Nonlinearities, Smoothing and Countercyclical Monetary Policy

Authors	Laura E. Jackson, Michael T. Owyang, and Daniel Soques
Working Paper Number	2016-008A
Creation Date	May 2016
Citable Link	https://doi.org/10.20955/wp.2016.008
Suggested Citation	Jackson, L.E., Owyang, M.T., Soques, D., 2016; Nonlinearities, Smoothing and Countercyclical Monetary Policy, Federal Reserve Bank of St. Louis Working Paper 2016-008. URL https://doi.org/10.20955/wp.2016.008

Published In	Journal of Economic Dynamics & Control
Publisher Link	https://doi.org/10.1016/j.jedc.2018.08.007

Nonlinearities, Smoothing and Countercyclical Monetary Policy*

Laura E. Jackson†, Michael T. Owyang‡, Daniel Soques§

keywords: time-varying transition probabilities, Markov-switching, monetary policy

May 11, 2016

Abstract

Empirical analysis of the Fed's monetary policy behavior suggests that the Fed smooths interest rates– that is, the Fed moves the federal funds rate target in several small steps instead of one large step with the same magnitude. We evaluate the effect of countercyclical policy by estimating a Vector Autoregression (VAR) with regime switching. Because the size of the policy shock is important in our model, we can evaluate the effect of smoothing the interest rate on the path of macro variables. Our model also allows for variation in transition probabilities across regimes, depending on the level of output growth. Thus, changes in the stance of monetary policy affect the macroeconomic variables in a nonlinear way, both directly and indirectly through the state of the economy. We also incorporate a factor summarizing overall sentiment into the VAR to determine if sentiment changes substantially around turning points and whether they are indeed important to understanding the effects of policy.

[JEL codes: C24, E32]

*The authors benefitted from conversations with Michael McCracken, Valerie Ramey, Martin Sola, participants at the St. Louis Fed Applied Econometrics Workshop, the San Francisco Fed Research Department Seminar Series, and at the Bentley University Research Seminar Series. Diana A. Cooke, Hannah G. Shell, and Kate Vermann provided research assistance. Views expressed here are the authors' alone and do not reflect the opinions of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

†Corresponding author. Department of Economics. Bentley University. 175 Forest Street, Waltham, MA 02452. ljackson@bentley.edu.

‡Research Division. Federal Reserve Bank of St. Louis.

§Department of Economics and Finance. University of North Carolina, Wilmington.

1 Introduction

Empirical analysis of the Fed's monetary policy behavior suggests that the Fed smooths interest rates– that is, the Fed moves the federal funds rate target in several small steps instead of one large step with the same magnitude. Smoothing has been characterized as an optimal monetary policy response in models that incorporate the private sector's expectations of future policy [e.g., Woodford (1999)]. In these types of models, the monetary authority's method of credibly altering expectations is important in determining the efficacy of the stabilization policy.

Because monetary policy is countercyclical, its effectiveness is often measured by its ability to induce large responses in output growth and inflation. However, the models used to measure these responses (e.g., VARs) are often linear and may exaggerate the effectiveness of policy if the dynamics of the economy change across states.[1] Even when the VARs do incorporate some form of regime switching, the responses are often computed within-regime—i.e., the responses are computed assuming that the regime never changes. This assumption is problematic for evaluating countercyclical policy. In these models, during recessions, the Fed drops the funds target to raise output growth but has no effect on the duration of the recession.[2]

To evaluate the effectiveness of countercyclical policy, we estimate a VAR with regime switching. In our model, the probability of transitioning across regimes depends on the level of output growth.[3] Thus, changes in the stance of monetary policy affect the macroeconomic variables nonlinearly, both directly and indirectly through the state of the economy. To this end, the estimated responses of macro variables to monetary shocks can depend on (1) the current state of the economy[4], (2)

[1] The existing literature provides mixed empirical evidence of asymmetry. Cover (1992) finds that negative money supply shocks have larger effects on output than positive shocks. Ravn and Sola (1996) find symmetric responses once they account for a break in late 1970's. Morgan (1993) finds asymmetric responses of output to interest rate changes but the evidence is weaker when excluding the early 1980's when the Fed abandoned traditional rate targeting polices.

[2] Garcia and Schaller (2002) extend Hamilton's (1989) regime-switching model to allow monetary policy to affect the growth rate of output and the probability of switching between states. They find that changes in the fed funds rate have larger effects during recessions than during booms and that policy has substantial effects on the probability of switching between expansionary and recessionary regimes.

[3] In related work, Weise (1999) uses a smooth-transition VAR to examine the asymmetric effects of policy based on the three dimensions of interest: size, sign, and position in the business cycle. The author finds evidence of size, but not sign, asymmetries and different effects during periods of high or low growth. Monetary shocks have stronger output effects and weaker price effects when growth is initially low but have stronger price effects and weaker output effects when in a high growth state.

[4] For instance, Thoma (1994) finds that negative shocks to money growth have stronger effects on output during periods of high-growth in real activity than in low-growth periods while positive shocks have small, mostly insignificant effects regardless of the contemporary economic conditions.

the history of the economy, (3) future shocks, and (4) the size of the (current) monetary shock.[5] We incorporate consumer and producer sentiment into the VAR to determine if confidence and expectations change substantially around turning points and whether they are indeed important to understanding the effects of policy. In addition, because the size of the shock is important in our model, we can evaluate the effect of smoothing the interest rate on the path of macro variables.

We find empirically relevant differences between the macroeconomic responses to contractionary and expansionary policy shocks, depending on the underlying state of the economy at the time of the shock. Small expansionary policy shocks induce responses with substantial variation in high and low output growth environments, but show less variation in periods of high and low inflation. The responses to large expansionary shocks do not exhibit the same variation. We also find significant differences between gradual policy changes and one-time, large policy shocks, thus making a case for more aggressive policy intervention to combat recessions.

The balance of the paper is outlined as follows: Section 2 outlines the models. We start by fixing notation with the familiar single regime VAR. We then add the effects of sentiment, modeled by a latent factor, and Markov-switching. Finally, we augment the Markov-switching with time-varying transition probabilities. Section 3 describes the data and the methods used to estimate the model. Details for the full sampler are left to the Appendix. Section 3.3 compares the different methods to compute the impulse responses to evaluate the effectiveness of the shocks. In this section, we reiterate the importance of history, future, sign, and scale of the shock. Section 4 presents the baseline results. Section 5, in particular, focuses on the experiment comparing the effect of a net 25-basis-point change in the federal funds rate implemented in a single step or in multiple steps. Section 6 offers final thoughts.

2 Empirical Approach

One of the most commonly used models in the empirical analysis of monetary policy is the VAR. A simple example of a monetary VAR is a three-variable model with measures of output growth and prices and a monetary policy instrument. The effects of the policy shocks are determined by tracing out the impulse responses to identified shocks. In this section, we construct a VAR

[5] Lo and Piger (2005) find that policy actions taken during recessions have much larger effects than those taken during expansions. However, they find no evidence of asymmetries based on the size or sign of the policy shock.

that allows for asymmetric responses to shocks and differences in shock volatilities. In addition, we model economic sentiment through a latent factor that is allowed to affect or be affected by macroeconomic aggregates in different ways depending on the state of the economy.

2.1 The VAR

Let y_t represent the $N \times 1$ vector of period$-t$ variables of interest; then, the reduced-form VAR(P) is

$$y_t = \widetilde{B}(L)\, y_{t-1} + \widetilde{\varepsilon}_t, \tag{1}$$

where we have suppressed the constant and any trends, $\widetilde{\varepsilon}_t \sim N\left(\mathbf{0}, \widetilde{\Omega}\right)$ is the reduced-form innovation, and $\widetilde{\Omega}$ is left unrestricted. Inference on the effect of shocks is derived from the structural form of the VAR:

$$\widetilde{A}^{-1} y_t = \widetilde{A}^{-1} \widetilde{B}(L)\, y_{t-1} + \widetilde{u}_t, \tag{2}$$

which is obtained by pre-multiplying by $\widetilde{A}^{-1}$ which represents the contemporaneous effects of the structural shocks $\widetilde{u}_t \sim N\left(0, \widetilde{\Sigma}\right)$, where $\widetilde{\Sigma}$ is diagonal, and $\widetilde{A}\widetilde{A}' = \widetilde{\Omega}$. Because the decomposition $\widetilde{A}\widetilde{A}' = \widetilde{\Omega}$ is not unique, further identifying restrictions must be imposed to obtain the structural form of the VAR and determine the (impact) effects of the shocks. These restrictions can come in the form of imposing a causal ordering on the variables in the VAR, assuming zero contemporaneous effects across variables [Christiano, Eichenbaum, and Evans (2000)], imposing zero restrictions on the long-run (or long-horizon) effects of certain shocks [Blanchard and Quah (1989)], predetermining the signs of the responses [e.g., Uhlig (2005)], or some combination of these [Arias, Rubio-Ramirez, and Waggoner (2014)].

2.2 Modeling Sentiment

Our desire is to augment the VAR with a broad measure of sentiment regarding the current strength of and outlook for the economy. Sentiment, however, is not easily quantifiable. We opt to include a factor (or vector of factors) F_t representing overall sentiment in the VAR. Then, the $(N+1) \times 1$ vector of variables of interest can be defined as $Y_t = [F_t, y_t']'$ and the VAR rewritten as

$$Y_t = B(L) Y_{t-1} + \varepsilon_t,$$

where the reduced form shocks $\varepsilon_t \sim N(\mathbf{0}, \Omega_t)$ are now an $(N+1) \times 1$ vector and we have imposed autoregressive dynamics on the factor. The factor F_t summarizes the information in M series collected in a vector $\mathbf{X}_t$ that contains observable information about consumer and producer sentiment. The factor is related to $\mathbf{X}_t = [X_{1t}, ..., X_{Mt}]'$ by

$$X_{mt} = \lambda_m F_t + \varsigma_{mt}, \tag{3}$$

where $\varsigma_{mt} \sim iidN\left(0, \sigma_m^2\right)$, which assumes that the innovations to the elements of $\mathbf{X}_t$ are uncorrelated. This assumption imposes that the correlation across series are a result of the factor alone and is relatively common in the factor literature.

2.3 The Markov-Switching VAR

Recently, studies have investigated whether monetary policy has time-dependent effects—for example, depending on the state of the economy.[6] For example, one could ask whether monetary policy has differing effects in recessions and expansions, when the Fed tightens or eases, or when the change in the fed funds target rate is large or small, etc.[7] One popular model used to determine the state-dependent effects of monetary policy is the Markov-switching VAR, which has a reduced-form:

$$Y_t = [1 - S_t] B_0(L) Y_{t-1} + S_t B_1(L) Y_{t-1} + \varepsilon_t, \tag{4}$$

where $S_t = \{0, 1\}$ follows an irreducible first-order Markov process with (constant) transition probabilities $p = \Pr[S_t = 1 | S_{t-1} = 1]$ and $q = \Pr[S_t = 0 | S_{t-1} = 0]$, $\varepsilon_t \sim N(0, \Omega_t)$, and regime-dependent heteroskedastic covariance matrix

[6] See Hamilton (2015) for a detailed overview of regime-switching modeling techniques and applications within macroeconomics.

[7] In recent work, Angrist, Jorda, and Kuersteiner (2013) find that contractionary policy can achieve reductions in output, employment, and inflation but expansionary policy produces very little stimulus. Barnichon and Matthes (2014) find that contractionary policy shocks have strong adverse effects on output while expansionary shocks do not have significant effects unless the shocks are large and occur specifically during recessions.

$$\Omega_t = [1 - S_t]\,\Omega_0 + S_t\Omega_{1.} \tag{5}$$

In this case, the economy takes on two alternative dynamics, dictated by the realization of the underlying state S_t. When $S_t = 1$, the economy has $B_1(L)$ dynamics and when $S_t = 0$, the economy has $B_0(L)$ dynamics. Thus, the model is linear, conditional on S_t being known.[8] The shock processes—by assumption—follow the same regime-switching process as the reduced-form VAR coefficients, making the contemporaneous effects of the shocks regime-dependent.[9] The shocks are identified using similar methods as above or, additionally, exploiting the regime-dependence [e.g., Rigobon and Sack (2004)].

2.4 Time-Varying Transition Probabilities

One drawback of the constant probability Markov-switching VAR is that the underlying regime is invariant to the model variables. Countercyclical policy then cannot affect—either directly or indirectly—the state of the economy, making both the regimes and the impulse responses to changes in policy difficult to interpret.[10] One way to ameliorate this problem is to allow the state of the economy to depend, in part, on variables in the VAR. For example, if we want to interpret the state variable as business cycle regimes, we can make the transition probabilities functions of output growth.

We can accomplish this by assuming that the state process S_t has time-varying, rather than constant, transition probabilities. Moreover, we assume that changes in the underlying state of the economy (and, thus, underlying changes in the dynamic responses to monetary shocks) are driven by (lags of) a variable z_t. If, as in our case, z_t is a variable in the VAR, shocks to the policy instrument affect z_t which, in turn, feed back into the regime.[11] Thus, more accommodative monetary policy in a recession can stimulate output and increase the probability of switching back

[8] Based on multiple Lagrange Multiplier tests for linearity, Weise (1999) finds that when using lagged output growth as the switching variable, the data prefer the non-linear model with time-variation in the coefficients of all equations in the VAR to a standard linear VAR with constant parameters. Furthermore, the parameter governing the speed of the transition between regimes is very large. This suggests a sharp transition between regimes and justifies the use of a discrete regime-switching model.

[9] It is straightforward to extend the model to allow the covariance switching process to vary from the coefficient switching process. The main drawback is that independent processes increases the number of regimes geometrically.

[10] The constant transition probability model is also of limited use for forecasting. Conditional on the past regime being known, no additional data improves the forecast of the regime.

[11] Potter (1995) called the system in which the transition variable is also in the VAR self-exciting.

to expansionary dynamics. We assume that the transition probabilities follow a logistic formulation:

$$p_{ji}\left(z_{t-d}\right) = \Pr\left[S_t = j | S_{t-1} = i\right] = \frac{\exp\left(\overline{\gamma}_{ji} + \gamma_{ji} z_{t-d}\right)}{\sum_k \exp\left(\overline{\gamma}_{ki} + \gamma_{ki} z_{t-d}\right)} \tag{6}$$

for each of the regimes with $\sum_k p_{ki}\left(z_{t-d}\right) = 1$ for all i, t. We define $S_t = 0$ as the reference state; thus, all parameters governing the transition into expansion ($\overline{\gamma}_{0i}$ and γ_{0i} for $i = 0, 1$) are normalized to 0.

We consider lagged output growth as the transition variable and set the delay parameter, d, to 1. Thus, output growth in the previous period will affect the probability of switching between expansion and recession in the current period. In order to identify the two separate regimes, we impose that the coefficient on lagged output growth influencing the transition from expansion to recession, γ_{10}, is negative. Therefore, if $S_{t-1} = 0$ (expansion) and output growth is above average, the probability that $S_t = 1$ (recession) falls.

3 Empirical Analysis

In this section, we describe how the model is estimated, the data used in the estimation, and the methods for which we compute the impulse responses.

3.1 The Sampler

The model parameters, factors, regimes, and transition probabilities are estimated using the Gibbs sampler. Let the full set of parameters, including the regimes and the factors, be represented by:

$$\Theta = \left\{ B_0\left(L\right), B_1\left(L\right), \Omega_0, \Omega_1, \overline{\gamma}, \boldsymbol{\gamma}, \left\{\lambda_m, \sigma_m^2\right\}_{m=1}^{M} \right\},$$

$\mathbf{S}_T = \{S_t\}_{t=1}^T$, and $\mathbf{F}_T = \{F_t\}_{t=1}^T$. The Gibbs sampler draws elements of Θ, $\mathbf{S}_T$, and $\mathbf{F}_T$, conditional on the previous draw of each other elements. We sample from five blocks: (1) the VAR coefficients and covariance matrices; (2) the regimes; (3) the transition function parameters; (4) the factor; and (5) the factor loadings and residual variances. The joint posterior distribution of all the model parameters and the factors are obtained from these draws from the conditional distribution after discarding some draws to allow for convergence.

The Gibbs sampler is a Bayesian method and requires a prior. We assume a multivariate normal-inverse Wishart prior for the VAR parameters, a multivariate normal prior for the transition function parameters, independent normal-inverse Gamma priors for each of the factor loadings and their associated residual variance. Table 1 shows the hyperparameters of the prior distributions.

Given the prior and the data and conditional on the sequence of regimes and the factor, the posterior for each regime's VAR parameters is conjugate normal-inverse-Wishart. The regimes are drawn from the Hamilton filter, modified to account for time-variation in the transition probabilities. The parameters of the transition function are drawn employing the difference in random utility model described in Kaufmann (2015). The factor is drawn from an application of the Kalman filter; conditional on the factor, the loadings and variances of the factor equations have normal-inverse-Gamma posterior densities. The blocks of the sampler and the derivation of the posterior distributions are described in detail in the Appendix.

3.2 Data

Our sample period runs from 1960:1 to 2008:12, when the federal funds rate approaches the zero lower bound. We exclude the zero lower bound period because of the difficulty in assessing the stance of monetary policy in a single policy instrument.[12] The data for the baseline VAR are monthly and consist of a measure of output, prices, and policy. We use the change in the log of the Conference Board Coincident Indicators Index (ZCOIN), the change in the log of the personal consumption expenditures price level index (PCEPI), and the effective federal funds rate.

The model also requires data that proxy for overall sentiment in the form of a factor. We utilize a small unbalanced panel of monthly data that includes multiple surveys and indices. We include the Conference Board Consumer Confidence Index (CBCCI), the University of Michigan Consumer Sentiment Index (UMCSI), the Organization for Economic Cooperation and Development Consumer Confidence Index (OECDCCI), and the Institute for Supply Management Purchasing Managers Index (PMI).[13]

We order the sentiment factor first in the VAR, allowing the macro variables and the policy

[12] One alternative that has been proposed is the shadow short rate of Krippner (2013) and Wu and Xia (2016). The shadow short rate exploits the Gaussian affine term structure model and changes in the long rate to estimate the level of a hypothetical short rate that is allowed to fall below the zero lower bound.

[13] All sentiment data are normalized to have mean zero and unit standard deviation.

rate to respond contemporaneously to shocks to overall (consumer and producer) sentiment. This restriction implies that the factor itself responds to policy shocks with a lag. The results are qualitatively similar and the overall conclusions unchanged if the factor is instead ordered last, after the policy rate, allowing it to respond to contemporaneous policy shocks.

3.3 Computing Impulse Responses

The effects of monetary policy shocks from VARs are typically summarized using impulse responses. Nonlinearity in the VAR complicates computation of the impulse responses. In a nonlinear model, the response can depend on the level (rather than the change) of all of the variables; thus, computation of the conditional expectation depends on the initial condition (i.e., the history of all of the innovations up until time t) and the future path of the variables (i.e., the sequence of shocks from $t+1$ to $t+h$). For example, the responses can vary depending on whether the economy starts in recession or expansion and can vary depending on whether a policy action is followed by successive positive or negative shocks. Moreover, in the nonlinear model the response can depend on the sign and magnitude of the shock.

The Markov-switching VAR is linear, conditional on knowing the regime. Regime-dependent impulse responses (RDIR) can be obtained from each of the two conditionally linear VARs as suggested by Ehrmann, Ellison, and Valla (2003) by assuming that the regime at the time of the shock lasts forever. Of course, these RDIRs have the limitation that they are constructed under the extreme counterfactual assumption that the state of the world does not change after the incidence of the shock. The linear responses have the advantage of being invariant to the history of shocks up through time t, the sequence of the shocks after time t, and the size of the shock. Moreover, the responses are symmetric to the sign of the shock.

While these (conditionally) linear responses are often used to distinguish between the dynamics across the regimes, they do not take into account the future possibility that the economy exits the initial regime. In this sense, they can overestimate the differences between shocks that are incident in the different regimes. Alternatively, Krolzig (2006) shows that simple constant probability transitions across regimes can be accounted for by computing the response as a weighted average of the two regime-dependent responses. The weight at any horizon is a function of the transition probabilities and the responses are computed conditional on the period$-t$ regime. For more complicated models with time-varying transition probabilities, we need to account for the response of the transition probability to the shock.

In our case, the transition probabilities depend on the variables in the VAR. Thus, simply propagating the transition probabilities out over time is insufficient to obtain any inferences about the effect of shocks. One alternative is the generalized impulse response functions (GIRFs) suggested by Koop, Pesaran, and Potter (1996). They argue that an impulse response at horizon h can be viewed as the difference between two conditional expectations, one conditional on the (structural) shock $u_t = \delta$ occurring at time t and one conditional on no shock at time t:

$$IR\left(h\right) = E_t\left[Y_{t+h}|u_t = \delta\right] - E_t\left[Y_{t+h}|u_t = 0\right].$$

In the linear model, the difference in the conditional expectation is invariant to the history up until time t and the future sequence of shocks up through $t + h$. In addition, the magnitude of δ acts only as a scaling factor and the response is symmetric with respect to the sign of δ. To compute the expectations, we average the expected paths of Y over all histories $\mathbf{Y}_{t-1}$ that correspond to a Gibbs draw of $S_t = i$. Thus, we are computing the averages of separate responses for average shocks that occur in different regimes.

Let $R_i^{[g]}$ represent the number of incidences of $S_t = i$ for the gth Gibbs iteration. In addition to the histories, the responses depend on the future sequence of shocks. We can account for variation in future shocks by computing the average response over Q draws of future shock paths. Finally, we average over a subsample of the Gibbs draws. The generalized response at horizon h is

$$IR_i\left(h\right) = \frac{1}{G}\frac{1}{R_i^{[g]}}\frac{1}{Q}\sum_g^G\sum_r^{R_i^{[g]}}\sum_q^Q\left\{\begin{array}{c}\left[Y_{t+h}|\mathbf{Y}_{t-1}, \Theta^{[g]}, S_t^{[g]} = i, u_t = \delta, \left\{u_{t+l}^{[q]}\right\}_{l=1}^{h}\right] \\ -\left[Y_{t+h}|\mathbf{Y}_{t-1}, \Theta^{[g]}, S_t^{[g]} = i, u_t = 0, \left\{u_{t+l}^{[q]}\right\}_{l=1}^{h}\right]\end{array}\right\} \qquad (7)$$

for each history starting with $S_t = i$, $i = 0, 1$, and the superscript g indicates the gth Gibbs iteration. The error bands for the impulse responses can be constructed by computing the appropriate coverage over the G Gibbs draws.

In addition to the policy shocks, a change in regime can cause a response in the macroeconomic variables both through a change in the regime-dependent mean growth rate and a change in the dynamics. We can compute the response to a change in the regime at time t. In this case, we do not shock the system as in the GIRFs; the only difference in the two conditional expectations is the change in regime.

We compute the response Y_{t+h} of a change from $S_{t-1} = j$ to $S_t = i$ by simulating the errors out to horizon h:

$$RR_{ij}\left(h\right) = \frac{1}{G}\frac{1}{R_i^{[g]}}\frac{1}{Q}\sum_g^G\sum_r^{R_i^{[g]}}\sum_q^Q \left\{ \begin{array}{c} \left[Y_{t+h}|\mathbf{Y}_{t-1}, \Theta^{[g]}, S_t^{[g]} = i, S_{t-1}^{[g]} = j, \left\{u_{t+l}^{[q]}\right\}_{l=1}^h\right] \\ -\left[Y_{t+h}|\mathbf{Y}_{t-1}, \Theta^{[g]}, S_t^{[g]} = j, S_{t-1}^{[g]} = j, \left\{u_{t+l}^{[q]}\right\}_{l=1}^h\right] \end{array} \right\} \quad (8)$$

for all histories in each Gibbs iteration for which $S_{t-1}^{[g]} = j$.

4 Results

To assess the effects of monetary policy in different phases of the business cycle, we compute the impulse responses to a shock to the federal funds rate under various model assumptions. Our baseline model is a monthly TVTP-FAVAR(12) with a single factor, where the transition variable is the lag of monthly output (ZCOIN) data, normalized around its mean and standardized to have unit variance. To account for the diminished variability of output after the Great Moderation, we allow for a structural break in the mean and standard deviation of ZCOIN after 1984 when standardizing the data.

Results are computed with 8000 draws of the Gibbs sampler, discarding the first 2000 draws to ensure convergence. We compute the mean and 68-percent posterior coverage of the resulting GIRFs and the responses to a change in regime. The GIRFs are computed over 400 equally-spaced draws (thinning every 20th draw) from the posterior distributions.

4.1 Baseline Results

Figure 1 plots the posterior probability of recessions ($S_t = 1$) for the full sample with the posterior mean and 68-percent coverage of the factor, filtered from the unbalanced panel of data described in Section 3.2. The NBER recessions are shaded in grey for comparison. The results are consistent with many other empirical models of monetary policy in a VAR environment with switching. The posterior probability of a recession is generally high during NBER recessions and the estimated regimes are persistent. The average across Gibbs iterations of the correlation between the estimated recessions and the NBER recessions is 0.44. The model appropriately identifies the NBER recessions in the pre-Great Moderation period. However, unlike the economic contractions of 2001 and 2007 which elicit a spike in the posterior probability of recession, the 1991 recession does not appear to be associated with a contractionary period that causes a variation in the effects of monetary policy.

The factor captures sentiment about current economic conditions as well as forward-looking projections of future economic activity. Prior to all of the NBER recessions in the sample, the factor declines substantially. Additionally, the factor begins to recover in the months leading up to and through the official end of each recession. The recession in 1980 elicits the deepest decline in sentiment, indicative of a severe economic contraction. This recession also produces the most clear identification of the recessionary regime, based upon the consistently high posterior probability of $S_t = 1$ during this time. Likewise, the factor declines substantially more around the financial crisis and Great Recession beginning in 2007, also associated with very high posterior probability of the $S_t = 1$ regime. In order to identify the scale and sign of the factor, we restrict the loading on the Conference Board Consumer Confidence Index to be equal to positive one. The estimated loadings on all sentiment series are similar with loadings slightly larger than one on the University of Michigan Consumer Sentiment Index and the Organization for Economic Cooperation and Development Consumer Confidence Index and slightly less than one on the Institute for Supply Management Purchasing Managers Index (PMI). The posterior mean and 68-percent coverage intervals for the loading estimates are presented in the top panel of Table 2.

The bottom panel of Table 2 provides the posterior means and 68-percent coverage intervals for the TVTP coefficient estimates.[14] The transition probabilities consist of a time-invariant component, $\overline{\gamma}_{ji}$, and a time-varying component, γ_{ji}, which represents the effects of lagged output on the regime. The posterior mean estimates of $\overline{\gamma}_{10}$ and $\overline{\gamma}_{11}$ are -4.08 and 1.22, suggesting that the expansionary regime is more persistent than the recessionary regime. We impose that lagged output's effect on the transition from expansion to recession is negative (estimated to be equal to -1.02), which reduces the probability of switching from expansion to recession if output growth is above

[14] The full set of parameter estimates are available from the authors upon request.

average. Additionally, the posterior mean estimate of the coefficient affecting the persistence of recessions, γ_{11}, is also negative (-0.67). Therefore, an increase in output growth will reduce the probability of remaining in recession from one period to the next. The time-varying effects are significant, suggesting that lagged output growth is an important indicator for determining the transition between the two regimes.

4.1.1 Comparison to Linear FAVAR

To establish a basis for comparison and to gauge the overall value-added of allowing the model parameters to vary across regimes, we also estimated a linear FAVAR without Markov-switching in any of the components.[15] Figure 2 plots the posterior mean of the sentiment factors filtered from both the linear and MS models. The factor series extracted from the model allowing for regime-dependence in the model parameters exhibits more drastic variation, rising more during expansions and falling more during recessions. This variation results from the time-varying propagation of innovations to the series within the VAR. Table 3 compares the root mean-squared-error of in-sample fitted values using the two models. For the ZCOIN and PCE inflation the RMSE's are essentially the same for both the linear and MS-TVTP models. However, for the federal funds rate series, the RMSE from the MS-TVTP is lower than that from the linear model. Therefore, it appears that the MS-TVTP model better describes the behavior of the policy rate, moving between regimes of expansionary and contractionary policy based upon the prevailing economic conditions.

These comparisons indicate that much of the non-linearity and regime-switching nature of the data are driven by variation in the volatility and covariance of the data series in recessionary and expansionary phases. While we do uncover differences in the systematic VAR parameters when $S_t = 1$ versus $S_t = 0$, most of the separate regime identification comes through the regime-dependent heteroskedastic covariance matrices Ω_0 and Ω_1.[16] In both the pre- and post-Great Moderation subperiods, the magnitudes of all reduced-form variance and covariance terms are larger in the recessionary regime.

[15] As we did with the MS-TVTP-FAVAR, we allow for a one-time structural break in the covariance matrix in 1984 to accommodate the Great Moderation.

[16] This result is consistent with Sims and Zha (2006) who estimate a variety of structural VAR model specifications to describe the potentially regime-switching behavior of monetary policy and its effects on the economy. They find that the model which best fits the data is one in which only the variances of structural innovations change across regimes.

4.2 Generalized Impulse Responses to Shocks of Varying Size and Sign

Figure 3 illustrates the GIRFs across a range of shock sizes, conditional on being in a given regime at the time of the shock. To model the likely behavior of the Fed at different points in the business cycle, we compute the responses of all macro variables in the VAR to contractionary shocks (6.25, 12.5, and 25 basis points) during the expansionary regime (top row of Figure 3) and expansionary shocks (-6.25, -12.5, and -25 basis points) during the recessionary regime (bottom row of Figure 3). For all variables, we find some evidence of multiplicative scaling in the effects of shocks of varying sizes. The responses to ± 25 basis point shocks are approximately four times the magnitude of those to ± 6.25-basis-point shocks, similarly for ± 12.5-basis-point shocks. This supports the conclusion of Lo and Piger (2005) who find no evidence of asymmetries based on the size or sign of the policy shock. Instead, they find evidence of asymmetries depending on the state of the business cycle at the time of policy action, with larger effects during recessions than during expansions.

Figure 3 suggests that the behavior of the sentiment factor is similar in both regimes. The peak response to all shocks is reached after 9 months. In both regimes, the peak effect on output (ZCOIN) growth is reached 12 months after the shock. With regards to inflation in both regimes and to all shock sizes, the peak effect is reached after 4 to 5 months. We see more volatility in the projected future path of inflation but the response is not significantly different from zero. The peak response of the policy rate is reached more quickly in recessions (2 months) than in the expansions (4 months).

We focus specifically on the responses of the sentiment factor and output growth to the shocks of 25 basis points, the shock size most commonly analyzed in the literature. Figure 4 illustrates the posterior mean GIRFs in recession (left column) and expansion (right column), the 68-percent posterior coverage interval, and the posterior mean response from the linear model without regime-switching. While the GIRFs of the sentiment factor in recession and expansion are similar, they both highlight that the regime-switching model suggests a larger response to policy shocks than that identified by the linear model. The GIRFs of output growth suggest less variation between the linear response and that produced in either regime. This result is due to the fact that the recessionary regime is rather short-lived and the GIRF simulations quickly switch from the recessionary regime into the expansionary regime. The RDIR for the expansionary regime closely resembles that of the linear VAR.

4.3 Generalized Impulse Responses to a Change in Regime

If we think of $S_t = 0$ and $S_t = 1$ as two steady states, we can compute the transition path between them. The GIRFs to a change in regime represent the behavior of macroeconomic variables in the model, conditional on a difference in regimes at time t. Figure 5 plots the mean response and the 68-percent posterior coverage intervals for these GIRFs. The left column of Figure 5 illustrates the effects of switching from expansion to recession. The subsequent months see a slight decline in output growth and little noticeable change in inflation. Additionally, the federal funds rate exhibits a shift upward in the mean but then declines following the regime shift. Sentiment falls at the time of the change and then takes more than two years to recover back to a level path.

The right column of Figure 5 depicts the macroeconomic behavior given a switch from the recession to the expansion regime. Due to the limited number of periods in recession, conditioning on this history when computing these GIRFs results in less-precise estimates. Based on the posterior mean path, output growth increases slightly at the time of the switch. In the subsequent months, the mean path of the federal funds rate rises and sentiment adjusts slightly downward. This could be indicative of precautionary behavior during the early stages of moderate recoveries.

4.4 Conditioning on Economic Conditions

In addition to computing GIRFs based on the economy either being in state $S_t = 0$ or $S_t = 1$ at the time of the shock, we can perform policy experiments that condition on the specific economic climate at the time of the policy action. For example, a recession during which inflation is far above target may witness different policy effects than a recession during which inflation is controlled. We examine the state-dependent effects of expansionary policies taken during recessions characterized by a variety of inflation and output growth values.

Panel (A) of Figure 6 plots the responses of the sentiment factor and output growth to 25- and 6.25-basis-point reductions in the federal funds rate (left and right columns, respectively) during recessions in which output growth was either: (1) less than 1-standard-deviation below average, (2) between 0- and 1-standard-deviation below average, (3) between 0- and 1-standard-deviation above average, and (4) greater than 1-standard-deviation above average. We compute these responses at the posterior mean estimate of all model parameters. As seen in Figure 6, a small expansionary shock of 6.25 basis points results in responses with considerable variation, depending on the state at the time of the shock. This variation decreases with the size of the shock and is less apparent with the 25-basis-point rate cut.

Panel (B) of Figure 6 plots the responses of the sentiment factor and output growth to 25- and 6.25-basis-point reductions in the federal funds rate (left and right columns, respectively) during recessions in which inflation was either above or below 3% at the time of the shock. The responses do not seem to show much variation depending on the level of inflation when policymakers took action. Interestingly, the most noticeable differences are seen for the sentiment factor when inflation is low. When inflation is less than 3%, expansionary policy shocks are more persistent over the medium- and long-term horizons and thus produce a larger boost to sentiment.

5 Sequential Shocks

Empirical evidence on Taylor rules and reaction functions suggests that the Fed smooths interest rates. For example, the Fed may anticipate that it will reduce the federal funds rate in the face of a recession. It can do so in one large move or make a series of smaller moves. We have argued before that, in the linear model, the response is invariant to the size of the shock, up to a scalar multiple—that is, a 25-basis-point shock produces the same response as a 1-basis-point shock multiplied by 25. Thus, a 25-basis-point shock produces a response equivalent to four consecutive 6.25-basis-point shocks, except for the slight variation in timing. On the other hand, altering the magnitude of the shock in the nonlinear model does not produce a scalar multiple response. Thus, there is no guarantee that the 25-basis-point shock will produce anything similar to a sequence of four 6.25-basis-point shocks.

5.1 Sequential Shock Responses

In order to evaluate the effect of smoothing the shocks, we compare the responses of the economic variables to two sets of shocks: (i) a 25-basis-point change in the federal funds rate and (ii) four consecutive 6.25-basis-point changes in the federal funds rate. We then measure the expected paths of the macroeconomic variables, including the latent state, integrating over the histories, future shocks, and Gibbs iterations:

$$IR_i(h) = \frac{1}{G}\frac{1}{R_i^{[g]}}\frac{1}{Q}\sum_{g}^{G}\sum_{r}^{R_i^{[g]}}\sum_{q}^{Q}\left\{\begin{array}{c} \left[Y_{t+h}|\mathbf{Y}_{t-1},\Theta^{[g]},S_t^{[g]}=i,u_t=100,\left\{u_{t+l}^{[q]}\right\}_{l=1}^{h}\right] \\ -\left[Y_{t+h}|\mathbf{Y}_{t-1},\Theta^{[g]},S_t^{[g]}=i,\left\{u_{t+p-1}=u_{t+p}^{[q]}+25\right\}_{p=1}^{4},\left\{u_{t+l}^{[q]}\right\}_{l=p+1}^{h}\right]\end{array}\right\}.$$

Notice that we are conditioning on the same (structural) shocks for period $t+1$ to $t+h$ even though we have additional shocks for the second term in periods $t+1$ to $t+3$. Thus, the innovation to the federal funds rate can be thought of as a 6.25-basis-point shock above and beyond the set of Monte Carlo structural shocks. This conditioning ensures the shocks to both terms are the same except for the innovations that we are interested in.

Figure 7 plots the GIRFs based on either a single 25-basis-point change or four sequential 6.25-basis-point changes. The left column portrays the responses if the federal funds rate is increased 25 basis points at time t or in four consecutive 6.25-basis-points moves at times t, $t+1$, $t+2$, and $t+3$ during an expansion and the sentiment factor is listed first in the VAR. These two contractionary policy sequences induce significantly different behavior in the factor. Even after the four months it takes to fully implement both policy prescriptions, the factor is still significantly lower after the one-time, large contractionary shock. The large shock results in a slightly deeper contraction in output growth, but this is not persistent. There is little discernible difference in the response of inflation. The differences in the paths of the federal funds rate suggest that a series of smaller rate hikes leads to a higher path for the policy rate over the medium- to longer-term horizons, as represented by the GIRF taking on negative values after the four months it takes to implement the smoothed policy approach.

The right column of Figure 7 shows the responses if the federal funds rate is decreased in a single 25-basis-point move or four sequential 6.25-basis-point moves during a recession. Under these conditions, when the factor is listed first in the VAR, the difference in responses of the factor stays positive after the large shock for longer than the four months witnessing small, incremental shocks. This result suggests a longer-lasting, more favorable response of the sentiment factor after a large policy accommodation. Congruently, the large shock induces a slightly bigger boost to output growth but, again, little variation in the response of inflation. After four months, by the time both policies have been fully implemented, any difference between the paths for output growth and inflation disappears. Therefore, the larger stimulus initially provides an immediate, stronger boost to output growth that is not surpassed by the smooth policy approach. By construction, for the first four months in which the sequential policy is enacted, the decline in the policy rate is more substantial after the initial large shock. Once both policies have had time to induce the same systematic changes in the federal funds rate, the difference in paths becomes positive. This result suggests that, following the large rate cut, the federal funds rate takes on larger values in the medium term than if the Fed enacts a series of smaller rate cutes to achieve its target.

5.2 Sequential Shocks and Conditioning on Economic Conditions

Section 4.4 above illustrated how the effects of countercyclical policy can depend on the prevailing economic conditions at the time of the policy shock. We found that smaller shocks induce responses which are more sensitive to the level of inflation or output growth when the policy is enacted. In this section, we extend this experiment to look at the difference in responses to the single 25-basis-point shock and the four sequential 6.25-basis-point shocks when inflation is above and below target or when output growth is strong or weak. These GIRFs use the posterior mean estimates of all model parameters.

Figure 8 plots the GIRFs of the sentiment factor and output based on the single or sequential shocks, conditional on the relevant levels of output growth or inflation. The left column shows the GIRFs during recessions in which output growth was either: (1) less than 1-standard deviation below average, (2) between 0- and 1-standard deviation below average, (3) between 0 and 1 standard deviation above average, and (4) greater than one standard deviation above average. Any substantial differences in the responses are seen after the four months it takes to cut the federal funds rate the full 25 basis points using incremental steps. When output growth is negative, the response of the sentiment factor exhibits greater persistence throughout the one-and-a-half years following the first four months of policy changes. Additionally, while we find less variation in the responses of output growth, we also see slightly greater persistence in the responses when output growth is negative. The right column shows the GIRFs based on whether inflation was above or below 3% at the time of the initial policy shock. We find almost no variation in the GIRFs conditioning on inflation levels and the responses appear similar to those using the full history of recessions identified in the sample, as in the right column of Figure 6.

6 Conclusions

We estimate a self-exciting, TVTP-VAR in which lagged output growth affects the underlying state of the economy. As a result, countercyclical policy affecting the variables within the VAR also affects the latent state explaining the transition between expansionary and recessionary regimes. Additionally, we extract a factor representing overall sentiment regarding the health and outlook of the economy. We find that this factor declines in the months preceding each of the NBER-dated recessions and recovers in the months leading up to the trough. Our model appropriately identifies NBER recessions in the pre-Great Moderation subperiod but does not identify as much variation in the dynamics of the model in these predetermined expansion or recession periods in the post-Great Moderation period.

We find empirically relevant differences in the effects of policy between the two regimes, as well as variation depending on the type of policy enacted at various points in the business cycle. The effects of small policy changes are sensitive to the levels of output growth and inflation at the time of the shocks, but large policy shocks have relatively similar effects in these different environments. Finally, smoothing of policy rates in order to enact gradual adjustments induces different effects than large, one-time policy shocks which ultimately result in changes in the policy rate of the same magnitude. The greater stimulus to overall sentiment and output from large policy shocks may suggest that more aggressive policy intervention, without as much emphasis on smoothing, may be appropriate to combat recessions.

References

[1] Angrist, Joshua D., Oscar Jorda, and Guido M. Kuersteiner (2013). "Semiparametric Estimates of Monetary Policy Effects: String Theory Revisited," Working paper, Federal Reserve Bank of San Francisco.

[2] Arias, Jonas E., Juan F. Rubio-Ramirez, and Daniel F. Waggoner (2014) "Inference Based on SVARs Identified with Sign and Zero Restrictions: Theory and Applications," Working Paper, Federal Reserve Bank of Atlanta.

[3] Barnichon, Regis and Christian Matthes (2014). "Measuring the Nonlinear Effects of Monetary Policy," Working paper, Federal Reserve Bank of Richmond.

[4] Blanchard, Olivier Jean and Danny Quah (1989). "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *The American Economic Review*, Vol. 79, No. 4, pp. 655-673.

[5] Christiano, Lawrence, Martin Eichenbaum and Charles Evans "Monetary Policy Shocks: What Have We Learned and to What End?," in Taylor and Woodford, *Handbook of Macroeconomics*.

[6] Chib, Siddhartha (1993). "Bayes Estimation of Regressions with Autoregressive Errors: A Gibbs Sampling Approach," *Journal of Econometrics*, Vol. 58, pp. 275-296.

[7] Lo, Ming Chien and Jeremy Piger (2005). "Is the Response of Output to Monetary Policy Asymmetric? Evidence From a Regime-Switching Coefficients Model," *Journal of Money, Credit, and Banking*, Vol. 37, No. 5, pp. 865-886.

[8] The Conference Board, Inc (2011). "Consumer Confidence Survey Technical Note," Retrieved from https://www.conference-board.org/data/consumerconfidence.cfm.

[9] Cover, James Peery (1992). "Asymmetric Effects of Positive and Negative Money-Supply Shocks," *The Quarterly Journal of Economics*, Vol. 107, No. 4, pp. 1261-1282.

[10] Ehrmann, Michael, Martin Ellison and Natacha Valla. (2003) "Regime-Dependent Impulse Response Functions in a Markov-Switching Vector Autoregression Model." *Economics Letters,* 78(3), pp. 295–299.

[11] Garcia, René and Huntley Schaller. (2002). "Are the Effects of Monetary Policy Asymmetric?," *Economic Inquiry*, Vol. 40, No. 1, pp. 102-119.

[12] Hamilton, James (2015). "Macroeconomic Regimes and Regime Shifts," forthcoming in Taylor and Uhlig, *Handbook of Macroeconomics*, Vol. 2.

[13] Hamilton, James (1989). "A new Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica* 57, 357-384.

[14] Kaufmann, Sylvia (2015). "K-State Switching Models with Time-Varying Transition Distributions–Does Loan Growth Signal Stronger Effects of Variables on Inflation?," *Journal of Econometrics*, Vol. 187, No. 1, pp. 82-94.

[15] Koop, Gary, Hashem M. Pesaran and Simon M. Potter. (1996) "Impulse Response Analysis in Nonlinear Multivariate Models." *Journal of Econometrics*, Elsevier,Vol. 74(1), pp. 119-147.

[16] Krippner, Leo (2013). "Measuring the Stance of Monetary Policy in Conventional and Unconventional Environments," Working Paper, Centre for Applied Macroeconomic Analysis (forthcoming).

[17] Krolzig, Hans-Martin (2006), "Impulse-Response Analysis in Markov Switching Vector Autoregressive Models," Presented at ESEM.

[18] Morgan, Donald P. (1993). "Asymmetric Effects of Monetary Policy," *Economic Review*, Federal Reserve Bank of Kansas City, pp. 21-33.

[19] OECD (2015), Consumer Confidence Index (CCI) (indicator). doi: 10.1787/46434d78-en (Accessed on 05 May 2015), Retrieved from https://data.oecd.org/leadind/consumer-confidence-index-cci.htm.

[20] Potter, Simon M. (1995). "A Nonlinear Approach to US GNP," *Journal of Applied Econometrics*, Vol. 10, No. 2, pp. 109-125.

[21] Ravn, Morten and Martin Sola (1996). "A Reconsideration of the Empirical Evidence on the Asymmetric Effects of Money-Supply Shocks: Positive vs. Negative or Big vs. Small?," Unpublished manuscript, University of Aarhus.

[22] Rigobon, Roberto and Brian Sack (2004). “The Impact of Monetary Policy on Asset Prices,” *Journal of Monetary Economics*, Vol. 51, No. 8, pp. 1553-1575.

[23] Sims, Christopher A. and Tao Zha (2006). “Were There Regime Switches in U.S. Monetary Policy?,” *The American Economic Review*, Vol. 96, No. 1, pp. 54-81.

[24] Thoma, Mark A. (1994). “Subsample Instability and Asymmetries in Money-Income Causality,” *Journal of Econometrics*, Vol. 64, pp. 279-306.

[25] Uhlig, Harald (2005). “What Are the Effects of Monetary Policy on Output? Results from an agnostic identification procedure,” *Journal of Monetary Economics*, Vol. 52, pp. 381-419.

[26] University of Michigan Survey of Consumers (2015). “About the Survey,” Retrieved from http://press.sca.isr.umich.edu/press/about_survey.

[27] Woodford, Michael (1999). “Optimal Monetary Policy Inertia,” *Proceedings*, Federal Reserve Bank of San Francisco.

[28] Wu, Jing Cynthia and Fan Dora Xia (2016). "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," *Journal of Money, Credit and Banking,* Vol. 48, No. 2-3, pp. 253-291.

[29] Weise, C. L. (1999). “The Asymmetric Effects of Monetary Policy: A Nonlinear Vector Autoregression Approach,” *Journal of Money, Credit and Banking*, Vol. 31, No. 1, pp. 85-108.

A Sampler Details

The following subsections describe the draws of the estimation method.

A.1 Drawing $B_0(L), B_1(L), \Omega_0, \Omega_1$ conditional on $\Theta_{-\{B_0(L),B_1(L),\Omega_0,\Omega_1\}}, \mathbf{S}_T, \mathbf{F}_T$

Conditional on the factor and the parameters of $\phi(.)$, the VAR model parameters are simply conjugate N-IW. Let $\mathbf{Y}_{T-p} = [Y_{T-p}, ..., Y_{1+P-p}]'$, $\mathbf{S}_{T-p} = [S_{T-p}, ..., S_{1+P-p}]'$, $\mathbf{X}_t^* = [\mathbf{1}_{T-P}, \mathbf{Y}_{T-1}, ..., \mathbf{Y}_{T-P}, \mathbf{S}_{T-P}, \mathbf{S}_{T-1} \odot \mathbf{Y}_{T-1}, ..., \mathbf{S}_{T-P} \odot \mathbf{Y}_{T-P}]'$, and $\mathbf{X}^*$ represent the vector of stacked $\mathbf{X}_t^*$'s.

Then, given the prior, a draw of $\{B_0(L), B_1(L)\}$ can obtained from $B_0(L), B_1(L) | \Theta_{-\{B_0(L),B_1(L)\}}, \mathbf{F}_T \sim N(\mathbf{b}, \mathbf{B})$, where

$$\mathbf{B} = \left(\mathbf{B}_0^{-1} + \mathbf{X}^{*\prime}\mathbf{X}^*\right)^{-1},$$

$$\mathbf{b} = \mathbf{B}\left(\mathbf{B}_0^{-1}\mathbf{b}_0 + \mathbf{X}^{*\prime}\mathbf{Y}_T\right).$$

Let $\boldsymbol{\varepsilon}_T$ reflect the stacked vector of errors; then, given the prior, we can draw Ω from

$$\Omega^{-1} \sim \mathbf{W}(\nu, \varpi),$$

where $\nu = \nu_0 + T/2$ and $\varpi = (\varpi_0 + \boldsymbol{\varepsilon}_T\boldsymbol{\varepsilon}_T')/2$.

A.2 Drawing $S_t|\Theta, \mathbf{Y}$

Let $\Omega_t = \{y_\tau : \tau \leq t\}$ collect all the data up to time t. From Chib (1993), the conditional density for $\mathbf{S}$ is

$$p(\mathbf{S}|\Theta, \mathbf{Y}) = p\left(S_T|\otimes_T, \boldsymbol{\mu}, \sigma^2, \boldsymbol{\gamma}\right) \prod_{t=1}^{T-1} p\left(S_t|S_{t+1}, \otimes_t, \boldsymbol{\mu}, \sigma^2, \boldsymbol{\gamma}\right). \tag{9}$$

The density $p(S_t|\Theta, \mathbf{Y})$ is computed by Hamilton's modification of the Kalman filter, the last iteration yielding $p(S_T|\Theta, \mathbf{Y})$. From Bayes Law, we have

$$p(S_t|S_{t+1}, \Theta, \mathbf{Y}) = \frac{p_{S_{t+1},S_t} p\left(S_t|\otimes_t, \boldsymbol{\mu}, \sigma^2, \boldsymbol{\gamma}\right)}{\sum_{j=1}^{3} p_{S_{t+1},j}\, p\left(S_t = j|\otimes_t, \boldsymbol{\mu}, \sigma^2, \boldsymbol{\gamma}\right)},$$

where $p_{j,i}$ is the (time varying) transition probability. Combined, this allows us to generate S_t recursively.

A.3 Drawing $\boldsymbol{\gamma}|\Theta_{-\boldsymbol{\gamma}}, \mathbf{Y}, \mathbf{S}$

The transition parameters are drawn using the difference random utility model described in Frühwirth-Schnatter and Frühwirth (2010) and Kaufmann (2015). Under this specification, the regime variable has an underlying continuous utility representation, $U_{m,t}$. The period t latent state utility for regime k is

$$U_{kt} = \mathbf{Z}_t' \boldsymbol{\gamma}_k + v_{k,t}, \qquad k = 0, 1$$

where

$$\mathbf{Z}_t = \left[z_{t-d}(1 - S_{t-1}), z_{t-d} S_{t-1}, (1 - S_{t-1}), S_{t-1} \right]',$$

$$\boldsymbol{\gamma}_k = \left[\gamma_{k0}, \gamma_{k1}, \bar{\gamma}_{k0}, \bar{\gamma}_{k1} \right],$$

and $v_{k,t}$ follows a Type 1 extreme value distribution. We assume the regime with the maximum utility at time t is the observed regime:

$$S_t = j \Longleftrightarrow U_{j,t} = \max_{k=0,1} U_{k,t}.$$

Differences in utility are given by

$$\omega_{k,t} = \begin{cases} U_{0,t} - U_{1,t} & \text{if } k = 0 \\ U_{1,t} - U_{0,t} & \text{if } k = 1 \end{cases},$$

where, analogous to the case above, the observed regime is the one with the highest utility

$$S_t = j \Longleftrightarrow \omega_{j,t} = \max_{k=0,1} \omega_{k,t}.$$

We can rewrite the state utilities as

$$U_{k,t} = \log(\zeta_{k,t}) + v_{-k,t},$$

where

$$\zeta_{k,t} = \exp\left(\mathbf{Z}_t' \gamma_k\right),$$

$$\zeta_{-k,t} = \begin{cases} \zeta_{1,t} & \text{if } k = 0 \\ \zeta_{0,t} & \text{if } k = 1 \end{cases}.$$

Similarly, the difference in state utilities can be rewritten as

$$\begin{aligned} \omega_{k,t} &= \mathbf{Z}_t' \gamma_k - \log(\zeta_{-k,t}) + v_{k,t} - v_{-k,t} \\ &= \mathbf{Z}_t' \gamma_k - \log(\zeta_{-k,t}) + \epsilon_{k,t}, \qquad \epsilon_{k,t} \sim \mathit{Logistic}. \end{aligned}$$

For normalization purposes, we impose $k = 0$ to be the reference regime. This implies the restriction $\boldsymbol{\gamma}_0 = [0, 0, 0, 0]'$. Thus, it is only necessary to draw the transition parameters for the regime $k = 1$. Practically, there are three substeps to the sampling technique for $\boldsymbol{\gamma}_1$. The first substep is to sample the latent state utility differences outlined above for all time periods:

$$\omega_{1,t} = \mathbf{Z}_{1,t}' \gamma_1 + \tilde{\epsilon}_t,$$

where

$$\tilde{\epsilon}_t = \log\left[S_t + W_t\left(1 - S_t - \frac{\zeta_{1,t}}{1 + \zeta_{1,t}}\right)\right] - \log\left[1 - S_t - W_t\left(1 - S_t - \frac{\zeta_{1,t}}{1 + \zeta_{1,t}}\right)\right],$$

$$W_t \sim U\left(0, 1\right).$$

Next, we estimate the logistic distribution of the true errors, ϵ, by a mixture of normal distributions with six components. The components, R_t, are sampled from the distribution

$$p(R_t = r) \propto \frac{w_r}{s_r} \exp\left[-0.5\left(\frac{\omega_{1,t} - \mathbf{Z}_t' \gamma_1}{s_r}\right)^2\right], \qquad r = 1, \ldots, 6,$$

where the component weights, w_r, and component standard deviation, s_r, are given in Table 1 of Frühwirth-Schnatter and Frühwirth (2010).

Finally, given the prior $\boldsymbol{\gamma}_1 \sim N(\mathbf{g}_0, \mathbf{G}_0)$, we generate the draw of $\boldsymbol{\gamma}_1$ from the normal posterior distribution $\boldsymbol{\gamma}_1 \sim N(\mathbf{g}, \mathbf{G})$, where

$$g = G\left(\mathbf{G}_0^{-1}\mathbf{g}_0 + \sum_{t=1}^{T}\frac{\mathbf{Z}_t\omega_{1,t}}{s_{R_t}^2}\right),$$

$$G = \left(\mathbf{G}_0^{-1} + \sum_{t=1}^{T}\frac{\mathbf{Z}_t\mathbf{Z}_t'}{s_{R_t}^2}\right)^{-1}.$$

A.4 Drawing λ_m, σ_m^2 conditional on $\Theta_{-\{\lambda_m,\sigma_m^2\}}, \mathbf{S}_T, \mathbf{F}_T$

Conditional on the factor, the factor equation parameters are N-IG. Given the prior, a draw of λ_m can obtained from $\lambda_m|\Theta_{-\lambda_m}, \mathbf{F}_T \sim N(\mathbf{d}_m, \mathbf{D}_m)$, where

$$\mathbf{D} = \left(\mathbf{D}_0^{-1} + \mathbf{F}_T'\mathbf{F}_T\right)^{-1},$$

$$\mathbf{d} = \mathbf{D}\left(\mathbf{D}_0^{-1}\mathbf{d}_0 + \mathbf{F}_T'\mathbf{X}_{mT}\right),$$

and $\mathbf{X}_{mT} = [X_{m1}, ..., X_{mT}]'$.

Define $\mathbf{X}_T = [\mathbf{X}_{1T}, ..., \mathbf{X}_{MT}]'$. Then, we can sample σ_m^{-2} from a gamma posterior $\sigma_m^{-2}|\Psi_{-\mathbf{\Omega}} \sim \mathbf{G}(r_m, \rho_m)$, where $r_m = (r_0 + T)/2$ represents the degrees of freedom and the scale parameter is $\rho_m = (\rho_0 + \mathbf{u}_T\mathbf{u}_T')/2$.

A.5 Drawing $\mathbf{F}_T$ conditional on $\Theta, \mathbf{S}_T$

Conditional on the VAR parameters, the transition function, and the factor equation parameters, the factor can be filtered using a linear Kalman filter. It will be convenient to rewrite the model in its state-space representation. Define the state $\varsigma_t = \left[\Xi_t', , ...\Xi_{t-p+1}'\right]'$, where $\Xi_t = \left[\left[1-\phi(z_{t-d})\right]\xi_t', \phi(z_{t-d})\xi_t'\right]'$. Then,

$$\begin{aligned} Z_t &= H\varsigma_t + e_t, \\ \varsigma_t &= W\varsigma_{t-1} + v_t, \end{aligned} \tag{10}$$

where $Z_t = [Y_t', X_t']'$, $e_t = [\mathbf{0}_N', u_t']'$, $v_t = \left[\varepsilon_t', \mathbf{0}_{2(N+1)(P-1)\times 1}'\right]'$. The state-space coefficient matrices are

$$H = \begin{bmatrix} \mathbf{I}_N & \mathbf{0}_{N\times 1} & \mathbf{0}_{N\times 2(N+1)(P-1)} \\ \mathbf{0}_{M\times N} & \boldsymbol{\lambda} & \mathbf{0}_{M\times 2(N+1)(P-1)} \end{bmatrix}$$

and

$$W = \begin{bmatrix} B_{01} & \cdots & B_{0P} & B_{11} & \cdots & B_{1P} \\ \mathbf{I}_{2(N+1)(P-1)} & & \mathbf{0}_{2(N+1)(P-1)\times 2(N+1)} & & & \end{bmatrix}, \tag{11}$$

where each B_{ip} is a $(N+1\times N+1)$ matrix collecting the pth lag coefficients for the ith regime.

Given a set of starting values of $\varsigma_{0|0}$ and $P_{0|0}$, the filter iterates prediction and update steps forward for $t = 1,..,T$. The prediction step computes a projection of the period$-t$ state variable based on information available at time $t-1$. The prediction density is typically written as

$$\varsigma_{t|t-1} = W\varsigma_{t-1|t-1},$$

$$P_{t|t-1}^x = WP_{t-1|t-1}^x W' + Q,$$

where $Q = E\left[\varpi\varpi'\right]$.

From the prediction density, we can update the state vector using the next period realization of the data. Define the prediction error as

$$\eta_{t|t-1} = Z_t - H\varsigma_{t|t-1}, \tag{12}$$

where the variance can then be written as:

$$P_{t|t-1}^y = H\varsigma_{t|t-1}P_{t|t-1}^x H' + V.$$

The covariance with $\varsigma_{t|t-1}$ as

$$P^{xy}_{t|t-1} = P^{x}_{t|t-1} H'.$$

The updated state vector density is then

$$\varsigma_{t|t} = \varsigma_{t|t-1} + P^{xy}_{t|t-1} P^{-1}_{t|t-1} \eta_{t|t-1},$$

with variance

$$P^{x}_{t|t} = P^{x}_{t|t-1} - P^{xy}_{t|t-1} P^{-1}_{t|t-1} P^{xy\prime}_{t|t-1}.$$

From these, we can retain $\left\{\varsigma_{t|t}\right\}_{t=1}^{T}$, $\left\{P^{x}_{t|t}\right\}_{t=1}^{T}$, $\left\{\varsigma_{t|t-1}\right\}_{t=1}^{T}$ and $\left\{P^{x}_{t|t-1}\right\}_{t=1}^{T}$. In a single-move Gibbs sampler, we would draw ς_t from $N\left(\varsigma_{t|T}, P^{x}_{t|T}\right)$, which requires smoothing. A standard backward smoother yields:

$$\varsigma_{t|T} = \varsigma_{t|t} + P^{x}_{t|t} W' \left(P^{x}_{t+1|t}\right)^{-1} \left(\varsigma_{t+1|T} - \varsigma_{t+1|t}\right) \tag{13}$$

with variance

$$P^{x}_{t|T} = P^{x}_{t|t} + P^{x}_{t|t} W' \left(P^{x}_{t+1|t}\right)^{-1} \left(\left(P^{x}_{t+1|T}\right)^{-1} - \left(P^{x}_{t+1|t}\right)^{-1}\right) \left(P^{x}_{t+1|t}\right)^{-1} W P^{x}_{t|t}.$$

An alternative is to use a multi-move sampler (Carter and Kohn (1994)), which draws the entire state vector at once from $p\left(\varsigma_t | \Omega_t, \varsigma^{*}_{t+1}\right)$, where Ω_t represents the data known at time t and the superscript $*$ indicates the truncation of the state vector due to the singular covariance matrix. We then draw F_t from $N\left(F_{t|t,\varsigma_{t+1}}, P^{x}_{t|t,\varsigma_{t+1}}\right)$, where

$$F_{t|t,\varsigma_{t+1}} = \left[\varsigma_{t|t} + P^{x}_{t|t} W^{*\prime} \left(W^{*} P^{x}_{t|t} W^{*\prime} + Q^{*}\right)^{-1} \left(\widehat{\varsigma}^{*}_{t+1} - \varsigma_{t+1|t}\right)\right] \mathbf{e}_N, \tag{14}$$

$$P^{x}_{t|t,\varsigma_{t+1}} = \mathbf{e}'_N \left[P^{x}_{t|t} + P^{x}_{t|t} W^{*\prime} \left(W^{*} P^{x}_{t|t} W^{*\prime} + Q^{*}\right)^{-1} W^{*} P^{x}_{t|t}\right] \mathbf{e}_N,$$

and $\mathbf{e}_N$ is a vector with a 1 as the $(N+1)$th element and 0's everywhere else. The main difference between (13) and (14) is that the former generates all the posterior distributions simultaneously while the latter forms them recursively, conditional on the $t + 1$ period draw.

B Tables and Figures

Table 1: Priors for Estimation

Parameter	Prior Distribution	Hyperparameters
$[B_0, B_1]$	$N(\mathbf{b}_0, \mathbf{B}_0)$	$\mathbf{b}_0 = [GDP_0, 0_{N\times P}, GDP_1 - GDP_0, 0_{N\times P}]'^{(1)}$
		$\mathbf{B}_0$: See note (2).
$\Omega_0^{-1}, \Omega_1^{-1}$	$\mathbf{W}(\nu_0, \varpi_0)$	$\nu_0 = N+2$; $\varpi_0 = I_{N+1}$
$\boldsymbol{\gamma}_1$	$N(\mathbf{g}_0, \mathbf{G}_0)$	$\mathbf{g}_0 = [-4, -4, -4, 4]'$; $\mathbf{G}_0 = diag(4, 4, 1, 1)$
λ_m	$N(\mathbf{d}_0, \mathbf{D}_0)$	$\mathbf{d}_0 = 0$; $\mathbf{D}_0 = 1$
σ_m^2	$IG(r_0, \rho_0)$	$r_0 = 1$; $\rho_0 = 1$

Table 1: (1) We set GDP_0 equal to the mean value of ZCOIN growth in NBER expansions and GDP_1 equal to the mean of ZCOIN growth in NBER recessions. (2) $\mathbf{B}_0$ imposes unit variance on the constant in each equation and shrinkage on higher lags of Y_t. Consider the components of $\mathbf{B}_0$ corresponding to the VAR coefficients for lag p in equation n. We assign the variance on the coefficient on variable n's own lag to be $\frac{0.5}{p^2}$ and the variance of the coefficients on the other variables to be $\frac{0.25}{p^2}$.

Table 2: Select Parameter Estimates

Factor Loadings				
	Posterior Mean	**68% Posterior Coverage**		**Sentiment Series**
λ_1	1.00	·	·	CBCCI
λ_2	1.15	1.10	1.20	UMCSI
λ_3	1.09	1.04	1.14	OECDCCI
λ_4	0.83	0.78	0.88	PMI

TVTP Coefficients			
	Posterior Mean	**68% Posterior Coverage**	
$\overline{\gamma}_{10}$	-4.08	-4.58	-3.62
$\overline{\gamma}_{11}$	1.22	0.72	1.72
γ_{10}	-1.02	-1.34	-0.69
γ_{11}	-0.67	-1.08	-0.25

Table 3: Root Mean-Squared Errors

	MS-TVTP-FAVAR	**Linear-FAVAR**
ZCOIN Growth	0.26	0.26
PCE Inflation	0.11	0.11
Federal Funds Rate	0.39	0.48

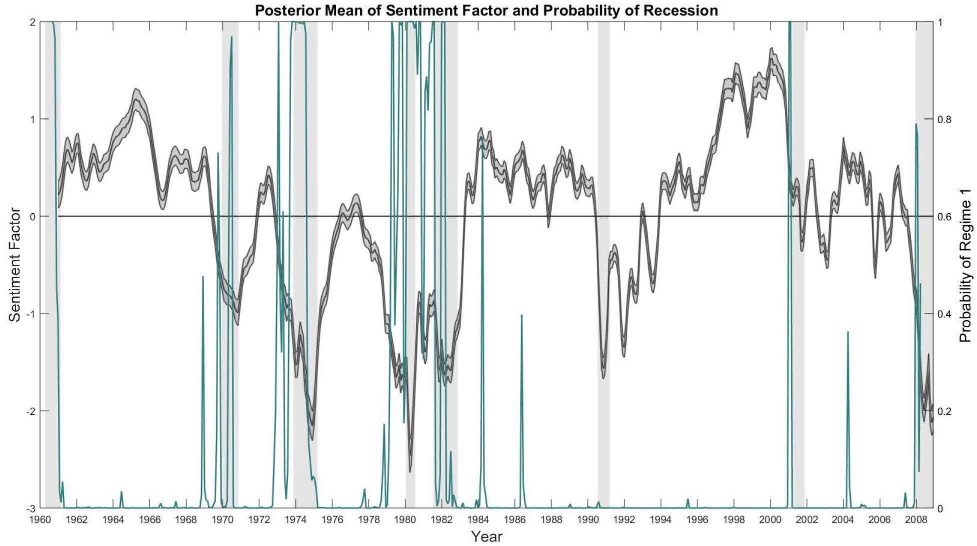

Figure 1: Left Axis: Posterior mean and 68% posterior coverage interval of factor extracted from unbalanced panel of sentiment data including: Conference Board (CB) Consumer Confidence Index, the University of Michigan Consumer Sentiment Index, the Organization for Economic Cooperation and Development (OECD) Consumer Confidence Index, and the Institute for Supply Management (ISM) Purchasing Managers Index (PMI). Right Axis: Posterior Mean Probability of Recession Regime - Four Variable FAVAR of Coincident Index growth, PCE Inflation, the Federal Funds Rate, and the Sentiment Factor. The posterior means are computed over 8000 draws from the Hamilton filter within the Gibbs sampler.

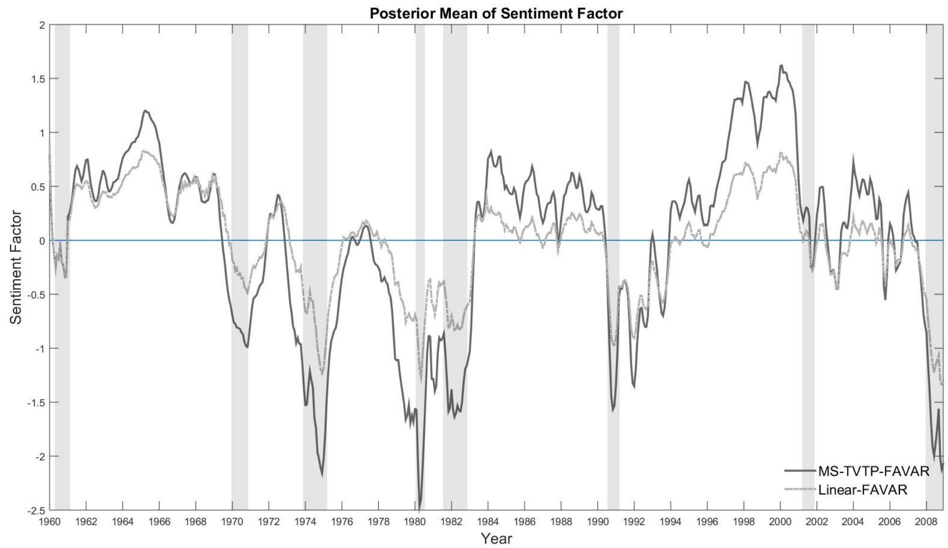

Figure 2: Factors extracted from both the TVTP-MS-FAVAR and the Linear FAVAR models. Both factors summarize information from the same unbalanced panel of sentiment data including: Conference Board (CB) Consumer Confidence Index, the University of Michigan Consumer Sentiment Index, the Organization for Economic Cooperation and Development (OECD) Consumer Confidence Index, and the Institute for Supply Management (ISM) Purchasing Managers Index (PMI). The posterior mean is computed over 8000 draws of the Kalman filter within the Gibbs sampler.

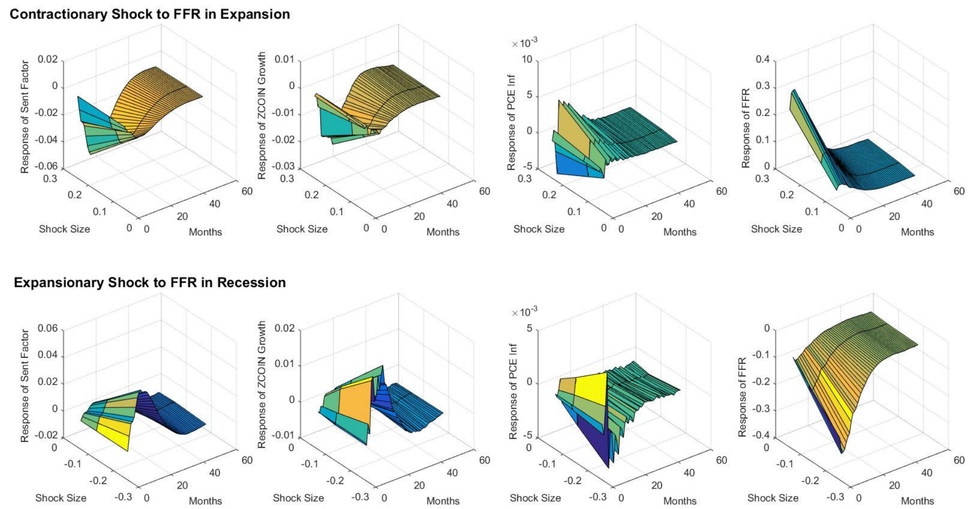

Figure 3: Generalized Impulse Responses: The responses are to a range of shocks to the Federal Funds Rate between +6.25 and +25 basis points in expansion or -25 and -6.25 basis points in recession. We condition on being in the expansionary regime (top row) or recessionary regime (bottom row) at the time of the shock.

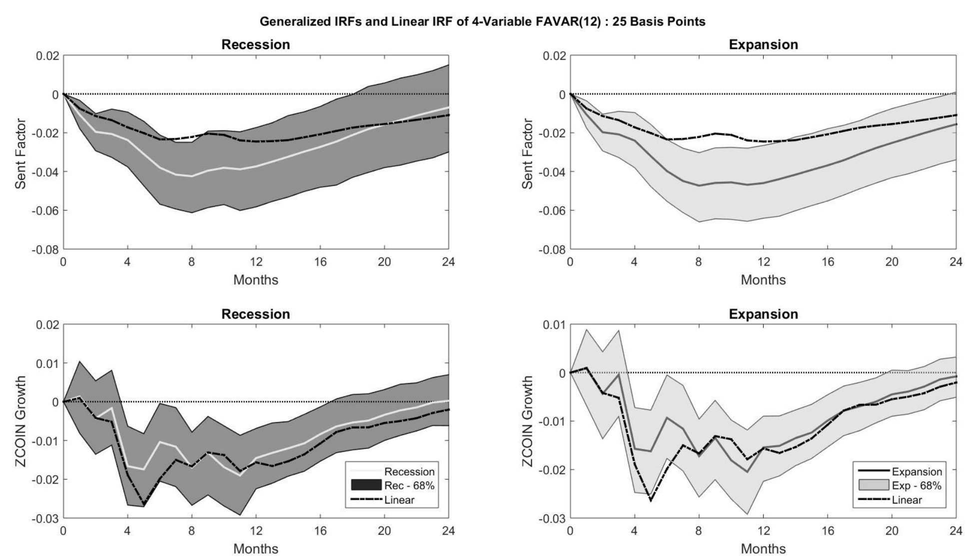

Figure 4: GIRF and Linear IRF of the sentiment factor and output growth in response to a 25 basis point shock to the Federal Funds Rate. For the GIRF's, we condition on being in the expansionary regime or recessionary regime at the time of the shock. The plots show the posterior mean and 68% posterior coverage intervals.

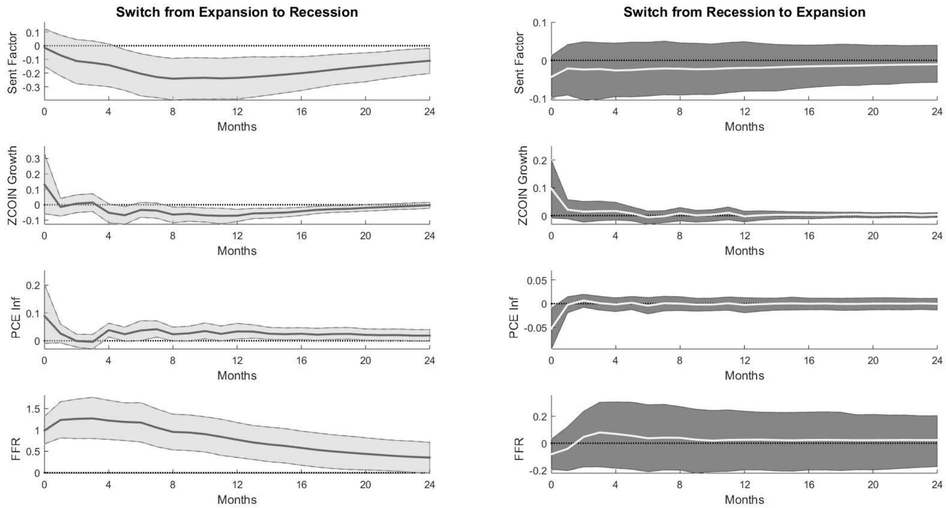

Figure 5: Generalized impulse responses to either a switch from the expansionary to the recessionary regime (left column) or a switch from the recessionary to the expansionary regime (right column). The Sentiment factor is listed first and thus responds to policy shocks with a lag while all macro variables respond to sentiment shocks contemporaneously. The plots show the posterior mean and 68% posterior coverage intervals, computed over 400 equally-spaced draws (thinning every 20th draw) from the posterior distributions.

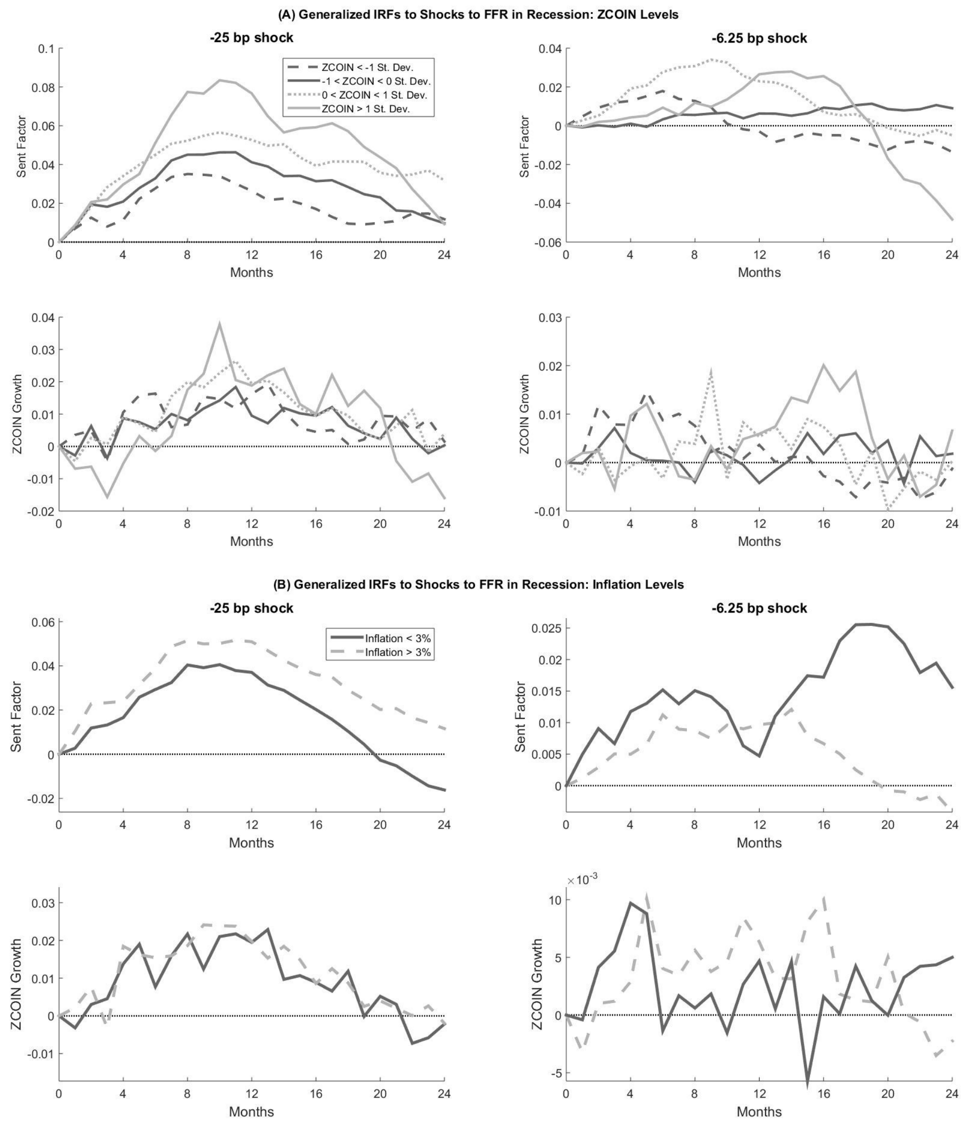

Figure 6: Generalized impulse responses to a 6.25 and 25 basis point reduction in the federal funds rate, conditional on being in recession with various levels of output growth at the time of the shock. The Sentiment factor is listed first and thus responds to policy shocks with a lag while all macro variables respond to sentiment shocks contemporaneously. The responses are computed at the posterior mean estimates of the model parameters.

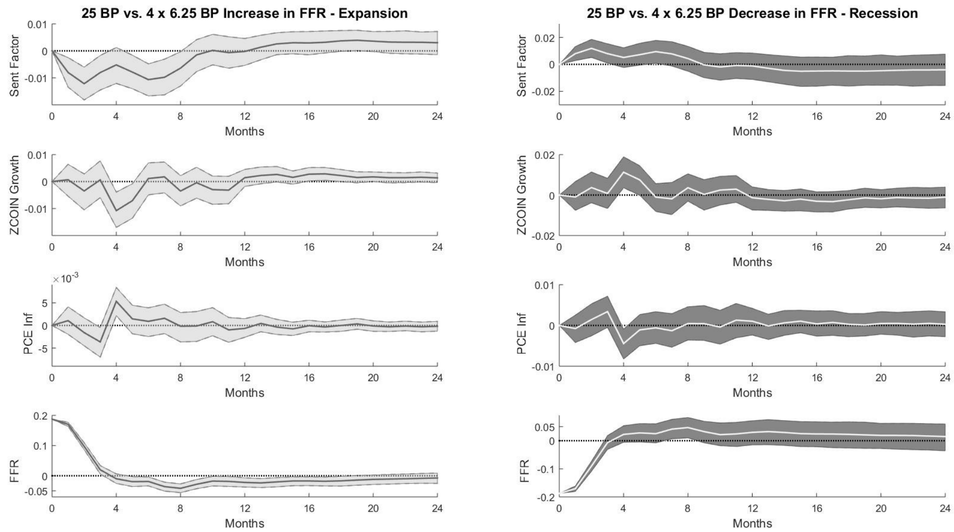

Figure 7: Generalized impulse responses to a single 25 basis point reduction (increase) in the Federal Funds Rate compared with 4 sequential 6.25 basis point cuts (increases), conditioning on being in recession (expansion) at the time of the policy shock. The Sentiment factor is listed first and thus responds to policy shocks with a lag while all macro variables respond to sentiment shocks contemporaneously. The plots show the posterior mean and 68% posterior coverage intervals, computed over 400 equally-spaced draws (thinning every 20th draw) from the posterior distributions.

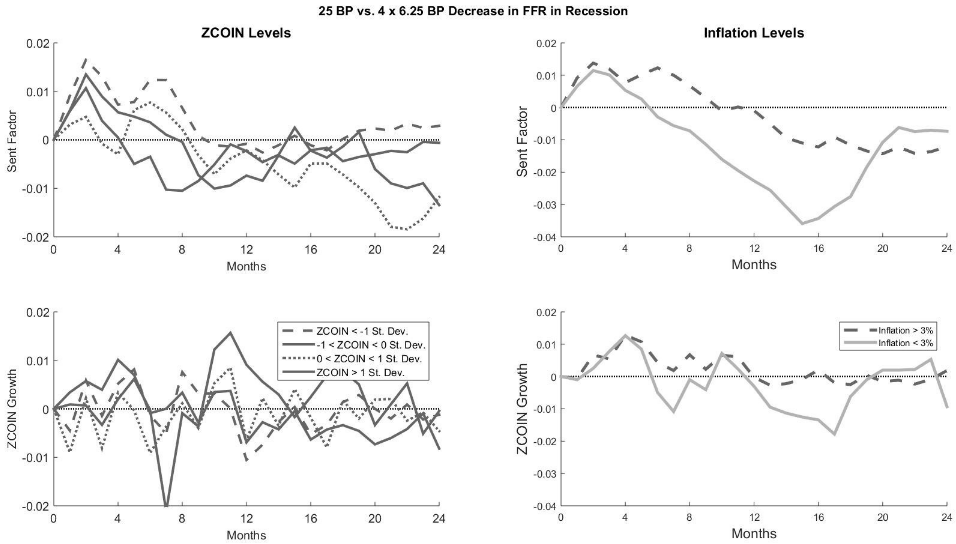

Figure 8: Generalized impulse responses to a single 25 basis point reduction in the Federal Funds Rate compared with 4 sequential 6.25 basis point cuts, conditional on being in recession with various levels of output growth at the time of the first shock. The Sentiment factor is listed first and thus responds to policy shocks with a lag while all macro variables respond to sentiment shocks contemporaneously. The responses are computed at the posterior mean estimates of the model parameters.