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INCOMPLETE CREDIT MARKETS AND MONETARY POLICY

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Abstract

We study monetary policy when private credit markets are incomplete. The macroeconomy we study has a large private credit market, in which participant households use non-state contingent nominal contracts (NSCNC). A second, small group of households only uses cash, supplied by the monetary authority, and cannot participate in the credit market. There is an aggregate shock. We find that, despite the substantial heterogeneity, the monetary authority can provide for optimal risk-sharing in the private credit market and thus overcome the NSCNC friction via a counter-cyclical price level rule. The counter-cyclical price level rule is not unique. To pin down a unique monetary policy rule, we consider two secondary goals for the monetary authority, (i) expected inflation targeting and, (ii) nominal GDP targeting. We examine the impact of each of these approaches on the price level rule and other nominal variables in the economy.

Keywords: Monetary policy, incomplete credit markets, non-state contingent nominal contracts, life cycle economies, heterogeneous households, nominal GDP targeting. *JEL codes:* E4, E5.

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1 Introduction

Following the financial crisis of 2007-2009, monetary policymakers have placed greater focus on private credit markets and the interaction of households with these markets.¹ This is in part because households' presence in private credit markets is large, and preceding the crisis, this presence increased substantially.² Very often, these financial transactions are carried out in nominal terms. A typical transaction might involve a relatively younger household borrowing to purchase a house, as well as, through intermediary services, a relatively older household saving for retirement. Apart from being nominal, these contracts are usually not contingent on future income realizations in the way economic theory would recommend. This is a form of market incompleteness of private credit markets that is often ignored in analyses of monetary policy. In this paper we study how the non-state-contingent nominal contracting (NSCNC) friction in credit markets impacts the design of monetary policy and what kind of policy can ensure a smoothly operating credit market.

In recent times, the Federal Reserve faced an additional constraint while conducting monetary policy: The short-term nominal interest rate targeted by policymakers in the U.S. effectively hit the zero lower bound.³ In or-

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²For example, Mian and Sufi (2011) document that the 1995 U.S. household debt-to-income ratio was about 1.15, but that by 2005, it was approximately 1.65.

³The FOMC's policy rate range was kept slightly greater than zero, and the Committee

der to provide further policy accommodation subsequent to this event, the Federal Reserve embarked on two types of policies. One of these is “forward guidance”—a promise by the central bank to hold interest rates at the zero lower bound beyond the time when the zero lower bound is actually binding. The other is “quantitative easing”—outright purchases of both privately-issued and publicly-issued debt where the central bank changes the size of its balance sheet.⁴ Both of these types of monetary policy responses have been implemented in several other large economies with policy rates constrained by the zero (or effective) lower bound. In our model, when a sufficiently large and persistent negative aggregate shock hits the economy the zero lower bound on nominal interest rates may threaten to bind. One goal of this paper is to study how to best conduct monetary policy in this situation under the NSCNC friction.

1.1 What we do

We consider a simple and stylized $T + 1$ period general equilibrium life cycle model of movements in private debt levels, interest rates, and inflation.⁵ One-period privately-issued household debt and currency are the only two assets. We divide the population into two groups, a large number of credit market participants (*a.k.a.*, “credit users”) and a small number of credit market non-participants (*a.k.a.*, “cash users”). Therefore, the first friction in our model is market segmentation. The second friction is in the credit market. Debt contracts in this market are specified and paid off in nominal terms,

did not attempt a negative nominal interest rate policy.

⁴The literature on these two policies is already very extensive, and a complete summary is beyond the scope of this paper. To list a few, see for instance Eggertsson and Woodford (2006) and Filardo and Hofmann (2014) for forward guidance. For theoretical analysis of quantitative easing see Curdia and Woodford (2011), Del Negro, Eggertsson, Ferrero and Kiyotaki (2017), Williamson (2012), Woodford (2012) among others and Hamilton and Wu (2012), Krishnamurthy and Vissing-Jorgensen (2011), and Neely (2015) for empirical evidence on quantitative easing.

⁵While any integer $T \geq 2$ will suffice for the general points we wish to make, we think it is quite useful to consider the quarterly frequency ($T = 240$) so that the findings can be appropriately compared to results from other models. The interest rates in such a case will all have a three-month interpretation.

and are not state-contingent. We call this the non-state contingent nominal contracting, or NSCNC, friction, and we discuss it extensively in the main text.

There is a stochastic income growth process—an aggregate shock. In particular, aggregate labor productivity growth follows a first-order autoregressive process. Because the real interest rate will always be equal to the aggregate rate of labor productivity growth in the equilibrium we study, we can think of this shock as a shock to the natural rate of interest. This provides a parallel to the New Keynesian monetary policy literature.

Participant households supply one unit of labor inelastically in each period, but their labor productivity varies over their life cycle. We study a stylized model in which participant households’ life cycle productivity endowment (*a.k.a.* “efficiency units”) is symmetric and peaks exactly in the middle period of the life cycle.⁶ These households sell their labor productivity units on an open market at the prevailing competitive per efficiency unit stochastic wage. The real rate of growth in wages is also the real rate of growth of output in the equilibrium of this economy. The credit-using households issue debt on net during the first portion of the life cycle and hold positive net assets during the second portion.⁷

The relatively small group of credit market non-participants, the cash users, are precluded from the credit market altogether. Their productivity endowment profile is flat and intermittent, so that they can earn income only sporadically (facing the same stochastic wage per productivity unit as the participant households). These agents consume only at times when income is unavailable.⁸ To smooth consumption, the non-participant households use

⁶There is no idiosyncratic uncertainty—the only source of uncertainty is the aggregate shock.

⁷While the model is simple and abstract, much of the borrowing that occurs can be thought of as mortgage debt, intended to move the consumption of housing services earlier in the life cycle.

⁸This segment of society can be roughly viewed as the unbanked sector. Some estimates suggest that about 8 percent of US households are unbanked, and as many as 20 percent are underbanked (they have a bank account but use alternative financial services). See Burhouse and Osaki (2012).

currency. The central bank supplies the currency and therefore effectively controls the price level through this channel. Also, at each date, the central bank rebates the currency seigniorage back to the cash users who are consuming at that date.

In this model the credit market participants who hold positive net assets—the “savers”—could use either cash or credit. However, in the equilibrium that we consider in this paper, the debt issued by relatively young credit market participants will pay a higher real return and so the savers will prefer to hold this privately-issued debt rather than the publicly-issued currency. This means the net nominal interest rate is positive.⁹

Given this framework, we consider a planner’s problem in which the planner confronts the NSCNC friction, but does not address the market segmentation friction. We then consider price level rules of the monetary authority that implement optimal risk-sharing.

1.2 Main findings

We first study the equilibrium of the non-stochastic economy. In this equilibrium, the real interest rate equals the aggregate productivity growth rate, which, in turn, is the aggregate real growth rate of the economy. In spite of heterogenous income across cohorts, the well-functioning private credit market ensures that each participant household consumes an equal portion of the total real income in the credit sector at each date. Therefore, each credit market participant has an *equity share* in the income of the credit sector of the economy earned at that date. Similarly, in the cash-using segment of the economy, after seigniorage transfers, the cash users consuming at that date also have an equity share in the income of the cash sector of the economy at that date. Equity share contracting is known to be optimal given the homothetic preferences we use.

⁹For simplicity, we assume throughout that the gross nominal interest rate, R^n , is greater than unity. The weaker condition would be $R^n \geq 1$. At $R^n = 1$, participant saver households would be indifferent between holding bonds and cash. This would alter the optimization problem of the credit users and the asset market clearing conditions but would leave the core findings of the paper unchanged.

We next turn to study the stochastic economy. The solution to the planner's problem shows how the planner would completely mitigate the NSCNC friction when there is an aggregate shock.¹⁰ We find that the planner, as in the non-stochastic economy, allocates consumption to the credit users and cash users which is an equal portion to the total income in their respective sectors, maintaining the equity share contracting feature present in the non-stochastic economy. But since there are now productivity shocks, the consumption of all credit and even-dated cash users rises or falls depending on the realization of the shock ensuring that there is perfect risk-sharing in each market.

We then show that the monetary authority can provide for this same optimal risk-sharing in the private credit market and thus overcome the NSCNC friction via a counter-cyclical price level rule. However, the counter-cyclical price level rule is not unique: Many counter-cyclical price level policy rules can appropriately provide the otherwise missing state-contingency. Consequently, we consider two possible secondary goals for the monetary authority: (i) expected inflation targeting (Policy 1) and, (ii) strict nominal GDP targeting (Policy 2).¹¹

When the monetary authority pursues Policy 1, for certain sufficiently negative and persistent shock realizations the net nominal interest rate required to implement the optimal risk-sharing allocation may threaten to encounter the zero lower bound. The policy intervention as prescribed by Policy 1 then involves a promise to engineer an increase in the price level one period in the future sufficient to keep the net nominal interest rate positive.¹² As

¹⁰We see this result as analogous to the New Keynesian literature. As in the simplest New Keynesian optimal monetary policy literature, the planner solution completely mitigates the nominal friction. However, welfare losses due to the real distortion are not mitigated. In the New Keynesian literature this would be losses due to monopolistic competition, while here it is losses due to market segmentation.

¹¹Note that in our stylized economy, inflation has no impact on the real allocations of either the credit users or the cash users. Also, in this paper there could potentially be other policies that satisfy alternative secondary objectives.

¹²If the zero bound is threatening to be encountered in subsequent periods, the same policy action has to be repeated.

additional shocks hit the economy, the zero lower bound situation will eventually dissipate and special policy actions will prove temporary. When the monetary authority pursues Policy 2—strict nominal GDP targeting—the zero lower bound is never a concern for the policymaker as the policy credibly generates sufficient inflation to keep the nominal interest rate positive regardless of particular shock realizations.¹³

We conclude that in this economy where the key nominal friction is NSCNC in household credit markets, the monetary policymaker can overcome the nominal friction and provide optimal risk sharing via countercyclical price level movements. A secondary monetary policy objective is required to uniquely pin down the monetary policy rule.

1.3 Recent related literature

Financial market incompleteness due to the NSCNC friction has a long history in discussions of monetary-fiscal policy interactions. Bohn (1988), for instance, presented a theory in which a government can use inflation to change the real value of the nominal government debt in response to shocks as a substitute for changing distortionary tax rates. Chari, Christiano, and Kehoe (1991), Chari and Kehoe (1999), Schmitt-Grohe and Uribe (2004), and Siu (2004) debated the extent of inflation volatility required to complete markets, coming to differing conclusions in models with and without sticky prices. In the current paper, we have flexible prices but no taxation, nominal government debt, or fiscal policy. Also, the extent of inflation volatility required to complete markets here is not large and is within observed inflation variance in G7 economies.

¹³Note that in models with NSCNC friction, such as Koenig (2013), Sheedy (2014) and Policy 2 in our analysis here, if one sets the nominal interest rate equal to the nominal income growth rate, the zero lower bound on the nominal interest never binds as long as there is a positive nominal income growth rate target. In the New Keynesian literature, however, the zero lower bound on the nominal interest rate adds an additional constraint on the optimization problem of the central bank. As a result of this constraint, the monetary authority is unable to lower the nominal interest rate when there is a threat of deflation or falling output.

Recent papers such as Koenig (2013), Sheedy (2014) and Garriga, Kydland, and Sustek (2017) primarily focus on monetary policy alone in economies where the NSCNC friction plays a key role in private credit markets. Koenig (2013) considers a two-period economy, and the mechanism used to achieve risk-sharing is essentially the same as the one outlined in this paper. Sheedy (2014) provides an extensive background discussion on the NSCNC friction. Sheedy (2014) also considers a quantitative-theoretic version of his model in which both sticky price and NSCNC frictions are present, and argues that the NSCNC friction is the more important of the two in a calibrated case by a factor of nine.¹⁴ Garriga, Kydland, and Sustek (2017) consider the effect of the NSCNC in housing markets on equilibrium allocations. Their analysis is quantitative-theoretic with an exogenously given monetary policy. They find the non-state contingent nominal contracting friction can be quite significant, and suggest that the nature of mortgage contracting has important implications for the impact of monetary policy on the economy.

Relative to this existing literature, we find that even when we incorporate greater heterogeneity among households—a life cycle model with 241 cohorts of overlapping generations at each date—we still find that monetary policy, via an appropriate price level rule, can provide optimal risk sharing when there are aggregate shocks.¹⁵ Also, in this paper the monetary authority can determine the price level endogenously by setting the growth rate of money. Finally, relative to the existing literature, this model allows the zero lower bound to be encountered endogenously. In particular, when a sufficiently large and persistent negative aggregate shock hits the economy the zero lower bound on nominal interest rates may threaten to bind (it does not actually bind in equilibrium due to choices of the policymaker). This last feature has some precedent. In Buera and Nicolini (2015), if the shock to the collateral

¹⁴Bullard (2014) and Werning (2014) provided discussant commentary on Sheedy (2014). Both of these discussions considered the question of how results might or might not extend to economies with additional heterogeneity.

¹⁵Bullard and DiCecio (2018), using a related framework, extend this result to include inter-cohort heterogeneity along the productivity profile dimension and still find that a price level rule with counter-cyclical price level movements can complete markets.

constraint that causes the recession is sufficiently large, the equilibrium real interest rate becomes negative for several periods. Therefore, in their model the economy may hit the zero lower bound temporarily in situations where the economy is stressed by negative shocks.

The general equilibrium life cycle model we use has recently been used to analyze issues related to monetary policy and the zero lower bound by Eggertsson, Mehrotra and Robbins (2018). Their model, like ours, takes advantage of the natural credit market that exists in the life cycle framework, and they use it to study deleveraging, debt dynamics, and issues related to the zero lower bound. They focus on sticky prices as the key friction, whereas we concentrate on NSCNC.

The present paper follows in a tradition of monetary theory that emphasizes asset market participation and non-participation. The superior rate of return that can be earned by asset market participant savers then generates a positive nominal interest rate in the economy, and risk sharing can be a key concern of policymakers. This literature includes Alvarez, Lucas, and Weber (2001) and Zervou (2013). The monetary features of models related to the one presented in this paper have been studied by Azariadis, Bullard, and Smith (2001) among others.

Our paper is also related to growing literature that examines the welfare performance of nominal GDP targeting in a New Keynesian setting. Garin, Lester, and Sims (2016) compare the welfare cost of alternative monetary policies and find that in most specifications nominal GDP targeting outperforms both inflation targeting and a simple Taylor rule. Billi (2017) finds that nominal GDP level targeting leads to larger falls in nominal GDP than strict price level targeting when the zero lower bound episodes are driven by persistent demand shocks and the central bank's policy operates under optimal discretion. See, for example, Woodford (2012) and Sumner (2014) for additional discussion of nominal GDP targeting.

The paper is organized as follows. Sections 2 and 3 describe our basic model and the planner's problem. Section 4 analyzes the role of monetary policy when credit markets are incomplete and the policymaker has an ad-

ditional secondary objective. Section 5 concludes.

2 Environment

The model has both real and nominal elements but is expressed in real terms for most variables. A key exception is net asset holding a which is expressed as a nominal quantity.

The economy features households and a monetary authority. Households are of two types, “participants” and “non-participants.” We also refer to these two types as “credit users” and “cash users,” respectively.¹⁶ Both participant and non-participant household cohorts are atomistic, identical, and have mass $(1 - \omega)$ and ω , where $0 < \omega < 1$. Households live in discrete time for $T + 1$ periods with integer $T > 2$. To interpret this model as a quarterly model in which households begin economic life with zero assets in their early 20s and continue until their 80s, $T + 1$ could correspond to 241 periods. A new cohort of households enters the economy each period replacing the exiting cohort and there is no population growth, and so we will think of each cohort as having a unit mass of identical households. The economy itself continues from the infinite past into the infinite future, so that $-\infty < t < +\infty$. The only assets in the economy are nominally denominated loans in the credit market and currency. Loan contracts are for one period, non state-contingent and expressed in nominal terms. We call this the non-state contingent nominal contracting friction, or NSCNC.¹⁷ Prices are flexible.

¹⁶There are no borrowing constraints, and debt is always fully repaid. There is no role for collateral. For alternative theories that emphasize collateral and come to different conclusions, see Williamson (2016) and Araujo, Schommer, and Woodford (2015).

¹⁷In Sheedy (2014), debt contracts can have long maturities. See also Garriga, Kydland, and Sustek (2017).

2.1 Stochastic structure and production technology

There is a linear aggregate production technology that uses only labor as input given by

$$Y(t) = Q(t) L(t) \quad (1)$$

where $Y(t)$ is aggregate real output, $L(t)$ is the aggregate labor input, and $Q(t)$ is the aggregate level of technology, or equivalently the average level of labor productivity. Since labor supply is inelastic in this model the aggregate labor supply will be the sum of the productivity endowments across the various households in the model as described below.¹⁸ We assume that the economy is growing due to technological improvement via

$$Q(t+1) = \lambda(t, t+1) Q(t) \quad (2)$$

where $Q(0) > 0$ and $\lambda(t, t+1)$ represents the gross growth rate of technology between dates t and $t+1$. We assume $\lambda(t, t+1)$ follows a standard autoregressive process given by

$$\lambda(t, t+1) = (1 - \rho) \bar{\lambda} + \rho \lambda(t-1, t) + \sigma \eta(t+1), \quad (3)$$

where $\bar{\lambda} > 1$ represents the average gross growth rate, $\rho \in (0, 1)$, $\sigma > 0$, and the shocks $\eta(t+1)$ have a truncated normal distribution with bounds $\pm b$ where b is set such that the zero lower bound on the nominal interest rate can threaten to bind for some realizations of the shock η and the persistence of the gross growth of technology ρ , but where b is also sufficiently restrictive that $\lambda(t, t+1)$ never takes on a negative value.¹⁹ It follows that the marginal product of labor is $Q(t)$ and therefore that the real wage per efficiency unit $w(t)$ follows the law of motion

$$w(t+1) = \lambda(t, t+1) w(t), \quad (4)$$

with $w(0) > 0$.

¹⁸See Bullard and Singh (2019) for a closely related model with endogenous labor supply.

¹⁹See Section 4.3 where we calibrate the productivity process to discuss the performance of nominal variables under Policy 1 and Policy 2.

2.2 Participant households

The productivity endowments of the credit market participant households are given by $e = \{e_s\}_{s=0}^T$. This notation means that each household entering the economy has productivity endowment e_0 in their first period of activity, e_1 in the second, and so on up to e_T . For a 241 period model we will use the endowment profile given by

$$e_s = f(s) = \mu_0 + \mu_1 s + \mu_2 s^2 + \mu_3 s^3 + \mu_4 s^4 \quad (5)$$

such that $f(0) = 0$, $f(60) = 57/100$, $f(120) = 1$, $f(180) = 57/100$, and $f(240) = 0$. Solving these five equations yields the values for μ_i , $i = 0, \dots, 4$. This stylized endowment profile is displayed in Figure 1.

Credit market participant households supply their life-cycle productivity units inelastically at the competitive real wage $w(t)$ per efficiency unit. As a result, at any point in time, income varies considerably in this economy. The total real income in the credit sector at date t is given by $w(t) \sum_{s=0}^T e_s$. The bulk of participant income is earned in the middle portion of life. Since we assume that the productivity profile is symmetric, in this economy there is an exact balance between the need for saving into relative old age and the need for borrowing in relative youth in the credit sector.

The timing protocol in the credit market is as follows. At any period t , agents enter with one-period nominal debt carrying an interest rate $R^n(t-1, t)$. Nature moves first and draws a value of $\eta(t)$ implying a value of $\lambda(t-1, t)$, the productivity growth rate between date $t-1$ and date t . The monetary policymaker moves next and chooses a value for its monetary policy instrument. Given these choices, credit-using households make decisions to consume and save via non-state contingent nominal consumption loan contracts for the following period, carrying a nominal interest rate $R^n(t, t+1)$.

Let $c_i(t)$ denote the real value of consumption of the credit market participant cohort i at date t . The cohort entering the economy at date $i = t$

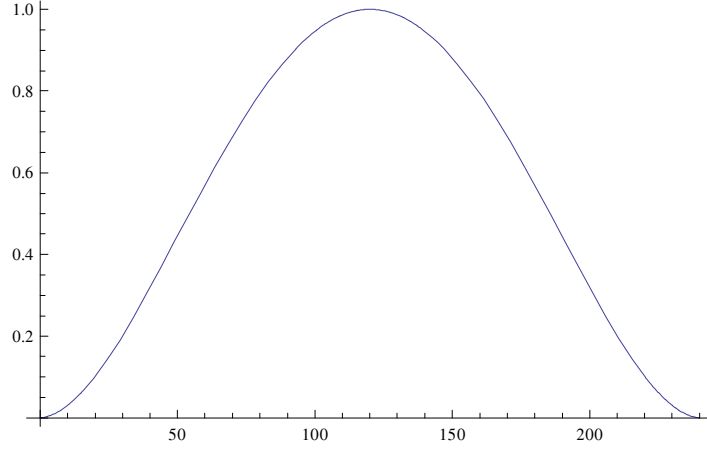


Figure 1: A schematic productivity endowment profile for credit market participant households. The profile is symmetric and peaks in the middle period of the life cycle. Total real income in the credit sector at date t is this profile multiplied by $w(t)$. About 50 percent of the households earn 75 percent of the income in the credit sector.

maximizes expected utility

$$\max_{\{c_t(t+s)\}_{s=0}^T} E_t \sum_{s=0}^T \ln c_t(t+s) \quad (6)$$

subject to a sequence of budget constraints expressed in real terms

$$\begin{aligned} c_t(t) &\leq e_0 w(t) - \frac{a_t(t)}{P(t)}, \\ c_t(t+1) &\leq e_1 w(t+1) + R^n(t, t+1) \frac{a_t(t)}{P(t+1)} - \frac{a_t(t+1)}{P(t+1)}, \\ &\dots \\ c_t(t+T) &\leq e_T w(t+T) + R^n(t+T-1, t+T) \frac{a_t(t+T-1)}{P(t+T-1)}, \end{aligned}$$

where $R^n(t, t+1)$ is the one-period gross nominal rate of return on loans originated at date t and maturing at date $t+1$ in the credit sector of the economy and $P(t)$ is the price level at date t .²⁰ The net nominal loan amounts

²⁰We use the notational convention throughout this paper that R represents gross real returns in the credit market and that other interest rates are differentiated by a superscript.

of the participant cohort i at date t is denoted by $a_i(t)$, and we interpret negative values as borrowing.

Note that in our model participants can hold both cash and credit. However the participant households holding positive assets (“savers”) will not hold currency because the real rate of return on currency will be lower than or equal to the real rate of return on private debt in all states of the world in the stationary equilibria we study in this paper.

The standard consolidated budget constraint of the participant is therefore given by

$$\begin{aligned} c_t(t) + \frac{P(t+1)}{P(t)} \frac{c_t(t+1)}{R^n(t, t+1)} \\ + \dots + \frac{P(t+T)}{P(t)} \frac{c_t(t+T)}{R^n(t, t+1) \cdot \dots \cdot R^n(t+T-1, t+T)} \\ \leq e_0 w(t) + \frac{P(t+1)}{P(t)} \frac{e_1 w(t+1)}{R^n(t, t+1)} \\ + \dots + \frac{P(t+T)}{P(t)} \frac{e_T w(t+T)}{R^n(t, t+1) \cdot \dots \cdot R^n(t+T-1, t+T)}. \end{aligned} \quad (7)$$

We can rewrite the right hand side of (7) as

$$\begin{aligned} \Xi_t(t) = e_0 w(t) + \frac{P(t+1)}{P(t)} \frac{e_1 w(t+1)}{R^n(t, t+1)} \\ + \dots + \frac{P(t+T)}{P(t)} \frac{e_T w(t+T)}{R^n(t, t+1) \cdot \dots \cdot R^n(t+T-1, t+T)}. \end{aligned} \quad (8)$$

From the participant households’ optimization problem and by rearranging the Euler equation, the non-state contingent nominal interest rate, $R^n(t, t+1)$, is given by

$$R^n(t, t+1)^{-1} = E_t \left[\frac{c_t(t)}{c_t(t+1)} \frac{P(t)}{P(t+1)} \right]. \quad (9)$$

The E_t operator indicates that households use information available as of the end of period t before the realization of $\eta(t+1)$.²¹ All cohorts have the same expectation of the aggregate nominal growth rate, so that equation (9)

²¹For further discussion of this, see Chari and Kehoe (1999).

suffices to determine the nominal interest rate for each cohort. For example, for agents entering the economy in any period $t - j$, the nominal interest rate is given by

$$R^n(t, t+1)^{-1} = E_t \left[\frac{c_{t-j}(t)}{c_{t-j}(t+1)} \frac{P(t)}{P(t+1)} \right], \quad (10)$$

but in the equilibrium we study, all households' consumption will grow at the same rate, so that this expectation is identical to the one for the household entering the economy at date t . The nominal interest rate depends jointly on the expected behavior of consumption as well as the expected policy rule for the price level, and therefore depends on expected nominal GDP growth.

2.3 Non-participant households

Non-participant households are precluded from the credit market. Like their participant agent counterparts, they live $T + 1$ periods. Let the stage of life of cash users be denoted by $s = 0, 1, \dots, T$. In $s = 0$, these agents are inactive. They do not consume, nor do they earn labor income. In odd-dated stages of life, these agents have a productivity endowment $\gamma \in (0, 1)$. We assume that γ is fairly low—in addition, there is no life cycle aspect to the value of γ .

The households entering the economy at date t earn income $\gamma w(t + s)$, $s > 0$, $s = 1, 3, 5, \dots, T - 1$. In the even-dated stages of life, the non-participant households consume. The period utility for households born at date t in these periods is $\ln c_t^{np}(t + s)$, $s = 2, 4, 6, \dots, T$. In each odd stage of life, these households solve a two-period problem and fully discount all future periods.²² Since the non-participant agents earn and consume in different periods, they save all income earned by holding currency, and then consume everything before working again in the following period.

The aggregate real demand for currency at date t , denoted by $h^d(t)$, equals $\frac{\gamma^T}{2} w(t)$. In this segment of the economy, at any date t the even-

²²This assumption is mainly to simplify the analysis of the cash-users and generate an interest-inelastic demand for money. Alternative assumptions can be made, for example a constrained productivity process, such that the cash users always solve a two-period problem.

dated cash users will use their cash and transfers from the central bank to buy consumption from the odd-dated cash users. This stylized design of the cash-using segment of the economy will deliver a conventional money demand, buffeted by the aggregate shock to productivity. The price level will be determined in this sector of the economy.

2.4 The monetary authority

The policymaker supplies currency, $H(t)$, to the non-participant households—the cash users. The total real value of currency outstanding in the economy at date t is given by $H(t)/P(t)$. We normalize the date 0 currency level to $H(0) = 1$.

The central bank chooses the growth rate of currency between any two dates $t - 1$ and t , $\theta(t - 1, t)$, written as

$$H(t) = \theta(t - 1, t) H(t - 1).$$

which implies, since money demand equals money supply at each date, that

$$\theta(t - 1, t) = \frac{P(t)}{P(t - 1)} \frac{w(t)}{w(t - 1)}. \quad (11)$$

From equation (11) it can be seen that at date t , $P(t - 1)$ and $w(t - 1)$ are known. The timing protocol for the economy means that nature moves first and chooses a growth rate $\lambda(t - 1, t)$ and hence a value for $w(t)$. This means that the central bank, moving after nature, can choose the gross rate of currency creation $\theta(t - 1, t)$ to set a value for $P(t)$.²³ This choice of $P(t)$ is sufficient to characterize equilibrium in the cash-using sector of the economy.²⁴ At each date the seigniorage earned by the monetary policymaker is transferred to even-dated cash users.

²³In this model the policymaker influences the price level without any control error, so that in effect the policymaker can simply choose the price level at each date. This aspect of the model is of course unrealistic, but the point here is to demonstrate what the optimal monetary policy would look like if such precise control were feasible. Keeping this type of assumption in place is akin to the analysis in the simplest versions of New Keynesian models in which shocks can be offset perfectly by the policymaker through appropriate adjustment of the nominal interest rate.

²⁴The central bank's price rule will also determine the gross inflation rate in the economy

3 Mitigating the nominal friction

In this section we first consider a non-stochastic economy where productivity growth rate is predictable and hence the NSCNC friction does not play any role. We then turn to the stochastic economy where the consumption allocations of credit users and cash users are determined by a planner.

3.1 The non-stochastic economy

An important benchmark in this economy is the non-stochastic balanced growth path. Let $\sigma = 0$. In addition assume that the monetary authority chooses $P(t) = P(t-1) = 1 \forall t$.

We conjecture that the gross real interest rate along the balanced growth path is $R = \bar{\lambda}$. Normalizing $w(0) = 1$, we have $w(t) = \bar{\lambda}^t w(0) = \bar{\lambda}^t$. When $R = \bar{\lambda}$, equation (8) simplifies to $\Xi_t(t) = w(t) \sum_{i=0}^T e_i$. This is the total real income earned in the credit sector of the economy at date t . This means that the household entering the economy at date t chooses to consume $(1/(T+1))w(t) \sum_{i=0}^T e_i$. All other households alive at date t will also choose to consume this amount. The consumption across the $T+1$ households exhausts total income in the credit sector. As a result the sum of asset holding across these households is zero. Therefore $R = \bar{\lambda}$ is the gross real interest rate along the non-stochastic balanced growth path of the economy.

Figures 2 and 3 plot the asset holding, household income and consumption by cohort along the non-stochastic balanced growth path. The private credit market completely solves the point-in-time (cross-sectional) income inequality problem for this economy.

Similar to Sheedy (2014), all credit sector cohorts choose to consume $(1/(T+1))w(t) \sum_{i=0}^T e_i$ and therefore have an “equity share” in the credit sector of the economy—they split up the total available real income at date t as equal real per capita consumption. Consumption grows at the same rate as the real wage, that is, the real growth rate of the economy, for all

and hence the gross real rate of return to currency holding, $R^m(t)$ at each date t in the currency-holding portion of the economy.

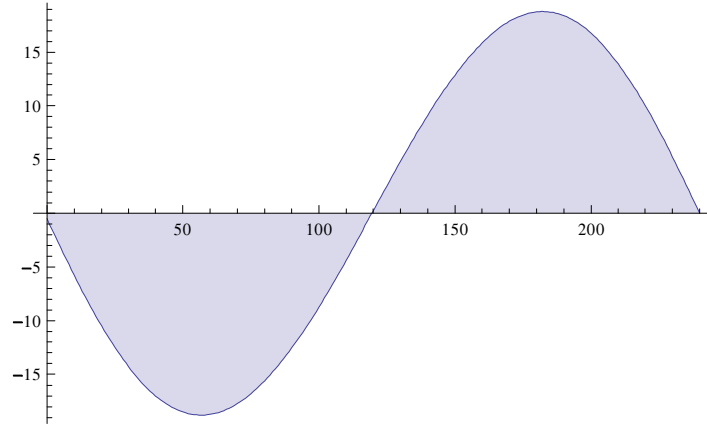


Figure 2: Net asset holding by cohort along the non-stochastic balanced growth path. Borrowing, the negative values to the left, peaks at stage 60 of the life cycle, roughly age 35, while positive assets peak at stage of life 120, roughly age 65. About 25 percent of the population holds about 75 percent of the assets.

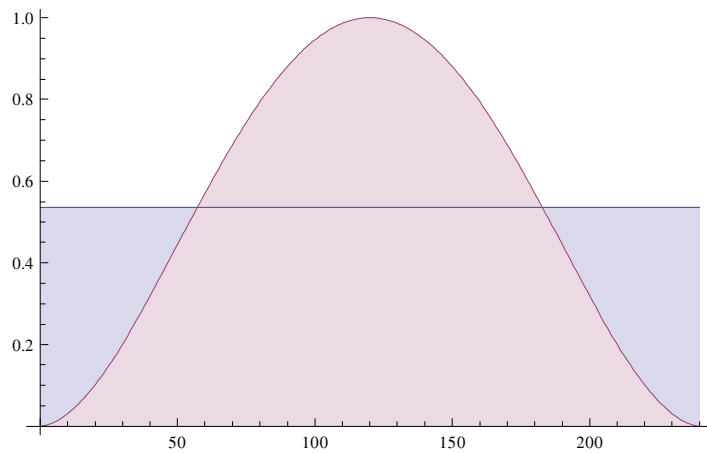


Figure 3: Schematic representation of consumption, the flat line, versus income, the bell shaped curve, by cohort along the non-stochastic balanced growth path with $w(t) = 1$. The private credit market completely solves the point-in-time (cross-sectional) income inequality problem.

households.

What about the non-participant, cash-using households? Equation (11) indicates that given price stability, the gross growth rate of currency is given by $\theta = \bar{\lambda}$, and the gross nominal interest rate, equation (9) simplifies to $R^n = \bar{\lambda} > 1$, so the net nominal interest rate would always be positive. After seigniorage transfers, the total consumption of even-dated cash users at date t is therefore $\frac{\gamma^T}{2} \bar{\lambda} w(t-1) = \frac{\gamma^T}{2} w(t)$.

A version of these results will carry over to the stochastic economy.

3.2 Aggregate shocks and the planner

In this subsection, we return to the case with $\sigma > 0$, enabling aggregate shocks. We assume that the planner takes market segmentation as given and chooses a consumption allocation that maximizes the utility of the households in their respective markets. Therefore, the planner overcomes the NSCNC friction in an economy with aggregate productivity shocks.

At date t , in the credit market all $T + 1$ cohorts consume while in the cash-using segment only $\frac{T}{2}$ even-dated cash users consume. Recall that the fraction of credit users is $1 - \omega$ and ω is the fraction of cash users. Therefore, at each date the planner maximizes the utility of all the agents consuming at that date such that

$$U^* = (1 - \omega) (\ln c_t(t) + \ln c_{t-1}(t) + \dots + \ln c_{t-T}(t)) \\ + \omega (\ln c_{t-2}^{np}(t) + \ln c_{t-4}^{np}(t) + \dots + \ln c_{t-T}^{np}(t)) \quad (12)$$

subject to the resource constraint in the credit market

$$c_t(t) + c_{t-1}(t) + \dots + c_{t-T}(t) \leq (e_o + e_1 + \dots + e_T)w(t), \quad (13)$$

and the resource constraint in the cash market

$$c_{t-2}^{np}(t) + c_{t-4}^{np}(t) + \dots + c_{t-T}^{np}(t) \leq \frac{T}{2} \gamma w(t). \quad (14)$$

Given this simple problem, the planner equalizes the marginal utilities of all the consumers in their respective markets at date t . Consequently, each

cohort in the credit sector receives an equity share of the total income $(1/(T+1)) w(t) \sum_{i=0}^T e_i$ and the consumption is equalized among the participants. Similarly, in the cash-using segment of the economy, each even-dated consumer receives $\gamma w(t)$. These allocations are stochastic but depend on $w(t)$ alone. At date $t+1$, the planner faces a similar problem. Therefore consumption of each participant cohort and the even-dated cash user cohort will grow at rate $\lambda(t, t+1)$ between date t and date $t+1$.

This consumption allocation is the same as the consumption in the non-stochastic economy described in Section 3.1. However, since there are now productivity shocks η in each period, the consumption of all credit and even-dated cash users rises or falls depending on the realization of the shock ensuring that there is perfect risk-sharing in each market.

See Appendix A for more details about the solution to the stochastic model.

4 Incomplete markets, monetary policy implementation and the zero lower bound

In our analysis the primary goal of the monetary policymaker is to implement the solution to the planner's problem in the economy with NSCNC friction and aggregate shocks. This is achieved by rebating the currency seigniorage to the even-dated cash users at that date and setting the price level to be counter-cyclical— $P(t)$ is set in such a way that it is inversely related to the growth rate of the real wage.²⁵ The generic form of this price level rule at date t is given by

$$P(t) = \frac{R^n(t-1, t)}{\lambda(t-1, t)} P(t-1). \quad (15)$$

Intuitively, the monetary authority credibly makes the price level at date t contingent on the value of $\lambda(t-1, t)$ and therefore provides the otherwise missing private sector state-contingency under the NSCNC friction. An important consequence of the generic price level rule is that the equilibrium

²⁵Note that, by lagging by one period, equation (9) gives $R^n(t-1, t)^{-1}$.

gross real interest rate is always equal to the (stochastic) gross growth rate of productivity.²⁶ This can be interpreted as the Wicksellian natural rate of interest—it is the real rate of interest that would occur in the competitive equilibrium of the credit sector of the economy without any frictions at all—and thus we find that the optimal monetary policy in our stylized life cycle model is in this respect very similar to the optimal policy that would arise in the simplest New Keynesian setting. In sum, the nominal friction causes the real interest rate in the economy to be distorted, and monetary policy should therefore be conducted in such a way that the Wicksellian natural rate of interest is restored.

The general policy advice coming from the solution to the planner’s problem can be implemented by a monetary policymaker generically using (15), but because of the interdependence of the nominal interest rate and the price level there are actually many counter-cyclical price level rules with the general form of (15), all of which can complete the credit market.

To select a particular counter-cyclical price level rule and therefore provide a more explicit description of policy, we consider two possible secondary objectives for the monetary policymaker. A first possibility is to set expected inflation equal to the target inflation rate. We call this “Policy 1.” A second possibility is to set the nominal interest rate equal to a target nominal interest rate. We call this “Policy 2.” Another description of this second possibility is “strict nominal GDP targeting,” because under this policy nominal GDP will never deviate from the targeted path. While both policies deliver the consumption allocations derived in Section 3 in each period, they have different implications for the nominal interest rate, the behavior of the policymaker when the zero lower bound threatens, and for the volatility of inflation.

²⁶Note that gross real interest rate in our analysis refers to the ex post gross real return from one-period nominal debt.

4.1 Policy 1: Expected inflation targeting

4.1.1 Stationary equilibrium

Policy 1 will have different nominal implications in this economy relative to Policy 2. Accordingly, let us denote the price level under Policy 1 by $\tilde{P}(t)$ and the gross nominal interest rate in effect from date $t - 1$ to date t under Policy 1 by $\tilde{R}^n(t - 1, t)$.

Given the timing protocol described above, a stationary equilibrium of the incomplete markets economy is a sequence of prices and nominal interest rates, $\left\{ \tilde{P}(t), \tilde{R}^n(t - 1, t) \right\}_{t=-\infty}^{+\infty}$ along with the associated consumption and asset allocations each period in which the monetary policymaker rebates seigniorage to even-dated cash users, targets expected inflation at π^* and credibly adheres to a counter-cyclical price level rule. In this equilibrium both types of households, participant and non-participant, maximize utility subject to their respective constraints and the policy rule, and markets clear.

Appendix B shows that the price rule at date t that achieves the secondary goal of the policymaker is given by

$$\tilde{P}(t) = \left(\frac{\pi^*}{E_{t-1}(\lambda(t - 1, t)^{-1})} \right) \frac{1}{\lambda(t - 1, t)} \tilde{P}(t - 1). \quad (16)$$

In the following subsections we now turn to describe how the price rule given by equation (16) ensures that the primary objective of the monetary policymaker is satisfied.

4.1.2 Participant and non-participant households

In the stationary equilibrium, the gross real interest rate $R(t - 1, t)$, $\forall t$ is always equal to the realized gross rate of wage growth $\lambda(t - 1, t)$ and the date t nominal interest rate equals

$$\tilde{R}^n(t, t + 1)^{-1} = E_t \left[\frac{c_t(t)}{c_t(t + 1)} \frac{\tilde{P}(t)}{\tilde{P}(t + 1)} \right]. \quad (17)$$

Each participant household consumes $(1/(T + 1)) w(t) \sum_{i=0}^T e_i$, an “equity share” in the real output of in the credit sector at date t and the

credit markets clear. For the stochastic economy, Figures 2 and 3 also depict schematically the cross-section of asset holding, household income and consumption by cohort along the stochastic balanced growth path at each date. The figures are drawn for $w(t) = 1$ but scale appropriately for other values of $w(t)$. Aggregate as well as individual consumption changes each period depending on the realized value of $w(t)$, but proportionately for all agents at that date. Consumption growth rates are equated for all participant households at date t . Accordingly, net asset holding also rises and falls for each cohort as time progresses, but in proportion to the value of $w(t)$ at that date.

If the price rule is given by equation (16) and the central bank fully rebates the seigniorage to the even-dated cash users each period, then the consumption of all the even-dated cash users at date t is $\gamma w(t)$. See Appendix A for more details.

4.1.3 Zero lower bound and Policy 1

When a relatively large negative shock is drawn by nature, consumption in the current period will fall. This, by itself, is not a concern for the equilibrium we have described above. However, if the serial correlation of the shock is high enough, the zero lower bound on the nominal interest rate is expected to be encountered as can be seen from equation (9). In this situation, the price rule in equation (16) will no longer be able to implement the planner's allocation. If the zero lower bound was actually violated, the nominal interest rate would be negative and participant saver households (households on the right hand side of Figure 3) would no longer want to hold the paper issued by relatively young participant households, because the real return to holding government-issued cash would be higher.

To avoid such a possibility, the price rule in equation (16) must be modified. In particular, the central bank announces that if a large negative shock hits the economy at any date t such that the agents expect at date t that the gross nominal interest rate $\tilde{R}^n(t, t+1) \leq 1$, the central bank reacts by credibly promising to create a higher than usual price level at date $t+1$ such

that the zero lower bound condition on the net nominal interest rate does not bind. The policy rule therefore can be described as

$$\tilde{P}(t+1) = \begin{cases} \left(\frac{\pi^*}{E_t(\lambda(t,t+1)^{-1})} \right) \frac{1}{\lambda(t,t+1)} \tilde{P}(t) & \text{if } \left(\frac{\pi^*}{E_t(\lambda(t,t+1)^{-1})} \right) > 1, \\ \left(\frac{\pi^*[1+\vartheta_p(t+1)]}{E_t(\lambda(t,t+1)^{-1})} \right) \frac{1}{\lambda(t,t+1)} \tilde{P}(t) & \text{if } \left(\frac{\pi^*}{E_t(\lambda(t,t+1)^{-1})} \right) \leq 1, \end{cases} \quad (18)$$

where $\vartheta_p(t+1) > 0$ is such that $\pi^*[1+\vartheta_p(t+1)]/E_t(\lambda(t,t+1)^{-1}) = 1^+$, and 1^+ represents a value just larger than unity. Participant households still consume an “equity share” in the real output in the credit market segment of the economy and the consumption of the even dated cash-users is also not impacted by the one-time increase in the price level.²⁷

Therefore, in our model the intervention of the policymaker is such that the gross nominal interest rate equals 1^+ only in periods where the productivity shocks are such that nominal interest rate is expected to be less than unity. The findings of the literature that uses a New Keynesian framework to examine optimal monetary policy when the net nominal interest rate is constrained to be non-negative are different from our results. Jung, Teranishi, Watanabe (2005) find that it is optimal for the central bank to continue a zero interest rate policy even after the natural interest rate returns to a positive level, generating inflationary expectations and thereby effectively lowering the real interest rates. They consider a one-time large negative shock to the natural rate of interest. Adam and Billi (2006, 2007) also study optimal monetary policy under commitment and discretion, respectively, when the nominal interest rate encounters the lower bound. In their set-up, since agents expect the nominal interest rate to encounter the lower bound in the

²⁷This policy achieves the inflation target π^* if the zero lower bound is never encountered. In periods when the zero lower bound threatens, inflation will be higher than otherwise, and so average inflation over the very long run will be somewhat higher than the inflation target. If the policymaker knows the stochastic process for the economy so that the number of expected encounters with the zero lower bound could be reliably estimated, this effect could be taken into account appropriately, for example by setting a slightly lower inflation target at normal times, so that in the very long run, the inflation target π^* would be achieved.

future, the central bank's optimal response is a preemptive easing of the nominal interest rate. Eggertsson and Woodford (2003) also find that optimal policy involves committing to creating an output boom once the natural interest rate is positive therefore creating expectation of future inflation. Such an expectation stimulates current demand.²⁸ See also Gali (2018) for an excellent overview.

In our model, when the agents expect nominal interest rate $\tilde{R}^n(t, t+1) \leq 1$, the central bank promises to create a higher than usual price level at date $t+1$. This implies that the pace of money creation, equation (11) will be higher next period. This is quite distinct from how quantitative easing has been considered in Del Negro, Eggertson, Ferrero and Kiyotaki (2017) for example. In the context of their New Keynesian model with both real and nominal frictions, they examine the quantitative implications of a liquidity shock that causes the nominal interest rate to encounter the zero lower bound. In response to this financial shock, the central bank provides liquidity by exchanging liquid assets for illiquid assets. By calibrating their model to the Great Recession in the U.S. economy, they find that the effects of the shock and the liquidity provided by the Federal Reserve can be quantitatively large. In our framework, both assets are equally liquid and only differ in their rates of return. Therefore, the NSCNC friction makes our results not directly comparable to some of the theoretical literature on quantitative easing.

4.2 Policy 2: Nominal GDP targeting

Policy 2 will have different nominal implications in this economy relative to Policy 1. Accordingly, let us denote the price level under Policy 2 by $\hat{P}(t)$ and the gross nominal interest rate in effect from date $t-1$ to date t under Policy 2 by $\hat{R}^n(t-1, t)$.

The hallmark of Policy 2 is that it is a constant nominal interest rate pol-

²⁸In Eggertsson and Woodford (2003), to implement the optimal policy the central bank chooses the nominal interest rate to achieve a target price level. In our model, the nominal interest rate is determined in the credit market and the central bank chooses the money growth rate in order to implement a particular price level.

icy. Equivalently, the expected rate of nominal GDP growth never changes, and actual nominal GDP simply follows the targeted nominal GDP path associated with the mean real growth rate of the economy $\bar{\lambda}$ and the exogenously given inflation target π^* .

A stationary equilibrium of the incomplete markets economy under Policy 2 is a sequence $\left\{ \hat{P}(t), \hat{R}(t-1, t) \right\}_{t=-\infty}^{+\infty}$ along with the associated consumption and asset allocations each period in which the monetary policymaker rebates seigniorage to even-dated cash users, keeps the gross nominal interest rate constant at $\bar{\lambda}\pi^* > 1$ and credibly adheres to a counter-cyclical price rule which determines $\hat{P}(t)$. In this equilibrium both types of households, participant and non-participant, maximize utility subject to their respective constraints and the policy rule, and markets clear. The price rule is given by

$$\hat{P}(t) = \frac{\bar{\lambda}\pi^*}{\lambda(t-1, t)} \hat{P}(t-1). \quad (19)$$

The consumption of both types of households is the same as under Policy 1. This is because the two policies only differ in terms of the nominal variables such as the nominal interest rate, the price level and inflation. Policy 2 is a strict nominal GDP targeting rule (see Appendix C for more details) and nominal income grows at rate $\bar{\lambda}\pi^*$.²⁹ In fact, as in Sheedy (2014), the nominal income in this case is non-state contingent.

4.3 Discussion

Policies 1 and 2 represent methods by which a monetary policymaker in this model could provide optimal risk-sharing and ensure that consumption allocations are the same as in the solution to the planner's problem. Nevertheless, these two policies differ along a few dimensions: (i) While both policies avoid the zero lower bound, Policy 1 requires an increase in the price level whenever the zero lower bound threatens to bind; (ii) By implementing

²⁹Note that this would imply that $\theta(t-1, t) = \pi^*\bar{\lambda}$ where the central bank chooses a *fixed currency stock growth rule*, a policy recommendation often associated with Milton Friedman.

Policy 1, the monetary authority sets expected inflation equal to its target (time invariant) each period. We stress that inflation has no impact on real allocations in our stylized economy—nevertheless one might be interested in at least knowing what type of policy rule would produce the lower inflation volatility because it is a practical concern and also because it might be an important consideration in a related but more elaborate economy in which the welfare cost of inflation is positive. In our calibrated cases, Policy 1 produces lower inflation variability.

Figures 4, 5 and 6 plot the nominal interest rate and inflation for different calibrations of the shock process for the two policy implementations. In plotting these figures, we use $\rho = 0.9$, $\eta \sim N(0, 1)$ and $\bar{\lambda} = \pi^* = 1.02$ and we vary the standard deviation of the shock to the productivity growth rate. In Figure 4, the standard deviation is set to $\sigma = 0.007$, in Figure 5 it is 0.01 and in Figure 6 it is 0.02.

By implementing Policy 2, the monetary authority targets nominal GDP and therefore nominal GDP grows at gross rate $\bar{\lambda}\pi^*$. However, under Policy 1, the monetary authority only partially returns nominal GDP towards its target after a shock. Therefore this policy is not a strict, but instead a partial, nominal GDP targeting policy.³⁰ As noted in Sheedy (2014), Policy 2—a policy of strict nominal GDP targeting—is generally in conflict with inflation targeting, because any fluctuations in the level of productivity, and hence output, lead to fluctuations in inflation of the same size and in the opposite direction. Policy 1 in fact mitigates these unnecessarily large fluctuations in inflation.³¹

³⁰We note that this model is unlikely to fit macroeconomic data from recent decades, since the monetary policy supporting the stationary equilibrium here has not been the one in use in the largest economies in recent years. Central banks around the world have mostly adopted policies emphasizing stable prices. The historically-observed price stability policy is inappropriate in the economy studied in this paper.

³¹Using these calibrated values, the average variability, over 5000 simulations, of inflation is 4-5 times more under Policy 2 compared to Policy 1.

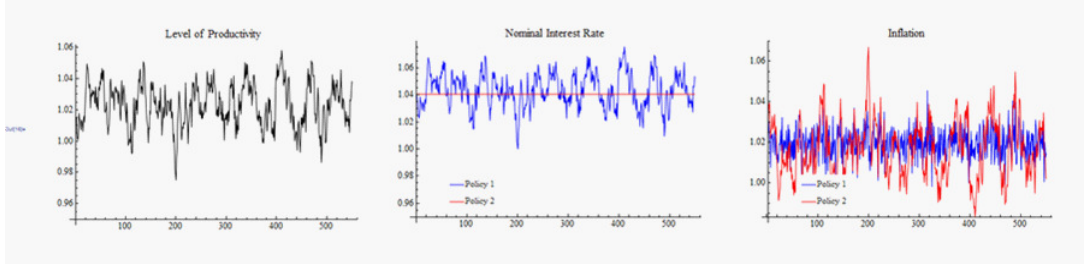


Figure 4: The figure plots the level of productivity, nominal interest rate and inflation target under Policy 1 (blue line) and Policy 2 (red line). The calibrated values of the shock process are: $\rho = 0.9$, $\bar{\lambda} = \pi^* = 1.02$ and $\sigma = 0.007$.

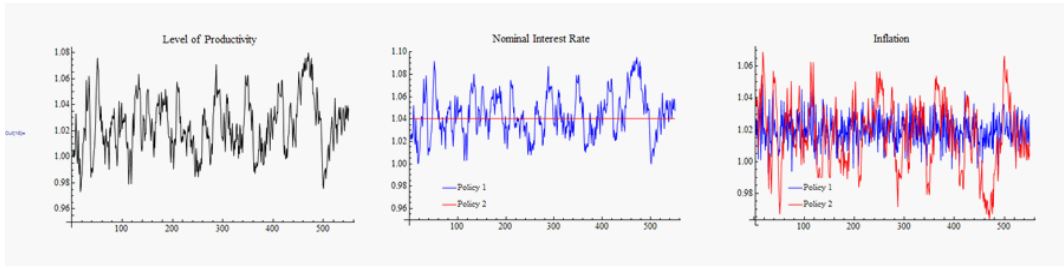


Figure 5: The figure plots the level of productivity, nominal interest rate and inflation under Policy 1 (blue line) and Policy 2 (red line). The calibrated values of the shock process are: $\rho = 0.9$, $\bar{\lambda} = \pi^* = 1.02$ and $\sigma = 0.01$.

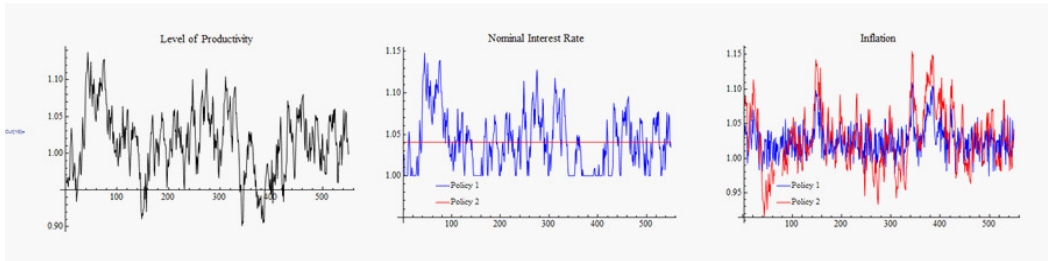


Figure 6: The figure plots the level of productivity, nominal interest rate and inflation under Policy 1 (blue line) and Policy 2 (red line). The calibrated values of the shock process are: $\rho = 0.9$, $\bar{\lambda} = \pi^* = 1.02$ and $\sigma = 0.02$.

5 Conclusions

This model has some ability to address core issues concerning recent monetary policy, which, because of the financial crisis of 2007-2009, has become more focused on private credit market behavior. The model has substantial income and financial wealth heterogeneity, which gives rise to a large and active private credit market with some realistic features, including relatively young households wishing to pull consumption forward in the life cycle, relatively old households saving for the later stages of life, and cash-using households that are precluded from the credit market. The net nominal interest rate is positive at all times, which ensures that cash is only used by the credit market non-participants in the equilibria that we consider in this paper. In one monetary policy scenario, a relatively large and persistent negative aggregate shock (that is, a big recession) means that this nominal interest rate can sometimes be expected to endogenously encounter the zero lower bound.

The key friction in the model is non-state contingent nominal contracting (NSCNC) in the credit sector. The non-state contingency means that credit market equilibrium will feature inefficient risk sharing if there is no intervention.³² However, the fact that the borrowing and lending in our model is in nominal terms means that the monetary authority may be able to replace the missing state-contingency with appropriate price level movements. We find that in the stationary equilibria we study, counter-cyclical price level movements along with currency seigniorage rebates to the cash users provides optimal risk-sharing in the credit markets and insulates cash users from changes in their consumption due to price level movements. We also find that there are multiple price level monetary policies that can implement this consumption allocation.

To pin down the monetary policy more explicitly, we consider two possible secondary monetary policy objectives that could be used to implement the

³²In our analysis we have not considered an equilibrium where in fact policy is sub-optimal and there is inefficient risk sharing. Bullard and Singh (2019) examine this in a model with elastic labor supply.

optimal allocations coming from the planner’s problem. One sets expected inflation to target inflation each period and prevents the nominal interest rate from encountering the zero lower bound through a commitment to a one-time increase in the price level. The other is a policy of strict nominal GDP targeting in which nominal GDP never deviates from a prescribed path. In our stylized model, both of these implementations will deliver the same consumption allocations, and so, strictly speaking, the two policies cannot be ranked. However, each policy has its practical advantages. For example, in variations of this model where inflation volatility played an important role for welfare, Policy 1 may be more appropriate.

Overall this paper contributes to the small but growing literature on optimal monetary policy with incomplete markets, heterogeneous agents and nominally-denominated debt. It also contains results relating to the large New Keynesian literature on the zero (or effective) lower bound. We think it would be interesting to include a meaningful welfare cost of inflation and examine how that might impact the results of our analysis and we hope to address that issue in future research.

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A Details of model solution

The model features heterogeneous households and an aggregate shock, so that the evolution of the asset-holding distribution in the economy is part of the description of the equilibrium. This would normally require numerical computation. However, our simplifying assumptions, including the symmetry of the productivity profile and log preferences, allow solution by “pencil and paper” methods. In this appendix we outline this solution in some detail.

A key feature of the solution is that the asset-holding distribution is linear in the current real wage $w(t)$, and therefore that it simply shifts up and down with changes in $w(t)$. Another key feature of the solution is that the ex post real rate of return on asset-holding is equal to the ex post real output growth rate period-by-period.

To find the solution we employ the guess-and-verify solution strategy and proceed as follows.

Step 1: We first propose the generic state-contingent policy rule for the price level P .

Step 2a: We then solve the problem of a participant household entering the economy at date t under the proposed policy and determine their state-contingent plan for consumption and asset holding.

Step 2b: We solve the problem of the other participant households entering the economy at earlier dates with shorter horizons and asset holdings from the previous period and determine how they adjust their asset holdings as the real wage evolves.

Step 3: We establish the equality of per capita consumption of the participant households—the optimal “equity share” contract.

Step 4: We verify that under the proposed policy rule and the derived participant household behavior, the loan market clearing condition is satisfied in the credit sector when the real rate of return on asset holding is equal to the output growth rate (equivalently, the productivity growth rate).

Step 5: Finally, we determine the consumption of the cash users given Step 1 and the assumption that the central bank fully rebates the seigniorage to the cash users.

Step 1. The household entering the economy at any date face uncertainty about income over their life cycle because it does not know what the real wage level is going to be in the future. In addition, these households can borrow and lend using one period NSCNC debt. The proposed generic policy rule is such that it makes the price level state contingent and is given by

$$P(t) = \frac{R^n(t-1, t)}{\lambda(t-1, t)} P(t-1) \quad (\text{A.1})$$

for all t , with $P(0) > 0$. Note that the gross nominal interest rate, $R^n(t-1, t) = \hat{R}^n(t-1, t) = \left(\frac{\pi^*}{E_{t-1}(\lambda(t-1, t)^{-1})} \right)$ when the central bank pursues Policy 1 and $R^n(t-1, t) = \tilde{R}^n(t-1, t) = \pi^* \bar{\lambda}$ when the central bank pursues Policy 2. The nominal variables differ for the two policies and are plotted in Figures 4, 5 and 6 in the main text. However, as noted in the main text, the consumption allocations of the credit users and the cash users are the same under the two policies.

Step 2a.

First consider the optimization problem of the participant households

entering the economy at date t

$$\max_{\{c_t(t+s)\}_{s=0}^T} E_t \sum_{s=0}^T \ln c_t(t+s) \quad (\text{A.2})$$

subject to life-time budget constraint

$$\begin{aligned} c_t(t) + \sum_{s=1}^T \left(\frac{P(t+s)}{P(t)} \frac{c_t(t+s)}{\prod_{j=0}^{s-1} R^n(t+j, t+j+1)} \right) \\ \leq e_0 w(t) + \sum_{s=1}^T \left(\frac{P(t+s)}{P(t)} \frac{e_s w(t+s)}{\prod_{j=0}^{s-1} R^n(t+j, t+j+1)} \right). \end{aligned} \quad (\text{A.3})$$

Substitution of the proposed generic state-contingent policy rule into the budget constraint yields

$$c_t(t) + \sum_{s=1}^T \left(\frac{c_t(t+s)}{\prod_{j=0}^{s-1} \lambda(t+j, t+j+1)} \right) \leq w(t) \sum_{s=0}^T e_s. \quad (\text{A.4})$$

Under the timing protocol we have assumed, $w(t)$ is known by the household at the time when this problem is solved. Let μ be the multiplier on the life-time budget constraint. The sequence of first order conditions with respect to consumption are

$$\frac{1}{c_t(t)} = \mu, \quad (\text{A.5})$$

for $s = 0$ and

$$1 = E_t \left[\frac{\mu c_t(t+s)}{\prod_{j=0}^{s-1} \lambda(t+j, t+j+1)} \right] \quad (\text{A.6})$$

for $s = 1, 2, \dots, T$. Substituting the first order conditions back into the life-time budget constraint implies that

$$c_t(t) = w(t) \frac{1}{T+1} \sum_{j=0}^T e_j. \quad (\text{A.7})$$

We conclude that the choice for the first period consumption, $c_t(t)$, depends on today's wage $w(t)$ alone and not on expectations of future real wage rates. This is because the policymaker is providing insurance to the household. The household has a state-contingent consumption plan and chooses to consume

$$c_t(t+s) = \left[\prod_{j=0}^{s-1} \lambda(t+j, t+j+1) \right] c_t(t) \quad (\text{A.8})$$

at each stage of life $s = 1, 2, \dots, T$, which depends on the realizations of future shocks to the productivity growth rate λ . Individual consumption will therefore grow at the growth rate of output.

The participant household also carries real assets into the next period. Recall that asset holding a is expressed in nominal terms in our notation. The date t real value of these assets is therefore given by

$$\frac{a_t(t)}{P(t)} = e_0 w(t) - c_t(t) \quad (\text{A.9})$$

$$= e_0 w(t) - \frac{1}{T+1} w(t) \sum_{j=0}^T e_j \quad (\text{A.10})$$

$$= w(t) \left[e_0 - \frac{1}{T+1} \sum_{j=0}^T e_j \right]. \quad (\text{A.11})$$

Step 2b.

There are also participant households that entered the economy at date $t-1, t-2, \dots, t-T$ that solve a similar problem. These households bring nominal asset holdings $a_{t-1}(t-1), a_{t-2}(t-1), \dots, a_{t-T}(t)$, respectively, into the current period, and have a shorter remaining horizon in their life cycle. Here we solve for the solution to a household problem for a household

that entered the economy at date $t - 1$. In particular, we show that asset holdings $a_{t-1}(t)$ are still linear in the current real wage $w(t)$. We then infer solutions for all of the other household problems for households entering the economy at dates $t - 2, \dots, t - T$.

The participant household entering the economy at date $t-1$ with $a_{t-1}(t-1)$ solves the following problem at date t :

$$\max_{\{c_{t-1}(t+s-1)\}_{s=1}^T} E_t \sum_{s=1}^T \ln c_{t-1}(t+s-1)$$

subject to life-time budget constraint

$$\begin{aligned} c_{t-1}(t) + \sum_{s=2}^T \left(\frac{P(t+s-1)}{P(t)} \frac{c_{t-1}(t+s-1)}{\prod_{j=0}^{s-2} R^n(t+j, t+j+1)} \right) \\ \leq e_1 w(t) + \sum_{s=2}^T \left(\frac{P(t+s-1)}{P(t)} \frac{e_s w(t+s-1)}{\prod_{j=0}^{s-2} R^n(t+j, t+j+1)} \right) \\ + \frac{R^n(t-1, t) a_{t-1}(t-1)}{P(t)}. \quad (\text{A.12}) \end{aligned}$$

The last term in this “remaining lifetime” budget constraint is given by equation (A.9) in Step 2a. The nominal value of $a_{t-1}(t-1)$ is

$$a_{t-1}(t-1) = P(t-1) w(t-1) \left[e_0 - \frac{1}{T+1} \sum_{s=0}^T e_s \right]. \quad (\text{A.13})$$

Using the proposed generic policy rule and the law of motion for $w(t)$, we

can write

$$a_{t-1}(t-1)R^n(t-1, t) = R^n(t-1, t) \frac{P(t)\lambda(t-1, t)}{R^n(t-1, t)} \frac{w(t)}{\lambda(t-1, t)} \left[e_0 - \frac{1}{T+1} \sum_{s=0}^T e_s \right] \quad (\text{A.14})$$

$$= P(t)w(t) \left[e_0 - \frac{1}{T+1} \sum_{s=0}^T e_s \right]. \quad (\text{A.15})$$

Therefore, the last term in the budget constraint can be written as

$$\frac{R^n(t-1, t) a_{t-1}(t-1)}{P(t)} = w(t) \left[e_0 - \frac{1}{T+1} \sum_{s=0}^T e_s \right]. \quad (\text{A.16})$$

The remaining life budget constraint therefore simplifies to

$$c_{t-1}(t) + \sum_{s=2}^T \left(\frac{c_{t-1}(t+s-1)}{\prod_{j=0}^{s-2} \lambda(t+j, t+j+1)} \right) \leq w(t) \left(\sum_{s=1}^T e_s + e_0 - \frac{1}{T+1} \sum_{s=0}^T e_s \right). \quad (\text{A.17})$$

Let's now turn to the first order condition for this problem. We have

$$\frac{1}{c_{t-1}(t)} = \mu, \quad (\text{A.18})$$

and for $s = 2, \dots, T$ we have

$$1 = E_t \left[\frac{\mu c_{t-1}(t+s-1)}{\prod_{j=0}^{s-2} \lambda(t+j, t+j+1)} \right] \quad (\text{A.19})$$

This yields

$$c_{t-1}(t) = w(t) \frac{1}{T+1} \left[\sum_{j=0}^T e_j \right]. \quad (\text{A.20})$$

This is linear in $w(t)$. This date t households' net assets equal:

$$\begin{aligned} \frac{a_{t-1}(t)}{P(t)} &= e_1 w(t) - c_{t-1}(t) + \frac{R^n(t-1, t) a_{t-1}(t-1)}{P(t)} \\ &= w(t) \left[e_0 + e_1 - \frac{2}{T+1} \sum_{s=0}^T e_s \right]. \end{aligned} \quad (\text{A.21})$$

For all other households who entered the economy at date $t-2, t-3, \dots, t-T$, consumption and assets at date t will also be linear in $w(t)$.

Step 3.

Equations (A.7), (A.20) and similarly for other cohorts show that date t consumption is equalized among participant households. The consumption for $j = 0, 1, \dots, T$ is given by

$$c_{t-j}(t) = w(t) \frac{1}{T+1} \sum_{s=0}^T e_s, \quad (\text{A.22})$$

Step 4.

To show that the asset market clears at the conjectured real interest rate which equals the growth rate of productivity, consider the asset market clearing condition given by

$$A(t) = \sum_{s=0}^{T-1} a_{t-s}(t) = 0.$$

For simplicity, let $T+1 = 3$. Dividing throughout by $P(t)$, we can rewrite the asset market clearing condition as

$$\frac{a_{t-1}(t)}{P(t)} + \frac{a_t(t)}{P(t)} = 0 \quad (\text{A.23})$$

which is

$$\frac{a_{t-1}(t)}{P(t)} + \{e_0 w(t) - c_t(t)\} = 0. \quad (\text{A.24})$$

The term $\frac{a_{t-1}(t)}{P(t)}$ can be written as

$$\frac{a_{t-1}(t)}{P(t)} = R(t-1, t) \frac{a_{t-1}(t-1)}{P(t-1)} + \{e_1 w(t) - c_{t-1}(t)\}, \quad (\text{A.25})$$

where $\frac{a_{t-1}(t-1)}{P(t-1)}$ is

$$\frac{a_{t-1}(t-1)}{P(t-1)} = e_0 w(t-1) - c_{t-1}(t-1). \quad (\text{A.26})$$

The entire expression is

$$\begin{aligned} \frac{a_{t-1}(t)}{P(t)} + \frac{a_t(t)}{P(t)} &= R(t-1, t) \{e_0 w(t-1) - c_{t-1}(t-1)\} \\ &\quad + \{e_1 w(t) - c_{t-1}(t)\} + \{e_0 w(t) - c_t(t)\} = 0. \end{aligned} \quad (\text{A.27})$$

The term $c_{t-1}(t)$ can be written as

$$c_{t-1}(t) = R(t-1, t) c_{t-1}(t-1),$$

and the consumption of the two cohorts are under the proposed policy rule are

$$c_t(t) = \frac{w(t) [e_0 + e_1 + e_2]}{3}, \quad (\text{A.28})$$

and

$$c_{t-1}(t-1) = \frac{w(t-1) [e_0 + e_1 + e_2]}{3}. \quad (\text{A.29})$$

Substituting consumption and imposing the symmetry of the endowment profile such that $e_0 = e_2$, we find that $A(t) = 0$ meaning that the asset market for the $T+1 = 3$ period economy clears at the conjectured real interest rate. A similar analysis holds for any integer $T > 2$.

Step 5.

To show that the consumption of all the even-dated cash users at date t is $\gamma w(t)$ if the price rule is given by equation (A.1) and the central bank fully rebates the seigniorage, we first consider the monetary authority.

The policymaker supplies currency, $H(t)$, to the non-participant households—the cash users. The total real value of currency outstanding in the economy at date t is given by $H(t)/P(t)$. We normalize the date 0 currency level to $H(0) = 1$.

The date t total demand for currency (by the odd-dated cash users) is given by

$$H(t) = \frac{T}{2} P(t) \gamma w(t). \quad (\text{A.30})$$

The central bank chooses growth rate of currency between any two dates $t-1$ and t , $\theta(t-1, t)$, such that the currency supply at date t can be written as

$$H(t) = \theta(t-1, t) H(t-1). \quad (\text{A.31})$$

Equating the demand and supply of currency we get

$$\frac{\gamma T}{2} w(t) P(t) = \theta(t-1, t) \frac{\gamma T}{2} w(t-1) P(t-1) \quad (\text{A.32})$$

which can be written as

$$\theta(t-1, t) = \frac{P(t)}{P(t-1)} \frac{w(t)}{w(t-1)} = R^n(t-1, t), \quad (\text{A.33})$$

where the last equality is after we incorporate the proposed price rule.

The real seigniorage, $S(t)$, earned by the monetary policymaker at date t is

$$S(t) = [\theta(t-1, t) - 1] \frac{T}{2} \frac{P(t-1) \gamma w(t-1)}{P(t)} \quad (\text{A.34})$$

$$= \frac{T}{2} \gamma w(t) - \frac{T}{2} \frac{\gamma w(t)}{R^n(t-1, t)}. \quad (\text{A.35})$$

The entire seigniorage is equally redistributed to the $T/2$ even-dated cash-users and therefore each of them receives

$$= \gamma w(t) - \frac{\gamma w(t)}{R^n(t-1, t)}. \quad (\text{A.36})$$

Given the policy rule, the consumption of the even-dated cash users, for $s = 2, 4, \dots, T$, is

$$c_{t-s}^{np}(t) = \frac{\gamma w(t-1) P(t-1)}{P(t)} + \gamma w(t) - \frac{\gamma w(t)}{R^n(t-1, t)} = \gamma w(t) \quad (\text{A.37})$$

B Derivation of the price rule for Policy 1

Consider a monetary policy given by the following equation

$$P(t) = \frac{\pi^* N(t-1)}{\lambda(t-1, t)} P(t-1), \quad (\text{B.1})$$

where $\pi^* \geq 1$ and $N(t-1) > 0$ denotes a (possibly time-varying) parameter. With this price rule, the central bank achieves its primary goal of providing optimal risk sharing in the credit market, but can now choose a value of $N(t-1)$ based on a secondary criterion.

Let's suppose that at any date $t-1$, the objective of the central bank is to choose $N(t-1)$ such that expected inflation at date t equals the target π^* or on average gross inflation equals π^* . This would simply imply setting

$$E_{t-1} \frac{\pi^* N(t-1)}{\lambda(t-1, t)} = \pi^* \quad (\text{B.2})$$

$$N(t-1) = \frac{1}{E_{t-1} [\lambda(t-1, t)^{-1}]}. \quad (\text{B.3})$$

Therefore the price rule under Policy 1 is given by equation (12) when the agents do not expect the zero lower bound on the nominal interest rate to bind. If given the optimal $N(t-1)$ in equation (B.3), the zero lower bound on the nominal interest rate threatens to bind, then the monetary policymaker can temporarily increase the price level given by equation (18) in the main text.

C Nominal GDP targeting

Policy 2 implies strict nominal GDP targeting while Policy 1 implies partial nominal GDP targeting. In this model, nominal GDP at date t

$$Y^n(t) = P(t) w(t) \left[\sum_{i=0}^T e_i + \frac{T\gamma}{2} \right] \quad (\text{C.1})$$

Then the target nominal GDP at date t , assuming $P(0) = 1$ and $w(0) = 1$ simplifies to

$$Y^{n,*}(t) = (\pi^* \bar{\lambda})^t \left[\sum_{i=0}^T e_i + \frac{T\gamma}{2} \right], \quad (\text{C.2})$$

and the target at date $t + 1$ can be written as

$$Y^{n,*}(t+1) = P(t+1)w(t+1) \left[\sum_{i=0}^T e_i + \frac{T\gamma}{2} \right] \quad (\text{C.3})$$

$$= \pi^* \bar{\lambda} P(t) w(t) \left[\sum_{i=0}^T e_i + \frac{T\gamma}{2} \right] \quad (\text{C.4})$$

which is also equal to the actual nominal GDP at date $t + 1$ under Policy 2 where the price level is given by $\hat{P}(t)$.

However, in the case of Policy 1, consider the nominal GDP at date $t + 1$

$$\tilde{Y}^n(t+1) = \tilde{P}(t+1) w(t+1) \left[\sum_{i=0}^T e_i + \frac{T\gamma}{2} \right] \quad (\text{C.5})$$

$$\tilde{Y}^n(t+1) = \tilde{P}(t+1) w(t+1) \left[\sum_{i=0}^T e_i + \frac{T\gamma}{2} \right] \quad (\text{C.6})$$

$$= \frac{\pi^*}{E_t(\lambda(t, t+1)^{-1}) \lambda(t, t+1)} \tilde{P}(t) \lambda(t, t+1) w(t) \left[\sum_{i=0}^T e_i + \frac{T\gamma}{2} \right] \quad (\text{C.7})$$

$$= \frac{\pi^*}{E_t(\lambda(t, t+1)^{-1})} \tilde{P}(t) w(t) \left[\sum_{i=0}^T e_i + \frac{T\gamma}{2} \right]. \quad (\text{C.8})$$

Comparison of (C.3) and (C.8) indicates that the monetary policy returns nominal GDP partially toward target depending on the value of ρ . So Policy 1 is in fact “partial” nominal GDP targeting.