9 is labeled the marginal value of labor: the increase in the value of the firm following a marginal increase in employment. When varying the transitory (permanent) shock, the permanent (transitory) shock is set to zero.

When shocks are positive, the marginal value of employment is always much higher for a permanent shock than for a transitory shock of the same magnitude. And when shocks are negative, the opposite result holds. Hence, the benefits of creating or destroying jobs after a transitory shock are on average smaller than when the shock is permanent.

If there is a positive transitory shock, the benefit of adding workers is relatively low because it is costly to create jobs and it will be costly to destroy jobs after the shock expires. Then, the firm decides to create only a few jobs. The marginal worker is more valuable this period and so the wage rises.

If there is a positive permanent shock, the firm pays the cost to create additional jobs. Since there are decreasing returns to labor, this offsets the shock, so that the marginal worker is not much

Figure 2: The marginal value of labor



more valuable than before the shock. Thus, the wage does not rise much but there is a substantial rise in employment.

The argument follows the same logic for negative shocks. Note that labor productivity is equal to output divided by employment such that the variations in labor productivity are smaller for a permanent shock compared with a transitory shock of the same magnitude.

Three assumptions are needed for the results to hold in this framework: (1) decreasing return to labor, (2) Nash bargaining with multiple workers as in Stole and Zwiebel (1996), and (3) costly employment adjustments. First, if the return to labor is constant instead of decreasing, the model is a search model with a linear technology and Nash bargaining.<sup>14</sup> Adding a worker to the firm has no impact on the productivity of the other workers. As result, persistent and transitory shocks have the same impact on wages. Second, if wages are set competitively, the model is similar to the model of Bentolila and Bertola (1990) and to that of Hopenhayn and Rogerson (1993), where shocks to idiosyncratic firms productivity have no impact on wages unless the shocks are correlated across firms. Third, in a frictionless labor market, employment fully adjusts to shocks. Then, labor productivity and wages are constant. Idiosyncratic shocks have no impact on wages and affect employment independently of their persistence.

To illustrate the last point, the dotted line in Figure 2 shows the marginal value of labor without adjustment costs. It does not depend on the persistence of the shocks. This is because in Equation (9), the term that differs between transitory and permanent shock  $E(V_n(A\eta, \epsilon', n + d))$  is always

<sup>14</sup>See Mortensen and Pissarides (1999) or Rogerson et al. (2005) for surveys.

set to zero in the absence of adjustment costs. Employment reacts similarly to both kinds of shocks. Because employment fully adjusts to shocks, labor productivity is constant. Because wages are a function of labor productivity, they are also constant. This shows that some form of adjustment costs is necessary for permanent and transitory shocks to have a differential effect on wages and employment.

I have just provided a simple model of firm employment and wage response to shocks of different persistence. Several assumptions were made to keep the model simple. Some could be relaxed without affecting the main mechanism. It is, for instance, intuitively robust to the introduction of aggregate shocks or a more flexible specification of adjustment costs.

### 3 Data and Descriptive Statistics

This section describes the data and then implements a simple econometric specification.

#### 3.1 Data Source

I use a balanced panel data of firms that covers the 1994-2000 period. This panel contains employment as well as standard accounting information on total compensation costs, value added, current operating surplus, gross productive assets, and so on. It was constructed by Jean-Marc Robin when he was a member of the Center for Research in Economics and Statistics. I refer the reader to Cahuc et al. (2006), who provide additional details on the sources and the construction of the data. It is exhaustive of all private companies with a sales turnover of more than 3.5 million francs (around 530,000 euros) and liable to corporate taxes under the standard regime and includes some other smaller firms. It corresponds to the profits declared by firms whose commercial, industrial or craft-work activity is conducted for lucrative purposes. It accounts for approximately 60% of the number of firms and 94% of the turnover.

The employment variable is defined as the arithmetic average of the number of workers in the firm at the end of each quarter of the fiscal year. The observed variable includes some part-time and temporary workers and rounding can be expected. Output is measured by value added, which is defined in the dataset as the difference between production and intermediate consumption net of all variations in stocks. Average labor compensation per firm is calculated as the ratio of the total real wage bill and the employment variable.

I drop firms with fewer than 30 employees since the sales threshold for inclusion in the sample is likely to be binding for many of these firms. Finally, I trim observations with average labor costs, labor productivity, or employment growth at the 99th quantile and the 1st quantile. I end up with a panel of 28,606 firms: 11,375 in the manufacturing sector, 3,330 in the construction section, 6,479 in the trade sector, and 7,422 in the service sector. The selected sample corresponds to 80% of total

employment, and 86% of total output. It covers only 20% of the total number of firms.<sup>15</sup>

### 3.2 Joint Dynamics of Output and Average Wages

This section examines a prediction of the theoretical model: a transitory shock to output has a stronger effect on wages than a permanent shock. Suppose the logarithm of value added  $\log O$  can be decomposed into a permanent component  $P$ , a serially uncorrelated component  $\nu^o$ , and classical measurement error  $r^o$ . The process for each firm  $i$  is

$$\log O_{it} = Z_{it}\varphi_t + P_{it} + \nu_{it}^o + r_{it}^o, \quad (10)$$

where  $t$  indexes time and  $Z_{it}\varphi_t$  is the interaction of a set of sectoral and time dummies. The permanent component  $P_{it}$  follows a martingale process of the form

$$P_{it} = P_{it-1} + \zeta_{it}^o, \quad (11)$$

where  $\zeta_{it}^o$  is serially uncorrelated. Logged output net of predictable firm components  $\log o_{it} = \log O_{it} - Z_{it}\varphi_t$  is in first differences:

$$\Delta \log o_{it} = \zeta_{it}^o + \Delta \nu_{it}^o + \Delta r_{it}^o. \quad (12)$$

Assume the (unexplained) growth of wages is

$$\Delta \log w_{it} = \tau \zeta_{it}^o + \phi \Delta \nu_{it}^o + \Delta r_{it}^w + \zeta_{it}^w, \quad (13)$$

where  $\zeta_{it}^w$  is a permanent wage shock independent of output and  $r_{it}^w$  is the wage measurement error. Permanent productivity shocks  $\zeta_{it}^o$  have a permanent impact on wages with a loading factor of  $\tau$ , and transitory productivity shocks  $\nu_{it}^o$  have an impact on wages with a loading factor of  $\phi \in [0, 1]$ . This econometric specification is borrowed from Blundell et al. (2008), who analyze the response of consumption to changes in income. It is possible to point-identify  $\sigma_{\zeta^o}^2$ ,  $\sigma_{\zeta^w}^2$ , and  $\tau$ <sup>16</sup>

$$\begin{aligned} \sigma_{\zeta^o}^2 &= E [\Delta \log o_{it} (\Delta \log o_{it-1} + \Delta \log o_{it} + \Delta \log o_{it+1})] \\ \sigma_{\zeta^w}^2 &= E [\Delta \log w_{it} (\Delta \log w_{it-1} + \Delta \log w_{it} + \Delta \log w_{it+1})] - \tau^2 \sigma_{\zeta^o}^2 \\ \tau &= \frac{E [\Delta \log w_{it} (\Delta \log o_{it-1} + \Delta \log o_{it} + \Delta \log o_{it+1})]}{E [\Delta \log o_{it} (\Delta \log o_{it-1} + \Delta \log o_{it} + \Delta \log o_{it+1})]}, \end{aligned}$$

<sup>15</sup>See Gourio and Roys (2014) for an analysis of small-firm dynamics in France.

<sup>16</sup>The theoretical moments are derived in Appendix A.

Table 1: Transmission of Output Shocks into Average Wages

	Full Sample	Manufact.	Construct.	Trade	Services	Small Firms	Large Firms
<b>Permanent</b>							
$\tau$	0.0356 (0.0060)	0.0361 (0.0083)	0.0157 (0.0217)	0.0328 (0.0115)	0.0428 (0.0145)	0.0409 (0.0145)	0.0210 (0.0067)
Share of wage growth variance							
$\tau^2 \sigma_{\zeta_o}^2$	0.14%	0.05%	0.03%	0.27%	0.20%	0.1%	0.18%
<b>Transitory</b>							
$\phi \geq$	0.5492 (0.0312)	0.5064 (0.0412)	0.4936 (0.0586)	0.4784 (0.0730)	0.7043 (0.0666)	0.5607 (0.0293)	0.5401 (0.0370)

Note: Values are pooled over all years and firms. Bootstrapped standard errors are reported in parenthesis below the coefficient estimate.

It is not possible to separately identify measurement error in output from transitory shocks to output. Only the sum is identified:

$$\sigma_{\nu_o}^2 + \sigma_{r_o}^2 = -E(\Delta \log o_{it} \Delta \log o_{it+1});$$

$\phi$  is not point-identified. However, a lower bound is given by

$$\begin{aligned} \phi &\geq \frac{E(\Delta \log w_{it} \Delta \log o_{it+1})}{E(\Delta \log o_{it} \Delta \log o_{it+1})} \\ &= \phi \frac{\sigma_{\nu_o}^2}{\sigma_{\nu_o}^2 + \sigma_{r_o}^2}. \end{aligned}$$

Table 1 displays the pooled estimates.<sup>17</sup> It also displays the variance decomposition of wage growth.

The estimates of  $\tau$  are marginally significant and the effect is economically small: A 1% permanent output shock induces a 3.56% permanent change in wages over the whole sample. Further, the transmission of permanent shocks to output explains less than 1% of wages growth.

Conversely, average wages respond strongly to transitory shocks to value added. Because of the presence of measurement errors, I can provide only a lower bound at this stage. But the loading factor is larger than 54% over the whole sample. There are differences in the transmission of transitory shocks by sector. It is particularly intuitive to observe a high transmission in the services sector (70.43%), where tips and other forms of variable compensation are prevalent. Finally, I split the sample into two subsamples depending on whether a firm's average employment over the period is above or below the median (equal to 51). There seems to be slightly more transmission of both

<sup>17</sup>The equally weighted minimum distance estimates are very similar.

transitory and permanent shocks in smaller firms but the differences are not large.

These empirical results are in line with the model's predictions. Yet the parameters  $\phi$  and  $\tau$  are reduced-form parameters: They are used to capture the important features of the joint distribution of the observed output and wage paths. In the next section, I use the information provided by these parameters (and others) to pin-down the structural parameters and evaluate the quantitative performance of the model. I then use these to infer the misallocation of labor resulting from adjustment costs.

## 4 Quantitative Evaluation

This section proposes a simple estimation of my model using indirect inference that requires that the model to account for a number of salient features of firm-level data.

### 4.1 Indirect Inference

Like calibration, indirect inference works by selecting a set of statistics of interest, which the model is asked to reproduce. These statistics are called sample auxiliary parameters  $\hat{\Psi}$  (or target moments). For an arbitrary value of  $\theta$ , I use the model to generate  $S$  statistically independent simulated datasets and compute simulated auxiliary parameters  $\Psi^s(\theta)$ . The parameter estimate  $\theta$  is then derived by searching over the parameter space to find the parameter vector that minimizes the criterion function:

$$\hat{\theta} = \arg \min_{\theta \in \Theta} \left( \hat{\Psi} - \frac{1}{S} \sum_{s=1}^S \Psi^s(\theta) \right)' W \left( \hat{\Psi} - \frac{1}{S} \sum_{s=1}^S \Psi^s(\theta) \right),$$

where  $W$  is a weighting matrix and  $\Theta$  the estimated parameters space. This procedure generates a consistent estimate of  $\theta$ . Following Blundell et al. (2008), I use a diagonal weighting matrix  $W = \text{diag}(V^{-1})$ , where  $V$  is the variance-covariance matrix of the sample auxiliary parameters. This weighting scheme allows for heteroskedasticity and has better finite sample properties than the optimal weighting matrix (see Altonji and Segal, 1996). The minimization is performed using the Nelder-Mead simplex algorithm. I used 5,000 different starting values to find the global minima. To simulate the model, I draw from the stationary distribution. Firms exit exogenously to insure the existence of a stationary distribution of employment and profitability in an economy with permanent shocks. They restart at a productivity level such that the power law exponent in the model and in the data coincide for firms with more than 170 employees. The exogenous exit rate is set so that the average firm has 7.5 employees, as is the case in France.

I incorporate classical measurement errors in (log) employment, output and the wage bill; the standard deviations are, respectively  $\sigma_{MRN}$ ,  $\sigma_{MRO}$ ,  $\sigma_{MRW}$ . Since the observed average wage variable is calculated as the ratio of the wage bill and employment, simulated average wages incorporate

measurement errors both in wages and in employment.

The full set of estimated parameters is  $\theta = (\alpha, b, c, \gamma, \sigma_\eta, \sigma_\epsilon, \sigma_{MRN}, \sigma_{MRO}, \sigma_{MRW})$ . I assume that adjustment costs are symmetric,  $c = \underline{c} = \bar{c}$ . The main reason is that identifying asymmetries proved difficult in practice. Intuitively, a shrinking plant compares the benefit of the last job retained with both the cost of destroying this job and the expected cost of creating the same job again in the future. For an expanding plant, the cost of the last job created is compared with the job creation cost and the expected cost of destroying that job in the future. In other words, a firm always takes into account both the cost of job creation and destruction regardless of whether it creates or destroy job. The relative importance given to these costs depends on the discount factor and expectations over the future. Since the discount factor is not well identified (this is a generic problem in this class of model), I keep adjustment costs symmetric. Estimating the model with either only job creation costs or only job destruction costs leads to a very similar fit and policy experiments.

The standard errors are obtained using 500 bootstrap repetitions. In each bootstrap repetition, a new set of data is produced by randomly selecting blocks of observations.<sup>18</sup> In the  $b$ th bootstrap repetition, auxiliary parameters  $\hat{\Psi}^b$  are calculated using the new set of data. An estimator  $\hat{\theta}^b$  is found by minimizing the weighted distance between the recentered bootstrap auxiliary parameters  $(\hat{\Psi}^b - \hat{\Psi})$  and the recentered simulated auxiliary parameters  $(\frac{1}{S} \sum_{s=1}^S \Psi^s(\theta^b) - \frac{1}{S} \sum_{s=1}^S \Psi^s(\hat{\theta}))$ :

$$\begin{aligned} \hat{\theta}^b = \arg \min_{\theta^b \in \Theta} & \left( \left( \hat{\Psi}^b - \hat{\Psi} \right) - \left( \frac{1}{S} \sum_{s=1}^S \Psi^s(\theta^b) - \frac{1}{S} \sum_{s=1}^S \Psi^s(\hat{\theta}) \right) \right)' \\ & \times W \left( \left( \hat{\Psi}^b - \hat{\Psi} \right) - \left( \frac{1}{S} \sum_{s=1}^S \Psi^s(\theta^b) - \frac{1}{S} \sum_{s=1}^S \Psi^s(\hat{\theta}) \right) \right). \end{aligned}$$

## 4.2 Identification

Identification of the model's parameters is achieved by a combination of functional form and distributional assumptions. It is difficult to prove, but the intuition is straightforward.

Average labor productivity and wages identify  $\alpha$  in the production function and the value of home production  $b$ . To understand the mechanism at work, consider a firm facing no adjustment costs  $c = 0$  and with a constant wage  $\gamma = 0$ . Its decision consists of equalizing at every period the marginal productivity of labor with wages  $w = b$ . Simple calculations show that labor productivity is then

$$p = \frac{O}{n} = \frac{A^{1-\alpha} n^\alpha}{n} = \frac{b}{\alpha}$$

Obviously, a high wage rate reduces employment and increases labor productivity. A high  $\alpha$  pushes firms to expand, which reduces labor productivity. In the full model, numerical simulation of the

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<sup>18</sup>See Hall and Horowitz (1996) for more details on the block-bootstrap. The sampling is random across firms but is done in block over the time dimension.



model for different values of  $\alpha$  and  $b$  shows the same patterns. The relationship between wages and  $\alpha$  is ambiguous. The less concave demand and the production technology, the lower the rate of diminishing returns as workers are added and, as a result, these workers are able to negotiate a larger share of the surplus. On the other hand, the less concave demand and the production technology, the larger the size of the firm conditional on  $A$  and the lower the productivity of labor.

Workers' bargaining power  $\gamma$  affects the correlation between output growth and wage growth. It has a negative impact on the reallocation rate: When wages are more flexible, firms hire (fire) less when productivity goes up (down) because wages simultaneously go up (down).

To separately identify adjustment costs  $c$  from the variance of idiosyncratic shocks (both transitory and permanent), consider the reallocation rate  $E(|\Delta \log n|)$ , which measures the number of jobs created or destroyed, and the inaction rate  $E(I\{\log n = \log n_{-1}\})$ , which counts periods in which employment stays constant. Adjustment costs decrease the propensity to create and destroy jobs: The reallocation rate is low when adjustment costs are high. Similarly, adjustment costs increase inaction. The volatility of shocks has the opposite effect on these two moments: A highly volatile environment triggers more job reallocation and less inaction. Hence, it helps to have another moment that goes in the same direction when either parameter increases. The variance of labor productivity  $E(\Delta \log p)^2$  achieves exactly this since it increases when either  $c, \sigma_\epsilon$  or  $\sigma_\eta$  increase.

To separately identify the variance of permanent and transitory shocks, I use the statistic  $E\left(x_{it} \sum_{j=-1}^1 y_{it+j}\right)$  where  $x$  and  $y$  are employment growth, wages growth or output growth as in Section 3.2. It captures the correlation between  $x$  and  $y$  due to permanent shocks and not transitory shocks.  $E(x_{it}y_{it-1})$  captures the correlation between  $x$  and  $y$  due to transitory shocks and not permanent shocks. To separately identify the variance of measurement error in output  $\sigma_{MRO}$  from true transitory variation in profitability  $\sigma_\epsilon$ , I use the fact that measurement errors in output affect neither the reallocation rate nor the correlation between wages and labor productivity while transitory shocks to profitability do. The approach is similar for measurement errors in employment and wages.

### 4.3 Structural Parameter Estimates

Some parameters are calibrated. The discount factor  $\beta$  is set to 0.9452 so that the annual real interest rate is 5 percent and the exogenous exit rate is 0.75 percent. The probability of finding a job  $f$  is set to 0.35 which is an average of the annual transition from unemployment to employment observed over the period of observations (the data are from Enquete Emploi, a survey of about 1/300th of the French population conducted annually by the Institute of Statistics and Economic Studies in France, INSEE). Table 2 reports the parameter estimates.

The 25 sample auxiliary parameters  $\hat{\Psi}$  are listed in Table 3. The standard deviations are obtained through bootstrapping on the original panel. The last column in Table 3 reports the simulated moments. Most moments were discussed in Section 3 and Section 4.2. The inaction rate

Table 2: Structural Parameter Estimates

Parameters	Estimate	Standard Error
$\alpha$	0.3697	(0.0060)
$b$	0.1409	(0.0009)
$c$	0.0244	(0.0013)
$\gamma$	0.5201	(0.0054)
$\sigma_\eta$	0.1979	(0.0046)
$\sigma_\epsilon$	0.2880	(0.0099)
$\sigma_{MRN}$	0.0222	(0.0006)
$\sigma_{MRO}$	0.0071	(0.0001)
$\sigma_{MRW}$	0.0705	(0.0056)

is defined as the fraction of employment growth lower than 1% in absolute value.

Adjustment costs per worker are estimated to be around 17% of the average worker wage rate or about two months of the annual wage bill per worker. In other words, a firm has to pay an extra year of average wages for every six jobs created or destroyed. While significant, it is lower than what could be expected given the stringent labor market regulation in France. An examination of the data on worker flows provides an explanation. There are two types of regular employment contracts in France: indefinite-term contracts (CDIs) and fixed-duration contracts (CDDs).<sup>19</sup> Almost 60% of the exit from employment can be attributed to the end of CDDs which are by definition flexible. Less than 7% of the exits from employment are made through a layoff procedure and less than 1% through a layoff for economic reasons. Hence by using CDDs, French firms can destroy jobs at relatively low costs. And, CDDs are also the most common method of hiring, representing more than 2/3 of all hires. With almost 15% of workers on CDDs, firms can potentially eliminate a significant share of their labor force without institutional costs.<sup>20</sup> While not directly comparable to other structural estimates of adjustment costs because countries and methods differ, the magnitudes seem in line.<sup>21</sup> However, the values are lower than the typical calibrated value of firing costs in macroeconomic models.<sup>22</sup> I return to this issue in the next section devoted to policy experiments. For now, note that despite of fairly modest estimated adjustment costs, the model reproduces the observed reallocation rate (0.0901 in the model and 0.0986 in the data) and inaction rate (0.1576

<sup>19</sup>A CDD can last up to 18 months. Firms must justify their creation by invoking, for instance, an increase in activity or the replacement of a temporarily sick worker.

<sup>20</sup>A caveat is that firms cannot terminate a CDD before its expiration date for economic reasons. The worker can quit if he finds a CDI.

<sup>21</sup>For instance, Rota (2004) estimates the median level of fixed (not linear) costs to be around 15 months labor cost in Italy. Using Compustat, Bloom (2009) estimates linear adjustment costs in the U.S. of about 1.8% of annual wages, and a fixed cost of around 2.1% of annual revenue with no quadratic adjustment costs. Aguirregabiria and Alonso-Borrego (2014) find that the linear component is the most important part of labor adjustment costs: Around 15% of a worker's annual salary for hiring and around 50% for firing. All these papers assume wages do not change with idiosyncratic profitability shocks.

<sup>22</sup>See Hopenhayn and Rogerson (1993).

Table 3: Auxiliary Parameters

Moments	Data	Std. Err.	Model
$E\left(\frac{o}{n}\right)$	0.2899	(0.0008)	0.2834
$E(w)$	0.1455	(0.0003)	0.1578
$E\left(\frac{wn}{o}\right)$	0.5621	(0.0030)	0.5652
Reallocation	0.0986	(0.0005)	0.0901
Inaction	0.1601	(0.0010)	0.1576
$E(\Delta \log n \Delta \log n)$	0.0321	(0.0004)	0.0224
$E(\Delta \log o \Delta \log o)$	0.0676	(0.0016)	0.1045
$E(\Delta \log w \Delta \log w)$	0.0338	(0.0010)	0.0257
$E(\Delta \log n \Delta \log n_{-1})$	0.0023	(0.0016)	0.0035
$E(\Delta \log o \Delta \log o_{-1})$	-0.0119	(0.0007)	-0.0344
$E(\Delta \log w \Delta \log w_{-1})$	-0.0137	(0.0006)	-0.0125
$E\left(\Delta \log n \sum_{-1}^1 \Delta \log n_i\right)$	0.0258	(0.0004)	0.0294
$E\left(\Delta \log o \sum_{-1}^1 \Delta \log o_i\right)$	0.0369	(0.0007)	0.0359
$E\left(\Delta \log w \sum_{-1}^1 \Delta \log w_i\right)$	0.0053	(0.0002)	0.0006
$E(\Delta \log w \Delta \log o)$	0.0146	(0.0006)	0.0355
$E(\Delta \log n \Delta \log o)$	0.0161	(0.0004)	0.0348
$E(\Delta \log w \Delta \log n)$	-0.0156	(0.0003)	0.0058
$E(\Delta \log w \Delta \log o_{-1})$	-0.0065	(0.0003)	-0.0132
$E(\Delta \log n \Delta \log o_{-1})$	0.0041	(0.0002)	0.0049
$E(\Delta \log n \Delta \log w_{-1})$	0.0053	(0.0002)	0.0010
$E\left(\Delta \log n \sum_{-1}^1 \Delta \log w_i\right)$	-0.0036	(0.0002)	0.0005
$E\left(\Delta \log o \sum_{-1}^1 \Delta \log n_i\right)$	0.0214	(0.0004)	0.0306
$E\left(\Delta \log o \sum_{-1}^1 \Delta \log w_i\right)$	0.0013	(0.0002)	0.0023
$\frac{E(\Delta \log w \Delta \log o_{-1})}{E(\Delta \log o \Delta \log o_{-1})}$	0.5462	(0.0291)	0.5771
$\frac{E(\Delta \log o \sum_{-1}^1 \Delta \log w_i)}{E(\Delta \log o \sum_{-1}^1 \Delta \log o_i)}$	0.0356	(0.0061)	0.0485

in the model and 0.1601 in the data).

The estimated elasticity of output with respect to employment  $\hat{\alpha}$  is 0.3697. Yet,  $\alpha$  is not the share of labor cost in output, which depends on workers' bargaining power  $\gamma$ . To understand the intuition, consider a firm that faces no adjustment costs:  $c = 0$ . There the share of labor cost in output is

$$\frac{wn}{o} = \frac{\alpha}{1 - \gamma(1 - \alpha)}.$$

In the full model, this expression would not hold, but the share of labor cost in output would still be increasing in both  $\alpha$  and  $\gamma$ . Hence, the elasticity of output with respect to employment  $\alpha$  is lower than the share of labor in output unless workers have no bargaining power  $\gamma = 0$ . Indeed, the model is able to match the average value of labor productivity and wages. Despite the low elasticity of output with respect to employment, the model predicts a share of labor cost in output of 0.5584, which is close to the share of labor cost in output of 0.5632 in the data. This is because, as explained above, it also depends on workers' bargaining power and workers get about half of the marginal surplus  $\hat{\gamma} = 0.5201$ .<sup>23</sup>

Using the estimates of the shocks process, a variance decomposition attributes 18.96% of profitability growth variations to permanent shocks and the remaining to transitory shocks. A similar decomposition for output growth reveals that 54.57% of the variance of output growth can be attributed to permanent shocks. To understand the intuition, note that the variance of output growth may be written as

$$\begin{aligned} \text{Var}(\Delta \log o) &= (1 - \alpha)^2 (\sigma_\eta^2 + 2\sigma_\epsilon^2) + \alpha^2 \text{Var}(\Delta \log n) + 2\sigma_{\text{MRO}}^2 \\ &\quad + (1 - \alpha) \alpha (2\text{Cov}(\Delta \log n, \log \eta) + 2\text{Cov}(\Delta \log n, \Delta \log \epsilon)) \end{aligned}$$

Even though transitory shocks to profitability explain a larger fraction of profitability variance, they explain a smaller fraction of output variance because employment variations amplify permanent shocks more than transitory shocks.

The variance of measurement errors in employment, wages and output is significant for all. Consider the following decomposition. Let  $\log x = \log x^{\text{model}} + \text{MR}$  where  $x$  is observed employment, wages, or output and  $\text{MR}$  is measurement error. Then  $\text{Var}(\Delta \log x) = \text{Var}(\Delta \log x^{\text{model}}) + 2\sigma_{\text{MR}}^2$  and the variance explained by the model is the ratio  $\frac{\text{Var}(\Delta \log x^{\text{model}})}{\text{Var}(\Delta \log x)}$ . According to the model's estimates, the model explains 96.92, and 70.59%, respectively, of the variance of employment growth, output growth, and wages growth. We hence find that the model attribute a very small share of the variance of employment and output growth to measurement errors. By contrast, a much larger

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<sup>23</sup>Using a related model, Rota (2004) estimates an even lower coefficients of 0.12 with a panel of Italian firms. In the literature,  $\alpha$  is often calibrated to 2/3, which corresponds to the share of labor in value-added in national accounts.

amount of measurement is required to fit wages growth variations. It could appear surprising because wages are typically a better recorded variable. In the model, the only source of variations in wages is labor productivity. In reality, several factors outside the model presumably contribute to observed wages variations and they are attributed to measurement error in the model.

## 4.4 Model Fit

### 4.4.1 Matched Moments

Overall the model matches the auxiliary parameters fairly well. It correctly predicts a large impact of transitory profitability shocks on wages: A measure of the transmission of transitory profitability shocks into wages  $\frac{E(\Delta w \Delta o_{-1})}{E(\Delta o \Delta o_{-1})}$  is 0.54 in the data and 0.57 in the model. The model also correctly predicts a weak impact of permanent profitability shocks on wages: A measure of the transmission of permanent profitability shocks into wages  $\frac{E(\Delta \log o \sum_{i=-1}^1 \Delta \log w_i)}{E(\Delta \log o \sum_{i=-1}^1 \Delta \log o_i)}$  is 0.04 in the data and 0.05 in the model.

The model overestimates the variance of output growth  $E(\Delta \log o \Delta \log o_i)$  to be 0.10, while it is 0.07 in the data. This is because there is a trade-off between reproducing this variance and the variance of employment growth  $E(\Delta \log n \Delta \log n_i)$ , which is equal to 0.03 in the data and 0.02 in the model. The model does a reasonable job of capturing the covariances between the different variables. Yet, they tend to be larger in the model than in the data. Increasing the variance of measurement error in employment reconciles the model and the data for these moments. While this would allow the model to fit most covariances well, it would substantially worsen the fit of both the inaction rate and the variance of employment growth. When the measurement error in employment increases, the fraction of employment variation smaller than 1% decreases noticeably. The introduction of quadratic adjustment costs could allow me to simultaneously match both of these statistics. This is because quadratic adjustment costs provide an incentive for the firm to make only small adjustments to their labor force. However, this would substantially complicate the numerical solution of the model.

### 4.4.2 Bivariate Correlations

This section examines the ability of the model to reproduce the observed correlations among employment, output, and wages in level and in first differences at various lags. Table 4 reports the results. The model, despite its parsimony, does a reasonably good job of matching the different correlations even though some were not directly targeted in the estimation.

The correlations in first differences are in line with the dynamics of employment and wages implied by the model. The correlation between output changes and employment changes at different lags is positive and slowly decays because employment changes are persistent with adjustment costs. The correlation between output and wages is positive initially. In the aftermath of a positive

Table 4: Bivariate Correlations

		First-difference				Level
		$t$	$t - 1$	$t - 2$	$t - 3$	
Output and employment	Data	0.3451	0.0936	0.0371	0.0164	0.9073
	Model	0.7160	0.0897	0.0646	0.0393	0.9986
Output and wages	Data	0.3047	-0.1192	-0.0009	0.0052	0.3911
	Model	0.6878	-0.3952	-0.0157	-0.0142	0.0948

Table 5: Distribution of (Absolute) Employment and Output Growth Rate

Bins	Output		Employment	
	Data	Model	Data	Model
$\geq 0.20$	31.4	52.0	25.6	14.7
$[0.10, 0.20)$	21.1	22.7	15.5	12.4
$[0.01, 0.1)$	41.9	22.8	51.0	59.4
$< 0.01$	5.6	2.5	7.9	13.5

profitability shock and before labor adjustment, the rise in labor productivity transmits into wages. Yet, there is a tendency for wages to decrease as employment reacts, which explains the negative correlation observed both in the data and in the model at  $t - 1$  and  $t - 2$ . Both contemporaneous correlations are positive but higher in the model than in the data. These were directly targeted in the estimation so it is the outcome of matching jointly the auxiliary parameters instead of matching only that particular moment. One factor, as discussed above, is that the estimated exogenous process for profitability has a higher variance than needed to explain the output growth rate.

The correlations in level support the model predictions regarding permanent shocks. The correlation between output and employment in level is 0.9 in the data and 0.99 in the model, which suggests that permanent output shocks are eventually transmitted to employment. This correlation is too strong in the model, which illustrates the limitation of using the same production production for all firms while there exist some permanent differences in the capital to labor ratio across firms. On the other hand, the correlation in level between output and wages is positive but significantly lower and equal to 0.39. This is consistent with the idea that after a shock, wages might be temporarily high or low but they eventually return to their initial level. I return to the link among wages, employment, and output in Section 6.1.

#### 4.4.3 Distribution of Employment and Output Changes

Table 5 reports the distribution of net employment growth and net output growth at the firm level both in the data and in the model. It shows the fraction of firms with net absolute variations above 20%, between 20% and 10%, between 10% and 1%, and below 1%.

Two facts stand out from the distribution of employment changes compared with output changes. First, there is a significant amount of relatively small net employment adjustment: 16% of firms have absolute employment changes of less than 1%, while the corresponding number is below 6% for output. Hence, employment growth rates display high spikes around zero compared with the smooth patterns of sales variations observed. Second, these small adjustments are complemented by significant bursts of job creation and destruction: More than a third of firms either contract or expand employment by more than 10% in a given year.

Our parsimonious model does a reasonable job of reproducing these patterns. As shown in Section 2.2, it may be optimal to maintain the same number of employees with linear adjustment costs. And employment changes tend to be concentrated in a single period. Yet, the model does not produce enough large employment adjustment compared with the data. Adding lumpy adjustment costs and a richer shock process could potentially improve the model fit in both dimensions. Yet, it would substantially complicate the numerical solution as explained in Section 2.2.

#### 4.4.4 Labor Income Share and Productivity

The model has particular implications for the relationship between the share of labor in value added and labor productivity, which differs from other models of firm dynamics. To be clear, the share of labor costs in output is  $\frac{wn}{o}$  while labor productivity is  $\frac{o}{n}$ . I run a regression of the latter (in log) against the former (in log) in the real data, in the simulated data from the baseline model, and finally in the simulated data from a model with constant wages. I obtain  $-0.4891$  in the data,  $-0.4843$  in the baseline model, and  $-0.9749$  in the competitive model.

A frictionless model of firm dynamics à la Hopenhayn (1992) would imply a constant marginal productivity of labor across firms and a common competitive wage rate  $w$ . The implied observed coefficient should then be zero as deviations from the mean can only be attributed to noise. A model with adjustment cost à la Hopenhayn and Rogerson (1993) and competitive wages (or more generally rigid) would lead to a coefficient of precisely minus one. This coefficient is slightly attenuated because of measurement errors.

Finally, the model with bargaining attenuates this coefficient toward 0. Indeed,  $w$  comoves with labor productivity, and a positive productivity shock that is not fully absorbed by employment, leads to temporarily high wages, which dampen the one-to-one relationship between the share of labor in output and labor productivity. The model does a surprisingly good job of matching this coefficient in the data even though it was not directly targeted in the estimation.

#### 4.4.5 Joint Dynamics of Profitability and Average Wages

Section 3.2 examined a key prediction of the theory above. A permanent shock should impact wages more than a transitory one. However, it was so by estimating the reduced-form response of output shocks. Using the structural parameter estimates, I can recover an estimate of profitability and

Table 6: The Impact of Output and Profitability Shocks on Average Wages

	Output	Profitability A	Profitability B
<b>Permanent Shocks</b>			
$\tau$	0.0356 (0.0061)	0.1098 (0.0081)	0.1126 (0.0089)
<b>Transitory Shocks</b>			
$\phi$ (lower bound)	0.5491 (0.0291)	0.5707 (0.0240)	0.5267 (0.0210)

estimate the impact of permanent and transitory shocks. Using observed output, and employment, and  $\hat{\alpha}$ , I recover an estimate of  $(1 - \alpha) (\log A_{it} + \log \epsilon_{it}) - \alpha m r n_{it} + m r o_{it}$  and estimate its impact on average wages as in Section 3.2. I also calculate a Solow residual by subtracting from the previous estimate  $\frac{1}{3} \log k$ , where  $k$  is defined as gross productive assets in the data. It is an attempt to account for physical capital variations, even though it is not explicitly modeled. The results are reported in Table 6. I label the first and second measures, respectively, Profitability A and Profitability B.

The results of Section 3.2 based on output and the one based on profitability reported here are consistent. The estimated transmission of transitory shocks is above 50% in all three specifications. The stability of the estimates was expected since employment should react little to transitory shocks. Transitory output variations are consequently a good proxy for transitory profitability variations. The transmission of permanent shocks is significantly higher for profitability shocks than output shocks. It is multiplied by a factor of 3 to reach about 10%. This also makes sense since permanent profitability shocks are magnified by the response of employment. Hence, when I was used as a proxy for profitability, I overestimated the variance of permanent shocks and, as a result, underestimated the response of average wages to permanent profitability shocks.

## 5 Policy Experiments

In this section, I use the model estimates to infer the aggregate effects of adjustment costs. From the point of view of a social planner, they misallocate labor across firms and hence reduce productivity.

### 5.1 Main Results

I perform the following experiment. I solve the allocation problem,

$$\begin{aligned}
 & \max_{\{n(A, \epsilon)\}_{\epsilon=-\infty}^{\infty}} \int (A\epsilon)^{1-\alpha} n(A, \epsilon)^{\alpha} d\Psi(A, \epsilon) \\
 s.t. \quad & \int n(A, \epsilon) d\Psi(A, \epsilon) \leq N,
 \end{aligned}$$



Table 7: Policy Experiment: Steady-State Effects of Removing Adjustment Costs

	(1) Baseline	(2) Transitory (only)	(3) Permanent (only)	(4) Constant Wage
Reallocation gains	0.9720	1.4387	0.2053	0.5676
St. Dev. log labor prod.	0.1774	0.2058	0.0846	0.1402
Reallocation rate	0.0986	0.0173	0.1814	0.1347
Reallocation rate, $c = 0$	0.3624	0.3950	0.2865	0.3624

where  $n(A, \epsilon)$  is the employment of firms with productivity  $A$ , transitory shock  $\epsilon$ , and  $\Psi$  is the stationary distribution of permanent and transitory productivities. Finally,  $N$  is total employment in the baseline economy. This can be interpreted as the pure productivity gain from reallocation - how much of an increase in output can be obtained, holding total employment constant, by reallocating labor across firms.<sup>24</sup> The results are reported in Table 7.

The gain in total output, holding total labor constant, is almost 1%, which is significant. This effect is lower than the results reported in Hopenhayn and Rogerson (1993) and most of the subsequent literature. An important reason is that most of this work considers much larger labor adjustment costs based on official measures of firing costs, while I chose a level of adjustment costs (and other parameters) to reproduce moments of output, employment and average wages of a firm-level dataset. For instance, Hopenhayn (2014) finds that a firing cost of two years of salaries leads to a TFP cost of 2.8% using a very similar model. I perform a similar calculation by setting  $c = b$  and  $\gamma = 0$  so that wages are set competitively and job creation and destruction are equal to a year of wages.<sup>25</sup> I obtain a productivity gain of 2.15%, which is lower but close to that of Hopenhayn (2014).

Removing adjustment costs in the model leads to equalizing the marginal productivity of labor across firms. How much of the observed dispersion can the model explain? I decompose the logged labor productivity variance into a between-firm variance (that remains constant over time) and a within-firm variance (that varies over time). The model speaks only to within-firm dispersion of labor productivity since firms allow labor productivity to fluctuate around a fixed interval. Formally, I consider a simple variance components model of the form:

$$\log p_{it} = \mu + u_i + v_{it},$$

where  $\mu$  is an intercept,  $u_i \sim I.I.D(0, \sigma_u^2)$ ,  $v_{it} \sim I.I.D(0, \sigma_v^2)$ , and  $u_i$  and  $v_{it}$  are independent of each

<sup>24</sup>When I remove adjustment costs, the results are unchanged if I replace the bargaining solution with competitive wages. This is because the over-hiring effect affects all firms the same way. In the frictionless economy, the ratio of employment with competitive wages compared with Stole-Zwiebel wage bargaining is equal to  $(1 - \gamma(1 - \alpha))^{\frac{1}{1-\alpha}}$ , which is independent of firm productivity.

<sup>25</sup>I let  $c$  equal one year of wages (instead of two) since Hopenhayn and Rogerson (1993) and Hopenhayn (2014) consider economies without job creation costs, while I consider symmetric adjustment costs.

other.  $\sigma_u^2$  corresponds to differences that remain constant over time (permanent differences between firms);  $\sigma_v^2$  are differences that vary randomly over time and units (residual variance). I estimate that the variance of within-firm logged productivity dispersion is 0.0368.<sup>26</sup> The model is thus able to explain 85% of the within-firm variance. It is actually possible to increase labor adjustment costs (the parameter  $c$ ) to match exactly the within-firm variance of labor productivity. But this would substantially deteriorate the fit of the other moments. For example, it would imply implausibly low reallocation rates and high inaction rates.

Despite being fairly modest, adjustment costs reduce job reallocation a lot and imply substantial inaction: The reallocation rate increases from 9.86% to 36.24% and the inaction rate decreases from 15.76% to 1.68% when I remove adjustment costs. It is fairly common to find that small frictions have large effects on investment decisions.

Overall, the current framework finds significant but not large effects of adjustment costs on efficiency. This contrasts with the perceived lack of reallocation in the French labor market. There is an important caveat to this result. The analysis does not account for the existence of match quality effects, which might be important since most hires are made through short-term contracts. Indeed, some firms may have to terminate good matches by non-renewing some short-term contracts and retain bad matches for employees who are protected by stringent regulations for layoff procedures.

## 5.2 Persistence of Shocks

To understand better the results, column (2) and column (3) of Table 7 report, respectively, the gains of removing adjustment costs in a economy where firms experience only transitory shocks and where firms experience only permanent shocks. I adjust the variance of shocks so that the different economies considered all have the same variance of output growth with adjustment costs. This isolates the effect of the persistence of shocks from the effect of the variance of shocks. The gains of removing adjustment costs are much higher when shocks are purely transitory: The reallocation gains are about 50% higher than in the baseline economy. In an economy with only permanent shocks, the reallocation gains are an order of magnitude lower or about 80% lower than in the baseline economy. One way to understand this effect is to calculate the dispersion of labor productivity, which is a commonly used metric to assess the extent of misallocation.<sup>27</sup> Using this metric, I would indeed conclude that transitory shocks lead to more misallocation than permanent shocks. This is intuitive: Transitory shocks are absorbed into labor costs more than into employment, leading to temporarily too high or too low labor productivity. Permanent shocks lead to more employment adjustments and as a result labor productivity is less volatile when shocks are very persistent.

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<sup>26</sup>The between-firm logged productivity dispersion is 0.1152. It is hence four times larger than the within-firm component. There are several ways to generate between-firm productivity dispersion in the model. One possibility is to add labor overhead to the production function  $(A\epsilon)^{1-\alpha} (n-f)^\alpha$ , where  $f$  is labor overhead. Another is to allow for mark-ups heterogeneity and consider the production function  $(A\epsilon)^{1-\alpha-\xi} n^{\alpha+\xi}$ , where  $\xi$  differs across firms.

<sup>27</sup>See Hsieh and Klenow (2009).

As a result, the reallocation gains from removing adjustment costs are higher in economies with transitory shocks.

### 5.3 Wage Bargaining

The last variant considers an economy where wages are constant across time and across firms by setting the Nash bargaining parameter to zero  $\gamma = 0$ . The results are reported in the last column of Table 7. To keep average wages equal to expected wages in the benchmark economy, I increase the value of home production  $b$ . The benefits of removing adjustment costs are about 40% lower in the economy where wages are constant.

To understand the result, it is useful to examine the marginal value of labor  $V_n$  in the baseline model, dropping the arguments of each function for the sake of clarity:

$$V_n = \alpha (\epsilon A)^{1-\alpha} n^{\alpha-1} - w - w_n n + \beta E [V'_n].$$

The static part of  $V_n$  is composed of three elements: the marginal productivity of labor  $\alpha (\epsilon A)^{1-\alpha} n^{\alpha-1}$ , the cost of one unit of labor  $w$ , and finally, the variations in labor cost caused by adding a worker times the size of the workforce  $w_n n$ . Only the latter element differs between the bargaining economy where  $w_n < 0$  and the constant wage economy where  $w_n = 0$ . If a worker rejects the wage offer and quits, the marginal product of the remaining workers increases. With bargaining, this enables any employee to hold up the firm for a higher wage, claiming a share of the infra margins of production.

This mechanism affects the firm's decision to create and destroy jobs. Recall that the optimal policy of the firm is such that  $V_n \in [-c, c]$ . Or,

$$-c \leq \alpha (\epsilon A)^{1-\alpha} n^{\alpha-1} - w - w_n n + \beta E [V'_n] \leq c.$$

The previous inequalities show that even though the range of variation in the marginal value of labor is the same in both economies, the range of variation in the marginal productivity of labor is greater in the economy with bargaining because of the additional term  $-w_n n$ . Intuitively, labor costs rise in good years and decline in bad years, which dampens the response of employment to shocks. In other words, bargaining reduces the incentives to adjust the workforce. A negative shock weakens workers bargaining position which reduces the need for reallocating labor. Similarly, a positive shock improves workers bargaining position and limits the magnitude of job creation.<sup>28</sup>

Hence, with wage bargaining, the economy reallocates less labor across firms in response to the shocks. Precisely, the reallocation rate is 21% higher when wages are rigid. This lack of reallocation is responsible for the larger efficiency losses from adjustment costs in the economy with bargaining.

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<sup>28</sup>If  $\beta = 0$ , the implied range of variation is equal to  $2 \frac{1-\gamma(1-\alpha)}{1-\gamma} c$  and  $\frac{1-\gamma(1-\alpha)}{1-\gamma} > 1$ .

## 6 Discussion of the Model's Assumptions

This section provides some support for the model assumptions. First, I discuss the result that wages are an affine function of labor productivity. Second, I relate this paper to several empirical papers that find very limited response of individual wages to firm-level shocks. Third, I provide some support for the assumption that profitability can be written as the sum of a random walk and a transitory shock.

### 6.1 Some Justification of the Wage Determination Equation

The wage function in Equation (8) is, admittedly, highly stylized. And one may worry this is not a good representation of wage determination in France given the high coverage of collective bargaining (about 95%) and the high fraction of workers paid at the minimum wage (about 15%). However, analyses of French wage data have shown that individual and firm effects explain a very high share of wage differentials (Abowd et al., 1999; Cahuc et al., 2006).<sup>29</sup>

While the model assumes a high degree of wage flexibility, the quantitative exercise uses data on wage variations from one year to the next. Avouyi-Dovi et al. (2013) show that the typical duration of a wage agreement at the firm-level is one year in France. Further, I conjecture that the component of labor compensation that changes over time is likely to be bonuses: Bonuses represented 13% of total labor compensation in France during the time period studied. Using detailed information on labor costs across firms in France, Biscourp et al. (2005) show that this is an important margin of adjustment. Using quarterly survey data from France, Le Bihan et al. (2014) observe the wage of one to three representative employees within each establishment. Even though it excludes bonuses, they find that the frequency of hourly base wage changes is 38% per quarter and 88% per year. And the average change in hourly wage is 2.2 percent. A simple calculation shows that it could be consistent with my estimates. I estimate the standard deviation of transitory shock is 0.29 (see Table 2) and the transmission into wages is 0.53 (see Table 6). Using a back-of-the-envelope calculation, it implies that bonuses respond more than one-to-one to idiosyncratic shocks. However, Le Bihan et al. (2014) data are very likely to miss many new hires or short-term workers, whom I expect to bear most of the adjustment (as discussed in subsection 6.2). A more definitive answer to this question would require detailed micro data that separate bonuses and base wages.

Another source of concern is the variations in working hours. The well-known difficulty of accurately measuring overtime hours complicates the assessment of this margin.<sup>30</sup> In France, Desplatz et al. (2003) and Biscourp et al. (2005) show that overtime compensation represents 1% of earnings.

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<sup>29</sup>For international evidence on these issues, see Dickens et al. (2007). Using 31 datasets from 16 countries, they document the existence of substantial wage variations across years. They also construct a measure of wage rigidity across countries. France does not appear to be characterized by an unusual degree of rigidity compared with other European countries or the United States.

<sup>30</sup>See, for instance, Kahn and Mallo (2007) for evidence from the United States or Bell and Hart (1999) for evidence from Germany.

Table 8: Average wages, employment, output and labor productivity

	(1)		(2)		(3)		(4)	
	OLS	FE	OLS	FE	OLS	FE	OLS	FE
Labor Product.	0.511 (467.49)	0.403 (313.10)						
Employment			-0.500 (-400.37)	-0.489 (-284.12)	0.026 (34.63)	-0.180 (-105.73)		
Output			0.510 (465.47)	0.386 (299.35)			0.113 (180.81)	0.166 (132.73)

$t$  statistics are listed in parentheses. FE, fixed effects; OLS, ordinary least squares.

Cahuc and Carcillo (2014) show that a major reform that reduced the taxation of overtime hours in France in 2007 has not had any impact on hours worked. And, Chemin and Wasmer (2009) (and the references therein) find that a major reform of workweek regulation in France had no impact on employment. Nonetheless, it is plausible employers use other forms of compensation for unreported overtime work. Consider the following simple model where I abstract for clarity from the wage mechanism in this paper. The production function is  $(\epsilon A)^{1-\alpha} (nh)^\alpha$ , where  $h$  is hours worked. Each household has the utility function  $\sum_{t=0}^{\infty} \beta^t \left( \log C_t - B \frac{h_t^{1+\phi}}{1+\phi} \right)$ .  $C_t$  is consumption,  $B$  reflects the preference for leisure and  $\phi$  is the inverse of the Frisch elasticity of hours supply. Consider a steady-state stationary equilibrium where there is no aggregate variation since a law of large numbers applies and macroeconomic aggregates are constant. Hours supply satisfies the first-order condition  $BCh_t^\phi = w$  with  $w$  the competitive wage per hour. Assume adjustment costs are sufficiently high so that employment  $n$  does not react to transitory shocks at all. It follows that the elasticity of wages to transitory shocks is  $1 - \alpha + \phi$ . I set  $\phi = \frac{1}{0.33}$ , which is in the lower range of estimates reported in Chetty (2012). There exists a range of mechanisms that restricts the use of overtime hours in France, which justifies the use of a low number. With  $\alpha = 0.37$  (see Table 2), the elasticity of hours to transitory shocks equals 0.27. Given the elasticity of at least 0.53 reported (see Table 6), at least half of wages variations are left unexplained by hours in this framework. Finally, the evidence in Guiso et al. (2005) suggests that for stayers, hours do not respond to transitory shocks. The adjustment would have to be born entirely by hours of nonstayers which seems unlikely if there are production complementarities across workers.

Finally, to provide further evidence for the mechanism highlighted in this paper, I consider different regressions of average wages on employment, output, and average labor productivity. The results are reported in Table 8. Each equation is estimated both by ordinary least squares (OLS) controlling for sector and time dummies and by fixed effects (FE). Column (1) controls for labor productivity. Column (2) controls for both employment and output. Columns (3) and (4) control for, respectively, employment and output alone.

According to the theory above, the marginal product of labor is the relevant determinant of wages and is proportional to the average given the Cobb-Douglas assumption. Therefore, column (1) in Table 8 is the most direct test of the theory. I find that more productive firms pay higher average wages. The effect is attenuated once I control for a firm FE. It is indeed plausible that some firms pay higher wages for other factors that are correlated with labor productivity. Column (2) shows that the direct effect of employment on average wages is negative once I control for output. This is precisely the mechanism at play in the theory.

## 6.2 Average Wages at the Firm Level and Individual Wages

This paper examines the behavior of wages and employment at the firm level in response to shocks of different persistence. A natural question is how individual-level wages react to shocks. Several empirical papers based on the work of Guiso et al. (2005) find a relative insensitivity of individual-level wages to firm-level shocks. This section explores some explanations that could reconcile the behavior of wages at the firm level versus the individual level. A useful extension of my model that sheds some light on this issue is the following. Consider a firm composed of two types of workers both homogeneous. Some workers have long-term contracts with the firm  $f$  and some workers have spot contracts as considered in this paper  $n - f$ . The firm's problem is

$$V(A, \epsilon, n) = \max_d \left\{ (\epsilon A)^{1-\alpha} (n + d - f)^\alpha - C(d) - w(A, \epsilon, n + d)(n + d - f) - w_f f \right. \\ \left. + \beta E [V(A\eta, \epsilon', n + d)] \right\}. \quad (14)$$

I claim that this model is consistent (at least qualitatively) with both the relative constancy of wages for some (but not all) individual workers and a significant response of a firm's average wage to transitory shocks.

Guiso et al. (2005) only use stayers, workers who stay with the firm over at least two consecutive years, in their estimation and discard the rest of the sample. An interpretation of the difference between my paper and that of Guiso et al. (2005) is that their model applies to the variable  $f$  and my model applies to the rest of the workforce  $n - f$ . Reality is likely to be somewhere in between. While more detailed data would be necessary to be conclusive on this issue, it is instructive to think that the average firm wage at time  $t$ , denoted  $\bar{w}_t$ , aggregates according to the model just laid out, two wages weighted by the share of workers with a flexible contract  $s_t$ . Thus,

$$\bar{w}_t = w_t s_t + w_f (1 - s_t).$$

It follows that a firm’s average wage variations from one year to the other are equal to

$$\Delta \bar{w}_t = \Delta w_t s_t + (w_{t+1} - w_f) \Delta s_t.$$

According to the equation above, a constant wage for stayers and a fluctuating average firm wage could arise for two reasons. First, a positive transitory shock leads to an increase in the flexible contract wage  $\Delta w_t$  through the mechanism explored in this paper. Second, the flexible wage is higher than the wage of stayer  $w_{t+1} > w_f$ , and  $\Delta s_t$  covaries with the idiosyncratic shock.

What elasticity of non-stayers’ wages is needed to reconcile the results of Guiso et al. (2005) with mine? I do not have data on  $\Delta s_t$  nor  $(w_{t+1} - w_f)$ , but I expect both variations to create wages variations that move in the same direction as my mechanism. Conditional on performing the same tasks, workers with CDDs are possibly representing a higher labor cost because of the extra training they need and to compensate them for the additional labor market risk they bear. Indeed, French firms are required by law to give a bonus for insecurity of employment of 10 percent in addition to a short-term worker’s remuneration. Also, about 60% of workers are hired on a CDDs whose average duration is about 5 months. Their share of the workforce is likely to covary with shocks. Guiso et al. (2005) start with a matched sample of 4 worker/year observations and end up with 45,446 observations by only “retaining workers with stable employment and tenure patterns.” I use these numbers to set  $s = 0.17$ . Combined with my estimate of 0.53%, the implied elasticity of nonstayers’ wages is 0.64.

The empirical evidence is quite scarce on this elasticity. However, the idea that some workers are exposed to a substantial risk while others are perfectly insured has also been shown relevant in other contexts. For instance, Haefke et al. (2013) find that the wage of newly hired workers responds almost one-to-one to changes in labor productivity. However, some debate exists on the magnitude, mainly because of composition effects. And the numbers are not directly comparable since aggregate shocks and idiosyncratic shocks differ in both their nature and their magnitude.

Finally, another difference is the presence of an additional AR(1) term in Guiso et al. (2005). As shown in the next subsection, the autocovariances of output decay quickly in the French data. I thus did not include an autoregressive component in the model.

### 6.3 Modeling Shocks

This section provides some justification for assuming that profitability is the sum of a permanent component and a transitory component. To do so, I investigate the presence of permanent shocks, transitory shocks, and measurement errors in output, employment and wages. It is not a formal test because output, employment, and wages are endogenous variables. Using the parameter estimates, it is possible to validate *ex post* whether this is a reasonable assumption by deriving an estimate of profitability.

Following the same logic as Section 3.2, suppose the logarithm of value added, employment, or wages, denoted  $\log X$ , can be decomposed into a permanent component  $P$  and a mean-reverting transitory component  $\nu$ . The process for each firm  $i$  is

$$\log X_{it} = Z_{it}\varphi_t + P_{it} + \nu_{it}. \quad (15)$$

Instead of assuming that  $\nu_{it}$  is serially uncorrelated, I assume that  $\nu_{it}$  follows an  $MA(q)$  process, where the order  $q$  is to be established empirically:

$$\nu_{it} = \sum_{j=0}^q \theta_j \epsilon_{it-j} \quad (16)$$

with  $\theta_0 = 1$ . The logged variable net of predictable individual components,  $\log x_{it} = \log X_{it} - Z_{it}\varphi_t$ , is in first differences:

$$\Delta \log x_{it} = \zeta_{it} + \Delta \nu_{it}. \quad (17)$$

Assume that  $\zeta_{it}$  and  $\nu_{it}$  are uncorrelated at all leads and lags. Assume the variance of the innovations is constant over time. Then the parameters to estimate are  $\sigma_\zeta^2, \sigma_\epsilon^2, q, \theta_1, \dots, \theta_q$ .

Identification of these parameters is straightforward. If  $\nu$  is an  $MA(q)$  process,  $cov(\Delta \nu_t, \Delta \nu_{t+s})$  is zero whenever  $s \geq q + 1$ . Then those covariances identify  $q, \theta_1, \dots, \theta_q$ , and  $\sigma_\epsilon^2$ . The key moment condition that identifies the variance of the permanent shock is

$$E \left[ \Delta \log x_{it} \left( \sum_{j=-(1+q)}^{(1+q)} \Delta \log x_{it+j} \right) \right] = \sigma_\zeta^2. \quad (18)$$

This was derived by Meghir and Pistaferri (2004). This exploits the structure of the  $MA$  process to cancel terms out.

To describe the structure of transitory shocks, I estimate the autocovariances of  $\Delta \log x_{it}$  using standard methods (Abowd and Card, 1989). The test statistic equals the squared autocovariance divided by its respective variance. It is distributed as a chi-square distribution, with a degree of freedom equal to the number of time periods available for estimation. To test for the absence of permanent shocks, the test statistic is equal to the pooled estimate of the variance of the permanent shock divided by its standard error. It is asymptotically (for large  $N$ ) distributed as a standard normal. The standard errors are computed using a block bootstrap procedure (see Hall and Horowitz, 1996). In this way, I account for serial correlation of arbitrary form, heteroskedasticity, as well as for the use of pre-estimated residuals.

Table 9 presents estimates of the autocovariances up to order three along with the test of zero restrictions for the null hypothesis that  $cov(\Delta \log x_{it}, \Delta \log x_{it+s}) = 0$  with  $1 \leq s \leq 3$ . It also presents an estimate of the variance of the permanent shock along with the fraction of the variance



Table 9: Autocovariances and Permanent Shocks

	Autocovariances Order				Permanent Shock	
	0	1	2	3	Estimate	Variance Share
Output	0.0676 (0)	-0.0119 (0)	-0.0005 (0.3518)	0.0008 (0.0127)	0.0369 (0.0007)	54.57%
Wages	0.0338 (0)	-0.0137 (0)	-0.0011 (0)	0.0007 (0)	0.0053 (0.0002)	15.55%
Employment	0.0321 (0)	0.0023 (0)	0.0003 (0)	0.0012 (0)	0.0258 (0.0004)	80.51%
Residual	0.0601 (0)	-0.0149 (0)	-0.0016 (0.0081)	0.0057 (0.1998)	0.0246 (0.0005)	40.95%

of each variables growth it explains.

Only first-order correlations are present for residual output growth. Therefore, the data do not reject the existence of an i.i.d transitory component. For residual average wage growth, all the autocovariances are significant but are very small, indicating than an i.i.d process is a reasonable approximation. Finally, residual employment growth follows a different pattern: Autocovariances of employment are positive and significant at all orders. This is consistent with the theory above where adjustment costs leads to persistence.

The variance of a permanent output shock is estimated to be 0.0369 (with a bootstrap standard error of 0.0007) for output and 0.0053 (with a bootstrap standard error of 0.0002) for wages. The hypothesis of no permanent shocks is strongly rejected in all cases. Permanent shocks explain half of the total variance of output growth residuals, while transitory shocks are the main source of variation of average wages, explaining around 85% of the total variance of wage growth.

While I provided some justification for the assumption that profitability is the sum of a random walk and an iid. shock, it is not a formal test because output is an endogenous variable. Using observed output and employment, and  $\hat{\alpha}$ , I recover an estimate of  $(1 - \alpha)(\log A_{it} + \log \epsilon_{it}) - \alpha m r n_{it} + m r o_{it}$ . I cannot reject the existence of a random walk component to profitability. Table 9 indicates that only first-order correlations are present. This validates the assumption made on the profitability process.

## 7 Conclusion

This paper analyzes the impact of permanent and transitory shocks to profitability on the reallocation of labor across firms. Firms produce with decreasing returns to labor and are subject to transitory and permanent shocks to profitability. They decide how many jobs to create and destroy at a constant cost per job. The wage is negotiated every year and workers are paid a share of the value of the marginal worker.

Quantitative evaluation of the model shows that fairly modest adjustment costs - about two months of average wages per job created or destroyed - can fit the data well. The model can account for about 85% of the within-firm log labor productivity dispersion. Removing adjustment costs leads to productivity gains close to 1% at the steady state. This effect is 50% larger in an economy with only transitory shocks and an order of magnitude lower in an economy with only permanent shocks. Bargaining dampens the reallocation of labor across firms, leading to larger efficiency losses from adjustment costs.

A number of issues remain that are left for future research. It remains open who is bearing the risk of the large idiosyncratic variance in profitability. Are some workers wages better insured than others? And, there may exist a trade-off between insuring wages and insuring employment. Another important issue is the aggregate effect of adjustment costs. This paper suggest that the efficiency losses are significant but small in the French labor market because there much job turnover due to short-term contracts. Yet by affecting match quality, these frictions might lead to larger losses.

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# Appendices

## A Theoretical Moments for Section 3.2

This section describes the theoretical moments used for the estimation in Section 3.2.

$$\begin{aligned} E[\Delta \log o_{it}(\Delta \log o_{it-1} + \Delta \log o_{it} + \Delta \log o_{it+1})] &= \sigma_{\zeta_o}^2 \\ E(\Delta \log o_{it} \Delta \log o_{it}) &= \sigma_{\zeta_o}^2 + 2(\sigma_{\nu_o}^2 + \sigma_{r_o}^2) \\ E(\Delta \log o_{it} \Delta \log o_{it+1}) &= -(\sigma_{\nu_o}^2 + \sigma_{r_o}^2) \end{aligned}$$

and  $E(\Delta \log o_{it} \Delta \log o_{it+s}) = 0, s \geq 2$ . Similarly,

$$\begin{aligned} E[\Delta \log w_{it}(\Delta \log w_{it-1} + \Delta \log w_{it} + \Delta \log w_{it+1})] &= \tau^2 \sigma_{\zeta_o}^2 + \sigma_{\zeta_w}^2 \\ E(\Delta \log w_{it} \Delta \log w_{it}) &= \tau^2 \sigma_{\zeta_o}^2 + 2(\phi^2 \sigma_{\nu_o}^2 + \sigma_{\nu_w}^2) + \sigma_{\zeta_w}^2 \\ E(\Delta \log w_{it} \Delta \log w_{it+1}) &= -(\phi^2 \sigma_{\nu_o}^2 + \sigma_{\nu_w}^2) \end{aligned}$$

and  $E(\Delta \log w_{it} \Delta \log w_{it+s}) = 0, s \geq 2$ . The covariance between output growth and wage growth is

$$\begin{aligned} E(\Delta \log w_{it} \Delta \log o_{it}) &= \tau \sigma_{\zeta_o}^2 + 2\phi \sigma_{\nu_o}^2 \\ E(\Delta \log w_{it} \Delta \log o_{it+1}) &= -\phi \sigma_{\nu_o}^2 \\ E[\Delta \log w_{it}(\Delta \log o_{it-1} + \Delta \log o_{it} + \Delta \log o_{it+1})] &= \tau \sigma_{\zeta_o}^2 \end{aligned}$$

and  $E(\Delta \log w_{it} \Delta \log y_{it+s}) = 0, |s| \geq 2$ .

## B Theoretical Model

This section describes the solution of the wage function and the numerical solution of the model. The technical appendix of Campbell and Fisher (2000) provides a rigorous treatment of a similar dynamic programming problem. I refer the reader to this technical appendix for the results on the existence of a solution and the properties of the policy function that apply to the current framework with some minor changes in the notation.

### B.1 Bargaining

Wages depend on the marginal value of labor  $V_n$ , which can be decomposed as follows:

$$V_n(A, \epsilon, n) = \begin{cases} \underline{c} & \text{if } n < \underline{n}(A, \epsilon) \\ \alpha (\epsilon A)^{1-\alpha} n^{\alpha-1} - w(A, \epsilon, n) - w_n(A, \epsilon, n)n + \beta E [V_n(A\eta, \epsilon', n+d)] & \text{if } \underline{n}(A, \epsilon) < n < \bar{n}(A, \epsilon) \\ -\bar{c} & \text{if } n > \bar{n}(A, \epsilon). \end{cases}$$

Let  $W(A, \epsilon, n)$  be the value of employment in a firm of size  $n$  with state  $(A, \epsilon, n)$ . The worker's surplus in an expanding firm is

$$\begin{aligned} W(A, \epsilon, \underline{n}(A, \epsilon)) - U &= \frac{\gamma}{1-\gamma} (V_n(A, \epsilon, \underline{n}(A, \epsilon)) + \bar{c}) \\ &= \frac{\gamma}{1-\gamma} (\underline{c} + \bar{c}). \end{aligned}$$

Similarly, in a contracting firm

$$\begin{aligned} W(A, \epsilon, \bar{n}(A, \epsilon)) - U &= \frac{\gamma}{1-\gamma} (V_n(A, \epsilon, \bar{n}(A, \epsilon)) + \bar{c}) \\ &= 0. \end{aligned}$$

Upon finding a job, the new job must be in a firm that is creating jobs. The value to a worker of unemployment is

$$U = b + \beta(1-f)U + \beta f E [W(A, \epsilon, \underline{n}(A, \epsilon)) | n < \underline{n}(A, \epsilon)].$$

Then,

$$(1-\beta)U = b + \beta f \frac{\gamma}{1-\gamma} (\underline{c} + \bar{c}).$$

It follows that workers' surplus is

$$\begin{aligned} W(A, \epsilon, n) - U &= w(A, \epsilon, n) - b + \beta \frac{\gamma}{1-\gamma} (P(n < \underline{n}(A\eta, \epsilon')) - f) (\underline{c} + \bar{c}) \\ &\quad + \beta \frac{\gamma}{1-\gamma} E [V_n(A\eta, \epsilon', n) + \bar{c} | \underline{n}(A\eta, \epsilon') < n < \bar{n}(A\eta, \epsilon')]. \end{aligned}$$

This must equal  $\frac{\gamma}{1-\gamma} [V_n(A, \epsilon, n) + \bar{c}]$ . Then,

$$\begin{aligned} w(A, \epsilon, n) &= (1-\gamma)b \\ &\quad + \gamma \alpha (\epsilon A)^{1-\alpha} n^{\alpha-1} - \gamma w_n(A, \epsilon, n)n \\ &\quad + \bar{c}\gamma(1-\beta(1-f)) + \underline{c}\beta\gamma f. \end{aligned}$$

Solving this differential equation gives Equation 8 in the main text.  $w(A, \epsilon, n)$  is homogeneous of degree 0 in  $(A, n)$ .



## B.2 Homogeneity of the Value Function

It is easy to show that  $V$  is homogeneous of degree 1 in  $A$  and  $n$ . Define  $v(x, \epsilon) = V(1, \frac{n-1}{A}, \epsilon)$  with  $x = \frac{n-1}{A}$ .  $v$  satisfies the Bellman equation:

$$v(x, \epsilon) = \max_y \left\{ (\epsilon y)^\alpha - w(y, \epsilon)y - \underline{c}(y-x)^+ - \bar{c}(y-x)^- + \beta E \left[ \eta v\left(\frac{y}{\eta}, \epsilon'\right) \right] \right\}.$$

Define  $d(y) = E \left[ \eta v\left(\frac{y}{\eta}, \epsilon'\right) \right]$  so that  $d_y(y) = E \left[ v_y\left(\frac{y}{\eta}, \epsilon'\right) \right]$ . The optimal policy can be rewritten as

$$y(x, \epsilon) = \begin{cases} \underline{y}(\epsilon) & \text{if } x < \underline{y}(\epsilon) \\ x & \text{if } \underline{y}(\epsilon) < x < \bar{y}(\epsilon) \\ \bar{y}(\epsilon) & \text{if } x > \bar{y}(\epsilon) \end{cases}$$

with

$$\begin{aligned} \frac{\alpha(1-\gamma)}{1-\gamma(1-\alpha)} \epsilon^{1-\alpha} \underline{y}(\epsilon)^{\alpha-1} - ((1-\gamma)b + \bar{c}\gamma(1-\beta(1-f)) + \underline{c}\beta\gamma f) - \underline{c} + \beta d_y(\underline{y}(\epsilon)) &= 0 \\ \frac{\alpha(1-\gamma)}{1-\gamma(1-\alpha)} \epsilon^{1-\alpha} \bar{y}(\epsilon)^{\alpha-1} - ((1-\gamma)b + \bar{c}\gamma(1-\beta(1-f)) + \underline{c}\beta\gamma f) + \bar{c} + \beta d_y(\bar{y}(\epsilon)) &= 0. \end{aligned}$$

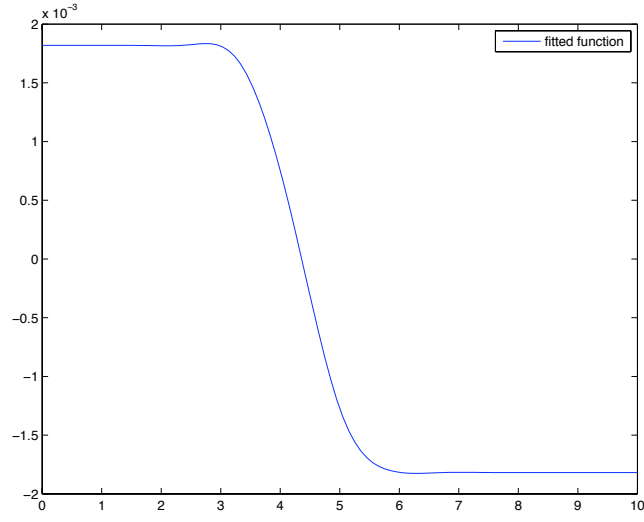
## C Numerical Solution

I use a collocation method as described in Judd (1998). Approximating the derivative of the value function  $v_y(y, \epsilon)$  is challenging: It has two kinks at the unknown thresholds. However, to find the optimal policy and simulate the model, it is sufficient to know the expected values  $d_y(y)$ . This expectation is a smooth function of  $y$ : The convolution of any integrable function and a normal density is analytic, a property of the exponential family of distributions (see Theorem 9 in Lehmann, 1959).

Figure 3 plots  $d_y$  as a function of  $y$ . For low (high) values of employment divided by profitability, the firm creates (destroys) jobs and the marginal value of the firm is equal to the costs of creating (destroying) jobs. For intermediate values, the firm is inactive and the marginal value of employment is a decreasing function of employment. Figure 3 displays clearly that the kinks at the threshold values are smoothed out by the expectation operator.

Let  $\hat{d}_y(y; p)$  be the function used to approximate  $d_y(y)$ , where  $p$  is a vector of parameters. I assume it can be written as a linear combination of a set of  $P$  known linearly independent basis

Figure 3:  $d_y$  as a function of  $y$



function  $B_1, \dots, B_n$ ,

$$\hat{d}_y(y; p) = \sum_{i=1}^P p_i B_i(y),$$

whose basis coefficients  $p_1, \dots, p_P$  are to be determined. I use B-splines as a basis function. I apply standard Chebyshev interpolation nodes and Gaussian quadrature to the contraction mapping that define  $d_y(y; p)$ .