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YiLi Chien,
Hanno Lustig
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Kanda Naknoi

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Research Division
P.O. Box 442
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Why Are Exchange Rates So Smooth? A Household Finance Explanation ☆

YiLi Chien[a], Hanno Lustig[b], Kanda Naknoi[c]

[a] *Research Division, Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63166; Email: yilichien@gmail.com*

[b] *Stanford Graduate School of Business, Stanford, CA, 94305; E-mail: hlustig@stanford.edu.*

[c] *Corresponding author: Department of Economics, University of Connecticut, 365 Fairfield Way, Unit 1063, Storrs, CT 06269; Email: kanda.naknoi@uconn.edu.*

Abstract

Empirical moments of asset prices and exchange rates imply that pricing kernels are almost perfectly correlated across countries. Otherwise, observed real exchange rates would be too smooth for high Sharpe ratios. However, the cross-country correlation among macro fundamentals is weak. We reconcile these facts in a two-country stochastic growth model with heterogeneous households and a home bias in consumption. In our model, only a small fraction of households trade domestic and foreign equities. We show that this mechanism can quantitatively account for the smoothness of exchange rates in the presence of volatile pricing kernels and weakly correlated macro fundamentals.

Keywords: Asset pricing, Market segmentation, Exchange rates, International risk sharing (JEL code: G15, G12, F31, F10)

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1. Introduction

A striking disconnect exists in international finance between the evidence gathered from asset prices and that gathered from quantities. When markets are complete, no arbitrage implies that the percentage rate of appreciation of the real exchange rate (RER) is given by the difference between the domestic and the foreign pricing kernels. We know from the data on RERs and asset prices that the volatility of RERs is much smaller than the volatility of pricing kernels. Hence, the evidence from asset markets implies that pricing kernels are highly correlated across countries. In fact, Brandt et al. (2006) conclude that the correlation of pricing kernels across countries is close to 1. However, the quantity data paint a different picture. In representative agent models with constant relative risk aversion (CRRA) preferences, the correlation of pricing kernels across countries is identical to that for aggregate consumption growth. Empirically, the correlation of aggregate consumption growth is well below 50% for most industrialized country pairs. In this paper, we address this disconnect between prices and quantities in international finance in a two-country model with heterogeneous portfolios.[1]

Household finance may hold the key to this disconnect. Standard macro-finance models assume that aggregate risk has been distributed across all households, but in our model, most households do not bear their share of aggregate risk. We introduce heterogeneity in trading technologies in international equity and bond markets into an otherwise standard Lucas (1982) two-country model. In particular, we allow only a small fraction of international equity and bond market participants to optimally adjust their portfolios every period. These active investors, the only ones to respond elastically to variation in state prices, are marginal in foreign exchange (FX) markets and determine the dynamics of exchange rates.

Our model features global and country-specific aggregate risk and household-specific idiosyncratic risk. In equilibrium, a large fraction of global aggregate risk is borne by the small pool of sophisticated investors who actively participate in both domestic and foreign equity and bond

[1] There is an ongoing debate on whether incomplete markets models help solve the puzzle caused by the disconnect between prices and quantities. When markets are incomplete, the percentage rate of depreciation of the RER may not be identical to the difference between the domestic and the foreign pricing kernels. Hence, the incomplete markets model may help to resolve the puzzle as suggested by Favilukis et al. (2015). Maurer and Tran (2016) argue that a model embedded with risk entanglement, a refinement concept of incomplete market, can successfully explain these puzzles in international finance. However, Lustig and Verdelhan (2015) show that market incompleteness cannot quantitatively resolve the puzzle without largely eliminating currency risk premia. Based on the empirical evidence of household finance, our view is that some households do not utilize the financial assets available to them and act as if in an incomplete-markets world.

markets in each period. These active domestic and foreign investors achieve a higher degree of risk sharing among themselves — across borders — than the average investors in these countries — within borders. Hence, the marginal investor's consumption growth is highly correlated across countries, but the average investor's is not. This mechanism can quantitatively account for the excess smoothness of the RERs with pricing kernels that satisfy the Hansen-Jagannathan bounds.

The other critical feature of our model result is a moderate home bias in consumption. If agents have an extreme home bias and time-additive preferences, then there will be little motive for international risk sharing, even for sophisticated investors, and RERs will become too volatile.[2] On the other hand, when agents have little home bias in consumption, frictionless trade yields too little volatility in the RERs. In the intermediate case of moderate home bias, a calibrated version of our model can match RER volatility in the data.

Our approach is firmly grounded in the empirical evidence on household finance. The evidence suggests that most households do not purchase all assets available on the menu (see, e.g., Guiso and Sodini (2013) for an excellent survey of this literature). In fact, the composition of household asset holdings varies greatly across households, even in a financially developed country like the United States. Only 50% of U.S. households participate in the equity market, according to the 2010 Survey of Consumer Finance. Obviously, the non-participants bear no aggregate risk. Even among the equity market participants, many of them report equity shares that are significantly lower than the corresponding share of equities as a fraction of the market, namely all marketable securities. In addition, most of the equity market participants trade very infrequently and do not rebalance their portfolios often in response to changes in investment opportunities (see the evidence reported by Ameriks and Zeldes (2004), Brunnermeier and Nagel (2008), Calvet et al. (2009), and Alvarez et al. (2012)). In sum, the heterogeneity in observed portfolio choices data implies a highly uneven distribution of risk across investors and across time.

In the quantitative exercise, we parameterize our model to match moments of the world economy in which the home country is the United States and the foreign country is the GDP weighted sum of France, Germany, Japan, and the United Kingdom. In our benchmark economy, we find that the international correlation of the pricing kernels exceeds 97%, while the international correlation of consumption growth is only 17%. Despite the high volatility of the pricing kernels as

[2] If the preference is not time additive, then agents could still have large incentives to trade despite the extreme consumption home bias.

observed in the data, our model produces a 9.4% standard deviation of the RER, which is close to the data. Without heterogeneity in household types, the pricing kernels become much less volatile and weakly correlated, resulting in a violation of the Hansen-Jagannathan bounds implied by the pricing data.

Most of the existing work on this puzzle modifies the preferences or the stochastic properties of endowment growth of an otherwise standard representative agent model, implicitly assuming that all risk-sharing opportunities within a country have been exhausted. Colacito and Croce (2011) endow the representative investor with recursive preferences that impute a concern about long-run risk in consumption. When long-run risks are highly correlated across countries, the pricing kernels become highly correlated even though aggregate consumption growth is not[3]. In contrast, Farhi and Gabaix (2016) rely on correlated disaster risk. Finally, Stathopoulos (2016) analyzes a model in which the representative investor has preferences with external habit persistence, which induces high correlation of the pricing kernels, despite the low correlation of current consumption growth.

Prior work has explored market segmentation to understand exchange rates. Alvarez et al. (2002) develop a Baumol-Tobin model of nominal exchange rates in which money and securities markets are segmented. Recently, Gabaix and Maggiori (2015) consider a different form of market segmentation: only financial institutions are active in international bond and FX markets, while retail investors are not. Our model does not have a financial sector, but the sophisticated investors do clear FX markets and earn an equilibrium risk premium in return, much like the large financial institutions in Gabaix and Maggiori (2015).

Our paper introduces the Chien et al. (2011, 2012) heterogeneous trading technologies in a two-country Lucas (1982) model. Chien and Naknoi (2015) use a two-country Lucas model with limited participation that is similar to ours to study global imbalances. Dou and Verdelhan (2015) explain volatility of international equity and bond flows in a two-country framework with segmented asset market. However, the key mechanism of their model relies on asymmetry in preferences and an alternative form of an incomplete market. There are two other recent studies directly related to our work, and they explore a segmented-market explanation. Zhang (2015) offers an explanation of the puzzlingly high correlation of stock market returns and low correlation of fundamentals across

[3]Gavazzoni and Santacreu (2015) find that a calibrated model with long-run risk originating from international technology diffusion can quantitatively explain the RER volatility puzzle.

countries, but her work does not address the exchange-rate puzzle. Kim and Schiller (2015) focus on RER volatility in a segmented-market model with trade frictions. Theirs is a standard limited-participation model without heterogeneity among equity market participants. Consequently, the concentration of the residual aggregate risk in their calibrated model is not high enough to reconcile the moments of asset prices and exchange rates.

Our main contribution to the literature is the integration of the micro evidence on household portfolio choices and trade frictions into a general equilibrium model to solve the exchange-rate volatility puzzle. We show that distinguishing marginal FX investors from less sophisticated equity market investors is essential for solving the puzzle and enhances the performance of the model. The quantitative results indicate that some degree of trade frictions in goods market is also essential. Therefore, international trade in both goods market and asset markets matters to the exchange rate determination. Hence, our model offers an initial step to closing the gap between the asset market approach and the goods market approach of exchange-rate determination. Moreover, our model does not require non-standard preferences or aggregate risk specifications. Instead, the mechanism in our model relies on the skewness of the cross-sectional distribution of aggregate risk, and this feature is strongly supported by the empirical evidence.

In addition, our study broadly contributes to the emerging literature that integrates international portfolio choices into international macroeconomics. Specifically, we demonstrate the importance of household portfolio heterogeneity in open economies, whereas the majority of open-economy macroeconomic models rely on a representative agent framework. Recent studies by Pavlova and Rigobon (2010) and Coeurdacier and Rey (2013) are prominent examples. Most international macroeconomic models assume either incomplete markets with only one asset or a complete market environment without portfolio heterogeneity. Although a complete menu of assets is traded by some households in our model, we emphasize the heterogeneity in household trading technologies, as in the data.

Nonetheless, our model has not resolved the Backus-Smith puzzle and the uncovered interest rate parity (UIP) puzzle. To be precise, our model predicts that the RER depreciates during a consumption boom and the RER is expected to depreciate when the bond interest rate is high relative to the trading partner's bond interest rate. These predictions contradict the empirical evidence, and they remain even after we modify our model to include recursive preferences, an alternative belief about shocks, and portfolio inertia. Solving these puzzles requires a more complex combination of frictions and shocks than that in our model, in order to increase the response of expected variation of the pricing kernel compared to the response of the risk-free rate while making them move in the opposite direction.

The rest of our study is organized as follows. The next section describes our model. The quantitative results and counterfactual exercises are detailed in Section 3. Finally, Section 4 concludes our study.

2. The Model

We consider an endowment economy with two countries, home and foreign. There are a large number of agents in each country with a unit measure. Each country is endowed with a nontraded good and an export good. For simplicity, we assume that home households consume the nontraded good and the foreign export good. Likewise, foreign households consume the nontraded good and the home export good. In the main context, we describe only the setup of the home country since it mirrors the description of the foreign country. The setup of the foreign country is described in the supplementary material.

2.1. Sources of Uncertainty

Time is discrete, infinite, and indexed by $t \in [0, 1, 2, ...)$. To have a stationary economy, we assume an identical average endowment growth rate for each country, while the actual growth rate may deviate from the average growth rate. More specifically, let $\ln m_t$ be the percentage deviation of the endowment from trend growth. Then, the home country's endowment, denoted by Y, in period t is $\ln Y_t = t \ln \overline{g} + \ln m_t$, where $\overline{g}$ is the average growth rate of the endowments of both countries. The output growth dynamic is therefore governed by the evolution of m, which follows the following AR(1) process:

$$\ln m_{t+1} = \rho \ln m_t + \varepsilon_{t+1},\ \ \varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2).$$

Similarly, output growth of the foreign country is subject to an aggregate shock denoted by ε_t^*. Define $z_t \equiv \{\varepsilon_t, \varepsilon_t^*\}$ as the aggregate shock in period t. Let z^t denote the history of aggregate shocks up to period t. In each country, a constant fraction λ of the endowment is the nontraded good and the rest is the export good: $Y_n(z^t) = \lambda Y(z^t)$ and $Y_x(z^t) = (1-\lambda)Y(z^t)$, where Y_n and Y_x denote endowments of home nontraded and home export goods, respectively.

As we show later, households are also subject to idiosyncratic income shocks. These shocks are i.i.d. across households and persistent over time in each country. We use η_t to denote the home idiosyncratic shock in period t and η^t to denote the history of idiosyncratic shocks to home households. We use $\pi(z^t, \eta^t)$ to denote the unconditional probability that state (z^t, η^t) will be realized. These shock processes are assumed to be independent among aggregate shocks and idiosyncratic shocks.

2.2. Preferences

The household derives utility from consuming composites of goods,

$$\sum_{t=1}^{\infty} \beta^t \sum_{(z^t,\eta^t)} \frac{c(z^t, \eta^t)^{1-\gamma}}{1-\gamma} \pi(z^t, \eta^t),$$

where $\gamma > 0$, $0 < \beta < 1$. The parameter γ denotes the coefficient of relative risk aversion, β denotes the time discount factor, and $c(z^t, \eta^t)$ denotes the consumption basket. The home consumption basket is a Cobb-Douglas composite of the nontraded good, c_n and the foreign export good, c_x^*:

$$c(z^t, \eta^t) = c_n(z^t, \eta^t)^\theta c_x^*(z^t, \eta^t)^{1-\theta}.$$

The parameter $\theta \in [0, 1]$ represents a home bias in consumption and governs the relative preferences over the nontraded and foreign export goods.

2.3. Aggregate Income

Aggregate income is the total value of exports and nontraded goods. Let the variables $q_n(z^t)$ and $q_x^*(z^t)$ denote the price of the home nontraded good in terms of the home consumption basket and the price of the home export good in terms of the foreign consumption basket, respectively. Then the total income at home, denoted $I(z^t)$, evaluated in terms of the home consumption basket is given by

$$I(z^t) = q_n(z^t) Y_n(z^t) + \frac{q_x^*(z^t)}{e_t(z^t)} Y_x(z^t),$$

where e_t denotes the RER, or the price of the home consumption basket relative to the foreign consumption basket.

2.4. Leverage and Asset Supply

The total income in each country is further divided into two parts: diversifiable income and nondiversifiable income. Claims to diversifiable income can be traded in financial markets, while claims to nondiversifiable income cannot.[4] In addition, the nondiversifiable component is subject to idiosyncratic stochastic shocks. We assume a constant share of nondiversifiable income, α, across countries and time. The share of diversifiable income is therefore $1 - \alpha$.

In each country, three types of assets are available: state-contingent claims on aggregate shocks, risky equities, and risk-free bonds. Note that we assume that the idiosyncratic risk is uninsurable and hence there are no state-contingent claims on idiosyncratic shocks. The aggregate state-contingent claims are in zero net supply. Both risky equities and risk-free bonds are claims to diversifiable income. Equities represent a leveraged claim to diversifiable income. The leverage ratio is constant over time and denoted by ϕ. Let $\overline{B}_t(z^t)$ denote the supply of a one-period risk-free bond in period t in the home country and $W_t(z^t)$ denote the price of a claim to the home country's total diversifiable income in period t. With a constant leverage ratio, the total supply of $\overline{B}_t(z^t)$ must be adjusted such that

$$\overline{B}_t(z^t) = \phi \left[W_t(z^t) - \overline{B}_t(z^t) \right].$$

By the previous equation, aggregate diversifiable income can be decomposed into interest payments to bondholders and payouts to shareholders. The total payouts, including cash dividends and net repurchases, denoted $\overline{D}_t(z^t)$, are

$$\overline{D}_t(z^t) = (1 - \alpha) I(z^t) - R^f_{t,t-1}(z^{t-1}) \overline{B}_{t-1}(z^{t-1}) + \overline{B}_t(z^t),$$

where $R^f_{t,t-1}(z^{t-1})$ denotes the home risk-free rate in period $t-1$. For simplicity, our model assumes that the supply of equity shares is constant. As a result, if a firm reissues or repurchases equity shares, it must be reflected by $\overline{D}_t(z^t)$ in our model.

Finally, we denote the value of total home equity or a claim to total payouts on $\overline{D}_t(z^t)$ as $V_t(z^t)$. The gross returns of home equities, $R^d_{t,t-1}(z^t)$, is therefore given by

$$R^d_{t,t-1}(z^t) = \frac{\overline{D}_t(z^t) + V_t(z^t)}{V_{t-1}(z^{t-1})}.$$

[4] The non-diversifiable and diversifiable incomes here work exactly like labor income and capital income in a production economy, respectively.

2.5. Correlation of Consumption Growth

Given the household preferences, the aggregate home (foreign) consumption basket becomes a composite of the home (foreign) nontraded good and the foreign (home) export good endowment:

$$C(z^t) = Y_n(z^t)^{\theta} Y_x^*(z^t)^{1-\theta}, \tag{1}$$

$$C^*(z^t) = Y_n^*(z^t)^{\theta} Y_x(z^t)^{1-\theta}. \tag{2}$$

As a result, the correlation of consumption growth is exogenously determined by the preferences parameter, θ, as well as the correlation of the endowment shock process. To see why, notice that the resource constraints in (1) and (2) together with the endowment imply that

$$\begin{aligned}\Delta \ln C(z^t) &= \theta \Delta \ln Y(z^t) + (1-\theta)\Delta \ln Y^*(z^t) \\ \Delta \ln C^*(z^t) &= \theta \Delta \ln Y^*(z^t) + (1-\theta)\Delta \ln Y(z^t),\end{aligned}$$

with an assumption of symmetric countries, $\sigma(\Delta \ln Y) = \sigma(\Delta \ln Y^*)$, where $\sigma(X)$ denotes the standard deviation of variable X. Let $\rho(X, X')$ denote the correlation between variables X and X'. Then, we can derive the correlation of consumption growth as

$$\rho(\Delta \ln C, \Delta \ln C^*) = \frac{2\theta(1-\theta) + (\theta^2 + (1-\theta)^2)\rho(\Delta \ln Y, \Delta \ln Y^*)}{(\theta^2 + (1-\theta)^2) + 2\theta(1-\theta)\rho(\Delta \ln Y, \Delta \ln Y^*)}. \tag{3}$$

The parameter θ governs the correlation of consumption growth in our model. If $\theta = 1$, then $\rho(\Delta \ln C, \Delta \ln C^*) = \rho(\Delta \ln Y, \Delta \ln Y^*)$. In this case, the preferences exhibit a complete home bias in consumption, and hence there is no goods trade between the two countries. We would like to think of this case as an approximation of having extremely high trade frictions, so that all goods trade is shut down and hence no international risk sharing. If $\theta = 0.5$, then $\rho(\Delta \ln C, \Delta \ln C^*) = 1$. The perfect correlation of consumption growth arises when there is no home bias in consumption. We interpret this case as an approximation of zero trade frictions and countries reaching full risk sharing.

2.6. Heterogeneity in Trading Technologies

There is significant portfolio heterogeneity not only across countries but also across investors within a country. To capture such heterogeneity, we implement the approach adopted by Chien, Cole, and Lustig (2011) and exogenously impose different restrictions on investor portfolio choices. These restrictions apply to the menu of assets that these investors can trade as well as the composition of household portfolios.

There are two classes of investors in terms of their asset-trading technologies. The first class of investors faces no restrictions on portfolio choices and the menu of tradable assets. Specifically, these investors trade a complete set of contingent claims on the domestic and foreign endowments. We call these investors "Mertonian traders". They optimally adjust their portfolio choices in response to changes in the investment opportunity set. Hence, they are marginal traders and price exchange rate risk in our model.

The second class of investors faces restrictions on their portfolios and are called "non-Mertonian traders". Specifically, their portfolio composition is restricted to be constant over time. We assume two types of non-Mertonian traders: non-Mertonian equity investors, who can trade domestic equities and domestic risk-free bonds, and non-participants, who invest in only domestic risk-free bonds. Even though the portfolio composition of non-Mertonian traders is exogenously given, they can still optimally choose how much to save and consume.

Non-Mertonian equity investors deviate from the optimal portfolio choices in two dimensions. First, they cannot change the share of equities in their portfolios in response to changes in the market price of risk, which indicates missed market timing. Second, their portfolio share in equities might deviate from the optimal share on average.

We denote the fraction of different types of investors in the home country and the foreign country by μ_j and μ_j^*, respectively, where $j \in \{me, et, np\}$ represents Mertonian traders, non-Mertonian equity traders, and non-participants, respectively.

2.6.1. Mertonian Traders

We start by considering a version of our economy in which all trade occurs sequentially. Securities markets are segmented. Only the Mertonian traders have access to all securities markets. A home Mertonian trader who enters the period with net financial wealth $a_t(z^t, \eta^{t-1})$ in node (z^t, η^t) has accumulated domestic claims worth $a_{ht}(z^t, \eta^{t-1})$ and claims on foreign investments worth $a_{ft}(z^t, \eta^{t-1})$:[5]

$$a_t(z^t, \eta^{t-1}) = a_{ht}(z^t, \eta^{t-1}) + \frac{a_{ft}(z^t, \eta^{t-1})}{e_t(z^t)},$$

where a_{ht} denotes the payoff of state-contingent claims in the home country expressed in terms of the home consumption basket and a_{ft} denotes the payoff on foreign state-contingent claims expressed in terms of the foreign consumption basket.

[5] The net financial wealth in node (z^t, η^t) does not depend on the realization of the idiosyncratic shock, η_t, because of uninsurable idiosyncratic risks.

$Q(z^{t+1}|z^t)$ and $Q^*(z^{t+1}|z^t)$ are the state-contingent prices in the home country and the foreign country expressed in units of the home and foreign consumption baskets, respectively. At the end of the period, home Mertonian traders go to securities markets to buy domestic and foreign state-contingent claims $a_{h,t+1}(z^{t+1},\eta^t)$ and $a_{f,t+1}(z^{t+1},\eta^t)$ and to the goods market to purchase $c(z^t,\eta^t)$ units of the home consumption basket, subject to the following one-period budget constraint:

$$\begin{aligned} & \sum_{z_{t+1}} Q(z^{t+1}|z^t)a_{h,t+1}(z^{t+1},\eta^t) + \sum_{z_{t+1}} Q^*(z^{t+1}|z^t)e_t(z^t)a_{f,t+1}(z^{t+1},\eta^t) + c(z^t,\eta^t) \\ \leq \;& a_t(z^t,\eta^{t-1}) + \alpha I(z^t)\eta_t, \;\; for \;\; all \;\; (z^t,\eta^t). \end{aligned}$$

Note that the numeraire is the home consumption basket. Investors can spend all of their nondiversifiable income and the accumulated wealth with which they entered the period.

Note that all investors are subject to non-negative net wealth constraints, given by $a_t(z^t,\eta^{t-1}) \geq 0$ and $a^*_t(z^t,\eta^{*,t-1}) \geq 0$, where a^*_t denotes the net asset holdings of foreign investors. The budget constraint of the foreign Mertonian households is similar.

FX Arbitrageurs. These Mertonian traders are FX arbitrageurs in our model. Only they enforce the no-arbitrage condition in the state-contingent claim market, which governs the evolution of exchange rates:

$$\ln \frac{e_{t+1}}{e_t} = \ln Q_{t+1} - \ln Q^*_{t+1}. \tag{4}$$

Hence, the percentage rate of depreciation of the RER is determined by the percentage difference between the home and foreign pricing kernels.

2.6.2. Non-Mertonian Traders

The non-Mertonian traders are restricted to fixed portfolio weights. Their total asset holdings at the beginning of period t are given by their asset position at the end of the previous period, denoted by $\widehat{a}_{t-1}(z^{t-1},\eta^{t-1})$, multiplied by the gross portfolio return, $R^p_{t,t-1}(z^t)$, which depends on their fixed portfolio. The non-Mertonian traders face the following budget constraint for all (z^t,η^t):

$$\widehat{a}_t(z^t,\eta^t) + c(z^t,\eta^t) \leq R^p_{t,t-1}(z^t)\widehat{a}_{t-1}(z^{t-1},\eta^{t-1}) + \alpha I(z^t)\eta_t,$$

where all variables are expressed in units of the home consumption basket. The gross return on the fixed portfolio is given by

$$R^p_{t,t-1}(z^t) = \omega R^d_{t,t-1}(z^t) + (1-\omega) R^f_{t,t-1}(z^{t-1}),$$

where ω denotes the fixed portfolio shares in domestic equities. In the case of non-participants, ω is zero. The budget constraint of foreign non-Mertonian households is similar.

In addition, all traders are subject to non-negative net wealth constraints, given by $\widehat{a}_t(z^t, \eta^t) \geq 0$ and $\widehat{a}^*_t(z^t, \eta^{*,t}) \geq 0$, where $\widehat{a}^*_t$ stands for the net asset holdings of foreign investors.

The details of the household problem and its associated Euler equations are described in the supplementary material.

2.7. Competitive Equilibrium

A competitive equilibrium for this economy is defined in the standard way. It consists of allocations of consumption; allocations of state-contingent claim, bond, and equity choices; and a list of prices such that (i) given these prices, the household's asset and consumption choices maximize the household's expected utility subject to the budget constraints, the non-negative net wealth constraints, and the constraints on portfolio choices and (ii) all asset markets clear.

2.7.1. Pricing Kernel

We use a recursive multiplier method to solve for equilibrium allocations and prices.[6] This approach has the advantage that we can express the household consumption share, which is consumption c relative to aggregate consumption C, in terms of a ratio of the household's recursive multiplier, ζ, to a single cross-sectional moment of the multiplier distribution, denoted h. To be precise,

$$\frac{c(z^t, \eta^t)}{C(z^t)} = \frac{\zeta(z^t, \eta^t)^{-\frac{1}{\gamma}}}{h_t(z^t)}, \tag{5}$$

where $\zeta(z^t, \eta^t)$ is the recursive Lagrangian multiplier of the domestic household and $h_t(z^t)$ is defined as a $-1/\gamma$ moment of $\zeta(z^t, \eta^t)$ across traders (see the supplementary material for details). Similarly, the same consumption-sharing rule is applied to foreign traders:

$$\frac{c^*(z^t, \eta^t)}{C^*(z^t)} = \frac{\zeta^*(z^t, \eta^{*,t})^{-\frac{1}{\gamma}}}{h^*_t(z^t)}.$$

[6] The use of cumulative multipliers in solving macro-economic equilibrium models was pioneered by Kehoe and Perri (2002), building on earlier work by Marcet and Marimon (1999). Chien et al. (2011) extend the methodology to an incomplete markets environment.

As a result, the home country's stochastic discount factor (SDF) is given by the standard Breeden-Lucas expression with a multiplicative adjustment:

$$Q_{t+1}(z^{t+1}|z^t) = \beta \left(\frac{C(z^{t+1})}{C(z^t)}\right)^{-\gamma} \left(\frac{h_{t+1}(z^{t+1})}{h_t(z^t)}\right)^{\gamma}, \tag{6}$$

which can be interpreted as the intertemporal marginal rate of substitution (IMRS) of an unconstrained domestic Mertonian trader. The foreign SDF of the foreign country is given by

$$Q^*_{t+1}(z^{t+1}|z^t) = \beta \left(\frac{C^*(z^{t+1})}{C^*(z^t)}\right)^{-\gamma} \left(\frac{h^*_{t+1}(z^{t+1})}{h^*_t(z^t)}\right)^{\gamma}, \tag{7}$$

which can be interpreted as the IMRS of an unconstrained foreign Mertonian trader. As a result, the percentage rate of depreciation of the RER is given by

$$\Delta \ln e_{t+1} = -\gamma(\Delta \ln C_{t+1} - \Delta \ln C^*_{t+1}) + \gamma(\Delta \ln h_{t+1} - \Delta \ln h^*_{t+1}).$$

Intuitively, the individual cumulative multiplier ζ can be thought of as a summary statistic for the individual past history measuring the effects on the individual consumption of the agent-specific trading technology as well as net wealth constraints. The aggregate cumulative multiplier h, a specific moment of the ζ distribution, then can be regarded as a summary statistic influencing the aggregate state-contingent prices. As a result, the growth rates of h and h^* become part of the SDFs as shown in equations (6) and (7). For a reasonable asset pricing result with a high and volatile market price of risk and with a low and stable risk-free rate, the growth rate of h has to be volatile and highly countercyclical and its conditional expectations need to be stable. This is achieved by the assumption of heterogeneous trading technologies, which effectively create an uneven loading of aggregate risk among different types of investors. The following section explains the key mechanism of our model and discusses its impact on pricing.

2.7.2. Segmentation Mechanism

The key mechanism of our model works through the concentration of aggregate risk among a small pool of sophisticated investors, the so-called — Mertonian traders —Ṫheir sophisticated trading technologies allow them to share country-specific risk with foreign traders. Thanks to better trading technologies, a small pool of Mertonian traders can accumulate a higher level of wealth and smooth consumption better than other traders. In contrast, a large pool of non-Mertonian traders hold home-biased portfolios and, consequently, their exposure to aggregate risk is small. Such heterogeneity of portfolio choices and the home bias in consumption affect asset pricing as follows.

First, the concentration of aggregate risk among a small group of Mertonian traders generates a high market price of risk. To be precise, high volatility of the SDF is achieved through high volatility of the h growth rate, thanks to the Mertonian traders' ability to respond to changes in aggregate risk. Second, the presence of non-participants, who do not bear aggregate risk, helps to stabilize the conditional expectation of the h growth rate. As a result, the risk-free rate becomes low and stable. Third, the equity home bias among non-Mertonian traders provides ample opportunities for risk sharing to Mertonian traders in both countries. Since they are marginal investors who price risk, sharing country-specific risk among themselves yields a high correlation of pricing kernels through the high correlation in the growth rates of h and h^*. Finally, international risk sharing at the aggregate level is restricted by a high degree of home bias in consumption.

3. Quantitative Results

We calibrate our model to evaluate the extent to which our model can account for the international correlation in pricing kernels, the volatility of pricing kernels and the volatility of RERs as observed in the data. Our benchmark model considers a symmetric two-country model in which both countries have identical preferences, portfolio restrictions, and shock processes. The benchmark model is calibrated to match several key features of data, including the data on trade in goods and assets. We then perform a number of counter-factual exercises to examine the effects of a home bias in consumption as well as heterogeneous portfolio choices on the dynamic behaviors of the RER and asset pricing.

In section 3.4, we demonstrate the effects of home bias in consumption by varying the parameter θ. In addition, in section 3.5, we consider changes in the trader pool to highlight the role of equity market participation. The last subsection documents the uncovered interest parity puzzle in our model.

3.1. Calibration

The home country in our model is parametrized to mimic the United States. The foreign country is an aggregation of four countries: France, Germany, Japan, and the United Kingdom. We collect annual data from International Financial Statistics from 1980 to 2012. The share of U.S. gross domestic product (GDP) in our hypothetical world economy is on average 52%, which is close to half. Thus, we assume equal sizes for the home and foreign economies. For simplicity and the demonstration of our mechanism, we set parameters such that the two economies are fully symmetric. Given that condition, all parameters are applied to both countries.

According to our data, the trade-to-GDP ratio in our hypothetical world is 0.32. Since there are only export and non-tradable goods in our model, we set the home bias parameter, θ, to 1 minus half of the trade-to-GDP ratio, which is 0.84. This calibration is based on the notion that θ is also the share of the home goods in the final consumption expenditure in our model. The innovation terms in the output shock process, or ε and ε^*, are calibrated into a Markov process to match the following statistics: (1) The consumption-growth correlation between the two countries is 0.17, (2) the average consumption growth of each country is 2.13% with a standard deviation of 2.36%, and (3) ρ is set to 0.95. Given θ and the calibrated aggregate consumption-growth correlation, the correlation between home and foreign endowment growth is pinned down to be -0.21, according to equation (3).

We also consider a two-state first-order Markov chain for idiosyncratic shocks. The first state is low, and the second state is high. Following Storesletten et al. (2004), we calibrate this shock process with two moments: the standard deviation of idiosyncratic shocks and the first-order autocorrelation of shocks, except that we eliminate the countercyclical variation in idiosyncratic risk. The Markov process for the log of nondiversifiable income, or $\log\eta$, has a standard deviation of 0.71 and an autocorrelation of 0.89. The transition probability is denoted by

$$\pi(\eta'|\,\eta) = \begin{bmatrix} 0.9450 & 0.0550 \\ 0.0550 & 0.9450 \end{bmatrix}.$$

The two states of the idiosyncratic shocks, whose means are each normalized to 1, are $\eta_L = 0.3894$ and $\eta_H = 1.6106$.

Following Mendoza et al. (2009), the fraction of nondiversifiable output is set to 88.75%. As shown in Section 2, equities in our model are simply leveraged claims to diversifiable income. Following Abel (1999) and Bansal and Yaron (2004), the leverage ratio parameter is set to 3. The model operates at an annual frequency. We set the time discount factor β to 0.95 to deliver the low risk-free rate. The risk-aversion rate γ is set to 5.5 to help to produce a high risk premium in our benchmark calibration.

As for the composition of trader types, we first set 50% of investors as non-participants (for both the home and foreign countries). This is based on the fact that one-half of U.S. households do not hold stocks, according to the 2010 Survey of Consumer Finance. The stock market participation rates are lower in general for other countries. Christelis et al. (2010) report stock market participation rates are 6.7%, 10.3% and 24.3% in the United Kingdom, France, and Germany, respectively, than in the United States. Iwaisako (2009) shows that the stock market participant rate of Japanese households is only 25% in 1999. Despite low participation rates in these countries, we maintain the symmetric country assumption for simplification. According to our model mechanism, a lower stock market participation rate for the foreign country could actually raise the volatility of foreign SDF. Therefore, setting non-participation to one-half is conservative in our opinion.

Second, to match a high market price of risk, a small fraction of Mertonian traders must absorb a large amount of aggregate risk. We therefore set the fraction of Mertonian traders to 5% for both countries. The small fraction of Mertonian traders is consistent with the empirical evidence. Previous studies have shown that most participants in the equity market trade infrequently and do not rebalance their portfolios often. Hence, only a small fraction of the stock market participants could be classified as Mertonian traders.

The remaining investors are non-Mertonian equity investors, and they represent 45%. Their portfolio is assumed to be a market portfolio of domestic assets. With a leverage ratio of 3, the home equities are 25% and home risk-free bonds are 75% of the market portfolio. To the best of our knowledge, we are not aware of empirical studies that provide a solid breakdown between domestic equities and bonds across households. Recall that the key mechanism that delivers highly correlated pricing kernels relies on a small pool of Mertonian traders who actively participate in international asset markets and adjust their portfolios frequently and efficiently. This mechanism operates in our calibrated economy as long as non-Mertonian equity traders hold a fixed and relatively small share of equities, including foreign equities. To simplify the computation, we shut down holdings of foreign equities among these traders.

3.2. Computation

The endowment processes of both countries share the same trend. Therefore, the ratio of aggregate consumption between the two countries is stationary. The RER is equal to the ratio of the marginal utility of consumption for unconstrained home and foreign Mertonian investors. Their intertemporal marginal utilities of consumption also determine the home and foreign pricing kernels. Therefore, the RER as well as pricing kernels are stationary if there is a non-zero measure of non-binding Mertonian traders for both countries in every possible state. We will assume that this is the case.

By equations (6) and (7), the stationarity of Q and Q^* imply that the growth rates of h and h^* are mean-stationary despite the processes of h and h^* themselves are not. Similarly, the log difference between h and h^* has to be stationary as a result of a stationary RER. In other words, h and h^* have to share the same stochastic trend in order to generate a stationary RER.

To solve our model, we use summary statistics for the aggregate history as state variables, denoted by $z_k \in \mathcal{A}$. More specifically, the percentage deviations of the endowments from the growth trend for both countries, $\ln m$ and $\ln m^*$, serve as a summary statistics for the aggregate history in our computation. Hence, for every possible state from z_k to z_k', the pricing kernels are

$$\begin{aligned} Q(z_k'; z_k) &= \beta \left(\frac{C(z_k')}{C(z_k)} \right)^{-\gamma} h_g(z_k'; z_k)^{\gamma} \\ Q^*(z_k'; z_k) &= \beta \left(\frac{C^*(z_k')}{C^*(z_k)} \right)^{-\gamma} h_g^*(z_k'; z_k)^{\gamma} \end{aligned}$$

where h_g and h_g^* are the growth rates of h and h^*, respectively.

The RER at a grid point $z_k \in \mathcal{A}$ can be derived as follows:

$$e_t(z_k) = \frac{C(z_k)^{-\gamma} h(z_k)^{\gamma}}{C^*(z_k)^{-\gamma} h^*(z_k)^{\gamma}} = \left(\frac{m_t^*(z_k)}{m_t(z_k)} \right)^{-\gamma(1-2\theta)} b(z_k)^{\gamma},$$

where $b(z_k)$ is the ratio of h and h^* conditional on the state z_k. Therefore, given the stationarity condition, the RER can be written only as a function of the summary statistics for the aggregate history.

In our computation, we record the growth rate of h and h^*. Given the stationarity of e_t, the growth rates of h at home and h^* abroad have to satisfy

$$\ln h_g(z_k'; z_k) - \ln h_g^*(z_k'; z_k) = b(z_k') - b(z_k),$$

which results from equation (4).

Given $\{b(z_k), h_g(z_k'; z_k), h_g(z_k'; z_k)\}$, we can completely characterize an equilibrium of this economy because we have the equilibrium prices, the RER, and the allocations. See the supplementary material for details.

3.3. Quantitative Results of the Benchmark Case

We compare the model statistics for the benchmark case with the data in Table 1. First, Panel (a) displays the key asset-pricing moments that are relevant to the puzzle examined in our study. Although the volatility of the SDF cannot be observed directly, its lower bounds can be inferred by using the Hansen-Jagannathan bounds (Hansen and Jagannathan (1991)). The post-war U.S. data on excess returns on equities, R^d, and risk-free rate, R^f, together with the Hansen-Jagannathan bounds imply[7]

$$\sigma(\ln Q_{t+1}) \geq \frac{E(R^d_{t+1} - R^f_t)}{\sigma(R^d_{t+1})E(R^f_t)} = \frac{0.08}{0.179(1.009)} = 0.443.$$

Similarly, the Hansen-Jagannathan bounds for foreign countries can also be obtained. The weighted average of equity returns and risk-free rates for our hypothetical foreign country (an aggregation of four countries) imply

$$\sigma(\ln Q^*_{t+1}) \geq \frac{E(R^{d*}_{t+1} - R^{f*}_t)}{\sigma(R^{d*}_{t+1})E(R^{f*}_t)} = \frac{0.10}{0.294(1.087)} = 0.313.$$

The 12.4% volatility of the RER data together with the Hansen-Jagannathan bounds above imply a high international risk-sharing index, which is defined as

$$1 - \frac{\sigma^2(\ln \frac{e_{t+1}}{e_t})}{\sigma^2(\ln Q_{t+1}) + \sigma^2(\ln Q^*_{t+1})} \geq 1 - \frac{0.124^2}{0.443^2 + 0.313^2} = 0.948.$$

The high risk-sharing index indicates that the correlation coefficient of the SDFs has to be highly correlated, as first pointed out by Brandt et al. (2006).

Our benchmark economy delivers high volatility and high correlation of pricing kernels, and these statistics are close to the observed statistics suggested by the pricing data. The standard deviation of the pricing kernels in the model is 0.42 and the correlation of the pricing kernels is 97.5%. In our model, these asset-pricing statistics are related to the volatility of the RER. Taking the standard deviation of equation (4) together with the assumption of symmetric countries, the standard deviation of RER appreciation is related to the moments of the SDFs as follows:

$$\sigma(\Delta \ln e) = \sigma(\ln Q)\sqrt{2(1 - \rho(\ln Q, \ln Q^*))} \tag{8}$$

As a result, the standard deviation of the RER is 9.4% in our model, whereas the observed volatility is 12.4%. Hence, our calibrated model is capable of producing reasonable volatility of the RER

[7]The data on asset pricing is obtained from Global Finance Data for the post-war period, 1950 to 2017.

and the pricing kernels, despite the low international correlation in aggregate consumption.

Our success in matching the asset-pricing moments relies on two mechanisms governing the two moments in (8). The first mechanism is the uneven distribution of aggregate risk across the population. Most of the global aggregate risk is borne by Mertonian investors. The concentration of risk among a small set of investors leads to high volatility of the pricing kernels. The second mechanism works through the ability of the Mertonian investors in the two countries to share country-specific components of aggregate risk among themselves. Therefore, their consumption tends to synchronize, and such synchronicity produces highly correlated pricing kernels.

As a group, the consumption and portfolio choices of Mertonian investors are less restricted by the presence of nontraded consumption than those of non-Mertonian investors because Mertonian investors represent only a small share of the population. However, if the international trade in goods is completely shut down, then there will be no reason for Mertonian investors to hold foreign assets. We demonstrate in the next subsection that international trade is essential for Mertonian investors to share risk across countries.

As a consistency check, the remaining panels in Table 1 report moments from the model and compare them with the data. In Panel (b), we show moments of the movements of the terms of trade and the international ratio of price of nontraded goods relative to that of traded goods. We are interested in these two variables because in theory we can decompose RER movements into movements of the terms of trade and those of the international ratio. (See the decomposition in the supplementary material. Evidently, their standard deviations in our model are very close to that in the data. Their correlation with the RER from our model is positive, as in the data. Furthermore, the correlation of RER appreciation with the international ratio is also close to that in the data. In addition, the moments related to the RER obtained from our benchmark model are in the 99% confidence interval constructed from the data for the United States and the four trading partners in our hypothetical world.

Next, we display business cycles properties of important variables in Panel (c) in Table 1. The international consumption-growth correlation in our model is calibrated to be 0.17, as in the data. However, the results related to cyclicality of some variables are mixed. Consumption is strongly procyclical, as in the data, but the consumption-output correlation from our model is much stronger than that in the data. Also, the trade-balance-to-GDP ratio is weakly procyclical, as in the data. On average, the correlation of this variable and output growth is weakly positive, but the correlation for the United States and the United Kingdom is actually negative. The terms of trade and the RER in our model are countercyclical, whereas in the data they are procyclical. In other words, the Backus-Smith puzzle, as first documented by Backus and Smith (1993), exists in our model.

Based on the first-order auto-correlation, consumption growth and RER appreciation in our model are much less persistent than in the data. The reason lies in the negative correlation between the home and foreign endowment shocks. The negative correlation ensures that the growth rate of world aggregate output converges to its long-run level. As a result, the growth rates of other macro variables also converge to their long-run level. As for volatility, output growth and the trade-balance-to-GDP ratio in our model are more volatile than those in the data.

In Table 2, we show the moments of consumption growth by investor group. As mentioned, our results are built on two key mechanisms: the concentration of aggregate risk within and the risk sharing among a small group of sophisticated investors. The first mechanism is reflected in Panel (a) of Table 2. The risk concentration leads to the high consumption growth volatility in the group of Mertonian traders, 7.8%. The corresponding numbers are only 3% and 1.1% for non-Mertonian equity traders and non-participants, respectively. The ranking of consumption volatility is consistent with the evidence in Parker and Vissing-Jorgensen (2009) that the consumption of rich households is more exposed to aggregate risk than that of poor households. Using the Consumption Expenditure Survey data from 1982 to 2004, they find that the top 5% of households (in terms of consumption) are estimated to be 4.5 times more exposed to aggregate consumption shocks than those in the bottom 80% (2.51 versus 0.56). In our benchmark model, the corresponding measure of risk exposure ratio between the top 5% and bottom 80% is lower at 2.13 (=1.98/0.93). This suggests that the concentration of aggregate risk in our model is mild compared to that in the data.

The second mechanism is confirmed in Panel (b). In particular, the international correlation of consumption growth between home and foreign Mertonian investors is almost perfect at 97.3%. This correlation is higher than the international correlation of home Mertonian traders with foreign non-Mertonian equity traders (52.5%), and foreign non-participants (40.8%). Moreover, the international correlations for non-Mertonian equity traders and non-participants are not only low but are also negative at -0.2 and -0.31, respectively. These statistics suggest that Mertonian traders load up aggregate risk and extensively share aggregate risk across borders among themselves.

In Panel (c), the cross-group correlation within the home country also reflect the extensive risk sharing across borders among the Mertonian traders. Their consumption-growth correlations with the other two groups are 68.8% and 57.1%, respectively. These correlations are lower than the international correlation among Mertonian traders in Panel (a) (97.3%). On the other hand, consumption growth of non-Mertonian equity traders is more strongly correlated with that of non-participants (95.3%) than that of Mertonian traders (68.8%).

Finally, Panel (d) displays the aggregate consumption-growth correlation with the consumption growth of each investor group. Thanks to the Mertonian traders' extensive risk sharing among themselves, they can smooth consumption better and the correlation for them (79.8%) is lower than those for the other two groups (98.1% and 92.5%, respectively). Moreover, the correlation with foreign consumption is lower than with domestic aggregate consumption for all groups, consistent with the consumption home bias. We further examine the impact of consumption home bias in the next subsection.

3.4. Impacts of Home Bias in Consumption

In this subsection, we investigate the role of a home bias in consumption. Intuitively, without a home bias in consumption and without the nontraded good, the law of one price holds and the RER is constant. In this case, investors achieve full risk sharing through international goods markets regardless of frictions in financial markets. To the contrary, if there is no trade due to either a complete home bias or prohibitively large trade frictions, then there will be no incentives for investors to hold external assets. Intuitively, we interpret a larger consumption home bias as increasing international trade frictions.

To explore the impacts of a home bias in consumption on RER volatility, we consider two exercises. First, we increase the share of home goods in the final consumption expenditure from 0.84 to 0.95, which significantly reduces the volume of trade in goods. The results are reported in Panel (a) of Table 3. There is virtually little change in the volatility of pricing kernels, while the correlation of pricing kernels drops from 0.975 to 0.892.

A higher degree of home bias in consumption implies that consumers are less inclined to consume the foreign good, and hence they reduce the willingness to share country-specific risk with foreign consumers. As a result, the international correlation of consumption growth falls from 16.9% to -10.9%. This fall can also be understood from equation (3): as θ approaches unity, the correlation of consumption growth approaches the correlation of endowment growth, which is

−21%.

Evidently, a high degree of home bias in consumption significantly reduces the correlation in the pricing kernels, even though only a small fraction of investors are sharing the country-specific risk. Given equation (8), conditioning on unchanged volatility of the pricing kernels, the sharp fall in the correlation in the pricing kernels produces a sharp increase in RER volatility, from 9.4% to 20.2%. The magnitude is more than twice of that in the benchmark case. This positive and large impact of a home bias in consumption on RER volatility is similar to that in the model by Warnock (2003), in which the RER is volatile as a result of nominal shocks.

The second exercise involves lowering the degree of home bias in consumption by setting the share of the nontraded goods in the consumption expenditure at 0.75. The last column in Panel (a) of Table 3 confirms the intuition that a reduction of the home bias makes the RER less volatile, although the pricing kernel is as volatile as in the benchmark case. In this case, the domestic pricing kernel is almost perfectly correlated with the foreign pricing kernel. A lower degree of home bias also improves the risk sharing at the aggregate level. The aggregate consumption-growth correlation increases to 44.5%. Therefore, according to (8), the almost perfect correlation in pricing kernels produces a lower standard deviation of the RER.

These two exercises demonstrate that the home bias in consumption is necessary for generating high RER volatility, although a too-high degree of home bias in consumption can generate higher RER volatility than in the data. The introduction of frictions in both international trade and finance in our model is novel compared to the existing models, which explain RER volatility as a result of international trade in assets either without international trade in goods, such as in Alvarez et al. (2002) and Colacito and Croce (2011), or with frictionless goods trade, such as in Colacito and Croce (2013), Colacito et al. (2018b), and Colacito et al. (2018a). Moreover, our approach is supported by the recent empirical evidence of Fitzgerald (2012). She finds that trade costs impede risk sharing among developed countries, but financial frictions do not impede risk sharing among them. Her finding suggests that international trade in goods is necessary for international risk sharing, as in our model.

3.5. Changes in the Composition of Traders

In this subsection, we vary the composition of traders to examine the impacts of the distribution of aggregate risk on the volatility of the RER, the volatility of the pricing kernels, the international correlation of the pricing kernels, and the international correlation of consumption growth.

First, we consider two special cases. The first one corresponds to the standard heterogeneous-agents economy, where all agents are marginal traders, such as in Krusell and Smith (1998). The second one corresponds to the standard segmented-market model, where all equity investors are marginal traders. These alternative models are characterized in our framework by different compositions of traders. In the first case, all the traders are set to Mertonian households and hence there is no heterogeneity in trading technologies (HTT). In the second case, given that a half of the population are non-participants according to the SCF data, we set the fraction of non-participation to 50%, as in our benchmark case, and the remaining 50% of traders are assumed to be Mertonian traders.

Panel (b) of Table 3 reports the result of both cases. In the case of no HTT, the RER volatility (13.1%) is close to the data, but the low volatility of the pricing kernels (11.9%) violates the Hansen-Jagannathan bound. In this case, all investors respond to changes in investment opportunities in every period by optimally adjusting their portfolios. Consequently, the gap of IMRS between home and foreign traders is low, and thus low volatility of the SDF discourages risk sharing across countries. Intuitively, since aggregate risk is evenly distributed over the population, the amount of country-specific risk borne by each trader becomes small. Hence, the incentive of international risk sharing at the individual level is substantially reduced and might not overcome goods market frictions. For this reason, the correlation of the pricing kernels significantly falls from 97.5% to only 38.7%.

In the last column of Panel (b), we depict the results from the standard segmented market model. In this case, one half of the population does not participate in the equity market and these non-participants create residual aggregate risk to be absorbed by the other half of the population. Due to the concentration of residual aggregate risk in one half of the population, volatility of the pricing kernels rises to 18.9%, from 11.9% in the previous case. Still, the volatility is far below the Hansen-Jagannathan bound. Compared with the previous case, higher volatility of the SDF caused by the smaller pool of Mertonian traders encourages a higher degree of risk sharing across borders. Thus, the correlation of pricing kernels increases to 66.8%, which is between the benchmark case and the no HTT case. This special case implies that it is quantitatively important to differentiate marginal traders from equity market participants, as in the empirical literature on household finance.

Having established the importance of marginal traders, next we vary the equity market participation rate by changing the pool of the non-participants and the non-Mertonian equity traders, holding the size of the Mertonian traders constant. Specifically, we decrease the size of the former to 30% and increase the size of the latter to 65%. The results are reported in the second column in Panel (c) of Table 3. Intuitively, as more traders participate in the equity market, country-specific risk becomes less concentrated among the Mertonian traders. Thus, the volatility of the pricing kernels falls. Quantitatively, a 20% increase in the equity market participation rate reduces the standard deviation of the pricing kernels by 6%.

As mentioned, lower SDF volatility reduces the international risk-sharing benefit for Mertonian traders and hence likely decreases the correlation of pricing kernels. However, the result shows that the SDF correlation actually increases slightly. This is because the equity portfolio held by the additional equity traders is biased toward the domestic equity. Their increasing participation enhances the demand for domestic assets and makes the foreign assets attractive to Mertonian traders. As a result, Mertonian traders become more willing to hold foreign assets and to share risk with foreign Mertonian traders. This offsets the effect caused by the reduced volatility of the SDF, and causes the correlation of the pricing kernels to increase slightly. However, the decrease in SDF volatility dominates the slight increase in the correlation of the pricing kernels. Overall, according to equation (8), RER volatility decreases. Quantitatively, a 20% increase in the equity market participation rate reduces the standard deviation of the RER by roughly 2%.

Finally, we vary the composition of equity market participants, holding the size of non-participants at the benchmark level. In the last column of Panel (c) in Table 3, we increase the size of the Mertonian traders to 20% of the population and decrease the size of the non-Mertonian equity traders to 30%. These changes reduce the concentration of aggregate risk among Mertonian traders, and hence the volatility of the pricing kernels falls from 42.3% to 26.2% and the correlation of the pricing kernels from 97.5% to 90.5%. According to equation (8), the fall in the correlation of the pricing kernels and the fall in their volatility have competing effects on RER volatility. Overall, the fall in the correlation of the pricing kernels dominates and the RER volatility increases by roughly 2%.

3.6. Uncovered Interest Parity Puzzle

In this subsection, we examine our model's prediction about the relationship between the exchange rate and interest rate differentials, as this relationship has been extensively studied in the empirical literature. Specifically, the empirical literature has found that a currency paying a high interest rate tends to appreciate relative to a currency paying a low interest rate. This finding is the opposite of the prediction of the UIP obtained in general equilibrium models. We refer to this failure of existing models as the UIP puzzle and examine whether our model solves the puzzle.

To put it differently, there will be no puzzle if the currency paying a high interest rate carries a sufficiently high risk premium and appreciates to reflect the high risk premium. Unfortunately, our benchmark economy does not produce a positive currency risk premium, and thus it exhibits the UIP puzzle. Despite our attempts to modify the model in several ways, the puzzle remains. To understand why, let us express the conditional expectation of RER appreciation implied by equation (4) as follows:

$$E_t \ln(e_{t+1}/e_t) = E_t \ln Q_{t+1} - E_t \ln Q^*_{t+1} = R^{f*}_t - R^f_t + \Omega^*_t - \Omega_t, \tag{9}$$

where Ω^*_t and Ω^*_t are defined as[8]

$$\begin{aligned} \Omega^*_t &= \ln E_t Q^*_{t+1} - E_t \ln Q^*_{t+1} \\ \Omega_t &= \ln E_t Q_{t+1} - E_t \ln Q_{t+1}. \end{aligned}$$

As mentioned, in order to solve the UIP puzzle, the high interest rate currency must appreciate on average. For example, if $R^{f*}_t < R^f_t$, then $E_t \ln(e_{t+1}/e_t)$ will have to be positive. According to equation (9), two conditions are necessary for generating a currency risk premium. First, the differential of the expected variation of the pricing kernels $(\Omega^*_t - \Omega_t)$ must move in the opposite direction from the risk-free rate differential $(R^{f*}_t - R^f_t)$. Second, the former must be more volatile than the latter.

However, both conditions are not satisfied in our benchmark model, as depicted in Table 4. In our benchmark model, the risk-free rate differential moves in the same direction as the differential of the expected variation of the pricing kernels with a 31% correlation. The volatility of the former and that of the latter are 1.56% and 0.66%, respectively. As a result, the currency risk premium in our benchmark model is negative at -0.259%.

Next, we demonstrate that the UIP puzzle remains even after we explore three modifications of our model. Specifically, we change the assumption related to the following aspects of the model, all

[8]Note that Ω^*_t and Ω^*_t are $Var_t(\ln Q^*_{t+1})/2$ and $Var_t(\ln Q_{t+1})/2$ if the pricing kernels, Q and Q^*, follow a log-normal distribution.

else equal: the utility function, agents' beliefs, and trading behavior. All three modifications have failed to satisfy the first condition, although they could significantly increase the time variation of the risk premium.

3.6.1. Modification I: Recursive Preferences

The first model modification is to change the utility into recursive preferences. The new form of preferences is given by

$$V_t = \left[(1-\beta)c_t(z^t, \eta^t)^{1-\rho} + \beta(\mathcal{R}_t V_{t+1})^{1-\rho}\right]^{1/(1-\rho)},$$

where

$$\mathcal{R}_t V_{t+1} = \left(E_t\left[V_{t+1}^{1-\alpha}\right]\right)^{1/(1-\alpha)}.$$

The time discount factor and risk-aversion rate continue to be denoted by β and α, respectively. The elasticity of intertemporal substitution (EIS) is denoted by $1/\rho$. The results are reported in Panel (b) of Table 4. Clearly, the introduction of recursive preferences alone does not resolve the UIP puzzle. In this case, the risk premium drops as the EIS increases, thanks to agents' willingness to substitute consumption over time. Hence, the same amount of aggregate risk concentrated in a small group of Mertonian traders results in a lower risk premium. For this reason, the differential of the expected variation of the pricing kernels is still positively correlated with the differential of the risk-free rate. Moreover, the volatility of the former is still dominated by that of the latter.

3.6.2. Modification II: Alternative Beliefs

The second modification of the model is to change agents' beliefs about the probability of aggregate shocks. This exercise is motivated by the following: In order to obtain stationarity in our two-country model, the average growth rate of each country must be identical. This assumption implies that a country experiencing a high-growth shock today will have a low expected growth rate of consumption, and hence this country will tend to have a high risk-free rate. In addition, a country that is more prone to a future low-growth shock is also more likely to have exchange rate appreciation in the future. For this reason, this mechanism may have potential to help us resolve the UIP puzzle.

We therefore modify the agents' beliefs such that all agents' expectation of the growth rate is identical to the average growth rate, regardless of the history of aggregate shocks. The results are reported in the last column of Table 4. It shows that this alternative belief helps to reduce the correlation between the differential of the expected variation of the pricing kernels and the differential of the risk-free rate. However, the reduction is not large enough to turn the correlation from a positive value to a negative one. Furthermore, the problem with low volatility of the differential of the expected variation of the pricing kernels persists.

3.6.3. Modification III: Portfolio Inertia

In a model similar to ours, Chien et al. (2012) show that infrequent portfolio adjustments could result in a significant increase of the variation of the market price of risk. The argument for infrequent adjustments is motivated by the inertia portfolio behaviors observed among most participants in the equity market. For this reason, we attempt to investigate whether portfolio inertia can help to resolve the UIP puzzle by producing large time varying risk premium. To alleviate the computational burden, we report the asset-pricing statistics for a one-country model in Panel (d) of Table 4. The results indicate that the large market price of risk, 25.22%, makes the volatility of the expected variation of the pricing kernels larger than that of the risk-free rate. Still, the correlation between the expected variation of the pricing kernels and the risk-free rate remains positive.

4. Conclusion

We use a general equilibrium model with asset-trading restrictions and home bias in consumption to demonstrate that RER volatility is related to frictions in both goods and financial markets. The asset-trading restrictions imposed in our model are in line with the empirical evidence in the household finance literature. With a realistic assumption that most investors do not actively participate in the domestic and foreign equity markets, we reconcile highly correlated and volatile pricing kernels with low cross-country correlation in consumption growth.

The insight from our model is that the high cross-country correlation in the pricing kernels is not necessarily evidence of a high degree of international risk sharing. In particular, international risk sharing is aggressively undertaken by a small fraction of sophisticated investors who face no restrictions on asset trading. These marginal investors are the arbitrageurs, and their portfolio adjustment determines RER volatility. In fact, their portfolio adjustment still generates a positive but far from perfect correlation between the RER and relative consumption growth, as in equation (4). Hence, our model has not solved the Backus-Smith puzzle. The UIP puzzle, which unrealistically predicts that a currency paying a high interest rate is expected to depreciate, also exists even after we modify the model to incorporate recursive preferences, an alternative belief about shocks, and portfolio inertia. Solving these puzzles requires a more complex combination of frictions and shocks than that in our model.

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List of Tables

Table 1: Benchmark Results

	Data	**Benchmark**
Panel (a): Asset Pricing Moments		
$\sigma(\ln Q)$	> 0.313	0.423
$\rho(\ln Q, \ln Q^*)$	> 0.948	0.975
$\sigma(\Delta \ln e)$	0.124 (0.002)	**0.094**
$E(R^d - R^f)$ (%)	9.015 (0.710)	7.824
$\sigma(R^d - R^f)$ (%)	23.520 (2.493)	26.752
$E(R^f)$ (%)	1.001 (0.101)	**2.340**
$\sigma(R^f)$ (%)	2.997 (0.423)	**0.996**
Panel (b): Moments of Components of RER appreciation		
$\sigma(\Delta \ln TOT)$	0.039 (0.012)	0.028
$\sigma(\Delta \ln(\frac{P_n P_x^*}{P_x P_n^*}))$	0.134 (0.011)	0.097
$\rho(\Delta \ln e, \Delta \ln TOT)$	0.361 (0.089)	0.679
$\rho(\Delta \ln e, \Delta \ln(\frac{P_n P_x^*}{P_x P_n^*}))$	0.920 (0.052)	0.870
Panel (c): Moments of Aggregate Variables		
$\rho(\Delta \ln Y, \Delta \ln C)$	0.809 (0.027)	**0.949**
$\rho(\Delta \ln Y, \frac{TB}{Y})$	0.055 (0.073)	0.148
$\rho(\Delta \ln Y, \Delta \ln TOT)$	0.241 (0.087)	$\mathbf{-0.588}$
$\rho(\Delta \ln Y, \Delta \ln e)$	0.046 (0.011)	$\mathbf{-0.774}$
$\rho(\Delta \ln e, \Delta \ln e')$	0.149 (0.087)	-0.036
$\rho(\Delta \ln C, \Delta \ln C')$	0.374 (0.084)	$\mathbf{-0.030}$
$\rho(\Delta \ln Y, \Delta \ln Y')$	0.352 (0.061)	$\mathbf{-0.050}$
$\sigma(\Delta \ln Y)$	0.022 (0.002)	**0.031**
$\sigma(TB/Y)$	0.017 (0.002)	**0.031**

Notes: The notations σ and ρ denote standard deviation and the correlation coefficient, respectively. The empirical moments are constructed from the average and the standard deviation of the five countries in our hypothetical world, namely, France, Germany, Japan, the United Kingdom and the United States. The equity return and risk-free interest rate statistics of panel (a) are from Global Finance Data for the post-war period, 1950 to 2017. As discussed in section 3.3, the lower bound of SDF volatility is inferred by the Hansen-Jagannathan bounds. Following Brandt et al. (2006), the cross-country SDF correlation is approximated by the international risk-sharing index, which is defined as $1-\sigma^2(\ln \frac{e_{t+1}}{e_t})/(\sigma^2(\ln Q_{t+1})+\sigma^2(\ln Q^*_{t+1}))$. For Panels (b) and (c), the statistics in boldface are outside the 99% confidence interval, and the numbers in the bracket are standard errors. They are also constructed from the average and the standard deviation of the five countries in our hypothetical world with the following exception. First, the standard error for $\rho(\Delta \ln C, \Delta \ln C^*)$ is based on the average and the standard deviation of the bilateral correlation between the U.S. vis-a-vis the other four countries. Next, the standard errors for the statistics of $\Delta \ln e$ and $\Delta \ln(\frac{P_n P_x^*}{P_x P_n^*})$ are based on the average and the standard deviation of U.S. series vis-a-vis the other four countries. The data are annual series from 1980 to 2012, and the aggregate for the rest of the world is the GDP weighted average of series for France, Germany, Japan and the United Kingdom. The simulated statistics are based on 18,000 agents for each type and 10,000 periods.

Table 2: Standard Deviation and Correlation of Group Consumption Growth in the Benchmark Economy

Panel (a): Standard Deviation of Consumption Growth	
$\sigma[\Delta \log(C_{me})]$	0.078
$\sigma[\Delta \log(C_{et})]$	0.030
$\sigma[\Delta \log(C_{np})]$	0.011
Panel (b): Cross-Country Correlation	
$\rho[\Delta \log(C_{me}), \Delta \log(C^*_{me})]$	0.973
$\rho[\Delta \log(C_{me}), \Delta \log(C^*_{et})]$	0.525
$\rho[\Delta \log(C_{me}), \Delta \log(C^*_{np})]$	0.408
$\rho[\Delta \log(C_{et}), \Delta \log(C^*_{et})]$	-0.200
$\rho[\Delta \log(C_{et}), \Delta \log(C^*_{np})]$	-0.263
$\rho[\Delta \log(C_{np}), \Delta \log(C^*_{np})]$	-0.308
Panel (c): Within-Country Correlation	
$\rho[\Delta \log(C_{me}), \Delta \log(C_{et})]$	0.688
$\rho[\Delta \log(C_{me}), \Delta \log(C_{np})]$	0.571
$\rho[\Delta \log(C_{et}), \Delta \log(C_{np})]$	0.953
Panel (d): Correlation with Aggregate Consumption	
$\rho[\Delta \log(C_{me}), \Delta \log(C)]$	0.798
$\rho[\Delta \log(C_{et}), \Delta \log(C)]$	0.981
$\rho[\Delta \log(C_{np}), \Delta \log(C)]$	0.925
$\rho[\Delta \log(C_{me}), \Delta \log(C^*)]$	0.670
$\rho[\Delta \log(C_{et}), \Delta \log(C^*)]$	-0.013
$\rho[\Delta \log(C_{np}), \Delta \log(C^*)]$	-0.110

Note: The simulation results are based on $18,000$ agents for each type and $10,000$ periods.

Table 3: Inspection of the Model Mechanism

Panel (a): Variation in Home-Bias in Consumption			
	Benchmark	Higher θ	Lower θ
Home-Bias Parameter θ	0.84	0.95	0.75
$\sigma(\ln Q) = \sigma(\ln Q^*)$	0.423	0.435	0.421
$\sigma(\Delta \ln e)$	0.094	0.202	0.053
$\rho(\ln Q, \ln Q^*)$	0.975	0.892	0.992
$\rho(\Delta \ln C, \Delta \ln C^*)$	0.169	−0.109	0.445

Panel (b): Two Special Cases			
	Benchmark	No HTT	Segmented Market
Population size of investors			
Mertonian	0.05	1	0.5
Non-Mertonian equity	0.45	0	0
Non-participants	0.50	0	0.5
$\sigma(\ln Q) = \sigma(\ln Q^*)$	0.423	0.119	0.189
$\sigma(\Delta \ln e)$	0.094	0.131	0.154
$\rho(\ln Q, \ln Q^*)$	0.975	0.387	0.668

Panel (c): Variation in the Trader Pool			
	Benchmark	More Non-Mertonians	More Mertonians
Population size of investors			
Mertonian	0.05	0.05	0.20
Non-Mertonian equity	0.45	0.65	0.30
Non-participants	0.50	0.30	0.50
$\sigma(\ln Q) = \sigma(\ln Q^*)$	0.423	0.360	0.262
$\sigma(\Delta \ln e)$	0.094	0.076	0.115
$\rho(\ln Q, \ln Q^*)$	0.975	0.978	0.905

Table 4: The Currency Risk Premium and UIP Puzzle

Panel (a): Benchmark	
$\frac{\sigma(Q)}{E(Q)}$	0.423
$Std\left[\frac{\sigma_t(Q)}{E_t(Q)}\right]$ (%)	9.435
$\rho(R_f^* - R_f, \Omega_t^* - \Omega_t)$	0.313
$Std(R_f^* - R_f)(\%)$	1.556
$Std(\Omega_t^* - \Omega_t)(\%)$	0.657
Panel (b): Recursive Preferences ($\sigma = 5.5, \rho = 1$)	
$\frac{\sigma(Q)}{E(Q)}$	0.139
$Std\left[\frac{\sigma_t(Q)}{E_t(Q)}\right]$ (%)	1.132
$\rho(R_f^* - R_f, \Omega_t^* - \Omega_t)$	0.593
$Std(R_f^* - R_f)(\%)$	0.647
$Std(\Omega_t^* - \Omega_t)(\%)$	0.123
Panel (c): Alternative Beliefs	
$\frac{\sigma(Q)}{E(Q)}$	0.322
$Std\left[\frac{\sigma_t(Q)}{E_t(Q)}\right]$ (%)	5.262
$\rho(R_f^* - R_f, \Omega_t^* - \Omega_t)$	0.087
$Std(R_f^* - R_f)(\%)$	1.226
$Std(\Omega_t^* - \Omega_t)(\%)$	0.766
Panel (d): Portfolio Inertia	
$\frac{\sigma(Q)}{E(Q)}$	0.411
$Std\left[\frac{\sigma_t(Q)}{E_t(Q)}\right]$ (%)	25.224
$\rho(R_f, \Omega_t)$	0.614
$Std(R_f)(\%)$	2.246
$Std(\Omega_t)(\%)$	6.674