Research Division
Federal Reserve Bank of St. Louis
Working Paper Series

Equilibrium Sovereign Default with Exchange Rate Depreciation

Sergey V. Popov
and
David G. Wiczer

Working Paper 2014-049A
http://research.stlouisfed.org/wp/2014/2014-049.pdf

November 2014

FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
P.O. Box 442
St. Louis, MO 63166

Equilibrium Sovereign Default with Exchange Rate Depreciation*

Sergey V. Popov†
Management School
Queen's University Belfast
s.popov@qub.ac.uk

David G. Wiczer
Research Department
FRB St Louis
wiczerd@stls.frb.org

November 24, 2014

Abstract

This study proposes and quantitatively assesses a *terms-of-trade penalty* for defaulting: defaulters must exchange more of their own goods for imports, which causes an adjustment to the equilibrium exchange rate. This penalty can take the place of an ad hoc fall in output: Facing only this penalty and temporary exclusion from debt markets, countries are willing to maintain borrowing obligations up to a realistic level of debt. The terms-of-trade penalty is consistent with the observed relationship between sovereign default and a country's trade flows and prices. The defaulter's currency depreciates while trade volume falls drastically. We demonstrate that a default episode can imply up to a 30% real depreciation, which matches observed crisis events in developing countries.

Keywords: endogenous default, exchange rate, trade balance.

JEL: F34, F11, F17.

*We are grateful to Dan Bernhardt, Juan Carlos Hatchondo, Makoto Nakajima, Juan Sanchez, Anne Villamil, and Rui Zhao for productive discussions and numerous comments. We also thank participants of the Missouri Economics Conference, Midwest Macro Meeting, Society of Economic Dynamics and seminars at the University of Illinois for useful suggestions. James Eubanks provided excellent research assistance on later drafts. All errors are ours. The views expressed in this work are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

†Corresponding author.

1 Introduction

Sovereign default corresponds to major changes to a country's trade relations: large fluctuations in traded quantities and relative prices. This is to say, currency crises and default crises often coincide. In our model, we explicitly link the two, positing that the sovereign default decision is influenced by the potential averse effect on trade. Our model incorporates this margin into a standard model of endogenous sovereign default. A simple *terms-of-trade penalty* for defaulters (1) is consistent with trade behavior after default and (2) does not require that a borrower's income to fall after default, while still acting as a sufficient deterrent to default that can allow countries to sustain reasonable levels of debt. We find the default decision is significantly sensitive to the international trade properties of the economy, which suggests that differences in import-export behavior might explain observed differences in countries' willingness to default. Specifically, countries that would trade more during normal times are less likely to jeopardize these flows by defaulting. This uncovers an interesting benefit from international integration: trade-loving countries also turn out to be better borrowers.

The motivation for our work comes from evidence that countries who have recently defaulted have worse terms of trade than those in good standing. Through our framework, we can study how trade penalties affect a country's decision to maintain debt obligations. Our qualitative statement is that that penalizing defaulting countries by worsening their terms of trade reduces the probability of default. Quantitatively, our model shows that a realistic deterioration in terms of trade is sufficient to maintain realistic quantities of foreign debt: between 20 and 50% of GDP depending on the size of the penalty.

To briefly outline our mechanism, after a default, the country finds imported goods more expensive and this penalty prevents default for some countries. When the cost of imports changes, the income effect reduces welfare and the total amount of imported goods is lower but it also increases the marginal value of domestic goods because they are imperfect substitutes. In turn, the real price of foreign goods, the exchange rate, is determined endogenously; thus, some of the adjustment to the penalty happens through prices, the depreciation, and some through quantities. In short, default makes importing more difficult, which changes the trade flows and prices.

Next, we review the related literature and present some empirical evidence consistent with our terms-of-trade penealty. Section 2 discusses the mechanism through which depreciation can deter sovereign default. Section 3 introduces the full infinite-horizon model, and then Section 4 quantifies the mechanism. Finally, we conclude.

1.1 Related Literature

Most of the current literature on endogenous defaults considers countries that are fundamentally self-sufficient for goods, but with access to international markets to smooth their income fluctuations. A country's income fluctuations determine its demand for credit, and defaulters face market exclusion and an ad hoc penalty to their income stream. Arellano (2008) exemplifies this type of model, in which endogenous default is a choice between servicing a sovereign's debt or defaulting and suffering a penalty to output. Bulow and Rogoff (1989) shows that some sort of penalty to defaulters is required to sustain a positive level of debt in equilibrium, as a purely reputational penalty — a temporary exclusion from credit markets — is not enough. As in Arellano (2008), most studies have interpreted the penalty to a defaulter as a loss of output. Modeling assumptions for the penalty and process for generating income are then tightly intertwined, as demonstrated by Aguiar and Gopinath (2006), which uses a process with a stochastic trend which allows the use of a simpler income penalty. Others have introduced richer debt markets because, as pointed out in Chatterjee and Eyigungor (2012), the maturity structure of debt relative to the persistence of shocks is important to the default decision; see also Hatchondo and Martinez (2009) on long-term bonds.

All of these studies rely upon an income penalty to address the problem raised by Bulow and Rogoff (1989). The motivation comes from an empirical finding that output on average falls after default. Tomz and Wright (2007), however, finds the link between "bad times" and default to be "surprisingly weak" — about 40% of the default episodes happen with above-trend output.

There are a few notable exceptions to this reliance on ad hoc output penalties, quite complementary to our own work is Mendoza and Yue (2012). It explicitly models production in the potentially defaulting country, which relies upon foreign borrowing to meet a working capital constraint.[1] Upon default, this working capital is unavailable and so the defaulter suffers lower output

[1] See also Guerrieri et al. (2012).

and reduced imports because it acquires fewer intermediate goods from abroad. Instead of explaining the mechanism of how foreign-dependent goods become more expensive, our study focuses on the *effect* of that change, and demonstrates that it has realistic implications for debt and default. Arellano (2005) used a version of terms-of-trade penalty too, imposing an 8% decrease to the efficiency of importing, but without the endogenous effect on relative prices we describe in Section 2. The results however are consistent with our own: Using that paper's baseline calibration in our model also produces a higher default probability simultaneously with larger sustainable default.

In the empirical literature, Rose (2005) documents that default reduces international trade by 8% for an extended period after default[2]. He speculates that trade suffers after default because trade partners will not extend short-term credit after a sovereign default, raising the cost of cross-border trade. This is consistent with Arteta and Hale (2008), which finds that credit becomes scarce for private firms after a sovereign default. In addition to disruptions to financing, the existing barriers may worsen. Exporters are at the whim of the other country's trade officers, who can greatly increase the cost of trading goods by overzealous customs inspections or other non-tariff trade barriers. The time cost can add 10-30% to the cost of imports, as estimated by Hummels (2001). Many of these costly bureaucratic barriers are fungible and can be made more stringent for less-favored trade partners. Furthermore, many developing economies depend on favorable trade agreements and sourcing partnerships with the developed world.[3] Without favored market access or the use of a trade partner transport infrastructure, the effect on the cost of exporting goods is similar to additional "iceberg" costs.

Finally, Martinez and Sandleris (2011) makes an empirical point against our terms-of-trade penalty: creditor countries do not lower their total trade (import plus export in US dollar value) with defaulters, but non-creditor countries do. They conclude that if a terms-of-trade penalty is present, it is not imposed by current creditors. They then posit that it is hard to devise an incentive-compatible penalizing scheme where creditors do nothing but other countries forgo trade to punish defaulters. We find the assumption that the total trade must decline to be too severe; true, depreciation and the terms-of-trade penalty together imply that imports of the punished country must

[2]Hence the 8% penalty in Arellano (2005).

[3]195 active bilateral trade agreements had been reported to the World Trade Organization as of January 2009, any of which could be terminated in response to a financial episode and would effectively increase tariffs faced by a country's exporters.

decline, but they do not prevent exports from going up, due to the price effect and as may occur in our model. These cancelling forces might produce a only small change in *total* bilateral trade.

1.2 Some Stylized Facts on Trade in Default

De Paoli and Hoggarth (2006) explores defaults since 1975 and finds a strong link between currency crises and sovereign default. The paper proposes an informal explanation for this observation related to nominal rigidities and potential central bank insolvency. Here, we expand on this finding: Table 1 shows that even when depreciation might not qualify as a crisis, defaulters usually experience a decline in their terms of trade. Though large changes to the nominal exchange rate sometimes occur, generally the real depreciation actually accounts for most of the change in relative prices. Our study considers only the real phenomena — defaulters face an endogenous depreciation of their real exchange rate, the price of foreign goods denominated in terms of domestic goods.

Table 1 presents the gross changes over a year for defaulters' nominal effective exchange rate (NEER), real effective exchange rate (REER), export price, and the fraction of expenditure on imports. Notice that the median defaulter has nearly the same depreciation of its real exchange rate as its nominal exchange rate after a year's time. With higher-frequency data, the NEER depreciation tends to be sharper, but the REER quickly catches up. In some cases, the nominal change is greater than the real, but in many of these instances narrative evidence suggests mismanagement by the monetary authority or financial market overreaction. Table 1 reveals that nominal appreciation is rather uncommon.

Countries in default suffer a consistent drop of about 10% in the price of their exports in foreign markets; the distribution of price changes is more tightly clustered than the distribution of depreciation. The REER weights their trading partners by volume of trade, which suggests that the fall in export prices is more pronounced with a country's more active trading partners. Defaulters receive fewer imports in return for the same number of export goods, a fall in export price, which might stimulate domestic households to substitute away from foreign goods, further affecting the exchange rate.

Note that we use relatively low-frequency data in this statistic, looking at annual changes and the year of default. The reasons are twofold: First, there is a great deal of nuance to defining

Gross change	NEER	REER	Export Prices	Imports/GDP
Mean	0.8492	0.8900	0.9184	0.8968
20% quantile	0.6835	0.7861	0.8291	0.7773
40% quantile	0.8729	0.8688	0.8984	0.8978
Median	0.9102	0.9220	0.9075	0.9211
60% quantile	0.9505	0.9429	0.9269	0.9436
80% quantile	1.0168	0.9755	1.0068	1.0070

Note: Exchange rate listed as foreign goods per home currency unit. Data on NEER and REER come from the IMF's IFS database; data on trade comes from the World Bank's WDI. Default episodes are from Beers and Chambers (2003) since 1975: defaults on both bond debt and bank debt, 53 observations in total.

Table 1: One year effect of sovereign default.

precisely when a country has declared default. Cruces and Trebesch (2013) does a very complete job of documenting different potential default dates, but which particular event in a default should matter most to international markets is unclear; hence, we use a wide time-window. Second high-frequency data are not consistently available for the national accounts data that we require for trade shares. The limitation this imposes, of course, is that we cannot say for sure that the change in terms-of-trade happened before or after the default, but Asonuma (2014) convincingly shows monthly changes in exchange rates before and after a selection of default episodes for which high-frequency data are available and dates are relatively certain. That paper's facts are consistent with considerable depreciation after an event that we feed into our model.

The data also reveal that households with bankrupt governments spend relatively less on imports, which have become more expensive following the price changes. In the model, this fact is an amalgam of two forces: the relative price of imports changes, meaning the substitution effect would decrease imports. This price change also has a negative income effect from the change and would diminish the demand for imports, with imports as normal goods. However, defaulting on debt saves the country debt service payments, which may have an opposing income effect. The falling share of imports combines these forces and disentangling them is quantitative. In our calibrated model as in the data, imports fall as a share of spending. Though Tomz and Wright (2007) suggests a surprisingly weak link between default and income, default does seem to change trade flows reliably.

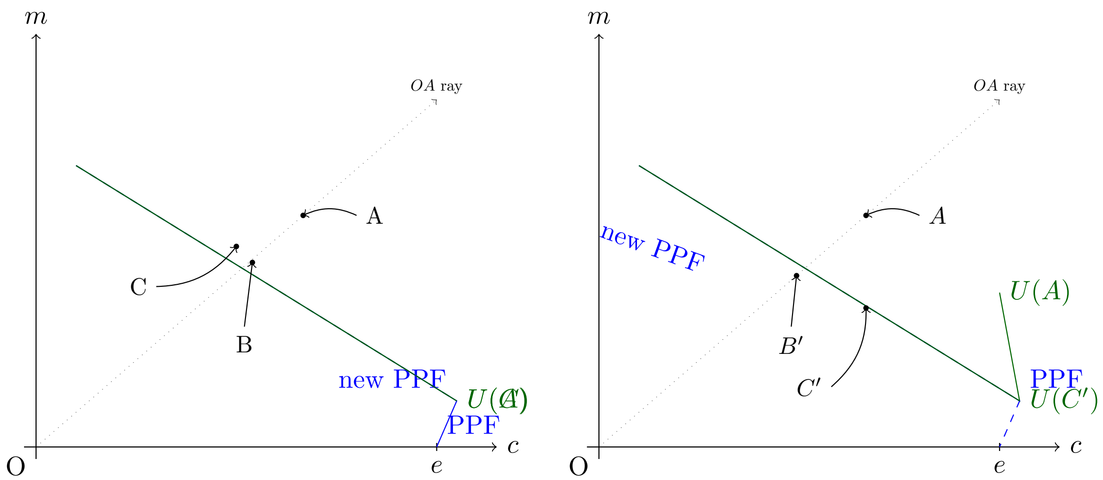
2 Terms-of-Trade Penalty: Partial Equilibrium

Bulow and Rogoff (1989) suggest that trade sanctions might sufficiently discourage default to support realistic levels of debt. They justify such trade disruptions by speculating that a country might rely on its reputation to maintain trade flows, which would be damaged by a default episode. They did not, however, model a trade channel penalty or make quantitative statements about its implications.

To specify this terms-of-trade penalty, we assume that sovereign default is punished by the temporary deterioration of the terms of trade. At its extreme, this would be an absolute embargo, which was not uncommon historically when countries refused to pay their creditors. In the 1861 Mexican default, creditors actually seized the port of Veracruz (see Todd (1991)): circumnavigating gunboats certainly increased the "iceberg costs" on Mexican exports. Though outright military force historically is only a minor force in punishing defaulters, our evidence suggests that *something* impedes trade for defaulters. When choosing to default, the sovereign understands that it is bringing higher trade costs upon its traders.

Our terms-of-trade penalty is a qualitatively important feature—one that allows the model to speak to an additional feature of the data, exchange rate depreciations. To see how this mechanism works, consider a simplified one-period model, where there is a consumer and two goods. The consumer can consume some of his endowment e, denoted by c, and use the "trading technology" to transform the rest into imported good m. The consumer has homothetic, monotone, and quasi-concave preferences over goods. The transformation technology $m = f(x)$ is concave and converts exports $x = e - c$ into imports m. The slope of indifference curve at the consumer's choice point is the real exchange rate. We want to compare the outcomes of different penalties: the negative shock to endowment of home production goods and the iceberg cost to importing. On Figure 1, solid lines represent the outcome before the penalty and dashed lines represent the outcome after the penalty.

Both panels of Figure 1 represent the same agent whose initial endowment is $(e, 0)$ and the choice before default is at point A. Because the preferences are homothetic, when the choice after penalty is above the OA ray, the slope of the indifference curve at tangent is steeper; the reverse is true when the new allocation is below the OA ray. We want to show that in the case of income penalty the new allocation is always in the upper region, and in the case of terms-of-trade allocation



(a) Income penalty

(b) Terms-of-trade penalty

Figure 1: Allocation change due to penalty

it's always in the lower region.

Figure 1a describes a situation when a lump sum is taken away from the endowment of the agent as a punishment for default. The new production possibility frontier (PPF) is a shift to the left of the old PPF. Let point B be the intersection of the new PPF and the homotheticity ray OA. At that point, the slope of new PPF is *bigger* than the slope of the indifference curve by concavity of f — the slope of new PPF here is equal to the slope of the old PPF at the point with the same ordinate, *below* point A along the old PPF. That means that the new optimal allocation, point C, will be *above* the homotheticity line, just like in Figure 1a. Therefore, the income penalty brings about currency appreciation.

Figure 1b represents a situation when a proportional imports penalty is imposed on the PPF, so the new PPF is the old PPF multiplied by $(1-\pi) < 1$. Analogously, the intersection of the new PPF and the homotheticity line OA is denoted by B'. The slope of a new PPF at $\hat{c}$ is π times the slope of the old PPF at the same abscissa, and therefore the slope of the new PPF is *smaller*. Every point along the old PPF to the left of A has a *smaller* slope of old PPF than at A by concavity of $f(\cdot)$. The slope of the old PPF at A is *equal* to the slope of the indifference curve at A by optimality. By homotheticity, the slope of indifference curve at B' is *equal* to the slope of the indifference curve at A. That means that at the point of $(\hat{c}, \hat{m})$ the slope of the new PPF is *smaller* than the slope of the indifference curve, which means that the consumer's choice will be *below* the homotheticity line along the new PPF, with smaller slope of the tangents at equilibrium, just like in Figure 1b. Therefore, the terms-of-trade penalty brings an exchange rate depreciation, as we observe in the data.

The existence of post-default exchange rate depreciation is a qualitative reason for using terms-of-trade penalty rather than a penalty directly to income. Naturally, there are also income effects at the default decision—agents do not have to repay both the debt and the interest—which might reverse the sign of exchange rate alteration. We verify the robustness of our argument by fitting a quantitative model's parameters to the data, which we present next.

3 The Model

Our model of default is a real business cycle model in the spirit of Eaton and Gersovitz (1981) and Arellano (2008), integrating a commodity space with both domestic and foreign goods. We describe a small open economy in which government internalizes its citizens' preferences over domestic goods, c_t, and imports, m_t. These goods are imperfect substitutes with constant elasticity, $\frac{1}{1-\kappa}$ in our formulation. The relative price of exports is the exchange rate, e_t. Imports are exchanged for exports, x_t by import firms according to $m_t = f(x_t)$. Their profit, Π_t, goes back to households.

Asset markets are incomplete with one period bonds serving as the only insurance against shocks to the income stream, y_t. The sovereign borrows b_t on behalf of its citizens from an international market paying a coupon q_t. Debt contracts are not enforceable, so the country may default. As punishment, the country suffers a deterioration in terms of trade and financial autarky for a random period of time.

The domestic country maximizes

$$U(c_t, m_t; \kappa, \alpha) \tag{1}$$

(where κ and α determine the relative preference for imports), subject to the income process and budget constraint

$$c_t + e_t m_t + b_t = y_t + q_t b_{t+1} + \Pi_t, \tag{2}$$

$$\log y_t = \rho \log y_{t-1} + \epsilon_t,\ \epsilon_t \sim \mathcal{N}(0, s^2). \tag{3}$$

The importer transforms exports to imports according to

$$m_t = f(x_t) \ : \ f'(\cdot) \geq 0, f''(\cdot) \leq 0. \tag{4}$$

In particular, we specify the import-export technology to be $f(x_t) = \theta_1(x_t - \theta_0)^\theta$. This technology is a reduced-form representation of the rest of the world's demand for domestic goods. For $\theta < 1$, the home country's exports receive diminishing returns in terms of imports. This is equivalent to assuming diminishing marginal utility of the country's goods to the foreign consumers. θ_0 allows for fixed costs to exporting, though in our calibration we find its value to be quite small.

Certainly, we could motivate the demand coming from the rest of the world (ROW) with a structural formulation for preferences. However, from the point of view of the small open economy, only the ROW demand can be observed, and this demand function is sufficient to pick the optimal quantity of exports. Both estimating and imposing an explicit form for the ROW utility seems equally treacherous. Calibrating the preferences of the ROW is daunting because one must aggregate structural preferences, but the weighting of the relative importance of a trade partner is only observed in equilibrium. Aesthetically, a full optimization by the ROW might be preferable, but we choose to keep our model focused and parsimonious; our contribution is not precisely how countries trade, but rather why they default.

Much of our analysis would go through with a simpler, linear technology, $f(x_t) = \theta_1 x_t$. With a linear technology, the trade channel penalty would cause depreciation in default, but the depreciation would be one-for-one with the size of the penalty. With a linear trade technology, we would essentially be directly imposing the size of depreciation, instead of allowing it to be determined endogenously by preferences. Our preferred form matches the stylized facts to a greater extent. In particular, with a linear technology the depreciation would be the same size every time, but Table 1 reveals that this is not the case. Furthermore, as we will show later, the trade data reject $\theta = 0$.

3.1 Timing

Time is infinite and discrete. At the beginning of each period, a country's state variables — income, borrowing to repay, and default status — are common knowledge. The first choice is whether to default; then the country chooses consumption, trade, and borrowing policies. If the country is in good standing, it faces a coupon price schedule from its creditors who will charge a premium over the fixed world interest rate based upon the probability of default in the next period.

If the country defaults, then it experiences a terms-of-trade deterioration before its trade policy is set. The equilibrium exchange rate clears the international trade markets. As in Eaton and Gersovitz (1981), countries in default are in financial autarky and their bond position reverts to zero. In subsequent periods, countries can leave this defaulted state with an exogenous probability of being "forgiven."

3.2 Representative Household and Government

The sovereign government internalizes the problem of the representative household. In particular, household preferences take the form $u(c_t, m_t; \kappa, \alpha) = [\alpha c_t^\kappa + (1-\alpha) m_t^\kappa]^{\frac{1-\sigma}{\kappa}}/(1-\sigma)$. We formulate the representative household problem recursively. Given income realization, y, bond position, b, and the indicator of whether it is being punished, taking as exogenous the price of bonds, $q(\cdot,\cdot)$ and imports, $e(\cdot)$, the government solves two subproblems: one if it defaults and one if it does not. The solution to the non-default problem is summarized by the value function $V(b, y)$ and the defaulter's problem has value function $W(y)$. The default decision is the argmax of these two functions, and the government's value function, $U(b, y)$, is the envelope over the subproblem value functions, $V(\cdot,\cdot), W(\cdot)$. Formally, the government solves the **Sovereign's Problem**:

$$U(b, y) = \max_{h \in \{0,1\}} \left\{ hW(y) + (1-h)V(b, y) \right\},$$

where $h \in \{0, 1\}$ indicates default. Conditional on not defaulting this period, the **Sovereign's Problem In Good Standing** is:

$$\begin{aligned} V(b, y) &= \max_{c,m,b'} u(c, m) + \beta EU(b', y'), \qquad (5) \\ \text{s.t. } c + em + b &= y + q(y, b')b' + \Pi, \\ \ln y' &= \rho \ln y + \epsilon,\ \epsilon \sim \mathcal{N}(0, s^2). \end{aligned}$$

The debt discount $q(y, b')$ adjusts the price of borrowing to accommodate the probability of default. By choosing $b' = b_0$, the country accepts the contract that gives it $q(y, b_0)b_0$ units of home good this period and takes away b_0 units of home goods next period. Using discounts instead of returns permits banks to decide not to lend at all, which may happen when default is nearly certain because of an extremely high level of debt, which implies an infinite interest rate.

If the country chooses to default, then its value function is an optimal solution to **The Sovereign's Problem In Default**:

$$\begin{aligned} W(y) &= \max_{c,m} u(c,m) + \beta E\left(\phi W(y') + (1-\phi)U(0,y')\right), \qquad (6) \\ \text{s.t. } c + em &= y + \Pi, \\ \ln y' &= \rho \ln y + \epsilon,\ \epsilon \sim \mathcal{N}(0,s^2). \end{aligned}$$

3.3 Importers

The importers face a trade demand given by equation (4) from the rest of the world. Note that the country does *not* face a perfectly elastic demand from the rest of the world. Instead, this is a "small country model" only in that the decisions of the country do not affect the demand of the rest of the world. In case of default, equation (4) is shifted to generate the terms-of-trade shock that serves as punishment.

Domestic importers take exports x from their countrymen at the price of the consumption good and exchange them for m, taking their price e as given. If the country is being punished for default, it receives fraction $1-\pi$ fewer imports. Their problem is summarized by **Importer's Problem**:

$$\begin{aligned} \Pi(x,m,h) &= em - x, \\ \text{s.t. } m &= (1-\pi)^h f(x). \end{aligned}$$

Notice that the trade channel penalty, $1-\pi$, enters as if there were a sudden increase in iceberg costs. This is a conscious choice, intended to expose defaulters to trade barriers that increase the cost of sending their goods abroad. For example, a country's exporters may find that its exports sit in customs longer after default, that foreign trade inspectors are more deliberate, or, more benignly, that favorable trade agreements are canceled; this can be an efficient way of adding real costs of exporting, and Hummels (2001) shows how more time en route can be a trade barrier. Alternatively, one can think of a country getting π less imports for the same amount of exports as a crude defaulted debt renegotiation process.

Figure 2 depicts the effect of the trade penalty, given the concave functional form of $f(x)$. The export quantity is a distance from disposable income to consumption of home-produced goods. The endogenous exchange rate, e, is the slope of the tangent.

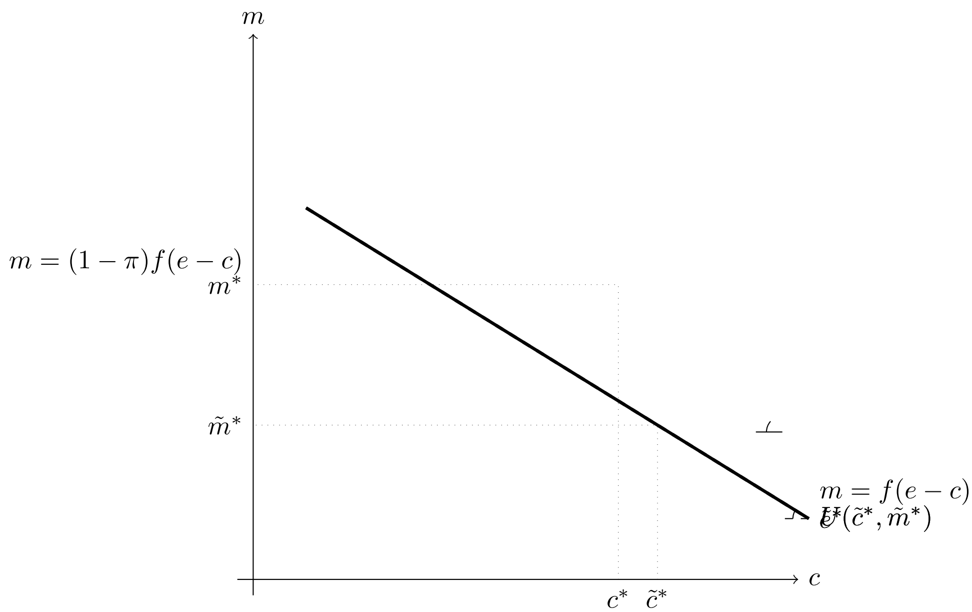


Figure 2: The export-import transformation function and the effect of the trade-channel penalty

3.4 International Financial Markets

Denote the default decision control $h(y, b) = \mathbb{I}(W(y) > V(y, b))$. It is equal to 1 when country announces default. World financial markets are risk-neutral but cannot enforce their debt contracts. They have perfect knowledge of the sovereign's problem, so they have zero profits and set $q(y, b')$ so that the expected return equals the international risk-free rate of return R. The credit market **Zero Profit Condition** is:

$$q(y, b') = \frac{1 - E[h(y', b')]}{1 + R}.$$

3.5 Recursive Equilibrium

The Recursive Competitive Equilibrium is a collection of

- choice functions $(c_V(y, b), b'_V(y, b), m_V(y, b))$ when the country is not in default,

- choice functions $(c_W(y,b), m_W(y,b))$ when the country is in default,
- default choice function $h(y,b)$,
- value functions $(V(y,b), W(y), U(y,b))$,
- choice functions when not in the default state $(x_{Vm}(y,b), m_{Vm}(y,b), \Pi_V(y,b))$
- choice functions in the default state $(x_{Wm}(y), m_{Wm}(y), \Pi_W(y))$,
- and prices $(e_V(y,b), e_W(y), q(y,b'))$

such that:

- $(c_V(y,b), b'_V(y,b), m_V(y,b))$ solve the **Sovereign's Problem In Default**, given $\Pi_V(y,b)$, $e_V(y,b)$ and $U(y,b)$, and $V(y,b)$ is the value function of this problem.
- $(c_W(y,b), m_W(y,b))$ solve the **Sovereign's Problem In Good Standing**, conditional on $\Pi_W(y)$, $e_W(y)$ and $U(y,b)$, and $W(y)$ is the value function of this problem.
- $(h(y,b))$ solve the Sovereign's Problem conditional on $V(y,b)$ and $W(y)$, and $U(y,b)$ is the value function of this problem.
- $(x_{Vm}(y,b), m_{Vm}(y,b))$ solve the Importer's Problem conditional on $e_V(y,b)$, and $\Pi_V(y,b)$ is the value function of this problem.
- $(x_{Wm}(y), m_{Wm}(y))$ solve the Importer's Problem conditional on $e_W(y)$, and $\Pi_W(y)$ is the value function of this problem.
- $q(y,b')$ satisfies the Zero Profit Condition given on $h(y,b)$.
- $e_V(y,b)$ is such that import market clearing condition $m_V(y,b) = m_{Vm}(y,b)$ holds.
- $e_W(y)$ is such that import market clearing condition $m_W(y) = m_{Wm}(y)$ holds.

Equilibrium exists for the same reason as in Arellano (2008); our problem is separable between borrowing and consumption. After the value of borrowing for the next period is chosen, the allocation of available income between consumption and imports is a maximizing monotone function upon compact set.

4 Quantitative Evaluations

To evaluate the predictive power of our model, we calibrate our baseline version to Argentine's historical data, following much related research. Argentina experienced three sovereign default events, in 1982, 1989, and 2002. None of the restructuring periods were particularly long. All these defaults were accompanied by nominal exchange rate depreciation and non-zero trade balances. The purpose of this numerical exercise is to find the size of depreciation at default implied by the structural estimates *without* using the depreciation as a moment in the estimation. Critically, an incorrectly specified model would result in either a too large or too small depreciation, and potentially rejecting the terms-of-trade penalty as a true penalty.

4.1 Data Description

Our parameter values come from our own estimates and Arellano (2008). Specifically, we estimated the parameters of income time series, the goods relative preference parameters, and the import-export relationship parameters; and we followed previous work for the values of R, β, ϕ and σ. We want to see what levels of borrowing and depreciation these parameters imply.

INDEC, National Institute of Statistics and Censuses, provides quarterly estimates of GDP composites, deseasonalized and in same-year prices, for 1993 through 2008. To calculate per capita values, we divide by the annual population of Argentina, obtained from the CIA Factbook. To convert the import data from peso expenditures into quantities of foreign goods, we used the real exchange rate taken from the European Central Bank website.[4]

The parameters that we used were estimated on the quarterly dataset from 1993 to 2008 and are provided in Table 2.

Most of our estimates are similar to previous estimates, though they could certainly be debated. For context on our trade parameters, especially the elasticity parameter κ, Ruhl (2003) is a nice discussion. Compared with other international business cycle models, our elasticity of substitution estimate is slightly high, which would tend to lessen the effect of our penalty and make it easier for countries to default.

[4] From Statistical Data Warehouse section, located at http://sdw.ecb.europa.eu/.

Parameter	Value	Explanation	Rationale
R	0.017	Unconditional expected rate of return required by international banking system.	Quarterly return on US 5 year bond
ρ	0.9849	Autocorrelation of log-output.	Estimated
s	0.0258	Standard deviation of log-output.	Estimated
π	Varying	Trade penalty for defaulting.	
ϕ	0.7180	Probability of not being forgiven in the next period.	Sandleris et al. (2004)
β	0.953	Subjective time discount factor.	Arellano (2008)
α	0.5859	Parameter of the instantaneous utility function; weight of home good consumption.	Estimated
κ	0.8447	Parameter of the instantaneous utility function; corresponds to 6.441 elasticity of substitution of export to import.	Estimated
σ	2	Parameter of the instantaneous utility function; corresponds to -1 elasticity of intertemporal substitution.	Arellano (2008)
θ	0.2082	Curvature of export-import transformation function.	Estimated
θ_0	0.0467	Relative position of export-import transformation function.	Estimated
θ_1	0.1959	Scale of export-import transformation function.	Estimated

Table 2: Parameter values, quarterly data.

For details on the estimation of our import-export parameters — $\theta, \theta_0, \theta_1$ — see Appendix B.3. We are not aware of another comparable estimate using macroeconomic data. However, Das et al. (2001), using firm-level data, found fixed costs for export supply close to zero. This is consistent with our value for θ_0. Hummels and Klenow (2005) analyzed the growth of exports from developing countries and found that up to 60% of the increase comes from the "extensive margin." Rather than deepening existing trade relationships, countries increase exports by diversifying their goods or partners. We take it as evidence that export markets quickly become satiated and justifies small θ.

4.2 Solving for the Recursive Equilibrium

Since competition in the home market is perfect, the solution to the equilibrium is the same as solving in the case of a centralized home economy in which the importer's problem is internal to the household. We call this the **Centralized Equilibrium** and describe it in Appendix A.

Every period the country faces an import-generating technology, $f(x)$ as in Equation 4, and allocates its consumption between imports and domestic production, as illustrated in Figure 2. Disposable income is $y + b - q(b'(y,b))b'(y,b)$ if the country can borrow and y if it cannot.

In the Argentine default example, the estimate of $\ln(1-\pi)$ is -0.6939, which implies $\pi \approx 0.5$. We take this estimate as illustrative, as there is no explicit international policy for punishing defaulters, so there is no guarantee that one estimate for π should hold in other contexts, or even be the same for the same country at different times. In otherwords, we do not want to interpret this value as structural. Instead, we evaluate the model's predictions at various punishment levels to obtain bounds on plausible outcomes . Varying the value of $\pi \in \{0.2, 0.5, 0.8\}$ also serves to demonstrate the effect of the trade channel punishment. We see that the country's default policies and behavior in default is sensitive to the magnitude of the trade channel punishment.

4.3 Equilibrium Results

We obtain three interesting equilibrium predictions. First, the country borrows more if it had a history of negative income shocks—i.e. high current borrowing. Second, the current account is countercyclical, in line with the data and with Aguiar and Gopinath (2006). Third, positive shocks to income depreciate the home currency. The borrowing policy result is immediate, as it is a consequence of risk aversion of a representative consumer. Most endogenous default papers have this result.

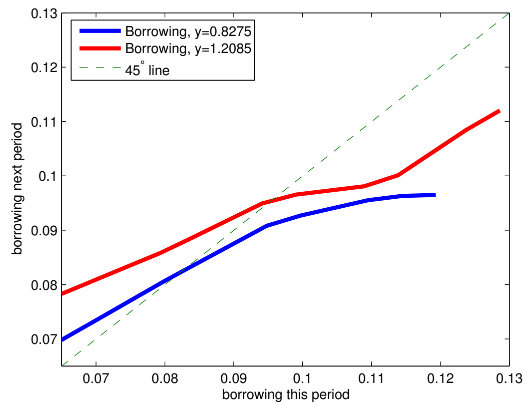


Figure 3: Borrowing policy. Curves stop on right at the point of default.

Our counter-cyclical current account (Figure 4) is consistent with data and, as Aguiar and Gopinath (2006) point out, nontrivial. In a partial equilibrium, positive shocks would induce savings and a positive current account; but if the interest rate is endogenous, it falls with positive income shocks (because the probability of default is counter-cyclical) and spurs borrowing. Arellano (2008) has a pro-cyclical current account, and Aguiar and Gopinath (2006) shows that in its framework, without demand for trade, the current account's correlation depends on the specification of the income process. In our model, the current account is governed by consumer preferences for foreign goods, the importing technology, and consumption smoothing. In this richer trade environment, the behavior of the current account does not just mirror the agents' borrowing policy. As our major contribution relies on the trading process that we model, we treat this result as a sign that our trading process properly approximates reality.

The counter-cyclical exchange rate is an uncommon result in the endogenous default literature as we discussed earlier, though others have established a similar result in data and Chari et al. (2002) shows that a fairly standard real business cycle can generate counter-cyclical real exchange rate movements. The intuition within our model is that with an increase in y, the country's possibility frontier in terms of (c, m) moves outwards. Consumption of both goods increases because the goods are normal. Since the marginal utility of consumers abroad goes down with additional exports, the exchange rate must rise with the increase in demand for foreign goods. It is important, however, to understand that foreign consumers in the model have no income shocks. To compare

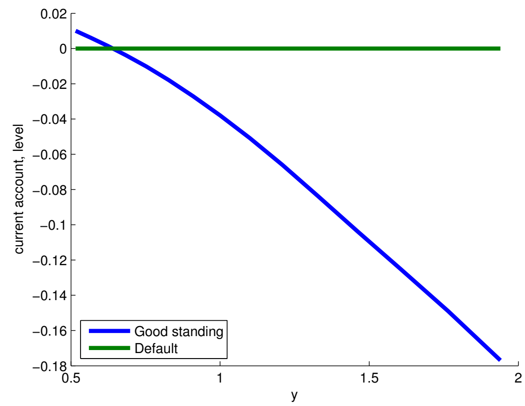


Figure 4: Current account conditional on output in stationary distribution.

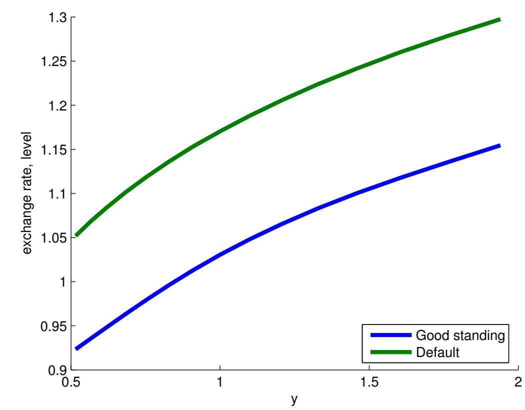


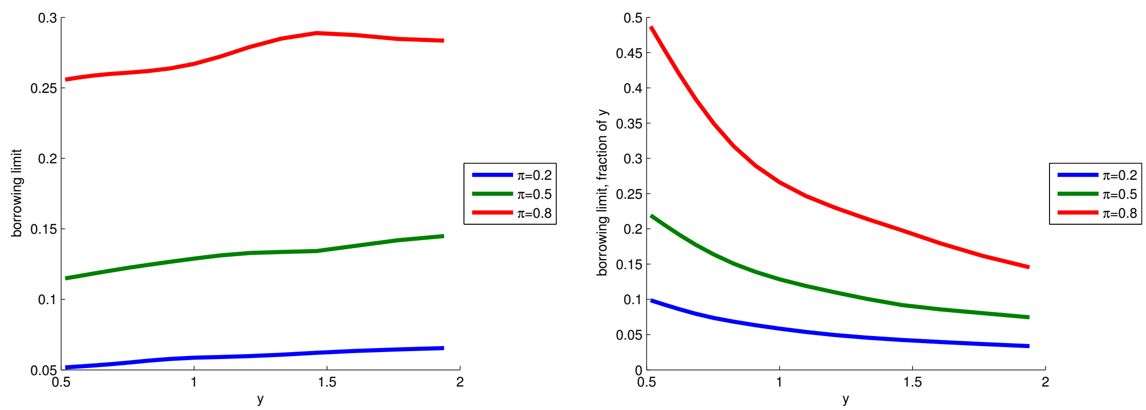
Figure 5: Exchange rate conditional on output in stationary distribution.

our result to the data, one must consider potential shifts in the world's demand function, which would manifest itself in Equation (4) and may shift the tangency point in Figure 2.

Notice also that the exchange rate is higher in default, though not $\frac{1}{1-\pi} = 2$ times higher, which happens because of the change in trade volumes due to substituting away from imports. The penalty shifts the line down, but the households are also able to adjust their consumption basket and, due to the curvature of Equation 4, this affects the exchange rate. As discussed earlier, we could generate a depreciation with linear trade technology, but the nonlinearity due to $\theta < 1$ allows for this additional freedom. As the data reveal, there is variation in the size of depreciation and trade volumes tend to shift, which are both consistent with our preferred parameterization.

4.4 The Effect Of Penalty Levels

Figure 6 depicts the default decision threshold: the debt is such that the country defaults if borrowing goes any higher. The small penalty case, $\pi = 0.2$, in which very little debt can be supported recalls the result of Bulow and Rogoff (1989). As in Aguiar and Gopinath (2006) and Arellano (2008), the sustainable debt increases in y which makes the interest rate spread counter-cyclical. The right panel of Figure 6 demonstrates that the borrowing limit does not increase as fast as y, so that an expansion does not imply that a borrower can support a higher percentage of debt.

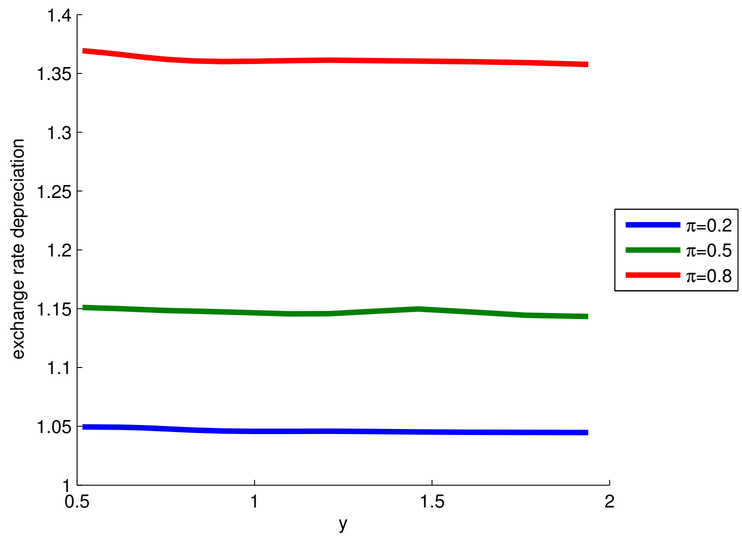


Note: Argentine has defaulted on debt of around 37% of GDP at the end of 2001; it's normalized detrended GDP was 0.95 at that time.

Figure 6: Default decision borrowing threshold.

The penalty value changes the exchange rate adjustment post default, as seen in Figure 7. The exchange rate change pictured is calculated as the difference between the exchange rate in a default state and the exchange rate just before the default. That is, if the country has a debt that leaves it indifferent between defaulting and remaining in good standing, what would be the difference of exchange rates in these two cases? This thought experiment about the moment of default is close to thinking about two otherwise equal countries, one slightly below the default threshold and one just above. Such a comparison isolates the change purely as a response to the default decision. The figure reveals that the level of penalty determines the size of the depreciation. However, the country moderates some welfare damage by substituting domestic product for import consumption. The amount of depreciation does not depend much on the country's position in the business cycle.

Figure 8 (left) illustrates the substitution behavior of domestic households. The country has



1.15 on this figure means that foreign goods cost 15% more after default than before default. Penn World Tables report 18% change of PPP index between 2001 and 2002 in Argentine.

Figure 7: Exchange rate drop due to default.

the same consumption policy at the moment of default independently of the level of π, which is why consumption policy before default is represented by a single line (this qualitatively holds for other parameterizations). As the trade channel penalty increases, the substitution effect becomes stronger. Figure 2 reveals the link between the change in the makeup of the consumption basket and the exchange rate that we highlighted earlier. Home good consumption is a normal good. In a default state, an increase in income leads to an increase in consumption; in the good standing state, higher income allows the country to borrow more, somewhat negating the decreasing returns to scale of the import-export relation. A positive shock to income in a defaulted country will lead to increased consumption of home goods; a positive shock in a non-defaulted country can lead to a more than one-for-one increase in exports.

Figure 8 (right) demonstrates the pattern of import's share in consumption. Decreasing returns to scale of the export-import relation make the dependence on y negative: countries in expansion export ever greater amounts to receive additional units of imports. The trade penalty has both a direct effect on the level of import (by construction) and an indirect effect through substitution. As in the data, proportional expenditure on imports falls following default; countries in expansion after default benefit from not servicing the debt; instead they can spend on consumption including im-

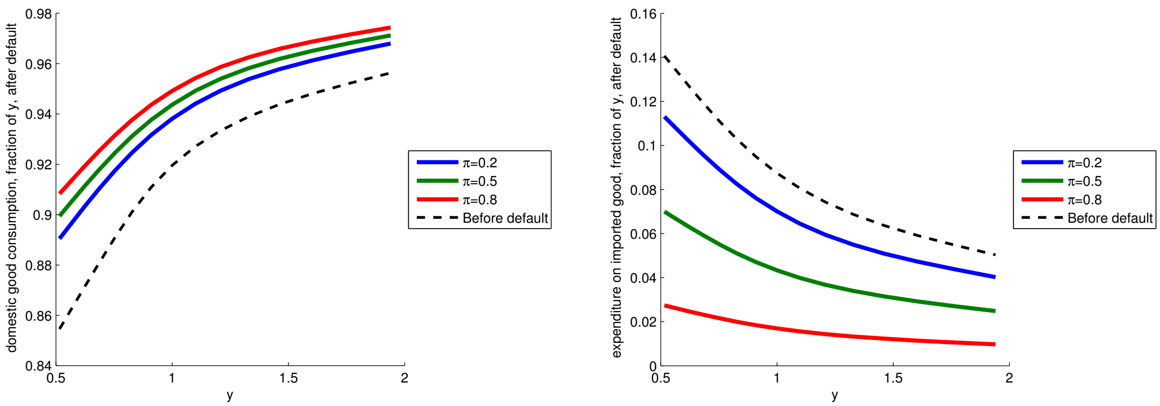


Figure 8: Domestic good consumption (left) and foreign good expenditure (right).

ports. Notice that the share of imports for high y countries exceeds $(1-\pi)$ times the corresponding value before default.

Figure 9 shows the change in capital account and trade balance at the moment of default decision. The trade balance improves more drastically in countries in downturn and the penalty level contributes significantly to the size of the change. The difference between the change in trade balance and change in capital account is the change in assets inflow. This might involve foreign currency reserves of the country's central bank. Particularly, it implies that a country that defaults in a cyclical expansion and faces $\pi = 0.8$ would have to come up with additional assets of up to 5% of GDP.

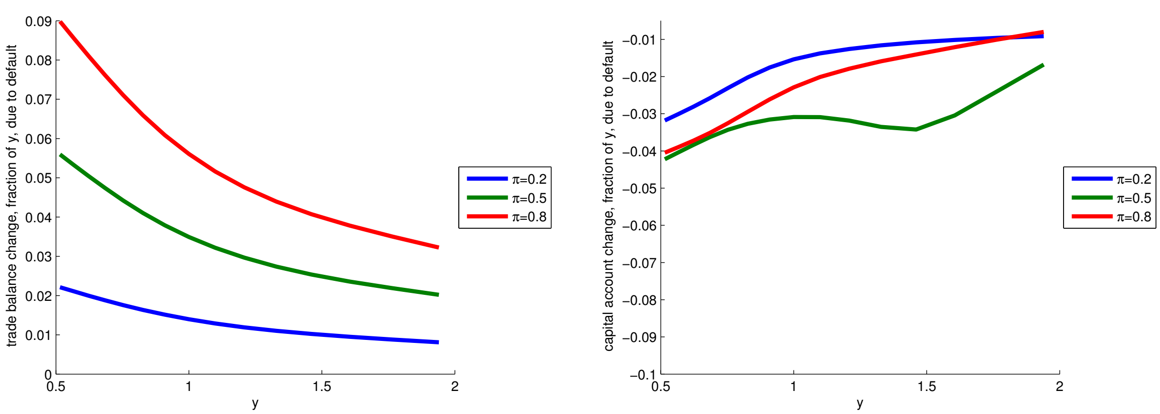


Figure 9: Trade balance (left) and capital account (right) changes at default.

To summarize, a higher trade channel penalty leads to: (a) greater household adjustment through the domestic consumption basket and trade quantities; (b) higher sustainable debt; and (c) greater exchange rate depreciation.

4.5 Comparative Statics

Figure 10 summarizes the differences in behavior as we look across parameters space. We consider are a change in preferences for imports (α decreased by 0.1), a change in ability to forgive (ϕ increased by 0.1), and a change in variance of income (s^2 doubled the value). The penalty π in the benchmark case is set at 0.5.

Changing α affects the willingness to default. A country that is less import-oriented defaults sooner and consumes fewer imports in all states. We can draw meaning from this finding in one of two ways: Countries with more reliance on imports might be able to sustain more debt, as default would be more painful. Or, this is a potential piece of some countries' motives to default, as a rapid change in tastes toward home consumption could precipitate default. Consider a government with debt just below the maximum allowable with a certain value for α; then, if this sovereign suddenly cared less about the import component of consumption, the level of allowable debt would shift down, possibly below the existing stock.

After default also, these parameters effect behavior. There is a stronger improvement in trade balance when α is lower, because a lower α country will forgo more home consumption in both relative and absolute terms. The lower the country's α, by consuming fewer imports, moves down the decreasing returns import-export technology and, in the aftermath of a default, can sell into a "steeper" market.

The variance of the income process does not notably affect the patterns of trade adjustment relation, either before or after default. However, it affects the default decision quite a bit. Countries in expansion are more motivated to sustain debt than countries near their average income both because they want to save to smooth consumption and also because imports are relatively more important. In recession, they highly value the ability to borrow. Thus, high-income-variance countries have a higher value from maintaining good standing and can sustain higher debt before defaulting.

Also, observe that the variance of the composition of import and home goods is higher when y is high, and therefore the impact of increased variance is stronger on the right tail of income distri-

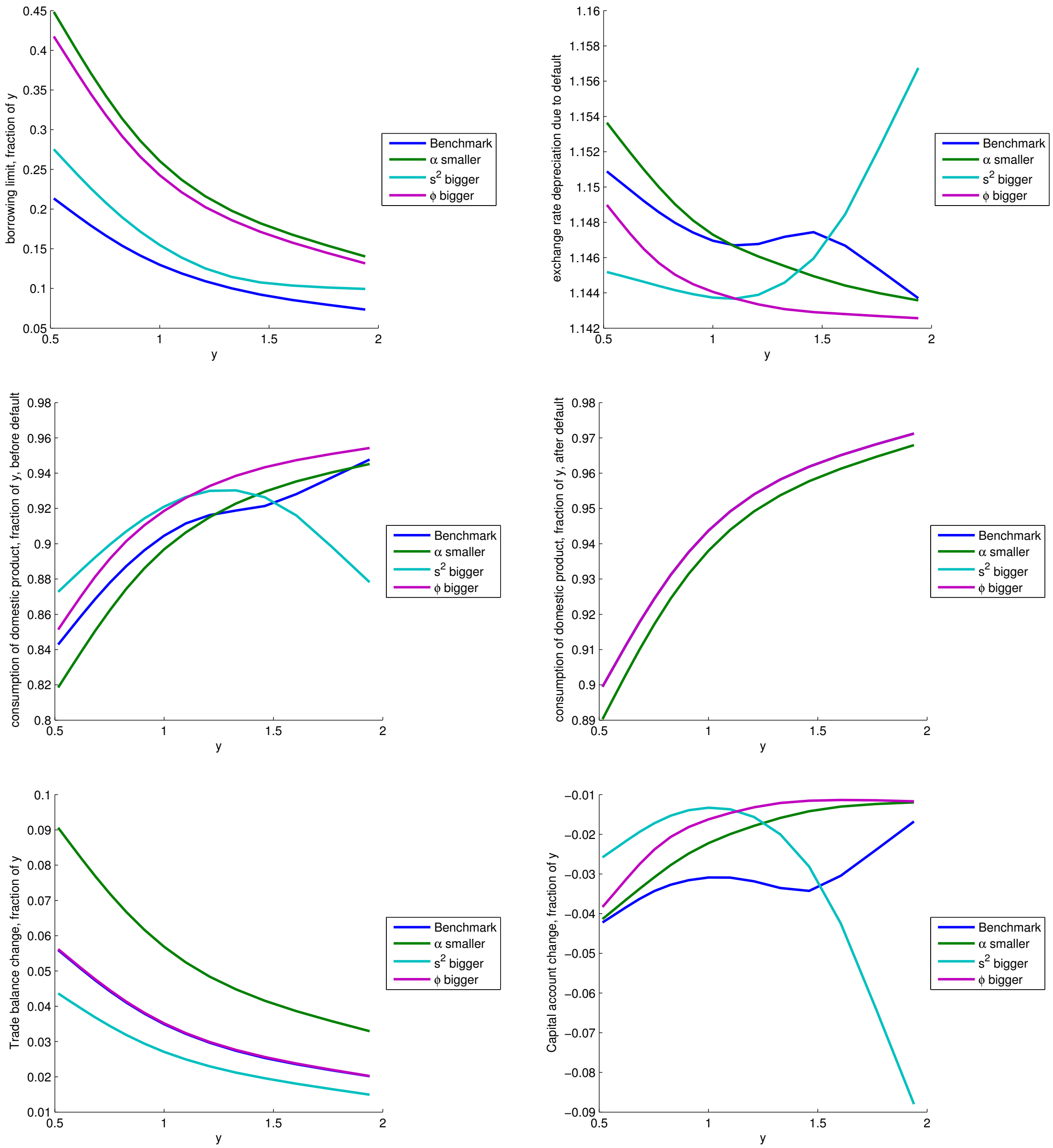


Figure 10: Comparative statics.

bution. That leads to greater aversion to financial autarky, and consequently, to bigger tolerance of debt. Therefore, countries with higher income volatility might actually be safer for investors than steady economies.

The value of ϕ affects the cost of default: The bigger is ϕ, the worse is a country's cost of default and this its aversion becomes even stronger if it is currently experiencing an expansion because imports and domestic goods are complementary. But, once the choice of defaulting has been made, ϕ does not affect other tradeoffs, and therefore does not change the policies significantly. It does not affect the consumption basket at all.

The top right plot of Figure 10 demonstrates that changing parameters do not change equilibrium depreciation much. In fact, only three parameters matter for depreciation: θ, κ and π. Other parameters, chosen to match Arellano (2008), do not affect equilibrium default depreciation, which arises the combination of CES utility function and the proportional penalty. This is basically a result of our separability: once assets and default have been chosen, then the trading and consumption basket are set.

In fact, all endogenous variables of the equilibrium are sensitive to the elasticity of substituion κ and to the productivity of the importing technology θ. The change in these parameters greatly change the default decision curve, which is almost unaffected by other parameters except π. It is clear why: when it is harder for consumers to substitute away from imported goods, the same size of π implies bigger welfare losses, and therefore the country agrees to respect the debt obligations.

5 Conclusion

Our paper explores the systematic linkage between sovereign default and real exchange rate depreciation. It introduces a model that links depreciation of the real exchange rate to the decision to default. In the model, countries interact with the rest of the world for two reasons: they borrow to smooth consumption over the business cycle and trade for goods that cannot be produced at home. They may default on their borrowing, but misbehaving on the international stage hurts their international trade. A country in default has to export more units of its own production for the same amount of imports. Consumers substitute away from the foreign good, and the new exchange

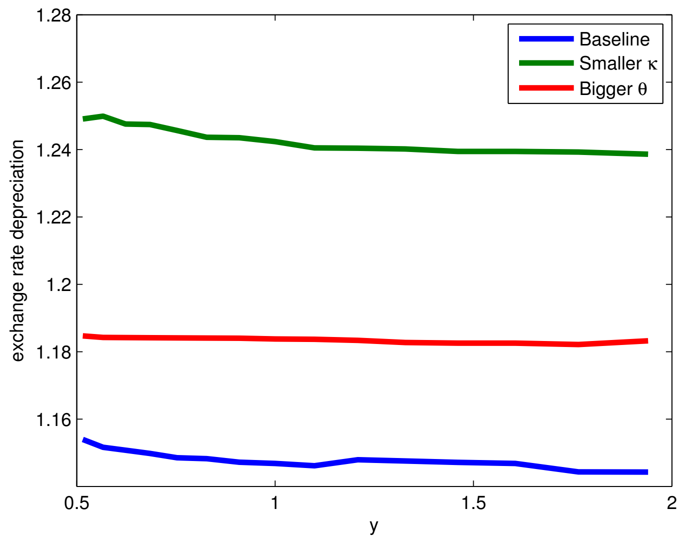


Figure 11: The effects of the changes in κ (-0.1) and θ $(+100\%)$, the curvatures of indifference curve and importing technology, on equilibrium depreciation at default.

rate, the equilibrium price of foreign goods, depreciates. Because trade disruption is costly in terms of welfare, it reduces a country's willingness to default and allows it to sustain positive debt.

Prior research has noted this empirical connection between default and trade, but we try to clarify its quantitative role in default. Other quantitative models capture the contribution of output fluctuations to the default strategy; however, they consider international markets only as imperfect insurance against stochastic income shocks, abstracting from the desire for differentiated goods from abroad. Here, we have highlighted the importance of international trade. Our trade channel penalty allows a country to support a realistic level of debt before default. Furthermore, the country's maximum borrowing limit is not necessarily increasing with income, a feature observed in the data but difficult to match with other models. We explore the model's response to changes in determinants such as income volatility and penalty duration. Greater reliance on trade and international borrowing markets both increase the debt a country is willing to sustain.

In our calibration exercise, we used to data from Argentina to demonstrate the predictions of our import-export mechanism. With the parameters from this calibration, the model generated reasonable predictions for the default decision and subsequent international adjustments. Future research might try to match other default episodes more closely. From this basic model, we can also integrate nominal fluctuations and financial flows to capture inflation and credit crunches. This paper, however, elides nominal concerns to emphasize and clarify the real factors behind the default-linked depreciation.

Hopefully, this paper can clarify how the level of international trade patterns affect the incentives to default. Within the model, more import-dependent countries default less frequently. From this result, that future work should more formally consider a mixed-model of default. A particularly promising avenue is to integrate parameter risk into endogenous variables, e.g. how does the probability of a political change effecting preferences for imported goods modify the interest rate's default premium? Our discussion also raises a new potential for models with heterogeneous agents and competing interests. Our simple model, considering trade and default, can be fruitfully combined with various other branches of the study of crises.

References

Aguiar, M. and G. Gopinath (2006): "Defaultable Debt, Interest Rates and the Current Account," *Journal of International Economics*, 69, 64–83.

Arellano, C. (2005): "Default Risk, the Real Exchange Rate and Income Fluctuations in Emerging Economies," in *2005 Meeting Papers*, Society for Economic Dynamics, 516.

——— (2008): "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 98, 690–712.

Arteta, C. and G. Hale (2008): "Sovereign debt crises and credit to the private sector," *Journal of International Economics*, 74, 53 – 69.

Asonuma, T. (2014): "Sovereign defaults, external debt and real exchange rate dynamics," MPRA Paper 55133, University Library of Munich, Germany.

Beers, D. T. and J. Chambers (2003): "Sovereign Defaults: Heading Lower into 2004," Research Report 09/2003, Standard and Poor's RatingsDirect.

Bulow, J. and K. Rogoff (1989): "Sovereign Debt: Is to Forgive to Forget?" *American Economic Review*, 79, 43–50.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2002): "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?" *Review of Economic Studies*, 69, 533–63.

Chatterjee, S. and B. Eyigungor (2012): "Maturity, indebtedness, and default risk," *American Economic Review*, 102, 2674–2699.

Cruces, J. J. and C. Trebesch (2013): "Sovereign Defaults: The Price of Haircuts," *American Economic Journal: Macroeconomics*, 5, 85–117.

Das, S., M. J. Roberts, and J. R. Tybout (2001): "Market Entry Costs, Producer Heterogeneity, and Export Dynamics," Nber working papers, National Bureau of Economic Research, Inc.

De Paoli, B. and G. Hoggarth (2006): "Costs of Sovereign Default," *Bank of England Quarterly Bulletin, Fall 2006.*

Eaton, J. and M. Gersovitz (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 48, 289 – 309.

Guerrieri, L., M. Iacoviello, and R. Minetti (2012): "Banks, Sovereign Debt and the International Transmission of Business Cycles," Working Paper 18303, National Bureau of Economic Research.

Hatchondo, J. C. and L. Martinez (2009): "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 79, 117–125.

Hummels, D. (2001): "Time as a trade barrier," Research report, Purdue University.

Hummels, D. and P. J. Klenow (2005): "The Variety and Quality of a Nation's Exports," *American Economic Review*, 95, 704–723.

Martinez, J. V. and G. Sandleris (2011): "Is it punishment? Sovereign defaults and the decline in trade," *Journal of International Money and Finance*, 30, 909–930.

Mendoza, E. G. and V. Z. Yue (2012): "A General Equilibrium Model of Sovereign Default and Business Cycles," *The Quarterly Journal of Economics*, 127, 889–946.

Rose, A. K. (2005): "One Reason Countries Pay Their Debts: Renegotiation and International Trade," *Journal of Development Economics*, 77, 189 – 206.

Ruhl, K. J. (2003): "Solving the Elasticity Puzzle in International Economics," mimeo, University of Minnesota.

Sandleris, G., G. R. Gelos, and R. Sahay (2004): "Sovereign Borrowing by Developing Countries: What Determines Market Access?" IMF Working Papers 04/221, International Monetary Fund.

Todd, W. F. (1991): "A History of International Lending," in *Research in Financial Services*, ed. by G. Kaufman, Greenwich, Conn.: JAI Press, vol. 3, 201–289.

Tomz, M. (2007): *Reputation and International Cooperation: Sovereign Debt across Three Centuries*, Princeton, NJ: Princeton University Press.

Tomz, M. and M. L. J. Wright (2007): "Do Countries Default in "Bad Times"?" *Journal of the European Economic Association*, 5, 352–360.

Wright, M. L. J. (2001): "Limited commitment: Theory with applications to repudiation risk," Ph.D. thesis, University of Chicago.

A Centralized Equilibrium

At the beginning of the game consumer chooses whether he wants to default or not is **Household's New Problem**:

$$U(b, y) = \max_{h \in \{0,1\}} hW(y) + (1-h)V(b, y).$$

The borrowers's problem conditional on not defaulting this period is **New Problem With No Default**:

$$\begin{aligned} V(b, y) &= \max_{c,x,m,b'} u(c, m) + \beta EU(b', y'), \\ \text{s.t.} & \\ c + x + b &= y + q(y, b')b', \\ m &= f(x), \\ \ln y' &= \rho \ln y + \epsilon, \ \epsilon \sim \mathcal{N}(0, s^2). \end{aligned} \tag{7}$$

If country chooses to default, then its value function is a solution to **New Problem In Default**:

$$\begin{aligned} W(y) &= \max_{c,x,m} u(c, m) + \beta E \left(\phi W(y') + (1-\phi) U(0, y') \right), \\ \text{s.t.} & \\ c + x &= y + \Pi, \\ m &= (1-\pi) f(x), \\ \ln y' &= \rho \ln y + \epsilon, \ \epsilon \sim \mathcal{N}(0, s^2). \end{aligned} \tag{8}$$

Combined with the Zero Profit Condition, solution to this problem will give the same values as the Equilibrium we want to study. However, it does not give the value of the exchange rate. We recover this from the first-order conditions to the New household choice problems:

$$e_V(y, b) = \frac{\partial u(c_V(y, b), m_V(y, b))}{\partial m} / \frac{\partial u(c_V(y, b), m_V(y, b))}{\partial c}$$

$$e_W(y) = \frac{\partial u(c_W(y), m_W(y))}{\partial m} / \frac{\partial u(c_W(y), m_W(y))}{\partial c}.$$

B Estimation

To test our model's implications, we estimated various parameters using Argentine data. In many cases, we could have better characterized the Argentine data generating process with different parametric functional forms, but this would distract from the model we introduce. Our goal is not to match Argentine perfectly, though there is a certain virtue in doing so, but we fear that additional complexity will impede one's intuition for the model. Rather we decided to stay in the simple world of AR(1) processes and (relatively) linear functions to provide a reasonable test of the quantitative implications of our model .

B.1 Output Time Series

Output y was calibrated to quarterly deseasoned per-capita GDP in constant prices, which was assumed to follow AR(1) process. It was not detrended because trend seems to be too small. Regression equation is

$$\ln y_t - m = \rho(\ln y_{t-1} - m) + \epsilon_t, \epsilon_t \sim N(0, s^2).$$

m is the normalizing coefficient. Estimates are following:

Variable	**Coefficient**	**(Std. Err.)**
m	9.0669*	(0.1464)
ρ	0.9878*	(0.0081)
s	0.0258	

Table 3: Estimation results, output dynamics, One star denotes 1% significance.

A Dickey-Fuller test does not reject a unit root hypothesis for this process, therefore, given standard deviations may be biased downward. Controlling a linear trend does not help to reject the unit root hypothesis, nor does it give much different estimates of other parameters.

B.2 Consumer's Utility Parameters

Utility function parameters were estimated from the first-order condition of consumer. c was taken to be equal to consumption (both private and public) plus investment from INDEC data. Nominal imports quantity was deduced from import value from INDEC data divided by the exchange rate, obtained from European Bank.

$$\ln e = \ln \frac{1-\alpha}{\alpha} + (\kappa - 1) \left(\ln \frac{c/y}{m/y} \right).$$

Estimates are following:

Table 4: Estimation results, output dynamics.

Variable	**Coefficient**	**(Std. Err.)**
$1-\kappa$	0.1553*	(0.0440)
$\ln \frac{1-\alpha}{\alpha}$	-0.3472*	(0.0972)

Within a 95% confidence interval, we can conclude that $\kappa < 1$, so goods are not perfect substitutes, and $\alpha > 0.5$. With the obvious transformations of the estimation in Table 4, $\kappa = 0.8447$ and $\alpha = 0.5859$. Again, the regression is not perfect. An ARIMA(2,1,1) model specification seems to perform better to correct for the nonstationarity for which exchange rates are notorious.

B.3 Import-Export Equation

The most interesting regression seems to be the trade equation. It was estimated by nonlinear least squares:

$$\ln m_t = \ln(1-\pi) I(\text{punished at } t) + \ln \theta_1 + \theta \ln (x_t - \theta_0) + \epsilon_t.$$

Here we decided to allow the import-export equation to be nonlinear (θ not necessarily equal to 1), have a fixed cost (θ_0 is not necessarily equal to 0). We do not explore why it happens that firms can earn positive profits in equilibrium; we just allow estimates to signal us about that. These degrees of freedom are not necessary, and they don't drive our main result. However, they certainly help to achieve better fit of default responses.

For numbers on imports and exports, we smoothed quarterly fluctuations on the INDEC data using an HP filter with smoothing parameter 400.

Table 5: Estimation results, import-export conditions.

Variable	**Coefficient**	**(Std. Err.)**
θ	0.2082*	(0.0763)
θ_0	0.0467*	(0.0070)
θ_1	0.1959*	(0.0409)
$\ln(1-\pi)$	-0.6939*	(0.0256)

The estimate of $\ln(1-\pi)$ suggests that π is equal to 0.5 with surprising precision. We will use this number in model outcome calculation; to make sure we don't fall victim of regression's non-robustness, we also solve for the model with $\pi = 0.8$ and $\pi = 0.2$, so whatever is the real value of π, the real outcome will be lying in between these two models' outcomes. As for values of θs, they are not too far away from the ones a person can get by estimating the 1993-2000 subsample of "no default."

We estimate this equation on a subsample of 1993-2003. The reason why we do not continue on the sample of 2004 and further is that it seems that Argentine does not have the same instant recovery of terms of trade after default as we have assumed in the model. Consider the time series of $u_t = \frac{\Delta m_t}{\Delta x_t}\frac{x_t}{m_t}$, a measure of elasticity of change in import with the change of export, presented on Figure 12. One can see an approximately constant elasticity during most of 1990s, then a decline, a series of definite changes in structure of equation, and an increase in 2004, with a stable more than 2 elasticity after default. The big jump down in 2001 is what we try to capture with $(1-\pi)$ multiplier. In this estimation, we allow for a period of adjustment, a process of re-establishing of connections lost in 2001. Imports do not only increase due to an increase in x, but also from a renewed efficiency of trading. Summarizing, we don't use the data after default because we believe that after forgiving the default import depends not only on export on the same period, but also on export on previous periods.

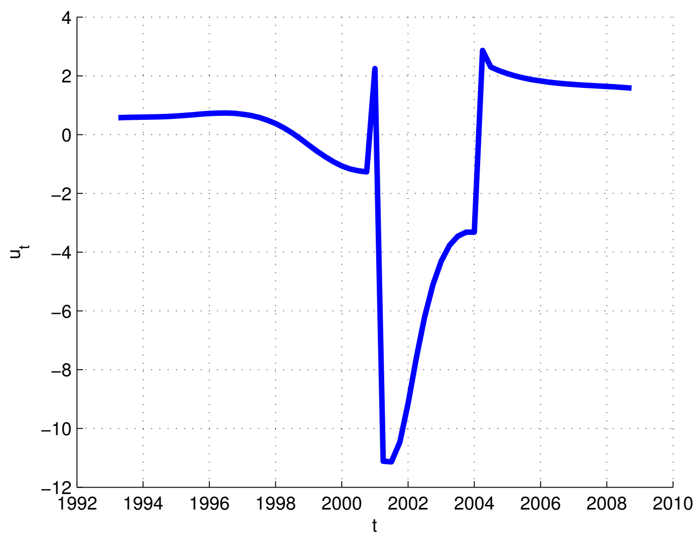


Figure 12: Time series of $u_t = \frac{\Delta m_t}{\Delta x_t} \frac{x_t}{m_t}$, "sample elasticity."