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Explaining Cross-Cohort Differences in Life-Cycle Earnings*

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Abstract

College-educated workers entering the labor market in 1940 experienced a 4-fold increase in their labor earnings between the ages of 25 and 55; in contrast, the increase was 2.6-fold for those entering the market in 1980. For workers without a college education these figures are 3.6-fold and 1.5-fold, respectively. Why are earnings profiles flatter for recent cohorts? We build a parsimonious model of schooling and human capital accumulation on the job, and calibrate it to earnings statistics of workers from the 1940 cohort. The model accounts for 99 percent of the flattening of earnings profiles for workers with a college education between the 1940 and the 1980 cohorts (52 percent for workers without a college education). The flattening in our model results from a single exogenous factor: the increasing price of skills. The higher skill price induces (i) higher college enrollment for recent cohorts and thus a change in the educational composition of workers and (ii) higher human capital at the start of work life for college-educated workers in the recent cohorts, which implies lower earnings growth over the life cycle.

JEL codes: E20, I26, J24, J31.
Keywords: Life-cycle earnings, flattening, skill price, education composition.

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1 Introduction

The labor earnings of college-educated workers reaching their 25th birthday in 1940 grew by a factor of 4 by the time they reached age 55. In contrast, the earnings of college-educated workers reaching their 25th birthday in 1980 grew by a factor of only 2.6. Figure 1 illustrates that the decline in life-cycle earnings growth was systematic across cohorts and was also experienced by high-school-educated workers. We use the term "flattening" to refer to this phenomenon. We measure flattening by the reduction in the 55-25 earnings ratio between two cohorts. In the case of college-educated workers, for instance, the ratio declined from 4 to 2.6, or the flattening was 34 percent between the 1940 and 1980 cohorts.

The data we use in Figure 1 are described in Appendix A. We illustrate a few additional points about the data in several figures in Appendix A. First, even though Figure 1 is about white men, we show that similar patterns emerge from the data for black men and for white and black women. Second, *earnings per hour* display similar flattening as earnings and this is true across race and gender cells. Third, distinguishing workers with 1-4 years of college from those with 5+ years of college does not alter the message that the life-cycle profiles of earnings and earnings per hour have flattened across cohorts. Given these observations, we focus the remainder of this paper on the flattening of the earnings profile of white men.

The flattening of earnings profiles has important implications for the evolution of cross-sectional inequality over time. In 1970, the ratio of the average 55-year-old worker's earnings to the average 25-year-old worker's earnings is slightly less than 2. This inequality ratio increases to about 2.5 in 2010. However, had there been no flattening in the earnings profiles, the inequality would have more than doubled: from 1970 to 2010, the inequality would have increased to 4.5.

We develop a parsimonious model based on Ben-Porath (1967), which is the workhorse framework in the life-cycle earnings literature (see, for example, Heckman et al., 1998; Huggett et al., 2011). The main addition in our model is that we have endogenous college enrollment. Each period a worker can allocate two inputs—his time and his stock of human capital—between work and accumulation of human capital on the job. The latter activity is subject to diminishing returns. We assume that workers differ in their ability to accumulate human capital, both in college and on the job, and that the distribution of ability is identical across cohorts. All workers are endowed with a high school education at the start of their lives; they have an initial stock of human capital that is increasing in ability. To model college enrollment we assume that a worker's human capital after college depends on ability, time spent in college, and goods spending. The goods spending represents a "quality" component of college that can be chosen. We show that, in each cohort, there is a threshold level of ability such that workers with higher ability choose a college education, while the others do not.

In our model, there is *only one* exogenous variable responsible for both the flattening of earnings profiles and the increase in college enrollment across cohorts: the skill price level, which we assume to be a deterministic and increasing function of time. A key aspect of our analysis, therefore, is the optimal response of college enrollment and human capital accumulation in each cohort to increases in the skill price. We calibrate the model to match some key statistics on the life-cycle earnings of the 1940 cohort and the time series of college enrollment in the United States. We then compare the profiles of life-cycle earnings of the post-1940 cohorts with the data. The calibrated model accounts for 52 percent of the flattening for high-school-educated workers between the 1940 and 1980 cohorts and for 99 percent of the flattening for college-educated workers. Between the 1940 and 1970 cohorts, the corresponding numbers are 41 percent and 73 percent.

To understand how the growth of the skill price flattens the earnings profiles across cohorts, suppose that the growth rate of the skill price is constant over time. The recent cohorts then start their lives facing a higher level of the skill price than older cohorts, but the same growth rate. This generates two key *endogenous differences* between the recent and the older cohorts: an intensive margin effect and a composition effect.[1]

[1] Since neither human capital nor skill price is observable, one can imagine constructing a skill price time series that accounts for all of the flattening under the assumption that all cohorts are identical and that human capital accumulation does not respond to skill price changes. Such an approach, however, contradicts a large literature that uses Ben-Porath (1967) as a model of human capital accumulation and life-cycle earnings (e.g., Heckman et al., 1998), where changes in skill price over the life cycle have first-order effects on human capital accumulation.

College Intensive Margin Effect A higher skill price implies that the marginal return to human capital is higher. Consider a worker with a level of ability such that it is optimal to attend college at both low skill price (old cohort) and high skill price (recent cohort). Such a worker in the recent cohort acquires more college human capital relative to the worker in the old cohort. Higher college human capital implies lower subsequent human capital accumulation on the job and lower earnings growth over the life cycle. This implication is due to: (i) human capital accumulation on the job is a function of only time and the stock of human capital and (ii) human capital accumulation is subject to diminishing returns.

College Composition Effect For the recent cohort, higher marginal return to human capital also implies that the ability threshold is lower (i.e., college enrollment is higher). Hence, the average ability among college-educated workers in the recent cohort is less than that in the old cohort. The lower average ability has two opposite consequences for the slope of earnings profiles. On the one hand, lower ability implies slower human capital accumulation on the job; hence, the earnings profile of college-educated workers in the recent cohort is flatter. On the other hand, lower ability also implies less college human capital, which induces faster accumulation and higher earnings growth for the recent cohort. In our calibrated model, the first effect dominates the second.

High School Intensive Margin Effect Consider now a worker with a level of ability such that college is not optimal in either the old or the recent cohorts. By assumption, such a worker starts working with exactly the same human capital in each cohort and, hence, experiences the same earnings growth, Again, this is because our human capital accumulation function on the job involves only time and the existing stock of human capital. Thus, the skill price increase has no effect on such workers.

High School Composition Effect Finally, the composition of high-school-educated workers changes in the recent cohort because of the lower ability threshold mentioned in the college composition effect. The average ability of the high-school-educated worker in the recent cohort is lower.

This, again, has two opposing effects on the slope of the earnings profile: lower ability implies slower human capital accumulation on the job and, hence, a flatter earnings profile; but lower ability also implies lower initial human capital and, hence, a steeper earnings profile. As in the case of the college composition effect, the ability effect dominates the human capital effect.

In our quantitative exercise, we consider a skill price process that exhibits a slowdown. In this case, the recent cohorts start with not only a higher level of skill price relative to the older cohorts but also face a lower growth rate of skill price over the life cycle. This generates some additional effects conducive to the flattening of earnings profiles. For instance, there will be a high school intensive margin effect in this case. A worker for whom college is not optimal in either cohort will accumulate human capital at a slower rate in the recent cohort.

In our model, individuals with sufficiently high ability enroll in college in all cohorts. The flattening of earnings profiles for such individuals is due to the slowdown of the skill price and the college intensive margin effect. While we cannot directly identify such individuals in the data, we suppose that such individuals in every cohort are workers in high-skill occupations with at least 5 years of college education (e.g., physicians and surgeons). Observed life-cycle earnings profiles for these workers follow the flattening pattern as in our model. We interpret this as indirect evidence of the intensive margin effect.

Our paper contributes to the literature on wage inequality in the United States. Closely related papers are Kambourov and Manovskii (2009), Hendricks (2015) and Jeong et al. (2015), who point out the flattening of earnings profiles of successive cohorts of workers. Their explanations involve demographic changes or changes in occupational mobility. Our analysis complements theirs since (i) our evidence includes a longer time horizon starting with the 1915 birth cohort and (ii) we propose a different, simple explanation that is not only consistent with the same set of facts but also consistent with the rising educational enrollment of successive cohorts of workers. Finally, our analysis is also consistent with the evolution of the cross-sectional inequality statistics pointed out by Katz and Murphy (1992) and Card and Lemieux (2001) for the period of time for which our and their analyses overlap. We provide the details in Section 5.

2 The Model

2.1 The Environment

Time is discrete. The economy is populated by overlapping cohorts of individuals. A unit mass of individuals is born each period and live for J periods. They are differentiated by their ability, a, to accumulate human capital. Their ability is exogenous and remains constant throughout their lives. We assume that $a \geq 0$ and that its cumulative distribution function (cdf), A, is the same across cohorts. An individual's initial human capital (at age 1) depends on his ability; we denote initial human capital by $h_1(a)$ for an age-1 individual with ability a.

Individuals can accumulate human capital through education and on the job. We consider two levels of education: high school and college. All age-1 individuals are endowed with a high school education, but they can choose whether or not to attend college. The cost of attending college is twofold: a time cost—individuals in college have no earnings for s periods—and a goods cost.

Individuals who do not attend college start working at age 1. Those who attend college start working at age $s+1$. Each period, workers can choose to allocate their time between renting their human capital at that period's price w and accumulating human capital.

We interpret an individual's age-1 human capital, $h_1(a)$, as human capital obtained from high school. The technology for accumulating human capital in college is described by the function $G(k, h_1(a), a)$, where k represents goods spending in college. Thus, $G(k, h_1(a), a)$ is the human capital at age $s+1$ (i.e., after s periods of college) for a worker of ability a with initial human capital $h_1(a)$ who invested k units of goods, in present value, in college education. Higher spending implies a higher quality of college education i.e., more human capital acquired in college. We assume that time spent in college is exogenous, while goods spending in college is a choice.

The technology for accumulating human capital on the job is described by the function $F(nh, a)$, where $n \in (0, 1]$ is time spent in human capital accumulation and h is human capital at the beginning of the period. Thus, $F(nh, a)$ is the additional human capital for a worker of ability a.[2]

We refer to w as the skill price and emphasize that it is the sole exogenous variable in the model. We assume that w is a deterministic function of time and that individuals perfectly forecast its future values. Finally, we assume that human capital depreciates at rate $\delta \in (0, 1)$ on the job and that workers can freely borrow and lend at the gross interest rate r.

2.2 Individual Choices

Let $W_{j,t}(h, a)$ denote the present value of earnings for a worker of age j and ability a, who starts period t with human capital h:

$$\begin{aligned} W_{j,t}(h,a) &= \max_n wh(1-n) + \frac{1}{r} W_{j+1,t+1}(h',a) && (1) \\ \text{s.t.} \quad & h' = (1-\delta)h + F(nh,a), && (2) \\ & W_{J+1,t+1} = 0. && (3) \end{aligned}$$

Equation (2) describes the law of motion of human capital and Equation (3) is a boundary condition. Earnings at date t are given by $wh(1-n)$.

For an individual born in period t with ability a, the value of being a worker with only a high school education is the value of starting his work life at age 1 with human capital $h_1(a)$. That is,

$$V^{\text{hs}}_{1,t}(a) = W_{1,t}(h_1(a), a). \tag{4}$$

Similarly, the value of becoming a college-educated worker for an individual born in period t is

$$V^{\text{col}}_{1,t}(a) = \max_k \frac{1}{r^s} W_{s+1,t+s}\left(G(k, h_1(a), a), a\right) - k. \tag{5}$$

[2] Note that n and h enter multiplicatively in F. Heckman et al. (1998) estimate production functions for human capital where they allow the elasticities with respect to time and human capital to differ. However, they cannot reject the hypothesis that these elasticities are the same.

Here the earnings accrue from age $s+1$ onward—that is, starting with calendar date $t+s$. Hence, the present value of earnings is measured by $W_{s+1,t+s}$ and discounted by r^s. College spending is measured in present value by k. To sum up, the value of attending college is the value of starting to work at age $s+1$ and date $t+s$ with human capital $G(k, h_1(a), a)$ net of the spending k.

The decision of whether to attend college or start working at age 1 is determined by

$$\max_{\text{hs,col}} \left\{ V_{1,t}^{\text{hs}}(a), V_{1,t}^{\text{col}}(a) \right\}. \tag{6}$$

2.3 Functional Forms

We assume that ability follows a Beta distribution in each cohort,

$$\frac{a}{\psi_0} \sim B(\psi_1, \psi_2),$$

where $\psi_0 > 0$ is a scale parameter, and ψ_1 and ψ_2 are the parameters of the Beta cdf.[3]

An individual's high school human capital, $h_1(a)$, depends on his ability according to

$$h_1(a) = z_H a, \tag{7}$$

where $z_H > 0$. We model the human capital technology in college, G, as

$$G(k, h_1(a), a) = (z_G k)^{\eta} \left(a h_1(a)\right)^{1-\eta}, \tag{8}$$

where $\eta \in (0,1)$ and $z_G > 0$. The presence of goods in Equation (8) (i.e., $\eta > 0$) helps us deliver the college intensive margin effect: Recent cohorts facing higher skill price choose to acquire more college human capital relative to older cohorts facing lower skill price. This, in turn, implies that more individuals in the recent cohort will enroll in college.

[3] The Beta distribution is defined over the unit interval. The parameter ψ_0 scales the domain of the distribution from the unit interval to $[0, \psi_0]$.

Human capital investment on the job, F, is

$$F(nh, a) = z_F a\,(nh)^{\phi}\,, \tag{9}$$

where $\phi \in (0,1)$ and $z_F > 0$.

3 Qualitative Analysis

In this section, we illustrate the qualitative implications of changes in skill price. In Section 3.1, we study a constant growth skill price process. With this process we can simplify the analysis and illustrate the key mechanisms of the model. In Section 3.2, we study a skill price process that displays a decreasing rate of growth.

3.1 Constant Growth of the Skill Price

In this section we assume that the skill price process is described by

$$w_{t+1} = g w_t,$$

with $g > 1$. That is, each individual from each cohort faces the same growth rate throughout his life. We provide and analyze the solution to an individual's problem (i.e., human capital accumulation on the job and schooling choice). We emphasize, in particular, the determination of cross-cohort differences in earnings growth.

A Worker's Life-Cycle Earnings In appendix B, we show that problem (1)-(3) admits an interior solution of the form

$$W_{j,t}(h, a) = \beta_{j,t} h + \alpha_{j,t}(a), \tag{10}$$

where $\beta_{j,t} = w_t + \beta_{j+1,t+1}(1-\delta)/r$ and $\beta_{J+1,t+1} = 0$. We focus the following discussion on this interior solution.[4] The term $\beta_{j,t}$ is the marginal return to human capital—that is, the increase in the present value of income resulting from an increase in the stock of human capital. It is convenient to express $\beta_{j,t}$, after solving forward, as

$$\beta_{j,t} = w_t \sum_{\tau=0}^{J-j} \left(g\frac{1-\delta}{r} \right)^{\tau}. \tag{11}$$

That is, the marginal return to human capital is the present value of the skill price, computed for the rest of the individual's life and adjusted for depreciation. Note that $\beta_{j,t}$ is linear in w_t with a slope that depends only on age. Importantly, conditional on age j, the slope is constant over time and, therefore, identical across cohorts. Finally, $\beta_{j,t}$ is independent of ability.

Using Equation (10), the first-order condition for the optimal choice of nh is

$$w_t = \frac{1}{r}\beta_{j+1,t+1}F_1(nh,a). \tag{12}$$

The left-hand side of Equation (12) is the marginal cost of increasing nh (i.e., the foregone earnings). The right-hand side is the discounted marginal benefit. It has two parts: the marginal value of human capital in the next period measured by $\beta_{j+1,t+1}$ and the marginal increase in human capital measured by the marginal product of nh, $F_1(nh,a)$.

Human capital accumulation amplifies the growth of the skill price. That is, a worker's earnings grow faster than w. To see this, recall that earnings are $wh(1-n)$. As long as h grows and n decreases over the life cycle, earnings grow faster than w. It is, in fact, a standard feature of the Ben-Porath model that n decreases with age and h increases until a certain age.

To determine the cross-cohort differences in earnings growth, recall that there are no cross-cohort differences in the skill price growth rate. The only source of cross-cohort differences is the skill

[4] In a corner—that is, when the optimal n equals 1—the value function is

$$W_{j,t}(h,a) = \frac{1}{r}W_{j+1,t+1}((1-\delta)h + F(h,a),a).$$

price level: recent cohorts face a higher skill price. Contemplate two cohorts: one recent and one old. Consider two workers with the same ability, one in each cohort. If they had the same human capital, then equations (11) and (12) imply that nh depends on age but does not depend on w. The life-cycle earnings profiles of these two workers are then parallel, with the higher profile being that of the worker in the recent cohort since the skill price is higher in the recent cohort.

If the human capital at the start of work life in the recent cohort happens to be higher, then equations (2) and (9) imply that human capital grows at a slower pace for this worker, implying a flatter life-cycle earnings profile. We show now that human capital at the start of work life is indeed higher in the recent cohort in our model.

College Human Capital We now determine the after-college human capital for individuals who enroll in college. (For high-school-educated workers, human capital at the start of the work life is exogenous, given by (7).) Problem (5) describes the investment decision of an individual with ability a born in period t, who enrolls in college. The optimal goods spending, k^*, satisfies

$$1 = \frac{1}{r^s}\beta_{s+1,t+s}G_1\left(k^*, h_1(a), a\right), \tag{13}$$

where the left-hand side is the marginal goods cost and the right-hand side is the marginal product of goods in the college human capital technology, $G_1\left(k^*, h_1(a), a\right)$, multiplied by the discounted marginal return to human capital, $\beta_{s+1,t+s}/r^s$. Note that a higher marginal return to human capital implies higher college spending and, therefore, higher college human capital.

Consider the old and recent cohorts again, and recall that the skill price level is higher for the recent cohort. This implies that the marginal return to human capital, $\beta_{j,t}$, is higher for the recent cohort. Equation (13) then implies that, conditional on enrolling in college, a worker with ability a from the recent cohort starts his work life with more human capital than a worker with the same ability from the old cohort.

College Enrollment To determine college enrollment in a given cohort, we compute an ability threshold such that a worker with this ability is indifferent between attending college or not—that is, we find a_t^* such that

$$V_{1,t}^{\mathrm{col}}(a_t^*) = V_{1,t}^{\mathrm{hs}}(a_t^*).$$

Note that the subscript t in a_t^* indicates *cohort t*—that is, the set of individuals of age 1 at calendar date t. In Appendix C, we show that this equation can be written as

$$(a_t^*)^{\phi/(1-\phi)} Z_1 + Z_2 = a_t^* w_t^{\eta/(1-\eta)} Z_3, \tag{14}$$

where Z_1, Z_2, and Z_3 are positive constants.

We now describe the case where the left-hand side of Equation (14) is convex, since this is the relevant case in our quantitative exercise (i.e., $\phi > 0.5$). When the skill price is sufficiently low, Equation (14) has no solution. The return to human capital can be so low that no individual finds it profitable to enroll in college. College enrollment is then zero.

For higher skill price levels, there are two ability thresholds, a_t^* and a_t^{**}, at which individuals are indifferent between college and high school. The choice of an individual with ability a is then

$$\begin{cases} \text{Attend college if} & a \in (a_t^*, a_t^{**}) \\ \text{Do not attend college if} & a \notin (a_t^*, a_t^{**}) \end{cases}$$

Individuals with $a < a_t^*$ do not enroll in college because their ability to accumulate human capital in college and on the job is not enough to offset the forgone earnings. Individuals with $a > a_t^{**}$ do not attend college because their ability is so high that accumulating human capital on the job is more profitable than attending college.

Remark 1 *In our quantitative section, the fraction of workers above a_t^{**} is negligible for every cohort. Hereafter, we abstract from this term to simplify the discussion and the notation.*

For the recent cohort, the higher skill price increases the slope of the right-hand side of Equation

(14). This is represented in Figure 2 as a rotation of the red line. The threshold ability falls from a^*_{old} to a^*_{recent}. It follows that the higher skill price faced by the recent cohort induces more people to attend college. The increase in college enrollment is entirely due to the presence of goods in the human capital technology in college. In the absence of goods in Equation (8) (i.e., when $\eta = 0$), college human capital and the ability threshold are the same across cohorts and do not depend on w (see Equation (14)).[5] In the presence of goods in Equation (8), college human capital is higher for the recent cohort. This is because a higher skill price in the recent cohort implies a higher marginal return to human capital and, hence, a higher goods spending and a higher college human capital (see Equation (13)). Even though a higher skill price implies higher forgone earnings, the higher college human capital offsets the higher opportunity cost for the recent cohort.

Differences in threshold ability across cohorts imply differences in the educational composition of workers. Put differently, the ability distribution and the human capital distribution, conditional on education, differ across cohorts. This generates composition effects that have implications for cross-cohort differences in earnings growth.

3.1.1 Cross-cohort Differences in Earnings Growth

The recent cohort has more individuals attending college relative to the old cohort: those with abilities in the interval $[a^*_{\text{recent}}, a^*_{\text{old}}]$ (see Figure 3). Hence, both the high-school- and college-educated workers have lower *average* ability in the recent cohort.

Figure 4 compares the decisions of two cohorts. The only difference between these two cohorts is that the recent cohort starts its life facing a higher skill price level. (Recall that the skill price growth is constant in this qualitative illustration.) The solid blue lines denote the old cohort facing a lower skill price; the red circles denote the recent cohort facing a higher skill price. We distinguish between three groups of ability (see Figure 3). The "always-high school" group, with $a \leq a^*_{\text{recent}}$, corresponds to those who decide to start working at age 1 under both skill price levels.

[5] When the growth rate of w is not the same across cohorts (Section 3.2), even with $\eta = 0$ college human capital and the ability threshold are different across cohorts.

The "switchers," with $a^*_{\text{recent}} \leq a \leq a^*_{\text{old}}$, are those who do not attend college under the low skill price (old cohort) but attend college under the high skill price (recent cohort). The "always-college" group, with $a \geq a^*_{\text{old}}$, corresponds to those who attend college under both the low and the high skill prices.

Panel A of Figure 4 illustrates the human capital at the start of work life and Panel B illustrates earnings growth. The human capital at the start of work life for the always-high school group is the same in each cohort. Human capital at age 1 for this group is exogenous, and accumulation on the job is independent of the skill price since the investment in human capital, nh, is the same in each cohort as noted in Equations (11) and (12). Thus, the earnings growth for this group is the same in both cohorts.

Panel A also reveals that human capital at the start of work life is higher for each member of the always-college group. This is the college intensive margin effect: The higher skill price implies that the marginal return to human capital is higher and, as implied by Equation (13), members of the always-college group in the recent cohort have more after-college human capital. Since they start their work life with higher human capital in the recent cohort, they experience lower earnings growth (see Panel B). This is a key mechanism in our model: A worker has more human capital at the start of his work life if he attends college and the incentives to accumulate human capital are decreasing in the stock of human capital.

Finally, members of the switchers group in the recent cohort have higher human capital at the start of their work life. This is because each member of the switchers group in the recent cohort decides to attend college and ends up with more human capital. Hence, the earnings growth for this group is less in the recent cohort.

Panel B of Figure 4 also shows that those with higher ability accumulate human capital faster and, hence, experience higher earnings growth. This is evident from the human capital accumulation technology (2). The discontinuity at a^*_{old} (or at a^*_{recent}) indicates, however, that the marginal worker accumulates human capital on the job at a slower pace if he is college educated than if he is not.

Note that the distribution of ability conditional on education is different across cohorts. For instance, the college-educated workers in the old cohort are those above a^*_{old} and the college-educated workers in the recent cohort are those above a^*_{recent}. So, when we compute average earnings growth among college-educated workers, we are averaging across different groups in the two cohorts (see Panel B of Figure 4). This is the college composition effect. There is a similar high school composition effect: The average earnings growth among high-school-educated workers in the old cohort includes those with ability less than a^*_{old}, whereas the earnings growth for the recent cohort includes only those with ability less than a^*_{recent}.

3.2 Slowdown of the Skill Price

Suppose that the skill price, w, does not grow at a constant rate. For the sake of exposition, and in line with our findings in Section 4, assume that (i) each cohort faces a constant, cohort-specific skill price growth rate; and (ii) the growth rate is lower for the recent cohort.[6] In this context there are several *additional* effects relative to Section 3.1.1. First, there is a direct effect. The lower growth of w implies a flatter earnings profile for the recent cohort, holding all else fixed. Second, the lower growth of w implies a slowdown in the pace of human capital accumulation and, hence, a flatter earnings profile for the recent cohort. Third, the lower growth of w implies a change in the distributions of ability and human capital conditional on education and generates additional intensive margin and composition effects.

To see the second effect, consider two workers, one from each cohort, with the same ability and human capital at age j. The lower skill price growth rate implies a lower return to human capital, β_j, for the recent cohort (see Equation (11)). Equation (12) then implies that the worker of the recent cohort allocates less time to human capital accumulation. Hence, the worker from the recent cohort experiences less earnings growth than the worker from the old cohort.

To see the third effect, the lower marginal return to human capital for the recent cohort generates

[6]The general case where the skill price growth rate decreases over the *life cycle* of any given cohort (Equation (15) in Section 4) does not lend itself to an easy qualitative analysis.

both intensive margin and composition effects. It implies that the college-educated workers in the recent cohort start their work lives with a lower level of human capital. It also implies that there are fewer college-educated workers in the recent cohort. However, these effects due to the lower skill price growth rate are countered by the effects due to the higher skill price *level* (since the skill price is growing over time). Higher level of the skill price implies higher marginal return to human capital, so the intensive margin and composition effects go in the opposite direction. Whether the earnings growth for the recent cohort is lower or higher depends on whether the effects due to the higher skill price level dominates the effects due to the lower skill price growth rate.

4 The Quantitative Exercise

4.1 Calibration

We assume that a model period is 1 year and that workers live for $J = 50$ periods (from age 18 to 68). College lasts for four periods, thus $s = 4$, and we set the annual rate of interest to 5 percent, thus $r = 1.05$. We follow Huggett et al. (2006) and set the annual rate of depreciation of human capital at 1.14 percent, thus $\delta = 0.0114$.

The skill price evolves according to

$$w_t = \exp\left(g_1(t - 1940) + g_2(t - 1940)^2\right), \tag{15}$$

where g_1 and g_2 are parameters to be determined. When $g_2 = 0$ the process for w exhibits constant growth with a growth factor $\exp(g_1)$. If $g_2 < 0$, then the skill price growth rate decreases over time. We normalize $w_{1940} = 1$. Note that this process is more general than the one discussed in Section 3.2 since the skill price growth rate is not constant throughout the life of any cohort.

The parameters to be determined are: the parameters of the ability distribution, ψ_0, ψ_1, and ψ_2; the curvature parameters in the human capital production functions for college and on the job, η

and ϕ; the scale parameters z_H, z_G, and z_F; and the parameters of the skill price process, g_1 and g_2. Let $\theta \equiv (\psi_0, \psi_1, \psi_2, \eta, \phi, z_H, z_G, z_F, g_1, g_2)'$.

We choose θ to minimize a distance between moments simulated from the model and their empirical counterparts. Let $p_t = 1 - A(a_t^*)$ denote the college enrollment for cohort t. Note that p_t depends on θ via two channels. First, the parameters of the skill price process, g_1 and g_2, determine the path of w_t, which in turn determines the ability of the marginal worker in any given cohort, a_t^*. Second, given a_t^*, college enrollment for a particular cohort depends upon the Cumulative Distribution Function A, which is determined by the parameters ψ_0, ψ_1, and ψ_2.

We denote by $E_{t,j}^i$ the average earnings for cohort t at age j, conditional on education $i \in \{\text{hs}, \text{col}\}$. We also define the conditional standard deviation $S_{t,j}^i$.[7] Let the bold letters $\mathbf{E}_{t,j}^i$ and $\mathbf{S}_{t,j}^i$ denote their empirical counterparts. We find θ by solving the following problem:

$$\min_{\theta} \sum_{i\in\{\text{hs,col}\}} \sum_{j=35,45,55} \left(\frac{E_{1940,j}^i / E_{1940,25}^i}{\mathbf{E}_{1940,j}^i / \mathbf{E}_{1940,25}^i} - 1 \right)^2 + \left(\frac{S_{1940,j}^i / E_{1940,j}^i}{\mathbf{S}_{1940,j}^i / \mathbf{E}_{1940,j}^i} - 1 \right)^2 + \sum_{i\in\{\text{hs,col}\}} \left(\frac{E_{1980,25}^i / E_{1940,25}^i}{\mathbf{E}_{1980,25}^i / \mathbf{E}_{1940,25}^i} - 1 \right)^2 + \sum_{t=1940,50,\ldots,80} \left(p_t / \mathbf{p}_t - 1 \right)^2 \quad (16)$$

There are four parts in this objective function. The first two parts target the growth of earnings experienced by the 1940 cohort and the dispersion (measured by the coefficient of variation) of earnings by age for this cohort. The third part targets the growth *over time* of the earnings of 25-year-old workers in each education group. Finally, the last part targets the time series of college enrollment of successive cohorts of workers from 1940 to 1980.

In the minimization problem (16), the earnings data pertain only to the 1940 cohort and to the time series of earnings for 25-year-old workers. No information pertaining to life-cycle earnings growth of cohorts other than the 1940 cohort is used. Thus, the calibration strategy does not target the existence and magnitude of the flattening of earnings profiles.

Table 1 reports the calibrated values of the parameters. We find ϕ, the elasticity parameter in

[7] Both $E_{t,j}^i$ and $S_{t,j}^i$ are computed by integrating earnings over the distribution of ability conditional on i.

the technology for human capital accumulation on the job, to be 0.56. This is within the range of estimates, 0.5 to 1, reported by Browning et al. (1999). Since the quadratic term, g_2, is negative there is a slowdown of the skill price: Over the 1940-70 period the skill price increases at an average rate of 1.3% per year, while over the 1980-2010 period it increases at 1.0% per year.

4.2 Results

Table 2 and Figure 5 illustrate the model's fit to the targeted moments. The model replicates well the earnings growth and dispersion statistics for the 1940 cohort, as well as the college enrollment time series.

Most of the earnings growth in our model is due to endogenous human capital accumulation, which amplifies the skill price growth. Absent human capital accumulation, with our calibrated skill price growth of 1.3 percent per year between 1940 and 1970, the earnings of high-school-educated workers in the 1940 cohort would have also grown by 1.3 percent on average between ages 25 and 55, instead of 4.6 percent (4.4 percent in the data, see Table 2). Similarly, the earnings of college-educated workers would have also grown by only 1.3 percent in the model instead of 4.7 percent.

Table 3 presents our main results. It shows the flattening in the life-cycle earnings of the 1950, 1960, 1970, and 1980 cohorts, relative to the 1940 cohort. Consider, for example, the college-educated workers of the 1940 and 1950 cohorts. In the 1940 cohort, the average earnings of this group grew by a factor of 4.0 between the ages of 25 and 55 in the data. In the 1950 cohort, they grew by a factor of 3.3. Thus, the ratio of earnings growth between the two cohorts is 0.83; or, the earnings profile is 17 percent flatter for the 1950 cohort relative to the 1940 cohort. In the model, a similar calculation implies a flattening of 11 percent. Thus, the model accounts for $11/17 = 67$ percent of the flattening between the 1940 and 1950 cohorts. Similarly, the model accounts for 59, 73, and 99 percent of the flattening between the 1940 and the 1960, 1970, and 1980 cohorts, respectively.

As Table 3 illustrates, for high-school-educated workers, the calibrated model accounts for 27 percent of the flattening between the 1940 and 1950 cohorts and 52 percent of the flattening between the 1940 and 1980 cohorts.

The model implies that earnings profiles are flatter for each new cohort of workers relative to the previous cohort. This flattening happens, as in the U.S. data, via lower earnings growth for the 55-year-old workers *over time* than for the 25-year-old workers. In the model, the average 25-year-old college-educated worker in the 1980 cohort earns 1.9 times as much as the corresponding worker in the 1940 cohort; in the data the figure is 1.7. The earnings of the average 55-year-old college-educated worker in the 1980 cohort are 1.2 times that of the corresponding worker in the 1940 cohort; in the data the figure is 1.1. The model's implications for all cohorts and all education categories are reported in Table 4.

4.2.1 Evidence

The key mechanisms in the model are the composition effect due to the increase in college enrollment in the recent cohort and higher college human capital (college intensive margin effect) in the recent cohort. As noted in Section 3.1, these effects only arise when the share of goods in the college human capital technology, η, is greater than zero. It is, therefore, important to verify that the implications of the model for spending in college are consistent with empirical evidence, even though this was not a target in the calibration. The college years for the cohorts in our model cover the calendar years 1929 to 1985, and the observed college expenditures per student increase at an annual rate of 1.2 percent over these years (see Carter et al., 2006, series Bc967). The college spending per student, in our model, increases at an annual rate of 1 percent.

In our model, the flattening of earnings for the always-college group results from the slowdown of the skill price and the college intensive margin effect (higher college human capital in the recent cohort). Composition plays no role by definition. What is the corresponding evidence on the intensive margin effect in the data? While we cannot directly identify the always-college group in the data, we suppose that small groups of workers in high-skill occupations with at least 5 years of college education are less affected by the composition issue than the bulk of college-educated workers. Consider, for instance, physicians and surgeons, a group of highly-skilled college-educated workers. This is a small group: in 2010, 57 percent of high-school educated workers aged 20-60 have a college education, but only 0.7 percent of them are physicians or surgeons.[8] Our conjecture is that rising college enrollment has a larger composition effect on the ability distribution of mid-level managers with a college education, than on the ability distribution of physicians and surgeons. Figure 6 shows earnings growth over the life cycle, by cohort, for physicians and surgeons. The figure shows flattening of earnings profiles (earnings as well as earnings per hour) until the 1970 cohort, followed by an increase for the 1980 cohort. This is the same pattern as in Figure 1.

Furthermore, the real earnings of physicians and surgeons at the start of their careers (age 30) increased at an average annual rate of 3.8 percent between 1940 and 1980. This is markedly higher than the average annual TFP growth over this period, so this is suggestive of an increase in college human capital for physician and surgeons. Figure 6 and the growth in start-of-the-career earnings are evidence suggestive of college intensive margin effect in our model: Higher college human capital in recent cohorts imply a slower rate of human capital accumulation on the job and, hence, less earnings growth.

4.2.2 Decomposing the results

How do the slowdown of the skill price, the composition effects, and the college intensive margin effect contribute to explaining the flattening of earnings profiles?

To answer this question we compare the 1940 and the 1980 cohorts. Following the discussion in Section 3.1.1, we partition the distribution of ability into three groups: always-college, always-high school, and switchers. Table 5 reports earnings growth for each group in the two cohorts. To understand the table, recall that the set of high-school-educated workers (blue italic type) includes always-high school and switchers groups in the 1940 cohort, but only the always-high school group

[8] The Census data we use groups individuals with 5+ years of college education together. In 2010, 11.7 percent of the 20-60 high-school educated workers have at 5+ years of college, implying that physician and surgeons represent only 6 percent of those with 5+ years of college ($100 \times 0.7/11.7 = 6\%$). Indeed it takes more than 11 years of college to become a surgeon (4 years of undergraduate studies, 4 years of medical school and 3 to 8 years of residency).

in the 1980 cohort. Similarly, the set of college-educated workers (red bold type) includes only the always-college group in 1940 but the always-college and switchers groups in 1980.

Start with the always-high school group. This group does not enroll in college in either the 1940 cohort or the 1980 cohort. By definition, the ability distribution is identical in each cohort and, since initial human capital is exogenous and proportional to ability, the distribution of human capital in this group is identical as well. The earnings growth for this group in the 1980 cohort is 0.84 times that in the 1940 cohort, so the earnings profile for this group is 16 percent flatter in 1980 relative to 1940. This flattening is due to (i) the direct effect of the lower skill price growth rate for the 1980 cohort relative to the 1940 cohort and (ii) the endogenous response of human capital accumulation to the lower growth rate. The skill price process directly flattens the earnings profile by 9 percent, all else equal.[9] But the pace of human capital accumulation is lower for the 1980 cohort, a feature of the model discussed in Section 3.2. The amplification in the model thus accounts for the remaining 7 percent of the flattening.

The average earnings profile of high-school-educated workers is 29 percent flatter for the 1980 cohort relative to the 1940 cohort (see Table 5). The difference between 16 percent flattening of the always-high school group and 29 percent for the high school group is due to composition (i.e., due to the switchers). Recall from Figure 3 that the switchers are those with higher ability among high-school-educated workers in the 1940 cohort who decided to enroll in college in the 1980 cohort. Since switchers are of higher ability than the always-high school workers, they have higher earnings growth in the 1940 cohort, a factor of 5.3 versus 3.2. Note that the change in composition between the two cohorts is also an endogenous response to the change in the skill price.

Turn now to the always-college group. Flattening for this group is 32 percent. This results from the direct effect of the exogenous skill price slowdown and the endogenous slowdown of human capital accumulation for the 1980 cohort, as in the case of the always-high school group. There is an additional effect on the always-college group since those in the 1980 cohort start their work

[9] As mentioned in Section 4.1, the annual growth rate of w is 1.3 percent from 1940 to 1970 and 1 percent from 1980 to 2010. The skill price growth for the 1980 cohort is therefore $(1.01/1.013)^{30} = 0.91$ times the growth of the 1940 cohort. Hence, the flattening is 9 percent.

lives with more human capital. This is because, quantitatively, the higher skill price level increases the return to human capital and this effect dominates the effect of the lower skill price growth rate (see the role of g in Equation (11)).

The average earnings profile of the college-educated workers is 35 percent flatter in the 1980 cohort. The difference between 32 percent for the always-college and 35 percent for the college group is due to composition. Since switchers are of lower ability than those in the always-college group, they have lower earnings growth in the 1980 cohort (a factor of 2.4 versus 2.7).

5 Discussion

In this section we consider our model's implications for other (untargeted) moments in the data and conduct a few robustness checks. Specifically, we explore the model's implications for cross-sectional inequality, the college premium, and the experience premium. We then examine the roles of observed changes in the price of college education, possible cross-cohort differences in the ability of high school graduates, and 1940 as a reference year for the calibration.

5.1 Implication for cross-sectional inequality

Much of the literature on wage inequality in the U.S. focuses on cross-sectional measures of inequality. In this section, we discuss our model's implications for the evolution of cross-sectional inequality and how they relate to previous findings. Katz and Murphy (1992) report a decline in the "relative age premium" since the late 1970s. That is, the wages of older workers increased relative to those of younger workers, conditional on high school education, but the wages of older workers relative to those of younger workers remained stable, conditional on higher levels of education.

To express this finding in the language of our model we consider two age groups: 35 and 55. We define the age premium at calendar date t as the ratio $E^{\mathrm{col}}_{t-55,55}/E^{\mathrm{col}}_{t-35,35}$ for college-educated workers, and $E^{\mathrm{hs}}_{t-55,55}/E^{\mathrm{hs}}_{t-35,35}$ for high school-educated workers. Katz and Murphy (1992)'s finding can be summarized as: The ratio

$$\frac{\mathbf{E}^{\text{col}}_{t-55,55}/\mathbf{E}^{\text{col}}_{t-35,35}}{\mathbf{E}^{\text{hs}}_{t-55,55}/\mathbf{E}^{\text{hs}}_{t-35,35}}, \tag{17}$$

that is the relative age premium, fell since the late 1970s. Katz and Murphy attributed the decline in the relative age premium to increases in the supply of college workers.

Card and Lemieux (2001) show that the return to college increased more for young than for old workers between the late 1970s and the 1990s. In our model's language the relative college premium

$$\frac{\mathbf{E}^{\text{col}}_{t-55,55}/\mathbf{E}^{\text{hs}}_{t-55,55}}{\mathbf{E}^{\text{col}}_{t-35,35}/\mathbf{E}^{\text{hs}}_{t-35,35}} \tag{18}$$

declined over time. Card and Lemieux attribute the decline in the relative college premium to increases in educational enrollment. Jeong et al. (2015) point out that the evidence on relative age premium by Katz and Murphy and the evidence on relative college premium by Card and Lemieux are equivalent. This is evident from comparing Equations (17) and (18).

In Figure 7 we report the relative college premium in Equation (18) computed from the Census data. The Figure shows a steep decline starting in the second half of the 1970s, consistent with the findings of both Katz and Murphy (1992) and Card and Lemieux (2001). We note, however, that in the period preceding the 1970s, this statistic is increasing.

Our model is consistent with the findings of Katz and Murphy (1992) and Card and Lemieux (2001) since it predicts a decline in the relative college premium. Quantitatively, the model counterpart of the statistic in Equation (18) exhibits a 3 percent decline between 1980 and 2010. This compares with the 8 percent decline observed during the same period. Thus, the model accounts for 37 percent (3/8) of the decline in the relative college premium. It should be noted that in the model the decline of the relative college premium is monotonic over time. Thus, our model does not account for the increase in the relative college premium before the 1970s reported in Figure 7.

5.2 The College Premium

Figure 8 shows the evolution of the college premium—that is, the ratio of the average earnings of college-educated workers to the average earnings of high-school educated workers—in the model and the data. The premium is normalized to 1 for the 1940 cohort in each panel. Apart from the premium for the 25-year-old workers, the college premium tends to be higher for each age group in subsequent cohorts. For example, the observed college premium is 25 percent higher for the 45-year-old in the 1980 cohort than for the 45-year-old in the 1940 cohort; the model predicts a 20 percent increase. The main message from Figure 8 is that the model is broadly consistent with the rise in the college premium observed in the U.S.

In the model, the rise in the college premium results from differences in human capital investment across cohorts, and not from different growth rates of the skill price for the college-educated- and high-school-educated workers. This is consistent with the findings by Bowlus and Robinson (2012), who attribute most of the rise in the college premium to human capital investment.

As it stands, the model cannot reproduce the *level* of the college premium for the 1940 cohort. This could be potentially resolved by using different *levels* of skill prices for the college-educated- and high-school-educated workers. In the next subsection we consider such a model.

5.2.1 Education-Specific Skill Prices

Let w_t^i, $i \in \{\mathrm{hs}, \mathrm{col}\}$, denote the college- or high school-specific skill price at date t. We adopt the following specification:

$$w_t^i = w_{1940}^i \exp\left(g_1^i(t-1940) + g_2^i(t-1940)^2\right), \tag{19}$$

where w_{1940}^{hs} is normalized to 1. Thus, relative to the baseline (equation 15), there are 3 more parameters: w_{1940}^{col}, g_1^{col}, and g_2^{col}. This specification allows the skill prices to follow two time-dependent quadratic processes where w_{1940}^{col} is the ratio between the college-specific and high school-specific skill price in 1940. Other than this change in specification, the model is the same as the baseline. The individual choices for the model with time series (19) are described in Appendix D.

To calibrate this model, we add some target moments to the ones in the baseline (see Section 4.1). Specifically, we add five more moments: the ratio of earnings between college- and high school-educated workers at age 35 in each of the five cohorts of interest. Table D.1 shows the calibrated parameters. Table D.2 shows the model's fit. Figure D.1 shows the model's implication for college enrollment. Figure D.2 shows the implications for the college premium. Again, consistent with Bowlus and Robinson (2012), the increase in college premium in the model is mostly due to human capital accumulation.

Table 8 indicates the model's implication for the flattening of the life-cycle earnings profiles. The main lesson from the table is that, relative to the baseline, the model accounts for a larger portion of the observed flattening for high-school-educated workers. For instance, the model with two skill prices accounts for 69 percent of the flattening between the 1940 and the 1980 cohort (versus 52 percent for the baseline, see Table 3). For college-educated workers, the model's performance is virtually the same as the baseline's. It is also the case that the model with two skill prices implies more flattening for high-school-educated workers than for college-educated workers, consistent with the data in Figure 1.

5.3 The Experience Premium

Katz and Murphy (1992) document that the average weekly earnings of workers with 1-5 years of experience changed by 0.07 log points during the period 1963-87 while for workers with 26-35 years of experience the change was 0.19 log points. We obtain a similar pattern after 1970 in our sample—the earnings of 55-year-old workers grew faster than those of 25-year-old workers. Before 1970, however, the observed pattern is opposite: The earnings of 25-year-olds increased by 0.77 log points between 1940 and 1970 and for 55-year-olds the increase was 0.60 log points.[10] Our model delivers the flattening of earnings profiles documented in Figure 1 for the 1915 to 1955 *birth cohorts*

[10]Hendricks (2015) also documents the u-shape of the return to experience.

whereas Katz and Murphy (1992) deliver the experience premium in the *cross-section* after 1963.

5.4 Changes in the Price of College Education

In our model, the relative price of college education is assumed to remain constant. In Equation (5), one unit of income purchases one unit of goods spent in college at all points in time. There has been, in fact, an increase in the relative price of higher education, as measured by the faster growth of the Higher Education Price Index relative to that of the Consumer Price Index, illustrated in Figure 9. The figure, however, shows that the difference in growth rates is significant only after 1985. Members of all cohorts in our model make their college enrollment decisions before 1985. Therefore, the recent increase in the price of higher education might be of second-order concern for our results.

5.5 Cross-Cohort Differences in Ability

All individuals in our model are endowed with a high school education. In our baseline calculations, we assume that the distribution of ability among high school graduates is constant across cohorts. The actual distribution might be different across cohorts for reasons noted by Hendricks and Schoellman (2014). For instance, in the United States, the fraction of individuals without a high school diploma in the 1940 cohort was 50 percent, whereas this fraction in the 1980 cohort was just 10 percent. To gauge the quantitative importance of cross-cohort differences in ability distribution for the flattening of life-cycle earnings, we compare the 1940 and 1980 cohorts assuming different distributions of ability.

Recall that our baseline distribution of ability is denoted by the density $A'(a)$. Let $B'_\lambda(a)$ denote the exponential density with parameter λ: $B'_\lambda(a) = \lambda e^{-\lambda a}$. We assume that the density of ability for the 1940 cohort is the baseline, $A'(a)$, and that for the 1980 cohort is $\zeta A'(a)B'_\lambda(a)$, where $\zeta > 0$ ensures that $\int \zeta A'(a)B'_\lambda(a)da = 1$. We consider three values of λ such that in the 1980 cohort the mass below the median of the baseline distribution is 52.5, 55, and 60 percent, respectively. Table

6 reports the results. The main message from the table is that differences in ability distributions across cohorts do not generate substantially more flattening relative to the baseline case.

Note that in Table 6 the alternative distribution of ability implies low college enrollment for the 1980 cohort, which means that the composition effect is not as strong as in the baseline. We can generate the same enrollment for the 1980 cohort as in the baseline case by increasing the skill price growth rate for this cohort. Such an experiment produces less flattening relative to the baseline. For example, when the mass below the baseline median ability is 52.5 percent, the flattening for high-school-educated workers is 39 percent (versus 52 percent in the baseline) and 77 percent for college-educated workers (versus 99 percent in the baseline).

5.6 Using 1950 as the Reference Year Instead of 1940

In our quantitative exercise, we use the 1940 cohort as a reference for both calibrating the model and measuring the flattening of earning profiles for subsequent cohorts. Our results do not depend critically on this choice. After calibrating the model to the life-cycle earnings data of the 1950 cohort, we find that the model generates significant flattening between the 1950 and subsequent cohorts. Table 7 presents the results. For instance, the earnings profile of high-school-educated workers in the 1980 cohort is 21 percent flatter than the profile in the 1950 cohort and the model explains 59 percent of the flattening.

6 Conclusion

The life-cycle earnings profiles of workers are becoming flatter with each new cohort. In this paper, we propose a quantitative theory of this phenomenon. We use a standard Ben-Porath model of human capital accumulation on the job and embed it into a schooling choice model. The model accounts for 52 percent of the observed flattening for high-school-educated workers and 99 percent for college-educated workers between the 1940 and the 1980 cohorts. The model is also consistent with the rise in college enrollment, the increase in the college premium over time, the decrease in the *relative* college premium since the 1970s, and the decrease in the *relative* age premium since the 1970s.

Our theory ascribes the flattening to only one exogenous variable: the skill price. The skill price level increases over time but its growth rate decreases. This skill price process affects the return to human capital and generates two effects: an intensive margin effect and a composition effect. First, our model implies that college-educated workers in the recent cohorts start their work lives with more human capital and, hence, invest less in human capital on the job and experience less earnings growth. Second, the increase in college enrollment of successive cohorts implies that the average ability of high-school- and college-educated workers is lower in recent cohorts. Since ability positively affects human capital accumulation and earnings growth on the job, the earnings profiles for recent cohorts are flatter.

The flattening of earnings profiles has potential implications for the macroeconomy. For example, when life-cycle profiles of earnings are steep, young agents will be net borrowers and old agents will be net savers, resulting in steep saving profiles over the life cycle. In contrast, with flat earnings profiles savings profiles will also be flat. Demographic changes then imply that the aggregate saving rate would be affected by the flattening of earnings profiles.

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Table 1: Calibrated parameters

Ability distribution	$\psi_0 = 77.65$, $\psi_1 = 4.46$, $\psi_2 = 325.97$
Initial human capital	$z_H = 10.22$
College technology	$\eta = 0.31$, $z_G = 0.56$
On-the-job technology	$\phi = 0.56$, $z_F = 0.23$
Skill price process	$g_1 = 0.014$, $g_2 = -3.817 \times 10^{-5}$
Life expectancy, college length	$J = 50$, $s = 4$
Interest rate, depreciation	$r = 1.050$, $\delta = 0.011$

Table 2: Calibration targets: model and data

	Model	Data	Model	Data
	HIGH SCHOOL		COLLEGE	
1940 Cohort				
Annual earnings growth 25-35 (%)	6.8	7.3	7.0	6.5
Annual earnings growth 25-45 (%)	5.5	5.8	5.7	6.0
Annual earnings growth 25-55 (%)	4.6	4.4	4.7	4.7
Coef. of variation at 35	0.41	0.41	0.52	0.49
Coef. of variation at 45	0.48	0.47	0.55	0.55
Coef. of variation at 55	0.52	0.53	0.56	0.62
Time series				
Annualized earnings growth of 25-year-old, 1940-1980 (%)	1.1	1.9	1.6	1.3

Source: IPUMS and authors' calculations.

Table 3: Accounting for the flattening in life-cycle earnings

	Cohorts			
	1950	1960	1970	1980
High school				
% flattening relative to 1940 (data)	36	52	59	59
% flattening relative to 1940 (model)	10	18	25	30
Model/data (%)	27	34	41	52
College				
% flattening relative to 1940 (data)	17	34	38	34
% flattening relative to 1940 (model)	11	20	28	34
Model/data (%)	67	59	73	99

Table 4: Model: Earnings at age 25 and 55, relative to 1940

	Cohorts				
	1940	1950	1960	1970	1980
High school: age 25	1.0	1.1	1.3	1.4	1.5
High school: age 55	1.0	1.0	1.0	1.1	1.1
College: age 25	1.0	1.2	1.4	1.6	1.9
College: age 55	1.0	1.1	1.1	1.2	1.2

Table 5: Earnings growth for three groups of workers

	Cohorts		
	1940	1980	Ratio
Always college	**4.0**	**2.7**	0.68
Switchers	*5.3*	**2.4**	0.45
Always high school	*3.2*	*2.7*	0.84
College	**4.0**	**2.6**	0.65
High school	*3.8*	*2.7*	0.71

Note: The numbers in the 1940 and 1980 columns are earnings growth between the ages of 25 and 55 for the subgroup corresponding to the row. The blue italic type denotes earnings growth for high-school educated workers and the red bold type denote earnings growth for college-educated workers. For instance, the earnings of the average 55-year-old switcher of the 1940 cohort are 5.3 times higher than the earnings of the average 25-year-old switcher in this cohort.

Table 6: Cross-cohort differences in the distribution of ability (%)

Population below baseline median	Flattening between 1940 and 1980 cohorts accounted by model		College enrollment of 1980 cohort
	High school	**College**	
50.0 (baseline)	52	99	50
52.5	52	100	48
55.0	53	101	46
60.0	54	102	40

Note: The first row repeats our baseline results noted in Table 3. The second and third columns report the flattening implied by the model between the 1980 and 1940 cohorts as a fraction of the flattening in the data. The last column reports the college enrollment of the 1980 cohort implied by the model.

Table 7: Accounting for the flattening in life-cycle earnings: Model calibrated to 1950 cohort

	Cohorts		
	1960	1970	1980
High school			
% flattening relative to 1950 (data)	25	37	36
% flattening relative to 1950 (model)	8	15	21
Model/data (%)	32	41	59
College			
% flattening relative to 1950 (data)	21	25	20
% flattening relative to 1950 (model)	9	17	24
Model/data (%)	46	68	116

Note: In this table, both data and model figures are relative to the 1950 cohort. Thus, these figures are not directly comparable with Table 3.

Table 8: Accounting for the flattening in life-cycle earnings: Education-specific skill prices

	Cohorts			
	1950	1960	1970	1980
High school				
% flattening relative to 1940 (data)	36	52	59	59
% flattening relative to 1940 (model)	13	24	33	40
Model/data (%)	36	46	56	69
College				
% flattening relative to 1940 (data)	17	34	38	34
% flattening relative to 1940 (model)	11	20	27	33
Model/data (%)	65	59	71	99

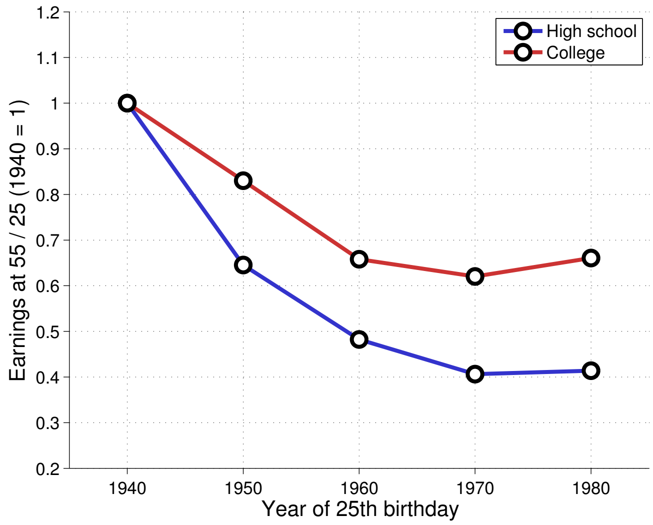


Figure 1: Growth in labor earnings from age 25 to 55 by cohort and educational attainment

Source: IPUMS.
Note: The data are for employed white men working for a wage. The earnings growth figures are normalized to 1 for the 1940 cohort.

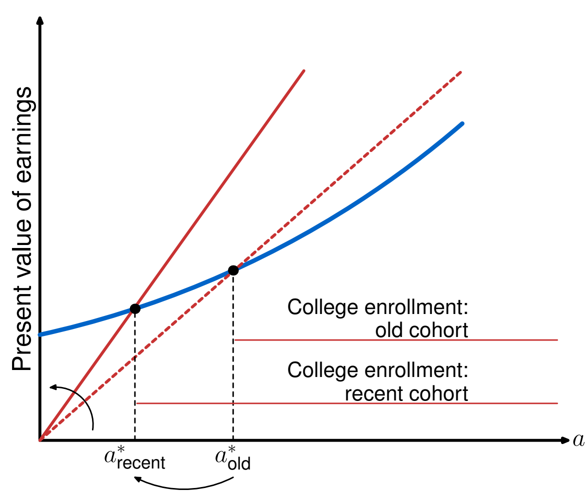


Figure 2: The effect of higher skill price on college enrollment

Note: The red lines from the origin represent the right-hand side of Equation (14). The blue, convex curve represents the left-hand side. An increase in the skill price implies an increase in the slope of the right-hand side of Equation (14) and, hence, a decrease in the ability threshold a^*.

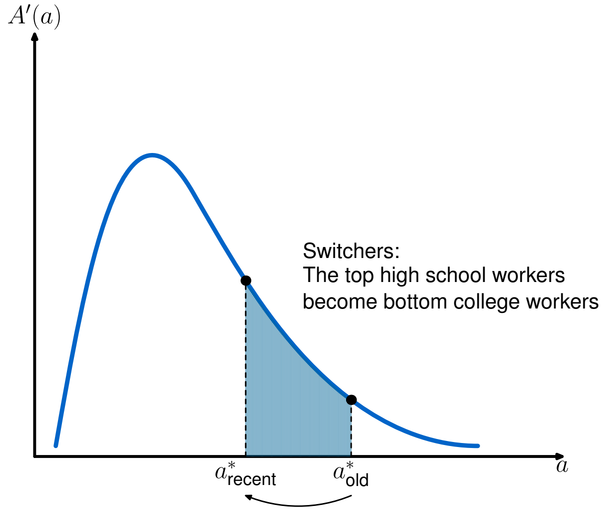
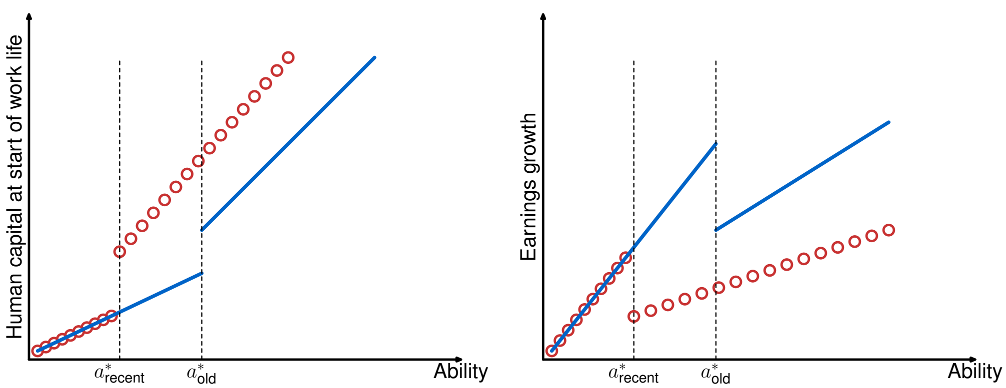


Figure 3: The changing composition of high school- and college-workers



A - Human capital at 25 by ability B - Earnings growth by ability

Figure 4: The effect of the skill price on the life-cycle profile of human capital and earnings growth

Note: These diagrams are stylized representations of the model's mechanics. The curves represented here need not be linear in the calibrated version of the model.

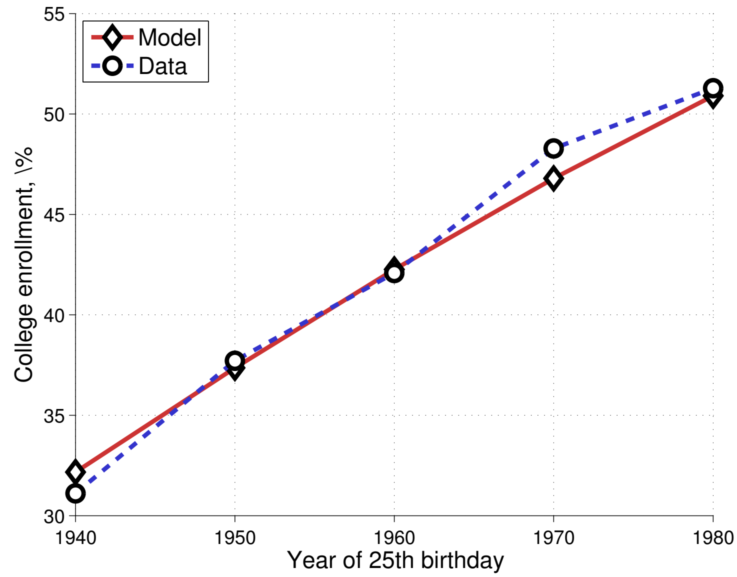
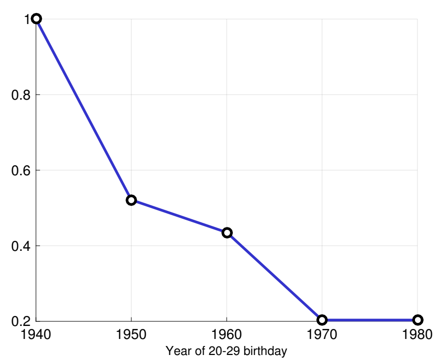
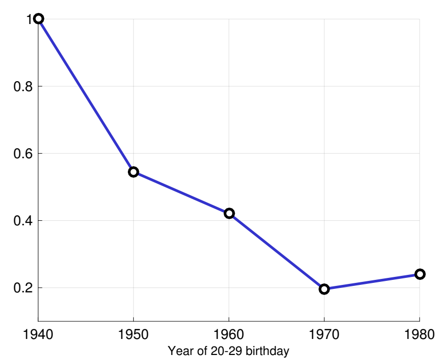


Figure 5: College enrollment: model and data



A - Growth in earnings



B - Growth in earnings per hour

Figure 6: Earnings growth for physicians and surgeons

Source: IPUMS.

Note: The data are for employed white men working for a wage. The growth figures are normalized to 1 for the 1940 cohort.

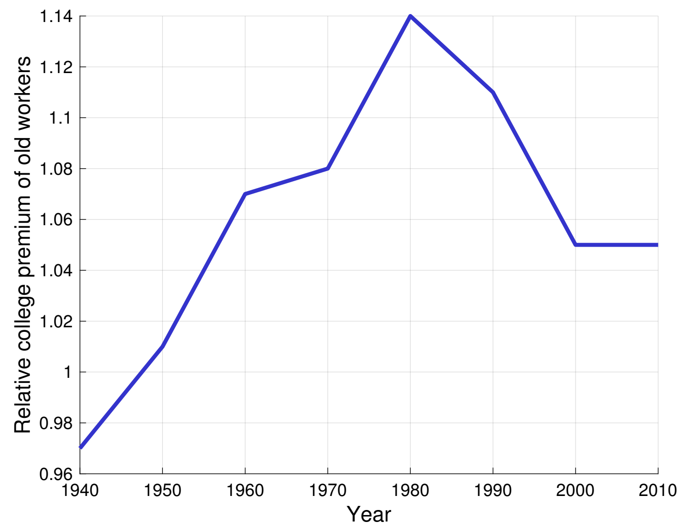


Figure 7: The relative college premium of 55 year-old workers (v. 35)

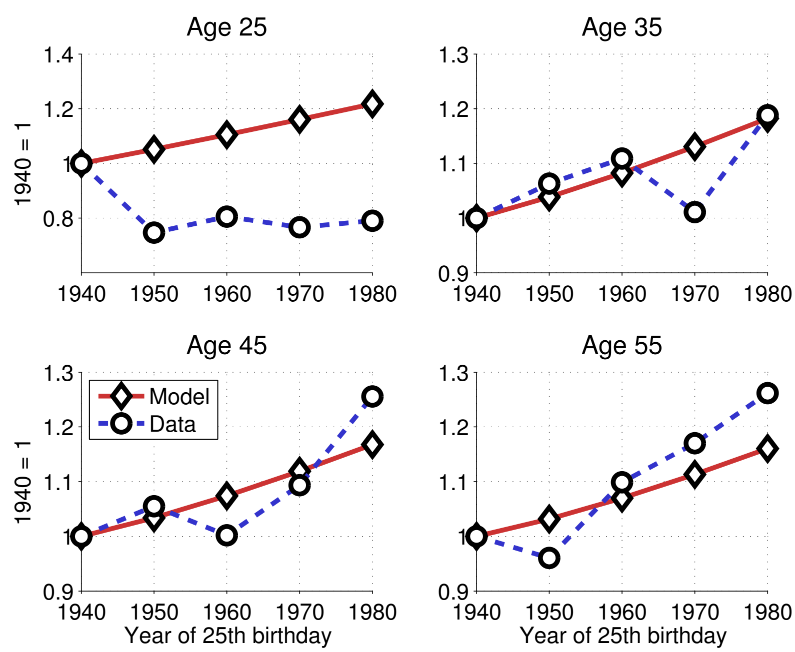


Figure 8: The change in the college premium: model and data

Note: These diagrams show the college premium by cohort at ages 25, 35, 45 and 55. The data and the model are normalized to 1 for the 1940 cohort. For example, the "Age 35" plot shows that the college premium for the 35 year-old of the 1960 cohort was 10% above the college premium of the 35 year-old of the 1940 cohort.

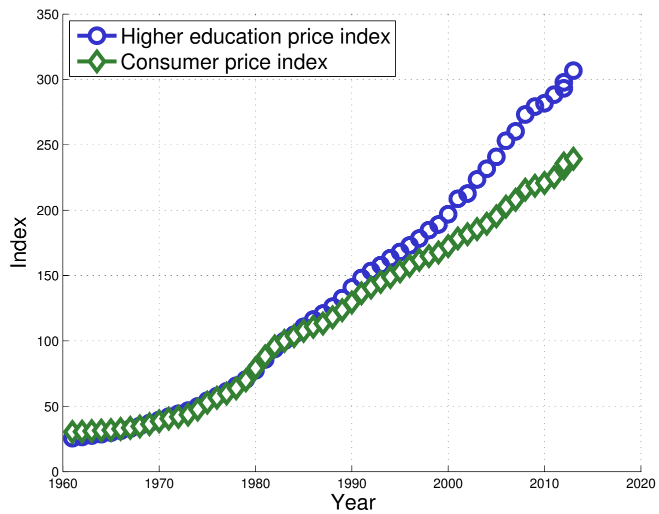


Figure 9: The higher education price index and the consumer price index

Source: Commonfund Institute, 2014 HEPI update.

A Data

We use data from the Integrated Public Use Microdata Series (IPUMS, www.ipums.org). The IPUMS consists of samples of the U.S. population from censuses. We use 1% samples (1940, 1950, 1970, 2010) and 5% samples (1960, 1980, 1990 and 2000). We extract the following variables.

- PERWT: person weight
- AGE: age in years, as of the last birthday
- RACE: race
- SEX: sex
- INCWAGE: wage and salary income
- EDUC: educational attainment
- EMPSTAT: employment status
- CLASSWKR: class of worker
- HRSWORK2 and UHRSWORK: hours worked last week (intervalled) and usual hours worked per week
- STATEICP, STATEFIP: sate codes
- OCC1950, OCC1990, OCC2010: occupation codes

For each year, we consider people who are employed (EMPSTAT=1) and working for a wage (CLASS-WKR=2). Besides sex and race, we group people into 6 age categories: 20-29, 30-39,...,70-79. So we define

$$j = \begin{cases} 1 & \text{if AGE} \in [20-29] \\ 2 & \text{if AGE} \in [30-39] \\ 3 & \text{if AGE} \in [40-49] \\ 4 & \text{if AGE} \in [50-59] \\ 5 & \text{if AGE} \in [60-69] \\ 6 & \text{if AGE} \in [70-79] \end{cases},$$

and

$$s = \begin{cases} 1 & \text{if SEX} = 1 \text{ (men)} \\ 2 & \text{if SEX} = 2 \text{ (women)}, \end{cases}, \quad r = \begin{cases} 1 & \text{if RACE} = 1 \text{ (white)} \\ 2 & \text{if RACE} = 2 \text{ (black)}. \end{cases}$$

We extract two measures of hours because the variable HRSWORK2 is available in the survey years before 1980 while UHRSWORK is available starting in 1980. From this raw data we construct a measure of hours, which we call HOURS, and which we define as

$$\text{HOURS} = \begin{cases} UHRSWORK & \text{if YEAR} \geq 1980 \\ (1+14)/2 & \text{if YEAR} < 1980 \text{ and HRSWORK2} = 1 \\ (15+29)/2 & \text{if YEAR} < 1980 \text{ and HRSWORK2} = 2 \\ (30+34)/2 & \text{if YEAR} < 1980 \text{ and HRSWORK2} = 3 \\ (35+39)/2 & \text{if YEAR} < 1980 \text{ and HRSWORK2} = 4 \\ (40+40)/2 & \text{if YEAR} < 1980 \text{ and HRSWORK2} = 5 \\ (41+48)/2 & \text{if YEAR} < 1980 \text{ and HRSWORK2} = 6 \\ (49+59)/2 & \text{if YEAR} < 1980 \text{ and HRSWORK2} = 7 \\ (60+70)/2 & \text{if YEAR} < 1980 \text{ and HRSWORK2} = 8. \end{cases}$$

The intervals for HRSWORK2 are $[1, 14], [15, 29], \ldots, [49, 59]$. The last interval is 60 hours or above. We replaced this by $[60, 70]$. We also construct an education variable as follows

$$e = \begin{cases} 1 & \text{if EDUC} = 6 \text{ (high school degree)} \\ 2 & \text{if EDUC} > 6 \text{ (some college).} \end{cases}$$

The variable INCWAGE is top coded. We remove the top-coded observations which are year-specific (and also state-specific for the year 2010.) We then convert INCWAGE to 2010 dollars using the Consumer Price Index provided by the Bureau of Labor Statistics (www.bls.gov). The 2010 value of a dollar in 1940, 1950,...,2010 are $\{15.58, 9.05, 7.37, 5.62, 2.65, 1.67, 1.27, 1.00\}$.

Building earnings profiles for synthetic cohorts

Define $\hat{\mathbf{E}}_{j,e,s,r,t}$ to represent the mean value of INCWAGE in age category j, education category e, sex category s, race category r in the census of year t. Define $\mathbf{E}_{j,e,s,r,c}$ to represent the mean value of earnings for cohort c. We define cohort by the census year during which a person is in age category 1. We then build earnings profiles for synthetic cohorts of worker as:

$$\begin{aligned} \mathbf{E}_{1,e,s,r,c} &= \hat{\mathbf{E}}_{1,e,s,r,c} \\ \mathbf{E}_{j+1,e,s,r,c} &= \hat{\mathbf{E}}_{j+1,e,s,r,c+10}. \end{aligned}$$

Similarly, let $\hat{\mathbf{W}}_{j,e,s,r,t}$ represent the mean value of INCWAGE/HOURS for a person in age category j, education category e, sex category s, race category r in the census of year t. We build earnings-per-hour profiles for synthetic cohorts of worker as:

$$\begin{aligned} \mathbf{W}_{1,e,s,r,c} &= \hat{\mathbf{W}}_{1,e,s,r,c} \\ \mathbf{W}_{j+1,e,s,r,c} &= \hat{\mathbf{W}}_{j+1,e,s,r,c+10}. \end{aligned}$$

We note that Kambourov and Manovskii (2009) use PSID data and also document the flattening of earnings profiles for cohorts of male workers entering the labor market in the late 1960s and later. Our work complements theirs since we consider cohorts entering the labor market since 1940. One difference, however, is that our use of Census data means that we construct synthetic cohorts, whereas they exploit the panel structure of the PSID.

Table A.1 shows $\mathbf{E}_{1,e,1,1,c}$ (age 25, white men) and $\mathbf{E}_{4,e,1,1,c}$ (age 55, white men) for the two education groups and all cohorts. The numbers in Table A.1 are the same as the one used in the construction of Figure 1.

Earnings growth statistics

Define earnings growth between age categories j and j' by:

$$\mathbf{G}^{E}_{e,s,r,c}(j, j') = \frac{\mathbf{E}_{j',e,s,r,c}}{\mathbf{E}_{j,e,s,r,c}}.$$

For earnings per hours, define

$$\mathbf{G}^{W}_{e,s,r,c}(j, j') = \frac{\mathbf{W}_{j',e,s,r,c}}{\mathbf{W}_{j,e,s,r,c}}.$$

Table A.1: Earnings at age 25 and 55 (2010 dollars) for high school and college workers

	Cohorts				
	1940	1950	1960	1970	1980
High school: age 25					
Earnings	15,102	22,341	29,465	35,619	31,715
Ratio relative to 1940	1.0	1.5	2.0	2.4	2.1
High school: age 55					
Earnings	54,480	52,017	51,270	52,213	47,334
Ratio relative to 1940	1.0	1.0	0.9	1.0	0.9
College: age 25					
Earnings	19,550	21,597	30,714	35,345	32,435
Ratio relative to 1940	1.0	1.1	1.6	1.8	1.7
College: age 55					
Earnings	78,226	71,730	80,882	87,717	85,747
Ratio relative to 1940	1.0	0.9	1.0	1.1	1.1

Source: IPUMS.

Figure A.9 and A.10 plot earnings (and earnings per hour) growth statistics (normalize to 1 for the 1940 cohort), from the age of 20-29 to the age of 50-59, by cohort, race, sex and education level. That is, for example, Figure A.1 plots

$$\frac{\mathbf{G}^E_{e,s,r,c}(\text{20-29}, \text{50-59})}{\mathbf{G}^E_{e,s,r,1940}(\text{20-29}, \text{50-59})},$$

for $c = 1940, 1950, \ldots$ and for all e, s and r. Similarly, Figure A.2 plots

$$\frac{\mathbf{G}^W_{e,s,r,c}(\text{20-29}, \text{50-59})}{\mathbf{G}^W_{e,s,r,1940}(\text{20-29}, \text{50-59})}.$$

Figures A.11 and A.12 consider earnings (and earnings per hour) growth between the age of 20-29 and 40-49. This opens up the possibility of considering one more cohort (the 1990 cohort.) Figures A.13 and A.14 consider earnings (and earnings per hour) growth between the age of 20-29 and 60-69. Figures A.15-A.20 plot the same statistics as Figures A.9-A.14. The difference is that 3 levels of education are considered. So, for the purpose of Figures A.15-A.20, the variable e is defined by

$$e = \begin{cases} 1 & \text{if EDUC} = 6 \text{ (high school degree)} \\ 2 & \text{if EDUC} \in [7, 10] \text{ (1-4 years of college completed)} \\ 3 & \text{if EDUC} = 11 \text{ (5+ years of college completed).} \end{cases}$$

Figure A.1: Growth of earnings from age 20-29 to age 50-59 by cohort

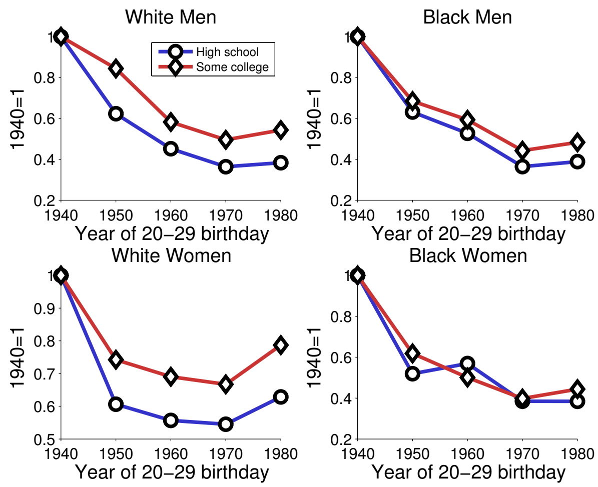


Figure A.2: Growth of earnings per hour from age 20-29 to age 50-59 by cohort

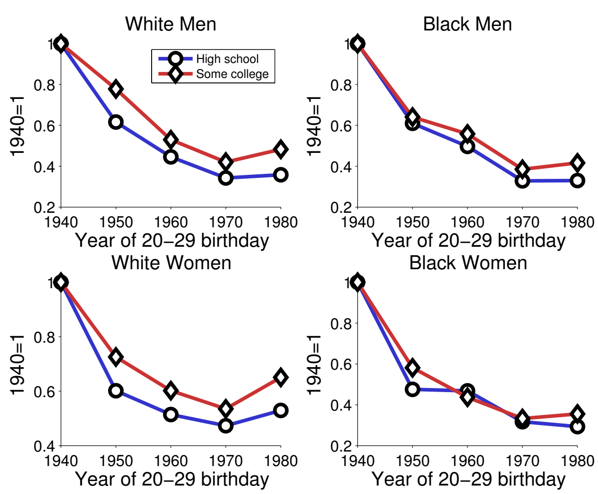


Figure A.3: Growth of earnings from age 20-29 to age 40-49 by cohort

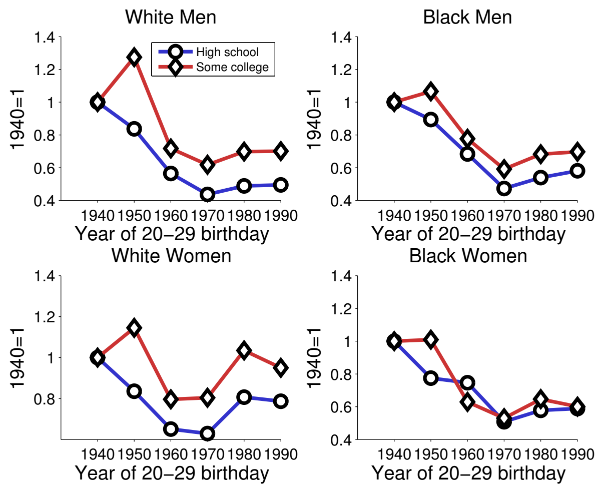


Figure A.4: Growth of earnings per hour from age 20-29 to age 40-49 by cohort

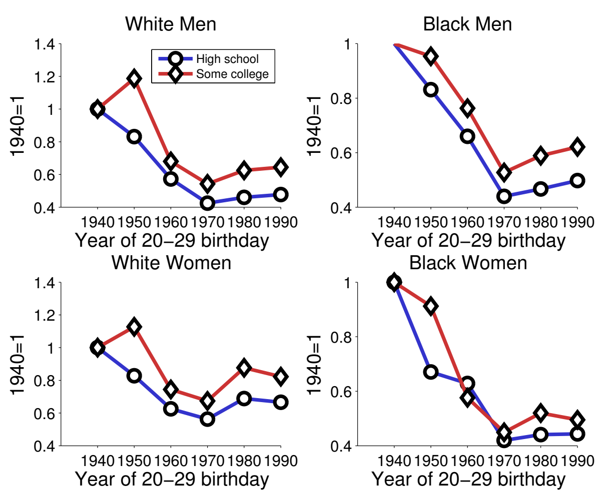


Figure A.5: Growth of earnings from age 20-29 to age 60-69 by cohort

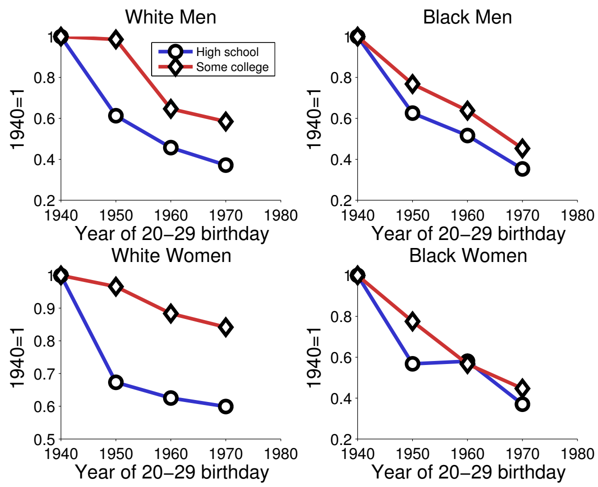


Figure A.6: Growth of earnings per hour from age 20-29 to age 60-69 by cohort

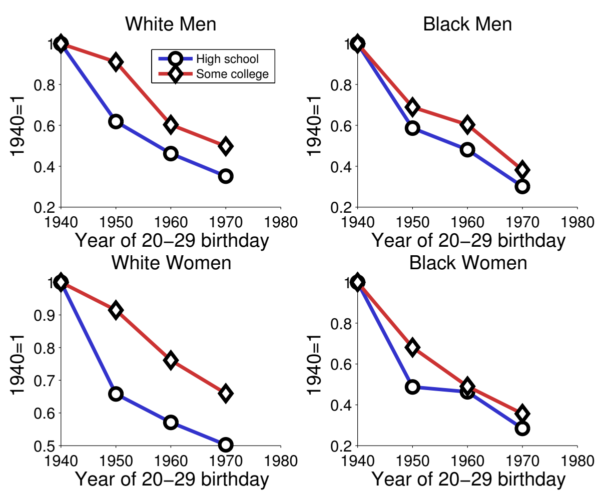


Figure A.7: Growth of earnings from age 20-29 to age 50-59 by cohort

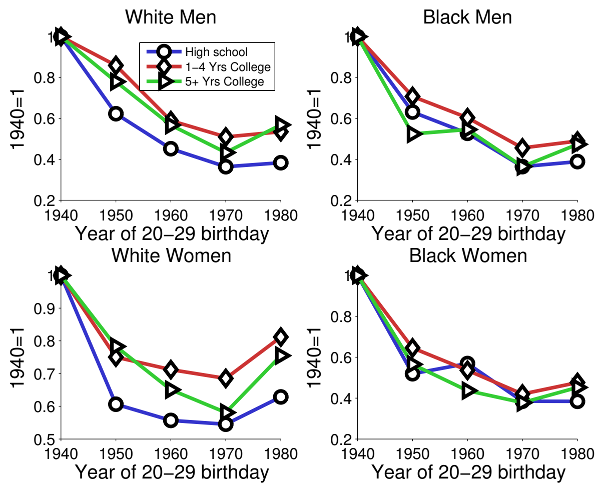


Figure A.8: Growth of earnings per hour from age 20-29 to age 50-59 by cohort

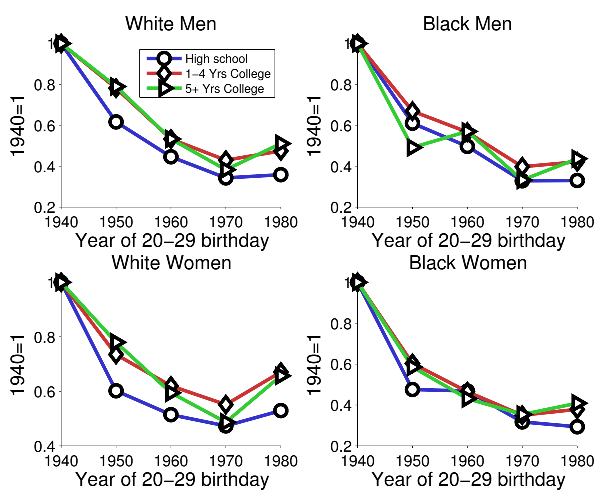


Figure A.9: Growth of earnings from age 20-29 to age 40-49 by cohort

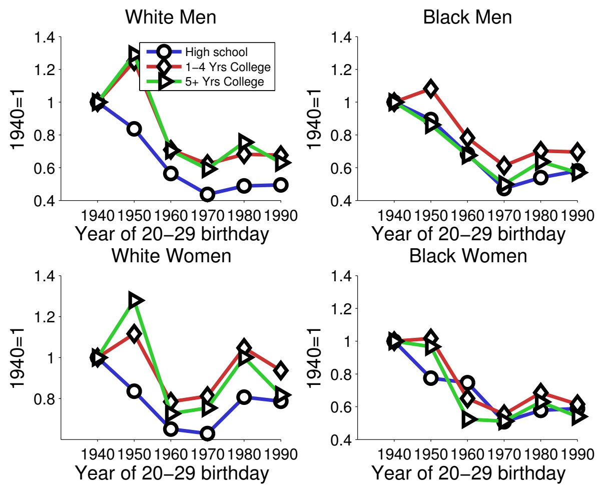


Figure A.10: Growth of earnings per hour from age 20-29 to age 40-49 by cohort

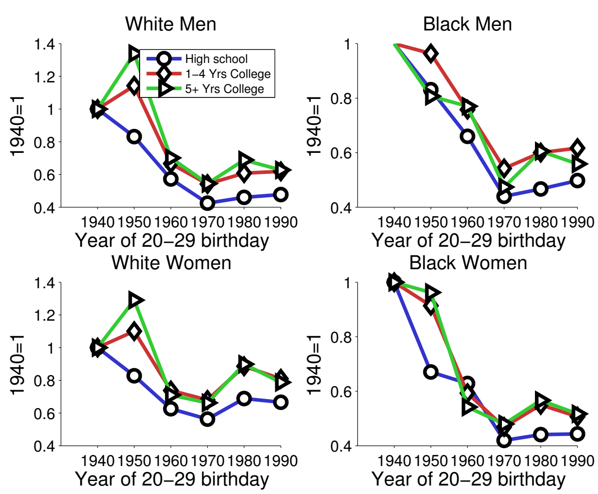


Figure A.11: Growth of earnings from age 20-29 to age 60-69 by cohort

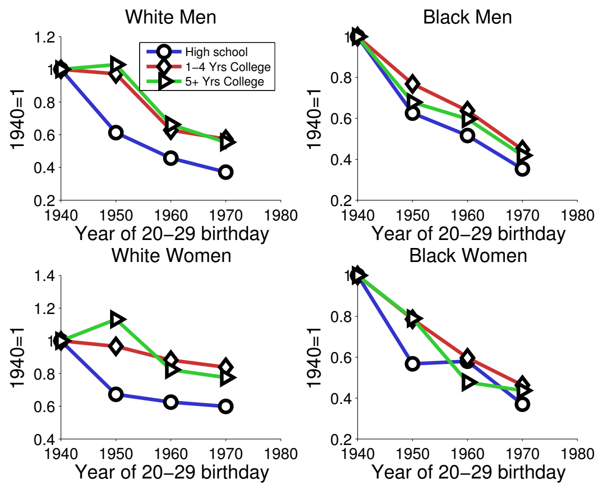
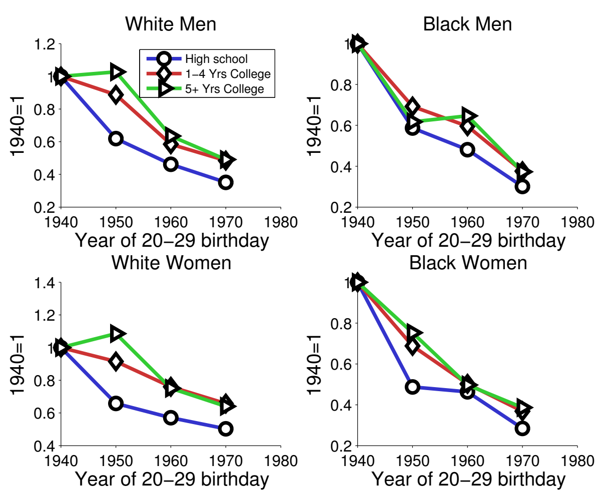


Figure A.12: Growth of earnings per hour from age 20-29 to age 60-69 by cohort



B Workers' Optimization

The optimization problem of a worker of age j and ability a at date t is

$$\begin{aligned} W_{j,t}(h,a) &= \max_n w_t h(1-n) + \frac{1}{r} W_{j+1,t+1}(h',a) \\ \text{s.t.} \quad & h' = (1-\delta)h + z_F a (nh)^{\phi}. \end{aligned}$$

Value Function

1. Age J
   It is immediate that

$$\begin{aligned} W_{J,t}(h,a) &= w_t h \\ &= \beta_{J,t} h + \alpha_{J,t} \end{aligned}$$

   where $\beta_{J,t} = w_t$ and $\alpha_{J,t} = 0$..

2. Age $J-1$
   The optimization problem reads

$$\max_n w_t h(1-n) + \frac{1}{r} w_{t+1} \left[(1-\delta) h + z_F a (nh)^{\phi} \right].$$

   At an interior solution, the solution for n satisfies the first-order condition: $w_t h = \frac{1}{r} w_{t+1} \phi z_F a n^{\phi-1} h^{\phi}$, implying

$$nh = \left[\frac{1}{r} \frac{w_{t+1}}{w_t} \phi z_F a \right]^{1/(1-\phi)}.$$

   Substituting into the objective function yields

$$W_{J-1,t}(h,a) = w_t h - w_t \left[\frac{1}{r} \frac{w_{t+1}}{w_t} \phi z_F a \right]^{1/(1-\phi)} + \frac{1}{r} w_{t+1} (1-\delta) h + \frac{1}{r} w_{t+1} z_F a \left[\frac{1}{r} \frac{w_{t+1}}{w_t} \phi z_F a \right]^{\phi/(1-\phi)}$$

   or,

$$\begin{aligned} W_{J-1,t}(h,a) &= h \left[w_t + \frac{1}{r} w_{t+1} (1-\delta) \right] + w_t \frac{1-\phi}{\phi} \left[\frac{1}{r} \frac{w_{t+1}}{w_t} \phi z_F a \right]^{1/(1-\phi)} \\ &= \beta_{J-1,t} h + \alpha_{J-1,t} \end{aligned}$$

   where

$$\begin{aligned} \beta_{J-1,t} &= w_t + \frac{1}{r} w_{t+1} (1-\delta), \\ \alpha_{J-1,t} &= w_t \frac{1-\phi}{\phi} \left[\frac{1}{r} \frac{w_{t+1}}{w_t} \phi z_F a \right]^{1/(1-\phi)}. \end{aligned}$$

3. Age j

Assume that $W_{j+1,t+1}(h,a) = \beta_{j+1,t+1}h + \alpha_{j+1,t+1}$. The optimization problem at age j and date t reads

$$\max_n w_t h(1-n) + \frac{1}{r}\beta_{j+1,t+1}\left((1-\delta)h + z_F a(nh)^{\phi}\right) + \frac{1}{r}\alpha_{j+1,t+1}$$

The first-order condition for n is $w_t h = \frac{1}{r}\beta_{j+1,t+1}\phi z_F a n^{\phi-1}h^{\phi}$, implying

$$nh = \left[\frac{1}{r}\frac{\beta_{j+1,t+1}}{w_t}\phi z_F a\right]^{1/(1-\phi)}.$$

Substituting into the objective function gives

$$\begin{aligned}W_{j,t}(h,a) = w_t h - w_t\left[\frac{1}{r}\frac{\beta_{j+1,t+1}}{w_t}\phi z_F a\right]^{1/(1-\phi)} \\ + \frac{1}{r}\beta_{j+1,t+1}(1-\delta)h + \frac{1}{r}\beta_{j+1,t+1}z_F a\left[\frac{1}{r}\frac{\beta_{j+1,t+1}}{w_t}\phi z_F a\right]^{\phi/(1-\phi)} \\ + \frac{1}{r}\alpha_{j+1,t+1}\end{aligned}$$

or,

$$\begin{aligned}W_{j,t}(h,a) &= \left(w_t + \frac{1}{r}\beta_{j+1,t+1}(1-\delta)\right)h + w_t\frac{1-\phi}{\phi}\left[\frac{1}{r}\frac{\beta_{j+1,t+1}}{w_t}\phi z_F a\right]^{1/(1-\phi)} + \frac{1}{r}\alpha_{j+1,t+1} \\ &= \beta_{j,t}h + \alpha_{j,t}\end{aligned}$$

where

$$\beta_{j,t} = w_t + \frac{1}{r}\beta_{j+1,t+1}(1-\delta), \tag{20}$$

$$\alpha_{j,t} = w_t\frac{1-\phi}{\phi}\left[\frac{1}{r}\frac{\beta_{j+1,t+1}}{w_t}\phi z_F a\right]^{1/(1-\phi)} + \frac{1}{r}\alpha_{j+1,t+1}. \tag{21}$$

Solving Equation (20) for $\beta_{j,t}$ yields

$$\beta_{j,t} = \sum_{\tau=0}^{J-j}\left(\frac{1-\delta}{r}\right)^{\tau} w_{t+\tau}. \tag{22}$$

Solving Equation (21) for $\alpha_{j,t}$ yields

$$\alpha_{j,t} = \sum_{\tau=0}^{J-j}\left(\frac{1}{r}\right)^{\tau} X_{j+\tau,t+\tau}, \tag{23}$$

where

$$X_{j,t} = w_t\frac{1-\phi}{\phi}\left[\frac{1}{r}\frac{\beta_{j+1,t+1}}{w_t}\phi z_F a\right]^{1/(1-\phi)}.$$

Constant Skill Price Growth

When w_t grows by the constant factor g each period (i.e., $w_{t+1} = gw_t$) Equations (22) and (23) become

$$\begin{aligned}
\beta_{j,t} &= w_t M_j, \\
\alpha_{j,t} &= w_t a^{1/(1-\phi)} N_j,
\end{aligned}$$

where

$$\begin{aligned}
M_j &= \sum_{\tau=0}^{J-j} \left(g \frac{1-\delta}{r} \right)^{\tau}, \\
N_j &= \frac{1-\phi}{\phi} \left(g \frac{\phi z_F}{r} \right)^{1/(1-\phi)} \sum_{\tau=0}^{J-j} \left(\frac{g}{r} \right)^{\tau} M_{j+\tau+1}^{1/(1-\phi)}.
\end{aligned}$$

C College Decision

Using the previous results, the value functions for high school and college can be written

$$\begin{aligned}
V_{1,t}^{\text{hs}}(a) &= \beta_{1,t} h_1(a) + \alpha_{1,t} \\
V_{1,t}^{\text{col}}(a) &= \max_k r^{-s} \beta_{s+1,t+s} (z_G k)^{\eta} (a h_1(a))^{1-\eta} + \alpha_{s+1,t+s} - k.
\end{aligned}$$

The first-order condition for k implies $k_t^* = \left[r^{-s} \beta_{s+1,t+s} \eta z_G^{\eta}\right]^{\frac{1}{1-\eta}} a h_1(a)$. Hence,

$$\begin{aligned}
V_{1,t}^{\text{col}}(a) = r^{-s} \beta_{s+1,t+s} &\left(z_G \left[r^{-s} \beta_{s+1,t+s} \eta z_G^{\eta} \right]^{\frac{1}{1-\eta}} a h_1(a) \right)^{\eta} (a h_1(a))^{1-\eta} \\
&+ \alpha_{s+1,t+s} - \left[r^{-s} \beta_{s+1,t+s} \eta z_G^{\eta} \right]^{\frac{1}{1-\eta}} a h_1(a)
\end{aligned}$$

or,

$$V_{1,t}^{\text{col}}(a) = \left(\frac{1}{\eta} - 1 \right) \left[r^{-s} \beta_{s+1,t+s} \eta z_G^{\eta} \right]^{\frac{1}{1-\eta}} a h_1(a) + \alpha_{s+1,t+s}. \tag{24}$$

An individual with ability a is indifferent between college and high school whenever $V_{1,t}^{\text{hs}}(a) = V_{1,t}^{\text{col}}(a)$ that is whenever

$$\left(\frac{1}{\eta} - 1 \right) \left[r^{-s} \beta_{s+1,t+s} \eta z_G^{\eta} \right]^{\frac{1}{1-\eta}} a h_1(a) + \alpha_{s+1,t+s} = \beta_{1,t} h_1(a) + \alpha_{1,t}.$$

When the skill price w_t grows by the constant factor g each period, this reads

$$a^{\phi/(1-\phi)} Z_1 + Z_2 = a w_t^{\eta/(1-\eta)} Z_3$$

where

$$\begin{aligned}
Z_1 &\equiv N_1 - g^s N_{s+1} > 0, \\
Z_2 &\equiv M_1 z_H > 0, \\
Z_3 &\equiv \frac{1-\eta}{\eta} \left[\eta r^{-s} g^s M_{s+1} z_G^{\eta} \right]^{1/(1-\eta)} z_H > 0.
\end{aligned}$$

D Model with Education-Specific Skill Prices

The present value of earnings for an age-j worker with ability a, human capital h in period t, and education $i \in \{\text{hs}, \text{col}\}$, denoted by $W^i_{j,t}(h,a)$, is:

$$\begin{aligned} W^i_{j,t}(h,a) &= \max_n w^i_t h(1-n) + \frac{1}{r} W_{j+1,t+1}(h',a) \\ \text{s.t} \quad & h' = (1-\delta)h + F(nh,a), \\ & W^i_{J+1,t+1} = 0. \end{aligned}$$

The value of a high school education is, therefore,

$$V^{\text{hs}}_{1,t}(a) = W^{\text{hs}}_{1,t}(h_1(a),a),$$

where $h_1(a)$ is the initial human capital for an individual with ability a. The value of a college education is

$$V^{\text{col}}_{1,t}(a) = \max_k \frac{1}{r^s} W^{\text{col}}_{s+1,t+s}\left(G(k,h_1(a),a),a\right) - k,$$

where G is the technology for accumulating human capital in college. The decision of whether to attend college or start working at age 1 is determined by

$$\max_{\text{hs,col}} \left\{V^{\text{hs}}_{1,t}(a), V^{\text{col}}_{1,t}(a)\right\}.$$

Table D.1: Calibrated parameters for model with education-specific skill prices

Ability distribution	$\psi_0 = 2.08$, $\psi_1 = 0.64$, $\psi_2 = 0.14$
Initial human capital	$z_H = 1.58$
College technology	$\eta = 0.31$, $z_G = 0.08$
On-the-job technology	$\phi = 0.56$, $z_F = 0.07$
Skill price process	$w^{\text{col}}_{1940} = 1.29$
	$g^{\text{col}}_1 = 0.010$, $g^{\text{col}}_2 = -7.04 \times 10^{-5}$
	$g^{\text{hs}}_1 = 0.013$, $g^{\text{hs}}_2 = -9.78 \times 10^{-5}$
Life expectancy, college length	$J = 50$, $s = 4$
Interest rate, depreciation	$r = 1.050$, $\delta = 0.011$

Table D.2: Calibration targets: model with education-specific skill prices and data

	Model	Data	Model	Data
	High School		College	
1940 Cohort				
Annual earnings growth 25-35 (%)	7.7	7.3	6.6	6.5
Annual earnings growth 25-45 (%)	5.9	5.8	5.2	6.0
Annual earnings growth 25-55 (%)	4.8	4.4	4.2	4.7
Coef. of variation at 35	0.50	0.41	0.43	0.49
Coef. of variation at 45	0.55	0.47	0.43	0.55
Coef. of variation at 55	0.57	0.53	0.43	0.62
Time series				
Annualized earnings growth of 25-year-old, 1940-1980 (%)	1.1	1.9	1.5	1.3

Source: IPUMS and authors' calculations.

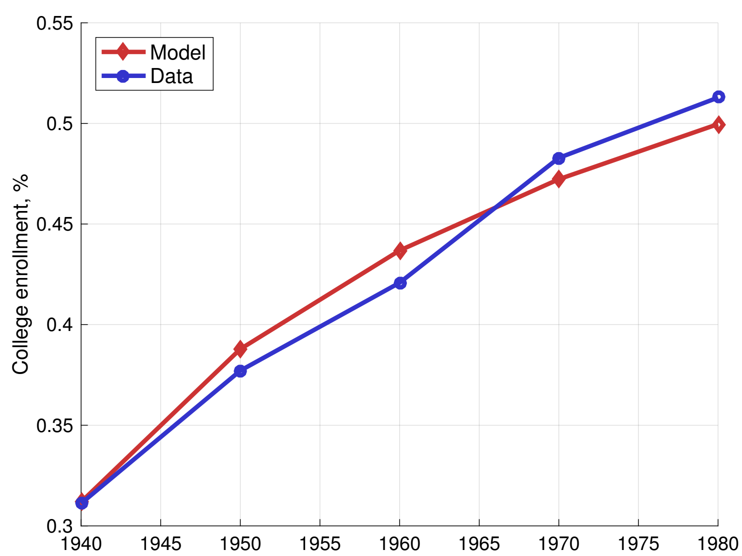


Figure D.1: College enrollment: model with two skill prices and data

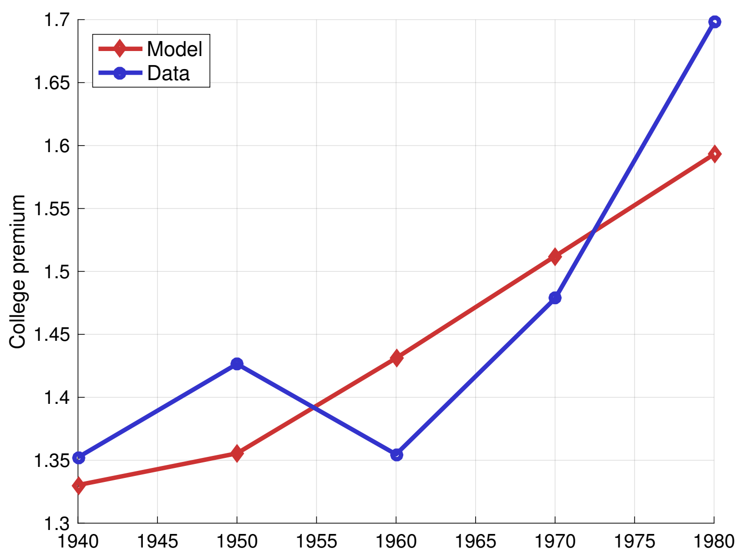


Figure D.2: College premium: model with two skill prices and data