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Nominal Exchange Rate Determinacy Under the Threat of Currency Counterfeiting*

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Abstract

We study the endogenous choice to accept fiat objects as media of exchange and their implications for nominal exchange rate determination. We consider a two-country environment with two currencies which can be used to settle any transactions. However, currencies can be counterfeited at a fixed cost and the decision to counterfeit is private information. This induces equilibrium liquidity constraints on the currencies in circulation. We show that the threat of counterfeiting can pin down the nominal exchange rate even when the currencies are perfect substitutes, thus breaking the Kareken-Wallace indeterminacy result.

JEL classication: D82, D83, E4, F4.

Keywords: Multiple Currencies, Counterfeiting Threat, Liquidity, Exchange Rates.

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1 Introduction

When agents have unrestricted access to currency markets and are free to use any currency as means of payment, Kareken and Wallace (1981) showed that the rate of return on the two currencies must be identical for both of them to circulate, e.g., they are perfect substitutes.[1] However, in this case the nominal exchange rate between these currencies is indeterminate.

Since Kareken and Wallace (1981), most standard models of international monetary economics have considered various departures from their original frictionless environment to generate determinate nominal exchange rates. These include currencies in the utility function, imposing restrictions on the use of currency for certain transactions, assuming differential transaction costs, or having differential terms of trade depending on the currency that is being used. By considering *asymmetric* treatment of currencies, the nominal exchange rate is determined as currencies then become imperfect substitutes. Our objective is to resolve the nominal exchange rate indeterminacy problem while treating the two currencies *symmetrically*.

In this paper we study the endogenous choice to accept different fiat objects as media of exchange, the fundamentals that drive their acceptance and the implications for their bilateral nominal exchange rate. In particular, agents in this economy have no restrictions on what divisible fiat currency can be used to settle transactions. The key aspect of our approach is that we introduce currency fraud as in Li, Rocheteau, and Weill (2012). More precisely, we allow both fiat currencies to be counterfeited at a fixed cost. Potential sellers of goods cannot distinguish genuine from counterfeit currencies; and, because of this private information problem, in equilibrium, they put a limit on how much of each currency they are willing to accept. The limits are chosen such that counterfeiting does not occur in equilibrium. The liquidity constraints are endogenous and depend on 1) the relative inflation rates and 2) the cost of counterfeiting. If one of the constraints is binding, the buyer spends all of that currency and uses some of the other currency to "top off" their desired purchase of goods. When both limits bind, the buyer gives up all of his currency holdings to purchase goods.

The most interesting case we study is when the inflation rates and the costs of counterfeiting are exactly the same for each currency. In this case, the only difference between the two currencies is a non-fundamental attribute, such as the color of the currencies. This corresponds exactly to the conditions underlying the Kareken and Wallace indeterminacy result. Nevertheless, in this case, we show that if the cost of counterfeiting is sufficiently low such that the limits bind, then the nominal exchange rate is determinate and equal to the ratio of the two money stocks. This is interesting because it is the same equilibrium

[1]When currencies are identical in every respect, they are perfect substitutes. Consequently, the composition of currency portfolios is indeterminate for all agents.

exchange rate that comes out of a standard two-country cash-in-advance model where domestic (foreign) goods must be purchased with domestic (foreign) currency. However, in our model, agents use both currencies to purchase goods. If the constraints do not bind, the nominal exchange rate is indeterminate.

While our model closely follows Li, Rocheteau, and Weill (2012) in terms of the counterfeiting problem, it differs in two key respects. First, they study counterfeiting of *real* assets and thus are able to use a finite horizon model. With fiat currencies an infinite horizon is required, which raises issues about the durability of counterfeit currency, which in Li et al. (2012) is not a concern as the economy ends in the same period that counterfeits are detected.[2] Moreover, because Li et al. (2012) study real assets, relative price indeterminacy does not occur, whereas this is obviously critical when analyzing fiat currencies.

Finally, allowing for counterfeiting is not standard in international monetary economics. While we have changed the environment by introducing a game of private information, we do so *symmetrically* such that no currency has a counterfeiting advantage over the other. After we present our main results, we describe how differing counterfeiting costs affect the equilibrium of the model.

Section 2 reviews the literature and Section 3 describes the model environment. Section 4 defines the monetary equilibrium and the implications of exchange rate determinacy. In Section 5, we present more general results for the cases when inflation rates and counterfeiting costs differ. Finally, Section 6 offers some concluding remarks. All proofs are given in the Supplementary Appendix.

2 Related literature

Models in mainstream international monetary economics typically pin down the value of a currency by imposing asymmetric assumptions on what objects may be used as media of exchange. For instance, Stockman (1980) and Lucas (1982), among others, assume that in order to buy a good produced by a particular country, the producing country's currency must be used to pay for goods. That is, in these environments, the demand for a specific fiat currency is solely driven by the demand for goods produced by that particular country. Other researchers have introduced local currency in the utility function as in Obstfeld and Rogoff (1984) and Devereux and Engel (2003) or have assumed differential trading cost advantages through network externalities as in Uribe (1997). Devereux and Shi (2013) study a trading post model under the assumption that there is only bilateral exchange at each trading post. Assumptions of this sort determine which currency is

[2]Li et al. (2012) consider an infinite horizon fiat money model in their original working paper. However, there is only 1 currency and they make the same assumptions that we do regarding the durability of counterfeits.

used to pay for goods. Consequently, they yield determinacy in agents' portfolio holdings of any two fiat currencies and therefore determinacy of the nominal exchange rate.

Another strand of the international finance literature studies endogenous currency pricing as in Gopinath et al (2010). In this literature firms choose whether to price their goods in domestic currency or foreign currency. The choice of pricing, in conjunction with sticky prices, has implications for the exchange rate movements. On the face of it, this sounds as if the firm is choosing which currency to accept as payment. This is not the case – the choice of currency *pricing* is not equivalent to the choice of currency *payment*. To illustrate, a firm could choose to price in dollars rather than euros and post a price of $1 for a unit of goods. But this does not say the firm *accepts only* dollars as payment. The firm may: 1) accept $1 as payment, 2) accept the foreign currency equivalent at the prevailing exchange rate, or 3) some combination of the two. Clearly, fluctuations in the exchange rate affect the quantity of the foreign currency that is paid but the firm is always receiving the equivalent payment of $1. This is exactly what Kareken and Wallace (1981) do – firms price in domestic currency but accept either currency as payment. For example, in New Keynesian international macroeconomic models, such as Clarida, Gali, and Gertler (2002), firms price in domestic currency. In these models the response of the nominal exchange to shocks in the economy is well defined. However, the steady state nominal exchange rate is indeterminate since there are no restrictions on currency payments.[3] So the choice of currency pricing in and of itself has no implications for nominal exchange rate determinacy.

In the early search theoretic models of money, agents are able to choose which currencies to accept and use for payment. This literature shows that multiple currencies can circulate even if one is dominated in rate of return and the nominal exchange rate is determinate [see Matsuyama, Kiyotaki, and Matsui (1993), Zhou (1997), Wright and Trejos (2001), Waller and Curtis (2003), Craig and Waller (2004), Camera, Craig, and Waller (2004)]. In these models, currency exchange can occur in bilateral matches if agents' portfolios are overly weighted towards one currency or the other. In fact, this leads to a distribution of determinate nominal exchange rates. However, these findings are driven solely by the decentralized nature of exchange, since agents never have access to a centralized market to rebalance their portfolios. Once agents have the ability to rebalance their currency holdings, be it by the large family assumption as in Shi (1997) or the periodic centralized market structure as in Lagos and Wright (2005), nominal exchange rates are indeterminate as in Kareken and Wallace (1981). To resolve this problem, Head and Shi (2003) consider an environment where the large household can hold a portfolio of currencies but individual buyers are constrained to hold only one currency. So although the household endogenously chooses a portfolio of currencies, bilateral exchange requires us-

[3] In the international New Keynesian models, the assumption of local currency pricing is often imposed. However, this does not imply payment has to be made in the local currency.

ing one currency or the other, but not both simultaneously. Similarly, Liu and Shi (2010) assume that buyers can offer any currency but sellers can accept only one currency. The main contribution of our paper relative to the previous search literature is that we include Walrasian markets with centralized exchange without assuming restrictions on currency exchange or differential pricing protocols—and yet we can obtain nominal exchange rate determinacy, even when the currencies are perfect substitutes.

The work closest in spirit to ours are Chapter 10.2 in Nosal and Rocheteau (2011) book and Zhang (2014). These authors have environments where private agents can freely choose which currency to transact with when trading among themselves. Nosal and Rocheteau (2011) consider a two country variation of the Lagos and Wright (2005) framework and consider a trading mechanism in decentralized markets whereby a buyer obtains better terms of trade in a country by using the domestic currency rather than the foreign one. Despite the asymmetric treatment of the currencies, there are no unexploited gains from trade in DM so that the outcome is pairwise Pareto efficient and nominal exchange rates are determined. In contrast to these authors, we consider an alternative trading protocol that does not give an advantage of one currency over the other. Finally, Zhang (2014) considers an open economy search model with multiple competing currencies and governments that require transactions to be made in a local currency. Buyers can always costlessly produce counterfeit currencies while sellers face a recognizability problem, as in Lester, Postlewaite, and Wright (2012). The recognizability problem is only in terms of foreign currencies, thus treating currencies asymmetrically and giving one currency an advantage over the other. In this paper sellers face a counterfeiting problem for both domestic and foreign currencies.

3 Model

Environment Consider a two-country model with each country having a non-tradable and tradable sector. Each country is labeled as either Home or Foreign and has a continuum of agents of measure 2. Time is discrete and indexed by $t \in \mathbb{N} := \{0, 1, 2, ...\}$. All agents discount the future at rate $\beta < 1$. For ease of understanding, we will describe the model environment from the perspective of the Home country keeping in mind that there is a symmetric Foreign counterpart.

As in Lagos and Wright (2005), in each country there is a sequential decentralized-then-centralized market (DM-CM) trading structure every period. In the DM, a perishable good is traded between home agents only.[4] As a result, we refer to this market as

[4]The restriction that only the home agents trade in the DM is for notational and presentational simplicity. A richer environment where foreign agents can trade with domestic one in DM does not change the private information game between buyers and sellers that is at the heart of our results. A formal argument that this is the case is shown in the online Appendix.

the non-tradable sector as in Gomis-Porqueras, Kam and Lee (2013). Traders in this market are either consumers of the DM good or producers of the DM good. DM types are permanent and both have measure 1. Consumers receive utility $u(q)$ from consuming q units of the DM good with $u(0) = 0$, $u'(q) > 0$ and $u''(q) < 0$. Producers incur the disutility cost $c(q)$ from producing q units of the DM good with $c(0) = 0$, $c'(q) > 0$, and $c''(q) \geq 0$. The first best quantity $q^* \in (0, \infty)$ satisfies $u'(q^*) = c'(q^*)$. Consumers and producers meet pairwise via random matching and bargain over the terms of trade.

Since all agents are anonymous in the DM, credit is not feasible. Hence, a medium of exchange is essential for trade in the DM. Towards this end, consumers can use either the home currency and/or foreign currency to pay for q. We label consumers in the DM as "buyers" and producers as "sellers". As will be discussed in more detail below, buyers have the ability to counterfeit each currency before trading in the DM and sellers do not have the ability to distinguish between counterfeit currency and genuine currency as in Li, et al. (2012). Through out the rest of the paper we assume that buyers can commit to the terms of trade prior to the counterfeiting decision. Solving this recognizability problem, characterizing the resulting allocations and explaining how nominal exchanges can be determined are the key contributions of our paper.

In the CM, both Home and Foreign agents can produce and consume a homogenous perishable good that is traded internationally in a perfectly competitive, frictionless market. Home agents get utility $\mathcal{U}(C)$ from consuming C units of the CM good and incur disutility $-N$ from producing N units of the CM good. Since this market is frictionless, agents can consume and produce the CM good and they are not anonymous, currencies in this market are not essential for exchange. Agents can adjust their currency portfolios in the CM by selling goods for cash, buying goods to reduce cash holdings, or trading currencies directly. In this sense, we refer the CM the tradable sector as in Gomis-Porqueras et al. (2013) which corresponds to the tradable sector in any standard international macro model. The goods price of domestic currency in the CM is denoted by $\phi = 1/P$ where P is the Home currency price of the CM good. Let e be the current nominal exchange rate which measures the value of one unit of Foreign currency (f) in units of the Home currency. Since there is frictionless trade in the CM, the law of one price holds. Thus we have that $P = eP^f$, where P^f is the Foreign currency price of the CM good. We can rewrite this as $\phi^f = e\phi$.

Finally, in order to make a meaningful distinction between genuine and counterfeit currencies, we follow the approach of Nosal and Wallace (2007) and assume that all counterfeit currency disintegrates at the end of the DM.[5] This makes it costly for DM-sellers to accept counterfeit currency as payment. Although this is an extreme assumption, it

[5]Alternatively, we could assume that all counterfeits are detected in the CM, confiscated, and destroyed. Our assumption allows us to avoid a discussion of the detection technology and ways of avoiding it.

simplifies the analysis greatly.

In the date-t CM, the DM-buyers in each country commit to a terms of trade in anticipation of the following DM trade, and, make their counterfeiting decisions. This is done before they enter their own country's DM in date $t+1$. Hereinafter, we denote these sequential markets, respectively, as CM(t) and DM($t+1$). Counterfeiting is costly, requiring a fixed disutility cost of $-\kappa$ to counterfeit the Home currency and $-\kappa^f$ to counterfeit the Foreign currency. While these costs can be different in general, for now we impose $\kappa = \kappa^f$ to eliminate any differences in counterfeiting costs. We will expand on the details of the counterfeiting decision later.[6]

Monetary Policy Let M and M^f denote the stock of Home and Foreign currency, respectively. We assume that the supply of the fiat currencies grow at a constant rate of γ and γ^f respectively. Lump sum transfers of Home and Foreign currencies, are made to the respective country's DM-buyers at the beginning of each CM $\tau_t = M_{t+1} - M_t = (\gamma - 1)\gamma^{t-1}M_0$ and $\tau_t^f = M_{t+1}^f - M_t^f = (\gamma^f - 1)(\gamma^f)^{t-1}M_0^f$, respectively.[7] The initial stocks M_0 and M_0^f are known. The gross real returns on each currency from CM(t) to CM($t+1$) are given by $R_t = \phi_{t+1}/\phi_t = P_t/P_{t+1} = 1/\Pi_t$ and $R_t^f = 1/\Pi_t^f$, where Π_t and Π_t^f are the CM inflation rates at home and abroad.

As with the counterfeiting costs, in general, the growth rates of the two currencies can be different; but for the purposes of obtaining our main result, we will impose $\gamma = \gamma^f$. This will ensure in equilibrium that the rates of return on the two currencies are the same.

3.1 Decentralized Market

At the beginning of each DM, a seller s is randomly matched with a buyer b. The buyer enters with a portfolio $A := \left(a, a^f\right)$ consisting of non-negative amounts of genuine Home currency, Foreign currency, and counterfeits, where $a = m + z$ is the sum of genuine and counterfeit Home currency and $a^f = m^f + z^f$ is the sum of genuine and counterfeit Foreign currency. The buyer makes a take-it-or-leave-it offer (TIOLI) $\omega := (q, d, d^f)$ that specifies the quantity q that a seller must produce in DM in exchange for d units of the

[6]We refer the reader to the manuscript in the Federal Reserve Bank of St Louis Working Paper series with No. 2015-028A to see how the nominal exchange rate is determined in a closed economy environment with two currencies.

[7]Observe that no transfers are made to DM-sellers as they would have no use for it in the immediate DM. These DM-sellers would just bring the additional money transfer with zero inflation cost into the CM, since this occurs within the same period. In turn, they can afford to work less in the CM without altering their consumption allocation since they have quasilinear preferences. Thus it is without loss to our result that we assume that transfers are made to DM-buyers only. Also, if we were to assume that seigniorage revenues were transferred to agents at the start of the CM, the effect of money supply growth would just wash out in the CM in terms of having any real effects since with quasilinear preferences, all agents would just work less in the CM.

Home currency (genuine and counterfeit) and d^f units of the Foreign currency (genuine and counterfeit).[8] It follows that $d = m_d + z_d$ and $d^f = m_d^f + z_d^f$. Feasibility requires $d \leq a$ and $d^f \leq a^f$.

The seller cannot recognize whether the buyer is offering genuine fiat currencies or not. Thus she must assign a belief as to the genuineness of the currencies. The seller chooses the probability of accepting the offer. The seller's problem is

$$\max_{\pi(\omega)} \pi(\omega) \left[-c(q) + W^s\left(\hat{\eta}(\omega)d, \hat{\eta}^f(\omega)d^f\right)\right] + (1 - \pi(\omega)) W^s(0, 0),$$

where $\hat{\eta}(\omega)$ and $\hat{\eta}^f(\omega)$ are the seller's beliefs that the Home and Foreign currencies are genuine and W^s denotes the CM value function for the seller.

Given an offer and counterfeiting strategy $(\omega, \eta(\omega), \eta^f(\omega))$ determined in a preceding CM, and given the buyer's belief $\hat{\pi}(\omega)$ about a seller's probability of accepting his offer, the induced beginning-of-DM value to a buyer with portfolio (a, a^f) is

$$\begin{aligned} V^b\left(a, a^f\right) = \sigma\, \hat{\pi}\left[u(q) + W^b\left[\eta(\omega)(a - d), \eta^f(\omega)\left(a^f - d^f\right)\right]\right] \\ + (1 - \sigma\widehat{\pi}(\omega))\; W^b\left[\eta(\omega)a, \eta^f(\omega)a^f\right]. \end{aligned} \tag{1}$$

Since DM sellers have no need for currency in the DM, their asset holdings are zero. Thus, the value function for a DM-seller is given by

$$V^s(0,0) = \max_{\pi(\omega)} \left\{\sigma\, \pi(\omega)\left[-c(q) + W^s\left(\hat{\eta}(\omega)\, d, \hat{\eta}^f(\omega)\, d^f\right)\right] + (1 - \sigma\pi(\omega)) W^s(0, 0)\right\}, \tag{2}$$

where $\pi(\omega)$ is the probability the seller accepts an offer ω from a randomly encountered buyer and $W^s(0, 0)$ is the payoff from walking away from a trade.

3.2 Centralized Market

To simplify exposition of the model we suppress definitions of strategies and the strategy space and refer the reader to our online working paper (Gomis-Porqueras et al., 2015) for more details. In what follows we denote period $t - 1$ variables with the subscript -1 and so on.

The DM-buyer's problem in the period t CM is given by

$$W^b\left(m, m^f\right) = \max_{C, N, \eta(\omega), \eta^f(\omega), a_{+1}, a_{+1}^f} \left[\mathcal{U}(C) - N - \kappa(1 - \eta(\omega)) - \kappa^f(1 - \eta^f(\omega)) + \beta V^b(a_{+1}, a_{+1}^f)\right]$$

$$s.t.\;\; C + \phi m_{+1} + \phi e m_{+1}^f = N + \phi m + \phi e m^f + \phi\tau,$$

$$(a_{+1}, a_{+1}^f) = \left(m_{+1} + z_{+1}, m_{+1}^f + z_{+1}^f\right),$$

[8] In our online Appendix, we also consider an alternative proportional bargaining protocol due to Kalai and Smorodinsky (1975). Our main result is robust to this change of trading protocol.

where m and m^f are the quantities of genuine currency brought into the CM, m_{+1} and m^f_{+1} are the quantities of genuine currencies acquired in the CM, and z_{+1} and z^f_{+1} are the quantities of counterfeits of each currency taken into $t+1$. If the buyer does not counterfeit then $z_{+1} = z^f_{+1} = 0$. Genuine *real* balances of home and foreign currency are given by ϕm and $\phi e m^f$, respectively.

The seller entering the CM with $\left(m_s, m^f_s\right)$ quantities of genuine currency faces the following problem

$$W^s\left(m_s, m^f_s\right) = \max_{C,N}\left[\mathcal{U}(C) - N + \beta V^s(0,0)\right] \quad s.t. \quad C = N + \phi m_s + \phi e m^f_s.$$

Substituting the budget constraint to eliminate N, we have that the CM value functions are linear in currency holdings brought into the CM. In particular, we have that

$$\begin{aligned} W^b(m, m^f) &= \phi(m + e m^f + \tau) + W^b(0,0), && (3)\\ W^s\left(m_s, m^f_s\right) &= \phi(m_s + e m^f_s) + W^b(0,0). && (4) \end{aligned}$$

It follows that the buyer's and seller's CM consumption satisfies $\mathcal{U}'(C^*) = 1$. Thus C^* is independent of the portfolio choice. Using (1) with (3) updated one period and rearranging terms yields

$$\begin{aligned} \max_{\omega, m_{+1}, m^f_{+1}, \eta(\omega), \eta^f(\omega)} \quad & \Bigg[\mathcal{U}(C^*) - \phi(m + e m^f + \tau) + \beta W^b(0,0) - \left(\frac{\phi}{\phi_{+1}} - \beta\right)\phi_{+1} m_{+1} \\ & - \left(\frac{\phi e}{\phi_{+1} e_{+1}} - \beta\right)\phi_{+1} e_{+1} m^f_{+1} - \kappa(1 - \eta(\omega)) - \kappa^f(1 - \eta^f(\omega)) \quad (5)\\ & + \beta\sigma\hat{\pi}_{+1}(\omega)\left(u(q_{+1}) - \eta(\omega)\phi_{+1} d_{+1} - \eta^f(\omega)\phi_{+1} e_{+1} d^f_{+1}\right)\Bigg]. \end{aligned}$$

The fourth and fifth term of this equation show the expected holding cost (equivalently inflation cost) of acquiring currencies in t for use in $t+1$. The marginal cost of acquiring a unit of genuine home currency (in real terms) is $\phi_{-1}/\phi - \beta > 0$ and $\phi_{-1} e_{-1}/\phi e - \beta > 0$ for a unit of real foreign currency. The rest of the terms are the expected total fixed cost of counterfeiting both currencies. In the third line we have the expected surplus from DM trade.

Similarly for the seller we have that

$$\begin{aligned} W^s\left(m_s, m^f_s\right) &= \mathcal{U}(C^*) - \phi(m_s + e m^f_s) + \beta W^s(0,0) + && (6)\\ &+ \max_{\pi_{+1}(\omega)}\left[\beta\sigma\, \pi_{+1}(\omega)\left(-c(q_{+1}) + \hat{\eta}(\omega)\phi_{+1} d_{+1} + \hat{\eta}^f(\omega)\phi_{+1} e_{+1} d^f_{+1}\right)\right]. \end{aligned}$$

3.3 Counterfeiting Game

Since the seller is unable to distinguish between counterfeits and genuine currency, whereas the buyer knows what he holds, we have a private information bargaining game. In the environment we examine, buyers are able to post offers in the CM and commit to honoring them in the ensuing DM. Based on these posted offers, the buyer first chooses the probability of counterfeiting. Then the buyer decides on her portfolio, $A(\omega)$ and CM consumption and effort, C and N. After buyers and sellers are matched in the DM, the seller can choose to trade with the buyer at the posted offer.[9] Below we describe the sequence of events in the counterfeiting game that buyers play with sellers.

1. A DM-buyer announces a TIOLI offer $\omega := (q_{+1}, d_{+1}, d^f_{+1})$ and commits to ω before making any other decisions in CM(t).
2. The buyer chooses the probability of counterfeiting the fiat currencies, $1-\eta(\omega), 1-\eta^f(\omega) \in [0,1]$.
3. The buyer then decides the portfolio $A_{+1}(\omega)$ and CM consumption and effort C and N.
4. The buyer enters DM($t+1$) and Nature randomly matches the buyer with a DM-seller with probability σ.
5. The DM-seller trades according to the proposed offer ω with probability $\pi(\omega) \in [0,1]$.

3.3.1 Solving the Private Information Game

In order to solve the game we proceed by backward induction from DM($t+1$) to CM(t). Again, we focus on the events in the Home country as similar conditions can be derived for the Foreign country.

In the last stage of the game, in DM($t+1$), the seller maximizes expected profit by playing a mixed strategy π, where

$$\pi_{+1}(\omega) \in \left\{ \arg \max_{\pi_{+1}(\omega)\in[0,1]} \pi_{+1}(\omega) \left[-c(q_{+1}) + \phi \left(\hat{\eta}(\omega) d_{+1} + \hat{\eta}^f(\omega) e_{+1} d^f_{+1} \right) \right] \right\}, \qquad (7)$$

taking as given a buyer's offer ω and the seller's beliefs about the buyer's counterfeiting probabilities, $(\hat{\eta}(\omega), \hat{\eta}^f(\omega))$.

[9] This extensive-form game is the same as the endogenous signaling game in Li et al (2012) when the reordering-invariant refinement proposed by In and Wright (2011) is used to construct the equilibrium. We thank one of the referees for making this point.

In the penultimate stage, in CM(t), the buyer chooses the counterfeiting lottery $(\eta(\omega), \eta^f(\omega))$ to solve the following cost-minimization problem

$$\Big\{-\kappa(1-\eta(\omega)) - \kappa^f(1-\eta^f(\omega)) - \beta\sigma\hat{\pi}_{+1}(\omega)\left(\eta(\omega)\phi_{+1}d_{+1} + \eta^f(\omega)\phi_{+1}e_{+1}d^f_{+1}\right) \\ - \left(\frac{\phi}{\phi_{+1}} - \beta\right)\phi_{+1}m_{+1} - \left(\frac{\phi e}{\phi_{+1}e_{+1}} - \beta\right)\phi_{+1}e_{+1}m^f_{+1}\Big\}, \tag{8}$$

given his earlier commitment ω and his beliefs about a seller's acceptance probability $\hat{\pi}(\omega)$.

The buyer chooses a TIOLI offer at the beginning of the game to maximize his payoff given the conjecture $(\hat{\eta}(\omega), \hat{\eta}^f(\omega), \hat{\pi}(\omega))$ of the continuation play. The buyer commits to an optimal offer $\omega:=(\hat{q}_{+1}, \hat{d}_{+1}, \hat{d}^f_{+1})$ to maximize

$$\Big\{-\kappa\left(1-\hat{\eta}(\omega)\right) - \kappa^f\left(1-\hat{\eta}^f(\omega)\right) + \beta\sigma\hat{\pi}_{+1}(\omega)\left[u(\hat{q}_{+1}) - \phi\left(\hat{\eta}(\omega)\hat{d}_{+1} + \hat{\eta}^f(\omega)e_{+1}\hat{d}^f_{+1}\right)\right] \\ - \left(\frac{\phi}{\phi_{+1}} - \beta\right)\phi_{+1}m_{+1} - \left(\frac{\phi e}{\phi_{+1}e_{+1}} - \beta\right)\phi_{+1}e_{+1}m^f_{+1}.\Big\}. \tag{9}$$

3.3.2 Equilibrium in the game

Having specified the seller's and buyer's respective problems, we can now characterize the resulting equilibrium in the private-information bargaining game in each country. We show this characterization for the Home country.[10] From here on, for ease of notation, we will lag the buyer's and seller's events by one period.

Proposition 1 *An equilibrium of the counterfeiting-bargaining game is such that*

1. *Each seller accepts with probability* $\hat{\pi}(\omega) = \pi(\omega) = 1$*;*

2. *Each buyer does not counterfeit:* $(\hat{\eta}(\omega), \hat{\eta}^f(\omega)) = (\eta(\omega), \eta^f(\omega)) = (1, 1)$*; and*

[10] A symmetric description can be written out for the Foreign country.

3. Each buyer's TIOLI offer $\omega := (q, d, d^f)$ *attains*

$$\max_{q,d,d^f} \left[-\left(\frac{\phi_{-1}}{\phi} - \beta\right)\phi m - \left(\frac{\phi_{-1}e_{-1}}{\phi e} - \beta\right)\phi e m^f + \beta\sigma\left[u(q) - \phi\left(d + ed^f\right)\right]\right]$$

$$\begin{aligned} & s.t. \\ (\zeta): &\quad \phi\left(d + ed^f\right) - c(q) \geq 0, \\ (\nu): &\quad 0 \leq d, \\ (\mu): &\quad d \leq m, \\ (\nu^f): &\quad 0 \leq d^f, \\ (\mu^f): &\quad d^f \leq m^f, \\ (\ \lambda): &\quad \phi d \leq \frac{\kappa}{\phi_{-1}/\phi - \beta(1-\sigma)} \\ (\lambda^f): &\quad \phi e d^f \leq \frac{\kappa^f}{\phi_{-1}e_{-1}/\phi e - \beta(1-\sigma)} \end{aligned} \tag{10}$$

and the equilibrium is unique.

The first five constraints of (10) are self explanatory. The last two, which are the key equilibrium constraints for our results, have the following intuition. Since the buyer does not counterfeit we have $z = z^f = 0$ and $d = m$, $d^f = m^f$ for $\phi_{-1}/\phi > \beta$ and $\phi_{-1}e_{-1}/\phi e > \beta$. Furthermore, there is no reason to incur the costs of acquiring genuine currencies and then make an offer that the seller will reject, so $\hat{\pi}(\omega) = 1$. If the buyer counterfeits, then the marginal cost of producing a counterfeit is zero while the marginal cost of acquiring genuine currency is positive. In this case $m, m^f = 0$. Similarly, there is no reason to incur the cost of counterfeiting if the offer is rejected by the seller, thus $\hat{\pi}(\omega) = 1$.

Slightly abusing notation, let $W^b\left[\eta, \eta^f\right]$ denote the value to a buyer under the pure strategies $\eta(\omega), \eta^f(\omega) \in \{0, 1\}$ where $\eta = \eta^f = 1$ represents "no counterfeiting of currencies." For no counterfeiting to be an optimal strategy, it must be the case $W^b[1,1] \geq W^b[0,1], W^b[1,0], W^b[0,0]$. Using the expression for the CM value function of the buyer, $W^b[1,1] \geq W^b[0,1]$ reduces to

$$\begin{aligned} -\left(\frac{\phi_{-1}}{\phi} - \beta\right)\phi d - \left(\frac{\phi_{-1}e_{-1}}{\phi e} - \beta\right)\phi e d^f + \beta\sigma\left[u(q) - \phi\left(d + ed^f\right)\right] \geq \\ -\kappa - \left(\frac{\phi_{-1}e_{-1}}{\phi e} - \beta\right)\phi e d^f + \beta\sigma\left[u(q) - \phi e d^f\right], \end{aligned}$$

which collapses to the second to the last constraint in (10). A similar exercise for $W^b[1,1] \geq W^b[1,0]$ yields the last constraint in (10). Finally, it is straightforward to show that if the previous two conditions are satisfied, then $W^b[1,1] \geq W^b[0,0]$ holds as well. We provide rigorous proof behind this intuition in our Online Appendix.

In words, in order to prevent counterfeiting, the seller must put a limit on how much of each currency he is willing to accept. If he is willing to accept more than this limit, then the cost of acquiring genuine currencies is too high so the buyer will choose to counterfeit. Alternatively, above these limits, the marginal cost to the buyer of increasing d and d^f is zero, hence the seller must make the marginal benefit of doing so equal to zero. This is achieved by not accepting an offer $\left(d, d^f\right)$ above these limits.

4 Monetary Equilibrium

We can now embed the equilibrium characterization of the private information games for the Home and Foreign countries into the monetary equilibrium of the two-country model. Since preferences are quasi-linear, the infinite history of past games between buyers and sellers does not matter for each current period agents' decision problems. This allows us to tractably incorporate the equilibrium characterization of the game previously described into the overall dynamic general monetary setting. Before we do so, we return to describing the Home agents' dynamic decision problems.

4.1 Home Agents' Recursive Problems

Using equation the characterization of the counterfeiting game, the DM-buyers' intertemporal problem, lagged one period, reduces to

$$\mathcal{Z}_{-1} + \max_{q,d,d^f m,m^f} \left[-\left(\frac{\phi_{-1}}{\phi} - \beta \right) \phi m - \left(\frac{\phi_{-1} e_{-1}}{\phi e} - \beta \right) \phi e m^f + + \beta \sigma \left(u(q) - \phi d - \phi e d^f \right) \right] \tag{11}$$

$$\begin{aligned}
\text{s.t.} \quad (\zeta): &\quad \phi\left(d + e d^f\right) - c(q) = 0, && (12)\\
(\nu): &\quad 0 \leq d, && (13)\\
(\nu^f): &\quad 0 \leq d^f, && (14)\\
(\mu): &\quad d \leq m, && (15)\\
(\mu^f): &\quad d^f \leq m^f, && (16)\\
(\ \lambda): &\quad \phi d \leq \frac{\kappa}{\phi_{-1}/\phi - \beta(1-\sigma)}, && (17)\\
(\lambda^f): &\quad \phi e d^f \leq \frac{\kappa^f}{\phi_{-1} e_{-1}/\phi e - \beta(1-\sigma)}. && (18)
\end{aligned}$$

where $\mathcal{Z}_{-1} = \mathcal{U}(C^*) - \phi(m_{-1} + e_{-1} m^f_{-1} + \tau_{-1}) + \beta W^b(0,0)$ and the Lagrangian multipliers for each constraint are listed in the parentheses.

As before, the first 5 constraints are straightforward. However, the private information bargaining game introduces the additional liquidity constraints (17) and (18) into the buyer's problem. The corresponding first-order conditions are given by

$$0 = \beta\sigma u'(q) - \zeta c'(q), \tag{19}$$

$$\beta\sigma = \zeta + \nu - \mu - \lambda, \tag{20}$$

$$\beta\sigma = \zeta + \nu^f - \mu^f - \lambda^f, \tag{21}$$

$$\mu = \frac{\phi_{-1}}{\phi} - \beta, \tag{22}$$

$$\mu^f = \frac{\phi_{-1}e_{-1}}{\phi e} - \beta, \tag{23}$$

$$\zeta \geq 0, \nu \geq 0, \nu^f \geq 0, \mu \geq 0, \mu^f \geq 0, \lambda \geq 0, \lambda^f \geq 0, \tag{24}$$

for every date $t \geq 1$.

Equation (19) corresponds to the first-order condition for DM output, which equates the marginal benefit of consuming and the marginal value of the payment to the seller. Since the buyer makes a TIOLI offers, the payment is equal to the seller's DM production cost. Equations (20) and (21) summarize the optimal choice with respect to the two nominal payments and equate the value of holding a particular fiat currency from one CM to the next versus trading it in DM. Finally, equations (22) and (23) describe the optimal accumulation of each currency, which of course depends on their implied rate of return.

4.2 Steady-State Monetary Equilibrium

We now focus on steady-state monetary equilibria where the nominal exchange rate can grow at a constant rate. In steady state, all real quantities are constant such that $\phi M = \phi_{-1}M_{-1}$ and $e\phi M^f = e_{-1}\phi_{-1}M^f_{-1}$. It then follows that $\Pi \equiv \frac{\phi_{-1}}{\phi} = \gamma = M/M_{-1}$ and the steady-state home currency (gross) depreciation/appreciation satisfies

$$\frac{e}{e_{-1}} = \frac{\gamma}{\gamma^f} = \frac{\Pi}{\Pi^f}. \tag{25}$$

Since we impose that the two currencies grow at the same rate, we have $\gamma = \gamma^f$ thus $e/e_{-1} = 1$ so there is no currency appreciation or depreciation. In what follows we explicitly assume $\Pi > \beta$. We will discuss the case of $\Pi = \beta$ later.

Since we assume $\Pi = \Pi^f > \beta$ it is straightforward to show that $d = m$ and $d^f = m^f$. In short, buyers do not acquire costly currency that they do not intend to use. As a result we have $\zeta > 0$, $\nu = \nu^f = 0$, $\mu = \mu^f > 0$. What remains to be determined is whether $\lambda = \lambda^f = 0$ or $\lambda = \lambda^f > 0$. We also impose $\kappa = \kappa^f = \tilde{\kappa} = \tilde{\kappa}^f$ so that the counterfeiting costs are the same across currencies and across countries. We can now state the main proposition of our paper.

Proposition 2 *If $\lambda = \lambda^f > 0$, then $e = M/M^f$.*

Proof: See the Appendix.

The key point of this Proposition is that even when both currencies are perfect substitutes, e.g., they have the same rates of return and counterfeiting costs, the nominal exchange rate is determinate when both liquidity constraints bind. Thus we have broken the nominal exchange rate indeterminacy result of Kareken and Wallace.[11] The fact that sellers face a game of private information generates an upper bound on how much of each currency they are willing to accept in exchange for goods. The no-counterfeiting restrictions in turn create endogenous liquidity constraints for the DM buyer. Note that if the cost of counterfeiting is low and/or the cost of holding genuine currency is high, then DM buyers are more willing to engage in counterfeiting. To offset this, DM sellers have to be more restrictive in the real quantities of each currency that they will accept. Thus, the buyer knows he must bring in a portfolio of currencies with aggregate value $2\kappa/\left[\Pi-\beta(1-\sigma)\right]$. The liquidity constraints prevent the buyer from freely substituting across currencies to achieve this aggregate value since he can bring in no more than $\kappa/\left[\Pi-\beta(1-\sigma)\right]$ of either currency. Thus, the liquidity constraints pin down the composition of the buyer's currency portfolio and thus the nominal exchange rate.[12]

It is worth noting that the determinate nominal exchange rate obtained above is the same as what comes out of a symmetric, two-country cash-in-advance (CIA) model as in Stockman (1980) and Lucas (1982). While one may be tempted to say that we have provided a "micro-foundation" for the two-country CIA model, this would be incorrect for two reasons. First, our result holds only for a limited set of parameter values. Second, in the standard CIA model only one of the currencies is used per transaction (by assumption), whereas here both currencies are used in the same transaction.

Another interesting aspect of this equilibrium is that, although there is no counterfeiting in equilibrium, the *threat* of counterfeiting is sufficient to generate determinacy of the nominal exchange rate. We find this interesting because it suggests counterfeiting may be an important factor affecting the equilibrium allocation and nominal exchange rate determinacy. In this sense, it is similar to Kehoe and Levine (1993) and the endogenous default literature in which default affects the equilibrium allocation but is not observed in equilibrium.

[11]The main insight of this paper is that when sellers face private information regarding the quality of the fiat money, endogenous liquidity constraints arise and the can help determine the nominal exchange rate. This equilibrium feature is not unique to buyer's take-it-or-leave-it offers. In the Appendix we consider proportional bargaining as an alternative trading protocol. We show then that when the the two liquidity constraints bind, the nominal exchange rate is also the ratio of the money supplies.

[12]Consider the case where $\kappa = \kappa^f$ and $\tilde{\kappa} = \tilde{\kappa}^f$ but $\kappa \neq \tilde{\kappa}$. In this case, although the currencies are perfect substitutes within a country's DM they are not perfect substitutes when comparing DMs. It is straightforward to show in this case that if all liquidity constraints bind we obtain $e = \kappa M/\tilde{\kappa}M^f$. As a result, the relative costs of counterfeiting directly influence the exchange rate even though no counterfeiting occurs in equilibrium.

In addition we also establish some results regarding efficiency. We can state the following result.

Corollary 3 *The Friedman rule* $\Pi = \beta$ *is not sufficient to generate the first best outcome.*

The proof is straightforward. Notice from (30) that if $\Pi = \beta$ but $\lambda > 0$ we have

$$\lambda = \beta\sigma \left[\frac{u'(q)}{c'(q)} - 1 \right] > 0,$$

which implies $q < q^*$ in this equilibrium. Since q solves $c(\bar{q}) = \kappa/\beta\sigma$ we simply need restrictions on the size of the parameter values to ensure that $\bar{q} < q^*$. The key takeaway from this corollary is that the threat of counterfeiting creates an additional friction on the demand for real balances that the Friedman rule does not necessarily overcome.

Having stated our key proposition on nominal exchange determinacy, we can now state the following.

Proposition 4 *If* $\lambda = \lambda^f = 0$, *then* e *is indeterminate.*

Proof: See the Appendix.

The crux of the problem is that we cannot pin down the *composition* of the seller's portfolio since buyers can perfectly switch from one currency to the other to acquire the DM good. Consequently, buyers are indifferent as to the quantities of real balances of Home and Foreign currency they hold. Thus, in the absence of binding liquidity constraints we get the standard indeterminacy result when the currencies are perfect substitutes. It is easy to show that this equilibrium occurs if it is very costly to counterfeit currencies. The key point of this proposition is that nominal exchange rate determinacy does not hold for all parameter configurations of the model as it does in a CIA model or a money in the utility function model.[13]

5 General Results on Determinacy

Our main objective was to show that the nominal exchange rate can be determinate even though the currencies are perfect substitutes, differing only by a non-fundamental attribute such as color. However, in general, we can consider cases where the counterfeiting costs and the inflation rates differ across countries. Doing so admits a wide variety of equilibria outcomes. While we do not delve into the details here, we can state the following Proposition.[14]

[13] For further analysis of determinacy regions of the nominal exchange rate, we refer the reader to the manuscript in the Federal Reserve Bank of St Louis Working Paper series with No. 2015-028A.

[14] We refer the reader to our online working paper at the Federal Reserve Bank of St Louis Working Paper series with No. 2015-028A for the proof of the following proposition.

Proposition 5 (Equilibria and Coexistence) *Depending on the relative inflation rates of the two fiat currencies, there are three cases characterizing a steady-state monetary equilibrium.*

1. *When currency f dominates in rate of return* ($\Pi > \Pi^f$) *and*

   (a) *when neither liquidity constraints bind* ($\lambda = \lambda^f = 0$)*, or when only the liquidity constraint on the dominated fiat currency binds* ($\lambda > 0, \lambda^f = 0$)*, then a monetary equilibrium exists with the unique outcome where only the low-inflation currency circulates; or,*

   (b) *the liquidity constraint on currency f binds* ($\lambda^f > \lambda = 0$)*, then there exists a monetary equilibrium with a unique outcome where the currencies coexist as media of exchange and the nominal exchange rate is determinate*

$$e = \frac{M}{M^f} \frac{\kappa^f}{c(q)\left[\Pi^f - \beta(1-\sigma)\right] - \kappa^f};$$

   where q solves

$$\frac{\Pi - \beta}{\sigma\beta} = \frac{u'(q) - c'(q)}{c'(q)}.$$

   (c) *both liquidity constraints bind* ($\lambda^f > 0, \lambda > 0$)*, then there exists a unique monetary equilibrium where the currencies coexist and the nominal exchange rate is determinate*

$$e = \frac{\kappa^f M}{\kappa M^f} \frac{\Pi - \beta(1-\sigma)}{\Pi^f - \beta(1-\sigma)}.$$

2. *When currency f is dominated in rate of return* ($\Pi^f > \Pi$)*, the coexistence results are the symmetric opposite to those of Case 1.*

The intuition for the results follows directly from the special case we look at the previous Propositions. In 1(b) and 1(c) the exchange rate depreciates according to (25). These results show, as in standard international monetary models, that the higher is the home country's inflation rate, the faster its currency depreciates. We also see that if it is relatively harder to counterfeit the home currency, then it will be more valuable relative to the foreign currency.

An interesting feature of 1(b) is that even if the home currency is dominated in rate of return, it still circulates as a medium of exchange. In this equilibrium, it is costly to counterfeit the home currency (κ is large) but not the foreign currency (κ^f is small). As a result, even though it has a better rate of return, sellers restrict the quantity of the foreign currency they are willing to accept because they are concerned about counterfeiting. In equilibrium, sellers accept the foreign currency first but only up to a point, and further purchases of goods are done with the home currency. This describes countries that are "dollarized" – the home currency has a worse rate of return yet the home currency is not driven out of circulation by the stronger foreign currency.

Although we have taken κ and κ^f to be exogenous, one could interpret them as being responsive to policy actions. If the home country undertakes policies to make its currency harder to counterfeit, then equilibrium 1(c) above suggests that this will cause the home currency to appreciate in value (a fall in e). Since it is harder to counterfeit the home currency, sellers are willing to accept a larger quantity of the home currency. This in turns raises the demand for home currency relative to the foreign currency.

6 Conclusion

In this paper we present a search theoretic model of two fiat currencies to study the properties of nominal exchange rates when agents face private information. Agents have no restrictions on what divisible fiat currency can be used to settle transactions. Buyers may counterfeit both fiat currencies at a fixed cost while sellers cannot distinguish between counterfeit and genuine fiat currencies. This informational problem gives rise to endogenous liquidity constraints that specify a seller's upper bound on how much fiat currency they are willing to accept. The private information problem faced by sellers yields some endogenous liquidity constraints. These liquidity constraints have the property that the marginal liquidity value of an additional unit of currency beyond the endogenous binding liquidity constraint is zero. The binding nature of these liquidity constraints is key in determining nominal exchange rates.

When endogenous liquidity constraints on both currencies are binding and the currencies are identical in every respect, we obtain the surprising result that the nominal exchange rate is the ratio of the two money stocks, thus breaking the nominal exchange rate indeterminacy of Kareken and Wallace (1981). When the foreign currency has a higher rate of return but a lower counterfeiting cost, then the buyer will first pay with the foreign currency up to the bound and use domestic currency to pay for the remainder of the goods purchased. Because of this, both currencies can circulate even though one currency is dominated in its rate of return. Also, because of this, we can have equilibrium cases in which only one currency emerges as an international currency, while the other currency circulates only locally. The private information problem explored in this paper allows us to break the Kareken and Wallace indeterminacy result and provides a rationalization of why currencies with dominated rates of return remain in circulation (apart from obvious explanations in terms of legal restrictions) as media of exchange.

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— Online Appendix —

This online appendix contains omitted proofs and variations or extensions to the main model presented in the paper.

A Proofs

A.1 Proof of Proposition 2

For $\lambda = \lambda^f > 0$ the solution to the buyer's problem is given by

$$\begin{aligned}
\phi m &= \phi e m^f = \frac{\kappa}{\Pi - \beta(1-\sigma)} && (26)\\
c(\hat{q}) &= \frac{2\kappa}{\Pi - \beta(1-\sigma)} && (27)\\
\zeta &= \beta\sigma\frac{u'(\hat{q})}{c'(\hat{q})} && (28)\\
\mu &= \Pi - \beta && (29)\\
\lambda &= \beta\sigma\left[\frac{u'(\hat{q})}{c'(\hat{q})} - 1\right] + \beta - \Pi. && (30)
\end{aligned}$$

Equation (26) shows the real balances are the same and equal to the liquidity bounds. Given these solutions, (27) yields the solution for $\hat{q}$. The last three are the solutions for the multipliers. Finally, to solve for m and m^f we use the market clearing conditions

$$\begin{aligned}
M &= m + \tilde{m}\\
M^f &= m^f + \tilde{m}^f.
\end{aligned}$$

In an equilibrium where the two countries are identical in every respect, buyers in each country face the same liquidity constraints. Thus we have $\phi m = \phi\tilde{m} = \kappa/\left(\Pi - \beta(1-\sigma)\right)$. It then follows that $m = M/2, m^f = M^f/2$. Substituting these expressions into (26) yields

$$\phi = \frac{2\kappa}{\left[\Pi - \beta(1-\sigma)\right]M} \quad \text{and} \quad e = \frac{M}{M^f}. \tag{31}$$

The only thing that is left to do is choose parameter values such that the solutions are valid. From (30) we need $\hat{q} < q^*$. As a result, the solution from (27) for $\hat{q}$ must satisfy this restriction. Since q^* is independent of κ and Π, it is clear that this condition is satisfied for a sufficiently small cost of counterfeiting and/or a sufficiently high inflation rate.

A.2 Proof of Proposition 4

For this equilibrium the solution to the buyer's problem yields

$$\begin{aligned}
\Pi - \beta &= \beta\sigma\left[\frac{u'(\hat{q})}{c'(\hat{q})} - 1\right] \\
c(\hat{q}) &= \phi m + \phi e m^f \\
\zeta &= \beta\sigma\frac{u'(\hat{q})}{c'(\hat{q})} \\
\mu &= \Pi - \beta.
\end{aligned}$$

In this case, $\hat{q}$ is pinned down by the first equation. The second equation then gives us the total real value of the buyer's currency portfolio. Using the second equation for both countries, in conjunction with the currency market clearing conditions, we obtain

$$\phi\left(M + eM^f\right) = 2c(\hat{q}),$$

which gives us one equation in two unknowns, ϕ and e. Thus, for any value of ϕ there is a nominal exchange rate that solves this expression. As a result, the nominal exchange rate is indeterminate.

B International DM trades

The key takeaway from this section is the following: Even if we generalize the environment in the main paper to allow agents to shop internationally in the DM—i.e., a Home buyer can also buy from a Foreign seller and vice-versa—we still get the same characterization in terms of the equilibrium liquidity constraints. As a consequence the main insight regarding the determinacy of the nominal exchange (having binding liquidity constraints) is robust to a more general setting where DM buyers can also buy from Foreign DM sellers.

Now in each DM, a Home DM-buyer has a probability $\xi \in (0,1]$ to be in Home DM. With probability $1-\xi$ a Home DM-buyer is to relocated to the Foreign DM. Ex ante, at the end of the CM (and prior to the DM), the buyer must take this possibility into account. The buyer at the end of CM will make a plan for what terms of trade to offer to a Home DM-seller, which we denote by $\omega := (q_{+1}, d_{+1}, d^f_{+1})$, and, what terms of trade to offer a Foreign DM-seller, denoted by $\omega^\star := (q^\star_{+1}, d_{+1}, d^f_{+1})$. The outcome of the plan would be contingent on the ex post realization of whether the buyer will shop in the Home DM or in the Foreign DM. For simplicity, assume that the probability of a match between a buyer and a seller in both Home and Foreign DM is identical and given by σ.

Given a commitment to the plan $(\omega, \omega^\star)$, and before the DM opens, the buyer then decides on his counterfeiting mixed strategy, $(\eta, \eta^f) := (\eta(\omega, \omega^\star), \eta^f(\omega, \omega^\star))$. As in the main paper, the buyer conditions his strategy on his beliefs about a Home (or Foreign) and the seller's probability of accepting his payment, $\hat{\pi}_{+1} := \hat{\pi}_{+1}(\omega, \omega^{\star})$ (or $\hat{\pi}_{\star,+1} := \hat{\pi}_{\star,+1}(\omega, \omega^{\star})$). From a seller's point of view, the Home (or Foreign) seller will ex-post play a mixed strategy $\pi_{+1} := \pi_{+1}(\omega)$ (or $\pi_{\star,+1} := \pi_{\star,+1}(\omega^{\star})$).

B.1 DM-buyers and DM-sellers

Since the model does not have aggregate uncertainty and agents have perfect foresight, let us re-write the game and work backwards from a DM(t) and then to a preceding date's CM($t-1$). Given a fixed strategy $(\omega, \omega^{\star}, \eta, \eta^f)$, which is determined at the end of the preceding CM, the induced beginning-of-DM value to a buyer with portfolio (a, a^f) is given by

$$\begin{aligned} V^b\left(a, a^f\right) = & \ \xi\sigma\hat{\pi}\left\{u(q) + W^b\left[\eta(a-d), \eta^f(a^f - d^f)\right]\right\} \\ & + (1-\xi)\sigma\hat{\pi}_{\star}\left\{u(q^{\star}) + W^b\left[\eta(a-d, \eta^f(a^f - d^f)\right]\right\} \\ & + \xi\left(1-\sigma\hat{\pi}\right)W^b\left(\eta a, \eta^f a^f\right) \\ & + (1-\xi)\left(1-\sigma\hat{\pi}_{\star}\right)W^b\left(\eta a, \eta^f a^f\right). \end{aligned} \tag{32}$$

Since DM sellers are do not face a probability of being relocated, the seller's problem is the same as that in the main paper. The difference now is that in each period a Home (Foreign) DM-seller may end up meeting with either a Home or a Foreign DM-buyer. Nevertheless, the seller's optimal strategy will still be the same given any buyer's offer.

After some algebra, and as a consequence of quasilinearity of all agent's per-period payoff functions, in the last stage of the counterfeiting-bargaining game (in the Home DM(t)), the seller maximizes expected profit by playing a mixed strategy π such that

$$\pi \in \left\{\arg\max_{\pi_{\star}\in[0,1]} \pi_{\star}\left[-c(q) + \phi\left(\hat{\eta}d + \hat{\eta}^f e d^f\right)\right]\right\}, \tag{33}$$

taking as given a buyer's offer ω and the seller's beliefs about the buyer's counterfeiting probabilities, $(\hat{\eta}, \hat{\eta}^f)$. One can also write down a corresponding problem for a Foreign DM seller where the mixed strategy is denoted by $\pi_{\star}$:

$$\pi_{\star} \in \left\{\arg\max_{\pi\in[0,1]} \pi\left[-c(q^{\star}) + \phi^f\left(\hat{\eta}d/e + \hat{\eta}^f d^f\right)\right]\right\}. \tag{34}$$

Note that since the law of one price holds in terms of the CM good, the Foreign seller's problem equivalently yields,

$$\pi_{\star} \in \left\{\arg\max_{\pi\in[0,1]} \pi\left[-c(q^{\star}) + \phi\left(\hat{\eta}d + \hat{\eta}^f e d^f\right)\right]\right\}. \tag{34'}$$

In the penultimate stage, in CM($t-1$), the buyer chooses the counterfeiting lottery

(η, η^f) to solve the following cost-minimization problem:

$$\max \Big\{ -\kappa(1-\eta) - \kappa^f(1-\eta^f) - \xi\beta\sigma\hat{\pi}\left(\eta\phi d + \eta^f \phi e d^f\right) \\ -(1-\xi)\beta\sigma\hat{\pi}_\star\left(\eta\phi_{+1} d + \eta^f \phi e d^f\right) \\ -\left(\frac{\phi_{-1}}{\phi} - \beta\right)\phi m - \left(\frac{\phi_{-1} e_{-1}}{\phi e} - \beta\right)\phi e m^f \Big\}, \tag{35}$$

given his earlier commitment ω and his beliefs about a Home (or Foreign) seller's acceptance probability $\hat{\pi}$ (or $\hat{\pi}_\star$).

The buyer chooses a TIOLI offer at the beginning of the game to maximize his payoff given his belief functions $(\hat{\eta}, \hat{\eta}^f, \hat{\pi}, \hat{\pi}_\star)$ about the continuation play. The buyer commits to an optimal plan of contingent offers, ω and $\omega^\star$, to maximize

$$-\kappa(1-\hat{\eta}) - \kappa^f\left(1-\hat{\eta}^f\right) + \xi\beta\sigma\hat{\pi}\left[u(\hat{q}) - \phi\left(\hat{\eta}\hat{d} + \hat{\eta}^f e\hat{d}^f\right)\right] \\ -(1-\xi)\beta\sigma\hat{\pi}_\star\left[u(q^\star) - \phi\left(\hat{\eta} d + \hat{\eta}^f e d^f\right)\right] \\ -\left(\frac{\phi_{-1}}{\phi} - \beta\right)\phi m - \left(\frac{\phi_{-1} e_{-1}}{\phi e} - \beta\right)\phi e m^f. \tag{36}$$

Given these descriptions, we may proceed directly to a generalization of the private-information bargaining game's description (as in Proposition 1).

Proposition 6 *An equilibrium of the counterfeiting-bargaining game is such that*

1. *Each Home seller accepts with probability* $\hat{\pi} = \pi = 1$ *and each Foreign seller accepts with probability* $\hat{\pi}_\star = \pi_\star = 1$*;*
2. *Each buyer does not counterfeit:*

$$(\hat{\eta}, \hat{\eta}^f, \hat{\eta}, \hat{\eta}^f) = (\eta, \eta^f, \eta, \eta^f) = (1, 1, 1, 1);$$

and

3. *Each buyer's travel-contingent TIOLI offer* $\omega := (q, d, d^f)$ *and* $\omega^\star := (q^\star, d, d^f)$

attains

$$\max\Bigg\{\left(\frac{\phi_{-1}}{\phi}-\beta\right)\phi m-\left(\frac{\phi_{-1}e_{-1}}{\phi e}-\beta\right)\phi e m^f+\xi\beta\sigma\left[u(q)-\phi\left(d+ed^f\right)\right]$$
$$+(1-\xi)\beta\sigma\left[u(q^\star)-\phi\left(d+ed^f\right)\right]$$

$$\begin{aligned}
&s.t.\\
&(\zeta): \quad \phi\left(d+ed^f\right)-c(q)\geq 0,\\
&(\zeta^f): \quad \phi\left(d+ed^f\right)-c(q^\star)\geq 0,\\
&(\nu): \quad 0\leq d,\\
&(\mu): \quad d\leq m,\\
&(\nu^f): \quad 0\leq d^f\\
&(\mu^f): \quad d^f\leq m^f,\\
&(\lambda): \quad \phi m\leq\frac{\kappa}{\phi_{-1}/\phi-\beta(1-\sigma)}\\
&(\lambda^f): \quad \phi e m^f\leq\frac{\kappa^f}{\phi_{-1}e_{-1}/\phi e-\beta(1-\sigma)}\Bigg\}
\end{aligned} \tag{37}$$

and the equilibrium is unique.

One can also write a symmetric characterization for the corresponding Foreign country. The intuitive explanation of this resulting characterization, and in particular, its endogenous liquidity constraints are similar to that in the main paper. Below we provide the detailed proof of this result. Note that by setting $\xi = 1$, we also have the proof of Proposition 1. A similar proof for this special case discussed in the main paper can also be found in our working paper Gomis-Porqueras et. al. (2015).

Proof. Denote the maximum value of the program in (64), when $\hat{\pi} = \pi = 1$, $\hat{\pi}_\star = \pi_\star = 1$, and $(\hat{\eta}, \hat{\eta}^f) = (\eta, \eta^f) = (1, 1)$, as $(U^b)^*$. The aim is to show that an equilibrium strategy yields the same value as $(U^b)^*$, and it satisfies the characterization in Proposition 6 (Case 1); and that any other candidate strategy under beliefs $\hat{\pi} \neq 1$, $\hat{\pi}_\star \neq 1$, and/or $(\hat{\eta}, \hat{\eta}^f) \neq (1, 1)$ will induce a buyer's valuation that is strictly less than $(U^b)^*$, and therefore cannot constitute an equilibrium (Cases 2-5).

Consider the subgame following an offer plan $(\omega, \omega_\star)$. Let $\rho(\chi, \chi^f)$ denote the joint probability measure on events $\{(\chi, \chi^f)\}$, where the pure actions over counterfeiting are $(\chi, \chi^f) \in \{0, 1\}^2$. Denote $P := 2^{\{0,1\}^2}$ as the power set of $\{0, 1\}^2$. By the definition of probability measures, it must be that $\sum_{\{z\}\in P} \rho(z) = 1$.

Consider the subgame where we have reached the seller's problem. At this stage, the event of a particular buyer going to the Foreign DM, or staying in the Home DM, is already realized. Therefore, without loss, let us focus on a DM-seller's problem in the

Home DM. The seller's problem in (61) is equivalent to:

$$\pi \in \left\{\arg\max_{\pi' \in [0,1]} \pi' \left[\phi\left([1-\hat{\rho}(1,0) - \hat{\rho}(1,1)]d \right.\right.\right.$$
$$\left.\left.\left. + [1-\hat{\rho}(0,1) - \hat{\rho}(1,1)]ed^f \right) - c(q)\right]\right\}. \tag{38}$$

This is a linear programming problem in π, given the seller's rational belief system $\hat{\rho}$ and buyer's offer ω. Thus the seller's best response satisfies:

$$\begin{aligned} &\left(\phi\left([1-\hat{\rho}(1,0)-\hat{\rho}(1,1)]d + [1-\hat{\rho}(0,1)-\hat{\rho}(1,1)]ed^f\right) - c(q) \begin{cases} > 0 \\ < 0 \\ = 0 \end{cases} \right) \\ &\Rightarrow \left(\pi(\omega) \begin{cases} = 1 \\ = 0 \\ \in [0,1] \end{cases} \right). \end{aligned} \tag{39}$$

We can also write down a similar characterization for an ex-post Foreign DM seller who meets a Home buyer. The conclusion would be the same.

Now consider the preceding stage, where a buyer has already committed to some travel-contingent plan of offer $(\omega, \omega_\star)$, i.e., before the buyer knows which DM (Home or Foreign) he has to travel to. At this stage, a buyer is deciding on counterfeiting choices and has to compare alternative payoffs from mixing over pure counterfeiting strategies. We will need to define some convenient notation here: Let $U^b_{\{z\}} \equiv U^b[\omega, \{z\}, \hat{\pi}|\mathbf{s}_{-1}, \phi, e]$ denote the buyer's expected payoff from *realizing* pure actions (χ^h, χ^f), given offer ω and rational belief system $\hat{\pi}, \hat{\pi}_\star \in [0,1]$, where $\{z\} \in P$. From the pure counterfeiting strategy induced payoffs, we can construct those arising from non-degenerating mixed strategies below.

We have the following possible payoffs following each non-empty (pure-strategy) counterfeiting event $\{z\} \in P$:

$$
\begin{aligned}
U^b_{\{(0,0)\}} &= -\left(\frac{\phi_{-1}}{\phi} - \beta\right)\phi d - \left(\frac{\phi_{-1}e_{-1}}{\phi e} - \beta\right)\phi e d^f \\
&\quad + \xi\beta\sigma\hat{\pi}\left[u(q) - \phi\left(d + ed^f\right)\right] + (1-\xi)\beta\sigma\hat{\pi}_\star\left[u(q^\star) - \phi\left(d + ed^f\right)\right]; \qquad (40) \\
U^b_{\{(0,1)\}} &= -\kappa^f - \left(\frac{\phi_{-1}}{\phi} - \beta\right)\phi d \\
&\quad + \xi\beta\sigma\hat{\pi}\left[u(q) - \phi d\right] + (1-\xi)\beta\sigma\hat{\pi}_\star\left[u(q^\star) - \phi d\right]; \qquad (41) \\
U^b_{\{(1,0)\}} &= -\kappa - \left(\frac{\phi_{-1}e_{-1}}{\phi e} - \beta\right)\phi e d^f \\
&\quad + \xi\beta\sigma\hat{\pi}\left[u(q) - \phi e d^f\right] + (1-\xi)\beta\sigma\hat{\pi}_\star\left[u(q^\star) - \phi e d^f\right]; \qquad (42) \\
U^b_{\{(1,1)\}} &= -\kappa^f - \kappa + \xi\beta\sigma\hat{\pi}u(q) + (1-\xi)\beta\sigma\hat{\pi}_\star u(q^\star) \qquad (43)
\end{aligned}
$$

Observe that

$$U^b_{\{(0,1)\}} + U^b_{\{(1,0)\}} = U^b_{\{(0,0)\}} + U^b_{\{(1,1)\}}. \qquad (44)$$

There are five cases to consider.

Case 1. Suppose there is a set of candidate equilibria such that $\rho(0,0) = 1$ and $\rho(z) = 0$, for all $\{z\} \in P$ and $z \neq (0,0)$. Then, we have $U^b_{\{(0,0)\}} > \max\{U^b_{\{(1,0)\}}, U^b_{\{(0,1)\}}, U^b_{\{(1,1)\}}\}$. Since $U^b_{\{(0,0)\}} > U^b_{\{(1,0)\}}$ and $U^b_{\{(0,0)\}} > U^b_{\{(0,1)\}}$, then, from (40)-(43) we can derive that

$$\phi m < \frac{\kappa}{\frac{\phi_{-1}}{\phi} - \beta(1 - \sigma\hat{\pi})}, \qquad (45)$$

and,

$$\phi e m^f < \frac{\kappa^f}{\frac{\phi_{-1}e_{-1}}{\phi e} - \beta(1 - \sigma\hat{\pi})}, \qquad (46)$$

where $m = d$ and $m^f = d^f$. (Since inflation is costly, and since utility is quasilinear, the portfolio (m, m^f) is such that the balance carried into the DM is exactly equivalent to the value of payments offered by the buyer's plan.)

The interpretation from (45) and (46) is that the liquidity constraints on either currencies are slack. Therefore the buyer's expected payoff in this case can be evaluated from (40). If $\hat{\pi} < 1$ or $\hat{\pi}_\star < 1$, then from the Home or Foreign seller's decision rule (39), or its Foreign equivalent, we can deduce $\omega \equiv (q, d, d^f)$ and $\omega_\star \equiv (q^\star, d, d^f)$ must be such that the seller's participation/incentive constraint binds. That is, if it is the Home seller, then

$$c(q) = \phi(d + ed^f). \qquad (47)$$

If it is the Foreign seller who will be the Home buyer in the DM, then

$$c(q^\star) = \phi(d + ed^f). \qquad (48)$$

Since (47) or (48) must hold ex post, all we need to do is verify the buyer's payoff. Since, the buyer's liquidity constraints (45) and (46) do not bind at $\hat{\pi} < 1$ and/or $\hat{\pi}_\star < 1$, a small increment in either payment offered, d or d^f if the buyer shops at Home, and $d^\star$ or $d^{f,\star}$ if the buyer shops abroad, relaxes (47) and (48). This serve to raise $\hat{\pi}$ and $\hat{\pi}_\star$—i.e., the buyer's rational belief that sellers in either contingency will be more likely to accept his offer—and thus the buyer's payoff (40). The maximal payoff to the buyer, keeping the seller in participation, is when the sellers' best responses are consistent with the buyer's belief system: $\pi = \hat{\pi} = 1$ and $\pi_\star = \hat{\pi}_\star = 1$, and the offer plan $(\overline{\omega}, \overline{\omega}_\star)$ is such that

$$
\begin{aligned}
\overline{U}^b \equiv & U^b_{\{(0,0)\}}[\overline{\omega}, \overline{\omega}_\star | \pi = \hat{\pi} = \pi_\star = \hat{\pi}_\star = 1] \\
= & \sup_{\omega} \Bigg\{ U^b_{\{(0,0)\}}[\overline{\omega}, \overline{\omega}_\star | \pi = \hat{\pi} = \pi_\star = \hat{\pi}_\star = 1] : \phi m \leq \frac{\kappa}{\frac{\phi_{-1}}{\phi} - \beta(1-\sigma)}, \\
& \qquad \phi e m^f \leq \frac{\kappa^f}{\frac{\phi_{-1} e_{-1}}{\phi e} - \beta(1-\sigma)}, c(q) \leq \phi(d + e d^f), c(q^\star) \leq \phi(d + e d^f) \Bigg\}.
\end{aligned}
$$

Then it is easily verified that this maximal value coincides with the maximum value of the program given in (64) in Proposition 7, i.e. $\overline{U}^b = (U^b)^*$, since the payoff function is continuous, and the constraints also define a nonempty, compact subset of the feasible set. Since a seller has no incentive to deviate from $\pi = 1$ or $\pi_\star = 1$, then a behavior strategy $(\omega, \omega_\star, (\eta, \eta^f), (\pi, \pi_\star)) = \langle \overline{\omega}, \overline{\omega}_\star, (1,1), (1,1) \rangle$ inducing the TIOLI payoff $\overline{U}^b$ is a PBE.

Case 2. Note that in any equilibrium, a seller will never accept an offer if $\rho(1,1) = 1$, and, a buyer will never counterfeit both assets with probability 1—counterfeiting for sure costs $\kappa + \kappa^f$ and the buyer gains nothing. Therefore, $\rho(1,1) < 1$ is a necessary condition for an equilibrium in the subgame following ω. Likewise, all unions of disjoint events with this event of counterfeiting all assets—i.e. $\{(\chi, \chi^f)\} \in \{(0,1)\} \cup \{(1,1)\}$ or $\{(\chi, \chi^f)\} \in \{(1,0)\} \cup \{(1,1)\}$—such that $\rho(0,1) + \rho(1,1) = 1$ or $\rho(1,0) + \rho(1,1) = 1$, respectively, cannot be on any equilibrium path.

Case 3. Suppose instead we have equilibria in which $\rho(0,0) + \rho(1,0) = 1$, $\rho(1,0) \neq 0$, and $\rho(1,1) + \rho(0,1) = 0$, so $U^b_{\{(1,0)\}} = U^b_{\{(0,0)\}} > \max\{U^b_{\{(0,1)\}}, U^b_{\{(1,1)\}}\}$.

Given this case, and from (44), we have $U^b_{\{(0,1)\}} = U^b_{\{(1,1)\}}$. From $U^b_{\{(1,0)\}} = U^b_{\{(0,0)\}}$, and (40) and (42), respectively, we have:

$$
\tilde{\pi} \equiv \xi \hat{\pi} + (1-\xi)\hat{\pi}_\star = \frac{\kappa - (\phi_{-1}/\phi - \beta)\phi d}{\beta \sigma \phi d}, \tag{49}
$$

and,

$$
\phi e d^f < \frac{\kappa^f}{\frac{\phi_{-1} e_{-1}}{\phi e} - \beta(1 - \sigma \tilde{\pi})}. \tag{50}
$$

If $\hat{\pi} < 1$, then from the Home seller's decision rule (39) we can deduce $\omega \equiv (q, d, d^f)$ must be such that the seller's participation/incentive constraint binds:

$$\begin{aligned} c(q) &= \phi[(1-\rho(1,0)-\rho(1,1))d + (1-\rho(0,1)-\rho(1,1))ed^*] \\ &= \phi[(1-\rho(1,0))d + ed^*]. \end{aligned} \tag{51}$$

Likewise, if $\hat{\pi}_\star < 1$, then a Foreign seller's participation constraint will be binding.

The buyer's payoff can be evaluated from (42). If $\hat{\pi} < 1$ and/or $\hat{\pi}_\star < 1$, then reducing d infinitesimally will increase $\tilde{\pi}$ in (49), and this uniformly increases the buyer's payoff in (42). The buyer would like to attain $\hat{\pi} = \hat{\pi}_\star = 1$ since the sellers' participation constraint will still be respected:

$$c(q) \leq \phi[(1-\rho(1,0))d + ed^f], \qquad c(q^\star) \leq \phi[(1-\rho(1,0))d + ed^f]. \tag{52}$$

Let the maximum of the buyer's TIOLI value (42) such that the constraints (49), (50) and (52) are respected, in this case be $(U^b)^\dagger$. However, since $\rho(1,0) \neq 0$, it is easily verified that

$$\begin{aligned} (U^b)^\dagger &< U^b_{\{(0,0)\}}[\overline{\omega}|\pi = \hat{\pi} = \pi_\star = \hat{\pi}_\star = 1; \rho(1,0) = 0] \\ &= \sup_{\omega, \rho(1,0)} \{U^b_{\{(1,0)\}}|(49),(50),(52)\} = (U^b)^*, \end{aligned}$$

in which the last equality is attained when $\rho(1,0) = 0$. This contradicts the claim that $\rho(0,0) + \rho(1,0) = 1$ and $\rho(1,0) \neq 0$ is a component of a PBE.

Case 4. Suppose there are equilibria consisting of $\rho(0,0) + \rho(0,1) = 1$ with $\rho(0,1) \neq 0$, and $\rho(1,0) = \rho(1,1) = 0$. The buyer's payoff is such that $U^b_{\{(0,1)\}} = U^b_{\{(0,0)\}} > \max\{U^b_{\{(1,0)\}}, U^b_{\{(1,1)\}}\}$. Given this assumption, we have from (44) that $U^b_{\{(1,0)\}} = U^b_{\{(1,1)\}}$. From (40) and (41), we can derive

$$\tilde{\pi} \equiv \xi\hat{\pi} + (1-\xi)\hat{\pi}_\star = \frac{\kappa^f - (\phi_{-1}e_{-1}/\phi e - \beta)\phi ed^f}{\beta\sigma\phi ed^f}. \tag{53}$$

From the case that $U^b_{\{(0,0)\}} > U^b_{\{(1,0)\}}$ and (40)-(42), we have:

$$\phi d < \frac{\kappa}{\frac{\phi_{-1}}{\phi} - \beta(1-\sigma\tilde{\pi})}. \tag{54}$$

The buyer's payoff can be evaluated from (41). If $\hat{\pi} < 1$ or $\hat{\pi}_\star < 1$, from (39), we can deduce that the Home seller's participation constraint is binding. Again, the same goes for the Foreign seller. If $\tilde{\pi} < 1$, then reducing d^f infinitesimally will increase these acceptance probabilities in (53) and thus $\tilde{\pi}$; and this serves to increase the buyer's payoff in (41). The buyer would like to attain $\hat{\pi} = \hat{\pi}_\star = 1$ since the sellers' participation constraint will still be respected at that point:

$$c(q) \leq \phi[d + (1 - \rho(0,1))ed^f], \qquad c(q^\star) \leq [\phi d + e(1 - \rho(1,0))d^f]. \tag{55}$$

Let the maximum of the buyer's TIOLI value (41) such that the constraints (53), (54) and (55) are respected, in this case be $(U^b)^{\dagger\dagger}$. However, since $\rho(1,0) \neq 0$, it is easily verified that $(U^b)^{\dagger\dagger} < U^b_{\{(0,0)\}}[\overline{\omega}|\pi = \hat{\pi} = \pi_\star = \hat{\pi}_\star = 1; \rho(0,1) = 0] = \sup_\omega\{U^b_{\{(0,1)\}}|(53),(54),(55)\} = (U^b)^*$, in which the last equality is attained when $\rho(0,1) = 0$. This contradicts the claim that $\rho(0,0) + \rho(0,1) = 1$ and $\rho(0,1) \neq 0$ is a component of a PBE.

Case 5. Suppose a candidate equilibrium is such that $\sum_{\{z\}\in P} \rho(z) = 1$, $\rho(z) \neq 0$ for all $\{z\} \in P$, and that $U^b_{\{(0,1)\}} = U^b_{\{(0,0)\}} = U^b_{\{(1,0)\}} = U^b_{\{(1,1)\}}$. Then from (41) and (43), and from (42) and (43), respectively, we can derive

$$\tilde{\pi} \equiv \xi\hat{\pi} + (1 - \xi)\hat{\pi}_\star = \frac{\kappa^f - (\phi_{-1}e_{-1}/\phi e - \beta)\phi e d^f}{\beta\sigma\phi e d^f} = \frac{\kappa - (\phi_{-1}/\phi - \beta)\phi d}{\beta\sigma\phi d}. \tag{56}$$

If the payment offered (d, d^f) are such that $\hat{\pi} < 1$ or $\hat{\pi}_\star < 1$, then from the seller's decision rule (39) we can deduce $\omega \equiv (q, d, d^f)$ and $\omega \equiv (q^\star, d, d^f)$ must be such that the Home seller's participation/incentive constraint binds:

$$c(q) = \phi[(1 - \rho(1,0) - \rho(1,1))d + (1 - \rho(0,1) - \rho(1,1))ed^f], \tag{57}$$

Likewise, for a Foreign seller. However, the buyer can increase his expected payoff in (43) by reducing both (d, d^*), thus raising $\hat{\pi}$ and $\hat{\pi}_\star$ in (56) while still ensuring that the sellers participate, until $\hat{\pi} = \hat{\pi}_\star = 1$, where

$$c(q) \leq \phi[(1 - \rho(1,0) - \rho(1,1))d + (1 - \rho(0,1) - \rho(1,1))ed^f]. \tag{58}$$

Let the maximum of the buyer's TIOLI value (43) such that the constraints (56) and (58) are respected, in this case be $(U^b)^\ddagger$. However, since $\rho(1,0), \rho(0,1), \rho(1,1) \neq 0$, it is easily verified that $(U^b)^\ddagger < U^b_{\{(1,1)\}}[\overline{\omega}|\pi = \hat{\pi} = \pi_\star = \hat{\pi}_\star = 1; \rho(0,0) = 1] = \sup_\omega\{U^b_{\{(1,1)\}}|(56),(55)\} = (U^b)^*$, in which the last equality is attained when $\rho(0,0) = 1$. This contradicts the claim that $\sum_{\{z\}\in P} \rho(z) = 1$, $\rho(z) \neq 0$ for all $\{z\} \in P$, is a component of a PBE.

Summary. From Cases 1 to 5, we have shown that the only mixed-strategy Nash equilibrium in the subgame following an offer ω must be one such that $\langle\rho(0,0), \pi\rangle = \langle 1, 1\rangle$, and that the offer ω satisfies the program in (64) in Proposition 6.

Finally, since $u(.)$ and $-c(.)$ are strictly concave functions and the inequality constraints in program (64) define a convex feasible set, the program (64) has a unique solution. ■

We can conclude that in this richer setting we obtain similar liquidity constraints than the ones presented in the paper. The main mechanism to pinned down the nominal exchange rate is then robust to having DM buyers also trade with Foreign DM sellers.

C Proportional Bargaining Trading Protocol

Let us consider an alternative trading protocol: The proportional bargaining solution of Kalai and Smorodinsky (1975). The buyer now proposes $\omega := (q_{+1}, d_{+1}, d^f_{+1})$ and commit to ω before making any (C, N) decisions in CM(t). Note that the underlying private information problem faced by the seller is still present under this new trading protocol. This is the case as the seller still can not differentiate between genuine and counterfeited currencies.

The buyers' payoff under this new trading protocol is given by

$$\begin{aligned} W^b\left(m, m^f\right) &= \max_{\omega, m_{+1}, m^f_{+1}, \eta(\omega), \eta^f(\omega)} \Big[\mathcal{U}(C^*) - \phi(m + em^f + \tau) + \beta W^b(0,0) \\ &\quad - \left(\frac{\phi}{\phi_{+1}} - \beta\right)\phi_{+1} m_{+1} - \left(\frac{\phi e}{\phi_{+1} e_{+1}} - \beta\right)\phi_{+1} e_{+1} m^f_{+1} - \kappa(1 - \eta(\omega)) \\ &\quad - \kappa^f(1 - \eta^f(\omega)) + \beta\sigma\hat{\pi}_{+1}(\omega)\left(u(q_{+1}) - \eta(\omega)\phi_{+1} d_{+1} - \eta^f(\omega)\phi_{+1} e_{+1} d^f_{+1}\right)\Big] \end{aligned} \tag{59}$$

while the seller's payoff is given by

$$\begin{aligned} W^s\left(m_s, m^f_s\right) &= \mathcal{U}(C^*) - \phi(m_s + em^f_s) + \beta W^s(0,0) + \\ &\quad + \max_{\pi_{+1}(\omega)} \left[\beta\sigma\pi_{+1}(\omega)\left(-c(q_{+1}) + \hat{\eta}(\omega)\phi_{+1} d_{+1} + \hat{\eta}^f(\omega)\phi_{+1} e_{+1} d^f_{+1}\right)\right] \end{aligned} \tag{60}$$

where the only difference with respect to the buyer's TIOLI is the payment (d, d^f) and DM quantitate traded (q).

The seller plays a mixed strategy π to maximize her expected payoff, which is given by

$$\pi(\omega) \in \left\{\arg\max_{\pi\in[0,1]} \pi(\omega)\left[-c(q) + \phi\left(\hat{\eta}(\omega) d + \hat{\eta}^f(\omega) ed^f\right)\right]\right\} \tag{61}$$

taking the posted offer and the buyer's belief about the seller's best response $\hat{\pi}(\omega)$ as given. To determine the exact terms of trade, the buyer solves the following problem:

$$\begin{aligned} &\max_{\eta(\omega),\eta^f(\omega)\in[0,1]} -\kappa(1-\eta(\omega)) - \kappa^f(1-\eta^f(\omega)) + \beta\sigma\hat{\pi}_{+1}(\omega)\left(\eta(\omega)\phi_{+1} d_{+1} + \eta^f(\omega)\phi_{+1} e_{+1} d^f_{+1}\right) \\ &- \left(\frac{\phi}{\phi_{+1}} - \beta\right)\phi_{+1} m_{+1} - \left(\frac{\phi e}{\phi_{+1} e_{+1}} - \beta\right)\phi_{+1} e_{+1} m^f_{+1}. \end{aligned} \tag{62}$$

Taking the counterfeiting probabilities as given, the buyer then chooses the proportional bargaining offer at the beginning of the game to maximize his payoff given the conjecture $(\hat{\eta}(\omega), \hat{\eta}^f(\omega), \hat{\pi}(\omega))$ of the continuation play. Consequently, the buyer commits to an optimal offer $\omega \equiv (\hat{q}, \hat{d}, \hat{d^f})$ solving

$$\max \Big\{ -\kappa \left(1 - \hat{\eta}\left(\omega\right)\right) - \kappa^f \left(1 - \hat{\eta}^f\left(\omega\right)\right) + \beta\sigma\hat{\pi} \left[u(\hat{q}) - \phi\left(\hat{\eta}\left(\omega\right)\hat{d} + \hat{\eta}^f\left(\omega\right)e\hat{d^f}\right)\right]$$
$$- \left(\frac{\phi}{\phi_{+1}} - \beta\right)\phi_{+1}m_{+1} - \left(\frac{\phi e}{\phi_{+1}e_{+1}} - \beta\right)\phi_{+1}e_{+1}m^f_{+1}\Big\} \tag{63}$$

where under proportional bargaining we have that the buyer and seller surpluses satisfy[15]

$$B_s \equiv u(\hat{q}) - \phi\left(\hat{\eta}\left(\omega\right)\hat{d} + \hat{\eta}^f\left(\omega\right)e\hat{d^f}\right) = \frac{\theta}{1-\theta}\left[\phi\left(\hat{\eta}\hat{d} + \hat{\eta}^f e\hat{d^f}\right) - c(\hat{q})\right] \equiv S_s > 0.$$

This contrasts with the TIOLI case, where the seller's surplus is such that

$$\phi\left(\hat{\eta}\left(\omega\right)\hat{d} + \hat{\eta}^f\left(\omega\right)e\hat{d^f}\right) - c(\hat{q}) = 0.$$

Proposition 7 *An equilibrium of the counterfeiting-bargaining game is such that*

1. *Each seller accepts with probability* $\hat{\pi}\left(\omega\right) = \pi(\omega) = 1$*;*
2. *Each buyer does not counterfeit:* $\left(\hat{\eta}\left(\omega\right), \hat{\eta}^f\left(\omega\right)\right) = \left(\eta(\omega), \eta^f(\omega)\right) = (1, 1)$*; and*
3. *Each buyer's TIOLI offer* ω *is such that*

$$\omega \in \Bigg\{ \underset{\omega}{arg\max} \Bigg[-\left(\frac{\phi_{-1}}{\phi} - \beta\right)\phi m - \left(\frac{\phi_{-1}e_{-1}}{\phi e} - \beta\right)\phi e m^f$$
$$+ \beta\sigma\left[u(q) - \phi\left(d + ed^f\right)\right]\Bigg] \qquad s.t.$$
$$\begin{aligned} (\zeta):&\quad u(q) - \phi(d + ed^f) = \frac{\theta}{1-\theta}\left(\phi(d + ed^f) - c(q)\right) > 0,\\ (\nu):&\quad 0 \le d,\\ (\mu):&\quad d \le m,\\ (\nu^f):&\quad 0 \le d^f,\\ (\mu^f):&\quad d^f \le m^f,\\ (\ \lambda):&\quad \phi d \le \frac{\kappa}{\phi_{-1}/\phi - \beta(1-\sigma)}\\ (\lambda^f):&\quad \phi e d^f \le \frac{\kappa^f}{\phi_{-1}e_{-1}/\phi e - \beta(1-\sigma)} \Bigg\} \end{aligned} \tag{64}$$

and the equilibrium is unique.

---

[15]Upon a successful DM match between a buyer and a seller, the maximum total surplus is $T_s = u(q) - c(q)$. Under the proportional bargaining trading protocol, the buyer's (B_s) and seller's (S_s) maximal surpluses are a fraction of the total surplus and are given by:

$$B_s = \theta[u(q) - c(q)] \;\; S_s = (1-\theta)[u(q) - c(q)]$$

where $\theta \in [0, 1]$ represents the bargaining power of the buyer.

Since the buyer does not counterfeit, we have that $z = z^f = 0$ and $d = m$, $d^f = m^f$ for $\phi_{-1}/\phi > \beta$ and $\phi_{-1}e_{-1}/\phi e > \beta$. Furthermore, there is no reason to incur the costs of acquiring genuine currencies and then make an offer that the seller will reject. So $\hat{\pi}(\omega) = 1$. If the buyer counterfeits then the marginal cost of producing a counterfeit is zero while the marginal cost of acquiring genuine currency is positive. In this case $m, m^f = 0$. Similarly, there is no reason to incur the cost of counterfeiting if the offer is rejected by the seller. So, again set $\hat{\pi} = 1$. Let $W^b\left[\eta, \eta^f\right]$ denote the value function for the pure strategies $\eta(\omega), \eta^f(\omega) \in \{0,1\}$ where $\eta(\omega) = \eta^f(\omega) = 1$ represents "no counterfeiting of currencies." For no counterfeiting to be an optimal strategy, it must be the case $W^b[1,1] \geq W^b[0,1], W^b[1,0], W^b[0,0]$. Using (59), $W^b[1,1] \geq W^b[0,1]$ reduces to

$$
\begin{aligned}
-\left(\frac{\phi_{-1}}{\phi} - \beta\right)\phi d - \left(\frac{\phi_{-1}e_{-1}}{\phi e} - \beta\right)\phi e d^f + \beta\sigma\left[u(q) - \phi\left(d + e d^f\right)\right] \geq \\
-\kappa - \left(\frac{\phi_{-1}e_{-1}}{\phi e} - \beta\right)\phi e d^f + \beta\sigma\left[u(q) - \phi e d^f\right],
\end{aligned}
$$

which collapses to the second to the last constraint in (64). A similar exercise for $W^b[1,1] \geq W^b[1,0]$ yields the last constraint in (64). Finally, it is straightforward to show that if the previous two conditions are satisfied, then $W^b[1,1] \geq W^b[0,0]$ holds as well.

Relative to the TIOLI case, the only difference in the monetary equilibrium is the ζ constraint as the seller always obtains some strictly positive surplus. It is worth emphasising, however, that the same underlying mechanism to determine the nominal exchange rate follow the same logic as in the TIOLI presented in the main text of the paper. More precisely, the nominal exchange rate can not be determined unless the liquidity constraints bind.

Let us now focus on the stationary monetary equilibrium associated with an economy with $\Pi = \Pi^f > \beta$ and $\kappa = \kappa^f$. This implies that $d = m$ and $d^f = m^f$. The DM-buyer's problem can be written as follows:[16]

$$
\begin{aligned}
\mathcal{U}(C^*) - \phi(m_{-1} + e_{-1}m^f_{-1} + \tau_{-1}) + \max_q \Big[-(\Pi - \beta)\left((1-\theta)u(q) + \theta c(q)\right) + \beta\sigma\theta\left[u(q) - c(q)\right] \Big] \\
\text{s.t.}\ (\lambda): \quad \phi m \leq \frac{\kappa}{\phi_{-1}/\phi - \beta(1-\sigma)} \\
(\lambda^f): \quad \phi e m^f \leq \frac{\kappa^f}{\phi_{-1}e_{-1}/\phi e - \beta(1-\sigma)}
\end{aligned}
\tag{65}
$$

[16]Here we have substituted the various constraints so we can write the total payment in terms on DM output and imposed that buyers spend all their currency as holding it is costly.

where the buyer's total payment is such that

$$\phi(d + ed^f) = \phi(m + em^f) = (1 - \theta)u(q) + \theta c(q). \tag{66}$$

The corresponding first order condition is then given by

$$-(\Pi - \beta)\left[(1-\theta)u'(q) + \theta c'(q)\right] + \beta\sigma\theta\left[u'(q) - c'(q)\right] - (\lambda + \lambda^f)\left[(1-\theta)u'(q) + \theta c'(q)\right] = 0.$$

We can now establish a result analogous to Proposition 2 presented in the main text under TIOLI, but now for the case of proportional bargaining. In particular we have that

Proposition 8 *If $\lambda = \lambda^f > 0$, then $e = M/M^f$.*

Proof: For $\lambda = \lambda^f > 0$ we have that the various payments are such that

$$\phi m = \phi e m^f = \frac{\kappa}{\Pi - \beta(1-\sigma)}.$$

Using the total payment equation (8), the quantity traded in DM, q, satisfies the following implicit equation

$$(1-\theta)u(q) + \theta c(q) = \frac{2\kappa}{\Pi - \beta(1-\sigma)}.$$

Now using the first order condition of the DM-buyer's problem, we have that the Lagrange multiplier λ is given by

$$\lambda = \frac{\beta\sigma\theta}{2}\frac{u'(q) - c'(q)}{(1-\theta)u'(q) + \theta c'(q)} - \frac{(\Pi - \beta)}{2}.$$

Finally, to solve for m and m^f we use the market clearing conditions

$$\begin{aligned} M &= m + \tilde{m} \\ M^f &= m^f + \tilde{m}^f \end{aligned}$$

where $\tilde{m}$ denotes the foreign demand of the domestic currency.

In an economy where both countries are identical, buyers in each country face the same liquidity constraints. Thus we have $\phi m = \phi\tilde{m} = \kappa/\left(\Pi - \beta(1-\sigma)\right)$. It then follows that $m = M/2, m^f = M^f/2$. Substituting these expressions we have that

$$\phi = \frac{2\kappa}{\left[\Pi - \beta(1-\sigma)\right]M} \quad \text{and} \quad e = \frac{M}{M^f}. \tag{67}$$