ECONOMIC RESEARCH
FEDERAL RESERVE BANK OF ST. LOUIS

WORKING PAPER SERIES

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Authors	David Andolfatto, Fernando M. Martin, and Shengxing Zhang
Working Paper Number	2015-003D
Revision Date	July 2017
Citable Link	https://doi.org/10.20955/wp.2015.003
Suggested Citation	Andolfatto, D., Martin, F.M., Zhang, S., 2017; Rehypothecation and Liquidity, Federal Reserve Bank of St. Louis Working Paper 2015-003. URL https://doi.org/10.20955/wp.2015.003

Published In	European Economic Review
Publisher Link	https://doi.org/10.1016/j.euroecorev.2017.09.010

Rehypothecation and Liquidity*

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July 27, 2017

Abstract

We develop a dynamic general equilibrium monetary model where a shortage of collateral and incomplete markets motivate the formation of credit relationships and the rehypothecation of assets. Rehypothecation improves resource allocation because it permits liquidity to flow where it is most needed. The liquidity benefits associated with rehypothecation are shown to be more important in high-inflation (high interest rate) regimes. Regulations restricting the practice are shown to have very different consequences depending on how they are designed. Assigning collateral to segregated accounts, as prescribed in the Dodd-Frank Act, is generally welfare-reducing. In contrast, an SEC15c3-3 type regulation can improve welfare through the regulatory premium it confers on cash holdings, which are inefficiently low when interest rates and inflation are high.

Keywords: rehypothecation, money, collateral, credit relationship.

JEL codes: E4, E5

*We would like to thank the Editor and two anonymous referees for helpful comments. We thank conference participants at the 10th Annual Conference on General Equilibrium & its Applications at Yale University, the First African Search and Matching Workshop in Marrakech, the Society for Economic Dynamics Annual Meeting, the conference "On Money as a Medium of Exchange: KW at 25!" in Santa Barbara, and the Summer Workshop on Money, Banking, Payments and Finance. We would also like to acknowledge the useful feedback we received from seminar participants at Arizona State University and the University of Iowa. The views expressed in this paper do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors.

1 Introduction

An agent wanting to borrow money can acquire more of it and at better terms by pledging collateral to incent repayment. The practice of using collateral to secure a debt is called *hypothecation*. The same agent may further improve the quantity and terms of his loan by granting the creditor temporary use-rights over the pledged collateral. The practice of re-using pledged collateral is called *rehypothecation*.

Because much rehypothecation evidently occurs in the shadow-banking sector, the true scope of the activity is not easily measured. However, data available for primary dealers suggest that rehypothecation was large and growing prior to the 2008 financial crisis. And while the practice appears to have diminished since the financial crisis, the present value of rehypothecated assets remains measured in the trillions of dollars; see Singh and Aitken (2010, Figure 1) and Shkolnik (2015, Figure 11).

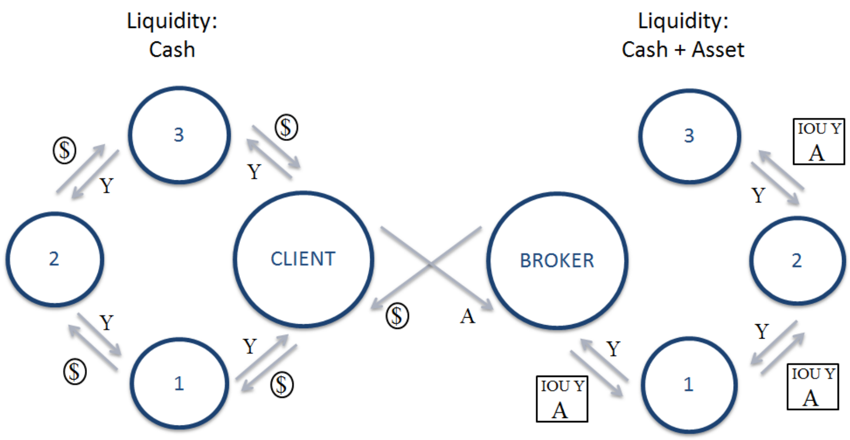
The role of collateral in lending arrangements is easy to understand. The question of why a debtor should prefer a collateralized loan over an outright sale (and subsequent repurchase) of collateral, on the other hand, is less straightforward, but is not the question we address here.[1] What we want to know is why–*given a collateralized lending arrangement*–an additional use-right over collateral is sometimes transferred to the creditor. Our answer is that–when collateral is *scarce* (in the sense of Caballero, 2006)–rehypothecation is a mechanism that increases the *effective* supply of collateral by permitting its reassignment to agencies in the best position to make use of it.

Selling a borrowed security (or re-using it as collateral) to exploit a trade that might not otherwise have happened sounds a lot like liquidity provision to us. Indeed, it is precisely this observation that motivates the title of our paper. In what follows, we seek to clarify the nature of rehypothecation, its connection to market liquidity, and how the practice may be affected by monetary and regulatory policies. To this end, we construct a dynamic general equilibrium model of monetary exchange with a security that can potentially serve as collateral in lending arrangements.

In our model, investors holding cash and securities gain random access to expenditure opportunities, some of which require cash financing and others for which securities can also be used. Investors with cash-only opportu-

[1] Monnet and Narajabad (2012) provide a framework for understanding the circumstances in which repurchase agreements are preferred to asset sales.

Figure 1: Rehypothecation



nities and investors with opportunities that can be financed more flexibly using cash or securities engage in a swap of assets to be reversed at a later date. In regions of the parameter space where both investors are liquidity constrained, it makes sense to have cash flowing to the cash investor (investor A) and securities flowing to the credit investor (investor B), with the exchange reversed (or otherwise settled) at a later date.

In reality, investor A could be a hedge fund and investor B a dealer bank. The hedge fund wants to borrow cash, offering government bonds as collateral.[2] Or investor A may be a retail investor holding a margin account with investor B, a discount broker. The retail investor wants to borrow cash to buy shares in a company, with the discount broker treating these shares as collateral for the cash loan.

In Figure 1, the client and broker exchange cash and an asset (denoted by A). The client uses borrowed cash to purchase a good, service, or security, denoted by Y. The cash potentially circulates in a chain of transactions and is ultimately returned. If the asset pledged as collateral can be rehypothecated, the broker is permitted to re-use it. In the figure, the broker issues an IOU for Y that is backed by A. As with cash, this security may conceivably circulate in a collateral chain before it is ultimately returned.[3]

[2] Again, we are not asking why a hedge fund in need of cash does not simply sell its security and reverse the transaction at a later date if so desired.

[3] We do not consider extended collateral chains in the formal model below, though such an extension can be easily incorporated without changing the flavor of our reported

The investors in our model require exchange media to facilitate profitable exchanges involving untrusting third parties. We model these third parties as workers and the profitable exchanges as consumption opportunities. That is, workers want to get paid in cash (sometimes securities) in exchange for their labor services. Lack of trust (i.e., the absence of fully enforceable credit) between workers and investors gives rise to inefficient outcomes, as is standard in the monetary literature. Note that a degree of realism could be gained by replacing workers in our model with agents who present investors with profitable investment opportunities, but our central conclusions are not sensitive to such a modification. Our framework of analysis, therefore, can be based on a relatively minor adaption of the Lagos and Wright (2005) and Geromichalos, Licari and Suárez-Lledó (2007) quasilinear models of money and asset exchange.

As far as we know, ours is the first dynamic general equilibrium model brought to bear on the question of rehypothecation. In our model economy, a low-return monetary instrument coexists with a high-return security because the former can be used in a wider array of transactions. The equilibrium real rate of return on money (the inverse of the inflation rate) is determined by monetary policy. The value of rehypothecation is shown to be higher in high inflation (high interest rate) economies. We calculate that permitting unfettered rehypothecation in a 10% inflation regime is worth over 1% of consumption in perpetuity. The value of rehypothecation is diminished in low-inflation, low-interest rate regimes and, indeed, the value of the practice vanishes at the Friedman rule. The intuition for this latter result is straightforward: at the Friedman rule, the opportunity cost of carrying idle money balances is zero, so that agencies become voluntarily flush with liquidity. Away from the Friedman rule, our calculations show that allowing unrestricted rehypothecation can in some cases lower the welfare cost of inflation up to 70%.

As an empirical matter, the volume of rehypothecation has diminished substantially in the low-inflation, low-interest rate environment that has characterized the U.S. economy since the Great Recession (see Shkolnik 2015, Figure 11). An unknown amount of this scaling down is no doubt attributable to increased risk perception and regulatory control. But as our theory suggests, the present low-inflation, low-interest rate environment is almost surely a contributing factor, as the opportunity cost of holding cash remains relatively low.

results.

Another contribution of our paper is to demonstrate how real-world policies designed to regulate rehypothecation can be modeled and studied in a dynamic general equilibrium framework. To identify the theoretical effect of regulatory interventions on rehypothecation, we study two general forms of regulation. The first policy has the flavor of SEC rule 15c3-3 for margin accounts, which restricts how much collateral a client borrows on margin can be rehypothecated by a broker-dealer. This policy targets the joint allocation of money and securities attached with the rehypothecation right. The amount of borrowed securities that can be rehypothecated depends on the amount of cash lent. The SEC rule 15c3-3 type of policy is equivalent to a regulatory haircut on collateral used in bilateral repo contracts.[4] The second policy we examine restricts the rehypothecation right on collateral without any reference to the cash flowing in the opposite direction. A form of this latter policy is a law that requires some collateral to be held in segregated accounts, much in the way the Dodd-Frank Act of 2010 restricts the rehypothecation of assets in credit derivatives markets.[5]

We show that only policies of the first type can be welfare-improving, while policies of the second type cannot improve welfare and in fact are typically welfare-reducing. Given the second-best nature of equilibrium outcomes in our model economy, it is perhaps not surprising to learn that trading restrictions can sometimes improve welfare. However, the mechanism through which this effect operates is specific to our model. A restriction along the lines of SEC rule 15c3-3 bestows a "regulatory premium" on cash, enhancing the demand for cash, which is inefficiently low in a high-inflation, high-interest rate economy. Specifically, investors demand more cash to relax regulatory restrictions on future rehypothecation.[6] The second policy, on the other hand, does not have a direct impact on the demand for real cash balances, so that the welfare consequences are quite different. Our model makes clear how the details of regulatory design related to the practice of rehypothecation can matter.

The outline of our paper is as follows. In Section 2, we describe the

[4]See, for example, International Capital Market Association (2012), for the equivalence of the initial margin and a haircut.

[5]In particular, swap contracts must now be cleared by central counterparties who are required to hold collateral in segregated accounts; see Monnet (2011).

[6]Our result resembles Farhi, Golosov and Tsyvinski (2009). In their paper, reserve requirements enhance the demand for real money balances, which leads to improved risk-sharing. They note that such a regulation is usually motivated by financial stability concerns but that, as in our paper, a regulation designed for one purpose may turn out to have unintended benefits along another dimension.

physical environment and characterize the set of Pareto optimal allocations. In Section 3, we describe the market structure, the frictions that make exchange media necessary, and monetary policy. We formalize the economic problems that agents solve in Section 4 and characterize a stationary monetary equilibrium. In Section 5, we study the properties of an unregulated economy, in particular, how the allocation depends on inflation and collateral supply. In Section 6, we evaluate the welfare consequences of real-world regulations designed to restrict rehypothecation. Section 7 presents a brief review of some related literature and offers a few concluding remarks.

2 Environment

In this section we describe the physical environment and characterize the set of Pareto optimal allocations. Time is discrete and the horizon is infinite, $t = 0, 1, 2, .., \infty$. Each date t is divided into three subperiods, which we label the *morning*, *afternoon*, and *evening*, respectively.

The economy is populated by two types of infinitely-lived agents labeled *investors* and *workers*. There is a continuum of each type of agent, with the population mass of each normalized to unity. All agents realize an idiosyncratic "location shock" in the morning which determines a subsequent travel itinerary for the remainder of the period. There are two spatially separated locations. Half of the population of investors and workers travels to each of the two locations in the afternoon.[7] There is no aggregate risk. All agents reconvene to a central location in the evening.

Agents have preferences defined over afternoon and evening goods. All goods are nonstorable. Let $(c_{j,t}, y_{j,t})$ denote the output consumed and produced in the afternoon in location $j = 1, 2$ at date t. Let $x \in \mathbb{R}$ denote expected consumption (production, if negative) of the evening good. An investor has preferences represented by

$$\sum_{t=0}^{\infty} \beta^t \left[0.5u(c_{1,t}) + 0.5u(c_{2,t}) + x_t^i\right] \tag{1}$$

where $u'' < 0 < u'$ and $u'(0) = \infty$ and $0 < \beta < 1$. Workers have linear

[7] This is a simplified way to capture trading frictions as in Duffie, Gârleanu and Pedersen (2005).

preferences, represented by

$$\sum_{t=0}^{\infty} \beta^t \left[-0.5y_{1,t} - 0.5y_{2,t} + x_t^w\right] \tag{2}$$

Thus, our model adopts the quasilinear preference and timing structure of Lagos and Wright (2005).

Finally, there is a single productive asset in fixed supply–a Lucas tree–that generates a constant nonstorable income flow $\omega > 0$ at the beginning of each evening.

A Pareto optimal allocation is a feasible allocation that maximizes a weighted sum of *ex ante* utilities (1) and (2). Given that u is strictly concave, efficiency dictates that c_1 is the same for all investors in location 1 and c_2 is the same for all investors in location 2. Feasibility requires $c_1 = y_1$ and $c_2 = y_2$. Since the disutility of production in the afternoon is linear for workers, we can interpret (y_1, y_2) as expected levels of production/disutility.

Clearly, the efficient allocation of afternoon consumption/production satisfies $c_1 = c_2 = y^*$ where $u'(y^*) = 1$. The resource constraint in the evening is given by

$$x^i + x^w = \omega \tag{3}$$

Since $\omega > 0$ is given, the choice of x^w serves only to distribute utility across investors and workers. Although x^w may be positive or negative, it will typically be positive, as workers need to be compensated in the evening for their afternoon effort. Workers themselves can produce in the evening in order to consume, but the net effect on utility is canceled out. In the aggregate, workers' evening net transfers are positive, $x^w > 0$, if investors do not consume all the dividend, i.e., $x^i < \omega$. Note that x^i may be negative, which would mean investors produce in the evening to further compensate workers. Since the total surplus is proportional to $u(y^*) - y^*$, the *ex ante* participation constraints are satisfied for any x^w such that $y^* \leq x^w \leq u(y^*)$.

3 Market structure and policy

The planning allocation characterized above hints at the pattern of trade that will prevail in a decentralized setting. Investors want to consume in the afternoon. They will want to acquire these desired goods and services from workers, who are in a position to deliver them. Workers must somehow be compensated for their travails. The requisite compensation can only happen in the evening, where workers can acquire x^w in the form of services from investors and/or from the income generated by the Lucas tree. The question is how these trade flows are to be financed.

Investors and workers do not trust each other. As explained in Gale (1978), the lack of trust necessitates the use of an exchange medium. Following Geromichalos, Licari and Suárez-Lledó (2007), we assume the existence of two exchange media: fiat money and a security that constitutes a claim to the Lucas tree. We assume that workers in location 1 only accept cash as payment, whereas workers in location 2 are willing to accept both cash and securities as payment.[8] This restriction on payments is simply a device intended to capture the fact that investors sometimes need cash to finance purchases and at other times are able to use securities as a means of finance.[9] In what follows, we relabel location 1 as the *cash market* and location 2 as the *credit market*.[10]

We assume that the afternoon cash and credit markets are competitive. Let (p_1, p_2) denote the price of output, measured in units of money, in the afternoon cash and credit markets, respectively. Let ϕ_2 denote the cum-dividend real price of securities in the afternoon credit market and let ϕ_3 denote the ex-dividend real price of securities in the evening. Finally, let p_3 denote the nominal price of output (transferable utility) in the evening.

Under the assumed market structure, trade flows are financed in the following manner. Afternoon purchases are financed with investor sales of money and securities. In the evening, money and securities flow back in the opposite direction. That is, workers spend their accumulated wealth on goods and services. Investors rebalance their depleted wealth portfolios in the act of worker compensation. This trade pattern repeats period after period. In what follows, we restrict attention to stationary equilibria and so

[8] See Lester, Postlewaite and Wright (2013) for a theory of asset liquidity.

[9] Note that while the purchases here are modeled as acquisitions of goods and services, the model could be extended to accommodate the fact that investors typically use cash and securities to finance acquisitions of different securities.

[10] The label "credit market" is chosen because the sale of assets here is equivalent to a fully collateralized lending arrangement. That is, investors could borrow output from workers in the afternoon, using the security as collateral that is legally seizable in the event of default. Note that technically, such a collateralized loan arrangement need not violate our lack of trust assumption. One could imagine, in particular, a mechanical protocol that executes collateralized loan arrangements among anonymous agents. In fact, the Bitcoin-related platform *Ethereum* is a protocol that permits exactly this type of exchange to take place.

we drop the time-subscript on variables.

Note that when investors are rebalancing their wealth portfolios in evening trade, they do not know beforehand which of the two locations they will be visiting the next afternoon. This would not be an issue if a well-functioning financial markets were available in the morning. In that case, investors could just dispose of securities in outright sales if they needed cash and vice-versa if they wanted to accumulate securities. Moreover, if they needed to borrow from other investors, they could potentially do so. We assume that these markets are unavailable to investors in the morning–a restriction meant to capture the fact that investors are not always in timely contact with centralized financial markets. Investors are therefore subject to a form of liquidity risk. Accumulating low-return cash will turn out to be useful because it hedges against a liquidity shock–the risk of visiting the cash market (i.e., the risk of needing cash to exploit a profitable expenditure opportunity).

Because investors are risk-averse, they will generally have an incentive to form risk-sharing arrangements. We assume that investors are grouped in pairs and that each such pairing represents an enduring relationship, where the two partners serve as if they were in a cooperative, seeking to maximize the value of their partnership. Moreover, for simplicity, we assume that when one partner travels to the cash market, the other partner travels to the credit market. That is, the idiosyncratic uncertainty associated with travel itineraries is perfectly negatively correlated across the two partners.[11] In this setup then, one investor is wanting cash and the other is wanting securities. The cash flowing to the cash investor is expected to be spent. Of course, exactly the same logic applies for the securities flowing to the credit investor. That is, since the two investors trust each other, securities play no role as collateral within the relationship. The only rationale for reassigning securities is so that they can be reused, just like cash.

We study two types of policies designed to regulate the rehypothecation of securities. The first regulatory restriction is modeled after SEC rule 15c3-3 for margin accounts in the United States. This rule specifies that the securities borrower (cash lender) can rehypothecate borrowed securities only in proportion to the amount of cash lent. So, for example, if a retail investor uses \$50 of margin to buy \$100 of Apple shares from a broker, the broker is permitted to rehypothecate no more than 140% of the cash loan, that is, \$70 worth of Apple shares in this example. The second regulatory restriction is

[11] On the other hand, note that because workers are risk-neutral, they do not value insurance.

modeled after a provision in the Dodd-Frank Act, in particular, Title VII, Section 724, which requires most swap contracts to be cleared through central counterparties with some pledged collateral kept in segregated accounts. That is, the rehypothecation of securities is in this case restricted.

We assume throughout that the supply of fiat money grows at a constant gross rate $\mu \geq \beta$ and that new money is injected (or withdrawn) via lump-sum transfers (or taxes) to the investors in the third subperiod. Let τ denote the real transfer per investor.

4 Decision-making

4.1 Workers

Because workers have linear preferences, their choices serve primarily to price money and securities via a set of no-arbitrage-conditions. In what follows, the only behavior we impose on workers is that they carry no wealth from an evening to the subsequent morning. This behavior is optimal when $\mu > \beta$ (when monetary policy is away from the Friedman rule) and without loss of generality when $\mu = \beta$.[12]

To acquire one dollar in the afternoon cash market, a worker must expend utility $1/p_1$. This dollar then buys $1/p_3$ units of output in the evening. Since there is no discounting across subperiods, a no-arbitrage-condition implies $1/p_1 = 1/p_3$ must hold in equilibrium.

To acquire one dollar in the afternoon credit market, a worker must expend utility $1/p_2$. This dollar then buys $1/p_3$ units of output in the evening. Since there is no discounting across subperiods, a no-arbitrage-condition implies $1/p_2 = 1/p_3$. Together, these results imply that $p_1 = p_2 = p_3 = p$.

To acquire one share of the security in the afternoon credit market, a worker must expend utility ϕ_2. These shares can be sold for evening output at price $\phi_3 + \omega$. Again, because there is no discounting across subperiods, a

[12] Note that workers have no transactional need for financial assets. Thus, given their linear preferences, for workers to be willing to acquire financial assets in the evening, they would have to earn a rate of return of at least $1/\beta$, to compensate for discounting across periods. Cash earns a rate of return of $1/\beta$ at the Friedman rule and less than that otherwise. As we shall see, when collateral is scarce, securities carry a liquidity premium and earn a rate of return lower than $1/\beta$.

no-arbitrage-condition implies

$$\phi_2 = \phi_3 + \omega$$

That is, the afternoon cum-dividend price of securities is equal to the ex-dividend evening price of securities plus the value of the dividend.

Since workers must be indifferent between being paid in money or securities in the afternoon credit market, the real rate of return on money and securities from afternoon to evening must be the same, $p_2/p_3 = (\phi_3+\omega)/\phi_2$. Since $p_2 = p_3$ and $\phi_2 = \phi_3 + \omega$, this condition is satisfied. Given these conditions, it is optimal for workers to passively supply whatever output is demanded from them in the afternoon, and to spend all their acquired wealth in each evening.

4.2 Investors

4.2.1 Morning

Let (m, a) denote the money and securities held by an investor at the beginning of the morning. Assuming that investors enter each morning with identical wealth portfolios, the consolidated assets of two investors in a trading relationship is $(2m, 2a)$. The location shock is realized in the morning, one investor in the relationship will have an opportunity to trade in the cash market and the other will have an opportunity to trade in the credit market. Before investors travel to their designated locations in the afternoon, they have an opportunity to rearrange money and securities between them.[13]

Let (m_2, a_2) denote the portfolio allocated to the investor traveling to the credit market, where $2m \geq m_2 \geq 0$ and $2a \geq a_2 \geq 0$. The portfolio allocated to the investor traveling to the cash market is given residually by $(m_1, a_1) = (2m - m_2, 2a - a_2)$.

Given our setup, we anticipate cash flowing to the cash-investor and securities flowing to the credit-investor, i.e., $m_2 \leq m$ and $a_2 \geq a$. Thus, the

[13] We assume here that investors do not belong to the same enterprise operating from a consolidated balance sheet. Instead, investors are involved in informal relationships. They have individual wealth portfolios but they seek to maximize the joint value of the relationship. In particular, they can commit to any feasible terms they strike in risk-sharing arrangements.

relevant non-negativity constraints are:

$$
\begin{aligned}
m_2 &\geq 0 \qquad (4)\\
2a - a_2 &\geq 0 \qquad (5)
\end{aligned}
$$

If $m_2 < m$, then the credit-investor is in effect sending $[m - m_2]$ dollars to the cash-investor. If $a_2 > a$, then the cash-investor is in effect sending $p\phi_2 [a_2 - a]$ dollars worth of securities to the credit-investor, where $p\phi_2$ is the nominal price of the security in the afternoon. If the value of what is exchanged is the same, then the transaction replicates what could have been accomplished through an outright purchase of securities by credit-investors in a morning securities market, if such a market existed. But because investors in a relationship trust each other, a degree of unsecured credit is possible. If $[m - m_2] < p\phi_2 [a_2 - a]$, then the cash-investor is a net creditor to the credit-investor (the money loan is overcollateralized). If $[m - m_2] > p\phi_2 [a_2 - a]$, then the credit-investor is a net creditor to the cash-investor (the money loan is undercollateralized).

One way to map our model into reality is to interpret the investor partnership as the type of relationship that is formed between different broker-dealers, or broker-dealers and other securities lenders (e.g., central banks, pension funds, insurers). Think of the investor traveling to the cash market as a client and the investor traveling to the credit market as a broker-dealer. The client wants to "borrow" $[m - m_2]$ dollars from his margin account held with the broker-dealer, and is willing to "pledge" securities worth up to $p\phi_2 [a_2 - a]$ dollars as "collateral." As is the case in reality, the broker-dealer agreement permits the rehypothecation of collateral for use in proprietary trades. Clearly, if the value of cash and securities passing hands is not the same, then some amount of unsecured credit is involved. We assume that broker-dealers and their clients can be trusted to repay unsecured debt. In reality, reputational concerns (the threat of punishments for default) can support a degree of unsecured credit. In any case, the credit arrangements described here are unwound each evening. If the broker-dealer rehypothecated the client's collateral in the afternoon, either in a short-sale or as collateral for a proprietary lending arrangement (with workers), then the collateral–or its value equivalent–is returned in the evening.

Investors in a trading relationship may face regulatory constraints on how they can use borrowed securities. The SEC rule 15c3-3 is modeled as follows,

$$\theta [m - m_2] \geq p\phi_2 [a_2 - a] \qquad \text{(R1)}$$

where $\theta \geq 0$ is a policy parameter. The regulatory constraint (R1) is a requirement on cash margin for borrowed securities with a rehypothecation right. In particular, it restricts the value of borrowed securities that can be rehypothecated by the credit-investor to be a multiple of the value of money lent to the cash-investor. In general, think of the cash-investor depositing all his securities with the credit-investor and placing a fraction of these in a segregated account. Securities in this segregated account may still serve as collateral for the cash loan but cannot be reused by the credit-investor, i.e., they do not carry rehypothecation rights. The regulatory constraint places a limit on the amount of securities that can be rehypothecated.[14]

Alternatively, a regulation in the form of Title VII, Section 724 of the Dodd-Frank Act is modeled here as

$$\vartheta a \geq [a_2 - a] \tag{R2}$$

for some $0 \leq \vartheta \leq 1$. If $\vartheta = 1$, then the creditor-investor may make full use of the securities he borrows from the cash-investor. If $\vartheta = 0$, then all of the borrowed securities are effectively held in a segregated account–they may not be spent (rehypothecated).

4.2.2 Afternoon

Recall that $(2m, 2a)$ represents the combined morning asset position of two investors in a relationship. Recall as well that (m_2, a_2) denotes the portfolio allocated to the investor traveling to the credit market in the afternoon. Let (m', a') denote the investors' combined asset position entering the evening.

Investors' combined expenditure on afternoon goods and services cannot exceed their combined wealth, net of what they wish to carry into the evening. Thus, the afternoon flow budget constraint for investors is given by:

$$2m - m' + p\phi_2(2a - a') - py_1 - py_2 \geq 0 \tag{6}$$

Individually, investors are subject to liquidity constraints depending on their travel itinerary. The cash-investor is subject to the following liquidity constraint:

$$2m - m_2 - py_1 \geq 0 \tag{7}$$

[14] In Canada, rehypothecation is apparently prohibited (Maurin, 2015) and so $\theta = 0$. In the U.K., there are apparently no legal limits to rehypothecation, in which case $\theta = \infty$. In the U.S., SEC15c3-3 sets $\theta = 1.4$.

while the credit-investor is subject to:

$$m_2 + p\phi_2 a_2 - py_2 \geq 0 \tag{8}$$

Condition (7) restricts the cash-investor's expenditures in the afternoon, py_1 so that they do not exceed his available cash, $m_1 = 2m - m_2$. Since money holdings cannot be negative, $m_1 \geq 0$. This, in turn, implies $2m \geq m_2$. Note that $m_1 = 0$ cannot be optimal in a monetary equilibrium (it would imply $y_1 = 0$) and hence, $2m > m_2$ as anticipated above when deriving (4). As well, $a_1 = (2a - a_2) \geq 0$ implies $2a \geq a_2$, as specified by (5). Since $y_2 \geq 0$, the liquidity constraint of the credit-investor (8) imposes a non-negativity restriction on his combined money and securities holdings. That is, afternoon expenditures, py_2 cannot exceed the total value of assets under his control, $m_2 + p\phi_2 a_2$. In addition to this we need to impose the non-negativity constraints $m_2 \geq 0$ and $a_2 \geq 0$, though only the former is potentially binding. Together, all these non-negativity constraints imply that consolidated money and securities holdings brought into the evening are also non-negative, i.e., $m', a' \geq 0$.

Since m' is the investors' consolidated cash position brought into the evening and $2m - m_2 - py_1$ is the cash brought into the evening by the cash-investor, the difference between these two objects represents the cash brought into the evening by the credit-investor. This object too must be non-negative,

$$m' - [2m - m_2 - py_1] \geq 0 \tag{9}$$

A similar argument applied to the credit-investor's securities holdings implies:

$$a' - [2a - a_2] \geq 0 \tag{10}$$

Recall that a' represents the investors' consolidated security holdings as they enter the evening. The difference $2a - a_2$ represents the (unspent) securities held by the cash-investor or, equivalently, the value of his securities deposited in a segregated account with the credit-investor, with no rehypothecation rights. Thus, (10) restricts the credit-investor's security holdings to be non-negative.

The following result allows us to omit (8) from the set of restrictions faced by the agent. (Note that proofs to all lemmas and propositions are available in Appendix B).

Lemma 1 *The liquidity constraint* (8) *is implied by restrictions* (6)*,* (7)*,* (9) *and* (10)

4.2.3 Evening

Let (m^+, a^+) denote the money and securities carried by an investor from the evening into the next period after investors settle the terms of their risk-sharing agreement reached in the morning. Again, since the relationship is assumed to solve the problem of maximizing joint welfare, and since the two investors are *ex ante* identical, symmetry demands that each investor enters the morning with an identical wealth portfolio. With this in mind, we can write each investor's evening budget constraint as follows,

$$x = (\phi_3 + \omega)\, a'/2 + (1/p)(m'/2 - m^+) - \phi_3 a^+ + \tau \tag{11}$$

Recall that τ is the real value of new money injections (or tax, if negative) per investor.

4.2.4 Investor choices

Let $B(2m, 2a)$ denote the joint value of an investor relationship in the morning with combined assets $(2m, 2a)$. Let $V(m', a')$ denote the joint value of this relationship entering the evening with combined assets (m', a'). The value functions $\{B, V\}$ must satisfy the recursion:

$$B(2m, 2a) \equiv \max_{y_1, y_2, m_2, a_2, m', a'} \left\{ u(y_1) + u(y_2) + V(m', a') \right\} \tag{12}$$

subject to (R1), (R2) (6), (7) and the non-negativity constraints (4), (5), (9) and (10).

Note that when $\vartheta = 1$, the non-negativity constraint (5) corresponds to regulatory constraint (R2). When $\vartheta < 1$, the non-negativity constraint (5) is implied by the regulatory constraint (R2), so that the former is redundant. Recall that by Lemma 1, (8) is implied by the other constraints.

In the evening, the joint problem solved by investors is given by,

$$V(m', a') \equiv \max_{m^+, a^+} \left\{ (\phi_3 + \omega)\, a' + (m' - 2m^+)/p - \phi_3 2a^+ + 2\tau + \beta B(2m^+, 2a^+) \right\} \tag{13}$$

There are also the non-negativity constraints $m^+, a^+ \geq 0$, but we anticipate that these will not bind for investors in the evening.[15] The optimality conditions associated with investor decisions are recorded in Appendix A.

[15] Investors will want to rebuild their asset positions in order to finance their consumption expenditures in the following afternoon.

4.2.5 Equilibrium conditions

We restrict attention to stationary allocations in which real quantities and prices remain constant while nominal quantities and prices grow at rate $\mu > \beta$. The Friedman rule monetary policy is written as $\mu = \beta$, but should be understood to mean $\lim_{\mu \searrow \beta} \mu$.[16]

The equilibrium is characterized mathematically in Appendix A. For convenience, we record the key economic restrictions in the body of this section. The first condition determines the level of economic activity in the afternoon cash market,

$$\mu = \beta[u'(y_1) + \theta p \chi_1/2] \tag{EQM1}$$

where $\chi_1 \geq 0$ denotes the Lagrange multiplier associated with the regulatory constraint (R1).

The second condition places a restriction on the equilibrium security price and the level of economic activity in the afternoon credit market,

$$\phi_3 = \beta(\phi_3 + \omega)[(1+\vartheta)u'(y_2) + (1-\vartheta) - \vartheta p \chi_1]/2 \tag{EQM2}$$

Let ζ_1 and ζ_2 denote the Lagrange multipliers associated with (9) and (10), respectively. That is, if $\zeta_1 > 0$ and $\zeta_2 > 0$, then the credit investor brings no cash or securities into the evening. The relevant economic restriction for afternoon economic activity in the credit market is given by,

$$u'(y_2) - 1 = p\zeta_1 = \zeta_2/\phi_2 \tag{EQM3}$$

Hence, either both (9) and (10) are slack or they both bind.

Let ζ_3 denote the Lagrange multiplier associated with (4). In Appendix A, we demonstrate that the following restriction applies,

$$p\zeta_3 = u'(y_1) - u'(y_2) + \theta p \chi_1 \tag{EQM4}$$

If $\zeta_3 > 0$, then optimal risk-sharing requires the credit investor to lend all his money to cash investor in the morning, i.e., $m_2 = 0$ and $m_1 = 2M$. Any remaining wedge in consumption between the cash and credit investor

[16] At the Friedman rule, the price level and aggregate real balances are indeterminate, which is why we focus on the equilibrium that arises in the limit, as μ approaches β from above.

will then be determined on whether the regulatory constraint (R1) binds or not.

Let χ_2 denote the Lagrange multiplier associated with (R2) which, recall, implies the non-negativity constraint (5). In Appendix A, we derive the following restriction,

$$p\chi_1 + \chi_2/\phi_2 = u'(y_2) - 1 \tag{EQM5}$$

Thus, if either of the regulatory constraints bind, activity in the afternoon credit market is constrained ($y_2 < y^*$). When $\vartheta = 1$, it is still possible for $\chi_2 > 0$ but in this case the constraint binds not for regulatory reasons, but rather because the non-negativity constraint (5) binds.

We now invoke the market-clearing conditions $m = M$ and $a = 1$. Cash-investors spend all of their cash (when $\mu = \beta$, they weakly prefer to do so). Thus, (7) holds with equality. Together with the market-clearing conditions, we have:

$$2M - m_2 = py_1 \tag{EQM6}$$

Finally, the regulatory constraints need to be satisfied in equilibrium. Using the market clearing conditions and (EQM5) we obtain equilbrium expressions for the regulatory constraints (R1) and (R2):

$$a_2 - 1 \leq (\theta/\phi_2)(y_1 - M/p) \tag{EQM7}$$

$$a_2 - 1 \leq \vartheta \tag{EQM8}$$

Clearly, both constraints cannot bind simultaneously, except in the non-generic case $\phi_2\vartheta = \theta(y_1 - M/p)$. Note that if $m_2 = 0$ (a typical case) then (EQM7) simplifies to $a_2 - 1 \leq (\theta/\phi_2)(y_1/2)$.

5 Unregulated economy

In this section, we describe analytically the properties of the equilibrium allocation in an unregulated economy. That is, we consider an economy in which the regulatory constraints on rehypothecation do not bind or are not imposed. In particular, assume that θ is high enough so that (EQM7) does not bind (i.e., $\chi_1 = 0$) and that $\vartheta = 1$. Now (EQM8) binds only when (5) binds, so that $\chi_2 > 0$ in this case reflects a binding short sale constraint, and not a binding regulatory constraint.

We start by stating two important properties of the unregulated economy, which are standard in monetary economies. Then, we characterize how the equilibrium is affected by the availability (or shortage) of collateral.

Proposition 1 *Operating monetary policy at the Friedman rule ($\mu = \beta$), implements the efficient allocation, $y_1 = y_2 = y^*$.*

In this economy, it is optimal to use lump-sum taxes to finance a deflation to a point that sets the nominal interest rate to zero. To see this, note that condition (EQM1) for $\chi_1 = 0$ implies $\mu = \beta u'(y_1)$ when $\chi_1 = 0$. Thus $y_1 = y^*$ when $\mu = \beta$. Condition (EQM3) implies $u'(y_2) - 1 = p\zeta_1 \geq 0$. Since $u'(y^*) = 1$, condition (EQM4) implies $p\zeta_3 = 1 - u'(y_2) \geq 0$. These latter two conditions imply $u'(y_2) = 1$, so that $y_2 = y^*$ and $\chi_2 = 0$.

Proposition 1 is important because it asserts that under an optimal monetary policy, investors are flush with liquidity so that the use of additional securities as exchange media is redundant. In particular, rehypothecation has no private or social value when the nominal interest rate is zero. By continuity, one would expect rehypothecation to have little value in relatively low-inflation (low-interest rate) regimes.

Proposition 2 *The level of economic activity in the cash-market y_1 is strictly decreasing in the rate of inflation μ.*

Because inflation acts as a tax on cash transactions, higher inflation means lower output in the cash-market. That y_1 is strictly decreasing in the inflation rate μ follows directly from condition (EQM1) when $\chi_1 = 0$.

The effect of inflation on y_2 depends on the supply of collateral securities, as indexed by the parameter ω. Consider the creditor-investor's liquidity constraint (8), $m_2 + p\phi_2 a_2 - py_2 \geq 0$. Let us assume $y_2 = y^*$ and then verify the conditions under which this result is valid.

Assume for the moment that securities are not rehypothecated, that is, $\vartheta = 0$ so that $a_2 = 1$. Moreover, assume (and later verify) that the creditor-investor lends all his money to the cash-investor, so that $m_2 = 0$. From the market-clearing condition (EQM6), $p = 2M/y_1$. From condition (EQM2), we see that when $\chi_1 = 0$ and $y_2 = y^*$, the security is priced at its fundamental value,

$$\phi_2^* = \phi_3^* + \omega = \omega/(1 - \beta) \tag{14}$$

Combining these results with the liquidity constraint (8) implies that the requisite condition is $\phi_2^* \geq y^*$, or $\omega \geq (1 - \beta)y^*$. Thus, if the income generated by the asset is sufficiently large (so that the market value of collateral securities is sufficiently high), then the creditor-investor can finance the efficient level of expenditure y^* exclusively with his own securities. This, in turn, implies that $m_2 = 0$ is part of an efficient risk-sharing arrangement as $\mu > \beta$ implies that the cash-investor remains liquidity constrained.

Definition 1 *Define $\omega^* \equiv (1-\beta)y^*$. An economy is said to be collateral-rich if $\omega \geq \omega^*$ and collateral-poor if $\omega < \omega^*$.*

Intuitively, a collateral-rich economy is one in which the market value of collateral securities is sufficiently high as to render rehypothecation superfluous (the efficient level of output $y_2 = y^*$ is achievable even when $\vartheta = 0$). In this model then, inflation can have no impact on the level of activity in the credit market when the economy is collateral-rich. Moreover, note that ω^* is independent of μ.

Let us now consider collateral-poor economies (and assuming $\mu > \beta$). We conjecture that $y_2 = y^*$ will continue to be implementable over some range $[\hat{\omega}, \omega^*)$. Assume (and later verify) that $m_2 = 0$, so that $p = 2M/y_1$. Since $y_2 = y^*$, we have $\phi_2 = \phi_2^*$. The creditor-investor's liquidity constraint (8) therefore is satisfied as a weak inequality if and only if $\phi_2^* a_2 \geq y^*$, or $\omega a_2 \geq (1-\beta)y^* = \omega^*$.[17] That is, $y_2 = y^*$ appears to be feasible even for $\omega < \omega^*$, *but only if rehypothecation is possible*, i.e., $a_2 > 1$. For lower values of ω in this range, greater levels of rehypothecation $(a_1 - 1)$ are needed to support a level of financing sufficient to support $y_2 = y^*$. In our model, which features only bilateral relationships, the maximum rehypothecation "multiplier" is $(a_2/a) = 2$. That is, since $a = 1$ in equilibrium, the maximum amount of rehypothecated securities is $a_2 = 2$. Thus, the creditor-investor's liquidity constraint will remain slack for any $\omega \geq (1/2)\omega^* \equiv \hat{\omega} < \omega^*$. Since $y_2 = y^*$ can be financed without cash, it remains privately optimal to send all money to the cash-investor, i.e., $m_2 = 0$. Note that $\hat{\omega}$ is also independent of μ.

Proposition 3 *If $\omega \geq \hat{\omega}$, then $y_2 = y^*$ for any $\mu \geq \beta$.*

For $\omega \in [\hat{\omega}, \omega^*)$, the quantity of rehypothecated securities $[a_2 - 1] = [\omega^*/\omega - 1] \geq 0$ is a decreasing function of ω. So, as we decrease ω from ω^* to $\hat{\omega}$, the volume of rehypothecation increases until its upper limit is reached $a_2 = 2$ at $\omega = \hat{\omega}$. For $\omega < \hat{\omega}$, the creditor-investor's liquidity constraint (8)

[17] By Lemma 1 and the derivations in Appendix A, this situation arises when constraints (9) and (10) do not bind. Note that constraint (6) always binds, while, in the absence of regulation, constraint (7) binds whenever $\mu > \beta$.

is satisfied with equality, i.e., constraints (9) and (10) bind, and $y_2 < y^*$. To see this, assume that $m_2 = 0$ ($\zeta_3 > 0$) which, from condition (EQM4) will be true so long as $y_2 > y_1$. Then, if (8) is satisfied with equality, the creditor-investor spends all of his securities in the afternoon, so that $\zeta_2 > 0$. By condition (EQM3), $\zeta_2 > 0$ implies $y_2 < y^*$.

Definition 2 *An economy is said to have a collateral shortage if $y_2 < y^*$.*

Economies for which $\omega \in [0, \hat{\omega})$ experience a collateral shortage. Note that collateral-poor economies in the range $\omega \in [\hat{\omega}, \omega^*)$ do not suffer from a collateral shortage because rehypothecation "stretches out" the limited supply of collateral in a manner sufficient to relax the afternoon liquidity constraint.

Let us now further examine the properties of the model when $\omega < \hat{\omega}$. Using condition (EQM2) together with $\chi_1 = 0$ and the fact that $u'(y_2) > 1$, we derive the equilibrium securities price,

$$\phi_2 = \left[\frac{1}{1 - \beta u'(y_2)}\right] \omega > \phi_2^* \text{ and } \phi_3 = \left[\frac{\beta u'(y_2)}{1 - \beta u'(y_2)}\right] \omega > \phi_3^* \qquad (15)$$

where ϕ_2^* and ϕ_3^* are defined in (14). Here we have the familiar result that a scarce collateral asset is priced above its fundamental value. The magnitude $[u'(y_2) - 1] \geq 0$ measures the liquidity premium on the security, vanishing only when $y_2 = y^*$. Note that (15) defines an equilibrium lower bound for y_2, i.e., $u'(y_2) < 1/\beta$.

Since $y_2 < y^*$ implies the constraints (9) and (10) bind, the creditor-investor's liquidity constraint (8) is satisfied with equality and we have $m_2/p + \phi_2 a_2 - y_2 = 0$. Here, $m_2 = 0$ as long as $\zeta_3 > 0$. By condition (EQM4) and $\chi_1 = 0$,

$$p\zeta_3 = u'(y_1) - u'(y_2) \geq 0 \qquad (16)$$

so that $\zeta_3 > 0$ as long as $y_1 < y_2$. Since the cash-investor has no use of the security, he lends it all to the creditor-investor ($a_2 = 2$). Using the pricing function for ϕ_2 in (15), the credit-investor's liquidity constraint implies that $y_2(\omega) < y^*$ is determined by,

$$\left[1 - \beta u'(y_2)\right] y_2 - 2\omega = 0$$

from which we derive

$$y_2'(\omega) = \frac{2}{[1 - \beta u'(y_2)] - y_2 \beta u''(y_2)} > 0 \qquad (17)$$

Thus, the credit investor's purchases y_2 decline as ω (and the price of securities) declines. Since $y_1(\mu)$, which is determined by condition (EQM1) for a given μ, is independent of ω, condition (16) suggests that there exists a critical value $0 < \omega_0(\mu) < \hat{\omega}$ such that $y_1(\mu) = y_2(\omega_0)$, in which case $\zeta_3 = 0$. Note that since $u(y_1) = \mu/\beta$ by (EQM1) and $u'(y_2) < 1/\beta$ by (15), the case $y_1(\mu) = y_2(\omega_0)$ can only exist when $\mu < 1$. Hence, when $\mu \geq 1$, we obtain $y_1(\mu) < y_2$ always.

For $\mu < 1$ and $\omega < \omega_0(\mu)$ the liquidity constraint for the credit investor begins to bind more tightly than the cash investor. To prevent this from happening, the optimal risk-sharing arrangement at this point now involves setting $m_2 > 0$ ($\zeta_3 = 0$). This is, it is now optimal for the creditor-investor to keep some cash on hand, rather than lending it all to the cash-investor. At this point, the liquidity constraint for the creditor-investor (8) is given by $m_2/p + \phi_2 a_2 - y_2 = 0$ with $m_2/p > 0$.

As long as $\mu < 1$, since $\zeta_3 \geq 0$ and $y_1(\mu)$ is fixed for a given μ, we have $y_2 = y_1(\mu)$ for all $\omega \leq \omega_0(\mu)$. Since the liquidity constraint (8) holds with equality, we have $m_2/p = y_1(\mu) - \phi_2 a_2$, where $a_2 = 2$. From (EQM6) we have $m_2/p = 2M/p - y_1(\mu)$. Together, these two restrictions imply

$$p = \frac{M}{y_1(\mu) - \phi_2} \tag{18}$$

where ϕ_2 is given by (15) with $y_2 = y_1(\mu)$. Thus, for $\omega \leq \omega_0(\mu)$, the effect of a lower ω is to lower the security price ϕ_2 and increase the price-level, without any effect on either y_1 or y_2 (the only effect of lower ω is to lower consumption in the evening). Since $m_2/p = y_1(\mu) - 2\phi_2$, the effect of a lower ω is to increase the real cash balances (m_2/p) allocated to the credit investor. In the limit, as $\omega = 0$, both investors become *de facto* cash-investors and both simply divide their cash evenly between themselves. Finally, note that the critical value $\omega_0(\mu)$ is decreasing in μ.

The results derived above are summarized in the following proposition.

Proposition 4 *If $\mu \in [\beta, 1)$, there exists $0 < \omega_0(\mu) < \hat{\omega}$ such that $y_2 = y_1(\mu) < y^*$ for all $\omega \in (0, \omega_0(\mu)]$. If $\mu \in [\beta, 1)$ and $\omega \in [\omega_0(\mu), \hat{\omega})$, or $\mu \geq 1$ and $\omega \in (0, \hat{\omega})$, then $y_1(\mu) < y_2 < y^*$, where y_2 is an increasing function of ω and is independent of μ. For $\omega \geq \hat{\omega}$, $y_1(\mu) < y_2 = y^*$ for any $\mu \geq \beta$.*

In the unregulated economy, the typical case is $y_1 < y_2$, which implies $m_2 = 0$, i.e., cash is not allocated to the credit-investor. Whether the credit-investor obtains the first-best level of consumption depends on whether the dividend is high enough. When monetary policy is deflationary and the dividend is low enough, the partnership allocates some cash to the credit-investor, $m_2 > 0$, and both investors obtain the same consumption, i.e., $y_1 = y_2$.

6 Regulating rehypothecation

In this section we study the effects of imposing regulatory constraints on rehypothecation, as modeled by (R1) and (R2). As noted above, both type of regulations cannot bind at the same time, so we consider each separately. As we shall see, these regulations bind only in certain regions of the parameter space.

Proposition 5 *If $\mu = \beta$ or $\omega \geq \omega^*$ then the regulatory constraints* (R1) *and* (R2) *do not bind* $(\chi_1 = \chi_2 = 0)$ *.*

Regulating rehypothecation can only be consequential in environments where the practice is essential. Since rehypothecation is not essential in collateral-rich economies or at the Friedman rule, we restrict attention to collateral-poor economies, $\omega < \omega^*$ and monetary policies for which $\mu > \beta$. Note that restricting attention to this region of the parameter space is only necessary and not sufficient to guarantee that one of the regulatory constraints will bind.

6.1 SEC15c3-3 regulation

We first study the effect of imposing the SEC15c3-3 type regulation as modeled in (R1). Set $\vartheta = 1$ so that χ_2 now becomes the Lagrange multiplier associated with the non-negativity constraint (5), i.e., $2a - a \geq 0$.

Consider a collateral-poor economy for which there is no collateral shortage, $\omega \in [\hat{\omega}, \omega^*)$. In this case (by Proposition 3) $y_2 = y^*$ in the absence of regulation and $[a_2 - 1] = [\omega^*/\omega - 1] \in [0, 2]$ measures the level of rehypothecation. Assume, for the moment, that $\chi_1 = \chi_2 = 0$. Since $y_2 = y^*$ and since $\mu > \beta$, we have $m_2 = 0$ and $p = 2M/y_1$. Moreover, $\phi_2 = \phi_2^*$. Combining these restrictions with (EQM7) yields the inequality,

$$\theta \geq \frac{2(\omega^* - \omega)}{(1 - \beta) y_1(\mu)} \equiv \Theta(\omega, \mu)$$

Thus, the regulatory constraint (R1) remains slack for any $\theta \geq \Theta(\omega, \mu)$. Notice that $\Theta(\omega, \mu)$ is decreasing in ω and increasing in μ. For higher levels of ω the value of collateral securities increases, permitting the regulatory constraint to tighten (θ to decline) without hampering activity in the credit market. A higher rate of inflation reduces the real demand for money balances y_1, which increases the price-level. As the *nominal* value of securities $p\phi_2$ increases and as the nominal supply of cash M is fixed at a point in time, the regulatory constraint must be relaxed to permit the same amount of money to support a higher nominal value of rehypothecation.

Suppose then that $\omega \in [\hat{\omega}, \omega^*)$ and that $\theta = \Theta(\omega, \mu)$, so that $y_1(\mu) < y_2 = y^*$. Now consider tightening the regulatory restriction on rehypothecation $\theta < \Theta(\omega, \mu)$ so that $\chi_1 > 0$, i.e., $\theta(M - m_2) - p\phi_2(a_2 - 1) = 0$. What effect does this binding regulation have on the allocation? To answer this question, we make use of the following result,

Lemma 2 *Let $\theta \geq 1$. If the regulatory constraint (R1) binds then the creditor-investor lends all his cash to the cash-investor* ($m_2 = 0$).

In words, the lemma above states that if the credit investor would like to rehypothecate more securities but is prevented by regulation from doing so then, if the constraint does not bind too tightly ($\theta \geq 1$), the optimal risk-sharing arrangement between the two investors entails the cash investor holding all available cash at the end of the morning. Note that $\theta \geq 1$ here is only sufficient and not necessary. In particular, the result may continue to hold for $\theta < 1$, but for θ is sufficiently small, the regulatory constraint binds to a point where leaving some cash with the credit investor ($m_2 > 0$) is desirable. Note that in the United States, SEC rule 15c3-3 stipulates $\theta = 1.4$.

Thus, assume $\theta \geq 1$ so that by Lemma 2, $m_2 = 0$ and $p = 2M/y_1$. Note that y_1 is now affected by the regulation through condition (EQM1). In particular, one effect of lowering θ (tightening the regulatory constraint) is to increase the demand for real money balances so that y_1 rises. Evidently, reducing the proportion of securities that can be rehypothecated relative to the cash loan induces investors to want more cash to relax the regulatory constraint. This form of regulation therefore results in a *regulatory premium* for cash. Given that the supply of money is fixed at any point in time, the effect is to put downward pressure on the price-level. Since $\chi_1 > 0$, by (EQM5), the effect of a binding regulatory constraint is to lower the level of economic activity in the afternoon credit market, $y_2 < y^*$.

The effect on economic welfare from this type of regulation is rather interesting. Away from the Friedman rule, risk-sharing is inefficient. For the parameterization considered above, $y_1 < y_2 = y^*$ when the regulatory constraint is slack. The effect of tightening the regulation here is to increase y_1 at the expense of y_2. Since y_2 is close to its efficient level, the welfare loss from a small decline in y_2 is of second-order. In contrast, because y_1 is far from optimal, an increase in y_1 has a first-order effect on welfare. Therefore, it is possible that legislation of this form has the effect of increasing *ex ante* investor welfare.

6.2 Title VII Section 724 Dodd-Frank regulation

We now study the effect of imposing the Dodd-Frank type regulation as modeled in (R2), when $\vartheta \in [0, 1)$. Since we do not consider both regulatory constraints operating at the same time (only one of them can bind), assume $\chi_1 = 0$ in what follows.

Let us begin by considering a collateral-poor economy for which there is no collateral shortage, $\omega \in [\hat{\omega}, \omega^*)$. In this case (by Proposition 3) $y_2 = y^*$ in the absence of regulation and $[a_2 - 1] = [\omega^*/\omega - 1] \in [0, 2]$ measures the level of rehypothecation. Combine this latter measure $[a_2 - 1] = [\omega^*/\omega - 1]$ with (EQM8) to derive the inequality,

$$\vartheta \geq \left[\frac{\omega^*}{\omega} - 1\right] \equiv \Gamma(\omega) \text{ for } \omega \in [\hat{\omega}, \omega^*]$$

Thus, $\Gamma(\omega)$ represents the most severe form the regulatory constraint can take without binding; that is, (R2) remains slack for any $\vartheta \geq \Gamma(\omega)$.

Notice that $\Gamma(\omega)$ is decreasing in ω and independent of μ. Moreover, $\Gamma(\hat{\omega}) = 1$ and $\Gamma(\omega^*) = 0$. For higher levels of ω the value of collateral increases, permitting a greater fraction of the outstanding market capitalization of collateral securities to be held in segregated accounts without hampering activity in the credit market.

When $\omega < \hat{\omega}$, the non-negativity constraint (5) binds in the unregulated economy. Thus, (R2) binds for any $\vartheta \in [0, 1]$.

In the following proposition we show that, as long as the dividend is high enough, tightening the Dodd-Frank regulatory constraint (decreasing ϑ) always reduces welfare. When there is deflation and dividends are low enough, so that both types of investors get the same allocation in the unregulated economy, the Dodd-Frank type regulation is innocuous.

Proposition 6 *If $\omega \in [\hat{\omega}, \omega^*)$ and $\vartheta < \Gamma(\omega)$, or if $\mu \in [\beta, 1)$ and $\omega \in (0, \hat{\omega})$, or $\mu \geq 1$ and $\omega \in (0, \hat{\omega})$, for any $\vartheta \in [0, 1]$, then $y_1 < y_2 < y^*$, with y_1 independent of ϑ and y_2 strictly decreasing in ϑ. If $\omega \in (0, \omega_0(\mu))$ and $\mu \in [\beta, 1)$ then $y_1 = y_2 < y^*$ and independent of ϑ.*

Unlike the SEC15c3-3 regulation studied earlier, the Dodd-Frank type regulation studied here has no impact on the demand for real money balances. That is, cash cannot be used to relax the legislated need to segregate collateral under (R2).

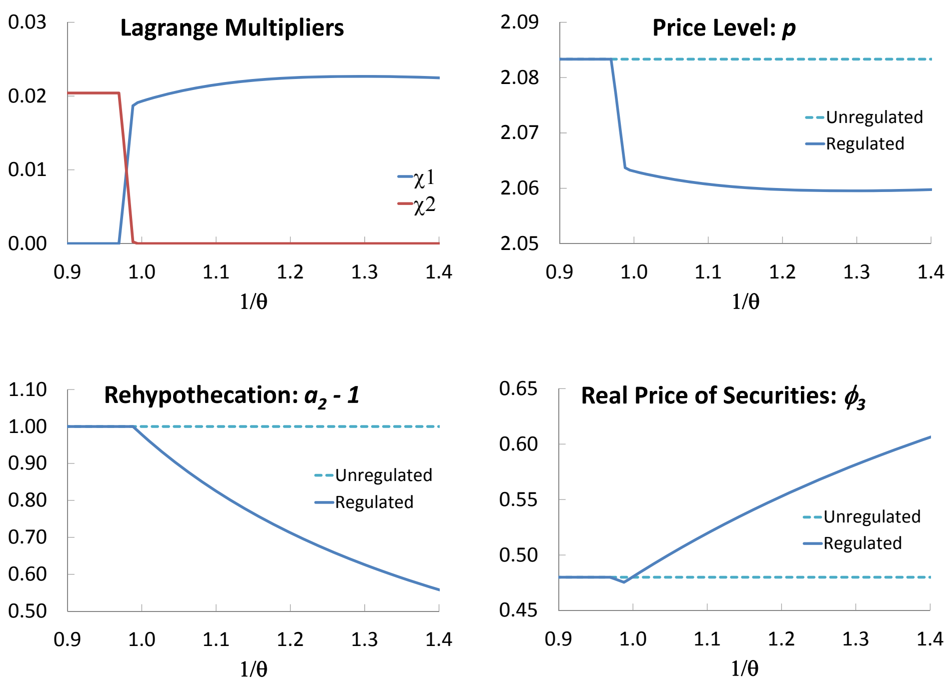
Finally, because the Dodd-Frank regulation confers no regulatory premium for cash, it does not relax the cash-constraint for the cash-investor; i.e., y_1 remains unaffected by ϑ. On the other hand, tightening the regulation (lowering ϑ) has either no effect or, more typically, contracts the level of economic activity in the afternoon credit market. Thus, in our environment, tightening the Dodd-Frank style regulation can never improve welfare. In fact, except in deflationary, low-dividend economies, the regulation always reduces welfare.

6.3 Numerical analysis

We further study the effects of the SEC15c3-3 regulation and its interaction with inflation using numerical methods. To this end, assume $U(c) = \ln c$, $\beta = 0.96$ and $\omega = 0.015$. This parameterization implies $\hat{\omega} = 0.02$, so we will be analyzing an economy with collateral-shortage. We vary regulatory tightness parameter θ assuming $\mu = 1$. When we vary the inflation rate μ, we fix $\theta = 1.05$. Throughout, we maintain $\vartheta = 1$. In all cases considered, we obtain $\zeta_3 > 0$ and thus, $m_2 = 0$. Welfare is measured as the equivalent afternoon-consumption compensation relative to the first-best allocation. That is, how much an investor would have to be compensated every period, in terms of afternoon consumption, in order to be indifferent between living in the corresponding economy and the first-best. Given log-utility the welfare cost simplifies to the proportion Δ solving $\ln(1+\Delta) = W(y^*, y^*) - W(y_1, y_2)$, where the function W is the investor's ex ante period-utility associated with an equilibrium allocation (y_1, y_2). Analytically, we obtain $W(y_1, y_2) \equiv 0.5[\ln y_1 + \ln y_2 - y_1 - y_2] + \omega$.

Figures 2 and 3 show the effects of tightening the regulatory constraint,

Figure 2: Effects of SEC15c3-3 Regulation on Rehypothecation and Prices

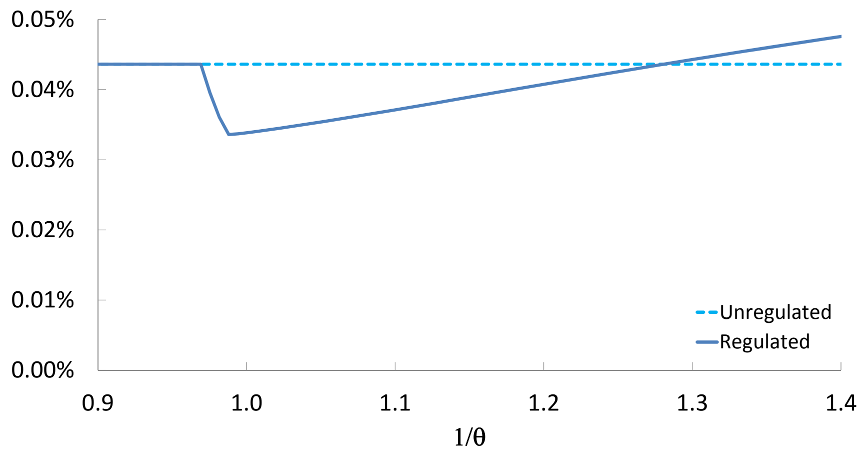


i.e., reducing θ. The x-axis of each figure is in terms of $1/\theta$, so that we tighten the constraint as we move to the right.

Figure 2 consists of four panels showing the effects of θ on (χ_1, χ_2), p, $a_2 - 1$ and ϕ_3, respectively.[18] For θ sufficiently high, the regulatory constraint does not bind ($\chi_1 = 0$); since this is a low-dividend economy, $y_2 < y^*$ and thus, $\chi_2 > 0$, that is, all securities are allocated to the credit-investor. For θ low enough, the regulatory constraint binds, $\chi_1 > 0$. Note that there is a range of values of θ, for which the regulatory constraint binds, while all securities remain assigned to the credit investor. That is, if θ is low, but not too low, it is possible for the restriction on rehypothecation to bind, while the volume of securities rehypothecated remains the same. This is possible since the value of securities ϕ_3 decreases, which allows the partnership to continue satisfying the regulatory constraint at $a_2 = 2$. In this region, ϕ_3 decreases as we lower θ since the premium on securities dictated by the

[18] Recall that, since $\vartheta = 1$, χ_2 now stands for the Lagrange multiplier on the non-negativity constraint (5) instead of the regulatory constraint (R2).

Figure 3: Welfare Cost of SEC15c3-3 Regulation

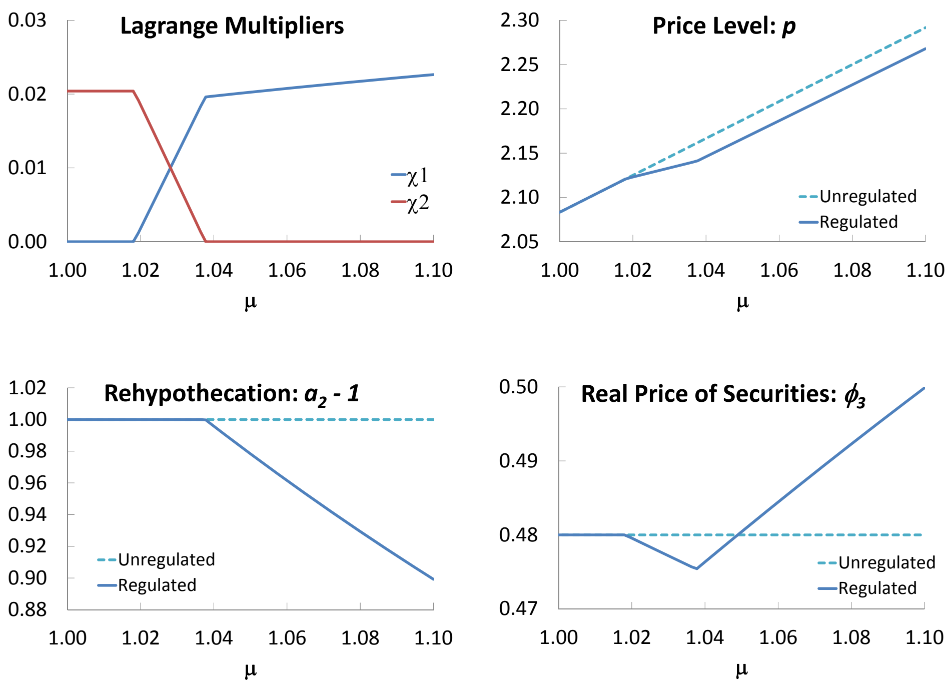


constraint $a - a_2 \geq 0$ becomes less important as the regulatory constraint becomes tighter (i.e., χ_2 decreases as θ decreases). Eventually, however, for θ sufficiently low, we obtain $\chi_2 = 0$ and $a_2 < 1$. That is, the regulatory constraint is tight enough that the value of the cash loan can no longer support allocating all the securities to the credit-investor. As we lower θ further the regulatory constraint binds more tightly and securities start carrying a higher premium.

Figure 3 shows welfare as a function of θ. There is always a welfare loss relative to the first-best due to monetary policy being away from the Friedman rule, $\mu > \beta$. As the regulation binds, welfare gets closer to the first-best, so that restricting rehypothecation for this parameterization improves welfare. As we discussed above, restricting rehypothecation brings the allocations of the cash-investor and the credit-investor closer together, improving risk-sharing. However, as implied by (EQM4), if the regulation is too tight (θ too low and thus, χ_1 too high), then it is possible to have $y_1 > y_2$, and so welfare can be lower than in the unregulated case.

Figure 4 shows the effects of increasing inflation. In an unregulated economy, monetary policy directly affects the consumption of the cash-investor: y_1 is decreasing in μ. However, given that all the cash is allocated to the cash-investor and all the securities to the credit-investor, consumption of the credit-investor is unaffected by inflation. In contrast, in an economy subject to restrictions on rehypothecation, the effects of increasing inflation

Figure 4: Effects of Inflation on Reypothecation and Prices



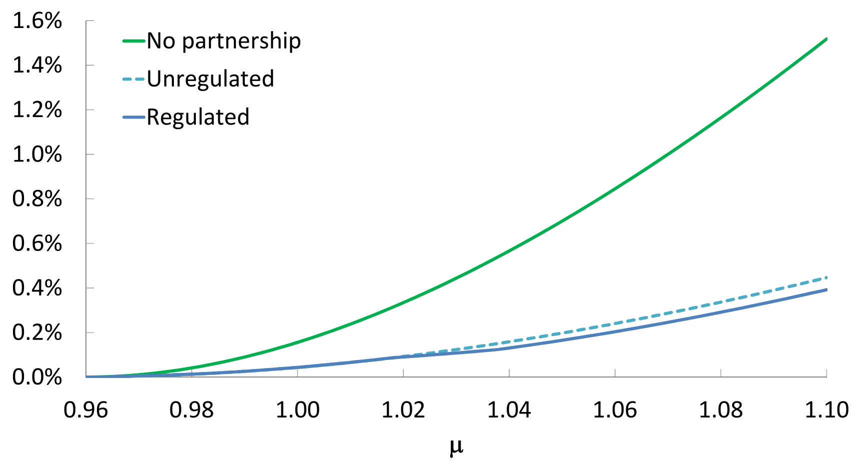
are similar to those of tightening the regulatory constraint: increasing μ raises the prices level, which in turn, tightens the regulatory constraint and thus, works similarly to lowering θ.

Figure 5 shows welfare for a given inflation rate relative to the first-best (which is implemented at the Friedman rule, $\mu = \beta$). It compares the cases of no partnership, unregulated partnership and regulated partnership.[19] The cost of 10% inflation in the no partnership case is in the range of costs derived in previous studies that also abstract from bargaining frictions (e.g., Lucas, 2000 and Rocheteau and Wright, 2004).[20] The value of allowing the practice of rehypothecation is roughly the distance between the no partnership and unregulated cases. As we can see, rehypothecation, which allows for risk-sharing between investors in a partnership, is especially useful in

[19]In the no partnership case, individual investors go at it alone. This case is equivalent to setting $\vartheta = 0$.

[20]Assuming bargaining over the terms of trade between investors and workers can increase the cost of inflation significantly—see Aruoba, Rocheteau and Waller (2007).

Figure 5: Welfare Cost of Inflation



high inflation economies. In our example, the welfare gain is about 1.1% of consumption when inflation is 10%. For most inflation rates, allowing for unrestricted rehypothecation reduces the welfare cost of inflation by about 70%. When μ is very close to β this reduction is a bit lower: it goes down to 50% when $\mu = \beta$

Why do the absolute gains from rehypothecation increase with inflation? In the no partnership case, both types of investors use cash to finance afternoon consumption. In some cases, the credit-investor has excess assets–in fact, for our parameterization, he consumes the first-best. When we allow for partnerships, cash flows to the cash-investor, who is in higher need of liquidity, at the cost of lowering the consumption of the credit-investor somewhat. This is why rehypothecation improves welfare, as explained above. Now, when we allow for rehypothecation, as we increase inflation, the marginal value of the extra unit of cash flowing to the cash-investor increases, while consumption for the credit-investor remains unaffected. Thus, the value of the partnership increases.

The absolute gains from restricting rehypothecation also increase with inflation, as the SEC15c3-3 type regulation mitigates the inefficiency of a low rate of return on cash. At 10% inflation, setting $\theta = 1.05$ yields a further 0.1% welfare gain. Although the welfare gain from restricting rehypothecation is an order of magnitude lower than altogether allowing the practice, the gain is still sizeable.

7 Discussion

Our paper is related to several other works studying the role of rehypothecation on improving the allocation of liquidity, such as Kahn and Park (2015) and Maurin, Monnet and Gottardi (2015). These authors study rehypothecation as an optimal contracting problem, stressing different frictions in the economy. For example, in Kahn and Park (2015), collateral is needed to solve an agency problem, whereas it is needed in our model to solve a commitment problem.[21] Kahn and Park (2015) emphasize the role of risk in collateral-return (repo fails). Maurin (2015) also demonstrates how rehypothecation risk can diminish the benefits of enhanced liquidity provision. Muley (2016) is the only other paper we are aware of that, like ours, tackles the question of how monetary policy interacts with the practice of rehypothecation. Among other things, he finds that using interest on reserves may be preferable to open market operations when collateral is scarce.[22]

Unlike the papers cited above, we study the role of rehypothecation on liquidity allocation in a dynamic monetary model. We think a monetary model is appropriate to study rehypothecation because the practice is fundamentally related to liquidity creation. In particular, the rehypothecation of private assets must compete with liquidity substitutes, primarily in the form of government money and debt. Among other things, we are able to use our model to address the question of how monetary policy might affect the practice and desirability of rehypothecation. Our theory predicts that rehypothecation is less valuable in the low-inflation, low-interest rate environment prevailing since 2008. Perhaps this is one reason why the practice has diminished in recent years, though obviously perceptions of risk and added regulatory controls must also have played a significant role.

The model is also well-suited to exploring the economic impact and welfare consequences of regulatory interventions designed to restrict rehypothecation. That regulatory restrictions on asset liquidity may in some cases improve overall liquidity is echoed in a related literature on asset liquidity in dynamic monetary models, such as Kocherlakota (2003), Shi (2008), Berentsen, Huber, Marchesiani (2014), Geromichalos and Herren-

[21] Although we do not model a lack of commitment within the risk-sharing arrangement here, we have done so in an earlier version of our paper: Andolfatto, Martin and Zhang (2015, section 6).

[22] In some interesting related work that abstracts from liquidity issues, Eren (2014) and Infante (2015) study aspects of the counterparty risk that is commonly associated with the practice of rehypothecation.

brueck (2016, 2017). None of these papers, however, specifically deal with the issue of rehypothecation.

It would be of some interest to extend the model developed here to incorporate aggregate uncertainty over returns on securities when commitment between investors is limited. Consider a version of our model where the cash-investor wants a U.S. treasury (UST) bond and the credit-investor wants a mortgage-backed-security (MBS). Imagine further that the conditional forecast over the future MBS dividend is subject to random "news shocks;" e.g., see Andolfatto and Martin (2013). Then a "bad news" shock realized in the evening would unexpectedly lower the price of MBS and raise the price of USTs through a portfolio rebalance effect, e.g., see Andolfatto (2015). The cash-investor is at this point is long MBS and short USTs, similar to the position of Lehman Brothers in the fall of 2008 (Mackintosh, 2008). The market revaluation of his position makes it more costly for him to reacquire the promised UST (or its value equivalent). If commitment is weak, then the UST borrower may strategically default. It would then be possible to investigate how monetary policy might be designed to accommodate spikes in "repo failures" in times of financial crisis (see Hördahl and King, 2008).

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Appendix

A Derivation of (EQM1)–(EQM8)

Here we derive the optimality conditions associated with the investor problem (12) and (13). Lagrange multipliers are assigned as follows. Let ψ be the multiplier associated with the budget constraint, (6); λ with the liquidity constraint, (7); χ_1 and χ_2 with the regulatory constraints, (R1) and (R2), respectively; ζ_1 and ζ_2 with the credit-investor's non-negativity constraints, (9) and (10), respectively; and ζ_3 with (4). Recall that (5) is implied by (R2).

The necessary first-order conditions for an optimum are:

$$\begin{aligned} u'(y_1) - p\left[\psi + \lambda - \zeta_1\right] &= 0 && (19)\\ u'(y_2) - p\psi &= 0 && (20)\\ -\lambda + \zeta_1 + \zeta_3 - \theta\chi_1 &= 0 && (21)\\ \zeta_2 - p\phi_2\chi_1 - \chi_2 &= 0 && (22)\\ V_m(m', a') - \psi + \zeta_1 &= 0 && (23)\\ V_a(m', a') - p\phi_2\psi + \zeta_2 &= 0 && (24) \end{aligned}$$

By the envelope theorem:

$$\begin{aligned} B_m(2m, 2a) &= \psi + \lambda - \zeta_1 + \theta\chi_1/2 && (25)\\ B_a(2m, 2a) &= p\phi_2\left[\psi + \chi_1/2\right] - \zeta_2 + (1+\vartheta)\chi_2/2 && (26) \end{aligned}$$

The demands for money and securities in the evening must satisfy:

$$\begin{aligned} 1/p &= \beta B_m(2m^+, 2a^+) && (27)\\ \phi_3 &= \beta B_a(2m^+, 2a^+) && (28) \end{aligned}$$

By the envelope theorem:

$$\begin{aligned} V_m(m', a') &= 1/p && (29)\\ V_a(m', a') &= \phi_3 + \omega && (30) \end{aligned}$$

We restrict attention to stationary allocations where all real variables are constant over time and nominal variables grow at rate μ. Combine (19), (25) and (27) to form,

$$\mu = \beta[u'(y_1) + \theta p\chi_1/2] \tag{31}$$

When (R1) is slack ($\chi_1 = 0$), we get the standard result that $\mu > \beta$ implies $y_1 < y^*$.

Now combine (20), (22), (24), (26) and (28) to form:

$$\phi_3 = \beta(\phi_3 + \omega)[(1+\vartheta)u'(y_2) + (1-\vartheta) - \vartheta p\chi_1]/2 \tag{32}$$

From conditions (19), (23) and (29), we have

$$\begin{aligned} p\lambda &= u'(y_1) - 1 && (33)\\ p\psi &= 1 + p\zeta_1 && (34) \end{aligned}$$

which imply $\lambda > 0$ iff $y_1 < y^*$ and $\psi > 0$ always.

Using (20), (23), (24), (29) and (30) we get

$$\begin{aligned} p\zeta_1 &= u'(y_2) - 1 && (35)\\ \zeta_2/\phi_2 &= u'(y_2) - 1 && (36) \end{aligned}$$

Note that if $y_2 = y^*$, then $u'(y_2) = 1$ and so (35)–(36) imply $\zeta_1 = \zeta_2 = 0$.

Finally, using (21), (22), (33)–(36) we get

$$\begin{aligned} p\zeta_3 &= u'(y_1) - u'(y_2) + \theta p\chi_1 && (37)\\ p\chi_1 + \chi_2/\phi_2 &= u'(y_2) - 1 && (38) \end{aligned}$$

Clearly, if $\chi_1 > 0$ or $\chi_2 > 0$ then $y_2 < y^*$.

We now invoke the market-clearing conditions $m = M$ and $a = 1$. Cash-investors spend all of their cash (when $\mu = \beta$, they weakly prefer to do so). Thus, (7) holds with equality. Together with the market-clearing condition, we have:

$$2M - m_2 = py_1 \tag{39}$$

Finally, the regulatory constraints need to be satisfied in equilibrium. Using the market clearing conditions and (39) we obtain equilbrium expressions for the regulatory constraints (R1) and (R2):

$$\begin{aligned} a_2 - 1 &\leq (\theta/\phi_2)(y_1 - M/p) && (40)\\ a_2 - 1 &\leq \vartheta && (41) \end{aligned}$$

Clearly, both constraints cannot bind simultaneously, except in the non-generic case $\phi_2\vartheta = \theta(y_1 - M/p)$.

B Proofs

Proof of Lemma 1. Given that (10) implies $a_2 \geq 2a - a'$, the flow budget constraint (6) implies $2m - m' + p\phi_2 a_2 - py_1 - py_2 \geq 2m - m' + p\phi_2(2a - a') - py_1 - py_2 \geq 0$. Note that (7) and (9) imply $m' \geq 0$. Thus, $2m + p\phi_2 a_2 - py_1 - py_2 \geq 0$. Given that(7) holds with equality (wlog when $\mu = \beta$), $2m - py_1 = m_2$, which yields (8). ∎

Proof of Proposition 1. Suppose $\mu = \beta$. In an unregulated economy, $\chi_1 = 0$. From (31), we get $y_1 = y^*$. Thus, (37) and (38) imply $1 - u'(y_2) \geq 0$ and $u'(y_2) - 1 \geq 0$, respectively. It follows that $u'(y_2) = 1$ and so $y_2 = y^*$. ∎

Proof of Proposition 2. Follows from (31) and $\chi_1 = 0$. ∎

Proof of Proposition 3 and 4. See Section 5. ∎

Proof of Proposition 5. From Proposition 1 we know that $y_1 = y_2 = y^*$ in an unregulated economy. Wlog, set $m_2 = M$ (which implies $(M/p = y^*)$ and $a_2 = 1$. Then, both (40) and (41) are trivially satisfied and so $\chi_1 = \chi_2 = 0$. The dividend threshold ω^* was defined as one that implements $y_2 = y^*$ for $\vartheta = 0$. Hence, wlog set $a_2 = 1$ when $\omega \geq \omega^*$ and so, both (40) and (41) are satisfied, which implies $\chi_1 = \chi_2 = 0$. ∎

Proof of Lemma 2. Let $\theta \geq 1$ and assume $\chi_1 > 0$. Suppose $\zeta_3 = 0$. From Proposition 5 we know that $\mu = \beta$ implies $\chi_1 = 0$, so for this case to be an equilibrium it must be that $\mu > \beta$. From (31) we have $y_1 < y^*$ and from (38) we have $y_2 < y^*$. Given $y_1 < y^*$, (33) implies $\lambda > 0$ and thus, (7) holds with equality. Given $y_2 < y^*$, (35) and (36) imply $\zeta_1 > 0$ and $\zeta_2 > 0$, respectively. Thus, $m' = 0$ and $a' = a - a_2$. In addition, since $\psi > 0$ always, (6) holds with equality. These results imply that (8) holds with equality as well. Given $\zeta_3 = 0$ and $\chi_1 > 0$, (37) implies $\theta p \chi_1 = u'(y_2) - u'(y_1) > 0$. Thus, $y_1 > y_2$. Using (7) and (8), both holding with equality, implies $2m - m_2 > m_2 + p\phi_2 a_2$, which in equilibrium can be rearranged as $2(M - m_2) > p\phi_2 a_2$. Since $\chi_1 > 0$, the regulatory constraint (40) holds with equality, which using (40) can be written as: $\theta(M - m_2) = p\phi_2(a_2 - 1)$. These two expressions put together imply $a_2(1 - \theta/2) > 1$. Given $a_2 \in [0, 2]$, we need $\theta < 1$ to satisfy this inequality, a contradiction with $\theta \geq 1$. ∎

Proof of Proposition 6. If $\omega \in [\hat{\omega}, \omega^*)$ and $\vartheta < \Gamma(\omega)$, or $\omega < \hat{\omega}$ for any $\vartheta \in [0, 1]$, then $\chi_2 > 0$ and (R2) is satisfied with equality: $a_2 = 1 + \vartheta$. By (38), $y_2 < y^*$. Thus, (33), (35) and (36) imply $\lambda > 0$, $\zeta_1 > 0$ and $\zeta_2 > 0$, i.e., (7), (9) and (10) are all satisfied with equality. By Lemma 1, (8) holds with equality as well. Given $\chi_1 = 0$, (31) implies $u'(y_1) = \mu/\beta$ and thus, $y_1 < y^*$ and independent of ϑ.

Suppose $\zeta_3 = 0$. Then, $m_2 \geq 0$ and by (35), $y_1 = y_2$. Thus, y_2 is also independent of ϑ. From Proposition 4, we know that $\zeta_3 = 0$ for $\vartheta = 1$ only if $\omega < \omega_0(\mu)$ and $\mu \in [\beta, 1)$. Thus, $\zeta_3 = 0$ for all $\vartheta \in [0, 1]$ only if $\omega < \omega_0(\mu)$ and $\mu \in [\beta, 1)$.

Assume now $\omega \in [\omega_0(\mu), \omega^*)$ and ϑ such that (R2) binds. Then $\zeta_3 > 0$, i.e., $m_2 = 0$ and by (35) $y_1 < y_2$. Since $m_2 = 0$ and (8) holds with equality, $y_2 = \phi_2(1+\vartheta)$. Given that $y_2/(1+\vartheta) = \phi_2 = \phi_3 + \omega$, we can rewrite (32) as

$$y_2 - (1+\vartheta)\omega = (\beta y_2/2)[(1+\vartheta)u'(y_2) + (1-\vartheta)]$$

Differentiate both sides by ϑ to obtain (after some rearrangement):

$$(1+\vartheta)(dy_2/d\vartheta)\left[(2\omega/y_2) - \beta y_2 u''(y_2)\right] = 2\omega + \beta y_2[u'(y_2) - 1]$$

Given $u'(y_2) - 1 > 0$ and $u''(y_2) < 0$, we obtain $dy_2/d\vartheta > 0$. ■