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TRADE AND LABOR MARKET DYNAMICS: GENERAL EQUILIBRIUM ANALYSIS OF THE CHINA TRADE SHOCK[1]

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We develop a dynamic trade model with spatially distinct labor markets facing varying exposure to international trade. The model captures the role of labor mobility frictions, goods mobility frictions, geographic factors, and input-output linkages in determining equilibrium allocations. We show how to solve the equilibrium of the model and take the model to the data without assuming that the economy is at a steady state and without estimating productivities, migration frictions, or trade costs, which can be difficult to identify. We calibrate the model to 22 sectors, 38 countries, and 50 U.S. states. We study how the rise in China's trade for the period 2000 to 2007 impacted U.S. households across more than a thousand U.S. labor markets distinguished by sector and state. We find that the China trade shock resulted in a reduction of about 0.55 million U.S. manufacturing jobs, about 16% of the observed decline in manufacturing employment from 2000 to 2007. The U.S. gains in the aggregate, but due to trade and migration frictions, the welfare and employment effects vary across U.S. labor markets. Estimated transition costs to the new long-run equilibrium are also heterogeneous and reflect the importance of accounting for labor dynamics.

KEYWORDS: Labor market dynamics, international trade, migration, internal trade, economic geography, mobility frictions, trade costs, input-output linkages, China's trade, welfare effects, general equilibrium, manufacturing employment.

1. INTRODUCTION

Understanding and quantifying the employment effects of trade shocks has been a central issue in recent research. A standard approach, relying on reduced-form analysis, has provided robust empirical evidence on the differential effects of trade shocks across local labor markets. These studies, however, say little about the effects on overall employment, welfare, or other aggregate outcomes and cannot be used to study counterfactual policies. In this paper we study the general equilibrium effects on U.S. labor markets of a surge in China's productivity, a shock that accounts for the increase in Chinese import penetration into the U.S. market.

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We develop a dynamic spatial trade and migration model to understand and quantify the disaggregate labor market effects resulting from changes in the economic environment. The model explicitly recognizes the role of labor mobility frictions, goods mobility frictions, geographic factors, input-output linkages, and international trade in shaping the effects of shocks across different labor markets. Hence, our model has intersectoral trade, interregional trade, international trade, and labor market dynamics.

In our economy, production takes place in spatially distinct markets. A market is a sector located in a particular region in a given country. In each market there is a continuum of heterogeneous firms producing intermediate goods à la Eaton and Kortum, 2002 (hereafter EK). Firms are competitive, have constant-returns-to-scale technology, and demand labor, local factors, and materials from all other markets in the economy. The supply side of the economy features forward-looking households choosing whether to be employed or non-employed in the next period and in which labor market to supply labor, conditional on their location, the state of the economy, sectoral and spatial mobility costs, and an idiosyncratic shock à la Artuç et al., 2010 (hereafter ACM). Employed households supply a unit of labor and receive the local competitive market wage; non-employed households obtain consumption in terms of home production. Incorporating these elements delivers a general equilibrium, dynamic discrete choice model with realistic geographic features and input-output linkages.

Taking a dynamic trade model with all these features to the data, and performing a counterfactual analysis, may seem unfeasible since it requires pinning down a large set of exogenous state variables (hereafter referred to as fundamentals), such as productivity levels across sectors and regions, bilateral mobility (migration) costs across markets, bilateral international and domestic trade costs, and endowments of immobile local factors.[1] Our methodological contribution is to show that, under perfect foresight, by expressing the equilibrium conditions in relative time differences we are able to solve the model and perform large-scale counterfactual analyses without needing to estimate the fundamentals of the economy. Aside from data that directly map into the model's equilibrium conditions, the only parameters we need in order to solve the full transition of the dynamic model are the trade elasticities, the migration elasticity, and the intertemporal discount factor.

Our method relies on conditioning on the observed allocations. The intuition is that the observed allocations are sufficient statistics for the fundamentals of the economy. Our result builds on Dekle et al., 2008 (hereafter DEK), who have shown a similar result in the context of a static trade model.[2] We show how to apply our method, which we

[1] Our model belongs to a class of dynamic discrete choice models in which estimation and identification of these large sets of fundamentals is, in general, challenging. For more details, see Rust (1987, 1994). For recent studies that estimate fundamentals in a similar context to ours, see ACM, and Dix-Carneiro (2014).

[2] Costinot and Rodríguez-Clare (2014) coin the term "exact hat algebra" and show that this technique also holds in a large variety of static trade models, even under the presence of fixed costs. What we are proposing is a "dynamic exact hat algebra." Other recent applications of the exact hat algebra method

label "dynamic hat algebra," to study the effects of actual or counterfactual changes to fundamentals using a dynamic discrete choice spatial trade model.[3] We illustrate this point by presenting several examples that highlight the type of questions that can be answered by applying our method. In addition, we show how to take the model to the data without assuming that the economy is at a steady state and discuss measurement requirements.

In our empirical section, we apply our model and solution method to study the effects of the rise in China's import competition on U.S. labor markets over the period 2000-2007. U.S. imports from China more than doubled from 2000 to 2007. During the same period, manufacturing employment fell considerably, while employment in other sectors, such as construction and services, grew. Several reduced-form studies (e.g., Autor et al., 2013, hereafter ADH; Acemoglu et al., 2016; Pierce and Schott, 2016) document that an important part of the employment loss in manufacturing was a consequence of China's trade expansion, either as a consequence of technological improvements in the Chinese economy or reductions in trade costs.[4]

We use our model to quantify how additional channels can also explain the employment loss in manufacturing sectors and how other sectors of the economy, such as construction and services, were also exposed to the China shock. More importantly, we use our model to compute the general equilibrium and welfare effects across labor markets over time. In summary, we account for the distributional effects across sectors and regions of the U.S. economy caused by the increase in Chinese competition as well as the aggregate effects.

We do this with a 38-country, 50-U.S.-state, and 22-sector version of our model.[5] We take data on the distribution of labor across markets in the U.S. economy and match our model to those of the years 2000 to 2007. We rely on the identification restriction suggested by ADH to measure China's shock; namely, we use the predicted changes in U.S. imports from China using as an instrument the change in imports from China by other high-income countries for the period 2000-2007. Using our model, we compute the changes in sectoral productivities in China between 2000 and 2007 that generate the same change in U.S. imports from China as predicted by the ADH regression. We label these changes in productivity the China trade shock and refer to them as such in the rest

are Caliendo and Parro (2015) and Burstein et al. (forthcoming). Eaton et al. (2016) show how to apply DEK in the context of multi-country trade model with capital accumulation.

[3]As we explain later, our method consists of solving the model in time differences and relative to a baseline economy. By doing so, we differentiate out the effects of a set of fundamentals on equilibrium allocations, and therefore this method can also be thought of as a structural difference in difference.

[4]ADH argue that structural reforms in the Chinese economy resulted in large technological improvements in export-led sectors. As a result, China's import penetration into the United States increased. Handley and Limão (2017) and Pierce and Schott (2016) argue that the United States' elimination of uncertainty about tariff increases on Chinese goods was another important reason why U.S. imports from China grew.

[5]It is worth noting that for an application of this dimension, not using our solution method will require estimating $N \times R \times J$ productivity levels, $N^2 \times R^2 \times J$ asymmetric bilateral trade costs, $N^2 \times R^2 \times J^2$ labor mobility costs, and $N \times R \times J$ stocks of local factors, where N, R, and J are countries, regions, and sectors, respectively.

of the paper.[6]

With our calibrated model and China trade shock, the main question we answer is the following: Imagine that agents anticipated all changes in fundamentals exactly as they occurred. This is the factual world we have lived in. Now, consider a counterfactual in which all of these changes in fundamentals occurred except there was no China shock; i.e., the estimated Chinese productivities did not change over time. What would have happened differently across U.S. labor markets?

We find that increased Chinese competition reduces the aggregate manufacturing employment share by 0.36 percentage points in the long run, which is equivalent to a reduction of about 0.55 million manufacturing jobs, or about 16 percent of the observed decline in manufacturing employment from 2000 to 2007.[7,8] We also find that workers relocate to construction and the services sectors, as these sectors expand due to the access to cheaper intermediate inputs from China. For instance, we find that about 50,000 jobs were created in construction as a result of the China shock.

With our model we can also quantify the relative contributions of different sectors, regions, and labor markets to the decline in manufacturing employment. We find that sectors in regions with higher exposure to import competition from China reduce the number of manufacturing jobs not only relative to the less exposed regions but also in absolute terms. The computer and electronics industry contributes about 25 percent of the decline in manufacturing employment, followed by the furniture, textiles, metal, and machinery industries, each contributing 10-15 percent to the total decline. Some sectors, such as food, beverage, and tobacco, experience much smaller employment effects, as they were less exposed to China and benefited from cheaper intermediate goods.

The fact that U.S. economic activity is not equally distributed across space, plus the differential sectoral exposure to China, implies that the impact of China's import competition varies across regions. We find that U.S. states with a larger concentration of sectors more exposed to China lose more manufacturing jobs. California, which by far accounts for the largest share of employment in computer and electronics (the sector most exposed to China's import competition), accounted for about 9 percent of the decline.

We also find that the change in employment shares across space is heterogeneous across industries. In particular, the reduction in local employment shares in manufacturing industries is more concentrated in a handful of states, while the increase in local

[6]Another way to interpret the China shock could be as a decline in trade costs that matches the predicted change in import competition from China as predicted by ADH. In the context of the model developed in the next section, which delivers a gravity structure, both are isomorphic as long as the decline in trade costs due to the China shock is only origin specific.

[7]The observed reduction in manufacturing employment in the United States from 2000 to 2007 was 3.4 millions according to the Department of Labor, Bureau of Labor Statistics.

[8]The 0.55 million is about 36% of the change in the aggregate manufacturing employment share unexplained by a secular trend. We compute the secular trend for the U.S. manufacturing employment share of total private employment as a linear trend from 1967 to 1999, the year before the China shock. The trend predicts a share of 12.83% for 2007, while the observed share was 11.85%. More details are provided in Section 5.

employment shares in non-manufacturing industries is spread more evenly across U.S. states.

Our framework also allows us to quantify the welfare effects of the increased competition from China on the U.S. economy. Our results indicate that the China shock increased U.S. welfare by 0.2%. Therefore, even when U.S. exposure to China decreases employment in the manufacturing sector, the U.S. economy is better off, as it benefits from access to cheaper goods from China. We also find a large dispersion in welfare effects across individual labor markets, ranging from about -0.8% to 1%. Larger welfare gains are generally in labor markets that produce non-manufacturing goods, as these industries do not suffer the direct adverse effects of the increased competition from China and benefit from access to cheaper intermediate manufacturing inputs from China used in production. Similarly, labor markets in states that trade more with the rest of the U.S. economy and purchase materials from sectors where Chinese productivity increases, tend to have larger welfare gains.

We also find that the welfare gains from the China shock take time to materialize due to relocation costs. For instance, while welfare increases in the manufacturing and non-manufacturing industries in the long run across all U.S. states, regional real wages in manufacturing industries decline during the China shock period, as workers in labor markets impacted by increased import competition cannot immediately relocate to other industries or states.

We compute the welfare effects in the rest of the world and find that all countries gain from the China shock, with some countries having larger welfare gains than the United States and others smaller. Since reaching the new steady state after the China shock takes time due to mobility frictions, we compute the transition or adjustment costs to the new steady state and find substantial variation across labor markets.

We also use our general equilibrium model to study other counterfactual questions related to the China shock. In particular, we ask the question: what would the employment effects across U.S. labor markets have been if the actual disability benefits would have been eliminated when the China shock occurred? To do so, we extend the model and introduce disability benefits that are financed by federal taxes, and we match the transition probabilities from non-employment into and out of the disability program. We find that the disability program amplified the decline in manufacturing employment by about 0.03 percentage points, that is, a reduction of about 50 thousand additional manufacturing jobs, and we also find an increase in the non-employment rate in the long run. The effects of the disability program on manufacturing employment tend to be larger in regions that are more concentrated in the manufacturing industries and where it is more difficult for workers to relocate to other industries.

We further extend our model in other dimensions by incorporating additional sources of persistence, CES preferences, and, elastic labor supply. We show that the dynamic hat algebra works in these alternative models and discuss their quantitative implications, which are similar to our baseline results. One extension that we do not consider in this paper is modelling the stochastic process of fundamentals. Such an extension would require departing from the perfect foresight assumption. Our approach will not necessarily fail if one were to relax the assumption of perfect foresight, but adding rational expectations would imply solving the model for every possible realization of fundamentals in the future, which in our application, with more than 1000 endogenous state variables, is a computational constraint.

Our paper relates to recent studies of the effects of Chinese import competition on labor markets in the United States, most notably ADH. We find that the differential employment effects across regions estimated with our method are in line with the ones by ADH. Our contribution relative to reduced-form studies is that in addition to the differential effects, we provide a general equilibrium quantification of the level of employment effects, as well as the aggregate and distributional welfare effects, discuss the important mechanisms, and perform policy evaluations.[9] These dimensions of the effects of the China shock are an important focus of our empirical analysis and complement the reduced-form approach.[10]

Our paper also relates to Acemoglu et al. (2016), who extend the ADH framework to study the employment effects of the China shock that operate through input-output linkages. Assuming that labor markets are small open economies without labor market dynamics as a source of adjustment, they find no employment gains in non-manufacturing industries. We find that the general equilibrium effects and the dynamic effects of the China shock on labor mobility are important drivers of the expansion of employment in non-manufacturing industries in the long run.[11]

Our approach also relates to a fast-growing strand of the literature that studies the impact of trade shocks on labor market dynamics.[12] The work most closely related to ours is Artuc and McLaren (2010) and ACM. We follow Artuc and McLaren (2010) and ACM in modeling the migration decisions of agents as a dynamic discrete choice problem. We depart from their assumption of a small open economy in partial equilibrium and introduce a multicountry, multiregion, multisector general equilibrium trade model with trade and migration costs. Our study is also complementary to Dix-Carneiro (2014), who

[9]Difference-in-difference estimates can only be used to infer how many more, or fewer, workers were employed in one labor market compared to another. In order to compute the aggregate employment effects using difference-in-difference estimates one needs an estimate of the employment impact in one particular labor market.

[10]More broadly, through the lens of our model, we can study the effects of changes in many economic conditions, for instance, how changes in trade costs, labor migration costs, local structures, productivity, non-employment benefits (or home production), and local policies affect the rest of the economy. In addition, we can analyze how aggregate changes in economic circumstances can have heterogeneous disaggregate effects.

[11]Our analysis not only differs from Acemoglu et al. (2016) due to general equilibrium effects, but also in terms of the level of detail in the industries used and the criteria for the sample selection of households.

[12]For instance, see Dix-Carneiro and Kovak (2017); Cosar (2013); Coşar et al. (2016); Kondo (forthcoming); and Menezes-Filho and Muendler (2011) and the references therein.

focuses on measuring the frictions that workers face to move across sectors and interprets their magnitude by simulating hypothetical trade liberalization episodes. Following Dix-Carneiro (2014), we use our general equilibrium model to quantify the dynamic effects of a trade shock across markets; but unlike him, we rely on our solution method to compute the general equilibrium effects at a more granular level.

Overall, we highlight three main departures of our paper from the previous literature. First, relative to other recent dynamic discrete choice models of labor reallocation, we include a wide range of general equilibrium mechanisms such as multiple countries, input-output linkages, multiple sectors, and multiple factors of production. The resulting framework allows us to study a wider range of policy experiments compared to previous work. Second, we provide a method to compute the model and study counterfactuals without the need to estimate exogenous constant and time-varying fundamentals, which is key in order to take the model to a highly disaggregated level as we do. Finally, our paper complements reduced-form studies on the effects of the China shock. We cannot only measure the differential impact across labor markets, but we can also compute employment effects and measure the welfare effects taking into account general equilibrium channels.

The paper is organized as follows. In Section 2 we present our dynamic spatial trade and migration model. In Section 3 we show how to solve the model and perform counterfactual analysis using the dynamic hat algebra. In Section 4 we explain how to take the model to the data and how we estimate the China shock. In Section 5 we use our model to quantify the effects of increased Chinese competition on different U.S. labor markets. We also present different extensions of the model and discuss additional results. Finally, we conclude in Section 6. All proofs are relegated to the appendix.

2. A DYNAMIC SPATIAL TRADE AND MIGRATION MODEL

We consider a world with N locations and J sectors. We use the indexes n or i to identify a particular location and index sectors by j or k. In each region-sector combination there is a competitive labor market. In each market there is a continuum of perfectly competitive firms producing intermediate goods.

Firms have a Cobb-Douglas constant-returns-to-scale technology, demanding labor, a composite local factor that we refer to as structures, and materials from all sectors. We follow EK and assume that productivities are distributed Fréchet with a sector-specific productivity dispersion parameter θ^j.

Time is discrete, and we denote it by $t = 0, 1, 2, \ldots$ Households are forward looking, have perfect foresight, and optimally decide where to move given some initial distribution of labor across locations and sectors. Households face costs to move across markets and experience an idiosyncratic shock that affects their moving decision. The household's problem is closely related to the sectoral reallocation problem in ACM and to the competitive labor search model of Lucas and Prescott (1974) and Dvorkin (2014).[13]

We first characterize the dynamic problem of a household deciding where to move conditional on a path of real wages across time and across labor markets. We then characterize the static subproblem to solve for prices and wages conditional on the supply of labor in a given market.

2.1. *Households*

At $t = 0$ there is a mass L_0^{nj} of households in each location n and sector j. Households can be either *employed* or *non-employed.* An *employed* household in location n and sector j supplies a unit of labor inelastically and receives a competitive market wage w_t^{nj}. Given the household's income, the household decides how to allocate consumption over local final goods from all sectors with a Cobb-Douglas aggregator. Preferences, $U(C_t^{nj})$, are over a basket of final local goods

$$C_t^{nj} = \prod\nolimits_{k=1}^{J} (c_t^{nj,k})^{\alpha^k}, \tag{1}$$

where $c_t^{nj,k}$ is the consumption of sector k goods in market nj at time t and α^k is the final consumption share, with $\sum_{k=1}^{J} \alpha^k = 1$. We denote the ideal price index by $P_t^n = \prod_{k=1}^{J} \left(P_t^{nk}/\alpha^k\right)^{\alpha^k}$, where P_t^{nk} is the price index of goods purchased from sector k for final consumption in region n, as defined below. As in Dvorkin (2014), *non-employed* households obtain consumption in terms of home production $b^n > 0$.[14] To simplify the notation, we represent sector zero in each region as non-employment; hence, $C_t^{n0} = b^n$.[15]

The household's problem is dynamic. Households are forward looking and discount the future at rate $\beta \geq 0$. Migration decisions are subject to sectoral and spatial mobility costs.

Assumption 1 *Labor relocation costs $\tau^{nj,ik} \geq 0$ depend on the origin (nj) and destination (ik) and are time invariant, additive, and measured in terms of utility.*

In addition, households have additive idiosyncratic shocks for each choice, denoted by ϵ_t^{ik}.

[13] Another related model of labor reallocation is Coen-Pirani (2010). Idiosyncratic preference shocks are widely used in the literature on worker reallocation. See, for example, Dix-Carneiro (2014); Kennan and Walker (2011); Monte (2015); Pilossoph (2014); and Redding (2016).

[14] Alternatively, one could assume that non-employed households use income to buy market goods. In this case, consumption of non-employed households in region n is given by b^n/P_t^n. We consider this alternative specification later on in our quantitative analysis. We also extend our model to include a particular form of non-employment insurance financed with labor income taxes.

[15] To simplify the notation, we ignore local amenities, which can vary by both sector and region. As it will become clear later, our exercise and results are invariant to including these amenities under the assumption that they enter the period utility additively and are constant over time. More general types of amenities, including congestion or agglomeration effects, can also be handled by the solution method we propose, but we abstract from them here.

The timing for the household's problem and decisions is as follows. Households observe the economic conditions in all labor markets and the realizations of their own idiosyncratic shocks. If they begin the period in a labor market, they work and earn the market wage. If they are non-employed in a region, they get home production. Then, both employed and non-employed households have the option to relocate. Formally,

$$\mathrm{v}_t^{nj} = U(C_t^{nj}) + \max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{ \beta E \left[\mathrm{v}_{t+1}^{ik} \right] - \tau^{nj,ik} + \nu \epsilon_t^{ik} \right\},$$

$$s.t.\ \ C_t^{nj} \equiv \begin{cases} b^n & \text{if } j = 0, \\ w_t^{nj}/P_t^n & \text{otherwise;} \end{cases}$$

where v_t^{nj} is the lifetime utility of a household currently in region n and sector j at time t and the expectation is taken over future realizations of the idiosyncratic shock. The parameter ν scales the variance of the idiosyncratic shocks. Note that households choose to relocate to the labor market that delivers the highest utility net of costs.

Assumption 2 *The idiosyncratic shock ϵ is i.i.d. over time and distributed Type-I Extreme Value with zero mean.*

Assumption 2 is standard in dynamic discrete choice models.[16] It allows for simple aggregation of idiosyncratic decisions made by households, as we now show.[17]

Let $V_t^{nj} \equiv E[\mathrm{v}_t^{nj}]$ be the expected lifetime utility of a representative agent in labor market nj, where the expectation is taken over the preference shocks. Then, given Assumption 2, we obtain (see Appendix A)

$$V_t^{nj} = U(C_t^{nj}) + \nu \log \left(\sum_{i=1}^{N} \sum_{k=0}^{J} \exp \left(\beta V_{t+1}^{ik} - \tau^{nj,ik} \right)^{1/\nu} \right). \tag{2}$$

Equation (2) reflects the fact that the value of being in a particular labor market depends on the current-period utility and on the option value to move into any other market in the next period.[18] V_t^{nj} can be interpreted as the expected lifetime utility of a household before the realization of the household preference shocks or, alternatively, as the average utility of households in that market.[19]

Using Assumption 2, we can also show that the share of labor that transitions across markets has a closed-form analytical expression. In particular, denote by $\mu_t^{nj,ik}$ the fraction of households that relocate from market nj to ik (with $\mu_t^{nj,nj}$ the fraction who choose

[16] For a survey on this literature, see Aguirregabiria and Mira (2010).

[17] In Appendix C.4, we extend our model for the case of an elastic labor supply. In particular, we incorporate labor-leisure decisions in each household's utility function, using alternative specifications.

[18] For an example of a model that delivers a similar expression, refer to Artuc and McLaren (2010), ACM, and Dix-Carneiro (2014). ACM also provide an economic interpretation of the different components of the option value to move across sectors.

[19] In our case, the measure of this representative agent evolves endogenously with the change in economic conditions. See Dvorkin (2014) for further details.

to remain in their original location); then (see Appendix A)

$$\mu_t^{nj,ik} = \frac{\exp\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta V_{t+1}^{mh} - \tau^{nj,mh}\right)^{1/\nu}}. \tag{3}$$

Equation (3) , which we refer to as the migration shares, has an intuitive interpretation. All other things being equal, markets with a higher lifetime utility (net of mobility costs) are the ones that attract more migrants. From this expression we can also see that $1/\nu$ can be interpreted as a migration elasticity.

Equation (3) is a key equilibrium condition in this model because it conveys all the information needed to determine how the distribution of labor across markets evolves over time. In particular,

$$L_{t+1}^{nj} = \sum_{i=1}^{N}\sum_{k=0}^{J} \mu_t^{ik,nj}\, L_t^{ik}. \tag{4}$$

The equilibrium condition (4) characterizes the evolution of the economy's state, the distribution of employment and non-employment across markets $L_t = \{L_t^{nj}\}_{n=1,j=0}^{N,J}$. Note that given our timing assumption, the supply of labor at each t is fully determined by forward-looking decisions at period $t-1$. Now, conditional on labor supplied at each market, we can specify a static production structure of the economy that allows us to solve for equilibrium wages at each time t such that labor markets clear. We now proceed to describe the production side of the economy.

2.2. *Production*

Production follows the multisector model of Caliendo and Parro (2015) and the spatial model of Caliendo et al. (2018). Firms in each sector and region are able to produce many varieties of intermediate goods. The technology to produce these intermediate goods requires labor and structures, which are the primary factors of production, and materials, which consist of goods from all sectors.[20] Total factor productivity (TFP) of an intermediate good is composed of two terms, a time-varying sectoral-regional component (A_t^{nj}), which is common to all varieties in a region and sector, and a variety-specific component (z^{nj}).

Intermediate Goods Producers: The output for a producer of an intermediate variety with efficiency z^{nj} is given by

$$q_t^{nj} = z^{nj}\left(A_t^{nj}\,(h_t^{nj})^{\xi^n}(l_t^{nj})^{1-\xi^n}\right)^{\gamma^{nj}} \prod_{k=1}^{J}(M_t^{nj,nk})^{\gamma^{nj,nk}},$$

[20] For example, one sector/industry is computer and electronic product manufacturing (NAICS 334 in the data), which is an aggregate of many varieties such as electronic computers (334111), audio and video equipment (33431), and circuit boards (NAICS 334412). Computer and electronic products are purchased by households for final consumption and by firms as materials for production. When we calibrate the model, we show how the share of expenditures by households and firms is guided by the data.

where l_t^{nj}, h_t^{nj} are labor and structures inputs, respectively, and $M_t^{nj,nk}$are material inputs from sector k demanded by a firm in sector j and region n to produce q units of an intermediate variety with efficiency z^{nj}. Material inputs are goods from sector k produced in the same region n. The parameter $\gamma^{nj} \geq 0$ is the share of value added in the production of sector j and region n, and $\gamma^{nj,nk} \geq 0$ is the share of materials from sector k in the production of sector j and region n. We assume that the production function exhibits constant returns to scale such that $\sum_{k=1}^{J} \gamma^{nj,nk} = 1 - \gamma^{nj}$. The parameter ξ^n is the share of structures in value added. Structures are in fixed supply in each labor market.

We denote by r_t^{nj} the rental price of structures in region n and sector j. The unit price of an input bundle is

$$x_t^{nj} = B^{nj} \left((r_t^{nj})^{\xi^n} (w_t^{nj})^{1-\xi^n} \right)^{\gamma^{nj}} \prod_{k=1}^{J} (P_t^{nk})^{\gamma^{nj,nk}}, \tag{5}$$

where B^{nj} is a constant and P_t^{nj} also applies to goods used as materials in production, as described below. Then, the unit cost of an intermediate good z^{nj} at time t is $\frac{x_t^{nj}}{z^{nj}\ (A_t^{nj})^{\gamma^{nj}}}$.

Trade costs are represented by $\kappa_t^{nj,ij}$ and are of the "iceberg" type. One unit of any variety of intermediate good j shipped from region i to n requires producing $\kappa_t^{nj,ij} \geq 1$ units in region i. If a good is nontradable, then $\kappa = \infty$. Competition implies that the price paid for a particular variety of good j in region n is given by the minimum unit cost across regions, taking into account trade costs, and where the vector of productivity draws received by the different regions is $z^j = (z^{1j}, z^{2j}, \ldots, z^{Nj})$. That is, using z^j to index varieties,

$$p_t^{nj}(z^j) = \min_i \left\{ \frac{\kappa_t^{nj,ij}\ x_t^{ij}}{z^{ij}(A_t^{ij})^{\gamma^{ij}}} \right\}.$$

Local Sectoral Aggregate Goods. Intermediate goods demanded from sector j and from all regions are aggregated into a local sectoral good denoted by Q and that can be thought as a bundle of goods purchased from different regions. In particular, let Q_t^{nj} be the quantity produced of aggregate sectoral goods j in region n and $\tilde{q}_t^{nj}(z^j)$ be the quantity demanded of an intermediate good of a given variety from the lowest-cost supplier. The production of local sectoral goods is given by

$$Q_t^{nj} = \left(\int (\tilde{q}_t^{nj}(z^j))^{1-1/\eta^{nj}}\ d\phi^j(z^j) \right)^{\eta^{nj}/(\eta^{nj}-1)},$$

where $\phi^j(z^j) = \exp\left\{ -\sum_{n=1}^{N} (z^{nj})^{-\theta^j} \right\}$ is the joint distribution over the vector z^j, with marginal distribution given by $\phi^{nj}(z^{nj}) = \exp\left\{ -(z^{nj})^{-\theta^j} \right\}$, and the integral is over R_+^N. For nontradable sectors, the only relevant distribution is $\phi^{nj}(z^{nj})$ since sectoral good producers use only local intermediate goods. There are no fixed costs or barriers to entry and exit in the production of intermediate and sectoral goods. Competitive behavior implies zero profits at all times.

Local sectoral aggregate goods are used as materials for the production of intermediate varieties as well as for final consumption. Note that the fact that local sectoral aggregate goods are not traded does not imply that consumers are not purchasing traded goods. On the contrary, both intermediate goods producers and households, via the direct purchase of the local sectoral aggregate good, purchase tradable varieties.

Given the properties of the Fréchet distribution, the price of the sectoral aggregate good j in region n at time t is

$$P_t^{nj} = \Gamma^{nj} \left(\sum_{i=1}^{N} (x_t^{ij} \kappa_t^{nj,ij})^{-\theta^j} (A_t^{ij})^{\theta^j \gamma^{ij}} \right)^{-1/\theta^j}, \tag{6}$$

where Γ^{nj} is a constant.[21] To obtain (6), we assume that $1+\theta^j > \eta^{nj}$. Following similar steps as earlier, we can solve for the share of total expenditure in market (n, j) on goods j from market i.[22] In particular,

$$\pi_t^{nj,ij} = \frac{(x_t^{ij} \kappa_t^{nj,ij})^{-\theta^j} (A_t^{ij})^{\theta^j \gamma^{ij}}}{\sum_{m=1}^{N} (x_t^{mj} \kappa_t^{nj,mj})^{-\theta^j} (A_t^{mj})^{\theta^j \gamma^{mj}}}. \tag{7}$$

This equilibrium condition reflects that the more productive market ij is, given factor costs, the cheaper the cost of production is in market ij and, therefore, the more region n purchases sector j goods from region i. In addition, the easier it is to ship sector j goods from region i to n (lower $\kappa^{nj,ij}$), the more region n purchases sector j goods from region i. This equilibrium condition resembles a gravity equation.

Market Clearing and Unbalanced Trade. With an eye toward our application and to accommodate for observed trade imbalances, we assume there is a mass 1 of rentiers in each region. Rentiers cannot relocate to other regions. They own the local structures, rent them to local firms, and send all their local rents to a global portfolio. In return, rentiers receive a constant share ι^n from the global portfolio, with $\sum_{n=1}^{N} \iota^n = 1$. The difference between the remittances and the income rentiers receive generates imbalances, which change in magnitude as the rental prices change and are given by $\sum_{k=1}^{J} r_t^{ik} H^{ik} - \iota^n \chi_t$, where $\chi_t = \sum_{i=1}^{N} \sum_{k=1}^{J} r_t^{ik} H^{ik}$ are the total revenues in the global portfolio. The local rentier owns this fraction of the global portfolio of structures and uses her income share from the global portfolio to buy goods produced in her own region using the consumption aggregator (1).

Let X_t^{nj} be the total expenditure on sector j good in region n. Then, goods market

[21] In particular, the constant Γ^{nj} is the Gamma function evaluated at $1 + \left(1 - \eta^{nj}/\theta^j\right)$.
[22] For detailed derivations, refer to Caliendo et al. (2018).

clearing implies

$$X_t^{nj} = \sum_{k=1}^{J} \gamma^{nk,nj} \sum_{i=1}^{N} \pi_t^{ik,nk} X_t^{ik} + \alpha^j \left(\sum_{k=1}^{J} w_t^{nk} L_t^{nk} + \iota^n \chi_t \right), \tag{8}$$

where the first term on the right-hand side is the value of the total demand for sector j goods produced in n used as materials in all sectors and regions in the economy, and $\alpha^j \sum_{k=1}^{J} (w_t^{nk} L_t^{nk} + \iota^n \chi_t)$ is the value of the final demand in region n.

Labor market clearing in region n and sector j is

$$L_t^{nj} = \frac{\gamma^{nj} \left(1 - \xi^n\right)}{w_t^{nj}} \sum_{i=1}^{N} \pi_t^{ij,nj} X_t^{ij}, \tag{9}$$

while the market clearing for structures in region n and sector j must satisfy

$$H^{nj} = \frac{\gamma^{nj} \xi^n}{r_t^{nj}} \sum_{i=1}^{N} \pi_t^{ij,nj} X_t^{ij}. \tag{10}$$

2.3. *Equilibrium*

The endogenous state of the economy at any moment in time is given by the distribution of labor across all markets L_t. The fundamentals of the economy are deterministic, some time varying and some constant. The time-varying fundamentals of the economy are sectoral-regional productivities $A_t = \{A_t^{nj}\}_{n=1,j=1}^{N,J}$ and bilateral trade costs $\kappa_t = \{\kappa_t^{nj,ij}\}_{n=1,i=1,j=1}^{N,N,J}$. Constant fundamentals are the labor relocation costs $\Upsilon = \{\tau^{nj,ik}\}_{n=1,j=0,i=1,k=0}^{N,J,J,N}$, the stock of land and structures across markets $H = \left\{H^{nj}\right\}_{n=1,j=1}^{N,J}$, and home production across regions $b = \{b^n\}_{n=1}^{N}$. We denote the time-varying fundamentals by $\Theta_t \equiv (A_t, \kappa_t)$ and constant fundamentals by $\bar{\Theta} \equiv (\Upsilon, H, b)$. The parameters in our model, assumed constant throughout the paper, are given by the value added shares (γ^{nj}); the labor shares in value added (1-ξ^n); the input-output coefficients ($\gamma^{nk,nj}$); the portfolio shares (ι^n); the final consumption expenditure shares (α^j); the discount factor (β); the trade elasticities (θ); and the migration elasticity (ν). We now proceed to formally define an equilibrium of the economy given the parameters of the model.

We first seek to find equilibrium wages $w_t = \{w_t^{nj}\}_{n=1,j=1}^{N,J}$ and the equilibrium allocations $\pi_t = \{\pi_t^{ij,nj}\}_{i=1,j=1,n=1}^{N,J,N}$, $X_t = \{X_t^{nj}\}_{n=1,j=1}^{N,J}$, given $\left(L_t, \Theta_t, \bar{\Theta}\right)$. We refer to this equilibrium as a *temporary* equilibrium. Formally,

Definition 1 *Given* $\left(L_t, \Theta_t, \bar{\Theta}\right)$ *, a* ***temporary equilibrium*** *is a vector of wages* $w\left(L_t, \Theta_t, \bar{\Theta}\right)$ *that satisfies the equilibrium conditions of the static subproblem,* (5) *to* (10) *.*

The temporary equilibrium of our model is the solution to a static multicountry interregional trade model.[23] Suppose that for any $\left(L_t, \Theta_t, \bar{\Theta}\right)$ we can solve the temporary

[23] It is important to emphasize that the temporary equilibrium described in Definition 1 is not specific

equilibrium.[24] Then the wage rate can be expressed as $w_t = w\left(L_t, \Theta_t, \bar{\Theta}\right)$, and given that prices are all functions of wages, we can express real wages as $\omega^{nj}\left(L_t, \Theta_t, \bar{\Theta}\right) = w_t^{nj}/P_t^n$. After defining the temporary equilibrium, we can now define the sequential competitive equilibrium of the model given a path of exogenous fundamentals $\{\Theta_t\}_{t=0}^{\infty}$ and given $\bar{\Theta}$. Let $\mu_t = \{\mu_t^{nj,ik}\}_{n=1,j=0,i=1,k=0}^{N,J,N,J}$ and $V_t = \{V_t^{nj}\}_{n=1,j=0}^{N,J}$ be the migration shares and lifetime utilities, respectively. The definition of a sequential competitive equilibrium is given as follows:[25]

Definition 2 *Given $\left(L_0, \{\Theta_t\}_{t=0}^{\infty}, \bar{\Theta}\right)$, a **sequential competitive equilibrium** of the model is a sequence of $\{L_t,\ \mu_t,\ V_t,\ w\left(L_t, \Theta_t, \bar{\Theta}\right)\}_{t=0}^{\infty}$ that solves equilibrium conditions (2) to (4) and the temporary equilibrium at each t.*

Finally, we define a stationary equilibrium of the model.

Definition 3 *A **stationary equilibrium** of the model is a sequential competitive equilibrium such that $\{L_t,\ \mu_t,\ V_t,\ w\left(L_t, \Theta_t, \bar{\Theta}\right)\}_{t=0}^{\infty}$ are constant for all t.*

A stationary equilibrium in this economy is a situation in which no aggregate variables change over time. It follows that in a stationary equilibrium, fundamentals need to be constant for all t. In such a stationary equilibrium, households may move from one market to another, but inflows and outflows balance.

3. DYNAMIC HAT ALGEBRA

Solving for all the transitional dynamics in a dynamic discrete choice model with this rich spatial structure is difficult, and it also requires pinning down the values of a large number of unknown fundamentals. Note from Definitions 1 to 3, that to solve for an equilibrium of the model, it is necessary to condition on Θ_t and $\bar{\Theta}$, namely, the level of the fundamentals of the economy (productivities, endowments of local structures, labor mobility costs, non-employment income, and trade costs) at each point in time. As we increase the dimension of the problem, for example, by adding countries, regions, or sectors, the number of fundamentals grows geometrically. We now show how to compute the counterfactual changes in all endogenous variables across markets and time as the solution to a system of non-linear equations. By employing dynamic hat algebra, we will not need to estimate the level of fundamentals.

to a multisector EK model, but it can also be the equilibrium of other trade models such as Melitz (2003). In other words, an economy has a temporary equilibrium if one can solve for equilibrium prices given the distribution of fundamentals and factors of production.

[24] In Appendix C.1, we present a one-sector version of our model that maps into the Alvarez and Lucas (2007) model. Alvarez and Lucas (2007) show existence and uniqueness of the equilibrium. For a proof and characterization of the conditions for existence and uniqueness of a more general static model than that of Alvarez and Lucas (2007), refer to Allen and Arkolakis (2014), and for a proof of existence and uniqueness of a static model more similar to our static subproblem, see Redding (2016).

[25] Proposition 8 from Cameron et al. (2007) shows the existence and uniqueness of the sequential competitive equilibrium of a simplified version of our model. Using the results from Alvarez and Lucas (2007) together with Proposition 8 from Cameron et al. (2007), there exists a unique sequential equilibrium of the one-sector model in Appendix C.1.

3.1. *Solving the Model*

We seek to use our model to perform various counterfactual experiments, i.e., to study the general equilibrium implications of a change in fundamentals relative to the fundamentals of a baseline economy. We now define formally the baseline economy.

Definition 4 *The* ***baseline economy*** *is the allocation* $\{L_t, \mu_{t-1}, \pi_t, X_t\}_{t=0}^{\infty}$ *corresponding to the sequence of fundamentals* $\{\Theta_t\}_{t=0}^{\infty}$ *and to* $\bar{\Theta}$.

We now show how to solve for the baseline economy in time differences. To ease the exposition, we denote by $\dot{y}_{t+1} \equiv (y^1_{t+1}/y^1_t, y^2_{t+1}/y^2_t, ...)$ the proportional change in any scalar or vector between periods t and $t+1$. We start by showing how to solve for a temporary equilibrium of the baseline economy at $t+1$ after a change in employment, $\dot{L}_{t+1}$, and fundamentals $\dot{\Theta}_{t+1}$, without needing estimates of Θ_t or $\bar{\Theta}$.

Proposition 1 *Given the allocation of the temporary equilibrium at* t, $\{L_t, \pi_t, X_t\}$, *the solution to the* ***temporary equilibrium*** *at* $t+1$ *for a given change in* $\dot{L}_{t+1}$ *and* $\dot{\Theta}_{t+1}$ *does not require information on the level of fundamentals at* t, Θ_t, *or* $\bar{\Theta}$. *In particular, it is obtained as the solution to the following system of non-linear equations:*

$$\dot{x}^{nj}_{t+1} = (\dot{L}^{nj}_{t+1})^{\gamma^{nj}\xi^n} (\dot{w}^{nj}_{t+1})^{\gamma^{nj}} \prod_{k=1}^{J} (\dot{P}^{nk}_{t+1})^{\gamma^{nj,nk}}, \tag{11}$$

$$\dot{P}^{nj}_{t+1} = \left(\sum_{i=1}^{N} \pi^{nj,ij}_t (\dot{x}^{ij}_{t+1} \dot{\kappa}^{nj,ij}_{t+1})^{-\theta^j} (\dot{A}^{ij}_{t+1})^{\theta^j \gamma^{ij}}\right)^{-1/\theta^j}, \tag{12}$$

$$\pi^{nj,ij}_{t+1} = \pi^{nj,ij}_t \left(\frac{\dot{x}^{ij}_{t+1} \dot{\kappa}^{nj,ij}_{t+1}}{\dot{P}^{nj}_{t+1}}\right)^{-\theta^j} (\dot{A}^{ij}_{t+1})^{\theta^j \gamma^{ij}}, \tag{13}$$

$$X^{nj}_{t+1} = \sum_{k=1}^{J} \gamma^{nk,nj} \sum_{i=1}^{N} \pi^{ik,nk}_{t+1} X^{ik}_{t+1} + \alpha^j \left(\sum_{k=1}^{J} \dot{w}^{nk}_{t+1} \dot{L}^{nk}_{t+1} w^{nk}_t L^{nk}_t + \iota^n \chi_{t+1}\right), \tag{14}$$

$$\dot{w}^{nj}_{t+1} \dot{L}^{nj}_{t+1} w^{nj}_t L^{nj}_t = \gamma^{nj}(1-\xi^n) \sum_{i=1}^{N} \pi^{ij,nj}_{t+1} X^{ij}_{t+1}, \tag{15}$$

where $\chi_{t+1} = \sum_{i=1}^{N} \sum_{k=1}^{J} \frac{\xi^i}{1-\xi^i} \dot{w}^{ik}_{t+1} \dot{L}^{ik}_{t+1} w^{ik}_t L^{ik}_t$.

Proposition 1 shows that given an allocation at time t, one can solve for the change in the temporary equilibrium given a change in labor supply $\dot{L}_{t+1}$ and fundamentals $\dot{\Theta}_{t+1}$

(recall that these represent any changes in productivity or trade costs), without requiring information on the levels of fundamentals at time t. Note that Proposition 1 does not impose any restrictions on $\dot{\Theta}_{t+1}$. In particular, Proposition 1 says that for any changes in fundamentals (one by one or jointly) across time and space, one can solve for the change in real wages resulting from $\dot{\Theta}_{t+1}$ given $\dot{L}_{t+1}$.

Of course, $\dot{L}_{t+1}$ is itself endogenous. However, building on this last result, we can now characterize the solution of the dynamic model. The next proposition shows that, given an allocation at $t=0$, $\{L_0, \pi_0, X_0\}$, the matrix of gross migration flows at $t=-1$, μ_{-1}, and a sequence of changes in fundamentals, one can solve for the sequential equilibrium in time differences without needing to estimate the levels of fundamentals. This result requires that the sequence of changes in fundamentals converges to one over time as the economy approaches the stationary equilibrium. Formally:

Definition 5 *A* ***converging sequence of changes in fundamentals*** *is such that* $\lim_{t\to\infty} \dot{\Theta}_t = 1$.

In what follows we impose further structure over the instantaneous utility of the agents. In particular,

Assumption 3 *Agents have logarithmic preferences,* $U(C_t^{nj}) \equiv \log(C_t^{nj})$.

To ease exposition, we denote $u_t^{nj} \equiv \exp(V_t^{nj})$. Moreover, we denote by $\dot{\omega}^{nj}(\dot{L}_{t+1}, \dot{\Theta}_{t+1})$ (for all n and j) the equilibrium real wages in time differences as functions of the change in labor $\dot{L}_{t+1}$ and time varying fundamentals $\dot{\Theta}_{t+1}$. Namely, $\dot{\omega}^{nj}(\dot{L}_{t+1}, \dot{\Theta}_{t+1})$ is the solution to the system in Proposition 1.

Proposition 2 *Conditional on an initial allocation of the economy,* $(L_0, \pi_0, X_0, \mu_{-1})$, *given an anticipated convergent sequence of changes in fundamentals,* $\{\dot{\Theta}_t\}_{t=1}^{\infty}$, *the solution to the sequential equilibrium in time differences does not require information on the level of the fundamentals* $\{\Theta_t\}_{t=0}^{\infty}$ *or* $\bar{\Theta}$ *and solves the following system of non-linear equations:*

$$\mu_{t+1}^{nj,ik} = \frac{\mu_t^{nj,ik}\left(\dot{u}_{t+2}^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_t^{nj,mh}\left(\dot{u}_{t+2}^{mh}\right)^{\beta/\nu}}, \tag{16}$$

$$\dot{u}_{t+1}^{nj} = \dot{\omega}^{nj}(\dot{L}_{t+1}, \dot{\Theta}_{t+1})\left(\sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{nj,ik}\left(\dot{u}_{t+2}^{ik}\right)^{\beta/\nu}\right)^{\nu}, \tag{17}$$

$$L_{t+1}^{nj} = \sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{ik,nj}\, L_t^{ik}, \tag{18}$$

for all $j, n, i,$ *and* k *at each* t, *where* $\{\dot{\omega}^{nj}(\dot{L}_t, \dot{\Theta}_t)\}_{n=1,j=0,t=1}^{N,J,\infty}$ *is the solution to the* *temporary equilibrium given* $\{\dot{L}_t, \dot{\Theta}_t\}_{t=1}^{\infty}$.

Proposition 2 is one of our key results. It shows that by taking time differences we can solve the model for a given sequence of changes in fundamentals using data for the initial period (i.e., the initial value of the migration shares and the initial distribution of households across labor markets) without knowing the levels of fundamentals. For instance, suppose we want to solve the model where all fundamentals are constant. In this case, the set of fundamentals are given by $\Theta_t \equiv (A, \kappa)$ and $\bar{\Theta} \equiv (\Upsilon, H, b)$ and in time differences are given by $\dot{\Theta}_t \equiv (1, 1)$ and $\dot{\bar{\Theta}} \equiv (1, 1, 1)$. Therefore, by computing the model in time differences, we do not need to identify any fundamentals of the economy. Of course, Proposition 2 can also be applied to compute the model with any sequence of fundamentals.

To gain intuition about how Proposition 2 works, consider the following example. Take migration shares (3) at time $t-1$. As we can see from (3), given β and ν, there are infinite combinations of values V_t^{ik} and migration costs $\tau^{nj,ik}$ that can reconcile a given migration flow. So, in principle, there is no way we can uniquely solve for V_t^{ik} without information on $\tau^{nj,ik}$. However, by taking differences over time, the evolution of the economy in changes is identified.[26] To take time differences, for example, consider migration flows for the same market at time t and take the relative time difference (3) between time t and $t-1$; namely,

$$\frac{\mu_t^{nj,ik}}{\mu_{t-1}^{nj,ik}} = \frac{\exp\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{1/\nu} \Big/ \exp\left(\beta V_t^{ik} - \tau^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J} \frac{\exp\left(\beta V_{t+1}^{mh} - \tau^{nj,mh}\right)^{1/\nu}}{\sum_{m'=1}^{N}\sum_{h'=0}^{J}\exp\left(\beta V_t^{m'h'} - \tau^{nj,m'h'}\right)^{1/\nu}}}.$$

Given the properties of the exponential function, the numerator of this last expression simplifies to $\exp\left(V_{t+1}^{ik} - V_t^{ik}\right)^{\beta/\nu} = (\dot{u}_{t+1}^{nj})^{\beta/\nu}$. Now multiply and divide each element of the sum in the denominator by $\exp\left(\beta V_t^{mh} - \tau^{nj,mh}\right)^{1/\nu}$ and use migration flows at time $t-1$ to obtain (16).[27] The procedure to derive equation (17) is similar and results from taking time differences between equation (2) expressed at time $t+1$ and at time t (see Appendix B).[28]

A couple of observations are noteworthy about the system of equilibrium conditions

[26] In Appendix G we show that by taking time differences, the evolution of the economy in changes is identified given a level of τ's even if we cannot identify separately the level of mobility frictions or initial lifetime utility.

[27] Another way to understand our method is by relating it to Hotz and Miller (1993) and Berry (1994). They show that choice probabilities provide information on payoffs and parameters, and by inverting choice probabilities it is possible to estimate the parameters. We show that by taking time differences of choice probabilities and inverting them, we can solve the model and perform counterfactuals without estimating the parameters.

[28] It is worth noting that given Assumption 3, we do not require information on the level of real wages, $\omega_t^{nj} = w_t^{nj}/P_t^n$, across markets in the initial period to solve the model. If instead we had linear utility, then equation (17) would be given by

$$\dot{u}_{t+1}^{nj} = \omega_t^{nj}\left(\dot{\omega}^{nj}(\dot{L}_{t+1}, \dot{\Theta}_{t+1}) - 1\right)\left(\sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{nj,ik}\left(\dot{u}_{t+2}^{ik}\right)^{\beta/\nu}\right)^{\nu},$$

(16), (17), and (18) in time differences. First, at the steady state, $\{\dot{u}_t^{ik} = 1\}_{i=1,j=0}^{N,J}$ for all t regardless of the level of the fundamentals. This is an advantage since it simplifies considerably the computation of the model given that there is no need to solve for the steady-state value functions. Second, we can use this system of equations conditioning on observables $(L_0, \pi_0, X_0, \mu_{-1})$ and solve for the equilibrium even if the economy is not initially in a steady state. To see this in a simple way, consider an economy with constant fundamentals $\{\dot{\Theta}_t = 1\}_{t=1}^{\infty}$ and let μ^* be the steady-state migration flow and L^* the steady-state employment distribution. Now suppose that $\mu_{-1} = \mu^*$, $L_0 = L^*$, and $\{\dot{u}_1^{ik} = 1\}_{i=1,j=0}^{N,J}$. From (16) note that since $\dot{u}_1^{ik} = 1$, then $\mu_0 = \mu_{-1} = \mu^*$. Then from (18) this implies that $L_1 = L_0 = L^*$ since μ^* is the steady-state migration flow; hence, $\dot{\omega}^{nj}(1,1) = 1$. Finally, given that $\{\dot{u}_1^{ik} = 1\}_{i=1,j=0}^{N,J}$, then only $\{\dot{u}_2^{ik} = 1\}_{i=1,j=0}^{N,J}$ solves (17). Now condition on observed data L_0 and μ_{-1}. If L_0 and μ_{-1} were at the steady state, then initiating the system at $\{\dot{u}_1^{ik} = 1\}_{i=1,j=0}^{N,J}$ should solve the system of equations. However, if L_0 is not the steady-state distribution of labor of the economy, then after applying μ_{-1} to L_0 we will obtain $\dot{L}_1 \neq 1$ and as a result $\dot{\omega}^{nj}(\dot{L}_1, 1) \neq 1$ and then $\{\dot{u}_2^{ik} \neq 1\}_{i=1,j=0}^{N,J}$ from (17). We use these observations to construct an algorithm that solves for the competitive equilibrium of the economy. In Appendix D, Part I, we present the algorithm.[29]

3.2. *Solving for Counterfactuals*

So far we have shown that we can take our model to the data and solve for the sequential competitive equilibrium of the economy. This might be interesting by itself; however, we also want to use the model to conduct counterfactuals. By counterfactuals we refer to the study of how allocations change across space and time, relative to a baseline economy, given a new sequence of fundamentals, which we denote by $\{\Theta_t'\}_{t=1}^{\infty}$.

From Proposition 2 we can solve for a baseline economy without knowing the level of fundamentals. Given this, we can then study the effects of a change in fundamentals from $\{\Theta_t\}_{t=1}^{\infty}$ to $\{\Theta_t'\}_{t=1}^{\infty}$ (where $\{\Theta_t\}_{t=1}^{\infty}$ is the sequence of fundamentals of a baseline economy and $\{\Theta_t'\}_{t=1}^{\infty}$ is the sequence of counterfactual fundamentals) without explicitly knowing the level of Θ_t and $\bar{\Theta}$. Of course, as in any dynamic model, when solving for the baseline economy, as well as for counterfactuals, we need to make an assumption of how agents anticipate the evolution of the fundamentals of the economy. For example, we can assume that the change in fundamentals is anticipated (or not) by agents at time 0. Consistent with our perfect foresight assumption, we follow the convention that, at the

which, as we can see, would require conditioning on the level of real wages ω_t^{nj} in the first period.

[29] It should be clear at this point that our solution method requires actual data on migration flows, trade, employment, and production to compute the model. In our quantitative application, we initialize the economy with data for production, trade, migration, and employment for the U.S. economy and the world in the year 2000. Therefore, we are not assuming the economy is in a steady state and our initial data reflect exactly the state of the U.S. economy in the year 2000, which is not necessarily a steady state.

beginning of the period in the baseline economy, agents anticipate the entire evolution of fundamentals. Then, to compute counterfactuals, we assume that agents at $t = 0$ do not anticipate the change in the path of fundamentals and that at $t = 1$ agents learn about the entire future counterfactual sequence of $\{\Theta'_t\}_{t=1}^{\infty}$. This timing assumption allows us to use information about agents' actions before $t = 1$ to solve for the sequential equilibrium, under the new fundamentals, in relative time differences.

The next proposition defines how to solve for counterfactuals from unexpected changes in fundamentals. It shows that conditioning on the allocation of the baseline economy $\{L_t, \mu_{t-1}, \pi_t, X_t\}_{t=0}^{\infty}$, we can solve for counterfactuals without information on $\{\Theta_t\}_{t=0}^{\infty}$ or $\bar{\Theta}$. For instance, imagine we do not know how the actual fundamentals changed over time, but want to know (e.g.) how an unexpected change in China's sectoral TFPs would have affected the U.S. economy? In what follows, we explain how to perform such counterfactuals without having to identify the actual evolution of fundamentals.

First, we introduce new notation. Let $\hat{y}_{t+1} \equiv \dot{y}'_{t+1}/\dot{y}_{t+1}$ be the ratio of time changes between the counterfactual equilibrium, $\dot{y}'_{t+1} \equiv y'_{t+1}/y'_t$, and the initial equilibrium, $\dot{y}_{t+1} \equiv y_{t+1}/y_t$. For instance, using this notation, $\hat{\Theta}_{t+1}$ refers to the counterfactual changes in fundamentals over time relative to the baseline economy; namely $\hat{\Theta}_{t+1} = \dot{\Theta}'_{t+1}/\dot{\Theta}_{t+1}$. Note that $\hat{\Theta}_{t+1} = 1$ does not mean that fundamentals are not changing, it means that fundamentals are changing in the same way as in the baseline economy; namely Θ'_{t+1}/Θ'_t $= \Theta_{t+1}/\Theta_t$.

Proposition 3 *Given a baseline economy, $\{L_t, \mu_{t-1}, \pi_t, X_t\}_{t=0}^{\infty}$, and a counterfactual convergent sequence of changes in fundamentals (relative to the baseline change), $\{\hat{\Theta}_t\}_{t=1}^{\infty}$, solving for the counterfactual sequential equilibrium $\{L'_t, \mu'_{t-1}, \pi'_t, X'_t\}_{t=1}^{\infty}$ does not require information on the baseline fundamentals $(\{\Theta_t\}_{t=0}^{\infty}, \bar{\Theta})$ and solves the following system of non-linear equations*:

$$\mu_t^{\prime nj,ik} = \frac{\mu_{t-1}^{\prime nj,ik}\,\dot{\mu}_t^{nj,ik}\left(\hat{u}_{t+1}^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_{t-1}^{\prime nj,mh}\,\dot{\mu}_t^{nj,mh}\left(\hat{u}_{t+1}^{mh}\right)^{\beta/\nu}}, \tag{19}$$

$$\hat{u}_t^{nj} = \hat{\omega}^{nj}(\hat{L}_t, \hat{\Theta}_t)\left(\sum_{i=1}^{N}\sum_{k=0}^{J}\mu_{t-1}^{\prime nj,ik}\,\dot{\mu}_t^{nj,ik}\left(\hat{u}_{t+1}^{ik}\right)^{\beta/\nu}\right)^{\nu}, \tag{20}$$

$$L_{t+1}^{\prime nj} = \sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{\prime ik,nj}\,L_t^{\prime ik}, \tag{21}$$

for all j, n, i, and k at each t, where $\{\hat{\omega}^{nj}(\hat{L}_t, \hat{\Theta}_t)\}_{n=1,j=0,t=1}^{N,J,\infty}$ is the solution to the temporary equilibrium given $\{\hat{L}_t, \hat{\Theta}_t\}_{t=1}^{\infty}$; namely, at each t, given $\left(\hat{L}_t, \hat{\Theta}_t\right)$, $\hat{\omega}^{nj}(\hat{L}_t, \hat{\Theta}_t) =$

$\hat{w}_t^{nj}/\hat{P}_t^n$ *solves*

$$\hat{x}_{t+1}^{nj} = (\hat{L}_{t+1}^{nj})^{\gamma^{nj}\xi^n} (\hat{w}_{t+1}^{nj})^{\gamma^{nj}} \prod_{k=1}^{J} (\hat{P}_{t+1}^{nk})^{\gamma^{nj,nk}}, \tag{22}$$

$$\hat{P}_{t+1}^{nj} = \left(\sum_{i=1}^{N} \pi_t^{\prime nj,ij} \dot{\pi}_{t+1}^{nj,ij} (\hat{x}_{t+1}^{ij} \hat{\kappa}_{t+1}^{nj,ij})^{-\theta^j} (\hat{A}_{t+1}^{ij})^{\theta^j \gamma^{ij}} \right)^{-1/\theta^j}, \tag{23}$$

$$\pi_{t+1}^{\prime nj,ij} = \pi_t^{\prime nj,ij} \dot{\pi}_{t+1}^{nj,ij} \left(\frac{\hat{x}_{t+1}^{ij} \hat{\kappa}_{t+1}^{nj,ij}}{\hat{P}_{t+1}^{nj}} \right)^{-\theta^j} (\hat{A}_{t+1}^{ij})^{\theta^j \gamma^{ij}}, \tag{24}$$

$$X_{t+1}^{\prime nj} = \sum_{k=1}^{J} \gamma^{nk,nj} \sum_{i=1}^{N} \pi_{t+1}^{\prime ik,nk} X_{t+1}^{\prime ik} + \alpha^j \left(\sum_{k=1}^{J} \hat{w}_{t+1}^{nk} \hat{L}_{t+1}^{nk} w_t^{\prime nk} L_t^{\prime nk} \dot{w}_{t+1}^{nk} \dot{L}_{t+1}^{nk} + \iota^n \chi_{t+1}^{\prime} \right), \tag{25}$$

$$\hat{w}_{t+1}^{nk} \hat{L}_{t+1}^{nk} = \frac{\gamma^{nj}(1-\xi^n)}{w_t^{\prime nk} L_t^{\prime nk} \dot{w}_{t+1}^{nk} \dot{L}_{t+1}^{nk}} \sum_{i=1}^{N} \pi_{t+1}^{\prime ij,nj} X_{t+1}^{\prime ij}, \tag{26}$$

where $\chi_{t+1}^{\prime} = \sum_{i=1}^{N} \sum_{k=1}^{J} \frac{\xi^i}{1-\xi^i} \hat{w}_{t+1}^{ik} \hat{L}_{t+1}^{ik} w_t^{\prime ik} L_t^{\prime ik} \dot{w}_{t+1}^{ik} \dot{L}_{t+1}^{ik}$.

Proposition 3 is another of our key results. It shows that we can compute counterfactuals from unanticipated changes to the baseline economy's fundamentals without knowing the levels or changes in fundamentals of the baseline economy. The baseline economy can contain either time-varying or constant fundamentals. For instance, if the baseline economy contains factual changes in fundamentals, the sequence of $\{\dot{L}_{t+1}, \dot{\mu}_t, \dot{\pi}_{t+1}, \dot{X}_{t+1}\}_{t=0}^{\infty}$ is exactly as in the data, while if the baseline economy contains constant fundamentals, the sequence of $\{\dot{L}_{t+1}, \dot{\mu}_t, \dot{\pi}_{t+1}, \dot{X}_{t+1}\}_{t=0}^{\infty}$ is computed using the results from Proposition 2. In any case, by computing the model in time differences and relative to a baseline economy, we do not need to identify any fundamentals of the baseline economy. As before, the proof of Proposition 3 is presented in Appendix B. In Appendix D, Part II is the algorithm we use to solve for counterfactuals—namely, for changes in fundamentals relative to the baseline.

It is worth emphasizing again that our solution method allows us to study the effects of changes in any fundamental without having to estimate the entire set. This method has two main advantages. First, by conditioning on observed allocations at a given moment in time, one disciplines the model by making it match all cross-sectional moments in the data. Second, after conditioning on data, one can use the model to solve for counterfactuals without backing out the fundamentals of the economy. If the goal is to study the effects of a change in fundamentals relative to an economy with constant fundamentals, Proposition 2 shows that solving for the baseline economy with constant fundamentals requires cross-sectional data at the initial period of analysis. If instead the goal is to study the effects of a change in fundamentals relative to an economy with actual changes in fundamentals, Proposition 3 shows that cross-sectional data for the entire period of analysis is required. Ultimately, the choice between conducting counterfactuals with constant or time-varying fundamentals will depend on the question being asked and the data availability. We now provide some illustrative examples of questions that can be answered with our method.

3.3. *Examples of Counterfactual Questions*

Our theoretical results can be applied to answer a wide variety of questions. In this subsection, we describe examples of counterfactual questions that can be answered using our dynamic hat algebra methodology and the measurements required to answer these questions. In an Online Appendix, we present the quantitative results for all the counterfactual questions discussed in this section.

Example 1: Dynamics with constant fundamentals. Suppose we want to study the following question: starting from an initial allocation, how would the economy evolve over time, and what would be its long-run features given the fundamentals in the initial period? This is one type of counterfactual that is possible to compute using the results from Proposition 2, which involves simulating the model assuming that starting in a given period no fundamentals ever change thereafter.

In terms of measurement, answering this question requires obtaining the initial observed allocations $(L_0, \pi_0, X_0, \mu_{-1})$, that is, employment, trade flows, expenditures, and gross migration flows for the first period. As we emphasized before, we do not need to impose that the economy starts in the steady state. This is one of the attractive features of Proposition 2, since once we express the model in differences relative to the initial observed allocations, we can solve for the trajectories to the steady state given the fundamentals in the initial period.[30]

Example 2: Unexpected change in fundamentals relative to constant fundamentals. A variant of the previous counterfactual is to assume that all agents expected no changes in fundamentals in a given period, but then a subset of fundamentals grow unexpectedly. For instance, we can think of questions such as: what would have happened to the U.S. economy if Chinese fundamental productivity, A_t^{ij}, in the manufacturing sectors surprisingly grew 20 percent but agents expected no changes?[31]

[30] For illustrative purposes, in the Online Appendix, we show the evolution of the U.S. economy to its steady state given year 2000 fundamentals and given year 2007 fundamentals.

[31] This is the type of counterfactual question answered in previous literature on labor dynamics such as ACM and Dix-Carneiro (2014). Relative to these papers, our methodology allows us to answer this

To answer this question, we need to solve for a counterfactual economy that contains constant fundamentals except for a 20 percent growth in Chinese productivity relative to a baseline economy with constant fundamentals. Since this counterfactual assumes constant fundamentals in the baseline economy, measurement only requires conditioning on the initial (first period) allocations of employment, gross migration flows, trade flows, and expenditures. We first apply the result of Proposition 2 with constant fundamentals like in the case of Example 1. After that, we use these results as the baseline economy and apply the results from Proposition 3. This proposition establishes that only the counterfactual changes in Chinese productivities relative to the actual ones are needed, and, therefore, it does not require measuring any actual change in fundamentals since in this case we are studying a hypothetical productivity change.

Example 3: Unexpected change in fundamentals relative to actual fundamentals. We can also apply the results from Proposition 2 and Proposition 3 to compute the effects of changes to fundamentals relative to an economy with time-varying fundamentals. For instance, we can study what the economy would look like if we assumed that a set of time-varying fundamentals evolves as they did over a given period of time while another subset of time-varying fundamentals changes in a different way relative to their true changes.

More concretely, suppose we want to answer the following question: what would have happened across U.S. labor markets if Chinese fundamental productivity, A_t^{ij}, instead of growing as it did, would have grown 20 percent less in each manufacturing sector per year from 2000 to 2007? To answer this question, we first compute the baseline economy with actual changes in fundamentals. Then, we use the baseline economy and Proposition 3 and solve for the counterfactual economy with a 20 percent lower Chinese productivity relative to the baseline economy. As Proposition 3 states, measurement requires time-series allocations of gross migration flows, trade flows, and expenditures for the factual baseline economy, which are sufficient statistics for the evolution of actual fundamentals. We also need to confront the fact that data ends in a given period and therefore we must compute the baseline economy thereafter. For instance, suppose we have data for the period 2000 to 2007. One can use the results from Proposition 2 and solve the baseline economy from 2007 forward given a convergent sequence of fundamentals. After this, one would obtain a baseline economy with actual changes in fundamentals from the years 2000 to 2007 and the computed baseline economy thereafter. Finally, given that in this example we want to study a hypothetical 20 percent lower productivity, Proposition 3 establishes that only the counterfactual changes in Chinese productivities relative to the actual ones are needed, and, therefore, it does not require measuring any actual change in fundamentals of the economy.

question without estimating a large set of parameters while at the same time solving a general equilibrium model with a higher dimensional state space.

Example 4: Studying the effects of actual changes in fundamentals. We can also use the methodology to study the effects of actual changes in fundamentals. For example, suppose we want to answer the question: what was the effect of the actual China shock? or, more concretely, what would have happened differently across U.S. labor markets if the China shock did not occur? This is the main question that we answer in the next sections. We refer to the China shock as the change in sectoral productivities in China that matches the increase in Chinese import penetration into the U.S. market during the period 2000-2007, as we explain further in the next section.

To answer this question, we first need to compute a baseline economy that contains information on the actual evolution of fundamentals, which requires time-series data as discussed in Example 3. We then compute the counterfactual economy in which agents were expecting the actual evolution of fundamentals but the China shock did not happen. In order to apply Proposition 3 to answer this question, we also need to measure the China shock since, different from Example 3, we are studying the effects of an actual change in fundamentals. The next sections describe how to take the model to the data, compute the baseline economy, and identify the China shock.

Finally, as our last illustrative counterfactual, suppose we are interested in studying the effects of an actual change in fundamentals relative to an economy where only a subset or no fundamentals change. For instance, suppose we ask the question: what would have happened if agents expected constant fundamentals and unexpectedly Chinese productivity grew as it did? In other words, what is the effect of China's productivity growth holding all else equal? To answer this question, we need to compute first a baseline economy with constant fundamentals, as we did in Examples 1 and 2, and measure the actual change in Chinese productivity over the period of analysis. In the Online Appendix, as an illustration, we present the results of a counterfactual where we study the effects of the China shock with all other fundamentals constant. We also discuss how to measure the China shock with constant fundamentals.

We now move to the empirical section of our paper, where we first describe how to take the model to the data, measure the China shock, and use our methodology to evaluate the effects of the China shock as described in the first part of Example 4.

4. TAKING THE MODEL TO THE DATA

Applying the solution method requires values of bilateral trade flows $\pi_t^{nj,ij}$, value added $w_t^{nj} L_t^{nj} + r_t^{nj} H_t^{nj}$, the distribution of employment L_t, and the migration flows across regions and sectors, $\mu_t^{nj,ik}$. We take the year 2000 as our initial period and match the model variables to the values observed in the data over the period 2000-2007. We also need to compute the share of value added in gross output γ^{nj}, the material shares $\gamma^{nj,nk}$, the share of structures in value added ξ^n, the final consumption shares α^j, and the global portfolio shares ι^n. Finally, we need estimates of the sectoral trade elasticities θ^j, the migration elasticity $1/\nu$, and the discount factor β. This section provides a summary of the data sources and measurements used to take the model to the data, with further details provided in Appendix E.

Regions, sectors, and labor markets. Our quantitative model has 50 U.S. states; 37 other countries, including China; and a constructed rest of the world. We consider 22 sectors, classified according to the North American Industry Classification System (NAICS). Of these 22, 12 are manufacturing sectors, 8 are service sectors, and we also include the construction sector and a combined wholesale and retail trade sector. In our analysis, we exclude the agriculture, mining, utilities, and public sectors. Our definition of a labor market in the U.S. economy is thus a state-sector pair, including non-employment, leading to 1150 markets. For other countries, we assume a single labor market in each country, but with the same set of productive sectors.

Trade and production data. We construct the bilateral trade shares $\pi_t^{nj,ij}$ for the 38 countries in our sample, including the U.S. aggregate, from the World Input-Output Database (WIOD). We discipline the different uses in the data as follows. The WIOD has information on trade flows across countries as well as data on input-output linkages (purchases of materials across sectors). The bilateral trade flows in the model include both traded goods for use as intermediates and traded goods for final consumption, and, therefore, they match all bilateral trade flows in the WIOD. The initial sectoral bilateral trade flows between the 50 U.S. states were constructed by combining information from the WIOD and the 2002 Commodity Flow Survey (CFS), which is the closest available year to 2000. From the WIOD we compute the total U.S. domestic sales for the year 2000 for our 22 sectors. From the 2002 CFS, we compute the bilateral expenditure shares across regions and sectors. These two pieces of information allow us to construct the bilateral trade flows matrix for the 50 U.S. states across sectors, where the total U.S. domestic sales match the WIOD data for the year 2000. We follow the same procedure to construct the time series of these bilateral trade flows, as described with more detail in Appendix E.

Bilateral trade flows between the 50 U.S. states and the rest of the countries in the world were constructed by combining information from the WIOD and regional employment data from the Bureau of Economic Analysis (BEA). In our model, local labor markets have different exposures to international trade shocks because there is substantial geographic variation in industry specialization. Regions with a high concentration of production in a given industry should react more to international trade shocks hitting that industry. Therefore, following ADH, our measure for the exposure of local labor markets to international trade combines trade data with local industry employment. Specifically, we split the bilateral trade flows at the country level, computed from the WIOD, into bilateral trade flows between the U.S. states and other countries by assuming that the share of each state in total U.S. trade with any country in each sector is determined by the regional share of total employment in that industry.

To construct the share of value added in gross output γ^{nj}, the material input shares $\gamma^{nj,nk}$, and the share of structures in value added ξ^n, we use data on gross output, value added, intermediate consumption, and labor compensation across sectors from the BEA for the U.S. states and from the WIOD for all other countries in our sample.

Finally, using the constructed trade and production data, we compute the final consumption shares α^j, as described in Appendix E, and we discipline the portfolio shares ι^n to match exactly the year 2000 observed trade imbalances.

Migration flows and the initial distribution of labor. The initial distribution of workers in the year 2000 by U.S. states and sectors (and non-employment) is obtained from the 5 percent Public Use Microdata Sample (PUMS) of the decennial U.S. Census for the year 2000. Information on industry is classified according to the NAICS, which we aggregate to our 22 sectors and non-employment.[32] We restrict the sample to people between 25 and 65 years of age who are either non-employed or employed in one of the sectors included in the analysis. Our sample contains almost 7 million observations.

TABLE I

U.S. INTERSTATE AND INTERSECTORAL LABOR MOBILITY

Probability	p25	p50	p75
Changing sector but not state	3.58%	5.44%	7.93%
Changing state but not sector	0.04%	0.42%	0.73%
Changing state and sector	0.02%	0.03%	0.05%
Staying in the same state and sector	91.4%	93.9%	95.8%

Note: Quarterly transitions. Data sources: ACS and CPS.

In our application, we abstract from international migration.[33] That is, we impose that $\tau^{nj,ik} = \infty$ for all j, k such that regions n and i belong to different countries. Given this assumption, we need to measure the gross flows only for the U.S. economy. A period in our model corresponds to one quarter. To construct quarterly mobility across our regions and sectors, we combine information from the Current Population Survey (CPS) to compute intersectoral mobility and from the PUMS of the American Community Survey (ACS) to compute interstate mobility. Table III in Appendix E shows the information provided by these two datasets in terms of transition probabilities.[34]

Table I shows some moments of worker mobility across labor markets computed from our estimated transition matrix for the year 2000. Our numbers are consistent with the estimates by Molloy et al. (2011) and Kaplan and Schulhofer-Wohl (2012) for interstate

[32] When we construct the mobility flows across our labor markets, all of the workers that are not employed in an industry (including workers that are either unemployed or not in the labor force) are part of the pool of non-employed workers.

[33] This simplification is a consequence of data availability. As we discussed previously, our model can accommodate international migration.

[34] In Appendix E, we compare our constructed migration flows with an alternative dataset from the Census Bureau's Longitudinal Employer-Household Dynamics (LEHD), in particular, the Job-to-Job Flows data (J2J). We find that the migration flows constructed using data from the ACS and CPS are highly correlated with the transition probabilities from the LEHD J2J data.

moves and Kambourov and Manovskii (2008) for intersectoral mobility.[35]

One important observation from Table I is the large amount of heterogeneity in transition probabilities across labor markets, which indicates that workers in some industries and states are more likely to switch to a different labor market than other workers. In particular, the 25th and 75th percentiles of the distribution of sectoral mobility probabilities by labor market are 40% lower and higher than the median, respectively. This dispersion is even larger for interstate moves. We interpret the observed low transition probabilities and their heterogeneity as evidence of substantial and heterogeneous costs of moving across labor markets, both spatially and sectorally.

Elasticities. We use a quarterly discount factor β of 0.99, implying a yearly interest rate of roughly 4%. The sectoral trade elasticities θ^j are obtained from Caliendo and Parro (2015). We estimate the migration elasticity, $1/\nu$, by adapting the method and data used in ACM. From their model, they derive an estimating equation that relates current migration flows to future wages and future migration flows. Then, they estimate the equation by GMM and instrument using past values of flows and wages.[36]

In order to adapt ACM's procedure to our model and frequency, we have to deal with two issues. First, in our model, agents have log utility, while in ACM preferences are linear; and second, ACM estimate an annual elasticity, while we are interested in a quarterly elasticity. Dealing with the first issue is not that difficult since from our model we obtain the analogous estimating equation to ACM's preferred specification but with log utility; namely,

$$\log\left(\mu_t^{nj,nk}/\mu_t^{nj,nj}\right) = \tilde{C} + \frac{\beta}{\nu}\log\left(w_{t+1}^{nk}/w_{t+1}^{nj}\right) + \beta\log\left(\mu_{t+1}^{nj,nk}/\mu_{t+1}^{nk,nk}\right) + \varpi_{t+1}, \tag{27}$$

where ϖ_{t+1} is a random term and $\tilde{C}$ is a constant. Intuitively, the cross-sectional migration flows contain information on expected values that depend on future wages and the option value of migration across markets, and future migration flows in this regression are sufficient statistics for these option values (see Appendix A). The relevant coefficient β/ν represents the elasticity of migration flows to changes in income, while in ACM it has the interpretation of a semi-elasticity. As pointed out by ACM, the disturbance term, ϖ_{t+1}, will in general be correlated with the regressors; thus, we require instrumental variables. As in ACM, our theory implies that past values of sectoral migration flows and wages are valid instruments; therefore, we use lagged flows and wages as instruments for the wage variable in (27).[37]

[35] Since our period is a quarter, our rates are not directly comparable with the yearly mobility rates for states and industries from these studies. Moreover, our sample selects workers from 25 to 65 years of age, who tend to have lower mobility rates than younger workers.

[36] ACM construct migration flow measures and real wages for 26 years between 1975-2000, using the U.S. CPS. We use ACM data in our estimation and do not proceed to disaggregate their data forward. Due to its small sample size, using the March CPS to construct interregional and intersectoral migration flows could bias downward the amount of mobility. For further details, see ACM and Appendix E.

[37] The exclusion restriction is that the error term, ϖ_{t+1}, is not correlated over time. Naturally, de-

Dealing with the second issue is more involved. As ACM discuss, Kambourov and Manovskii (2013) point out a difficulty in interpreting flow rates that come out of the March CPS retrospective questions. They conclude that although superficially it appears to be annual, the mobility measured by the March CPS is less than annual. ACM correct for this bias and conclude that the March CPS measures mobility at a five-month horizon. Then, they annualize the migration flow matrix by assuming that within a year the monthly flow rate matrix is constant. We transform the five-month migration flow matrices in ACM to quarterly matrices using the same procedure but adapted to convert to quarterly flows.

After dealing with these two issues, we obtain a migration elasticity of 0.2, implying a value of $\nu = 5.34$. This is our preferred estimate, and we use this number in our empirical section below. To the best of our knowledge, there is no benchmark value for this quarterly elasticity in the literature. Yet, to put it in perspective, our estimate is consistent with the intuition that this elasticity should be smaller, thus ν larger, at higher frequencies. In fact, the implied annual inverse elasticity in our model is 2.02 at an annual frequency and a larger value of 3.95 at a five-month frequency.[38]

4.1. *Identifying the China Trade Shock*

As discussed above, we want to apply our solution method to study the effects on the U.S. economy of the China trade shock. In particular, we want to study the welfare and employment effects on the U.S. labor markets if the world would have evolved as it did except for the China trade shock. To do so, we first need to measure the China trade shock. We proceed as follows.

In previous work, ADH and Acemoglu et al. (2016) argue that the increase in U.S. imports from China had asymmetric impacts across regions and sectors. In particular, labor markets with greater exposure to the increase in import competition from China saw a larger decrease in manufacturing employment. Given that the observed changes in U.S. imports from China are not necessarily the result of an exogenous shock to China (TFP or trade costs), we replicate the procedure of ADH to identify the component of imports from China that is driven by China. To do so, we compute the predicted changes in U.S. imports from China using the change in imports from China by other advanced economies as an instrument. This procedure is related to the first-stage regression of the two-stage least-squares estimation in ADH conducted under our definition of labor markets, that is, at our regional and sectoral disaggregation, and using trade flows from

pending on the context, this is a strong assumption that in some cases could be violated. For example, if there are unobservable serially correlated characteristics of some labor markets, they are going to be subsumed in the residual. We rely on ACM's strategy but note that future research should focus on finding a different instrument, or a different estimation strategy, that is not subject to this criticism. See ACM for a discussion on other strengths and weaknesses of this approach.

[38] As mentioned above, ACM's model has linear utility, and therefore $1/\nu$ is a semi-elasticity in ACM. They estimate $\nu = 1.88$ at an annual frequency.

the WIOD.[39]

We estimate the following regression:

$$\Delta M_{USA,j} = a_1 + a_2 \Delta M_{other,j} + u_j,$$

where here j is one of our 12 manufacturing sectors, $\Delta M_{USA,j}$ is the change in U.S. imports from China, and $\Delta M_{other,j}$ is the change in imports from China by other advanced economies between 2000 and 2007.[40]

We then use the changes in U.S. imports predicted by this regression to calibrate the size of the TFP changes for each of the manufacturing sectors in China. Concretely, we solve for the change in China's TFP in the 12 manufacturing industries $\{\hat{A}_t^{China,j}\}_{j=1,t=2000}^{12,2007}$ such that the model-predicted imports match the predicted imports from China from 2000 to 2007 given by $a_2 \Delta M_{other,j}$. To do so, we use our dynamic model that has a baseline economy with time-varying fundamentals that exactly match all observed moments in the data and a counterfactual economy where agents expected all fundamentals to evolve as in the data and the China shock did not happen.[41,42]

In Appendix F, Figure 15 shows the predicted change in U.S. manufacturing imports from China computed as in ADH and our measured sectoral productivity changes in China over the sample period.[43] The computer and electronics industry is the most exposed to import competition from China, accounting for about 45% of the predicted total change in U.S. imports from China, followed by the textiles industry with about 15%, and the metals industry with 7%. On the other hand, the nonmetallic, wood and

[39] See Appendix F for more details on the data construction and estimation. One might be concerned that with our data and at our level of disaggregation the specification from ADH might not deliver employment effects comparable to ADH. Therefore, in Appendix F we also run the second-stage regression in ADH with our data; the results we obtain are largely aligned with those in ADH.

[40] In particular, the set of countries used by ADH in the construction of $\Delta M_{other,j}$ are Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland. The coefficient a_2 in the regression is estimated to be 1.386 with a robust standard error of 0.033. The predictive power of the regressor is large, with an R-squared of 0.99. Including additional countries in the construction of $\Delta M_{other,j}$ has very small effects on the predicted values for $\Delta M_{USA,j}$. See Appendix F for further details.

[41] Since the change in U.S. imports from China is evenly distributed over this period, we interpolate an initial guess of sectoral TFP changes over 2000-2007 across all quarters and feed this sequence of TFP shocks into our dynamic model. We iterate over these changes in TFP and solve for the TFP changes that minimize a weighted-sum of squares of the difference between the change in the predicted U.S. imports from China over 2000-2007 (using ADH first-stage regression) and the ones from the dynamic model employing a non-linear solver. See Appendix F for more details.

[42] Our measurement of the China shock also assumes that all other fundamentals are orthogonal to the change in Chinese productivity.

[43] We compared our calibrated TFPs to other estimates in the literature. While sectoral TFP data for manufacturing sectors in China for our sample period are not available from statistical agencies, some studies have estimated TFP in China using micro and macro data. For instance, using firm-level data, Brandt et al. (2012) compute annual Chinese TFP growth in the manufacturing sector of about 8 percent over the period 1998-2007, while using macro data they estimate aggregate TFP growth of 13.4% per year. We obtain average (and aggregate) TFP growth in manufacturing of 11% over 2000-2007. In addition, they present estimates for selected industries (Table A.13 in their online appendix) that we map into some of our industries and obtain a correlation of 0.8.

paper, and petroleum industries are the least exposed, with each of them accounting for less than 1.5% of the predicted total change in U.S. imports from China.

5. THE EFFECTS OF THE CHINA TRADE SHOCK

In this section, we quantify the dynamic effects of China's import competition on the U.S. economy. In particular, the question that we address is the following. Imagine that agents anticipated all changes in fundamentals exactly as they occurred. This is the factual world we have lived in. Now, consider a counterfactual in which all of these changes in fundamentals occurred except there was no China shock. What would have happened differently across U.S. labor markets?

To answer this question, we first solve for the baseline economy with the actual evolution of fundamentals over 2000-2007 and use Proposition 2 to compute the baseline economy from 2007 forward assuming constant fundamentals thereafter. We then use the results from Proposition 3 and solve for the difference between our baseline economy and a counterfactual economy with the actual changes in fundamentals over the period 2000-2007 except for the China shock; i.e., the estimated Chinese productivities did not change over time. We first discuss the effects on aggregate, sectoral, and regional employment in Section 5.1 and then analyze the effects on welfare across markets in Section 5.2.

5.1. *Employment Effects*

Starting with sectoral employment, Figure 1 presents the dynamic response of sectoral shares of employment due to the China shock.[44] In particular, the figure shows the difference between our baseline economy with the actual changes in fundamentals and a counterfactual economy with the actual changes in fundamentals except for the estimated changes in productivities in China (the economy without the China shock). In Appendix H, we show separately the evolution of employment shares in the baseline economy and the counterfactual economy.

The upper-left panel in Figure 1 shows the transitional dynamics of manufacturing employment due to the China shock. The figure shows that import competition from China contributed to a decline in the share of manufacturing employment. Our results indicate that increased competition from China reduced the share of manufacturing employment by 0.36 percentage points after 15 years, which is equivalent to about 0.55 million jobs or about 36% of the change in manufacturing employment that is not explained by a secular trend.[45]

[44] Recall that in this study, the China shock is the change in manufacturing productivity in China from 2000 to 2007 computed to match the predicted change in U.S. imports from China using the ADH results. Of course, part of the observed contraction in manufacturing employment share may actually be caused by increases in productivity in China occurring in the 1980s and 1990s.

[45] The difference between the observed share of manufacturing employment in the U.S. economy in

As shown in the other three panels of Figure 1, increased import competition from China leads workers to relocate to other sectors; thus, the share of employment in services, wholesale and retail, and construction increases. The role of intermediate inputs and sectoral linkages is crucial to understanding these relocation effects. Import competition from China leads to decreased production among U.S. manufacturing sectors that compete with China, but it also affords the U.S. economy access to cheaper intermediate goods from China that are used as inputs in non-manufacturing sectors. Production and employment increase in the non-manufacturing sectors as a result. As an example, we find that in the long run, about 50 thousand jobs are created in construction as a result of the China shock.[46]

Our quantitative framework also allows us to further explore the decline in manufacturing employment caused by the China shock. In particular, we quantify the relative contribution of different sectors, regions, and local labor markets to the decline in the manufacturing share of employment.

Figure 2 shows the contribution of each manufacturing industry to the total decline in the manufacturing sector employment. Industries with higher exposure to import competition from China lost more employment. The computer and electronics industry contributed about 25 percent of the decline in manufacturing employment, followed by the furniture, textiles, metal, and machinery industries, each contributing 10-15 percent to the total decline. Industries less exposed to import competition from China, such as the food, beverage and tobacco, petroleum, and nonmetallic minerals industries, explain a smaller portion of the decline in manufacturing employment. In fact, these industries also benefit from access to cheaper intermediate goods from industries that experienced a substantial productivity increase in China.

The fact that the U.S. economic activity is not equally distributed across space, combined with its differential sectoral exposure to China, implies that the impact of import competition from China on manufacturing employment varies across regions.

Figure 3 presents the regional contribution to the total decline in manufacturing employment. States with a comparative advantage in industries more exposed to import competition from China lose more employment in manufacturing. For instance, Califor-

2007 and its predicted value using a simple linear trend on this share between 1965 and 2000 is 1%. In other words, the change in the U.S. manufacturing share that is unexplained by a linear trend is 1%. To compute the implied levels of manufacturing employment loss in 2007, we take data on total employment from the BEA for the year 2007 (Table SA25N: Total Full-Time and Part-Time Employment by NAICS Industries). To match the sectors in our model, we subtract employment in farming, mining, utilities, and the public sector, which yields a level of employment of 151.4 million. We multiply by our model's implied change in manufacturing employment share and get 0.55 million jobs.

[46]In Appendix C.2 we extend our model for the case of a CES utility function with an elasticity of substitution between manufacturing and non-manufacturing different from 1. Our main results are robust to changes in the value of this elasticity. For instance, we find that in the range of an elasticity of substitution between 0.1 and 2, the manufacturing employment share declines about 0.36 percentage points as a consequence of the China shock and aggregate welfare increases between 0.166 and 0.232 percent. The stability of these effects is due to the fact that the manufacturing expenditure share moves little in the counterfactual economy relative to the baseline economy.

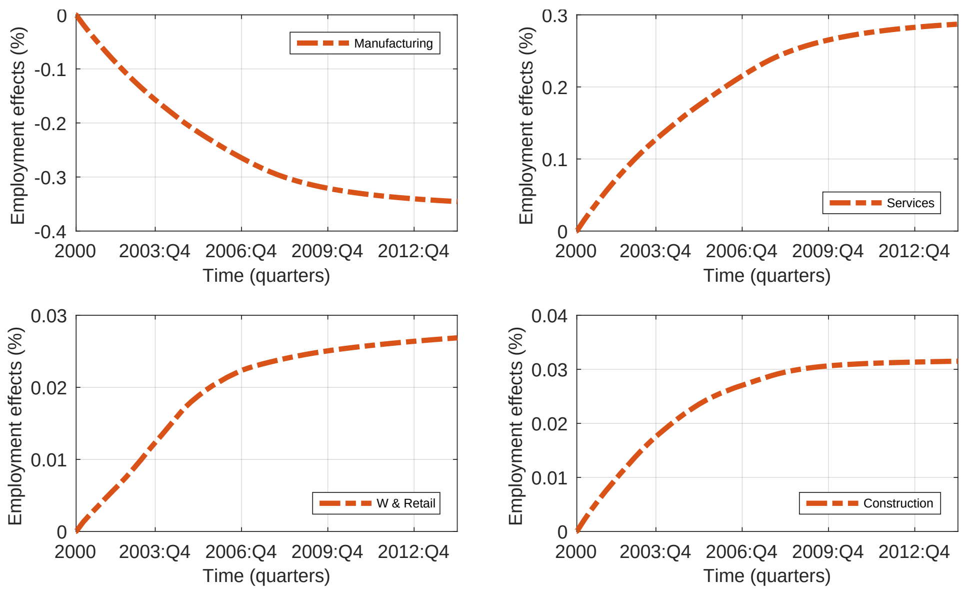

Figure 1: The effect of the China shock on employment shares

Note: The figure presents the effects of the China shock measured as the change in employment shares by sector (manufacturing, services, wholesale and retail, and construction) over total employment between the economy with all fundamentals changing as in the data and the economy with all fundamentals changing except for the estimated sectoral changes in productivities in China (the economy without the China shock).

nia alone accounted for 20% of all employment in the computer and electronics industry in the year 2000. For comparison, the state with the next-largest share of employment in this industry is Texas with 8%, while all other states had less than 2%. As a result, California contributed the most to the overall decline in manufacturing employment (about 9%), followed by Texas. States with a comparative advantage in goods that were less affected by import competition from China and states that benefited from the access to cheaper intermediate goods had the smaller reduction in manufacturing employment.

While Figure 3 shows the spatial distribution of the aggregate decline in manufacturing employment, it is also informative to study the local impact of the China shock in each state . For instance, even when larger regions, such as California, are more exposed to the China shock because they have a large fraction of U.S. employment in industries with high exposure to foreign trade, they also tend to be more diversified. That is, employment and production are also important in other sectors, such as services, with little direct exposure to trade. Therefore, although the contribution of larger regions to the aggregate decline in manufacturing is large, the local impact of the China shock could be mitigated compared with smaller and less diversified regions where manufacturing represents a higher share of local employment.

This local impact is shown in Figure 4, which displays the regional contribution to

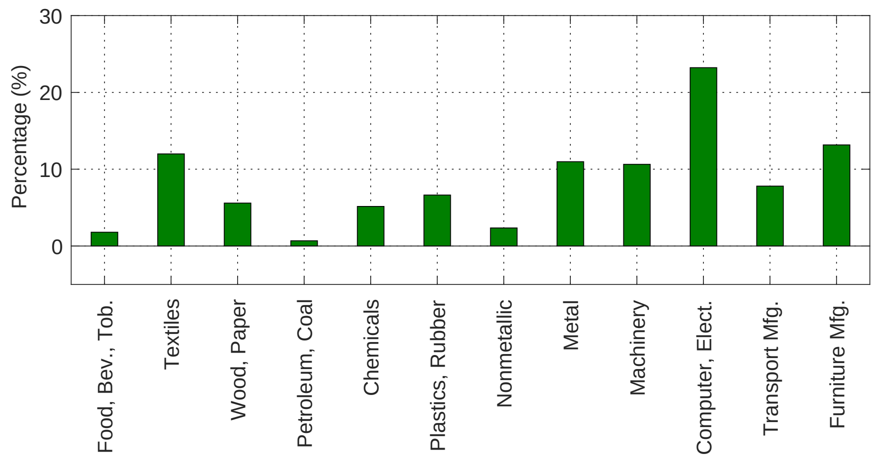

Figure 2: Manufacturing employment declines due to the China trade shock (percent of total)

Note: The figure presents the contribution of each manufacturing industry to the total reduction in the manufacturing employment due to the China Shock.

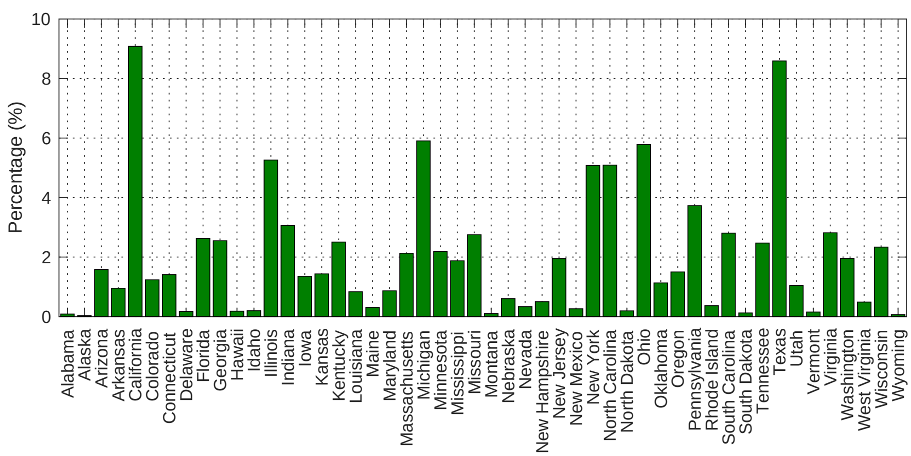

Figure 3: Regional contribution to U.S. aggregate manufacturing employment decline (percent)

Note: The figure presents the contribution of each state to the total reduction in manufacturing sector employment due to the China shock.

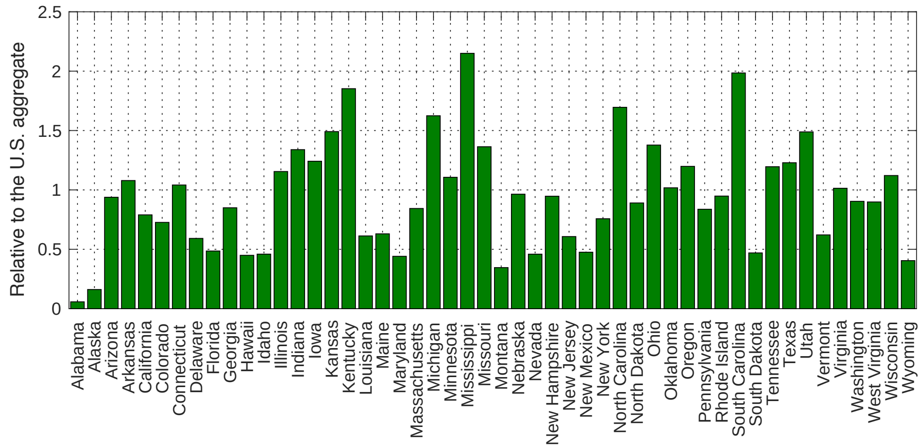

Figure 4: Regional contribution to U.S. aggregate manufacturing employment decline, normalized by regional employment share

Note: The figure presents the contribution of each state to the U.S. aggregate reduction in manufacturing sector employment due to the China shock, normalized by the employment of each state relative to the U.S. aggregate employment.

the total decline in manufacturing employment normalized by the employment share of the state in the U.S. economy. In the figure, a number greater than 1 means that the local change in manufacturing employment share is larger than the national change (-0.36 percentage points). As we can see from this figure, the local impact in manufacturing employment in some states, such as South Carolina, Mississippi, and Kentucky, was bigger than the impact for the whole U.S. economy. The figure also shows that in other larger and more diversified states, such as California and Texas, the decline in manufacturing employment as a share of the state employment is more similar to the aggregate U.S. decline in manufacturing employment.

We now turn to the sectoral and spatial distribution of the employment gains in the non-manufacturing industries due to the China shock. The sectoral contribution to the change in non-manufacturing employment is displayed in Figure 5. As we can see, all non-manufacturing industries absorbed workers displaced from manufacturing industries. In particular, besides the category other services, the health and education industries are the largest contributors among the service industries, accounting for about 20 percent and 10 percent, respectively, of the change in non-manufacturing employment, followed by construction and transport services with a bit less than 10 percent each. Figure 6 shows that U.S. states with a larger service sector contribute more to the increase in non-manufacturing employment, as they were able to absorb more workers displaced from the manufacturing industries. Specifically, California and New York are the largest contributors, accounting for about 12 percent and 8 percent of the total increase in non-manufacturing employment, respectively.

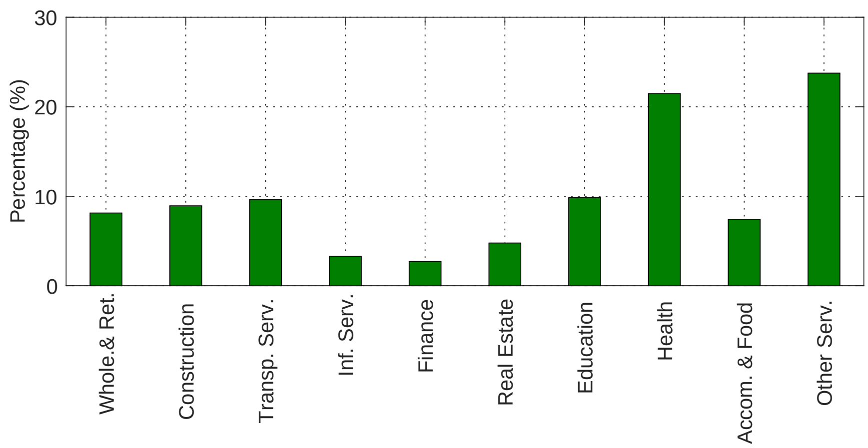

Figure 5: Non-manufacturing employment increases due to the China trade shock (percent of total)

Note: The figure presents the contribution of each non-manufacturing sector to the total increase in non-manufacturing employment due to the China shock.

Economic activity is unevenly distributed across space in the United States, and, therefore, the sectoral employment effects in Figures 2 and 5 can mask different distributional effects across space in different industries. To study the regional employment effects from the China shock in different industries, Figures 7 and 8 present U.S. maps that show the changes in regional employment by industry. The first column of each figure presents the contribution of each region to the U.S. aggregate change in industry employment as a consequence of the China shock (analogous to Figure 3). The second column presents the contribution of each region to the U.S. aggregate change in industry employment normalized by the employment share of the state (analogous to Figure 4). Figure 7 presents the results for three selected manufacturing industries, computer and electronics, machinery, and textiles, and Figure 8 presents the results for three selected non-manufacturing industries, construction, services, and wholesale and retail. In Appendix H we present the figures with the effects for all the other sectors.

From Figures 7 and 8 we can see the unequal regional effects from the China shock in different industries. For instance, the decline in employment in the computer and electronics industry (Figure 7, panel a.1), an industry highly exposed to Chinese import competition, is concentrated in California, while the decline in employment in machinery (Figure 7, panel b.1) is more concentrated in the midwestern states. Part of this concentration reflects that economic activity in these industries is mostly concentrated in these regions. After normalizing the contribution of each state by the employment share of the state in the U.S. economy, Figure 7 panels a.2, b.2, and c.2 reveal the regions where the China shock had a larger local impact relative to the aggregate impact on the United States. For example, panel c.2 shows that, as a consequence of the China shock,

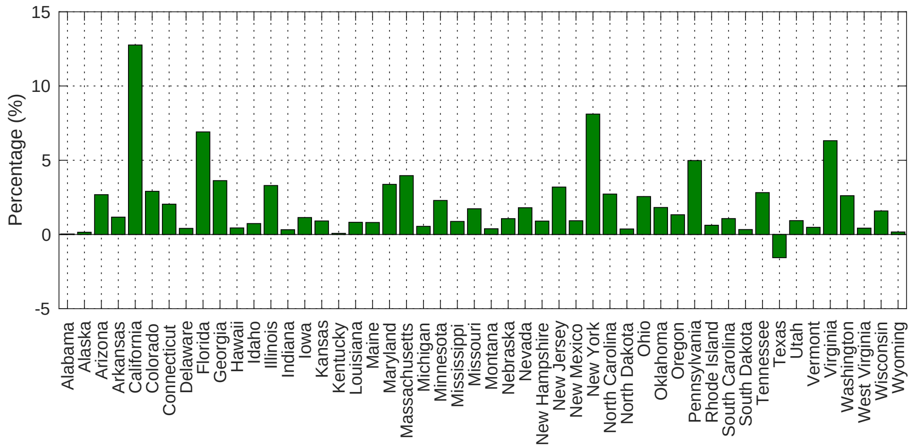

Figure 6: Regional contribution to U.S. aggregate non-manufacturing employment increase (percent)

Note: The figure presents the contribution of each state to the total rise in non-manufacturing employment due to the China shock.

South Carolina, North Carolina, Utah, Virginia, and Mississippi experienced a reduction in the employment share in the textile industry more than three times as large as that for the nation. Panel b.2 presents the case of the machinery industry. Even after controlling for size, the midwestern states experienced the largest reductions in state employment share in the machinery industry relative to the national reduction in employment for that industry.

Figure 8 presents the results for selected non-manufacturing industries. Recall from Figures 1 and 5 that non-manufacturing industries increased their employment share as a consequence of the China trade shock. We can see in Figure 8 panels a.1, b.1, and c.1 that, similar to the case of manufacturing industries, larger states such as California and New York are more important contributors to the overall change in employment. However, different from the manufacturing industries, after controlling for the relative size of the state, the local impact is much more evenly distributed across space. As shown, the reduction in local employment in manufacturing industries is more concentrated in a handful of states, while the increase in local employment in non-manufacturing industries is spread more evenly across U.S. states.

Finally notice that Figures 1, 2, 7, and 8 shed light on the contribution of each state/industry pair to the aggregate decline in manufacturing employment. For instance, Figure 7 shows that California contributes 14.2 percent to the decline in employment in the computer and electronics industry, while Figure 2 shows that the computers and electronics industry contributes to 23.2 percent to the decline in manufacturing employment. Given this, the computer and electronics industry in California accounts for about 3.3 percent of the total decline in manufacturing employment.

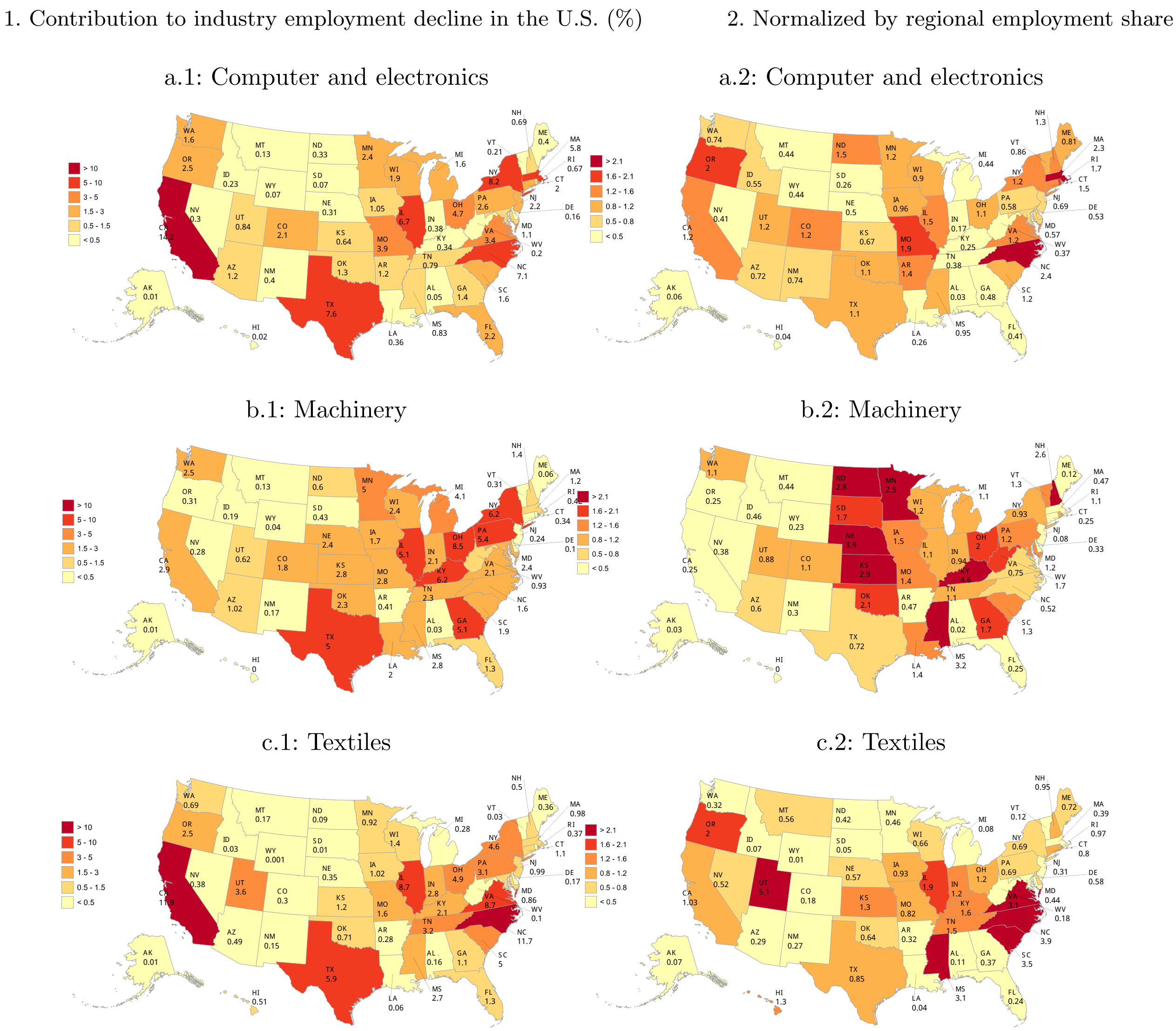

Figure 7: Regional employment declines in manufacturing industries

Note: This figure presents the reduction in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate reduction in industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate reduction in industry employment normalized by the employment size of each state relative to U.S. aggregate employment. Panels a present the results for the computer and electronics industry. Panels b present the results for the machinery industry. Panels c present the results for the textiles industry.

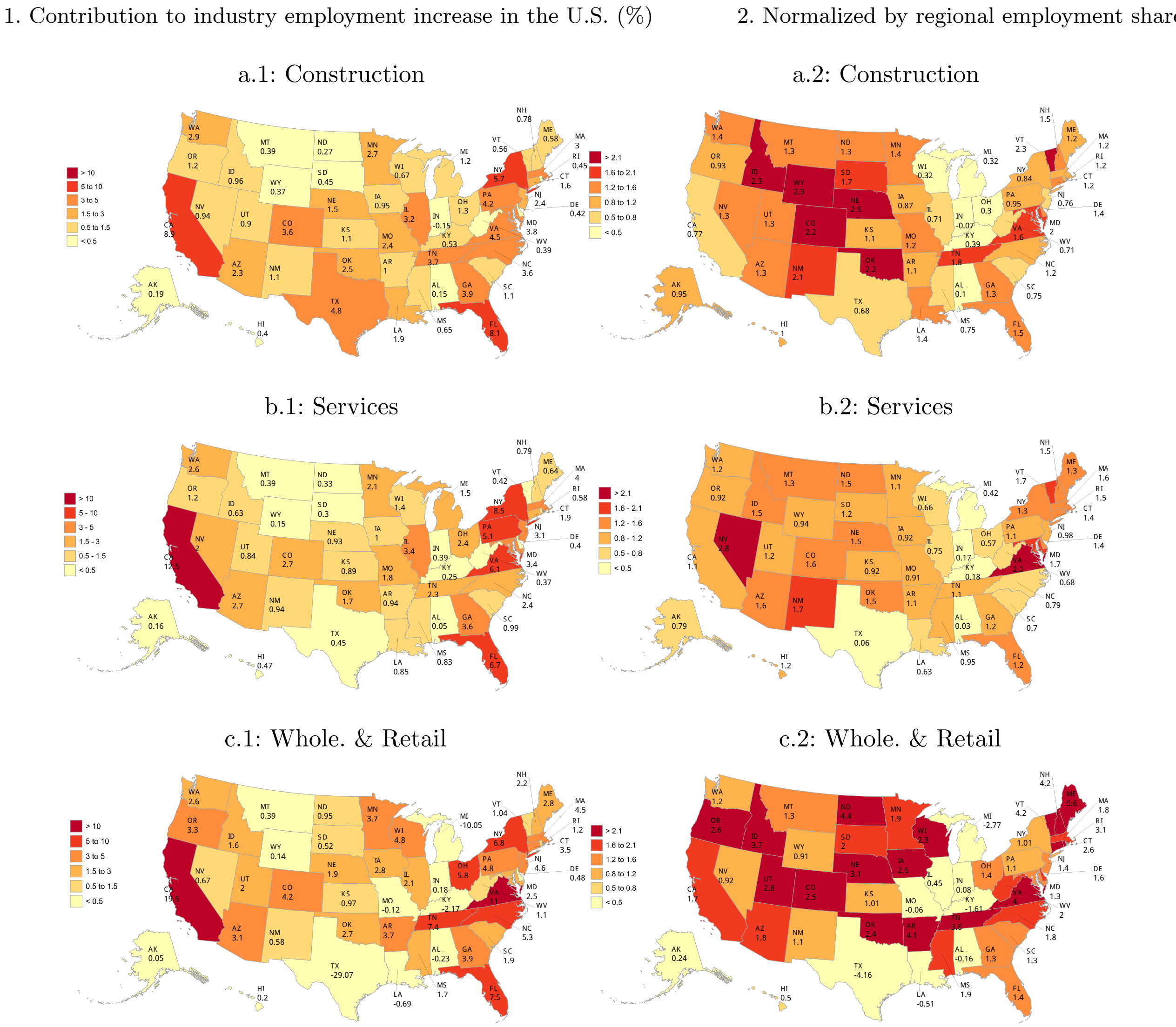

Figure 8: Regional employment increases in non-manufacturing industries

Note: This figure presents the rise in local employment in non-manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate increase in industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate increase in industry employment normalized by the employment size of each state relative to U.S. aggregate employment. Panels a present the results for construction. Panels b present the results for all services. Panels c present the results for wholesale and retail.

Overall, the contribution of each labor market to the total decline in manufacturing employment varies considerably across regions and industries. We find a decline in employment in most manufacturing labor markets, although employment increased in some. The computer and electronics industry in California was the labor market that contributed the most to the decline in manufacturing employment, accounting for 3.3 percent of the total decline. Employment increased in some labor markets, such as food, beverage, and tobacco in Connecticut, New Hampshire, Rhode Island, and Vermont; petroleum in California and Arkansas; and transportation equipment in New Hampshire and Rhode Island, among others.

We also find that the China shock reduced the U.S. non-employment rate by 0.22 percentage points in the long run (Figure 9). We find that the fall in non-employment is mainly due to a decline in the flow of households from non-manufacturing industries to non-employment, which is explained by the expansion of non-manufacturing industries after the China shock. We also find that the flow of households from manufacturing to non-employment increased in states with more concentration in manufacturing industries such as Alabama, Arkansas, Mississippi, Michigan, and Ohio, among others, but declined in larger and more diversified states, such as California, New York, Florida, Illinois, and Pennsylvania. The latter states have a relatively larger services sector that can more easily absorb workers displaced from manufacturing industries.[47] Later on, we extend our framework to analyze further the employment and non-employment effects of the China shock by introducing disability benefits and by modelling the flow of non-employed households into and out of the disability program.

Before turning to welfare effects, we finish this section by discussing how our employment effects relate to recent reduced-form approaches for studying the effects of the China shock, most notably ADH. This study provides robust evidence about the differential effects of the China shock across U.S. labor markets; namely, labor markets with larger exposure to import competition from China experience larger employment declines relative to less exposed labor markets. Our general equilibrium approach also delivers implications on the differential employment effects of the China shock across U.S. labor markets. It is important to note that since our baseline economy matches the factual economy, if we run the second-stage ADH regression in our baseline economy, by construction we will replicate the ADH regression results.[48] Yet, we can look at the differential employment effects predicted by our model from the changes in TFP in China

[47] The observed non-employment rate increased from 27.4% in 2000 to 29.1% in 2003 and then declined to 28.5% in 2007. These numbers are obtained using data from the ACS and using the same sample criteria as in our empirical analysis. ADH show evidence that higher exposure to Chinese imports in a labor market cause a larger increase in non-employment in that market. In our model, non-employment falls due to the China shock, but we construct a measure of import changes per worker in each U.S. state over the period 2000-2007 and find a positive relation between import penetration and non-employment in a labor market.

[48] In Appendix F, we run the second-stage regression of ADH and the results we obtain are largely aligned with those presented in ADH at a different level of aggregation.

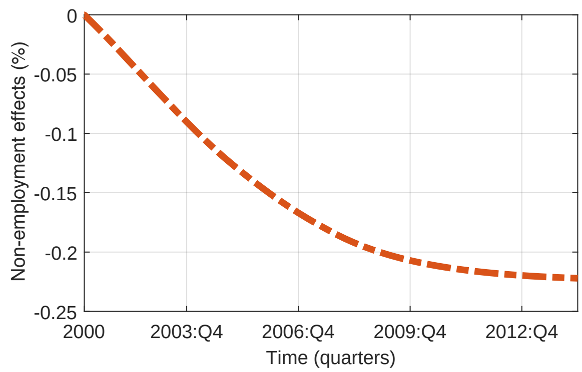

Figure 9: The effect of the China shock on non-employment shares

Note: The figure presents the effects of the China shock, measured as the difference in the non-employment shares between the economy with all fundamentals changing as in the data and the economy with all fundamentals changing except for the estimated sectoral changes in productivities in China (the economy without the China shock).

and compare them with the differential effects predicted by the second-stage ADH regression. We find that the differential employment effects in our model are aligned with those in ADH.[49] Beyond the relative employment effects across labor markets, our general equilibrium approach also complements the reduced-form evidence by providing a quantification of the aggregate and disaggregate effects on the level of employment across U.S. labor markets, as discussed in this section, and by quantifying welfare effects and enabling the study of policy changes, the focus of the next sections.

5.2. *Welfare Effects*

We now turn to the aggregate and disaggregate welfare effects on the U.S. economy of increased import competition from China. The change in welfare from a change in fundamentals, $\hat{W}^{nj}$, measured in terms of consumption equivalent variation, can be expressed as

$$\hat{W}^{nj} = \sum_{s=1}^{\infty} \beta^s \log\left(\frac{\hat{C}_s^{nj}}{\left(\hat{\mu}_s^{nj,nj}\right)^{\nu}}\right). \tag{28}$$

We compute the welfare effects of the China shock using equation (28).[50] In Appendix

[49] In particular, we compare $\Delta L_{it}^{m,ADH} = b_2\ \Delta IPW_{uit}$ with the model implied ΔL_{it}^{m}, where ΔIPW_{uit} is the change in imports per worker predicted by the first-stage ADH regression. The correlation between these two variables reveals that the differential effects are aligned, with a correlation of 0.7. Different from ADH, the implied b_2 in our model is labor-market specific and shaped by all the mechanisms in our model.

[50] In a one-sector model with no materials and structures, equation (28) reduces to $W^{nj} = \sum_{s=1}^{\infty} \beta^s \log \frac{(\hat{\pi}_s^{nn})^{-1/\theta}}{(\hat{\mu}_s^{nn})^{\nu}}$, which combines the welfare formulas in ACM and Arkolakis et al. (2012).

A we present the derivation of equation (28) and discuss the different mechanisms that shape the welfare effects of changes in fundamentals in our model in more detail.

We find that U.S. aggregate welfare increases by 0.2% due to China's import penetration growth.[51] The aggregate change in welfare masks, however, an important heterogeneity in the welfare effects across different labor markets. Figure 10 presents a histogram with the changes in welfare across 1150 U.S. labor markets. An important takeaway from the figure is that there is a very heterogeneous response to the same aggregate shock across labor markets; changes in welfare range from a decline of about 0.8 percent to an increase of 1 percent.

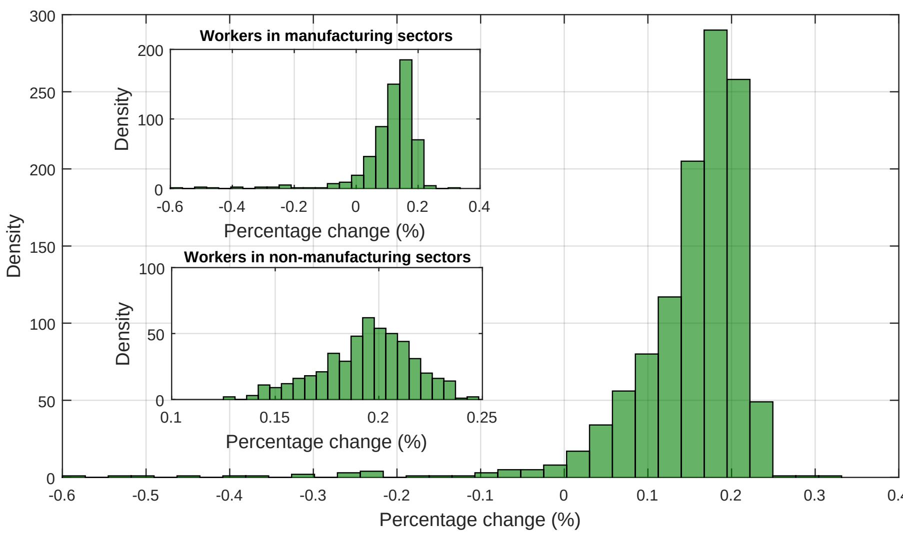

Figure 10: Welfare effects of the China shock across U.S. labor markets

Note: The figure presents the change in welfare across all labor markets (central figure), for workers in manufacturing sectors (top-left panel), and for workers in non-manufacturing sectors (bottom-left panel) as a consequence of the China shock. The largest and smallest 1 percentile are excluded in each figure. The percentage change in welfare is measured in terms of consumption equivalent variation.

Welfare effects are more dispersed across labor markets that produce manufacturing goods than those that produce non-manufacturing goods, as manufacturing industries have different exposure to import competition from China. Also, labor markets that produce service goods gain from the China shock, and welfare tends to be higher in those labor markets than in the manufacturing sectors. Labor markets that produce non-manufacturing goods do not suffer the direct adverse effects of increased competition from China and at the same time benefit from access to cheaper intermediate manufacturing

[51] We aggregate welfare across labor markets using the employment shares at the initial year. In other words, we use an utilitarian approach to aggregate welfare of heterogeneous workers.

inputs from China. Similarly, labor markets located in states that trade more with the rest of the U.S. economy and purchase materials from sectors in which Chinese productivity increased more tend to have larger welfare gains. For instance, all labor markets located in California gain, even though California is highly exposed to China. The reason is that California benefits more than other states from the access to cheaper goods purchased from the rest of the U.S. economy and China. In addition, larger states such as California, have more diversified production, so industries less affected by the China shock can more easily absorb workers from industries in the same state that are more affected by the China shock.[52]

Migration costs are also important to understanding the differences between welfare effects of the China shock in the short run and in the long run. In the short run, migration costs prevent workers in the labor markets most negatively affected by the China shock from relocating to other industries. Therefore, real wages fall where labor market conditions worsen. In the long run, workers are able to relocate to industries or states with higher labor demand and real wages. As a result, we find that while in the long run only about 4 percent of the labor markets experience welfare losses, real wages drop in about 47 percent of all labor markets when the China shock hits the U.S. economy.

To study this in further detail, Figures 11 and 12 present the welfare effects of the China shock in the short run and in the long run across U.S. states. Figure 11 panel a.1 presents the long-run welfare effects across U.S. states at the regional aggregate level, panel a.2 for the manufacturing industries, and panel a.3 for the non-manufacturing industries. In the long-run, aggregate welfare increases in all states due to the China shock, ranging from 0.12 in Michigan to 0.22 in Vermont. The other two panels show that all states experience welfare gains both in the manufacturing industries and in the non-manufacturing industries in the long run. We also find that in all states the welfare gains in the non-manufacturing industries are larger than in the manufacturing industries. As discussed above, non-manufacturing industries have no direct exposure to China and also benefit from the access to cheaper materials from the manufacturing industries.

However, even though the manufacturing industries across U.S. states are better off in the long run due to the China shock, they are worse off in the short run due to increased import competition and relocation costs. In Figure 12, the first panel shows the change in real wages in the manufacturing industries across U.S. states when the China shock hit the U.S. economy, and the second panel shows the change in real wages between 2000 and 2007. In the first panel, we can see that real wages fall when the China shock starts. In the second panel, we see that the real wage decline deepens over the China shock period as the magnitude of the China shock accumulates over time, more than

[52]We perform a series of robustness exercises where we recompute the allocation and welfare results using different values of ν, ranging from $\nu = 3$ to $\nu = 5.34$. We find that the effect of the China shock on manufacturing employment shares and aggregate welfare are very robust to the value of ν, although the value of this parameter has a moderate effect on the dispersion of the welfare effects across labor markets.

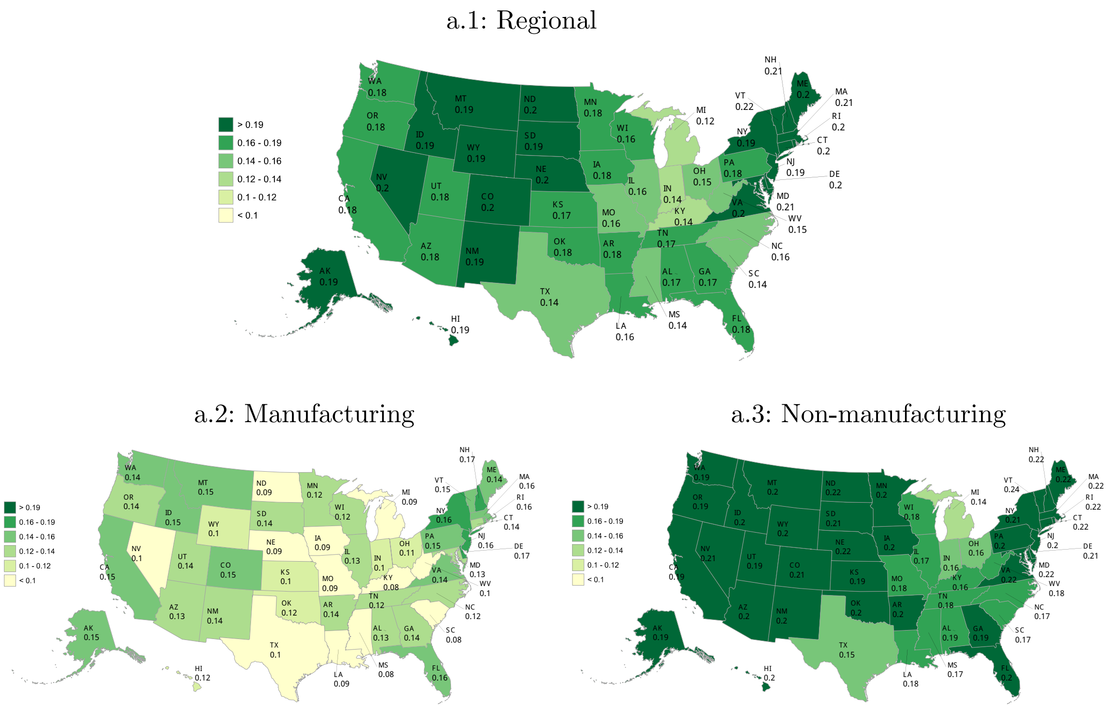

Figure 11: Regional welfare effects (percent)

Note: The figure presents the welfare effects across states in the U.S. Panel a.1 shows the regional effects in each state, panel a.2 presents the manufacturing welfare effects in each state, and panel a.3 presents the welfare effects in the non-manufacturing sectors in each state. We aggregate welfare across labor markets within a state using employment shares for the initial year.

offsetting the effect of some labor relocation during this period. The bottom line of these figures is that the relocation process after a trade shock takes time, and the welfare gains from increased competition only show up after this relocation process occurs. Therefore, taking into account the dynamic relocation process after the China shock is crucial to capturing the long-run welfare gains.

We also compute the welfare effects across countries. Figure 13 shows that all countries gain from the China shock, with some countries gaining more and others gaining less than the United States. Countries that are more open to trade, not only to China but to the world, such as Cyprus and Australia, experience bigger welfare gains, as they benefit from the access to cheaper intermediate goods from China as well as from purchasing cheaper goods from other countries that also benefit from purchasing cheaper intermediate goods from China.

a.1: One quarter after 2000 a.2: From 2000 to 2007

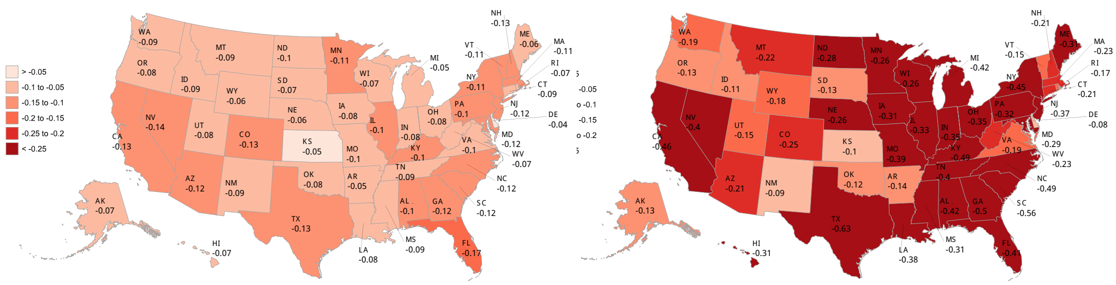

Figure 12: Regional real wage changes in the manufacturing sector (percent)

Note: The figure presents the change in real wages in the manufacturing sector across U.S. states Panel a.1 presents the change in real wages at impact, one quarter after the China shock started. Panel a.2 presents the change in real wages from 2000 to 2007, during the entire period of the China shock. We aggregate the changes in real wages across labor markets within a state using employment shares for the initial year.

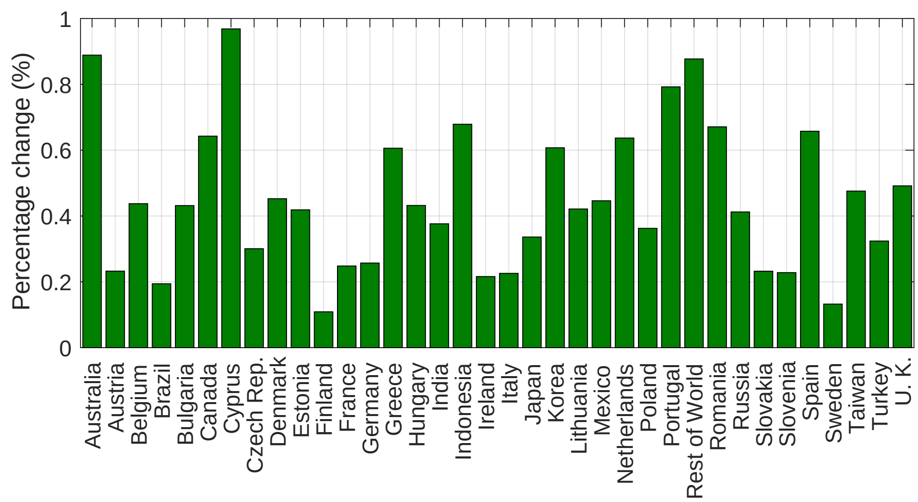

Figure 13: Welfare effects across regions

Note: The figure presents the change in welfare across countries in our sample from the effect of the China shock. The percentage change in welfare is measured as the percentage change in real consumption.

5.2.1. *Adjustment Costs*

Recent papers have highlighted the importance of the transitional dynamics for welfare evaluation, specifically, the fact that comparisons across steady-state equilibria can significantly overstate or understate welfare measures (i.e., Dix-Carneiro, 2014; Alessandria and Choi, 2014; Burstein and Melitz, 2011). In order to provide a measure that accounts for the transition costs to the new steady state, we follow Davidson and Matusz (2010)'s proposed measure of adjustment costs. Formally, to measure the adjustment cost for market nj, we use

$$AC^{nj} = \log\left(\frac{\hat{V}^{nj}_{SS}}{(1-\beta)\hat{W}^{nj}}\right).$$

In words, the adjustment cost formula compares the welfare changes due to the China shock if the steady state is reached instantaneously relative to the actual welfare change where there are transitional dynamics. We find that, on average, transition costs reduce steady-state welfare by about 4.7%. However, the variation across individual labor markets is substantial. Figure 14 presents a histogram of the adjustment costs across individual labor markets. The distribution has a long right tail, and several labor markets have adjustment costs substantially larger than the average transition cost. We also find that some labor markets have negative adjustment costs, as the welfare gains with transition dynamics overshoot the steady state.[53]

5.3. *Additional Results*

In this section, we discuss additional results of the China shock. In Section 5.3.1, we extend the model to include federally funded Social Security Disability Insurance (SSDI), and study the effects of policy counterfactuals. In Section 5.3.2, we extend the model to incorporate additional sources of persistence in the relocation decisions of workers and discuss the effects of the China shock in this alternative model.

5.3.1. *Adding Disability Insurance to the Model*

We now extend our model to include SSDI. With this new version of the model, we can apply our method to study the following question: what would have happened if during the China shock, agents expected SSDI benefits and they were not granted? With this counterfactual, we learn about the marginal employment and non-employment effects of the China shock due to SSDI. To do this, we need to take a stand on how we introduce SSDI into the model. In particular, we need to model how benefits are financed and the transitions of workers into (and out of) the disability program.

SSDI is a federal program of the Social Security Administration funded by payroll tax contributions from workers and employers. We model this by including a federal

[53]Part of the heterogeneity in the adjustment costs across labor markets might capture human capital specificities that might vary across sectors. For instance, some workers could experience a reduction in the market value of their skills because the same skills are embodied in cheaper labor in China. One way to think about this in our model is that the sectoral migration costs capture, in part, the skill composition in each industry, and, therefore, how costly it is for certain skill groups to switch across industries that require different human capital specificity.

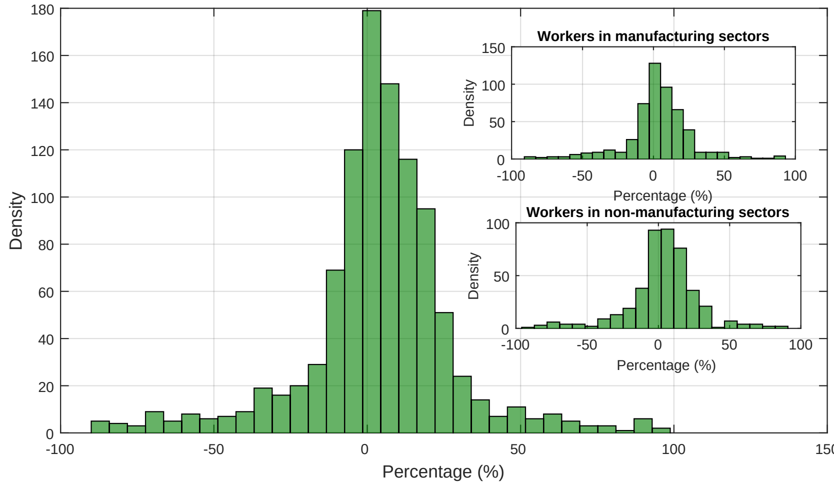

Figure 14: Adjustment costs

Note: The figure presents the transition costs across all labor markets (central figure), for workers in manufacturing sectors (top-right panel), and for workers in non-manufacturing sectors (bottom-right panel) due to the China shock. Labor markets with computed adjustment costs larger than 100 percent and smaller than -100 percent are excluded.

government in the model that finances the SSDI payments by levying taxes τ on the labor income of workers across all labor markets in the United States, denoted by N^{US}. The revenues from taxes are then used to pay SSDI to the fraction of people receiving the benefit in each region. We denote by L_t^{nD} the mass of workers on disability in region n at time t and by b^{DI} the SSDI benefits that a worker obtains. We assume taxes and per capita benefits do not vary over time, but the federal government revenue and expenditure on SSDI are time varying as a consequence of changes in labor income as well as changes in the number of beneficiaries. In order to allow for the federal government to balance total revenues and spending, we include a lump-sum transfer/tax G_t that applies to all rentiers located in the United States equally. Therefore, the per-period government budget constraint is given by

$$\sum_{n=1}^{N^{US}} \sum_{k=1}^{J} \tau w_t^{nk} L_t^{nk} + G_t = \sum_{n=1}^{N^{US}} b^{DI} L_t^{nD},$$

where $\sum_{n=1}^{N^{US}} \sum_{k=1}^{J} \tau \, w_t^{nk} L_t^{nk}$ is aggregate tax revenue from labor income and $\sum_{n=1}^{N^{US}} b^{DI} \, L_t^{nD}$ is the aggregate government expenditure on the SSDI program.

To be eligible for SSDI, households cannot be engaged in a "substantial gainful activity."[54] Therefore, we assume that all transitions into disability come from non-employment.

[54]The Social Security Administration determines what is considered a substantial gainful activity, and this varies with the nature of a person's disability. For more information, refer to

The way we model this is by assuming that non-employed households enter the SSDI program with probability δ. Being on SSDI tends to be a persistent state, and beneficiaries exit from the program mainly due to death, medical recovery, or conversion to retirement benefits. Given our sample selection, we abstract from retirement benefits. Below we discuss that the sum of exit probabilities due to death and medical recovery are somewhat constant for workers of different characteristics, thus we model the transition out of SSDI with the constant hazard rate $1-\rho^{DI}$. In addition, we assume that a transition out of disability is into non-employment only, and thus the subsequent transitions of workers out of disability and into employment are similar to the transitions of the non-employed.[55]

With the introduction of disability insurance in our model, the lifetime utilities of workers in non-employment (V_t^{n0}) and disability (V_t^{nD}) are given by

$$V_t^{n0} = \log b^n + \nu(1-\delta)\log\left[\sum\nolimits_{i=1}^{N}\sum\nolimits_{k=0}^{J}\exp\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{1/\nu}\right] + \delta\beta V_{t+1}^{nD},$$

$$V_t^{nD} = \log(b^{DI}/P_t^n) + (1-\rho^{DI})\beta V_{t+1}^{n0} + \rho^{DI}\beta V_{t+1}^{nD}.$$

Note that while nominal benefits do not vary across locations, real expenditures vary with the price of local goods, so households with SSDI located in n have a purchasing power given by b^{DI}/P_t^n. Finally, the law of motion of the mass of workers are then given by

$$\begin{aligned} L_{t+1}^{nD} &= \rho^{DI}L_t^{nD} + \delta L_t^{n0}, \\ L_{t+1}^{n0} &= (1-\rho^{DI})L_t^{nD} + \sum\nolimits_{i=1}^{N}\sum\nolimits_{k\neq 0}^{J}\mu_t^{ik,n0}L_t^{ik} + \sum\nolimits_{i=1}^{N}\mu_t^{i0,n0}(1-\delta)\,L_t^{i0}, \\ L_{t+1}^{nj} &= \sum\nolimits_{i=1}^{N}\sum\nolimits_{k\neq 0}^{J}\mu_t^{ik,nj}L_t^{ik} + \sum\nolimits_{i=1}^{N}\mu_t^{i0,nj}(1-\delta)\,L_t^{i0} \qquad for\ j\geq 1. \end{aligned} \tag{29}$$

As we can see, ρ^{DI} disciplines how persistent the SSDI state is in the economy. Finally, the rest of the equilibrium conditions of the model are given by the goods market clearing condition

$$X_t^{jn} = \sum\nolimits_{k=1}^{J}\gamma^{nk,nj}\sum\nolimits_{i=1}^{N}\pi_t^{ik,nk}X_t^{ik} + \alpha^j((1-\tau)\sum\nolimits_{k=1}^{J}w_t^{nk}L_t^{nk} + b^{DI}L_t^{nD} + \iota^n\chi_t - G_t/N^{US}),$$

and by equations (5) to (10), (2), and (3).

In Appendix C.5, we show how the equilibrium conditions can be expressed in relative time differences so that we can apply the results from Propositions 2 and 3 to compute the model and solve for counterfactuals.

To perform counterfactual analysis that involves changes in the generosity of SSDI, b^{DI}, we need to obtain values for L_{2000}^{nD}, b^{DI}, τ, ρ^{DI}, and δ. Appendix E presents in greater details the data and sources used for the calculations. Succinctly, we obtain

https://www.ssa.gov/oact/cola/sga.html.

[55] Note that the reemployment probabilities of workers in non-employment with SSDI are different than those of workers in non-employment without SSDI, as SSDI is a very persistent state.

L^{nD}_{2000} as the fraction of non-employed workers between 25 and 64 years old receiving SSDI in the year 2000. We discipline the per capita SSDI benefit, b^{DI}, using the average monthly payments from the years 2000 to 2007. We obtain an average quarterly benefit of $2,843$ US dollars. We set the tax rate to 0.9% ($\tau = 0.009$). This is the payroll tax rate applied by the federal government to finance the program. In order to obtain ρ^{DI}, we use estimates of recovery and death probability for workers on disability. This probability for households in our demographic group range between 29% and 34% cumulative over the first 9 years in the program, as computed by Raut (2017). Given these values, we set $\rho^{DI} = 0.991$, which implies that after 9 years, the probability of staying on disability is 73%. Note that with this formulation the SSDI re-employment transition rate in our model is consistent with the data. Finally, we calibrate the quarterly probability of non-employed households entering the SSDI program to $\delta = 0.003$. With data on L^{nD}_t and L^{n0}_t for the years 2000 to 2007 and with our estimate of ρ^{DI}, we solve for the value of δ that minimizes the distance between the data L^{nD}_t and the model implied L^{nD}_t using the equilibrium condition (29).

We use our extended model with SSDI to study the effects of the China shock with changes to the SSDI policy. In particular, we answer the question: what would have been the employment effects across U.S. labor markets if actual disability benefits would have been eliminated when the China shock occurred? To do so, we first compute the employment effects of the China shock in this model with SSDI using the results from Proposition 3 as we did before.[56] This counterfactual quantifies the employment effects of the China shock given the actual SSDI program. We then compute the employment effects in our baseline economy with the actual evolution of fundamentals and the actual level of SSDI relative to a counterfactual economy where the China shock did not happen and SSDI benefits were not granted. This second counterfactual quantifies the employment effects of the China shock and the effects of the SSDI benefits. Finally, the difference between the employment effects in the first counterfactual and the second one is the contribution of SSDI benefits to the decline in manufacturing employment due to the China shock. We find that the disability program amplified the decline in manufacturing employment by about 0.03 percentage points; that is, about 50 thousand additional manufacturing job were lost due to the disability program. We also find an increase in the non-employment rate in the long run. The effects of the disability program on manufacturing employment tend to be larger in regions that are more concentrated in the manufacturing sector and where it is more difficult for workers to relocate to other industries. We find that the regions that contribute more to the decline in manufacturing employment due to the disability benefits are Mississippi, South Carolina, and Tennessee, each accounting for about 4-5 percent of the total decline.

[56] Alternatively, note that in a model with constant SSDI, $\dot{b}^{1n}_t = 0$ and $\delta = 1$ is equivalent to a model where non-employed households spend all non-market income b^n on market goods. In such a model, we find that the China shock results in a 0.36 percent decline in manufacturing employment and aggregate welfare increases by 0.67 percent.

5.3.2. *Effect of the China Shock with Persistent Migration Decisions*

In our model, the *i.i.d.* nature of the idiosyncratic shocks, together with migration costs, generates a gradual adjustment toward the steady state. In this section, we extend the model to incorporate an additional source of persistence in workers' decisions and we quantify the effects of the China shock using this alternative model.[57] In Appendix C.3, we show how we derive all the equilibrium conditions and how to apply the dynamic hat algebra to this model.

Suppose that at each moment in time households are subject to a Poisson process that determines the arrival of a new draw of the idiosyncratic shock. In particular, with probability ρ the household does not receive a preference draw and stays in the same labor market, while with probability $1-\rho$ the household receives a new draw. We assume that the likelihood of these events are not location specific.[58] As before, let $V_t^{nj} = E[v_t^{nj}]$. The value function can be then written as

$$V_t^{nj} = U(C_t^{nj}) + \rho\beta V_{t+1}^{nj} + (1-\rho)\nu \log\left(\sum_{i=1}^{N}\sum_{k=0}^{J} \exp\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{1/\nu}\right),$$

and then the fraction of households that stay in market nj at time t is now given by

$$\mu_t^{nj,nj} = \rho + \frac{(1-\rho)\exp(\beta V_{t+1}^{nj})^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp(\beta V_{t+1}^{mh} - \tau^{nj,mh})^{1/\nu}},$$

while the fraction of workers that move to market ik is given by

$$\mu_t^{nj,ik} = \frac{(1-\rho)\exp(\beta V_{t+1}^{ik} - \tau^{nj,ik})^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp(\beta V_{t+1}^{mh} - \tau^{nj,mh})^{1/\nu}}.$$

As we can see from these new equilibrium conditions, the fraction of households that decide to stay in a particular market is larger than ρ given that some of the agents with a new draw still decide to stay. Also note that in the limit when $\rho = 1$, the economy becomes static, there is no migration, and we are back to a spatial trade model with no labor reallocation. On the other hand, when $\rho = 0$ the model collapses to the one we had before.

Crucially, in this new setup, both the migration elasticity $1/\nu$ together with ρ determine the flow of workers across markets. Recall from Section 4 that the cross-sectional variation in migration flows and wages are used to identify the migration elasticity $1/\nu$; but now

[57] One way to add persistence is by including preferences to local amenities that are time-invariant. In Appendix C.3, we extend the model by incorporating into households' moving decisions the preference for local amenities. We also show that all our quantitative results are robust to the presence of additive and time-invariant amenities.

[58] There is an alternative interpretation that can be given to this specification. Consider the model where households only take an idiosyncratic draw when they are born. In this model, at each moment in time a fraction ρ of agents survives to the next period, while a fraction $1-\rho$ is replaced with new agents (possibly the offspring of the agents that die) that take a new draw when born.

TABLE II

AGGREGATE EFFECTS ACROSS MODELS WITH DIFFERENT DEGREES OF PERSISTENCE

Model		Change in
ρ	ν	Mfg. emp. share
0	5.3436	-0.361
0.1	5.0369	-0.315
0.2	4.6973	-0.302
0.3	4.3189	-0.296

Note: This table presents long-run employment effects due to the China shock, under different values of ρ and ν.

this cross-sectional variation is also going to depend on ρ. Given this, ρ and ν cannot be separately identified from variation in wages and migration flows. Therefore, if we adjust the migration flow matrix by ρ, we can run regression (27) to identify $1/\nu$. In doing so, however, we need to condition on ρ. So, in order to evaluate how our results change as we add additional sources of persistence, we proceed to estimate three different values of ν conditioning on three different values of ρ. Specifically, we impose $\rho = 0.1$, $\rho = 0.2$, and $\rho = 0.3$. Given these values for ρ, we obtain $\nu_{\rho=0.1} = 5.0369$, $\nu_{\rho=0.2} = 4.6973$, and $\nu_{\rho=0.3} = 4.3189$ using our specification (equation 27), where we use $\tilde{\mu}_t^{nj,nj} \equiv \left(\mu_t^{nj,nj} - \rho\right)/(1-\rho)$ and $\tilde{\mu}_t^{nj,ik} \equiv \mu_t^{nj,ik}/(1-\rho)$ instead of $\mu_t^{nj,nj}$ and $\mu_t^{nj,ik}$ in order to be consistent with this new model.

Although the employment and welfare effects are similar to the model with $\rho = 0$, the manufacturing employment effect tends to be slightly smaller as the persistence parameter ρ increases. Table II summarizes the effects on aggregate manufacturing employment shares under different values of ρ and ν. As discussed above, conditional on receiving an idiosyncratic preference draw, the migration cost elasticity is higher in the model with persistent idiosyncratic shocks than in the model in Section 2. Therefore, the higher mobility persistence coming from the parameter ρ in the model is somehow offset by a higher migration elasticity $1/\nu$, and the resulting employment dynamic is similar to the one in the model with $\rho = 0$.

6. CONCLUSION

Aggregate trade shocks can have varying effects across labor markets. One source of variation is the exposure to foreign trade, measured by the degree of import competition across labor markets. Another source of variation is the extent to which trade shocks impact the exchange of goods and the reallocation of labor across and within sectors and locations. Moreover, since labor movement across markets takes time, and mobility frictions depend on local characteristics, labor market outcomes adjust differently across industries, space, and over time to the same aggregate shock. Therefore, the study of the effects of shocks on the economy requires understanding the impact of trade on labor market dynamics.

In this paper, we build on ACM and EK to develop a dynamic and spatial trade model. The model explicitly recognizes the role of labor mobility frictions, goods mobility frictions, geographic factors, input-output linkages, and international trade in determining allocations. We calibrate our model to 38 countries, 50 U.S. states, and 22 sectors to quantify the impact of increased import competition from China over the period 2000-2007 on employment and welfare across spatially different labor markets. Our results indicate that although exposure to import competition from China reduces manufacturing employment, aggregate U.S. welfare increases. Disaggregate effects on employment and welfare across regions, sectors, labor markets, and over time are shaped by all the mechanisms and ingredients mentioned previously.

We find that the China trade shock led to a decline in manufacturing employment of about 550 thousand workers, but aggregate U.S. welfare increased by 0.2%. We also find that workers mainly relocate to services, which benefit from access to cheaper materials from China. Industries with higher exposure to import competition from China lost more employment. We find that the computer and electronics industry is the largest contributor to the aggregate decline in manufacturing employment, accounting for about one-fourth of the total decline. Across space, we find that California is the region with the largest contribution to the total decline because of its large fraction of U.S. employment in the computer and electronics industry. Overall, the employment effects across space are heterogenous; they tend to be more localized across space in manufacturing industries and to be more evenly distributed across U.S. states in non-manufacturing industries. We also find that the actual SSDI program contributed to an additional 50 thousand manufacturing jobs lost.

Welfare effects are also heterogenous across labor markets, and the largest winners from the China shock are the non-manufacturing industries since they have no direct exposure to China but at the same time benefit from access to cheaper intermediate goods from China. We find that welfare gains take time to materialize due to mobility frictions; in the short run, all states experience a decline in real wages in the manufacturing industries, but they are better off in the long run.

We emphasize that our quantitative framework and solution method can be applied to an arbitrary number of sectors, regions, and countries. The framework can furthermore be used to address a broader set of questions, generating a promising research agenda. For instance, with our framework, we can study the impact of changes in trade costs, or productivity, in any region of any country in the world. The framework can also be used to explore the effects of capital mobility across regions; to study the economic effects of different changes in government policies, such as changes in taxes, subsidies, or non-employment benefits; or to study policies that reduce mobility frictions.[59]

Other interesting topics to apply this framework are the quantification of the effects

[59] There is a rapid and growing interest to answer these types of questions; see for instance, Fajgelbaum et al. (forthcoming), Ossa (2015), and Tombe and Zhu (2015).

of trade agreements and other changes in trade policy on internal labor markets and the impact of migration across countries. In addition, it can be used to study the transmission of regional and sectoral shocks across a production network when trade and factor reallocation is subject to frictions.[60] The model can also be computed at a more disaggregated level to study migration across metropolitan areas, or commuting zones, although the challenge in this case would be collecting the relevant trade and production data at these levels of disaggregation. Quantitative answers to some of these questions using dynamic models of the type developed here present an exciting avenue for future research.

Another important extension would be to depart from our perfect foresight assumption by modelling stochastic processes of fundamentals. This extension would widen the type of shocks that can be studied with our framework.

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[60] We can therefore extend the analysis of Acemoglu et al. (2012) to a frictional economy. Moreover, we could incorporate local natural disaster shocks and quantify their effect, as recently analyzed in Carvalho et al. (2016).

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APPENDIX A: DERIVATIONS

In this appendix, we first derive the lifetime expected utility (2) and the gross migration flows described by equation (3) . After doing so, we derive the welfare equation.

A.1. *Derivations*

The lifetime utility of a worker in market nj is given by

$$\mathrm{v}_t^{nj} = U(C_t^{nj}) + \max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{ \beta E\left[\mathrm{v}_{t+1}^{ik} \right] - \tau^{nj,ik} + \nu \epsilon_t^{ik} \right\},$$

Denote by $V_t^{nj} \equiv E[\mathrm{v}_t^{nj}]$ the expected lifetime utility of a worker, where the expectation is taken over the preference shocks. We assume that the idiosyncratic preference shock ϵ is *i.i.d.* over time and is a realization of a Type-I Extreme Value distribution with zero mean. In particular, $F(\epsilon) = \exp\left(-\exp\left(-\epsilon - \bar{\gamma}\right)\right)$, where $\bar{\gamma} \equiv \int_{-\infty}^{\infty} x \exp(-x - \exp(-x))dx$ is Euler's constant and $f(\epsilon) = \partial F / \partial \epsilon$. We seek to solve for

$$\Phi_t^{nj} = E\left[\max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{ \beta E\left[\mathrm{v}_{t+1}^{ik} \right] - \tau^{nj,ik} + \nu\ \epsilon_t^{ik} \right\} \right].$$

Let $\bar{\epsilon}_t^{ik,mh} = \frac{\beta(V_{t+1}^{ik} - V_{t+1}^{mh}) - \left(\tau^{nj,ik} - \tau^{nj,mh}\right)}{\nu}$, note that

$$\Phi_t^{nj} = \sum\nolimits_{i=1}^{N} \sum\nolimits_{k=0}^{J} \int_{-\infty}^{\infty} (\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu \epsilon_t^{ik}) f(\epsilon_t^{ik}) \prod_{mh \neq ik} F(\bar{\epsilon}_t^{ik,mh} + \ \epsilon_t^{ik}) d\epsilon_t^{ik},$$

Then substituting for $F(\epsilon)$ and $f(\epsilon)$, we obtain

$$\Phi_t^{nj} = \sum\nolimits_{i=1}^{N} \sum\nolimits_{k=0}^{J} \int_{-\infty}^{\infty} (\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu \epsilon_t^{ik}) e^{(-\epsilon_t^{ik} - \bar{\gamma})} e^{\left(-e^{(-\epsilon_t^{ik} - \bar{\gamma})} \sum_{m=1}^{N} \sum_{h=0}^{J} e^{(-\bar{\epsilon}_t^{ik,mh})} \right)} d\epsilon_t^{ik}.$$

Defining $\lambda_t^{ik} \equiv \log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp(-\bar{\epsilon}_t^{ik,mh})$ and considering the change of variables $\zeta_t^{ik} = \epsilon_t^{ik} + \bar{\gamma}$, we get

$$\Phi_t^{nj} = \sum_{i=1}^{N} \sum_{k=0}^{J} \int_{-\infty}^{\infty} (\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu \left(\zeta_t^{ik} - \bar{\gamma}\right)) \exp(-\zeta_t^{ik} - \exp(-(\zeta_t^{ik} - \lambda_t^{ik}))) d\zeta_t^{ik}.$$

Consider an additional change of variables : let $\tilde{y}_t^{ik} = \zeta_t^{ik} - \lambda_t^{ik}$. Hence, we obtain

$$\Phi_t^{nj} = \sum_{i=1}^{N} \sum_{k=0}^{J} \exp(-\lambda_t^{ik}) \left(\begin{array}{c} (\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu \left(\lambda_t^{ik} - \bar{\gamma}\right)) \\ +\nu \int_{-\infty}^{\infty} \tilde{y}_t^{ik} \exp(-\tilde{y}_t^{ik} - \exp(-\tilde{y}_t^{ik})) d\tilde{y}_t^{ik} \end{array} \right),$$

and using the definition of $\bar{\gamma}$, we get

$$\Phi_t^{nj} = \sum_{i=1}^{N} \sum_{k=0}^{J} \exp(-\lambda_t^{ik})(\beta V_{t+1}^{ik} - \tau^{nj,ik} + \nu\lambda_t^{ik}),$$

and replacing the definition of λ_t^{ik}, we get

$$\Phi_t^{nj} = \sum_{i=1}^{N} \sum_{k=0}^{J} \exp\left(-\log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp(-\bar{\epsilon}_t^{ik,mh})\right) \left(\begin{array}{c} \beta V_{t+1}^{ik} - \tau^{nj,ik} \\ +\nu \log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp(-\bar{\epsilon}_t^{ik,mh}) \end{array} \right).$$

Substituting the definition of $\bar{\epsilon}_t^{ik,mh}$, we get,

$$\Phi_t^{nj} = \nu \left(\log \sum_{m=1}^{N} \sum_{h=0}^{J} e^{\left(\beta V_{t+1}^{mh} - \tau^{nj,mh}\right)^{1/\nu}} \right) \sum_{i=1}^{N} \sum_{k=0}^{J} e^{\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{1/\nu}} \sum_{m=1}^{N} \sum_{h=0}^{J} e^{\left(\beta V_{t+1}^{mh} - \tau^{nj,mh}\right)^{1/\nu}},$$

which implies

$$\Phi_t^{nj} = \nu \left(\log \sum_{m=1}^{N} \sum_{h=0}^{J} \exp(\beta V_{t+1}^{mh} - \tau^{nj,mh})^{1/\nu} \right),$$

and therefore

$$V_t^{nj} = U(C_t^{nj}) + \nu \left(\log \sum_{i=1}^{N} \sum_{k=0}^{J} \exp(\beta V_{t+1}^{ik} - \tau^{nj,ik})^{1/\nu} \right).$$

We now derive equation (3). Define $\mu_t^{nj,ik}$ as the fraction of workers that reallocate from labor market nj to labor market ik. This fraction is equal to the probability that a given worker moves from labor market nj to labor market ik at time t; that is, the probability that the expected utility of moving to ik is higher than the expected utility in any other location. Formally,

$$\mu_t^{nj,ik} = \Pr\left(\frac{\beta V_{t+1}^{ik} - \tau^{nj,ik}}{\nu} + \epsilon_t^{ik} \geq \max_{mh \neq ik} \left\{ \frac{\beta V_{t+1}^{mh} - \tau^{nj,mh}}{\nu} + \epsilon_t^{mh} \right\} \right).$$

Given our assumptions on the idiosyncratic preference shock,

$$\mu_t^{nj,ik} = \int_{-\infty}^{\infty} f(\epsilon_t^{ik}) \prod_{mh \neq ik} F\left(\beta(V_{t+1}^{ik} - V_{t+1}^{mh}) - \left(\tau^{nj,ik} - \tau^{nj,mh}\right) + \epsilon_t^{ik}\right) d\epsilon_t^{ik},$$

From the above derivations, we know that

$$\mu_t^{nj,ik} = \int_{-\infty}^{\infty} \exp(-\epsilon_t^{ik} - \bar{\gamma}) e^{\left(-e^{(-\epsilon_t^{ik} - \bar{\gamma})} \sum_{m=1}^{N} \sum_{h=0}^{J} e^{(-\bar{\epsilon}_t^{ik,mh})}\right)} d\epsilon_t^{ik}.$$

Using the definitions from above, we get

$$\mu_t^{nj,ik} = \exp(-\lambda_t^{ik}) \int_{-\infty}^{\infty} \exp\left(-\tilde{y}_t - \exp(-\tilde{y}_t)\right) d\tilde{y}_t,$$

and solving for this integral we obtain

$$\mu_t^{nj,ik} = \frac{\exp\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp\left(\beta V_{t+1}^{mh} - \tau^{nj,mh}\right)^{1/\nu}}.$$

A.2. *The Option Value and Welfare Equations*

In this section, we discuss the welfare effects resulting from changes in fundamentals in our economy.

To begin, let $V_t^{\prime nj}$ be the present discounted value of utility at time t in market nj under the counterfactual change in fundamentals $\{\Theta_t'\}_{t=0}^{\infty}$, and let V_t^{nj} denote the same object for the case of the baseline economy given a sequence of fundamentals $\{\Theta_t\}_{t=0}^{\infty}$. Now, write the expected lifetime utility of being at market nj at time t as

$$V_t^{nj} = \log C_t^{nj} + \beta V_{t+1}^{nj} + \nu \log \left(\sum\nolimits_{i=1}^{N} \sum\nolimits_{k=0}^{J} \exp \left(\beta \left(V_{t+1}^{ik} - V_{t+1}^{nj} \right) - \tau^{nj,ik} \right)^{1/\nu} \right), \tag{30}$$

where the second term on the right-hand side of equation (30) is the option value. From equation (3) we know that

$$\mu_t^{nj,nj} = \frac{\exp\left(\beta V_{t+1}^{nj}\right)^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp\left(\beta V_{t+1}^{mh} - \tau^{nj,mh}\right)^{1/\nu}},$$

and therefore the option value is given by

$$\nu \log \sum\nolimits_{m=1}^{N} \sum\nolimits_{h=0}^{J} \exp \left(\beta \left(V_{t+1}^{mh} - V_{t+1}^{nh} \right) - \tau^{nj,mh} \right)^{1/\nu} = -\nu \log \mu_t^{nj,nj}.$$

Plugging this equation into the value function, we get

$$V_t^{nj} = \log C_t^{nj} + \beta V_{t+1}^{nj} - \nu \log \mu_t^{nj,nj}.$$

Finally, iterating this equation forward we obtain

$$V_t^{nj} = \sum_{s=t}^{\infty} \beta^{s-t} \log C_s^{nj} - \nu \sum_{s=t}^{\infty} \beta^{s-t} \log \mu_s^{nj,nj}.$$

Given this, we obtain that the expected lifetime utilities in the counterfactual and in the baseline economy are given by

$$\begin{aligned} V_t^{\prime nj} &= \sum_{s=t}^{\infty} \beta^{s-t} \log \left(\frac{C_s^{\prime nj}}{(\mu_s^{\prime nj,nj})^{\nu}} \right), \\ V_t^{nj} &= \sum_{s=t}^{\infty} \beta^{s-t} \log \left(\frac{C_s^{nj}}{(\mu_s^{nj,nj})^{\nu}} \right). \end{aligned}$$

We define the compensating variation in consumption for market nj at time $t = 0$ to be the scalar δ^{nj} such that

$$\begin{aligned} V_0^{\prime nj} &= V_0^{nj} + \sum_{s=0}^{\infty} \beta^{s} \log \left(\delta^{n,j} \right), \\ &= \sum_{s=0}^{\infty} \beta^{s} \log \left(\frac{C_s^{nj}}{(\mu_s^{n,j;n,j})^{\nu}} \; \delta^{n,j} \right). \end{aligned}$$

Re-arranging this we have that $\log\left(\delta^{n,j}\right) = (1-\beta)\left(V_0^{\prime nj} - V_0^{nj}\right)$, or

$$\log\left(\delta^{n,j}\right) = (1-\beta) \sum_{s=0}^{\infty} \beta^{s} \log \left(\frac{C_s^{\prime nj} / C_s^{nj}}{(\mu_s^{\prime nj,nj} / \mu_s^{nj,nj})^{\nu}} \right), \tag{31}$$

which can also be written as

$$\begin{aligned}\log\left(\delta^{n,j}\right) &= \sum_{s=0}^{\infty}\beta^{s}\log\left(\frac{C_s'^{nj}/C_s^{nj}}{(\mu_s'^{nj,nj}/\mu_s^{nj,nj})^{\nu}}\right)-\sum_{s=0}^{\infty}\beta^{s+1}\log\left(\frac{C_s'^{nj}/C_s^{nj}}{(\mu_s'^{nj,nj}/\mu_s^{nj,nj})^{\nu}}\right),\\ &= \log\left(\frac{C_0'^{nj}/C_0^{nj}}{(\mu_0'^{nj,nj}/\mu_0^{nj,nj})^{\nu}}\right)+\sum_{s=1}^{\infty}\beta^{s}\log\left(\frac{\left(C_s'^{nj}/C_s^{nj}\right)/\left(C_{s-1}'^{nj}/C_{s-1}^{nj}\right)}{\left((\mu_s'^{nj,nj}/\mu_s^{nj,nj})/(\mu_{s-1}'^{nj,nj}/\mu_{s-1}^{nj,nj})\right)^{\nu}}\right),\\ &= \log\left(\frac{C_0'^{nj}/C_0^{nj}}{(\mu_0'^{nj,nj}/\mu_0^{nj,nj})^{\nu}}\right)+\sum_{s=1}^{\infty}\beta^{s}\log\left(\frac{\hat{C}_s^{nj}}{\left(\hat{\mu}_s^{nj,nj}\right)^{\nu}}\right).\end{aligned}$$

Given that $C_0'^{nj}=C_0^{nj}$, and $\mu_0'^{nj,nj}=\mu_0^{nj,nj}$, we obtain

$$\log\left(\delta^{n,j}\right)=\sum_{s=1}^{\infty}\beta^{s}\log\left(\frac{\hat{C}_s^{nj}}{\left(\hat{\mu}_s^{nj,nj}\right)^{\nu}}\right),$$

which is our measure of consumption equivalent change in welfare in equation (28).

Note that the change in welfare in market nj from a change in fundamentals relative to the baseline economy is given by the present discounted value of the expected change in real consumption and the change in the option value. Equation (31) shows that the change in the option value is summarized by the change in the fraction of workers that do not reallocate, $\hat{\mu}_t^{nj,nj}$, and the variance of the taste shocks ν. The intuition is that higher $\hat{\mu}_s^{nj,nj}$ means that fewer workers in market nj move to a market with higher expected value. Notice that if the cost of moving to a different labor market is infinite, then $\hat{\mu}_t^{nj,nj}=1$ and the option value is zero.

In our model, the change in real consumption in market nj, $\hat{C}_s^{nj}$, is given by the change in the real wage earned in that market, $\hat{w}_t^{nj}/\hat{P}_t^{n}$, and can be expressed as[61]

$$\hat{C}_t^{nj}=\frac{\hat{w}_t^{nj}}{\prod_{k=1}^{J}(\hat{w}_t^{nk})^{\alpha^k}}\prod_{k=1}^{J}\left(\frac{\hat{w}_t^{nk}}{\hat{P}_t^{nk}}\right)^{\alpha^k}.$$

The first component denotes the unequal welfare effects for households working in different sectors within the same region n and reflects the fact that workers in sectors that pay higher wages have more purchasing power in that region. The second component is common to all households residing in region n and captures the change in the cost of living in that region. This second component is a measure of the change in the average real wage across labor markets in region n, weighted by the importance of each sector in the consumption bundle, and it is shaped by several mechanisms in our model. Specifically,

$$\prod_{k=1}^{J}\left(\frac{\hat{w}_t^{nk}}{\hat{P}_t^{nk}}\right)^{\alpha^k}=\sum_{k=1}^{J}\alpha^k\left(\log\left(\hat{\pi}_t^{nk,nk}\right)^{-1/\theta^k}+\log\frac{\hat{w}_t^{nk}}{\hat{x}_t^{nk}}\right) \tag{32}$$

The first term in equation (32) is the change in trade openness, $\log\left(\hat{\pi}_t^{nk,nk}\right)$, that gives households in region n access to cheaper imported goods. The second term in equation (32) is the change in factor prices, $\log\frac{\hat{w}_t^{nk}}{\hat{x}_t^{nk}}$, and captures the effects of migration, local factors, and intersectoral trade.

To fix ideas, consider the case where we abstract from materials in the model, $\log\frac{\hat{w}_t^{nk}}{\hat{x}_t^{nk}}=-\xi^n\log\left(\hat{L}_t^{nk}/\hat{H}^{nk}\right)$. Migration into region n may have a positive or negative effect on factor prices depending on how L_t^{nk} changes relative to the stock of structures H^{nk}. In our model, structures are in fixed supply, thus migration has a negative effect on real wages because the inflow of workers strains local fixed factors and raises the relative price of structures and the cost of living in region n. This is a congestion effect as in Caliendo et al. (2018).[62] Finally, material inputs and input-output linkages impact welfare through changes in the cost of the input bundle, as in Caliendo and Parro (2015).

[61] $\hat{C}_t^{n,0}=1$ if the household in region n at time t is non-employed.
[62] Dix-Carneiro (2014) studies the impact of capital mobility on the relocation of labor.

Now consider the case of a one-sector model (more details are presented in Appendix C.1) with N labor markets indexed by ℓ. Households in location ℓ consume local goods. In this setup, the welfare equation (31) takes the form

$$\hat{W}^{\ell} = \sum_{s=1}^{\infty} \beta^{s} \log \frac{\hat{w}_{s}^{\ell}/\hat{P}_{s}^{\ell}}{(\hat{\mu}_{s}^{\ell,\ell})^{\nu}}$$

and the change in real wages is given by $\log(\hat{w}_t^{\ell}/\hat{P}_t^{\ell}) = -(1/\theta^j\gamma)\log\hat{\pi}_t^{\ell,\ell} - \beta\log(\hat{L}_t^{\ell}/\hat{H}_t^{\ell})$. It follows then, that in a one-sector model with no materials and structures, the welfare equation reduces to

$$\hat{W}^{\ell} = \sum_{s=1}^{\infty} \beta^{s} \log \frac{(\hat{\pi}_{s}^{\ell,\ell})^{-1/\theta}}{(\hat{\mu}_{s}^{\ell,\ell})^{\nu}},$$

which combines the welfare formulas in ACM and Arkolakis et al. (2012).

APPENDIX B: PROOF OF PROPOSITIONS

This appendix presents the proofs of the propositions presented in the main text.

Proposition 1 *Given the allocation of the temporary equilibrium at t, $\{L_t, \pi_t, X_t\}$, the solution to the* ***temporary equilibrium*** *at $t+1$ for a given change in $\dot{L}_{t+1}$ and $\dot{\Theta}_{t+1}$ does not require information on the level of fundamentals at t, Θ_t, or $\bar{\Theta}$ and solve the following system of non-linear equations:*

$$\dot{x}_{t+1}^{nj} = (\dot{L}_{t+1}^{nj})^{\gamma^{nj}\xi^{n}} (\dot{w}_{t+1}^{nj})^{\gamma^{nj}} \prod_{k=1}^{J} (\dot{P}_{t+1}^{nk})^{\gamma^{nj,nk}}, \tag{33}$$

$$\dot{P}_{t+1}^{nj} = \left(\sum_{i=1}^{N} \pi_t^{nj,ij} (\dot{x}_{t+1}^{ij}\dot{\kappa}_{t+1}^{nj,ij})^{-\theta^{j}} (\dot{A}_{t+1}^{ij})^{\theta^{j}\gamma^{ij}} \right)^{-1/\theta^{j}}, \tag{34}$$

$$\pi_{t+1}^{nj,ij} = \pi_t^{nj,ij} \left(\frac{\dot{x}_{t+1}^{ij}\dot{\kappa}_{t+1}^{nj,ij}}{\dot{P}_{t+1}^{nj}} \right)^{-\theta^{j}} (\dot{A}_{t+1}^{ij})^{\theta^{j}\gamma^{ij}}, \tag{35}$$

$$X_{t+1}^{nj} = \sum_{k=1}^{J} \gamma^{nk,nj} \sum_{i=1}^{N} \pi_{t+1}^{ik,nk} X_{t+1}^{ik} + \alpha^{j} \left(\sum_{k=1}^{J} \dot{w}_{t+1}^{nk} \dot{L}_{t+1}^{nk} w_t^{nk} L_t^{nk} + \iota^{n}\chi_{t+1} \right), \tag{36}$$

$$\dot{w}_{t+1}^{nj} \dot{L}_{t+1}^{nj} w_t^{nj} L_t^{nj} = \gamma^{nj}(1-\xi^{n}) \sum_{i=1}^{N} \pi_{t+1}^{ij,nj} X_{t+1}^{ij}, \tag{37}$$

where $\chi_{t+1} = \sum_{i=1}^{N}\sum_{k=1}^{J} \frac{\xi^{i}}{1-\xi^{i}} \dot{w}_{t+1}^{ik}\dot{L}_{t+1}^{ik} w_t^{ik} L_t^{ik}$.

Proof: Let $\{L_t, \pi_t, X_t\}$ be the allocation of the temporary equilibrium associated with Θ_t and $\bar{\Theta}$. Consider a given change in L_t to L_{t+1} and $\Theta_t = \{A_t, \kappa_t\}$ to $\Theta_{t+1} = \{A_{t+1}, \kappa_{t+1}\}$. Denote these changes in time differences as $\dot{L}_{t+1}$ and $\dot{\Theta}_{t+1}$. First we show how to express the equilibrium conditions that define a temporary equilibrium under L_t and under L_{t+1} in time differences, namely we derive equations (33) to (37) . Recall that we have defined the operator "$\cdot$" over a variable y_{t+1} as $\dot{y}_{t+1} = \frac{y_{t+1}}{y_t}$.

From the first-order conditions of the intermediate goods producers' problem we obtain that $\frac{r_t^{nj}H^{nj}}{\xi^{n}} = \frac{w_t^{nj}L_t^{nj}}{1-\xi^{n}}$; and expressing this condition in time difference we obtain

$$\frac{\dot{r}_{t+1}^{nj}}{\xi^{n}} = \frac{\dot{w}_{t+1}^{nj}\dot{L}_{t+1}^{nj}}{1-\xi^{n}}. \tag{38}$$

Now use the definition of the input bundle (5) at time t (x_t^{nj}) and $t+1$ (x_{t+1}^{nj}). Taking the ratio of these expressions and substituting $\dot{r}^{nj}_{t+1}$ using (38) we obtain (33).

Use equilibrium conditions (6) and (7) at time t (P^{nj}_t and $\pi^{nj,ij}_t$) and at $t+1$ (P^{nj}_{t+1} and $\pi^{nj,ij}_{t+1}$) and express these conditions relative to each other; namely

$$\frac{P^{nj}_{t+1}}{P^{nj}_t} = \left(\sum\nolimits_{i=1}^{N} \frac{(x^{ij}_{t+1}\kappa^{nj,ij}_{t+1})^{-\theta^j}(A^{ij}_{t+1})^{\theta^j\gamma^{ij}}}{\sum_{m=1}^{N}(x^{mj}_t\kappa^{nj,mj}_t)^{-\theta^j}(A^{mj}_t)^{\theta^j\gamma^{mj}}}\right)^{-1/\theta^j}.$$

Now multiplying and dividing each element in the summation by $(x^{ij}_t\kappa^{nj,ij}_t)^{-\theta^j}(A^{ij}_t)^{\theta^j\gamma^{ij}}$ and then using $\pi^{nj,ij}_t$, we obtain

$$\frac{P^{nj}_{t+1}}{P^{nj}_t} = \left(\sum_{i=1}^{N}\pi^{nj,ij}_t\left(\frac{x^{ij}_{t+1}\kappa^{nj,ij}_{t+1}}{x^{ij}_t\kappa^{nj,ij}_t}\right)^{-\theta^j}\left(\frac{A^{ij}_{t+1}}{A^{ij}_t}\right)^{\theta^j\gamma^{ij}}\right)^{-1/\theta^j}.$$

Finally using the "·"notation, we arrive at (34).

Similarly, multiplying and dividing the numerator of $\pi^{nj,ij}_{t+1}$ by $(x^{ij}_t\kappa^{nj,ij}_t)^{-\theta^j}(A^{ij}_t)^{\theta^j\gamma^{ij}}$ and then multiplying and dividing each element in the summation of the denominator of $\pi^{nj,ij}_{t+1}$ by $(x^{ij}_t\kappa^{nj,ij}_t)^{-\theta^j}$ $(A^{ij}_t)^{\theta^j\gamma^{ij}}$ and then using $\pi^{nj,ij}_t$, we obtain

$$\pi^{nj,ij}_{t+1} = \frac{\pi^{nj,ij}_t\left(\frac{x^{ij}_{t+1}\kappa^{nj,ij}_{t+1}}{x^{ij}_t\kappa^{nj,ij}_t}\right)^{-\theta^j}\left(\frac{A^{ij}_{t+1}}{A^{ij}_t}\right)^{\theta^j\gamma^{ij}}}{\sum_{m=1}^{N}\pi^{nj,mj}_t\left(\frac{x^{mj}_{t+1}\kappa^{nj,mj}_{t+1}}{x^{mj}_t\kappa^{nj,mj}_t}\right)^{-\theta^j}\left(\frac{A^{mj}_{t+1}}{A^{mj}_t}\right)^{\theta^j\gamma^{mj}}}.$$

Now substituting the denominator with (34), we arrive at (35).

To derive (36), start with the market clearing at $t+1$,

$$X^{nj}_{t+1} = \sum\nolimits_{k=1}^{J}\gamma^{nk,nj}\sum\nolimits_{i=1}^{N}\pi^{ik,nk}_{t+1}X^{ik}_{t+1} + \alpha^j\left(\sum\nolimits_{k=1}^{J}w^{nk}_{t+1}L^{nk}_{t+1} + \iota^n\chi_{t+1}\right),$$

and now multiply and divide $\sum_{k=1}^{J}w^{nk}_{t+1}L^{nk}_{t+1}$ by $w^{nk}_tL^{nk}_t$ to obtain, $\sum_{k=1}^{J}\dot{w}^{nk}_{t+1}\dot{L}^{nk}_{t+1}w^{nk}_tL^{nk}_t$. Substitute this expression to obtain (36), where $\chi_{t+1} = \sum_{i=1}^{N}\sum_{k=1}^{J}\dot{r}^{ik}_t r^{ik}_t\ H^{ik}$, and using (38) we can express this as $\chi_{t+1} = \sum_{i=1}^{N}\sum_{k=1}^{J}\frac{\xi^i}{1-\xi^i}\dot{w}^{ik}_{t+1}\dot{L}^{ik}_{t+1}w^{ik}_tL^{ik}_t$.

Finally, to obtain (37), start with the labor market clearing condition at $t+1$,

$$w^{nj}_{t+1}L^{nj}_{t+1} = \gamma^{nj}(1-\xi^n)\sum_{i=1}^{N}\pi^{ij,nj}_{t+1}X^{ij}_{t+1},$$

and multiply and divide the left-hand side by $w^{nj}_{t+1}L^{nj}_{t+1}$ to obtain (37).

Now, inspecting equations (33) to (37), we see that with information on the allocation at t, $\{L_t, \pi_t, X_t\}$, we can solve for $\{\dot{w}^{nj}_{t+1}, \dot{x}_{t+1}, \dot{P}^{nj}_{t+1}, \pi^{nj,ij}_{t+1}, X^{nj}_{t+1}\}^{N,N,J}_{n=1,i=1,j=1}$, given $\dot{\Theta}_{t+1} = \{\dot{\kappa}^{nj,ij}_{t+1}, \dot{A}^{nj}_{t+1}\}^{N,N,J}_{n=1,i=1,j=1}$, without estimates of Θ_t and $\bar{\Theta}$.

Proposition 2 *Conditional on an initial allocation of the economy, $(L_0, \pi_0, X_0, \mu_{-1})$, given an anticipated sequence of changes in fundamentals, $\{\dot{\Theta}_t\}_{t=1}^{\infty}$, with $\lim_{t\to\infty}\Theta_t = 1$, the solution to the sequential equilibrium in time differences does not require information on the level of the fundamentals, $\{\Theta_t\}_{t=0}^{\infty}$ or $\bar{\Theta}$, and solves the following system of non-linear equations:*

$$\mu^{nj,ik}_{t+1} = \frac{\mu^{nj,ik}_t\left(\dot{u}^{ik}_{t+2}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu^{nj,mh}_t\left(\dot{u}^{mh}_{t+2}\right)^{\beta/\nu}}, \tag{39}$$

$$\dot{u}^{nj}_{t+1} = \dot{\omega}^{nj}(\dot{L}_{t+1}, \dot{\Theta}_{t+1})\left(\sum\nolimits_{i=1}^{N}\sum\nolimits_{k=0}^{J}\mu^{nj,ik}_t\left(\dot{u}^{ik}_{t+2}\right)^{\beta/\nu}\right)^{\nu}, \tag{40}$$

$$L_{t+1}^{nj} = \sum\nolimits_{i=1}^{N} \sum\nolimits_{k=0}^{J} \mu_t^{ik,nj} \; L_t^{ik}, \tag{41}$$

for all $j, n, i,$ and k at each t, where $\{\dot{\omega}^{nj}(\dot{L}_t, \dot{\Theta}_t)\}_{n=1,j=0,t=1}^{N,J,\infty}$ is the solution to the temporary equilibrium given $\{\dot{L}_t, \dot{\Theta}_t\}_{t=1}^{\infty}$.

Proof: Consider the fraction of workers who reallocate from market n, j to i, k at $t = 0$; that is, equilibrium condition (3) at $t = 0$:

$$\mu_0^{nj,ik} = \frac{\exp\left(\beta V_1^{ik} - \tau^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp\left(\beta V_1^{mh} - \tau^{nj,mh}\right)^{1/\nu}}.$$

Taking the relative time differences (between $t = -1$ and $t = 0$) of this equation, we get

$$\frac{\mu_0^{nj,ik}}{\mu_{-1}^{nj,ik}} = \frac{\frac{\exp\left(\beta V_1^{ik} - \tau^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta V_1^{mh} - \tau^{nj,mh}\right)^{1/\nu}}}{\frac{\exp\left(\beta V_0^{ik} - \tau^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta V_0^{mh} - \tau^{nj,mh}\right)^{1/\nu}}}.$$

Given that mobility costs do not change over time, this expression can be written as

$$\frac{\mu_0^{nj,ik}}{\mu_{-1}^{nj,ik}} = \frac{\exp\left(\beta V_1^{ik} - \beta V_0^{ik}\right)^{1/\nu}}{\frac{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta V_1^{mh} - \tau^{nj,mh}\right)^{1/\nu}\frac{\exp\left(\beta V_0^{mh} - \tau^{nj,mh}\right)^{1/\nu}}{\exp\left(\beta V_0^{mh} - \tau^{nj,mh}\right)^{1/\nu}}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta V_0^{mh} - \tau^{nj,mh}\right)^{1/\nu}}},$$

which is equivalent to

$$\frac{\mu_0^{nj,ik}}{\mu_{-1}^{nj,ik}} = \frac{\exp\left(V_1^{ik} - V_0^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_{-1}^{nj,mh}\exp\left(V_1^{mh} - V_0^{mh}\right)^{\beta/\nu}}.$$

Using the definition of u_t^{ik} we get

$$\mu_0^{nj,ik} = \frac{\mu_{-1}^{nj,ik}\left(\dot{u}_1^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_{-1}^{nj,mh}\left(\dot{u}_1^{mh}\right)^{\beta/\nu}},$$

where we express the migration flows at $t = 0$ as a function of data at $t = -1$. Following similar steps, we can express the migration flows at any t as

$$\mu_t^{nj,ik} = \frac{\mu_{t-1}^{nj,ik}\left(\dot{u}_{t+1}^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_{t-1}^{nj,mh}\left(\dot{u}_{t+1}^{mh}\right)^{\beta/\nu}}, \tag{42}$$

which is equilibrium condition (16) in the main text.

Now take the equilibrium condition (2) in time differences at region n and sector j between periods 0 and 1,

$$V_1^{nj} - V_0^{nj} = U(C_1^{nj}) - U(C_0^{nj}) + \nu \log\left[\frac{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta V_2^{mh} - \tau^{nj,mh}\right)^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta V_1^{mh} - \tau^{nj,mh}\right)^{1/\nu}}\right].$$

Multiplying and dividing each term in the numerator by $\exp\left(\beta V_1^{mh} - \tau^{nj,mh}\right)^{1/\nu}$ and using (3), we obtain

$$V_1^{nj} - V_0^{nj} = U(C_1^{nj}) - U(C_0^{nj}) + \nu \log\left(\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_0^{nj,mh}\exp\left(\beta V_2^{mh} - \beta V_1^{mh}\right)^{1/\nu}\right).$$

Taking exponential from both sides and using the definition of $u_{t+1}^{i,k}$ and Assumption 3, we obtain

$$\dot{u}_1^{nj} = \dot{\omega}^{nj}(\dot{L}_1, \dot{\Theta}_1)\left(\sum_{i=1}^{N}\sum_{k=0}^{J}\mu_0^{nj,ik}\left(\dot{u}_2^{ik}\right)^{\beta/\nu}\right)^{\nu},$$

where $\dot{\omega}^{nj}(\dot{L}_1, \dot{\Theta}_1) = \dot{w}^{nj}(\dot{L}_1, \dot{\Theta}_1)/\dot{P}^n(\dot{L}_1, \dot{\Theta}_1)$ solves the temporary equilibrium at $t = 1$. Finally, for all t we get,

$$\dot{u}_{t+1}^{nj} = \dot{\omega}^{nj}(\dot{L}_{t+1}, \dot{\Theta}_{t+1})\left(\sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{nj,ik}\left(\dot{u}_{t+2}^{ik}\right)^{\beta/\nu}\right)^{\nu}, \tag{43}$$

where $\dot{\omega}^{nj}(\dot{L}_{t+1}, \dot{\Theta}_{t+1}) = \dot{w}^{nj}(\dot{L}_{t+1}, \dot{\Theta}_{t+1})/\dot{P}^n(\dot{L}_{t+1}, \dot{\Theta}_{t+1})$ solves the temporary equilibrium at $t+1$.

Note that by Proposition 1, the sequence of temporary equilibria given $\dot{\Theta}_{t+1}$ does not depend on the level of Θ_t and $\bar{\Theta}$. The equilibrium conditions (42) and (43) do not depend on the level of Θ_t and $\bar{\Theta}$ either. Therefore, given a sequence $\{\dot{\Theta}_t\}_{t=1}^{\infty}$, with $\dot{\Theta}_\infty = 1$, the solution to the change in the sequential equilibrium of the model given $\dot{\Theta}_t$ does not require knowing the level of Θ_t and $\bar{\Theta}$.

Proposition 3 *Given a baseline economy, $\{L_t, \mu_{t-1}, \pi_t, X_t\}_{t=0}^{\infty}$, and a counterfactual convergent sequence of changes in fundamentals, $\{\hat{\Theta}_t\}_{t=1}^{\infty}$, solving for the counterfactual sequential equilibrium $\{L'_t, \mu'_{t-1}, \pi'_t, X'_t\}_{t=1}^{\infty}$ does not require information on the fundamentals $(\{\Theta_{1t}\}_{t=0}^{\infty}, \bar{\Theta})$ and solves the following system of non-linear equations*:

$$\mu_t^{\prime nj,ik} = \frac{\mu_{t-1}^{\prime nj,ik}\dot{\mu}_t^{nj,ik}\left(\hat{u}_{t+1}^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_{t-1}^{\prime nj,mh}\dot{\mu}_t^{nj,mh}\left(\hat{u}_{t+1}^{mh}\right)^{\beta/\nu}}, \tag{44}$$

$$\hat{u}_t^{nj} = \hat{\omega}^{nj}(\hat{L}_t, \hat{\Theta}_t)\left(\sum_{i=1}^{N}\sum_{k=0}^{J}\mu_{t-1}^{\prime nj,ik}\dot{\mu}_t^{nj,ik}\left(\hat{u}_{t+1}^{ik}\right)^{\beta/\nu}\right)^{\nu}, \tag{45}$$

$$L_{t+1}^{\prime nj} = \sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{\prime ik,nj}\,L_t^{\prime ik}, \tag{46}$$

for all j, n, i and k at each t, where $\{\hat{\omega}^{nj}(\hat{L}_t, \hat{\Theta}_t)\}_{n=1,j=0,t=1}^{N,J,\infty}$ is the solution to the temporary equilibrium given $\{\hat{L}_t, \hat{\Theta}_t\}_{t=1}^{\infty}$.

Proof: *Given a baseline economy,* $\{L_t, \mu_{t-1}, \pi_t, X_t\}_{t=0}^{\infty}$, we first show how to obtain real wages across labor markets, $\{\hat{\omega}^{nj}(\hat{L}_t, \hat{\Theta}_t)\}_{n=1,j=0,t=1}^{N,J,\infty}$, given $\{\hat{L}_t, \hat{\Theta}_t\}_{t=1}^{\infty}$. After this, we show how to obtain the equilibrium conditions (44), (45), and (46).

Take as given $\{\hat{L}_{t+1}, \hat{\Theta}_{t+1}\}$ for any given t. We want to obtain the solution to $\{\hat{\omega}^{nj}(\hat{L}_{t+1}, \hat{\Theta}_{t+1})\}_{n=1,}^{N,J}$, recalling that $\hat{\omega}^{nj}(\hat{L}_{t+1}, \hat{\Theta}_{t+1}) \equiv \hat{w}_{t+1}^{nj}/\hat{P}_{t+1}^{n}$. We now derive that the equilibrium conditions to solve for $\hat{w}_{t+1}^{nj}$ are given by

$$\hat{x}_{t+1}^{nj} = (\hat{L}_{t+1}^{nj})^{\gamma^{nj}\xi^n}(\hat{w}_{t+1}^{nj})^{\gamma^{nj}}\prod_{k=1}^{J}(\hat{P}_{t+1}^{nk})^{\gamma^{nj,nk}}, \tag{47}$$

$$\hat{P}_{t+1}^{nj} = \left(\sum_{i=1}^{N}\pi_t^{\prime nj,ij}\dot{\pi}_{t+1}^{nj,ij}(\hat{x}_{t+1}^{ij}\hat{\kappa}_{t+1}^{nj,ij})^{-\theta^j}(\hat{A}_{t+1}^{ij})^{\theta^j\gamma^{ij}}\right)^{-1/\theta^j}, \tag{48}$$

$$\pi_{t+1}^{\prime nj,ij} = \pi_t^{\prime nj,ij}\dot{\pi}_{t+1}^{nj,ij}\left(\frac{\hat{x}_{t+1}^{ij}\hat{\kappa}_{t+1}^{nj,ij}}{\hat{P}_{t+1}^{nj}}\right)^{-\theta^j}(\hat{A}_{t+1}^{ij})^{\theta^j\gamma^{ij}}, \tag{49}$$

(50)

$$X'^{nj}_{t+1} = \sum_{k=1}^{J} \gamma^{nk,nj} \sum_{i=1}^{N} \pi'^{ik,nk}_{t+1} X'^{ik}_{t+1} + \alpha^j \left(\sum_{k=1}^{J} \hat{w}^{nk}_{t+1} \hat{L}^{nk}_{t+1} w'^{nk}_t L'^{nk}_t \dot{w}^{nk}_{t+1} \dot{L}^{nk}_{t+1} + \iota^n \chi'_{t+1} \right),$$

where $\chi'_{t+1} = \sum_{i=1}^{N} \sum_{k=1}^{J} \frac{\xi^i}{1-\xi^i} \hat{w}^{ik}_{t+1} \hat{L}^{ik}_{t+1} w'^{ik}_t L'^{ik}_t \dot{w}^{ik}_{t+1} \dot{L}^{ik}_{t+1}$ and the labor market equilibrium is

$$\hat{w}^{nk}_{t+1} \hat{L}^{nk}_{t+1} = \frac{\gamma^{nj}(1-\xi^n)}{w'^{nk}_t L'^{nk}_t \dot{w}^{nk}_{t+1} \dot{L}^{nk}_{t+1}} \sum_{i=1}^{N} \pi'^{ij,nj}_{t+1} \; X'^{ij}_{t+1}. \tag{51}$$

Equilibrium condition (47) is derived by taking the ratio between equilibrium condition (33) in the counterfactual economy $\dot{x}'^{nj}_{t+1}$ and $\dot{x}^{nj}_{t+1}$ from the baseline economy, using the notation $\hat{x}^{nj}_{t+1} = \dot{x}'^{nj}_{t+1}/\dot{x}^{nj}_{t+1}$.

The equilibrium condition (48) requires more work. Start from the counterfactual evolution of prices

$$\dot{P}'^{nj}_{t+1} = \left(\sum_{i=1}^{N} \pi'^{nj,ij}_t (\dot{x}'^{ij}_{t+1} \dot{\kappa}'^{nj,ij}_{t+1})^{-\theta^j} (\dot{A}'^{ij}_{t+1})^{\theta^j \gamma^{ij}} \right)^{-1/\theta^j}.$$

Now multiply and divide each expression in the parenthesis by $\left(\dot{x}^{ij}_{t+1} \dot{\kappa}^{nj,ij}_{t+1} \right)^{-\theta^j} (\dot{A}^{ij}_{t+1})^{\theta^j \gamma^{ij}}$ and then use equilibrium condition (35) to rewrite $\left(\dot{x}^{ij}_{t+1} \dot{\kappa}^{nj,ij}_{t+1} \right)^{-\theta^j} (\dot{A}^{ij}_{t+1})^{\theta^j \gamma^{ij}} = \dot{\pi}^{nj,ij}_{t+1} \left(\dot{P}^{nj}_{t+1} \right)^{-\theta^j}$. It immediately follows that

$$\begin{aligned} \dot{P}'^{nj}_{t+1} &= \left(\sum_{i=1}^{N} \pi'^{nj,ij}_t \dot{\pi}^{nj,ij}_{t+1} \left(\dot{P}^{nj}_{t+1} \right)^{-\theta^j} (\hat{x}^{ij}_{t+1} \hat{\kappa}^{nj,ij}_{t+1})^{-\theta^j} (\hat{A}^{ij}_{t+1})^{\theta^j \gamma^{ij}} \right)^{-1/\theta^j}, \\ \dot{P}'^{nj}_{t+1} &= \dot{P}^{nj}_{t+1} \left(\sum_{i=1}^{N} \pi'^{nj,ij}_t \dot{\pi}^{nj,ij}_{t+1} (\hat{x}^{ij}_{t+1} \hat{\kappa}^{nj,ij}_{t+1})^{-\theta^j} (\hat{A}^{ij}_{t+1})^{\theta^j \gamma^{ij}} \right)^{-1/\theta^j}, \end{aligned}$$

and then we obtain (48).

To solve for (49), start from (35) for the case of the counterfactual economy, namely

$$\pi'^{nj,ij}_{t+1} = \pi'^{nj,ij}_t \left(\frac{\dot{x}'^{ij}_{t+1} \dot{\kappa}'^{nj,ij}_{t+1}}{\dot{P}'^{nj}_{t+1}} \right)^{-\theta^j} (\dot{A}'^{ij}_{t+1})^{\theta^j \gamma^{ij}},$$

and now multiply and divide the right-hand side by $\left(\dot{x}^{ij}_{t+1} \dot{\kappa}^{nj,ij}_{t+1} \right)^{-\theta^j} (\dot{A}^{ij}_{t+1})^{\theta^j \gamma^{ij}}$ and again use equilibrium condition (35) to rewrite $\left(\dot{x}^{ij}_{t+1} \dot{\kappa}^{nj,ij}_{t+1} \right)^{-\theta^j} (\dot{A}^{ij}_{t+1})^{\theta^j \gamma^{ij}} = \dot{\pi}^{nj,ij}_{t+1} \left(\dot{P}^{nj}_{t+1} \right)^{-\theta^j}$ to immediately obtain (49).

To obtain (50), start from (36) for the case of the counterfactual economy,

$$X'^{nj}_{t+1} = \sum_{k=1}^{J} \gamma^{nk,nj} \sum_{i=1}^{N} \pi'^{ik,nk}_{t+1} X'^{ik}_{t+1} + \alpha^j \left(\sum_{k=1}^{J} \dot{w}'^{nk}_{t+1} \dot{L}'^{nk}_{t+1} w'^{nk}_t L'^{nk}_t + \iota^n \chi'_{t+1} \right),$$

and now multiply and divide $\dot{w}'^{nk}_{t+1} \dot{L}'^{nk}_{t+1} w'^{nk}_t L'^{nk}_t$ by $\dot{w}^{nk}_{t+1} \dot{L}^{nk}_{t+1}$. Following this last step, one also obtains χ'_{t+1} and (51).

Note that (47) to (51) form a system of non-linear equations that given the baseline economy, $\left(\dot{\pi}^{nj,ij}_{t+1}, \dot{w}^{nk}_{t+1} \dot{L}^{nk}_{t+1} \right)$, the solution for the counterfactual economy at time t, $\left(w'^{nk}_t L'^{nk}_t \right)$ and the counterfactual change in fundamentals, $\left(\hat{\kappa}^{nj,ij}_{t+1}, \hat{A}^{ij}_{t+1} \right)$, can be used to solve for $\hat{w}^{nj}_{t+1}$, and hence $\hat{\omega}^{nj}(\hat{L}_{t+1}, \hat{\Theta}_{t+1}) \equiv \hat{w}^{nj}_{t+1}/\hat{P}^{n}_{t+1}$. Note that for the case of $t=0$, we have that $w'^{nk}_t L'^{nk}_t = w^{nk}_t L^{nk}_t$.

Now we show how to obtain (44), (45), and (46).

Start from (39) for the case of the counterfactual economy,

$$\mu'^{nj,ik}_{t+1} = \frac{\mu'^{nj,ik}_t \left(\dot{u}'^{ik}_{t+2} \right)^{\beta/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \mu'^{nj,mh}_t \left(\dot{u}'^{mh}_{t+2} \right)^{\beta/\nu}}.$$

Now take the ratio between this equilibrium condition and (39) to obtain

$$\frac{\mu_{t+1}^{\prime nj,ik}}{\mu_{t+1}^{nj,ik}}=\frac{\frac{\mu_t^{\prime nj,ik}\left(\dot{u}_{t+2}^{\prime ik}\right)^{\beta/\nu}}{\mu_t^{nj,ik}\left(\dot{u}_{t+2}^{ik}\right)^{\beta/\nu}}}{\frac{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_t^{\prime nj,mh}\left(\dot{u}_{t+2}^{\prime mh}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_t^{nj,mh}\left(\dot{u}_{t+2}^{mh}\right)^{\beta/\nu}}},$$

which can be written as

$$\mu_{t+1}^{\prime nj,ik}=\frac{\mu_t^{\prime nj,ik}\dot{\mu}_{t+1}^{nj,ik}\left(\hat{u}_{t+2}^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\frac{\mu_t^{\prime nj,mh}\left(\dot{u}_{t+2}^{\prime mh}\right)^{\beta/\nu}}{\sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{nj,ik}\left(\dot{u}_{t+2}^{ik}\right)^{\beta/\nu}}},$$

and take each expression in the summation term of the denominator and multiply and divide by $\mu_t^{nj,mh}\left(\dot{u}_{t+2}^{mh}\right)^{\beta/\nu}$ to obtain

$$\mu_{t+1}^{\prime nj,ik}=\frac{\mu_t^{\prime nj,ik}\dot{\mu}_{t+1}^{nj,ik}\left(\hat{u}_{t+2}^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\left(\frac{\mu_t^{\prime nj,mh}}{\mu_t^{nj,mh}}\right)\frac{\mu_t^{nj,mh}\left(\hat{u}_{t+2}^{mh}\right)^{\beta/\nu}}{\sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{nj,ik}\left(\dot{u}_{t+2}^{ik}\right)^{\beta/\nu}}\left(\dot{u}_{t+2}^{mh}\right)^{\beta/\nu}}.$$

Use (39) in the denominator to obtain

$$\mu_{t+1}^{\prime nj,ik}=\frac{\mu_t^{\prime nj,ik}\dot{\mu}_{t+1}^{nj,ik}\left(\hat{u}_{t+2}^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\left(\frac{\mu_t^{\prime nj,mh}}{\mu_t^{nj,mh}}\right)\mu_{t+1}^{nj,mh}\left(\hat{u}_{t+2}^{mh}\right)^{\beta/\nu}},$$

which gives us (44).

To obtain (45), start from (40) for the counterfactual economy,

$$\dot{u}_{t+1}^{\prime nj}=\dot{\omega}^{nj}(\dot{L}_{t+1},\dot{\Theta}_{t+1})'\left(\sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{\prime nj,ik}\left(\dot{u}_{t+2}^{\prime ik}\right)^{\beta/\nu}\right)^{\nu},$$

and take the ratio of this expression relative to (40) to obtain

$$\frac{\dot{u}_{t+1}^{\prime nj}}{\dot{u}_{t+1}^{nj}}=\frac{\dot{\omega}^{nj}(\dot{L}_{t+1},\dot{\Theta}_{t+1})'}{\dot{\omega}^{nj}(\dot{L}_{t+1},\dot{\Theta}_{t+1})}\left(\frac{\sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{\prime nj,ik}\left(\dot{u}_{t+2}^{\prime ik}\right)^{\beta/\nu}}{\sum_{i=1}^{N}\sum_{k=0}^{J}\mu_t^{nj,ik}\left(\dot{u}_{t+2}^{ik}\right)^{\beta/\nu}}\right)^{\nu}.$$

Using the "hat" notation

$$\hat{u}_{t+1}^{nj}=\hat{\omega}^{nj}(\hat{L}_{t+1},\hat{\Theta}_{t+1})\left(\sum_{i=1}^{N}\sum_{k=0}^{J}\frac{\mu_t^{\prime nj,ik}\left(\dot{u}_{t+2}^{\prime ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_t^{nj,mh}\left(\dot{u}_{t+2}^{mh}\right)^{\beta/\nu}}\right)^{\nu}.$$

Now multiply and divide each term in the summation of the right-hand side by $\mu_t^{nj,ik}\left(\dot{u}_{t+2}^{ik}\right)^{\beta/\nu}$ to obtain

$$\hat{u}_{t+1}^{nj}=\hat{\omega}^{nj}(\hat{L}_{t+1},\hat{\Theta}_{t+1})\left(\sum_{i=1}^{N}\sum_{k=0}^{J}\left(\frac{\mu_t^{\prime nj,ik}}{\mu_t^{nj,ik}}\right)\frac{\mu_t^{nj,ik}\left(\dot{u}_{t+2}^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_t^{nj,mh}\left(\dot{u}_{t+2}^{mh}\right)^{\beta/\nu}}\left(\hat{u}_{t+2}^{ik}\right)^{\beta/\nu}\right)^{\nu}$$

and use (39) to obtain

$$\hat{u}_{t+1}^{nj}=\hat{\omega}^{nj}(\hat{L}_{t+1},\hat{\Theta}_{t+1})\left(\sum_{i=1}^{N}\sum_{k=0}^{J}\left(\frac{\mu_t^{\prime nj,ik}}{\mu_t^{nj,ik}}\right)\mu_{t+1}^{nj,ik}\left(\hat{u}_{t+2}^{ik}\right)^{\beta/\nu}\right)^{\nu},$$

which is equivalent to (45).

The equilibrium condition (46) is simply the evolution of labor for the counterfactual economy, namely

(41) with the "prime" notation.

At $t = 1$, the equilibrium conditions are slightly different. This is the result of the timing assumption that the counterfactual fundamentals are unknown before $t = 1$. This means that at $t = 0$, $\hat{u}_0^{nj} = 1$, $\mu_0'^{nj,ik} = \mu_0^{nj,ik}$, and $L_1'^{nj} = L_1^{nj} = \sum_{i=1}^N \sum_{k=0}^J \mu_0^{ik,nj} L_0^{ik}$. To account for the unexpected change in fundamentals at $t = 1$, we need to solve for

$$\mu_1'^{nj,ik} = \frac{\vartheta_0^{nj,ik} \left(\hat{u}_2^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^N \sum_{h=0}^J \vartheta_0^{nj,mh} \left(\hat{u}_2^{mh}\right)^{\beta/\nu}} \tag{52}$$

and

$$\hat{u}_1^{nj} = \hat{\omega}^{nj}(\hat{L}_1, \hat{\Theta}_1) \left(\sum_{i=1}^N \sum_{k=0}^J \vartheta_0^{nj,ik} \left(\hat{u}_2^{ik}\right)^{\beta/\nu}\right)^\nu, \tag{53}$$

where

$$\vartheta_0^{nj,ik} \equiv \mu_1^{nj,ik} \left(\hat{u}_1^{ik}\right)^{\beta/\nu}.$$

To obtain this expression, take the lifetime utility at period $t = 0$ for the economy with no shock,

$$u_0^{nj} = (w_0^{nj}/P_0^n) \left(\sum_{m=1}^N \sum_{h=0}^J \left(u_1^{mh}\right)^{\beta/\nu} \exp\left(\tau^{nj,mh}\right)^{-1/\nu}\right)^\nu,$$

multiply and divide by $u_1'^{mh}$ to obtain

$$u_0^{nj} = (w_0^{nj}/P_0^n) \left(\sum_{m=1}^N \sum_{h=0}^J \left(\frac{u_1^{mh}}{u_1'^{mh}}\right)^{\beta/\nu} \left(u_1'^{mh}\right)^{\beta/\nu} \exp\left(\tau^{nj,mh}\right)^{-1/\nu}\right)^\nu,$$

define

$$\phi_1^{mh} \equiv \left(u_1^{mh}/u_1'^{mh}\right)^{\beta/\nu},$$

and then

$$u_0^{nj} = (w_0^{nj}/P_0^n) \left(\sum_{m=1}^N \sum_{h=0}^J \phi_1^{mh} \left(u_1'^{mh}\right)^{\beta/\nu} \exp\left(\tau^{nj,mh}\right)^{-1/\nu}\right)^\nu.$$

Take the lifetime utility at period $t = 1$ in the counterfactual economy,

$$u_1'^{mh} = (w_1'^{nj}/P_1'^n) \left(\sum_{m=1}^N \sum_{h=0}^J \left(u_2'^{mh}\right)^{\beta/\nu} \exp\left(\tau^{nj,mh}\right)^{-1/\nu}\right)^\nu,$$

and take the difference between $u_1'^{nj}$ and u_0^{nj} to get

$$u_0^{nj} = (w_0^{nj}/P_0^n) \left(\sum_{m=1}^N \sum_{h=0}^J \phi_1^{mh} \left(u_1'^{mh}\right)^{\beta/\nu} \exp\left(\tau^{nj,mh}\right)^{-1/\nu}\right)^\nu,$$

$$\frac{u_1'^{mh}}{u_0^{nj}} = \frac{(w_1'^{nj}/P_1'^n)}{(w_0^{nj}/P_0^n)} \left(\frac{\sum_{m=1}^N \sum_{h=0}^J \left(u_2'^{mh}\right)^{\beta/\nu} \exp\left(\tau^{nj,mh}\right)^{-1/\nu}}{\sum_{m=1}^N \sum_{h=0}^J \phi_1^{mh} \left(u_1'^{mh}\right)^{\beta/\nu} \exp\left(\tau^{nj,mh}\right)^{-1/\nu}}\right)^\nu. \tag{54}$$

Note that we can re-write $\mu_0^{nj,ik}$ as

$$\begin{aligned}
\mu_0^{nj,ik} &= \frac{\left(u_1^{ik}\right)^{\beta/\nu} \exp\left(\tau^{nj,ik}\right)^{-1/\nu}}{\sum_{m=1}^N \sum_{h=0}^J \left(u_1^{mh}\right)^{\beta/\nu} \exp\left(\tau^{nj,mh}\right)^{-1/\nu}} \\
&= \frac{\left(u_1^{ik}/u_1'^{ik}\right)^{\beta/\nu} \left(u_1'^{ik}\right)^{\beta/\nu} \exp\left(\tau^{nj,ik}\right)^{-1/\nu}}{\sum_{m=1}^N \sum_{h=0}^J \left(u_1^{mh}/u_1'^{mh}\right)^{\beta/\nu} \left(u_1'^{mh}\right)^{\beta/\nu} \exp\left(\tau^{nj,mh}\right)^{-1/\nu}} \\
&= \frac{\phi_1^{ik} \left(u_1'^{ik}\right)^{\beta/\nu} \exp\left(\tau^{nj,ik}\right)^{-1/\nu}}{\sum_{m=1}^N \sum_{h=0}^J \phi_1^{mh} \left(u_1'^{mh}\right)^{\beta/\nu} \exp\left(\tau^{nj,mh}\right)^{-1/\nu}}.
\end{aligned}$$

Given this, we can take equation (54) and multiply and divide each term in the summation by $\phi_1^{ik}\left(u_1^{\prime ik}\right)^{\beta/\nu}$ to obtain

$$\frac{u_1^{\prime mh}}{u_0^{nj}}=\frac{(w_1^{\prime nj}/P_1^{\prime n})}{(w_0^{nj}/P_0^{n})}\left[\sum\nolimits_{i=1}^{N}\sum\nolimits_{k=0}^{J}\left(\frac{\phi_1^{ik}\left(u_1^{\prime ik}\right)^{\beta/\nu}\exp\left(\tau^{nj,ik}\right)^{-1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\phi_1^{mh}\left(u_1^{\prime mh}\right)^{\beta/\nu}\exp\left(\tau^{nj,mh}\right)^{-1/\nu}}\right)\frac{\left(u_2^{\prime ik}\right)^{\beta/\nu}}{\phi_1^{ik}\left(u_1^{\prime ik}\right)^{\beta/\nu}}\right]^{\nu}.$$

We then substitute $\mu_0^{nj,ik}$ to obtain

$$\frac{u_1^{\prime mh}}{u_0^{nj}}=\frac{(w_1^{\prime nj}/P_1^{\prime n})}{(w_0^{nj}/P_0^{n})}\left(\sum\nolimits_{i=1}^{N}\sum\nolimits_{k=0}^{J}\frac{\mu_0^{nj,ik}}{\phi_1^{ik}}\left(\frac{u_2^{\prime ik}}{u_1^{\prime ik}}\right)^{\beta/\nu}\right)^{\nu}$$

and use the "dot" notation to obtain

$$\dot{u}_1^{\prime mh}=(\dot{w}_1^{\prime nj}/\dot{P}_1^{\prime n})\left(\sum\nolimits_{i=1}^{N}\sum\nolimits_{k=0}^{J}\frac{\mu_0^{nj,ik}}{\phi_1^{ik}}\left(\dot{u}_2^{\prime ik}\right)^{\beta/\nu}\right)^{\nu}.$$

This last step uses the fact that $(w_0^{\prime nj}/P_0^{\prime n})=(w_0^{nj}/P_0^{n})$ and $u_0^{\prime mh}=u_0^{mh}$. Now take this expression for $\dot{u}_1^{\prime mh}$ relative to the equilibrium condition for $\dot{u}_1^{mh}$, namely

$$\dot{u}_1^{mh}=(\dot{w}_1^{nj}/\dot{P}_1^{n})\left[\sum\nolimits_{i=1}^{N}\sum\nolimits_{k=0}^{J}\mu_0^{nj,ik}\left(\dot{u}_2^{ik}\right)^{\beta/\nu}\right]^{\nu},$$

to obtain

$$\frac{\dot{u}_1^{\prime mh}}{\dot{u}_1^{mh}}=\frac{(\dot{w}_1^{\prime nj}/\dot{P}_1^{\prime n})}{(\dot{w}_1^{nj}/\dot{P}_1^{n})}\left(\frac{\sum_{i=1}^{N}\sum_{k=0}^{J}\frac{\mu_0^{nj,ik}}{\phi_1^{ik}}\left(\dot{u}_2^{\prime ik}\right)^{\beta/\nu}}{\sum_{i=1}^{N}\sum_{k=0}^{J}\mu_0^{nj,ik}\left(\dot{u}_2^{ik}\right)^{\beta/\nu}}\right)^{\nu},$$

or

$$\hat{u}_1^{mh}=(\hat{w}_1^{nj}/\hat{P}_1^{n})\left(\sum\nolimits_{i=1}^{N}\sum\nolimits_{k=0}^{J}\frac{\mu_0^{nj,ik}}{\phi_1^{ik}}\frac{\left(\dot{u}_2^{\prime ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_0^{nj,mh}\left(\dot{u}_2^{mh}\right)^{\beta/\nu}}\right)^{\nu}.$$

Now multiply and divide each term in the summation by $\left(\dot{u}_2^{ik}\right)^{\beta/\nu}$ to obtain

$$\hat{u}_1^{mh}=(\hat{w}_1^{nj}/\hat{P}_1^{n})\left(\sum\nolimits_{i=1}^{N}\sum\nolimits_{k=0}^{J}\frac{\left(\dot{u}_2^{\prime ik}/\dot{u}_2^{ik}\right)^{\beta/\nu}}{\phi_1^{ik}}\frac{\mu_0^{nj,ik}\left(\dot{u}_2^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_0^{nj,mh}\left(\dot{u}_2^{mh}\right)^{\beta/\nu}}\right)^{\nu},$$

and use the equilibrium condition for $\mu_1^{nj,ik}$ to get

$$\hat{u}_1^{mh}=(\hat{w}_1^{nj}/\hat{P}_1^{n})\left(\sum\nolimits_{i=1}^{N}\sum\nolimits_{k=0}^{J}\frac{\mu_1^{nj,ik}}{\phi_1^{ik}}\left(\hat{u}_2^{ik}\right)^{\beta/\nu}\right)^{\nu}.$$

Finally note that $\left(\mu_1^{nj,ik}/\phi_1^{ik}\right)=\vartheta_0^{nj,ik}$ and that substituting this we obtain (53).

To obtain (52), take

$$\mu_1^{\prime nj,ik}=\frac{\left(u_2^{\prime ik}\right)^{\beta/\nu}\exp\left(\tau^{nj,ik}\right)^{-1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\left(u_2^{\prime mh}\right)^{\beta/\nu}\exp\left(\tau^{nj,mh}\right)^{-1/\nu}},$$

then use the equilibrium condition for $\mu_1^{nj,ik}$

$$\begin{aligned}\frac{\mu_1^{\prime nj,ik}}{\mu_1^{nj,ik}} &= \frac{\frac{\left(u_2^{\prime ik}\right)^{\beta/\nu}\exp\left(\tau^{nj,ik}\right)^{-1/\nu}}{\left(u_2^{ik}\right)^{\beta/\nu}\exp\left(\tau^{nj,ik}\right)^{-1/\nu}}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\frac{\left(u_2^{\prime mh}\right)^{\beta/\nu}\exp\left(\tau^{nj,mh}\right)^{-1/\nu}}{\sum_{i=1}^{N}\sum_{k=0}^{J}\left(u_2^{ik}\right)^{\beta/\nu}\exp\left(\tau^{nj,ik}\right)^{-1/\nu}}} \\ &= \frac{\left(u_2^{\prime ik}/u_2^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_1^{nj,mh}\left(u_2^{\prime mh}/u_2^{mh}\right)^{\beta/\nu}},\end{aligned}$$

and then multiply and divide the numerator and each expression in the summation of the denominator by $\left(u_1^{\prime ik}/u_1^{ik}\right)^{\beta/\nu}$ to obtain

$$\frac{\mu_1^{\prime nj,ik}}{\mu_1^{nj,ik}} = \frac{\left(u_1^{\prime ik}/u_1^{ik}\right)^{\beta/\nu}\left(\hat{u}_2^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_1^{nj,mh}\left(u_1^{\prime mh}/u_1^{mh}\right)^{\beta/\nu}\left(\hat{u}_2^{mh}\right)^{\beta/\nu}}.$$

Using the definition of $\vartheta_0^{nj,ik}$, we obtain (52).

APPENDIX C: EXTENSIONS

C.1. *The One-Sector Trade and Migration Model*

In this appendix, we present the one-sector model. To simplify our notation, we index the N labor markets by ℓ, m, and n. As in the main text, we let $\ell = 0$ denote non-employment status.

C.1.1. *Households*

The problem of the agent is as follows:

$$\mathrm{v}_t^{\ell} = \log(w_t^{\ell}/P_t^{\ell}) + \max_{\{m\}_{m=1}^{N}}\left\{\beta E\left[\,\mathrm{v}_{t+1}^{m}\right] - \tau^{\ell,m} + \nu\epsilon_t^{m}\right\}.$$

After using the properties of the Extreme Value distribution, we find that the expected lifetime utility of a worker is given by

$$V_t^{\ell} = \log(w_t^{\ell}/P_t^{\ell}) + \nu\log\left(\sum\nolimits_{m=1}^{N}\exp\left(\beta V_{t+1}^{m} - \tau^{\ell,m}\right)^{1/\nu}\right).$$

Similarly, the transition matrix, or choice probability, is given by

$$\mu_t^{\ell,m} = \frac{\exp\left(\beta V_{t+1}^{m} - \tau^{\ell,m}\right)^{1/\nu}}{\sum_{n=1}^{N}\exp\left(\beta V_{t+1}^{n} - \tau^{\ell,n}\right)^{1/\nu}}$$

and the evolution of the distribution of labor across markets is given by

$$L_{t+1}^{\ell} = \sum\nolimits_{m=1}^{N}\mu_t^{m,\ell}L_t^{m}.$$

C.1.2. *Production*

As in the main text, at each ℓ there is a continuum of perfectly competitive intermediate good producers with constant returns to scale technology and idiosyncratic productivity z^{ℓ} ∼Fréchet$(1,\theta)$. In particular, the problem of an intermediate good producer is as follows:

$$\min_{\{l_t^{\ell},M_t^{\ell}\}} w_t^{\ell}l_t^{\ell} + r_t^{\ell}h^{\ell} + P_t^{\ell}M_t^{\ell}, \text{ subject to } q_t^{\ell}(z^{\ell}) = z^{\ell}A^{\ell}\left(\left(h^{\ell}\right)^{\xi}\left(l_t^{\ell}\right)^{1-\xi}\right)^{\gamma}\left(M_t^{\ell}\right)^{1-\gamma},$$

where M_t^{ℓ} is the demand for material inputs, l_t^{ℓ} the demand for labor, h^{ℓ} the demand for structures, and A^{ℓ} fundamental TFP in ℓ. As shown shortly, material inputs are produced with intermediates from every other market in the world. Denote by P_t^{ℓ} the price of materials produce in ℓ. Therefore, the unit price of an input bundle is given by

$$x_t^{\ell} = B^{\ell}\left(\left(r_t^{\ell}\right)^{\xi}\left(w_t^{\ell}\right)^{1-\xi}\right)^{\gamma}\left(P_t^{\ell}\right)^{1-\gamma},$$

where B^{ℓ} is a constant.

The unit cost of an intermediate good z^{ℓ} at time t is

$$\frac{x_t^{\ell}}{z^{\ell}\,A^{\ell}}.$$

Competition implies that the price paid for a particular variety in market ℓ is given by

$$p_t^\ell(z) = \min_{m\in N} \frac{\kappa^{\ell,m} x_t^m}{z^\ell A^\ell}.$$

Final goods in ℓ are produced by aggregating intermediate inputs from all ℓ. Let Q_t^ℓ be the quantity of final goods in ℓ and $\tilde{q}_t^\ell(z)$ the quantity demanded of an intermediate variety such that the vector of productivity draws received by the different ℓ is $z = (z^1, z^2, \ldots, z^N)$. The production of final goods is given by

$$Q_t^\ell = \left(\int_{R_{++}^N} \left(\tilde{q}_t^\ell(z)\right)^{1-1/\eta} \, d\phi(z) \right)^{\eta/(\eta-1)},$$

where $\phi(z) = \exp\left\{-\sum_{\ell=1}^N (z^\ell)^{-\theta}\right\}$ is the joint distribution function over the vector z. Given the properties of the Fréchet distribution, the price of the final good ℓ at time t is

$$P_t^\ell = \Gamma \left(\sum_{m=1}^N \left(\frac{x_t^m \kappa^{\ell,m}}{A^m} \right)^{-\theta} \right)^{-1/\theta},$$

where Γ is a constant given by the value of a Gamma function evaluated at $1 + (1 - \eta/\theta)$ and we assume that $1 + \theta > \eta$. The share of total expenditure in market ℓ on goods from m is given by

$$\pi_t^{\ell,m} = \frac{\left(x_t^m \kappa^{\ell,m}/A^m\right)^{-\theta}}{\sum_{n=1}^N \left(x_t^n \kappa^{\ell,n}/A^n\right)^{-\theta}}.$$

C.1.3. *Market Clearing*

Let X_t^ℓ denote the total expenditure on final goods in ℓ. Then, the goods market clearing condition is given by

$$X_t^\ell = (1-\gamma) \sum_{m=1}^N \pi_t^{m,\ell} X_t^m + \frac{w_t^\ell L_t^\ell}{1-\xi}.$$

The labor market clearing condition in ℓ is

$$w_t^\ell L_t^\ell = (1-\xi)\gamma \sum_{m=1}^N \pi_t^{m,\ell} X_t^m,$$

and the structures market clearing condition in ℓ is

$$r_t^\ell H^\ell = \xi\gamma \sum_{m=1}^N \pi_t^{m,\ell} X_t^m,$$

where $H^\ell = h^\ell$.

We now provide a formal definition of the equilibrium together with the equilibrium conditions.

Definition *Given* (L_0, Θ), *a sequential competitive equilibrium of the one-sector model is a sequence of* $\{L_t,\, \mu_t,\, V_t,\, w\,(L_t, \Theta)\}_{t=0}^\infty$ *that solves*

$$V_t^\ell = \log(w_t^\ell / P_t^\ell) + \nu \log \left(\sum_{m=1}^N \exp\left(\beta V_{t+1}^m - \tau^{\ell,m}\right)^{1/\nu} \right),$$

$$\mu_t^{\ell,m} = \frac{\exp\left(\beta V_{t+1}^m - \tau^{\ell,m}\right)^{1/\nu}}{\sum_{n=1}^N \exp\left(\beta V_{t+1}^n - \tau^{\ell,n}\right)^{1/\nu}},$$

$$L_{t+1}^\ell = \sum_{m=1}^N \mu_t^{m,\ell} L_t^m,$$

where w_t^ℓ / P_t^ℓ *is the solution to the temporary equilibrium at each* t *and solves*

$$x_t^\ell = B^\ell \left(\left(r_t^\ell\right)^\xi \left(w_t^\ell\right)^{1-\xi} \right)^\gamma \left(P_t^\ell\right)^{1-\gamma},$$

$$r_t^\ell = w_t^\ell \frac{L_t^\ell}{H^\ell} \frac{\xi}{(1-\xi)},$$

$$P_t^\ell = \Gamma \left(\sum_{m=1}^N \left(B^m \left(w_t^m\right)^\gamma \left(P_t^m\right)^{1-\gamma} \right)^{-\theta} \left(\kappa^{\ell,m}/A^m \right)^{-\theta} \right)^{-1/\theta},$$

$$\pi_t^{\ell,m} = \frac{\left(x_t^m \kappa^{\ell,m}/A^m\right)^{-\theta}}{\sum_{n=1}^N \left(x_t^n \kappa^{\ell,n}/A^n\right)^{-\theta}},$$

$$w_t^m L_t^m = \sum_{\ell=1}^N \frac{\left(x_t^m \kappa^{\ell,m}/A^m\right)^{-\theta}}{\sum_{n=1}^N \left(x_t^n \kappa^{\ell,n}/A^n\right)^{-\theta}} w_t^\ell L_t^\ell.$$

C.2. *The Model with CES Preferences*

In this appendix, we extend the model to the case of a constant elasticity of substitution (CES) utility function. In particular, we allow for different degrees of substitutability across manufacturing and non-manufacturing industries. Preferences over the basket of final local goods is given by $U(C_t^{nj})$, where

$$C_t^{nj} = \left((\varkappa^{nj})^{1/\eta} (c_t^{nj,M})^{\frac{\eta-1}{\eta}} + (1-\varkappa^{nj})^{1/\eta} (c_t^{nj,S})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{55}$$

where $c_t^{nj,M}$ and $c_t^{nj,S}$ are Cobb-Douglas aggregates of consumption of manufacturing goods and non-manufacturing goods, respectively, in market nj at time t, given by

$$c_t^{nj,M} = \prod_{k \in M} (c_t^{nj,k})^{\alpha^k};\ c_t^{nj,S} = \prod_{k \in S} (c_t^{nj,k})^{\alpha^k},$$

with $\sum_{k \in M} \alpha^k = 1$; $\sum_{k \in S} \alpha^k = 1$. The price index of final goods in market nj is the given by

$$P_t^{nj} = \left(\varkappa^{nj} (p_t^{nj,M})^{1-\eta} + (1-\varkappa^{nj}) (p_t^{nj,S})^{1-\eta} \right)^{\frac{1}{\eta-1}},$$

$$p_t^{nj,M} = \prod_{k \in M} (p_t^{nj,k}/\alpha^k)^{\alpha^k};\ p_t^{nj,S} = \prod_{k \in S} (p_t^{nj,k}/\alpha^k)^{\alpha^k},$$

As in Section 2, the equilibrium of the economy is given by equations (5) to (10), and (2) to (4) subject to the utility function given by $U(C_t^{nj})$, with C_t^{nj} given by equation (55).

C.2.1. *Equilibrium Conditions in Relative Time Differences*

As before, we denote by $\dot{y}_{t+1} \equiv y_{t+1}/y_t$ the change in any variable between two periods of time in the baseline economy and by $\dot{y}'_{t+1} \equiv y'_{t+1}/y'_t$ the change in time in the counterfactual economy. The relative change in variable y between the counterfactual economy and the baseline economy is given by $\hat{y}_{t+1} \equiv \dot{y}'_{t+1}/\dot{y}_t$. Therefore, the relative change in the local price index between the counterfactual economy and the baseline economy is given by

$$\hat{P}_{t+1}^{nj} = \left(\alpha_t'^{nj,M} \dot{\alpha}_{t+1}^{nj,M} \left(\hat{p}_{t+1}^{nj,M} \right)^{1-\eta} + \alpha_t'^{nj,S} \dot{\alpha}_{t+1}^{nj,S} \left(\hat{p}_{t+1}^{nj,S} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}},$$

where $\alpha_t^{nj,M}$ and $\alpha_t^{nj,S}$ are the final expenditure share of manufacturing and non-manufacturing goods, respectively, given by

$$\alpha_t^{nj,M} \equiv \frac{p_t^{nj,M} c_t^{nj,M}}{P_t^{nj} C_t^{nj}} = \varkappa^{nj} \left(\frac{p_t^{nj,M}}{P_t^{nj}} \right)^{1-\eta},$$

$$\alpha_t^{nj,S} \equiv \frac{p_t^{nj,S} c_t^{nj,S}}{P_t^{nj} C_t^{nj}} = (1 - \varkappa^{nj}) \left(\frac{p_t^{nj,S}}{P_t^{nj}} \right)^{1-\eta},$$

with $\alpha_t^{nj,M} + \alpha_t^{nj,S} = 1$. It follows that $\alpha_t'^{nj,M} = \alpha_{t-1}'^{nj,M} \dot{\alpha}_t^{nj,M} \left(\frac{\hat{p}_t^{nj,M}}{\hat{P}_t^{nj}} \right)^{1-\eta}$ and that $\alpha_t'^{nj,S} = \alpha_{t-1}'^{nj,S} \dot{\alpha}_t^{nj,S} \left(\frac{\hat{p}_t^{nj,S}}{\hat{P}_t^{nj}} \right)^{1-\eta}$. Finally, we have

$$\hat{p}_{t+1}^{nj,M} = \prod_{k \in M} \left(\hat{p}_{t+1}^{nj,k} \right)^{\alpha^k}$$

$$\hat{p}_{t+1}^{nj,S} = \prod_{k \in S} \left(\hat{p}_{t+1}^{nj,k} \right)^{\alpha^k}.$$

The rest of the equilibrium conditions in relative time differences are the same as those derived in Section 3.

C.3. *Additional Sources of Persistence in the Model*

In the model developed in Section 2, the i.i.d taste shocks as well as the asymmetric migration costs are a source of persistence in the migration choice. There is, therefore, a gradual adjustment of shocks to the new steady state in the model. In this section, we extend the model to incorporate additional sources of persistence and, as a robustness exercise, we quantify the effects of the China shock in these alternative models. Importantly, we show how dynamic hat algebra can be applied to these alternative models.

C.3.1. *Persistence Due to Local Preferences (Amenities)*

In the first extension of our model, we add additional persistence by introducing a fixed individual heterogeneity to preferences. Concretely, we assume that the utility of residing in a particular location includes preferences for amenities, which are location specific and time invariant. Therefore, we now have that

$$U(C_t^{nj}, B^n) = \log(C_t^{nj}) + \log B^n,$$

where B^n is a local, time invariant amenity in location n. As we can see, this additional preference for a location adds more persistence to the migration decision, as agents are going to command a larger wage differential, and a larger idiosyncratic draw in order to find it optimal to migrate. Notice also that a model with fixed preferences over locations is isomorphic to the model in Section 2 if we apply a suitable renormalization of migration costs $\tau^{nj,ik}$. In particular, the value of a household in location nj at time t is now given by

$$v_t^{nj} = \log(C_t^{nj}) + \log B^n + \max_{\{i,k\}_{i=1,k=0}^{N,J}} \{\beta E[v_{t+1}^{ik}] - \tau^{nj,ik} + \nu \epsilon_t^{ik}\}.$$

We can now define $\bar{\tau}^{nj,ik} = \tau^{nj,ik} - \log B^n$, so that the value function becomes isomorphic to that in Section 2. The only distinction is that the implied level of migration costs in the model with fixed preferences for locations will be lower than those in the model of Section 2. This distinction is important when estimating the model in levels. However, the dynamic hat algebra will differentiate out the levels of $\tau^{nj,ik}$ and B^n so that all propositions in Section 3 still hold.

C.3.2. *Additional Source of Persistence in Household Choices*

An alternative extension of our model is to consider the case in which agents have a more persistent idiosyncratic shock; that is, their idiosyncratic preferences for locations do not change every period. We now proceed to characterize the problem allowing for a particular type of serial correlation of shocks. Consider the value of an agent located at nj, and assume that we start the economy with a given allocation of workers across markets. This initial allocation is assumed to be determined by an initial draw of idiosyncratic shocks ϵ_0^{ik}. Now suppose that at each moment in time agents are subject to a Poisson process that determines the arrival of a new draw of the idiosyncratic shock. In particular, we assume with probability ρ that the household does not receive a preference draw and therefore stays in the same labor market. On the other hand, we assume a probability of $1-\rho$ that the household receives a new draw, although not all agents with a new draw will migrate. We assume that the likelihood of these events are not location specific.

As before, let $V_t^{nj} = E[v_t^{nj}]$. The value function can be then written as

$$V_t^{nj} = U(C_t^{nj}) + \rho\beta V_{t+1}^{nj} + (1-\rho)\nu \log \left(\sum_{i=1}^{N} \sum_{k=0}^{J} \exp(\beta V_{t+1}^{ik} - \tau^{nj,ik})^{1/\nu} \right).$$

The fraction of households that stay in market nj at time t is now given by

$$\mu_t^{nj,nj} = \rho + \frac{(1-\rho)\exp(\beta V_{t+1}^{nj})^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp(\beta V_{t+1}^{mh} - \tau^{nj,mh})^{1/\nu}},$$

while the fraction of workers that move to market ik is given by

$$\mu_t^{nj,ik} = \frac{(1-\rho)\exp(\beta V_{t+1}^{ik} - \tau^{nj,ik})^{1/\nu}}{\sum_{m=1}^{N} \sum_{h=0}^{J} \exp(\beta V_{t+1}^{mh} - \tau^{nj,mh})^{1/\nu}}.$$

We then define the choice probabilities conditional on receiving a new idiosyncratic preference draw as

$$\tilde{\mu}_t^{nj,nj} = \frac{\mu_t^{nj,nj} - \rho}{1-\rho},$$

$$\tilde{\mu}_t^{nj,ik} = \frac{\mu_t^{nj,ik}}{1-\rho}.$$

The evolution of employment at market nj is given by

$$L_{t+1}^{nj} = \rho L_t^{nj} + (1-\rho) \sum_{i=1}^{N} \sum_{k=0}^{J} \tilde{\mu}_t^{ik,nj} L_t^{ik}.$$

This is the system of equations that defines the equilibrium of the household's dynamic system in a model with persistent idiosyncratic shocks. This equilibrium condition shows how adding persistence affects the evolution of the state variable of the economy. It is precisely from the fact that only a share $(1-\rho)$ of households have a new idiosyncratic draw that this is the share of agents that decide to reallocate across markets over time. Of course, not all of the agents with a new draw migrate. In fact, a fraction $(1-\rho)\tilde{\mu}_t^{nj,nj}$ decides to stay.

Note also that the value function can be re-expressed as

$$V_t^{nj} = U(C_t^{nj}) + \beta V_{t+1}^{nj} - (1-\rho)\nu \log \tilde{\mu}_t^{nj,nj}.$$

This equation shows how the persistent parameter ρ re-scales the option value of migration. Importantly, notice that in the model with this additional shock, $1/\nu$ is the migration elasticity conditional on receiving an idiosyncratic preference draw, while in the model where $\rho = 0$, $1/\nu$ is the unconditional migration elasticity. We now show these equilibrium conditions in relative time differences and that all propositions in Section 3 still hold.

C.3.3. *Equilibrium Conditions in Relative Time Differences*

As before, let $\hat{y}_{t+1} \equiv \dot{y}'_{t+1}/\dot{y}_{t+1}$ be the proportional change between the counterfactual equilibrium $\dot{y}'_{t+1} \equiv y'_{t+1}/y'_t$, and the baseline equilibrium $\dot{y}_{t+1} \equiv y_{t+1}/y_t$ across time. The expected value of a household in market nj at time t in a model with the additional source of persistence, expressed in relative time differences, is then given by

$$\hat{u}_t^{nj} = \hat{\omega}^{nj}(\hat{L}_t, \hat{\Theta}_t) \left(\hat{u}_{t+1}^{nj}\right)^{\beta\rho/\nu} \left(\sum\nolimits_{i=1}^{N} \sum\nolimits_{k=0}^{J} \tilde{\mu}_{t-1}^{\prime nj,ik} \dot{\tilde{\mu}}_t^{nj,ik} \left(\hat{u}_{t+1}^{ik}\right)^{\beta/\nu}\right)^{(1-\rho)\nu}.$$

The probability choice $\mu_t^{nj,ik}$ in relative time differences is given by

$$\tilde{\mu}_t^{nj,ik} = \frac{\tilde{\mu}_{t-1}^{\prime nj,ik} \dot{\tilde{\mu}}_t^{nj,ik} \left(\hat{u}_{t+1}^{ik}\right)^{\beta/\nu}}{\sum_{h=1}^{N} \sum_{m=0}^{J} \tilde{\mu}_{t-1}^{\prime nj,mh} \dot{\tilde{\mu}}_t^{nj,mh} \left(\hat{u}_{t+1}^{mh}\right)^{\beta/\nu}}.$$

The evolution of the state variable L_{t+1}^{nj} is given by

$$L_{t+1}^{nj} = \rho L_t^{nj} + (1-\rho) \sum\nolimits_{i=1}^{N} \sum\nolimits_{k=0}^{J} \tilde{\mu}_t^{ik,nj} L_t^{ik}$$

where $\hat{\omega}^{nj}(\hat{L}_t, \hat{\Theta}_t)$ solves the temporary equilibrium expressed in relative time differences as before. Given that we do not need to estimate levels of migration costs in this dynamic system, and that the equilibrium conditions of the static subproblem have not changed, all propositions of Section 3 still hold.

C.4. *Intensive Margin: Elastic Labor Supply*

In this appendix, we extend the model to allow for an elastic labor supply by each household. Specifically, we introduce labor-leisure decisions into each household's utility function. As before, we denote $\dot{y}_{t+1} \equiv y_{t+1}/y_t$ to be the change in any variable between two periods in time in the baseline economy and $\dot{y}'_{t+1} \equiv y'_{t+1}/y'_t$ to be the change in time in the counterfactual economy. The relative change in variable y between the counterfactual economy and the baseline economy is given by $\hat{y}_{t+1} \equiv \dot{y}'_{t+1}/\dot{y}_t$. We also define $\hat{U}_{t+1}^{\ell} = (U_{t+1:}^{\ell'} - U_{t:}^{\ell'}) - (U_{t+1:}^{\ell} - U_{t:}^{\ell})$ to the relative change in utility between the counterfactual economy and the baseline economy in labor maket ℓ. In the context of our model with inelastic labor supply, we obtain that

$$\hat{U}_{t+1}^{\ell} = \log \frac{\hat{w}_{t+1}^{\ell}}{\hat{P}_{t+1}^{\ell}}, \tag{56}$$

and the rest of the equilibrium condition in relative time differences are the same as those derived in Section 3.

In what follows, we present alternative specifications for the utility function that have been considered in the macro literature.

C.4.1. *Case 1*

Consider the following alternative utility function

$$U(C_t^{\ell}, l_t^{\ell}) = \log C_t^{\ell} - \frac{(l_t^{\ell})^{1+1/\phi}}{1+1/\phi},$$

where C_t^{ℓ} is the amount of consumption by households located in ℓ at time t. Households are endowed with one unit of labor; thus, $1 - l_t^{\ell}$ is the amount of leisure consumed in location ℓ at time t. The household's problem is given by

$$\max_{\{C_t^\ell, l_t^\ell\}} \log C_t^\ell - \frac{(l_t^\ell)^{1+1/\phi}}{1+1/\phi} \text{ s.t. } P_t^\ell C_t^\ell = w_t^\ell l_t^\ell, \text{ with } 0 \leq l_t^\ell \leq 1,$$

and the optimality conditions are given by

$$C_t^\ell = \frac{w_t^\ell}{P_t^\ell}, \text{ and } l_t^\ell = 1.$$

Using the optimality conditions, we can express the indirect utility as

$$U_{t:}^\ell = \log \frac{w_t^\ell}{P_t^\ell}.$$

The indirect utility in relative time differences is given by

$$\hat{U}_{t+1}^\ell = \log \frac{\hat{w}_{t+1}^\ell}{\hat{P}_{t+1}^\ell}.$$

C.4.2. *Case 2*

Consider the following utility function:

$$U(C_t^\ell, l_t^\ell) = \log C_t^\ell + \phi \log(1 - l_t^\ell).$$

In this case, the elasticity of utility with respect to leisure is given by ϕ. At each time t, households decide consumption and the amount of time devoted to leisure and the household's problem is then given by

$$\max_{\{C_t^\ell, l_t^\ell\}} \log C_t^\ell + \phi \log(1 - l_t^\ell) \text{ s.t. } P_t^\ell C_t^\ell = w_t^\ell l_t^\ell, \text{ with } 0 \leq l_t^\ell \leq 1.$$

The optimality conditions are given by

$$C_t^\ell = \frac{1}{1+\phi} \frac{w_t^\ell}{P_t^\ell},$$

$$l_t^\ell = \frac{1}{1+\phi}.$$

Using the optimality conditions, we can express the indirect utility as

$$U_{t:}^\ell = \log \frac{1}{1+\phi} \frac{w_t^\ell}{P_t^\ell} + \phi \log \frac{1}{1+\phi}.$$

The indirect utility in relative time differences is given by

$$\hat{U}_{t+1}^\ell = \log \frac{\hat{w}_{t+1}^\ell}{\hat{P}_{t+1}^\ell}.$$

C.4.3. *Case 3*

Consider the following alternative utility function:

$$U(C_t^\ell, l_t^\ell) = \log C_t^\ell - B l_t^\ell.$$

In this case, the household's problem is given by

$$\underset{\{C_t^\ell, l_t^\ell\}}{max} \log C_t^\ell - B l_t^\ell \text{ s.t. } P_t^\ell C_t^\ell = w_t^\ell l_t^\ell, \text{ with } 0 \leq l_t^\ell \leq 1,$$

and the optimality conditions are given by

$$C_t^\ell = \frac{1}{B} \frac{w_t^\ell}{P_t^\ell},\ l_t^\ell = \frac{1}{B}.$$

In this case, the indirect utility is given by

$$U_{t:}^\ell = \log \frac{1}{B} \frac{w_t^\ell}{P_t^\ell} + \log \frac{1}{B}.$$

The indirect utility in relative time differences is given by

$$\hat{U}_{t+1}^\ell = \log \frac{\hat{w}_{t+1}^\ell}{\hat{P}_{t+1}^\ell}.$$

C.5. *The Model with Social Security Disability Insurance*

In this appendix, we show that the lifetime utilities of workers in non-employment and disability can be rearranged in order to apply the results from Propositions 2 and 3 in the paper.

Recall that the value functions of non-employed workers and on disability, expressed in time differences, are given by

$$V_{t+1}^{n0} - V_t^{n0} = \log \dot{b}^n + \nu(1-\delta) \log \left[\sum\nolimits_{i=1}^N \sum\nolimits_{k=0}^J \mu_t^{nj,ik} \exp \left(V_{t+2}^{ik} - V_{t+1}^{ik} \right)^{\beta/\nu} \right] + \delta\beta \left(V_{t+2}^{nD} - V_{t+1}^{nD} \right),$$

$$V_{t+1}^{nD} - V_t^{nD} = \log \left(b^{DI} / P_t^n \right) + (1-\rho)\beta \left(V_{t+2}^{n0} - V_{t+1}^{n0} \right) + \rho\beta \left(V_{t+2}^{nD} - V_{t+1}^{nD} \right).$$

Rearranging these expressions to match those of Proposition 2, we obtain

$$\dot{u}_{t+1}^{n0} = \dot{b}^n \left[\sum\nolimits_{i=1}^N \sum\nolimits_{k=0}^J \mu_t^{nj,ik} \left(\dot{u}_{t+2}^{ik} \right)^{\beta/\nu} \right]^{\nu(1-\delta)} \left(\dot{u}_{t+2}^{nD} \right)^{\delta\beta}, \tag{57}$$

$$\dot{u}_{t+1}^{nD} = \frac{\dot{b}^{DI}}{\dot{P}_{t+1}^n} \left(\dot{u}_{t+2}^{n0} \right)^{(1-\rho)\beta} \left(\dot{u}_{t+2}^{nD} \right)^{\rho\beta}. \tag{58}$$

Rearranging these expressions to match those of Proposition 3, we obtain

$$\hat{u}_{t+1}^{n0} = \hat{b}^n \left[\sum\nolimits_{i=1}^N \sum\nolimits_{k=0}^J \mu_t'^{nj,ik} \dot{\mu}_{t+1}^{nj,ik} \left(\hat{u}_{t+2}^{ik} \right)^{\beta/\nu} \right]^{\nu(1-\delta)} \left(\hat{u}_{t+2}^{nD} \right)^{\delta\beta}, \tag{59}$$

$$\hat{u}_{t+1}^{nD} = \frac{\hat{b}^{DI}}{\hat{P}_{t+1}^n} \left(\hat{u}_{t+2}^{n0} \right)^{(1-\rho)\beta} \left(\hat{u}_{t+2}^{nD} \right)^{\rho\beta}. \tag{60}$$

APPENDIX D: SOLUTION ALGORITHM

Part I: Solving for the sequential competitive equilibrium

The strategy to solve the model given an initial allocation of the economy, $(L_0, \pi_0, X_0, \mu_{-1})$, and given an anticipated convergent sequence of changes in fundamentals, $\{\dot{\Theta}_t\}_{t=1}^\infty$, is as follows:

1. Initiate the algorithm at $t = 0$ with a guess for the path of $\left\{ \dot{u}_{t+1}^{nj\,(0)} \right\}_{t=0}^T$, where the superscript (0) indicates that it is a guess. The path should converge to $\dot{u}_{T+1}^{nj\,(0)} = 1$ for a sufficiently large T. Take as given the set of initial conditions L_0^{nj}, $\mu_{-1}^{nj,ik}$, $\pi_0^{ni,nj}$, $w_0^{nj} L_0^{nj}$, $r_0^{nj} H_0^{nj}$.

2. For all $t \geq 0$, use $\left\{\dot{u}_{t+1}^{nj\,(0)}\right\}_{t=0}^{T}$ and $\mu_{-1}^{nj,ik}$ to solve for the path of $\left\{\mu_t^{nj,ik}\right\}_{t=0}^{T}$ using equation (16).
3. Use the path for $\left\{\mu_t^{nj,ik}\right\}_{t=0}^{T}$ and L_0^{nj} to get the path for $\left\{L_{t+1}^{nj}\right\}_{t=0}^{T}$ using equation (18).
4. Solving for the temporary equilibrium:
   (a) For each $t \geq 0$, given $\dot{L}_{t+1}^{nj}$, guess a value for $\dot{w}_{t+1}^{nj}$.
   (b) Obtain $\dot{x}_{t+1}^{nj}$, $\dot{P}_{t+1}^{nj}$, and $\pi_{t+1}^{nj,ij}$ using equations (11), (12) , and (13) .[63]
   (c) Use $\pi_{t+1}^{nj,ij}$, $\dot{w}_{t+1}^{nj}$, and $\dot{L}_{t+1}^{nj}$ to get X_{t+1}^{nj} using equation (14).
   (d) Check if the labor market is in equilibrium using equation (15). If it is not, go back to step (a) and adjust the initial guess for $\dot{w}_{t+1}^{nj}$ until labor markets clear.
   (e) Repeat steps (a) through (d) for each period t and obtain paths for $\left\{\dot{w}_{t+1}^{nj}, \dot{P}_{t+1}^{nj}\right\}_{t=0}^{T}$.
5. For each t, use $\mu_t^{nj,ik}$, $\dot{w}_{t+1}^{nj}$, $\dot{P}_{t+1}^{nj}$, and $\dot{u}_{t+2}^{nj\,(0)}$ to solve backwards for $\dot{u}_{t+1}^{nj\,(1)}$ using equation (17). This delivers a new path for $\left\{\dot{u}_{t+1}^{nj\,(1)}\right\}_{t=0}^{T}$, where the superscript 1 indicates an updated value for u.
6. Take the path for $\left\{\dot{u}_{t+1}^{nj\,(1)}\right\}_{t=0}^{T}$ as the new set of initial conditions.
7. Check if $\left\{\dot{u}_{t+1}^{nj\,(1)}\right\}_{t=0}^{T} \simeq \left\{\dot{u}_{t+1}^{nj\,(0)}\right\}_{t=0}^{T}$. If it is not, go back to step 1 and update the initial guess.

Part II: Solving for counterfactuals

Denote by $\hat{y}_{t+1} \equiv \dot{y}'_{t+1}/\dot{y}_{t+1}$ the proportional change between the counterfactual equilibrium, $\dot{y}'_{t+1} \equiv y'_{t+1}/y'_t$, and the baseline economy, $\dot{y}_{t+1} \equiv y_{t+1}/y_t$ across time. With this notation, $\hat{\Theta}_{t+1}$ is the proportional counterfactual changes in fundamentals across time relative to the baseline economy; namely $\hat{\Theta}_{t+1} = \dot{\Theta}'_{t+1}/\dot{\Theta}_{t+1}$.

To compute counterfactuals, we assume that agents at $t = 0$ are not anticipating the change in the path of fundamentals and that at $t = 1$ agents learn about the entire future counterfactual sequence of $\{\Theta'_t\}_{t=1}^{\infty}$.

Take as given a baseline economy, $\{L_t, \mu_{t-1}, \pi_t, X_t\}_{t=0}^{\infty}$, and a counterfactual convergent sequence of changes in fundamentals, $\{\hat{\Theta}_t\}_{t=1}^{\infty}$.

To solve for the counterfactual equilibrium, proceed as follows:

1. Initiate the algorithm at $t = 0$ with a guess for the path of $\left\{\hat{u}_{t+1}^{n,j\,(0)}\right\}_{t=0}^{T}$, where the superscript (0) indicates it is a guess. The path should converge to $\hat{u}_{T+1}^{nj\,(0)} = 1$ for a sufficiently large T. Take as given the initial conditions L_0^{nj}, $\mu_{-1}^{nj,ik}$, $\pi_0^{nj,ij}$, $w_0^{nj} L_0^{nj}$, $r_0^{nj} H_0^{nj}$; the baseline economy, $\{\dot{L}_t, \dot{\mu}_{t-1}, \dot{\pi}_t, \dot{X}_t\}_{t=0}^{\infty}$; and the solution to the sequential competitive equilibrium of the baseline economy.
2. For all $t \geq 0$, use $\left\{\hat{u}_{t+1}^{nj\,(0)}\right\}_{t=0}^{T}$ and $\{\dot{\mu}_{t-1}\}_{t=0}^{\infty}$ to solve for the path of $\left\{\mu_t'^{nj}\right\}_{t=0}^{T}$ using the following equations:

   For $t = 0$,

$$\hat{u}_0^{nj\,(0)} = 1,$$

$$\mu_0'^{nj,ik} = \mu_0^{nj,ik}$$

$$L_1'^{nj} = L_1^{nj} = \sum\nolimits_{i=1}^{N} \sum\nolimits_{k=0}^{J} \mu_0^{ik,nj} L_0^{ik}.$$

[63]Notice that $\dot{w}_t^{nj} = \dot{w}_t^n = \dot{r}_t^{nj} = \dot{r}_t^n$ for all n such that $\tau^{nj,nk} = 0$ and $\dot{r}_t^{nj} = \dot{w}_t^{nj} \dot{L}_t^{nj}$ for all n such that $\tau^{nj,nk} \neq 0$.

For period $t = 1$,

$$\mu_1'^{nj,ik} = \frac{\vartheta_0^{nj,ik} \left(\hat{u}_2^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J} \vartheta_0^{nj,mh} \left(\hat{u}_2^{mh}\right)^{\beta/\nu}},$$

where

$$\vartheta_0^{nj,ik\ (0)} = \mu_1^{nj,ik} \left(\hat{u}_1^{ik\ (0)}\right)^{\beta/\nu}.$$

For period $t \geq 1$,

$$\mu_t'^{nj,ik} = \frac{\mu_{t-1}'^{nj,ik} \dot{\mu}_t^{nj,ik} \left(\hat{u}_{t+1}^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J} \mu_{t-1}'^{nj,mh} \dot{\mu}_t^{nj,mh} \left(\hat{u}_{t+1}^{mh}\right)^{\beta/\nu}}.$$

3. Use the path for $\left\{\mu_t'^{nj,ik}\right\}_{t=0}^{T}$ and $L_0'^{nj}$ to get the path for $\left\{L_{t+1}'^{nj}\right\}_{t=0}^{T}$ using equation (21) in the paper. That is,

$$L_{t+1}'^{nj} = \sum_{i=1}^{N}\sum_{k=0}^{J} \mu_t'^{nj,ik} L_t'^{ik}.$$

4. Solve for the temporary equilibrium as follows:

(a) For each $t \geq 0$, given $\hat{L}_{t+1}^{nj}$, guess a value for $\left\{\hat{w}_{t+1}^{nj}\right\}_{n=1,j=0}^{N,J}$.

(b) Obtain $\hat{x}_{t+1}^{nj}$, $\hat{P}_{t+1}^{nj}$, and $\hat{\pi}_{t+1}^{nj,ij}$ using

$$\hat{x}_{t+1}^{nj} = (\hat{L}_{t+1}^{nj})^{\gamma^{nj}\xi^n} (\hat{w}_{t+1}^{nj})^{\gamma^{nj}} \prod_{k=1}^{J} (\hat{P}_{t+1}^{nk})^{\gamma^{nj,nk}},$$

$$\hat{P}_{t+1}^{nj} = \left(\sum_{i=1}^{N} \pi_t'^{nj,ij} \dot{\pi}_{t+1}^{nj,ij} (\hat{x}_{t+1}^{ij} \hat{\kappa}_{t+1}^{nj,ij})^{-\theta^j} (\hat{A}_{t+1}^{ij})^{\theta^j\gamma^{ij}}\right)^{-1/\theta^j},$$

and

$$\pi_{t+1}'^{nj,ij} = \pi_t'^{nj,ij} \pi_{t+1}^{nj,ij} \left(\frac{\hat{x}_{t+1}^{ij}\hat{\kappa}_{t+1}^{nj,ij}}{\hat{P}_{t+1}^{nj}}\right)^{-\theta^j} (\hat{A}_{t+1}^{ij})^{\theta^j\gamma^{ij}}.$$

(c) Use $\pi_{t+1}'^{nj,ij}$, $w_t'^{nk}L_t'^{nk}$, $\dot{w}_{t+1}^{nk}\dot{L}_{t+1}^{nk}$, $\hat{w}_{t+1}^{nj}$, and $\hat{L}_{t+1}^{nj}$ to get $X_{t+1}'^{nj}$ using equation

$$X_{t+1}'^{nj} = \sum_{k=1}^{J} \gamma^{nk,nj} \sum_{i=1}^{N} \pi_{t+1}'^{ik,nk} X_{t+1}'^{ik} + \alpha^j \left(\sum_{k=1}^{J} \hat{w}_{t+1}^{nk}\hat{L}_{t+1}^{nk} w_t'^{nk} L_t'^{nk} \dot{w}_{t+1}^{nk} \dot{L}_{t+1}^{nk} + \iota^n \chi_{t+1}'\right),$$

where $\chi_{t+1}' = \sum_{i=1}^{N}\sum_{k=1}^{J} \frac{\xi^i}{1-\xi^i} \hat{w}_{t+1}^{ik}\hat{L}_{t+1}^{ik} w_t'^{ik} L_t'^{ik} \dot{w}_{t+1}^{ik}\dot{L}_{t+1}^{ik}$.

(d) Check if the labor market is in equilibrium using a slightly modified version of equation (15); namely

$$\hat{w}_{t+1}^{nk}\hat{L}_{t+1}^{nk} = \frac{\gamma^{nj}(1-\xi^n)}{w_t'^{nk}L_t'^{nk}\dot{w}_{t+1}^{nk}\dot{L}_{t+1}^{nk}} \sum_{i=1}^{N} \pi_{t+1}'^{ij,nj}\ X_{t+1}'^{ij}.$$

If it is not go back to step (a) and adjust the initial guess for $\left\{\hat{w}_{t+1}^{nj}\right\}_{n=1,j=0}^{N,J}$ until labor markets clear.

(e) Repeat steps (a) though (d) for each period t and obtain paths for $\left\{\hat{w}_{t+1}^{nj}, \hat{P}_{t+1}^{nj}\right\}_{n=1,j=0,t=0}^{N,J,T}$.

5. For each t, use $\mu_t'^{nj,ik}$, $\hat{w}_{t+1}^{nj}$, $\hat{P}_{t+1}^{nj}$, and $\hat{u}_{t+2}^{nj\ (0)}$ to solve for backwards $\hat{u}_{t+1}^{nj\ (1)}$ using the following equations:

For periods t where $t \geq 2$

$$\hat{u}_t^{nj(1)} = \left(\frac{\hat{w}_t^{nj}}{\hat{P}_t^n}\right)\left(\sum_{i=1}^N \sum_{k=0}^J \mu_{t-1}^{\prime nj,ik} \dot{\mu}_t^{nj,ik} \left(\hat{u}_{t+1}^{ik(0)}\right)^{\beta/\nu}\right)^\nu.$$

For period 1:

$$\hat{u}_1^{nj\ (1)} = \left(\frac{\hat{w}_1^{nj}}{\hat{P}_1^n}\right)\left(\sum_{i=1}^N \sum_{k=0}^J \vartheta_0^{nj,ik\ (0)} \left(\hat{u}_2^{ik}\right)^{\beta/\nu}\right)^\nu.$$

This delivers a new path for $\left\{\hat{u}_{t+1}^{nj\ (1)}\right\}$, where the superscript 1 indicates an updated value for $\hat{u}$.

6. Take the path for $\left\{\hat{u}_{t+1}^{nj\ (1)}\right\}$ as the new set of initial conditions.
7. Check if $\left\{\hat{u}_{t+1}^{nj\ (1)}\right\} \simeq \left\{\hat{u}_{t+1}^{nj\ (0)}\right\}$. If it is not, go back to step 1 and update the initial guess.

APPENDIX E: DATA

E.1. *Data Description*

List of sectors and countries. We calibrate the model to the 50 U.S. states, 37 other countries including a constructed rest of the world, and a total of 22 sectors classified according to the North American Industry Classification System (NAICS) for the year 2000. The list includes 12 manufacturing sectors, 8 service sectors, a combined wholesale and retail trade sector, and the construction sector. Our selection of the number of sectors and countries was guided by the maximum level of disaggregation at which we were able to collect the production and trade data needed to compute our model. The 12 manufacturing sectors are Food, Beverage, and Tobacco Products (NAICS 311–312); Textile, Textile Product Mills, Apparel, Leather, and Allied Products (NAICS 313–316); Wood Products, Paper, Printing, and Related Support Activities (NAICS 321–323); Petroleum and Coal Products (NAICS 324); Chemical (NAICS 325); Plastics and Rubber Products (NAICS 326); Nonmetallic Mineral Products (NAICS 327); Primary Metal and Fabricated Metal Products (NAICS 331–332); Machinery (NAICS 333); Computer and Electronic Products, and Electrical Equipment and Appliance (NAICS 334–335); Transportation Equipment (NAICS 336); Furniture and Related Products, and Miscellaneous Manufacturing (NAICS 337– 339). The 8 service sectors are Transport Services (NAICS 481-488); Information Services (NAICS 511–518); Finance and Insurance (NAICS 521–525); Real Estate (NAICS 531-533); Education (NAICS 61); Health Care (NAICS 621–624); Accommodation and Food Services (NAICS 721–722); Other Services (NAICS 493, 541, 55, 561, 562, 711–713, 811-814). We also include the Wholesale and Retail Trade sectors (NAICS 42-45) and the Construction sector, as mentioned earlier.

The countries in addition to the United States are Australia, Austria, Belgium, Bulgaria, Brazil, Canada, China, Cyprus, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Italy, Ireland, Japan, Lithuania, Mexico, the Netherlands, Poland, Portugal, Romania, Russia, Spain, the Slovak Republic, Slovenia, South Korea, Sweden, Taiwan, Turkey, the United Kingdom, and the rest of the world.

International trade, production, and input shares across countries. International trade flows across sectors and the 38 countries including the United States for the year 2000, $X_0^{nj,ij}$, where n, i are the 38 countries in our sample, are obtained from the World Input-Output Database (WIOD). The WIOD provides world input-output tables from 1995 onward. National input-output tables of 40 major countries and a constructed rest of the world are linked through international trade statistics for 35 sectors. For three countries in the database, Luxembourg, Malta, and Latvia, value added and/or gross output data were missing for some sectors; thus, we decided to aggregate these three countries with the constructed rest of the world, which gives us the 38 countries (37 countries and the United States) we used in the paper. From the world input-output table, we know total purchases made by a given country from any other country, including domestic sales, which gives us the bilateral trade flows.[64]

We construct the share of value added in gross output γ^{nj} and the material input shares $\gamma^{nj,nk}$ across countries and sectors using data on value added, gross output data, and intermediate consumption from the WIOD.

The sectors, indexed by ci for sector i in the WIOD database, were mapped into our 22 sectors as follows: Food Products, Beverage, and Tobacco Products (c3); Textile, Textile Product Mills, Apparel, Leather, and Allied Products (c4–c5); Wood Products, Paper, Printing, and Related Support Activities (c6–c7); Petroleum and Coal Products (c8); Chemical (c9); Plastics and Rubber Products (c10); Nonmetallic Mineral Products (c11); Primary Metal and Fabricated Metal Products (c12); Machinery (c13); Computer and Electronic Products, and Electrical Equipment and Appliances (c14); Transportation Equipment (c15); Furniture and Related Products, and Miscellaneous Manufacturing (c16); Construction (c18); Wholesale and Retail Trade (c19–c21); Transport Services (c23–c26); Information Services (c27); Finance and Insurance (c28); Real Estate (c29–c30); Education (c32); Health Care (c33); Accommodation and Food Services (c22); and Other Services (c34).

Interregional Trade Flows. The sectoral bilateral trade flows across the 50 U.S. states, $X_0^{nj,ij}$ for all $n, i = U.S.\ states$, were constructed by combining information from the WIOD database and the 2002 Commodity Flow Survey (CFS). From the WIOD database we compute the total U.S. domestic sales for the year 2000 for our 22 sectors. We use information from the CFS for the year 2002, which is the closest available year to 2000, to compute the bilateral expenditure shares across U.S. states, as well as the share of each state in total sectoral expenditure. The CFS survey for the year 2002 tracks pairwise trade flows across all 50 U.S. states for 43 commodities classified according to the Standard Classification of Transported Goods (SCTG). These commodities were mapped into our 22 NAICS sectors by using the CFS tables for the year 2007. The 2007 CFS includes data tables that cross-tabulate establishments by their assigned NAICS codes against commodities (SCTG) shipped by establishments within each of the NAICS codes. These tables allow for mapping of NAICS to SCTG and vice versa. Having constructed the bilateral trade flows for the NAICS sectors, we first compute how much of the total U.S. domestic sales in each sector is spent by each state. To do so, we multiply the total U.S. domestic sales in each sector by the expenditure share of each state in each sector. Then we compute how much of the sectoral expenditure of each state is spent on goods from each of the 50 U.S. states. We do so by applying the bilateral trade shares computed with the 2002 CFS to the regional total spending in each sector. The final product is a bilateral trade flows matrix for the 50 U.S. states across sectors, where the bilateral trade shares across U.S. states are the same as those in the 2002 CFS and the total U.S. domestic sales match those from the WIOD for the year 2000.

Regional production data and input shares. We compute the share of value added in gross output γ^{nj} and the material input shares $\gamma^{nj,nk}$ for all $n, i = U.S.\ states$ for each state and sector in the United States for the year 2000, using data on value added, gross output, and intermediate consumption. We obtain data on sectoral and regional value added from the Bureau of Economic Analysis (BEA). Value added for each of the 50 U.S. states and 22 sectors is obtained from the Bureau of Economic Analysis (BEA) by subtracting taxes and subsidies from GDP data. Gross outputs for the U.S. states in the 12 manufacturing sectors are computed from our constructed bilateral trade flows matrix as the sum of domestic sales and total exports.[65] With the value-added data and gross output data for all U.S. states

[64] In a few cases (12 of 30,118 observations), the bilateral trade flows have small negative values due to negative changes in inventories. Most of these observations involve bilateral trade flows between the constructed rest of the world and some other countries, and in two cases, bilateral trade flows of Indonesia. We input zero trade flows for these small negative bilateral trade flows. They represent a negligible portion of total trade.

[65] In a few cases (34 observations), gross output was determined to be a bit smaller than value added (probably due to some small discrepancies between trade and production data—for instance, a few missing trade shipments in the CFS database); in these cases we constrain value added to be equal to

and sectors, we compute the share of value added in gross output γ^{nj}. For the 8 service sectors, the wholesale and retail trade sector, and the construction sector, we have only the aggregate U.S. gross output computed from the WIOD database; thus, we assume that the share of value added in gross output is constant across states and equal to the national share of value added in gross output; that is, $\gamma^{nj} = \gamma^{USj}$ for each non-manufacturing sector j and $n = U.S.\ states$.

While material input shares are available by sector at the country level, they are not disaggregated by state in the WIOD database. We assume therefore that the share of materials in total intermediate consumption varies across sectors but not across regions. Note, however, that the material-input shares in gross output are still sector and region specific, as the share of total material expenditure in gross output varies by sector and region.

Trade between U.S. states and the rest of the world. The bilateral trade flows between each U.S. state and the rest of the countries in our sample were computed as follows. In our paper, local labor markets have different exposure to international trade shocks because there is substantial geographic variation in industry specialization. Labor markets which has a larger concentration of production in a particular industry should react more to international trade shocks that affect that industry. Therefore, our measure for the exposure of local labor markets to international trade combines trade data with local industry employment. Specifically, following ADH, we assume that the share of each state in total U.S. trade with any country in the world in each sector is determined by the regional share of total employment in that industry. The employment shares used to compute the bilateral trade shares between the U.S. states and the rest of the countries are constructed using employment data across sectors and states from the BEA.[66] Using this procedure, we obtain $X_t^{nj,ij}$ for all $n = U.S.\ states$, $i \neq U.S.\ states$ and $n \neq U.S.\ states$, $i = U.S.\ states$.

Bilateral trade shares. Having obtained the bilateral trade flows $X_t^{nj,ij}$ for all n, i, we construct the bilateral trade shares $\pi_t^{nj,ij}$ as $\pi_t^{nj,ij} = X_t^{nj,ij} / \sum_{m=1}^{N} X_t^{nj,mj}$.

Share of final goods expenditure. The share of income spent on goods from different sectors is calculated as follows:

$$\alpha^j = \frac{\sum_{n=1}^{N} X^{nj} - \sum_{n=1}^{N} \sum_{k=1}^{J} \gamma^{nk,nj} \sum_{i=1}^{N} \pi^{ik,nk} X^{ik}}{\sum_{n=1}^{N} \sum_{k=1}^{J} w^{nk} L^{nk} + \sum_{n=1}^{N} \iota^n \chi},$$

where $\sum_{n=1}^{N} X^{nj}$ is the total spending on sector j goods across all countries and regions, $\sum_{n=1}^{N} \sum_{k=1}^{J} \gamma^{nk,nj} \sum_{i=1}^{N} \pi^{ik,nk} X^{ik}$ denotes total spending in intermediate goods across all countries and regions, and $\sum_{n=1}^{N} \sum_{k=1}^{J} w^{nk} L^{nk} + \sum_{n=1}^{N} \iota^n \chi$ is the total world income.

Share of labor compensation in value added. Disaggregated data on labor compensation are generally very incomplete. Therefore, we compute the share of labor compensation in value added, $1 - \xi^n$, at the national level and assume that it is constant across sectors. For the United States, data on labor compensation and value added for each state for the year 2000 are obtained from the BEA. For the rest of the countries, data are obtained from the OECD input-output table for 2000 or the closest year. For India, Cyprus, and the constructed rest of the world, labor compensation data were not available. In these cases, we input the median share across all countries from the other 34 countries that are part of the rest of the world.

Local shares from global portfolio. To calibrate ι^n, we proceed as follows. Denote by D^n the imbalance of location (region/country) n. Data on D_n comes directly from bilateral trade data for the year 2000.

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gross output.

[66] In 22 cases, data are missing, and in these cases we search for employment data in the closest available year. Still, in three cases (Alaska in the plastics and rubber industry, and North Dakota and Vermont in the petroleum and coal industries) we could not find employment data. Thus, we input zero employment. The 19 cases in which we find employment data in years different from 2000 represent in total less than 0.01% of U.S. employment in 2000.

Using data on value added by sector and location, VA^{nk}, and labor compensation shares, $1 - \xi^n$, we solve for the local shares from the global portfolio as follows:

$$\iota^n = \frac{\sum_{k=1}^{J} \xi^n VA^{nk} - D^n}{\sum_{n=1}^{N} \sum_{k=1}^{J} \xi^n VA^{nk}}.$$

Note that trivially, $\sum_{n=1}^{N} \iota^n = 1$, since $\sum_{n=1}^{N} D^n = 0$.

The initial labor mobility matrix and the initial distribution of labor. To determine the initial distribution of workers in the year 2000 by U.S. states and sectors (and non-employment), we use the 5% Public Use Microdata Sample (PUMS) of the decennial U.S. Census for the year 2000. As we mentioned before, industries are classified according to the NAICS, which we aggregate to our 22 sectors and non-employment. We restrict the sample to people between 25 and 65 years of age who are either non-employed or employed in one of the sectors included in the analysis. Our sample contains almost 7 million observations.

We combine information from the PUMS of the American Community Survey (ACS) and the Current Population Survey (CPS) to construct the initial matrix of quarterly mobility across our states and sectors (μ_{-1}).[67] Our goal is to construct a transition matrix describing how individuals move between state-sector pairs from one quarter to the next (from t to $t+1$). The ACS has partial information on this; in particular, the ACS asks people about their current state and industry (or non-employment) and the state in which they lived during the previous year. We use the year 2001 since this is the first year for which data on interstate mobility at a yearly frequency are available.[68] After selecting the sample as we did before in terms of age range and the industries in our analysis, we have around 600,000 observations. We find that around 2% of the U.S. population moves across states in a year in this time period. Unfortunately, the ACS does not have information on workers' past employment status or the industries in which people worked during the previous period, so we resort to other data for this information.

We use the PUMS from the monthly CPS to obtain information on past industry of employment (or non-employment) at the quarterly frequency. The main advantage of the CPS is that it is the source of official labor market statistics and has a relatively large sample size at a monthly frequency. In the CPS, individuals living in the same address can be followed month to month for a small number of periods.[69] We match individuals with their survey conducted three months apart and compute their employment or non-employment status and work industry, accounting for any change between interviews as a transition.[70] The main limitation with the CPS is that individuals who move to a different residence, which of course includes interstate moves, cannot be matched to previous surveys. Our three-month match rate is close to 90%.[71] As the monthly CPS does not have information on interstate moves, we use this matched information to compute the industry and non-employment transitions within each state—that is, a set of 50 transition matrices, each with 23×23 cells. After restricting the sample as discussed earlier, in any given month we have around 12,800 observations for the entire United States. To more precisely estimate the transitions, we use all months from October 1998 to September 2001, leading to a sample of over 475,000 matched records. Since for this time period the CPS uses the Standard Industry Classification (SIC), we translate this classification into NAICS, using the crosswalk in Table IV.

[67] The ACS interviews provide a representative sample of the U.S. population for every year since 2000. For the year 2001, the sample consists of 0.5% of the U.S. population. The survey is mandatory and is a complement to the decennial Census.

[68] The 2000 Census asked people about the state in which they lived five years before but not the previous year; thus, we do not use the Census data despite the much larger sample.

[69] In particular, the CPS collects information on all individuals at the same address for four consecutive months, stops for eight months, and then surveys them again for another four months.

[70] We observe individuals three months apart using, on the one hand, their first and fourth interviews, and on the other, their fifth and eighth interviews.

[71] Mortality, residence change, and nonresponse rates are the main drivers of the 10% mismatch rate.

Table III summarizes the information used to construct a quarterly transition matrix across state, industry, and non-employment status. The letter x in the table denotes information available in the matched CPS, and the letter y denotes information available in the ACS. The information missing from the above discussion is the past industry history of interstate movers. To have a full transition matrix, we assume that workers who move across states and are in the second period in state i and sector j have a past industry history similar to workers who did not switch states and are in the second period in state i and sector j.[72]

TABLE III

Information Available on ACS and CPS

		State A				**State B**			
		Ind 1	Ind 2	...	Ind J	Ind 1	Ind 2	...	Ind J
State A	Ind 1	x	x	...	x				
	Ind 2	x	x	...	x				
	...	...	...	...	...				
	Ind J	x	x		x				
	Total	**y**	**y**	**...**	**y**	**y**	**y**	**...**	**y**
State B	Ind 1					x	x	...	x
	Ind 2					x	x	...	x
	...					...	...	...	...
	Ind J					x	x		x
	Total	**y**	**y**	**...**	**y**	**y**	**y**	**...**	**y**

As mentioned earlier, information on interstate mobility in the ACS is for moves over the year. To calculate quarterly mobility, we assume that interstate moves are evenly distributed over the year and we rule out more than one interstate move per year. In this case, our adjustment consists of keeping only one-fourth of the interstate moves and imputing three-fourths as non-moves. After this correction, we impute the past industry history for people with interstate moves from state i to state n and industry j according to the intrastate sectoral transition matrix for state n conditional on industry j.

Our computed value for the initial labor transition matrix is consistent with aggregate magnitudes of interstate and industry mobility for the yearly frequency estimated in Molloy et al. (2011) and Kambourov and Manovskii (2008). We obtain a mobility transition matrix with over 1.3 million elements.[73]

E.2. *Constructing the Actual Baseline Economy*

In this section of the appendix, we describe the data sources and assumptions used to construct the time-series data needed to compute the dynamic counterfactuals with time-varying fundamentals.

Trade, production, and input shares across countries. International trade flows across sectors and the 38 countries in our sample over the period 2000-2007 are obtained from the WIOD database.[74] To construct the sectoral bilateral trade flows across the 50 U.S. states over 2000-2007, we proceed as

[72] Mechanically, we distribute the interstate movers according to the intersectoral mobility matrix for the state in which they currently live.

[73] With the exception of one element, all zero transitions occur out of the diagonal. In fact, the diagonal of the matrix typically accumulates the largest probability transition values, which just reflects the fact that staying in one's current labor market is a high probability event. However, we do find that one of the estimated transition probabilities in the diagonal is zero. Only in this case we replace this value with the minimum value of the other elements in the diagonal and re-normalize such that the conditional transition probabilities add to one.

[74] Gross output data for Cyprus was not available for 2007 in the petroleum industry; thus we input its value for the year 2004, which is the closest year with available data.

TABLE IV

CONCORDANCE SIC87DD - NAICS

NAICS	NAICS Sector Description	SIC87dd Codes
1	Food, Beverage, and Tobacco Products	20**, 21**
2	Textiles and Apparel Products	22**, 23**, 31**
3	Wood, Paper, Printing and Related Products	24** exc. 241*, 26**, 274*-279*
4	Petroleum and Coal Products	29**
5	Chemicals	28**
6	Plastics and Rubber Products	30**
7	Nonmetallic Mineral Products	32**
8	Primary and Fabricated Metal Products	33**, 34**
9	Machinery	351*-356*, 3578-3599
10	Computer, Electrical, and Appliance	3571-3577, 365*-366*, 3812-3826, 3829, 386*-387*, 361*-364*, 367*-369*
11	Transportation Equipment	37**
12	Furniture and Miscellaneous Products	25**, 3827, 384*-385*, 39**
16	Information and Communication	271*-273*

Note: An entire broad group was mapped into the NAICS sector by substituting the last one or two digits with an asterisk. All intervals listed in the table are inclusive.

follows. The CFS releases sectoral bilateral trade data for the U.S. states every five years, therefore, we use the 2002 and 2007 releases to construct the bilateral trade flows for those years. We then interpolate the years 2003 through 2006 using linear growth. As explained above, and because of the lack of bilateral trade data in the CFS before 2002, we assume that the sectoral bilateral trade shares across U.S. states in 2000 are the same as in 2002 and, therefore, that the bilateral trade shares in 2001 are the same as in 2002. Finally, and as we did for the year 2000, to match the bilateral expenditures across states from the CFS with the aggregate U.S. domestic sales from the WIOD, we multiply the total U.S. sectoral domestic sales from the WIOD for every year over 2000-2007 by the expenditure share of each state in each sector. Then we compute how much of the sectoral expenditure of each state is spent on goods from each of the 50 U.S. states using the bilateral trade shares constructed for each year as explained above. The time series of the bilateral trade flows between each U.S. state and the rest of the countries in our sample were computed in the same way as we proceed for the year 2000. The employment shares used to compute U.S. states' exposure to international trade in each industry are constructed using employment data across sectors and states from the BEA for each year over the period 2000-2007.

Migration flows and employment. Migration flows for each quarter over the period 2000-2007 were constructed using the same procedure as the initial labor mobility matrix described before. With the time series of migration flows and the initial distribution of employment for the year 2000, we are able to recover the distribution of employment across U.S. labor markets for 2000-2007.

E.3. *Longitudinal Employer-Household Dynamics migration flow data*

As described in this appendix, we use multiple periods to construct some of our labor market flows data. We combine three years of monthly matched CPS records to obtain information on sectoral mobility patterns and flows in and out of non-employment. Our records are matched three months apart (one quarter). In any given month for the years 1998-2000, we have around 12,800 matched records, and when we pool three years of data, we have 475,440 individuals in the sample. Despite the relatively large sample size, measurement error and empty cells could still be a source of concern. To gain information on how our constructed transitions and labor market flows compare to the data, we construct a matrix of interstate and intersectoral transitions using data from the Census Bureau's Longitudinal Employer-Household Dynamics (LEHD), in particular, the Job-to-Job Flows data (J2J). The data we use can be obtained at http://lehd.ces.census.gov/data/j2j beta.html. As described by the Census Bureau, the Job-to-Job Flows data is a beta release of new national statistics on quarterly job mobility in the United States. The data include statistics on: (1) the job-to-job transition rate, (2) hires and separations to and from employment, and (3) characteristics of origin and destination jobs for job-to-job transitions. These statistics are available nationally and at the state level and contain origin and destination state as well as origin and destination industry. This J2J data is readily available to the public with no restrictions. The main advantage of the LEHD data is that it combines administrative data from state Unemployment Insurance programs, the Quarterly Census of Employment and Wages, and additional administrative data and data from censuses and surveys. As such, sample size is probably not an issue. However, these data present some limitations. (1) In the early 2000s, a large number of states are not included in the data. States have joined the LEHD program gradually over time, but even today data for Massachusetts is unavailable. (2) Manufactures are aggregated as a single sector and because access to the micro-data is restricted, individual industries cannot be identified. (3) There is very limited information on origin-destination for flows involving non-employment.

Due to these limitations, we prefer to use our own constructed flows. However, we use the J2J data to gauge how our transitions compare to those in the J2J. For this, we aggregate our manufactures as a single sector and do not compare transitions involving non-employment. Moreover, we compare only the flows for the groups of states that are available in the J2J data in the year 2000, since this is the year for which we construct our flows.[75]

We find that the migration flows constructed using data from the ACS and CPS are highly correlated with the transition probabilities from the LEHD J2J data. The overall correlation is 0.99, and the correlations across locations and across industries are both also 0.99. If we take out the non-movers, the correlations are still quite high; the overall correlation is 0.7, the correlation across locations is 0.81 and the correlation across industries is 0.96. Therefore, our computed mobility rates are very close to those in the LEHD J2J dataset. Finally, we want to highlight that we conducted robustness checks in which we add a very small number to any of our zero probability transitions. We find that our results remain largely unaltered. The reason is that these type of transitions typically involve a small labor market either as origin or destination (or both). Thus, quantitatively, as we aggregate results at the level of sectors or states, whether transitions are exactly zero or approximately zero do not seem to affect the results much.

E.4. *Comparing Migration Flows: Data Versus Model*

We evaluate if the *i.i.d.* assumption on preference shocks delivers too much mobility relative to the data. To do so, we simulatedata from our model and compare the outcomes with the data. In particular, we simulated from our model a panel of one million individuals over 120 quarters and kept track of their labor market history. The initial distribution of workers matches that of the year 2000 and the simulation is performed under our baseline economy (without the China shock).

TABLE V

ACTUAL AND SIMULATED MOBILITY RATES PERCENT

	Data	Model
Quarterly sector switching rate	6.1	5.4
Yearly state mobility rate	2.3	2.4

[75]We use four quarters of data in the J2J dataset, from 2000Q2 to 2001Q1.

Note: Model values are computed with simulated individual histories over 120 periods. Data on yearly state mobility rate computed using the ACS, 2001-2007. Data on quarterly sector mobility rate computed using matched CPS, 2000-2007. Sector mobility excludes non-employment

Table V shows the probability of a worker switching from one of the 22 sectors to another from one quarter to the next and the probability of the worker moving to a different state from one year to the next. The simulations are largely consistent with the data. Thus, while workers receive a shock every period, only a small fraction decide to move. The numbers reported in Table V align well with mobility rates computed in other studies in the literature, like Molloy et al. (2011) and Kaplan and Schulhofer-Wohl (2012) for interstate mobility and Kambourov and Manovskii (2008) for intersectoral mobility.

E.5. *Additional Data used for the Model with SSDI*

In this appendix, we provide further information on the calibration of the model with SSDI. As discussed in the main text, to compute the model with SSDI, we need data on the number of workers with SSDI over our sample period. We obtain this information from the Annual Statistical Report on the Social Security Disability Insurance Program (https://www.ssa.gov/policy/docs /statcomps / di_asr/) for each year from 2000 to 2007. The report presents tables with the number of disabled workers per year and by demographic group. We compute for each year the number of disabled workers between 25 to 65 years of age, which is the demographic in our sample. The annual reports for the years 2000 to 2002 do not contain information on the number of SSDI recipients older than 65. In principle, workers at full retirement age (FRA) are expected to receive benefits from other sources and not from SSDI. Yet, the number of SSDI workers older than 65 receiving benefits from 2003 to 2007 is not zero. For instance, in the year 2003, the number of SSDI recipients older than 65 is 0.65% of the total number of disabled workers. We apply this share to the years 2000 to 2002. We use this data to construct a time series of the number of workers with SSDI. To determine the share of workers with SSDI over non-employed workers, we take the share relative to the number of non-employed workers between 25 and 65 years of age computed from the ACS. With these figures we have a time series of L_t^{nD}; L_{2000}^{nD} is the one we use in our initial period.

To calculate the average benefit, b^{DI}, we proceed as follows. From the Annual Statistical Report on the Social Security Disability Insurance Program, the average monthly benefits for the years 2000 to 2007 vary from 824 to 1072 U.S. dollars. We calibrate our benefit to the average of this period, which corresponds to an average quarterly benefit of $b^{DI} = 2843$ U.S. dollars. We set the tax rate at 0.9%; that is, $\tau = 0.009$. This tax rate is obtained from the report of "Trends in the Social Security and Supplemental Security Income Disability Programs" elaborated by the Social Security Administration. This report can be found at https://www.ssa.gov/policy/ docs / chartbooks / disability_trends/.

In order to calculate ρ^{DI} we need an estimate of the exit rate from the SSDI. We use the estimates in Table 2 in Raut (2017). That study uses administrative data from the Social Security Administration and finds that the sum of recovery and death probability for workers in disability in our demographic group is between 29% and 34% cumulative over the last 9 years in the program. Given this, we use a quarterly $\rho^{DI} = 0.991$.

In order to obtain δ we proceed as follows. The equilibrium condition in the model implies that $L_{t+1}^D = \rho^{DI} L_t^D + \delta L_t^0$. We define the function $z(\delta)_t = \rho^{DI} L_t^D + \delta L_t^0 - L_{t+1}^D$ and then use time-series data for L_t^{nD} and L_t^{n0} for the years 2000 to 2007 and solve for δ such that it minimizes the sum square of the errors of the equation; namely $\delta = \arg\min \sum_{t=2000}^{2007} (z(\delta)_t)^2$. We obtain a value of $\delta = 0.003$.

APPENDIX F: ESTIMATION

F.1. *Predicting Import Changes from China*

To identify the China shock, we use international trade data for 2000 and 2007 obtained from the WIOD database as described in Section 4 and Appendix E. For our purposes, we use the data series that measure imports from China by the United States and from China by other advanced economies as in ADH. Using this data, we compute the changes in the level of imports from China between 2000 and 2007 by the United States and the other advanced economies. The change in U.S. imports from China during this period can, in part, be the result of domestic U.S. shocks, but we are looking for a measure of changes in imports that are mostly the result of shocks that originate in China. Inspired by ADH's instrumental-variable strategy, we run the following regression:

$$\Delta M_{USA,j} = a_1 + a_2 \Delta M_{other,j} + u_j,$$

where j is 1 of our 12 manufacturing sectors and $\Delta M_{USA,j}$ and $\Delta M_{other,j}$ are the changes in real U.S. imports from China and imports by the other advanced economies from China between 2000 and 2007, respectively.

The coefficient of the regression is estimated to be $a_2 = 1.386$ with a robust standard error of 0.033. We want to emphasize that our motivation for the choice of our sample of countries is to closely follow ADH, where the authors include 8 high-income countries (excluding the United States) to construct their instrument, Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland, to estimate the above regression. Figure 15 shows the actual and predicted change in U.S. manufacturing imports from China constructed with this set of countries.

This regression is related to the first-stage regression in ADH's two-stage least-squares estimation. Using this result, we construct the changes in U.S. imports from China for each industry that are predicted by the change in imports from China for other advanced economies. We then use the predicted changes in U.S. imports according to this regression to calibrate the size of the TFP changes for each of the manufacturing sectors in China that matches the change in imports in the model to the predicted change from the first-stage regression in ADH. Concretely, we solve for the change in China's TFPs in our 12 manufacturing industries, $\{\hat{A}_t^{China,j}\}_{j=1,t=2000}^{12,2007}$, such that the model-predicted imports match the predicted imports from China from 2000 to 2007 given by $a_2 \Delta M_{other,j}$. To do so, we use our dynamic model with time-varying fundamentals, that is, the model that has a baseline economy that contains information on the actual evolution of fundamentals and a counterfactual economy where agents expected all fundamentals to evolve as in the data and instead estimated Chinese TFP did not change. Since the change in U.S. imports from China is evenly distributed over this period, we interpolate an initial guess of TFP changes over 2000-2007 across all quarters and feed this sequence of TFP shocks into our dynamic model. We iterate over these changes in TFP and solve for the TFP changes that minimize a weighted-sum of squares of the difference between the change of the ADH-predicted U.S. imports from China over 2000-2007 and the ones from the dynamic model using a non-linear solver. Our estimated changes in TFPs explain more than 90 percent of the observed predicted U.S. exposure to import competition from China, and the correlation between the predicted changes in import competition in the model and the predicted changes in the data is 0.96.

Figure 15 also shows the implied sectoral productivity changes in China. In Figure 15, measured TFP is defined as $(A_t^{nj})^{\gamma^{nj}}/(\pi_t^{nj,nj})^{1/\theta^j}$, see Caliendo et al. (2018) for details. Our model estimates that TFP increased in all manufacturing industries in China. While our estimated changes in Chinese TFP are correlated with the changes in U.S. imports from China by sector, this correlation is not perfect.

F.2. *Reduced-Form Analysis*

In this appendix, we reproduce the reduced-form evidence on the differential employment effects across labor markets of increased import competition from China found in ADH, but under our definition of labor market, under our sample selection criteria and using our trade data constructed from the WIOD database.[76]

[76]That is, we use U.S. states instead of commuting zones, and we use 12 manufacturing sectors classified by NAICS instead of the 397 SIC manufacturing industries that ADH use. Moreover, we restrict the sample to people within ages 25 to 65 that are in the labor force, while ADH use people

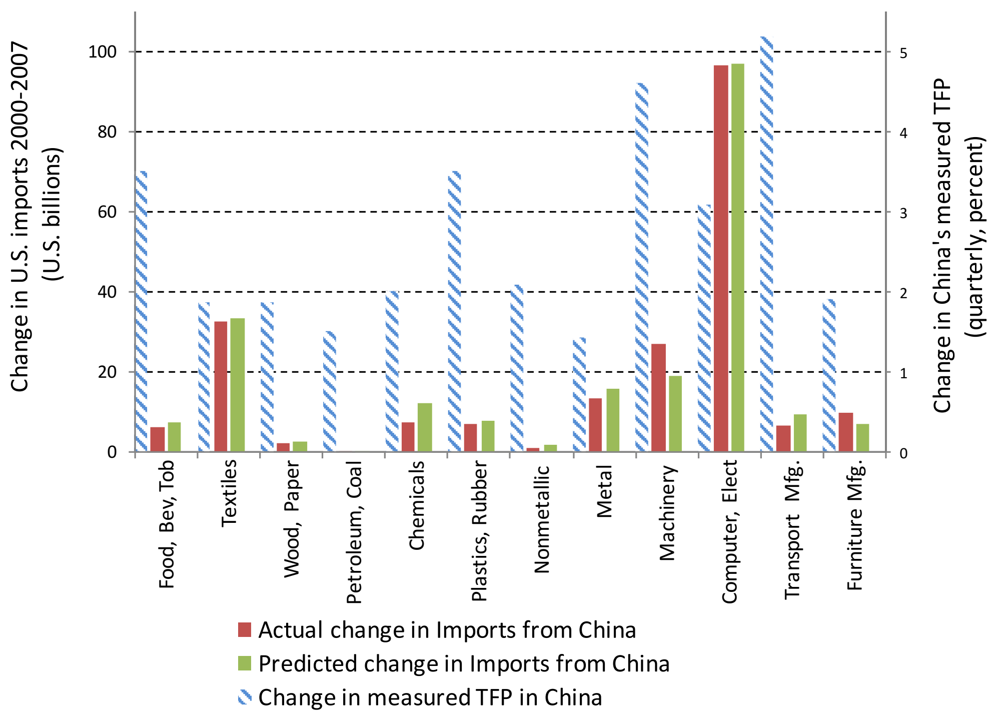


Figure 15: Actual and predicted changes in imports vs. China's TFP changes (2000-2007)

Note: The figure presents the actual changes in U.S. manufacturing imports from China, the predicted changes in manufacturing imports from using the ADH specification, and the CDP estimated changes in China's measured TFP by sector for the period 2000-2007.

To construct the second-stage regression, we follow the same methodology as ADH to impute the U.S. total imports to state-industry units, except where ADH use commuting zones and SIC codes, we use states and our 12 manufacturing sectors. Total U.S. manufacturing imports are allocated to states by weighting total imports according to the number of employees in a certain local industry relative to the total national employment. To do so, we use the employment data described in Appendix E. Once we have employment data for the 12 industries and the U.S. states, we allocate the national import data to the state level using the following formula proposed by ADH (see their equation 3):

$$\Delta IPW_{uit} = \sum_j \frac{L_{ijt}}{L_{ujt}} \frac{\Delta M_{ucjt}}{L_{it}}.$$

The expression above states that the change in U.S. imports per worker from China is defined based on each state's industry employment structure in the starting year. Following ADH's notation, L_{it} is the total employment in state i at time t, j is 1 of our 12 manufacturing sectors, and u stands for a U.S.-related variable (as opposed to a variable constructed using other countries imports, for which they use an o). For example, ΔM_{ucjt} is the change in U.S. imports from China for industry j at time t.[77]

We also followed ADH in constructing our dependent variable, the change in local manufacturing employment as a share of the working age population, but used our data (see description in Appendix E). With our variables, we run a regression relating the change in local manufacturing employment from 2000 to 2007 (ΔL^m_{it}) to the change in imports per worker (ΔIPW_{uit}):

$$\Delta L^m_{it} = b_1 + b_2\, \Delta IPW_{uit} + e_{it}.$$

In this regression the unit of observation is a U.S. state. As in ADH, we perform a two-stage least-squares regression instrumenting ΔIPW_{uit} with ΔIPW_{oit}, which is other advanced economies' change in imports from China per worker.[78]

Our estimate of b_2 is -0.67 with a robust standard error of 0.14. Our reduced-form results using our data are largely aligned with ADH, both in terms of the sign and significance. The differences between our's and ADH's point estimates stem from the use of different time periods (we use only changes between 2000 to 2007, while in several of ADH's specifications they use 1990 to 2007), additional controls in the regressions, different definitions of geographic areas and industries (we use U.S. states and NAICS sectors), and, sample selection criteria (population ages and labor force). Overall, our estimate of $b_2 = -0.67$ compares with their estimates in column 2 of their Table 2, which under their definitions of commuting zones and SIC industries delivers $b_2 = -0.72$ with their codes and data.

APPENDIX G: IDENTIFICATION OF THE TRAJECTORY OF VALUES AND LEVELS OF MOBILITY COSTS

Proposition 2 states that, given an initial allocation, and an anticipated sequence of changes in fundamentals, we can solve for the sequential equilibrium without knowing the level of fundamentals. In this appendix, we discuss whether the initial flows of workers across labor markets, as captured by the mobility matrix μ, is sufficient to identify the role of mobility frictions *vis a vis* the role of initial differences in expected utility across labor markets.

This problem becomes evident by looking at the following equation, which is just a rearrangement of

within ages 16 to 64 that worked the previous year.

[77]In ADH this equation varied over commuting zones (i) and SIC industries (j).

[78]Note that, as in ADH, the formula for ΔIPW_{oit} contains the imports from other advanced economies, but the employment of the different U.S. states and sectors. We calibrated our model with data on other countries from the WIOD. Unfortunately, the WIOD does not contain data from New Zealand and Switzerland. Therefore, our definition of other advanced economies uses data from Australia, Denmark, Finland, Germany, Japan, and Spain.

equations (2) and (3) in the paper:

$$V_t^{nj} = \log\left(\frac{w_t^{nj}}{P_t^n}\right) - \nu \log\left(\mu_t^{nj,ik}\right) + \beta V_{t+1}^{ik} - \tau^{nj,ik} \qquad \forall i,k.$$

In particular, we have

$$\log\left(\frac{\mu_t^{nj,ik}}{\mu_t^{nj,nj}}\right) = \frac{\beta}{\nu}\left(V_{t+1}^{ik} - V_{t+1}^{nj}\right) - \frac{\tau^{nj,ik}}{\nu}. \tag{61}$$

One concern is that there could be a different economy, for example, with a lower level of mobility costs and different expected utilities, for which the mobility matrix is the same. That is,

$$\log\left(\frac{\mu_t^{nj,ik}}{\mu_t^{nj,nj}}\right) = \frac{\beta}{\nu}\left(\check{V}_{t+1}^{ik} - \check{V}_{t+1}^{nj}\right) - \frac{\check{\tau}^{nj,ik}}{\nu}. \tag{62}$$

Note, however, that the previous expressions uses information only on flows going in one direction. Additional information is obtained by the flows of workers moving out of labor market ik and into nj. We can manipulate equation (61) to get,

$$\log\left(\frac{\mu_t^{nj,ik}}{\mu_t^{nj,nj}}\right) + \log\left(\frac{\mu_t^{ik,nj}}{\mu_t^{ik,ik}}\right) = -\frac{\tau^{nj,ik} + \tau^{ik,nj}}{\nu}.$$

This expression shows that differences in the intensity of the flows out of one labor market and into another must be explained by differences in the mobility costs only. In addition, it is clear that both alternative economies have the same level of mobility frictions up to some constant; that is, for example, $\tau^{nj,ik} = \check{\tau}^{nj,ik} + c^{nj,ik}$ and $\tau^{ik,nj} = \check{\tau}^{ik,nj} - c^{nj,ik}$. Then, using (61) and (62),

$$\begin{aligned}
\frac{\beta}{\nu}\left(V_{t+1}^{ik} - V_{t+1}^{nj}\right) - \frac{\tau^{nj,ik}}{\nu} &= \frac{\beta}{\nu}\left(\check{V}_{t+1}^{ik} - \check{V}_{t+1}^{nj}\right) - \frac{\check{\tau}^{nj,ik}}{\nu}, \\
\frac{\beta}{\nu}\left(V_{t+1}^{ik} - V_{t+1}^{nj}\right) &= \frac{\beta}{\nu}\left(\check{V}_{t+1}^{ik} - \check{V}_{t+1}^{nj}\right) + \frac{c^{nj,ik}}{\nu}. \qquad (63)
\end{aligned}$$

From this example, it is clear that it will be challenging to identify separately $\tau^{nj,ik}$ from $c^{nj,ik}$. Note however that, since this level is constant, the dynamic evolution of the *changes* in the variables in this two economies will be identical, which is a main message of our paper and highlights the usefulness of our dynamic hat algebra. This can be clearly seen in the following expressions, which are just intermediate steps in the proof of Proposition 2.

$$\check{\mu}_0^{nj,ik} = \frac{\exp\left(\beta\check{V}_1^{ik} - \check{\tau}^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta\check{V}_1^{mh} - \check{\tau}^{nj,mh}\right)^{1/\nu}}.$$

Taking the relative time differences (between $t=-1$ and $t=0$) of this equation, we get

$$\begin{aligned}
\frac{\check{\mu}_0^{nj,ik}}{\mu_{-1}^{nj,ik}} &= \frac{\frac{\exp\left(\beta\check{V}_1^{ik} - \check{\tau}^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta V_1^{mh} - \check{\tau}^{nj,mh}\right)^{1/\nu}}}{\frac{\exp\left(\beta\check{V}_0^{ik} - \check{\tau}^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta\check{V}_0^{mh} - \check{\tau}^{nj,mh}\right)^{1/\nu}}} \\
&= \frac{\frac{\exp\left(\beta\check{V}_1^{ik} - \tau^{nj,ik} + c^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta V_1^{mh} - \tau^{nj,mh} + c^{nj,mh}\right)^{1/\nu}}}{\frac{\exp\left(\beta\check{V}_0^{ik} - \tau^{nj,ik} + c^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\exp\left(\beta\check{V}_0^{mh} - \tau^{nj,mh} + c^{nj,ik}\right)^{1/\nu}}},
\end{aligned}$$

where $\mu_{-1}^{nj,ik}$ is common to both economies since is taken directly from the data (observed flows).

Given that mobility costs and c do not change over time, this can be expressed as

$$\begin{aligned}
\frac{\breve{\mu}_0^{nj,ik}}{\mu_{-1}^{nj,ik}} &= \frac{\exp\left(\breve{V}_1^{ik} - \breve{V}_0^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_{-1}^{nj,mh}\exp\left(\breve{V}_1^{mh} - \breve{V}_0^{mh}\right)^{\beta/\nu}} \\
&= \frac{\exp\left(V_1^{ik} - V_0^{ik}\right)^{\beta/\nu}}{\sum_{m=1}^{N}\sum_{h=0}^{J}\mu_{-1}^{nj,mh}\exp\left(V_1^{mh} - V_0^{mh}\right)^{\beta/\nu}} \\
&= \frac{\mu_0^{nj,ik}}{\mu_{-1}^{nj,ik}},
\end{aligned}$$

where the second-to-last equality comes from manipulating equation (63), taking changes over time, and noting that for both economies changes will be zero once they reach their respective steady state. We can proceed in the same way with the other model variables to show that across these two potentially different economies, the dynamic evolution in changes will be identical even if we cannot identify separately the *level* of mobility frictions or initial lifetime utility.

APPENDIX H: ADDITIONAL RESULTS

H.1. *Sectoral Employment Effects*

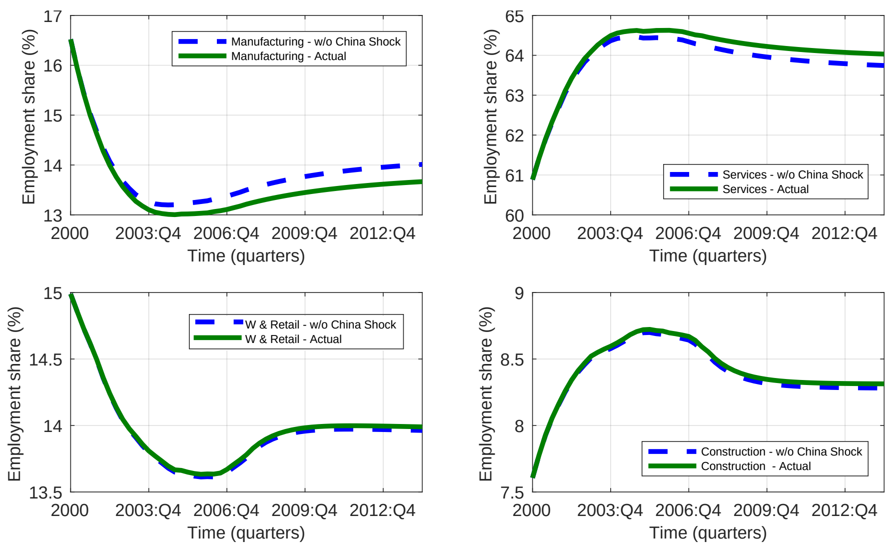


Figure 16: The Evolution of Employment Shares

Note: The figure presents the evolution of employment in each sector (manufacturing, services, wholesale and retail, and construction) over total employment. Total employment excludes farming, utilities, and the public sector. The dashed lines represent the shares from the economy without the China shock and all other fundamentals changing, while the lines represent the shares from the baseline economy with all fundamentals changing, denoted by “Actual”.

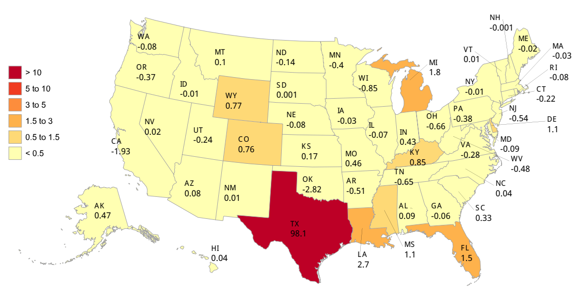
H.2. *Regional Employment Effects*

In this appendix, we present the U.S. states' contributions to the change in the employment share in different industries The key finding in these figures is the large spatial heterogeneity in the employment effects from the China shock across different industries.

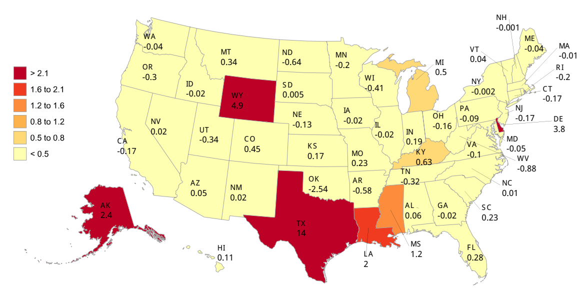
1. Contribution to industry employment decline in the U.S. (%)

2. Normalized by regional employment share

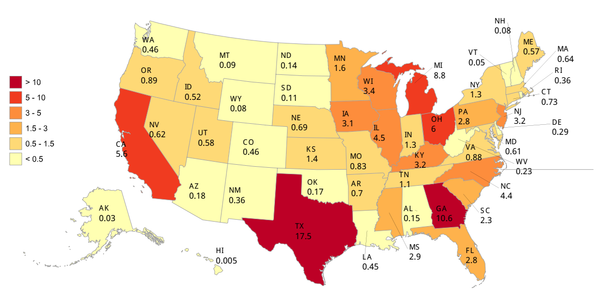
a.1: Petroleum, Coal



a.2: Petroleum, Coal



b.1: Wood, paper



b.2: Wood, paper

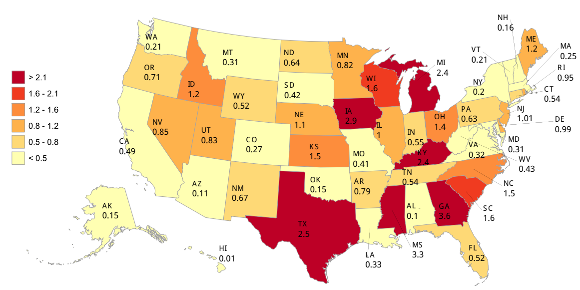


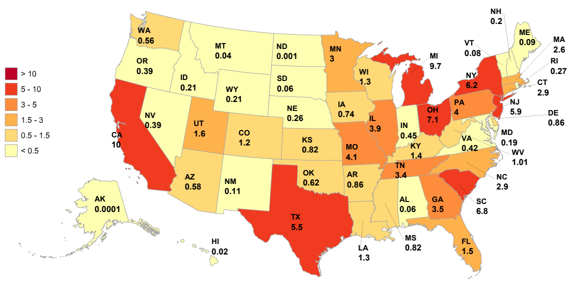
Figure 17: Regional employment declines in manufacturing industries

Note: This figure presents the reduction in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate reduction in industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate reduction the industry employment normalized by the employment size of each state relative to U.S. aggregate employment. Panels a present the results for the petroleum and coal industry. Panels b present the results for the wood and paper industry.

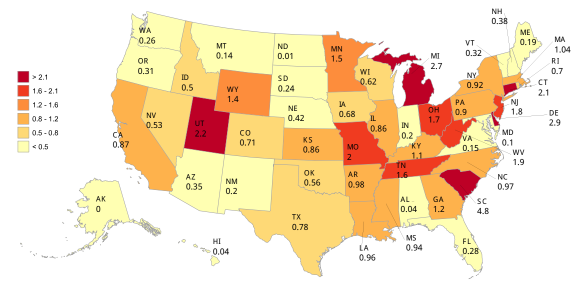
1. Contribution to industry employment decline in the U.S. (%)

2. Normalized by regional employment share

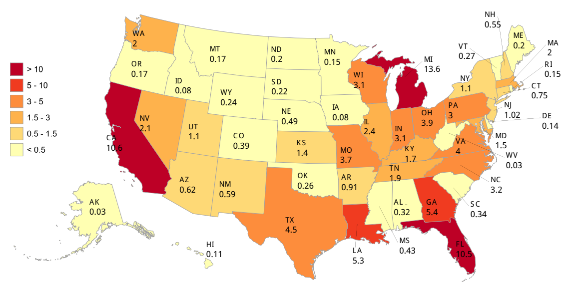
a.1: Chemicals



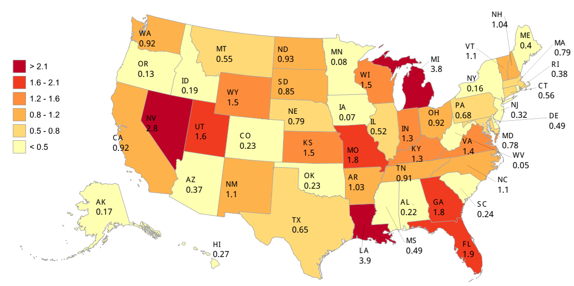
a.2: Chemicals



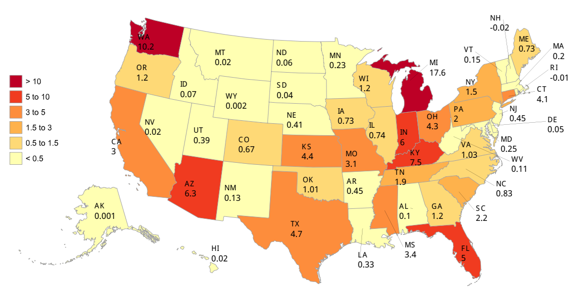
b.1: Nonmetallic



b.2: Nonmetallic



c.1: Transport Mfg.



c.2: Transport Mfg.

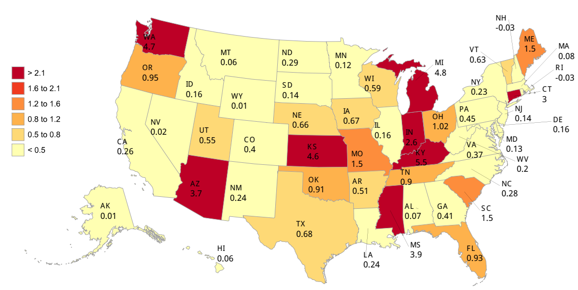


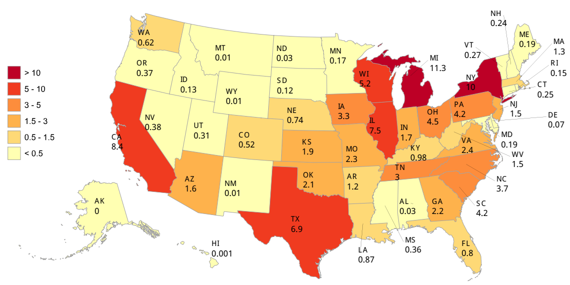
Figure 18: Regional employment declines in manufacturing industries

Note: This figure presents the reduction in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate reduction in industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate reduction in industry employment normalized by the employment size of each state relative to U.S. aggregate employment. Panels a present the results for the chemicals industry. Panels b present the results for the nonmetallic industry. Panels c present the results for the transport manufacturing industry.

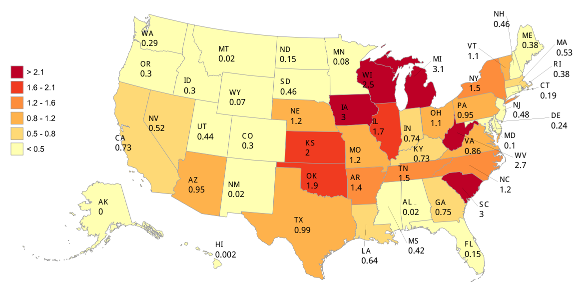
1. Contribution to industry employment decline in the U.S. (%)

2. Normalized by regional employment share

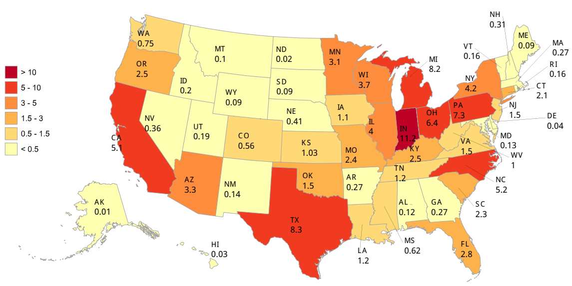
a.1: Plastics, Rubber



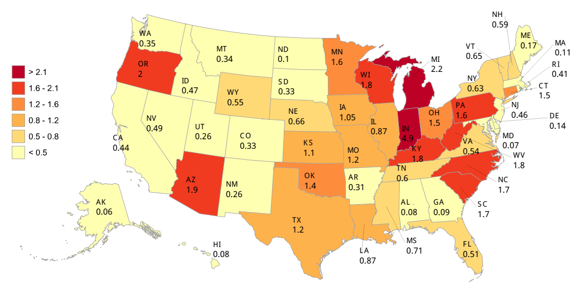
a.2: Plastics, Rubber



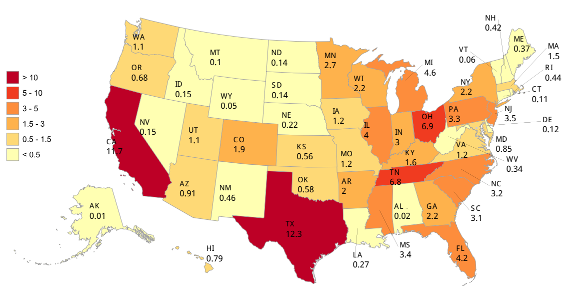
b.1: Metal



b.2: Metal



c.1: Furniture



c.2: Furniture

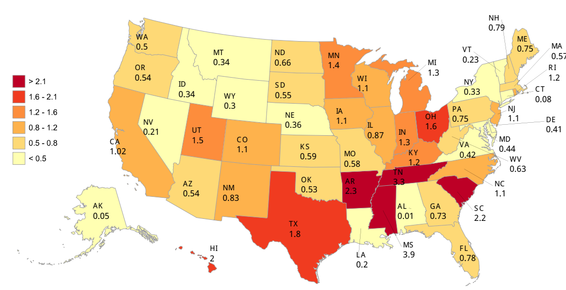


Figure 19: Regional employment declines in manufacturing industries

Note: This figure presents the reduction in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate reduction in industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate reduction in industry employment normalized by the employment size of each state relative to U.S. aggregate employment. Panels a present the results for the plastics and rubber industry. Panels b present the results for the metal industry. Panels c present the results for the furniture industry.

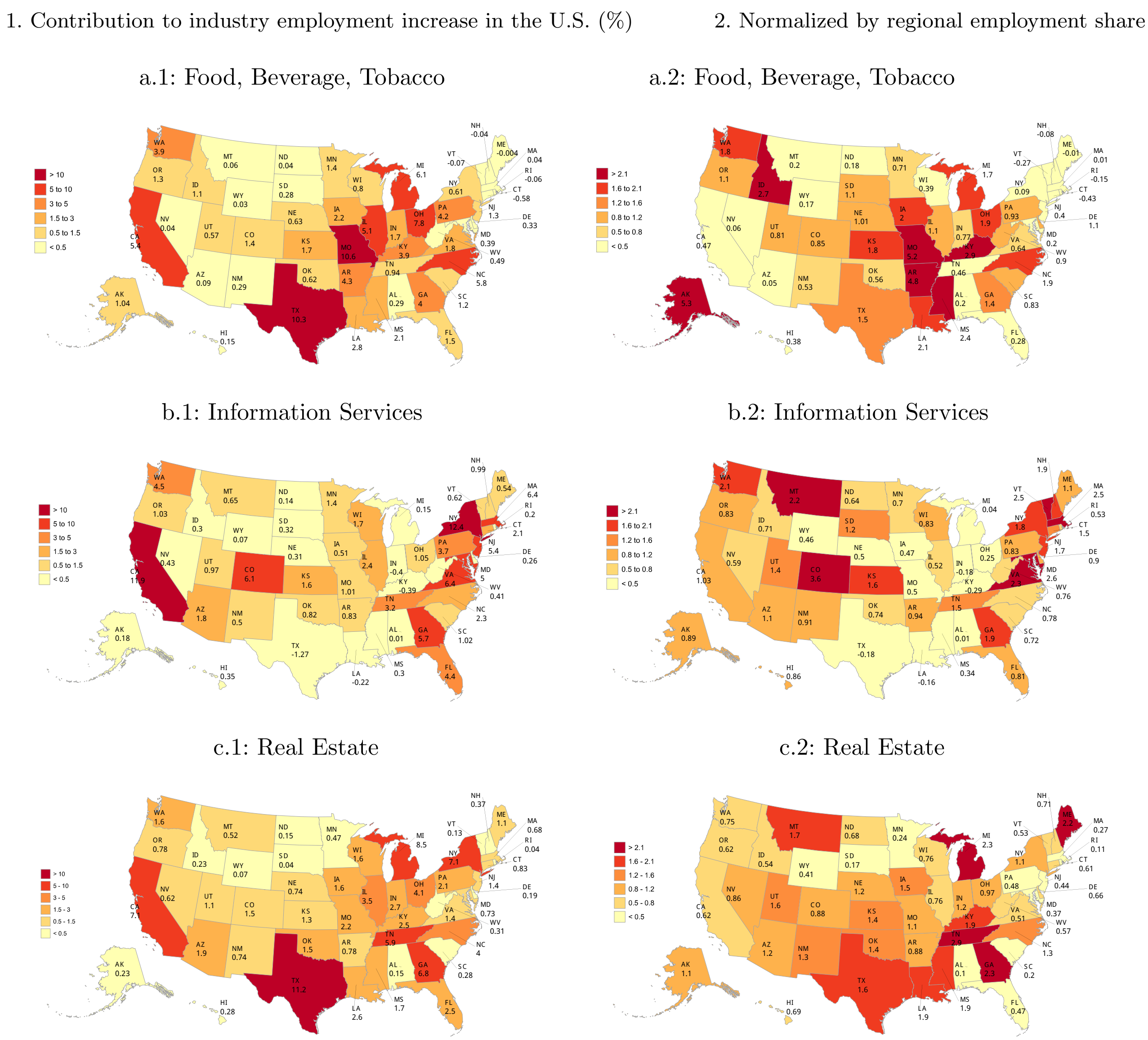


Figure 20: Regional employment increases in manufacturing and non-manufacturing industries

Note: This figure presents the rise in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate increase in industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate increase in industry employment normalized by the employment size of each state relative to U.S. aggregate employment. Panels a present the results for the food, beverage, and tobacco industry. Panels b present the results for the information services industry. Panels c present the results for the real estate industry.

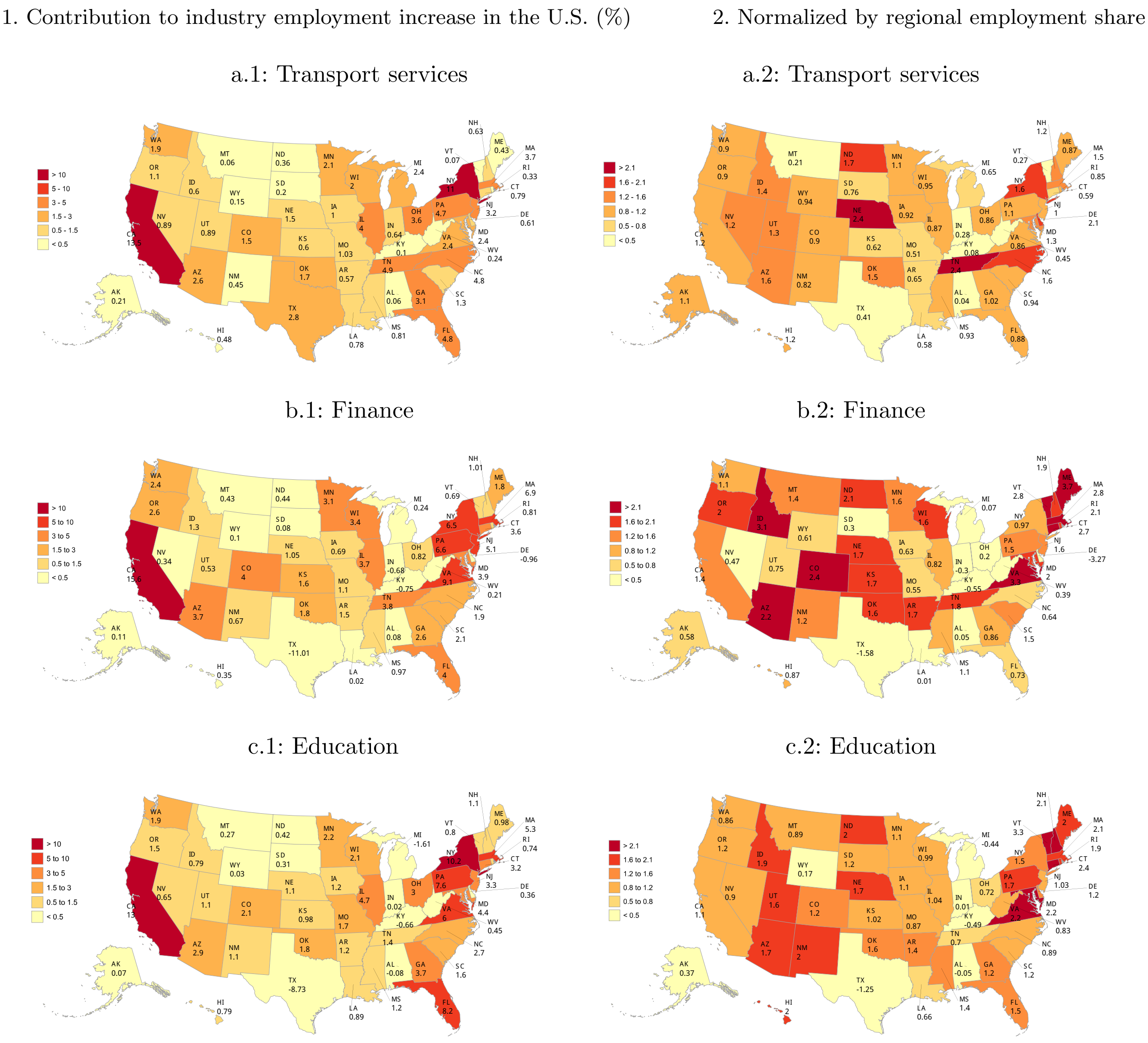


Figure 21: Regional employment increases in non-manufacturing industries

Note: This figure presents the rise in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate increase in industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate increase in industry employment normalized by the employment size of each state relative to U.S. aggregate employment. Panels a present the results for the transport services industry, Panels b present the results for the finance industry. Panels c present the results for the education industry.

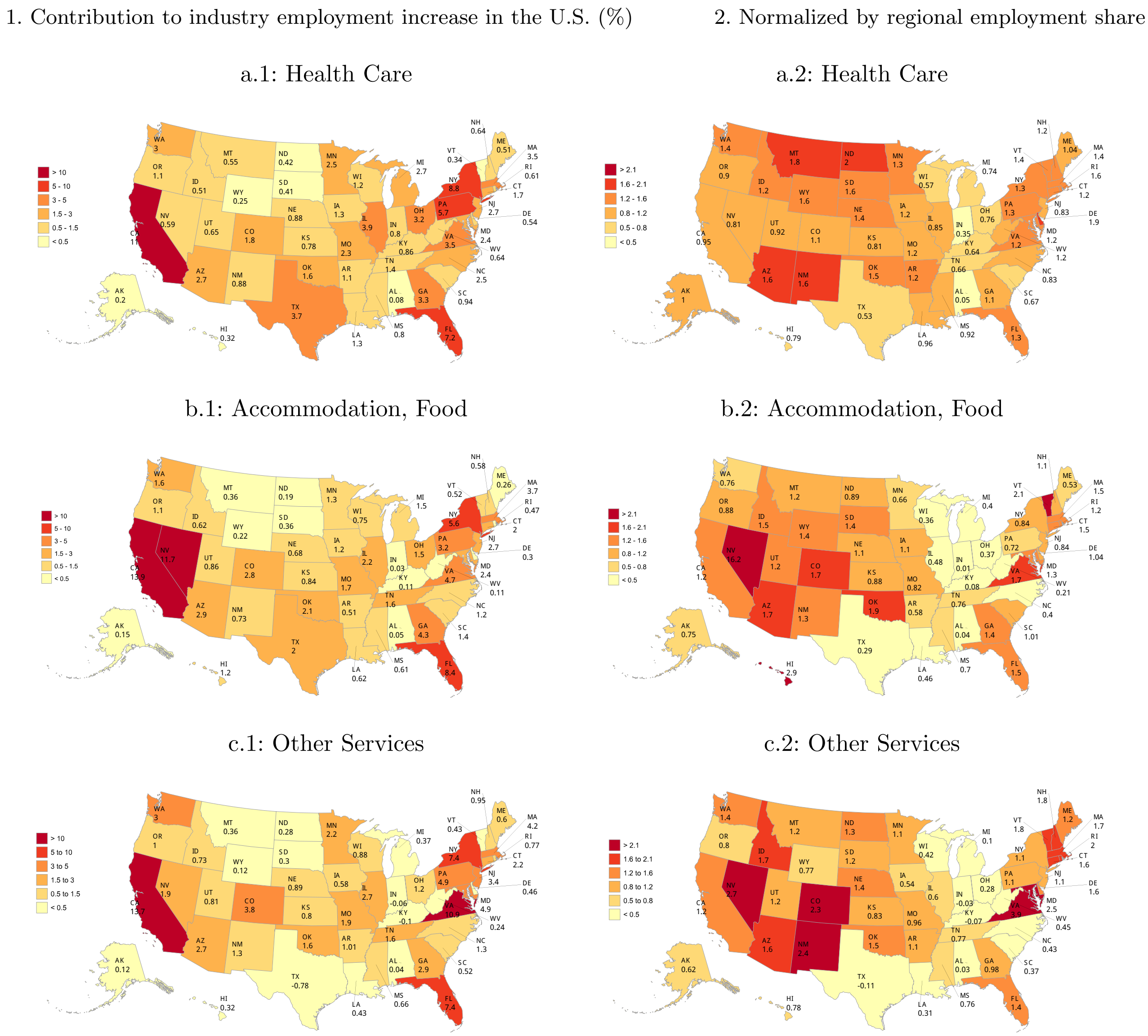


Figure 22: Regional employment increases in non-manufacturing industries

Note: This figure presents the rise in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate increase in industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate increase in industry employment normalized by the employment size of each state relative to U.S. aggregate employment. Panels a present the results for the health care industry. Panels b present the results for the accommodation and food industry. Panels c present the results for the other services industry.