ECONOMIC RESEARCH
FEDERAL RESERVE BANK OF ST. LOUIS

WORKING PAPER SERIES

Explaining Educational Attainment Across Countries and over Time

Authors	Diego Restuccia, and Guillaume Vandenbroucke
Working Paper Number	2014-048A
Creation Date	November 2014
Citable Link	https://doi.org/10.20955/wp.2014.048
Suggested Citation	Restuccia, D., Vandenbroucke, G., 2014; Explaining Educational Attainment Across Countries and over Time, Federal Reserve Bank of St. Louis Working Paper 2014-048. URL https://doi.org/10.20955/wp.2014.048

Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

Explaining Educational Attainment across Countries and over Time[†]

Diego Restuccia
University of Toronto[*]

Guillaume Vandenbroucke
Federal Reserve Bank of St. Louis[**]

November 2014

Abstract

Consider the following facts. In 1950, the richest countries attained an average of 8 years of schooling whereas the poorest countries 1.3 years, a large 6-fold difference. By 2005, the difference in schooling declined to 2-fold because schooling increased faster in poor than in rich countries. What explains educational attainment differences across countries and their evolution over time? We consider an otherwise standard model of schooling featuring non-homothetic preferences and a labor supply margin to assess the quantitative contribution of productivity and life expectancy in explaining educational attainment. A calibrated version of the model accounts for 90 percent of the difference in schooling levels in 1950 between rich and poor countries and 71 percent of the faster increase in schooling over time in poor relative to rich countries. These results suggest an alternative view of the determinants of low education in developing countries that is based on low productivity.

Keywords: schooling, productivity, life expectancy, labor supply.
JEL codes: O1, O4, E24, J22, J24.

[†]Forthcoming in the *Review of Economic Dynamics*. We thank Margarida Duarte, Chad Jones, Nicola Pavoni, Todd Schoellman, Gianluca Violante, and participants at several seminars and conferences for comments. All remaining errors are our own. Restuccia gratefully acknowledges the financial support from the Social Sciences and Humanities Research Council of Canada. The views expressed in this work are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

[*]Department of Economics, University of Toronto, 150 St. George Street, Toronto, ON M5S 3G7, Canada. E-mail: `diego.restuccia@utoronto.ca`.

[**]Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63166-0442, USA. E-mail: `guillaumevdb@gmail.com`.

1 Introduction

Human capital accumulation is believed to play a crucial role in understanding income differences across countries, although a precise assessment of this importance has been hindered by the lack of empirical measures of human capital.[1] A key component of human capital formation is investment in schooling. Cross-country data on average years of schooling indicates that although educational attainment is substantially lower in poor countries relative to rich countries, over time poor countries have increased schooling faster than rich countries. In particular, the cross-country income elasticity of schooling declined from 0.6 in 1950 to 0.3 in 2005, a drop by half in the conditional dispersion of schooling. To account for these facts, we develop an otherwise standard frictionless model of human capital accumulation extended to allow for non-homothetic preferences and an operating labor supply margin. We use the model to assess the quantitative significance of differences in productivity and life expectancy in explaining educational attainment across countries and over time.

We combine education data from Barro and Lee (2010) and income per capita data from the Conference Board (2010) to construct a panel dataset for 84 countries from 1950 to 2005. We emphasize three facts. First, there are large differences in schooling measures across countries, with average schooling being 8 years in rich countries and only 1 year in poor countries in 1950 (11 and 5 years in rich and poor countries in 2005). Second, average years of schooling have increased over time in all countries in our sample. Third, average years of schooling have increased more in poor than in rich countries. Hence, dispersion in schooling levels has decreased overtime. What can explain schooling differences across countries and their evolution over time?

To answer this question, we develop a model of human capital accumulation. The basic features of the model are standard and build closely from Bils and Klenow (2000).[2] Two features of the

[1] Important exceptions include Hendricks (2002) and Schoellman (2012). See also recent quantitative work by Manuelli and Seshadri (2006) and Erosa et al. (2010).

[2] The model builds broadly from the seminal works of Becker (1962), Ben-Porath (1967), and Mincer (1974).

model deserve special attention: a dominant income effect and endogenous labor supply. First, the dominant income effect in the model derives from non-homothetic preferences. We note, in Section 3, that the non-homotheticity per se is not essential. We adopt this specification because such preferences have become central in theories of the allocation of labor across sectors associated with structural change, and we argue are important in understanding the allocation of time between the production of goods and schooling across rich and poor countries.[3] Second, the endogeneity of labor supply is important because, as the available data from the International Labor Organization show, there are large cross-country differences in hours of work –average hours are lower in rich than in poor countries– thereby affecting schooling decisions. Broadly speaking, consider schooling as a time-allocation problem whereby a unit of time is allocated between the production of consumption goods, schooling, and leisure. Then, with non-homothetic preferences, increases in productivity lead to a reallocation of time away from the production of goods into schooling and leisure. Other things equal, abstracting from leisure over estimates the impact of increases in productivity on schooling (and hence over estimates the income elasticity of schooling). Other authors, such as Heckman (1976) and Blinder and Weiss (1976), have emphasized the importance of jointly modeling labor supply and human capital accumulation in bringing additional quantitative discipline to human capital models.

Our strategy to discipline the forces in the model is simple. We calibrate a benchmark version of our model to fit a long time series for schooling and hours in the United States. The calibration puts restrictions on the strength of the income effect in the model. In the data for the United States there is substantial variation over time in hours of work, schooling, and income so that our calibration strategy provides discipline to the income effect in the model.[4]

A recent quantitative version of these models applied to cross sectional inequality in the United States include for example Guvenen and Kuruscu (2010).

[3]For applications in development, see for instance Laitner (2000), Kongsamut et al. (2001), Gollin et al. (2002), and Duarte and Restuccia (2010). Other applications include the changing patterns of trade, e.g. Markusen (2010) and Fieler (2011); the study of broader transformations in an economy, e.g., Greenwood and Seshadri (2005) among others.

[4]Over the period we analyze, GDP per capita in the United States increased by a factor of 10 and average years of schooling increased by a factor of 2. The factor difference between the richest and poorest ten-percent of countries in 1950 is 16-fold in GDP per capita and 6-fold in average years of schooling.

We then perform a cross-country quantitative experiment to assess the ability of the model in explaining schooling levels across countries and their evolution over time. In our quantitative experiment, we allow the levels of productivity and life expectancy to vary over time and across countries. We discipline these elements by reproducing the cross-country distribution of GDP per capita in 1950 and 2005 as well as the cross-country relationship between life expectancy and income in 1950 and 2005. The main result is that the model is consistent with the three facts emphasized earlier in the cross-country and time-series data. In particular, the model generates substantial dispersion in schooling levels across countries: in 1950, the model accounts for 90 percent of the difference in schooling between rich and poor countries. In addition, the model implies a faster increase in schooling for poor countries than for rich countries and, therefore, is consistent with the convergence in schooling levels observed in the cross-country data.

Our results have important implications for at least two strands of the literature on education. First, the model implies that the faster increase in schooling in poor relative to rich countries is associated with a stronger decline in hours of work and hence, even though human capital increases as a result of schooling investment, per capita income may or may not increase relative to rich countries. This suggest that the empirical relationship between schooling and per capita income growth across countries, as pioneered for example by Benhabib and Spiegel (1994), does not provide an accurate assessment of the importance of human capital for development. Second, our results suggest that improving education and welfare in poor countries hinges more on solving their productivity gap with rich countries than pursuing often emphasized educational policies aimed at solving institutional and other frictions.[5] Even though there is room for other factors to be relevant such as credit constraints for investment in education, restrictions on school infrastructure, aid and policy, among others; our results stress the need for greater focus on the productivity problem in poor countries.

Our paper relates to a large literature in macroeconomics and development addressing the disparities in schooling levels across countries. Within this literature, the closest paper to ours

[5] See, for instance, Duflo (2001).

is Bils and Klenow (2000) who also emphasize differences in productivity and life expectancy. A key difference is that whereas Bils and Klenow (2000) mainly focus on the cross-sectional correlation of schooling and per-capita income growth found in the empirical literature, for instance Barro (1991), we focus on a broader dimension of the data, namely the level and time-series differences in schooling across countries. We also depart from Bils and Klenow (2000)'s framework by allowing for an income effect through non-homothetic preferences and for hours of work differences across countries. These departures are critical in understanding the convergence pattern in educational attainment across countries.[6] Broadly speaking, the main focus of this literature is on differences in schooling at a point in time and, as a result, most of the existing frameworks are not designed to address the evolution of schooling over time. One important exception is the study of Manuelli and Seshadri (2009) that looks at the evolution of education for a subset of Asian and Latin American countries. Manuelli and Seshadri (2009) consider a standard model of human capital –a version of the Ben-Porath framework– where increases in productivity levels lead to increases in average years of schooling. Hence, not surprisingly, these results are consistent with the evidence for East Asia but not for Latin America since for Latin America countries whereas schooling years have converged to the level of the United States, productivity levels have not.[7]

In emphasizing the connection between life expectancy, education, and growth, our paper relates to the literature on life expectancy and human capital accumulation.[8] By modeling labor supply, our results rationalize the findings in Acemoglu and Johnson (2007) of a strong relationship between life expectancy and education across countries but a much weaker relationship

[6]Cordoba and Ripoll (2013) consider a model where fertility, mortality, credit constraints, and access to public education drive schooling differences across countries at a point in time. We complement this work by assessing the quantitative contribution of productivity and life expectancy to schooling differences across countries and over time in a framework without frictions.

[7]Our paper also relates to a recent literature in macroeconomics assessing the role of human capital in development, for instance Manuelli and Seshadri (2006) and Erosa et al. (2010). The focus of this literature is on the amplification effect of human capital in explaining income gaps across countries. Our framework abstracts from features that generate amplification effects since this is not our focus. Incorporating amplification effects in the model would reduce the size of productivity gaps needed to reproduce income differences across countries in the quantitative experiments without altering our main findings.

[8]See for instance Soares (2005), Jayachandran and Lleras-Muney (2009), and Hazan (2009).

with income per capita. Our analysis emphasizes the importance of income effects (from non-homothetic preferences) in poor countries and as a result it relates closely to the time-series and cross-sectional implications of Engel's law within an economy, e.g. Ogaki (1992). While our paper focuses on the cross-country relationship across points in time using a representative agent framework, the mechanism emphasized in the model have cross-sectional implications as well. We do not pursue these implications in this paper but we know from our own subsequent work using a similar but simpler model in Restuccia and Vandenbroucke (2013a), which builds on the present paper, that these implications are consistent with more disaggregated observations in the United States such as hours of work differences for individuals across the wage distribution and hours and education differences across races. There is a large literature emphasizing the role of skill-biased technological progress in determining education and wage inequality in the second half of the twentieth century in the United States.[9] Cross-country differences in skill-biased technology or returns may be important in understanding education outcomes across countries. However, to maintain our focus on income effects in the cross-country analysis, we abstract from such issues in this paper.

The paper is organized as follows. In the next section, we present the facts from a panel dataset of 84 countries from 1950 to 2005 for a measure of educational attainment and income. Section 3 presents the model. In Section 4 we calibrate the model. Section 5 performs a quantitative analysis of cross-country differences productivity and life expectancy in explaining the patterns in the panel data. We conclude in Section 6.

2 Facts

Data We construct a panel dataset of schooling and income as follows. We obtain average years of schooling for the population aged 25 to 29 from Barro and Lee (2010). We restrict

[9] See for instance Acemoglu (2002) and Violante (2008) for surveys.

the sample to the narrow age population to minimize the impact of demographics and other changes on schooling measures across time and space. It is also the definition that is best suited for the historical data on educational attainment we use for the United States and for the model we consider in Section 3. The schooling data is available for a large set of countries from 1950 to 2005 in 5 year intervals. We obtain gross domestic product (GDP) per capita from the Conference Board (2010), Total Economy Database. We restrict the time frame of this data from 1950 to 2005. To abstract from short-run fluctuations in real GDP we filter it using the Hodrick and Prescott (1997) filter with $\lambda = 100$ for yearly observations and keep the trend component of these time series. When we merge these two sets of data, we end up with a sample of 84 countries that have available data for schooling and GDP per capita from 1950 to 2005.[10]

Facts We emphasize three sets of facts that arise from analyzing these data. First, schooling differences across countries are large at any point in time between 1950 to 2005. Second, schooling increases over time in all countries in our sample. Third, schooling differences across countries are smaller in 2005 than they were in 1950. The reduction in schooling differences across countries is systematic and occurs despite the fact that the income gap between rich and poor countries has not generally decreased. We now document these facts in detail.

1. There are large differences in educational attainment across countries.

   Consider Table 1 which decomposes our sample into ten groups of countries according to the 1950 distribution of GDP per capita –i.e., the countries in each decile are the same in 1950 and in 2005. For each decile, the table reports the average GDP per capita relative to the United States and the average years of schooling. In 1950 there is a 6-fold difference in schooling between the richest and poorest decile of the distribution. In 2005 there remains a noticeable 2-fold difference. These differences are not specific to the top

[10] Our sample of 84 countries comprises a fairly representative set of the world's income distribution. For instance, the factor difference in GDP per capita between the richest and poorest five-percent of countries is 25-fold which is comparable to many previous studies.

and bottom decile and/or to the initial and end year of our sample. Figure 1 documents them across all countries, for selected years, and shows that there has been a significant dispersion of schooling across all levels of income and at all dates. To put the magnitude of cross-country differences in schooling in perspective, consider that in 1900 in the United States a 35-year old had completed about 7.4 years of schooling. Hence, a 25-29 year old in 2005 in the average poor country still had 2 years less of schooling than a 35-year old in the United Sates in 1900.[11]

2. Educational attainment increased over time in all countries.

   Schooling increased between 1950 and 2005 for every country in our sample. Table 1 conveys an aggregated view of this fact since average years of schooling increase for each decile of the distribution. The increase in schooling between 1950 and 2005 is 37 percent for countries in the top decile and 299 percent for countries in the bottom decile. We note that the increase in educational attainment is positive for all deciles of the income distribution regardless of the initial income level or subsequent income growth relative to the United States. We expand on this fact next.

3. Differences in educational attainment across countries have been reduced substantially over time.

   Poor countries exhibit a tendency to increase their schooling faster than rich countries. In Table 1 this is evidenced by the tendency of the 2005-to-1950 ratio of schooling (last column) to decrease with relative income. This is a remarkable finding given that for some deciles, such as the second and the fourth, relative income did not change between 1950 and 2005. For deciles such as the third or the fifth relative income increased and for the tenth decile relative income decreased. Yet, each group of countries experienced a substantial increase in schooling. A more complete and systematic documentation of the decline in schooling dispersion across countries is to report for each year the cross-

[11] The figure 7.4 years of schooling for the average 35-year old is from Goldin and Katz (2008). Table 1 shows that the years of schooling for the average 25-29 year old in the average poor country is 5.07 in 2005.

sectional elasticity of schooling to income levels, as in Figure 2.[12] This elasticity decreases systematically over time. For instance, whereas two countries that differ in income per capita by one percent have in average a 0.6% difference in schooling in 1950, their schooling difference is reduced by half to 0.3% in 2005. The same declining pattern is observed for the time-series elasticity, that is the elasticity of income levels to schooling for each country over time, although with only 12 observations per country the pattern has more noise.

In summary, even though there are still large differences in educational attainment across countries, we find that these differences have been systematically reduced with poor countries increasing their educational attainment over time faster than rich countries. While we have reported these facts for individuals 25-29 years of age, we emphasize that the facts are robust to other age categories and for males and females. Moreover, the convergence pattern is also robust to a broader set of countries.[13]

3 The Model

Time is continuous. The world comprises a set of closed economies. In what follows we focus on a single economy to describe the model. At every moment a generation of homogeneous individuals of size one is born and lives for an interval of time of length T_τ. The index τ denotes a generation, that is the date at which an individual is of age 0. We use t to refer to calendar time. Individuals are endowed with one unit of productive time at each moment and no assets at birth. There is a worldwide rate of interest r, which we assume to be equal to the rate of time discount ρ, and at which individuals can freely borrow and lend. There is no uncertainty.

[12]In each year, we regress the log of average years of schooling (for people 25-29 years old) on a constant and the log of GDP per capita, and report the slope coefficient in Figure 2.

[13]In the online appendix associated with this paper, we report average years of schooling for people 25-29 for countries by deciles of the schooling distribution in 1950 using the entire sample in Barro and Lee (2010). The countries with lowest schooling in 1950 (Decile 1) increased their educational attainment from 0.3 to 4.1 years whereas those countries with the highest schooling in 1950 (Decile 10) increased their schooling from 8.7 to 11.7 years. Hence, the factor difference in educational attainment between these groups of countries declined from a 31-fold in 1950 to less than 3-fold in 2005.

Preferences

The preferences of an individual of generation τ are defined over lifetime sequences of consumption, $(c_{\tau,t})_{t=\tau}^{\tau+T_\tau}$, and leisure time, $(\ell_{\tau,t})_{t=\tau}^{\tau+T_\tau}$, as well as over time spent in school, s_τ. They are represented by

$$\int_{\tau}^{\tau+T_\tau} e^{-\rho t}\left[U\left(c_{\tau,t}\right)+\alpha V\left(\ell_{\tau,t}\right)\right]dt+\beta W\left(s_\tau\right), \tag{1}$$

where the functions U, V and W are increasing, concave and twice continuously differentiable. We use the following functional forms for U, V and W:

$$U\left(c_{\tau,t}\right)=\ln\left(c_{\tau,t}-\bar{c}\right),\ V\left(\ell_{\tau,t}\right)=\frac{\ell_{\tau,t}^{1-\mu}-1}{1-\mu},\quad \text{and}\quad W\left(s_{\tau,t}\right)=\ln\left(s_{\tau,t}\right),$$

where $\bar{c}\geq 0$ and $\mu\geq 0$.

The term $\beta W(s_\tau)$ is the present value utility derived from s_τ units of time spent in school. Depending upon the sign of β, which is determined in the calibration of Section 4, time spent in school is either a good or a nuisance. Introducing a utility value of school time is common in models of schooling choices such as Heckman et al. (1998), Bils and Klenow (2000) or Wolpin and Lee (2010) to cite just a few, and is also consistent with findings from a large literature in development economics –see Schultz (1963). Furthermore, there exists estimates of a substantial "psychic" component of attending school. Heckman et al. (2006) and the references therein present and discuss such evidence. Overall, our representation of preferences is close to that of Bils and Klenow (2000) except for our addition of a taste for leisure time. Bils and Klenow model the utility from schooling as a constant flow of utils, ξ, derived from each moment spent in school, i.e. they assume $W(s)=\xi\int_0^s e^{-\rho t}dt$ which is also increasing and concave in s.

We follow McGrattan et al. (1997) by assuming that consumption itself is an aggregate of market consumption $c_{\tau,t}^m$ and nonmarket consumption $c_{\tau,t}^n$:

$$c_{\tau,t}=\left[\phi\left(c_{\tau,t}^m\right)^\sigma+(1-\phi)\left(c_{\tau,t}^n\right)^\sigma\right]^{1/\sigma},$$

where $\phi \in (0, 1)$ and $\sigma < 1$. At this stage, a few observations are in order.

First, the parameter $\bar{c}$ introduces a non-homotheticity which has the standard interpretation of a subsistence level above which aggregate consumption must remain at every point in time. This feature of preferences plays an important role in both the theoretical and quantitative properties of the model because it introduces a positive income effect on leisure and schooling time at low levels of income. This is because, at low levels of income, a large fraction of time must be devoted to market and nonmarket activities to acquire enough consumption to provide at least $\bar{c}$. As income rises this requirement loosens, thus more time can be allocated to both leisure and schooling. In introducing this non-homothetic feature of preferences, we follow a tradition that is common to a broad literature: for example the literature emphasizing the shift in economic activity from agriculture to non-agriculture such as Gollin et al. (2002) and Duarte and Restuccia (2010); models of the allocation of hours such as Rogerson (2008), models of the dynamics of saving rates such as Christiano (1989), among many other applications. See Atkeson and Ogaki (1996) for empirical evidence from micro and macro data.

Second, we distinguish between market and nonmarket consumption. Nonmarket consumption is produced via a household technology where nonmarket time is the only input. We introduce the nonmarket sector into the model because the income elasticity of schooling and market hours are quite sensitive to the possibility of nonmarket reallocation at low levels of income. Moreover, it is found empirically that for the very poor countries, much of the production of food takes place in rural agriculture via subsistence home production. More specifically, to illustrate the sensitivity of the income elasticity of hours, consider the case where $\phi = 1$ so that nonmarket consumption is not valued. Then, market work increases exponentially at low levels of income because the time required to afford at least $\bar{c}$ rises exponentially when productivity decreases. Thus, the poorest country for which the model has a solution is one for which productivity is just high enough to afford subsistence when working near 100 percent of the endowment of time. We find in our empirical work that the poorest countries are in fact much less productive than that and yet workers in these countries spend noticeably less then 100 percent of their time on the market.[14] Thus, household production acts as an additional mean to meet the minimum consumption requirement in poor countries, even at very low levels of market productivity, by increasing both market and nonmarket hours in detriment of leisure and schooling.

Technologies

Individuals can acquire human capital by spending time in school and purchasing educational services. The human capital technology follows Bils and Klenow (2000) and is described by

$$H(s_\tau, x_\tau) = x_\tau^\gamma h(s_\tau) \equiv x_\tau^\gamma \exp\left(\frac{\theta}{1-\psi} s_\tau^{1-\psi}\right),$$

with $\gamma \in (0,1)$ and where x_τ represents purchases of educational services. These services are purchased up front. Hence, x_τ is appropriately described as the present value of educational services.

The nonmarket good is produced with nonmarket time in line with the technology

$$c_{\tau,t}^n = z_\tau^n n_{\tau,t}, \tag{2}$$

where z_τ^n is an exogenous variable representing the state of the household technology at the beginning of the individual's life, and $n_{\tau,t} \in (0,1)$ is the time allocated by the individual to nonmarket production.

[14] In Figure 3 the maximum hours worked are about $2800/52 = 54$ hours/week.

Optimization

An individual of generation τ chooses sequences of market consumption, nonmarket consumption, leisure, educational servcices and how much time to spend in school, in order to maximize the lifetime utility given by Equation (1) subject to the nonmarket technology described in (2) and an intertemporal budget constraint:

$$\int_{\tau}^{\tau+T_\tau} e^{-\rho t} c^m_{\tau,t} dt + x_\tau = z^m_\tau \int_{\tau+s_\tau}^{\tau+T_\tau} e^{(g^m-\rho)t} \left(1 - n_{\tau,t} - \ell_{\tau,t}\right) H\left(s_\tau, x_\tau\right) dt.$$

The left-hand side of the constraint measures the expenditures in consumption and educational services in present value at date τ. On the right-hand side, the individual's lifetime income depends upon a number of variables. First, it depends upon the level of market productivity at the beginning of life, z^m_τ, which we assume to grow at the constant rate g^m. Second, it depends upon the length of the individual's working life which extends from date $\tau + s_\tau$ until $\tau + T_\tau$. Note that working life decreases as time spent in school rises. There is, therefore, an opportunity cost of time spent in school. Third, the lifetime income depends upon labor supply during the individual's working life, $1 - n_{\tau,t} - \ell_{\tau,t}$. Finally, it depends upon the individual's human capital stock acquired through s_τ years of school, $H(s_\tau, x_\tau)$.

For simplicity, we abstract from life-cycle considerations by restricting choices to constant sequences of market work, nonmarket work, consumption and leisure. That is, we impose $c^i_{\tau,t} = c^i_\tau$ $(i = m, n)$, $\ell_{\tau,t} = \ell_\tau$, and $n_{\tau,t} = n_\tau$.[15] Thus, we re-write the optimization problem of

[15] With separable utility in market consumption and leisure, and the assumption that the rate of time discount equals the rate of interest an individual optimally chooses a constant path of market consumption. So the assumption of constant market consumption is innocuous. We impose constant leisure and nonmarket hours for simplicity. In addition, we do not have detailed data on the lifecycle behavior of labor supply for generations dating back to the 19th century and the changes in lifecycle labor supply for recent cohorts are small in comparison with the variation in hours over time. Hence, we think there is little benefit of modeling labor supply over the lifecycle since our focus is on changes across countries and over time.

an individual of generation τ as

$$\begin{aligned}
\max_{c_\tau^m, c_\tau^n, \ell_\tau, x_\tau, s_\tau} \quad & \int_0^{T_\tau} e^{-\rho t}\left[U(c_\tau) + \alpha V(\ell_\tau)\right] dt + \beta W\left(s_\tau\right), \\
\text{s.t.:} \quad & c_\tau^m \int_0^{T_\tau} e^{-\rho t} dt + x_\tau = z_\tau^m \left(1 - n_\tau - \ell_\tau\right) H(s_\tau, x_\tau) \int_s^{T_\tau} e^{(g^m - \rho)t} dt, \\
& c_\tau = \left[\phi \left(c_\tau^m\right)^\sigma + (1-\phi)\left(c_\tau^n\right)^\sigma\right]^{1/\sigma}, \\
& c_\tau^n = z_\tau^n n_\tau, \\
& H(s_\tau, x_\tau) = x_\tau^\gamma h(s_\tau).
\end{aligned}$$

This problem makes the various margins of time allocation explicit. At an "extensive" margin the individual chooses the length of working life by choosing how much time to spend in school, s_τ. At an "intensive" margin the individual chooses how much time to spend working at home and in the market, and how much leisure to enjoy at each point in time. The first order conditions for this problem are

$$\begin{aligned}
c_\tau^m \quad &: \quad 0 = a_\tau U'(c_\tau) c_\tau^{1-\sigma} \phi (c_\tau^m)^{\sigma-1} - \lambda a_T, \\
n_\tau \quad &: \quad 0 = a_\tau U'(c_\tau) c_\tau^{1-\sigma} (1-\phi)(c_\tau^n)^{\sigma-1} z_\tau^n - \lambda z_\tau^m x_\tau^\gamma h(s_\tau) d_\tau(s_\tau), \\
\ell_\tau \quad &: \quad 0 = a_\tau \alpha V'(\ell_\tau) - \lambda z_\tau^m x_\tau^\gamma h(s_\tau) d_\tau(s_\tau), \\
x_\tau \quad &: \quad 0 = \gamma z_\tau^m \left(1 - n_\tau - \ell_\tau\right) x_\tau^{\gamma-1} h(s_\tau) d_\tau(s_\tau) - 1, \\
s_\tau \quad &: \quad 0 = \beta W'(s_\tau) + \lambda z_\tau^m \left(1 - n_\tau - \ell_\tau\right) x_\tau^\gamma \left[h'(s_\tau) d_\tau(s_\tau) + h(s_\tau) d_\tau'(s_\tau)\right],
\end{aligned}$$

where λ is the Lagrange multiplier associated with the lifetime budget constraint and where $a_\tau = \int_0^{T_\tau} e^{-\rho t} dt$ and $d_\tau(s_\tau) = \int_{s_\tau}^{T_\tau} e^{(g^m - \rho)t} dt$ are discounting terms. We use the first order condition with respect to x_τ to write

$$x_\tau = \gamma z_\tau^m \left(1 - n_\tau - \ell_\tau\right) x_\tau^\gamma h(s_\tau) d_\tau(s_\tau),$$

implying that the fraction of lifetime income spent in educational services is γ. Then, the budget constraint can be written as $a_\tau c_\tau^m = (1-\gamma) z_\tau^m (1 - n_\tau - \ell_\tau) x_\tau^\gamma h(s_\tau) d_\tau(s_\tau)$. Using these expressions, we now analyze the first order conditions for schooling, leisure time and nonmarket time. We first re-write the first order condition for schooling as

$$\beta W'(s_\tau) = -\frac{\phi}{1-\gamma} a_\tau U'(c_\tau) c_\tau \left(\frac{c_\tau^m}{c_\tau}\right)^\sigma \left[\frac{h'(s_\tau)}{h(s_\tau)} + \frac{d_\tau'(s_\tau)}{d_\tau(s_\tau)}\right], \tag{3}$$

where the left-hand side measures the marginal benefit of schooling while the right-hand side measures the marginal cost. Note the term

$$A_\tau(s_\tau) \equiv \frac{h'(s_\tau)}{h(s_\tau)} + \frac{d_\tau'(s_\tau)}{d_\tau(s_\tau)},$$

on the right-hand side, which measures the marginal effect of schooling on lifetime income. First, suppose that $\beta = 0$ so that there are no utility benefits (or cost) of attending school. Then, the optimal level of schooling becomes the solution of $A_\tau(s_\tau) = 0$, which is income maximizing. In this case, changes in productivity and/or the level of income do not matter for the level of schooling. Only differences in life expectancy affect schooling decisions through the discounting term $d_\tau(s_\tau)$. Hence, for productivity to matter for schooling decisions it is necessary that $\beta \neq 0$. In the calibration of Section 4 we use time-series data on life expectancy and productivity in the United States to impose discipline on the parameters of the model, and in particular on β.

We now discuss the first order condition with respect to s_τ when $\beta > 0$ since this is the prevailing case in our empirical work. The first order condition reveals that $A_\tau(s_\tau) < 0$. This is because when $\beta > 0$ individuals have more schooling than necessary to maximize their lifetime income. Thus, increases in schooling have a negative effect on lifetime income at the optimal choice. Given the functional forms for h and d_τ, it is then immediate that the marginal cost of schooling is positive and increasing in s_τ while the marginal benefit is positive and decreasing. Consider now an increase in aggregate consumption c_τ. Given our specification for U, the term $U'(c_\tau) c_\tau$ decreases whenever $\bar{c} > 0$, implying a decline in the marginal cost of schooling and, thus an increase in schooling. The interpretation of this income effect is that the marginal utility of consumption, that is the cost of affecting resources away from consumption and into schooling, decreases fast as consumption rises. When $\bar{c} = 0$ and/or when aggregate consumption is large, this income effect becomes negligible since $U'(c)c = 1$ in both cases. We note that many utility functions share the property that $U'(c)c$ is decreasing. The function $U(c) = c^{1-\eta}/(1-\eta)$ with $\eta > 1$ is one example. Thus, the critical aspect of preferences that is needed for schooling to increase with productivity is that the marginal utility of consumption decreases faster than consumption increases. A non-homothetic term in preferences, such as the one we use, is a convenient and easy-to-interpret device to generate such pattern. In addition to the income effect, the composition of consumption may exacerbate or dampen the changes in the marginal cost of schooling. This transpires through the term $(c_\tau^m/c_\tau)^\sigma$ whenever $\sigma \neq 0$. In the calibration of Section 4 we use available estimates of σ in the literature which suggest a positive value, namely $\sigma = 0.4$. Thus, absent the income effect caused by $\bar{c}$, increases in market productivity leading to increases in market consumption relative to aggregate consumption raise the marginal cost of schooling because they increase its opportunity cost.

We next turn to the first order condition with respect to ℓ_τ, which we write as

$$\alpha V'(\ell_\tau) = \frac{\phi}{1-\gamma}\frac{1}{1-n_\tau-\ell_\tau}U'(c_\tau)\,c_\tau\left(\frac{c_\tau^m}{c_\tau}\right)^\sigma, \tag{4}$$

where the marginal benefit of leisure features on the left-hand side while the right-hand side is the marginal cost. We note that the marginal benefit is decreasing in ℓ_τ while the marginal cost is increasing, holding all else constant. There is an income effect affecting leisure time in the same way as for schooling. That is, increases in aggregate consumption lower the marginal cost of leisure and, thus, lead to increase in leisure time. This effect becomes negligible when $\bar{c} = 0$ and/or aggregate consumption is large relative to $\bar{c}$. We also note that the composition of consumption affects the marginal cost of leisure time as it did for the marginal cost of schooling. It may thus exacerbate or dampen the effect of growth in aggregate consumption on leisure.

Finally, we write the first order condition with respect to nonmarket time as

$$(1-\phi)\left(z_{\tau}^{n}\right)^{\sigma}\left(n_{\tau}\right)^{\sigma-1}\left(1-n_{\tau}-\ell_{\tau}\right)=\frac{\phi}{1-\gamma}(c_{\tau}^{m})^{\sigma}. \tag{5}$$

Note that the left-hand side of this equation is decreasing in n_τ since $\sigma < 1$. Thus, as market consumption increases nonmarket time tends to decrease. On the other hand, as nonmarket productivity increases, nonmarket time increases as well.

We conclude our description of the model by setting up a notation for the period income of an individual of generation τ at age 35:

$$y_{\tau}=z_{\tau}^{m}e^{35\times g^{m}}\left(1-n_{\tau}-\ell_{\tau}\right)H(s_{\tau},x_{\tau}).$$

Since we use this measure in our quantitative analysis, we emphasize how increases in productivity affect income. First, an increase in productivity raises income through three channels: a direct effect through z_τ^m; an indirect effect through increases in schooling s_τ; and another indirect effect through increases in expenditures in education x_τ and therefore human capital. Second, an increase in productivity induces a decline in market time and, thus, in labor income. The decrease in market time hinders the incentive to acquire education. We note this as an important property of the model to keep in mind when analyzing the results in light of the data since the model will imply large changes in schooling for poor countries relative to rich countries with relatively minor catch up in income per capita. The catch up in schooling does produce convergence in human capital but will also reduce market hours much more rapidly in poor countries tempering the effect of this catch up on per capita income. It is not surprising then that there is somewhat weak evidence on the effect on schooling on growth in income per-capita across countries (e.g. Benhabib and Spiegel (1994)).

4 Calibration

We calibrate a benchmark economy with exogenous variation in market productivity z^m, non-market productivity z^n, and life expectancy T to time-series data for the United States. The motivation for this strategy is that the United States has experienced a well-documented long-run increase in schooling followed by a flattening of the trend towards the end of the 20th century and a long-run decline in weekly hours followed by a flattening of the trend. Our calibration procedure exploits these trends to discipline the strength of the income effect that is central to the quantitative implications of the model across countries.

The schooling data were provided by Claudia Goldin and Larry Katz and serve as the basis of Figures 1.4 to 1.6 in their book.[16] The data is years of schooling by birth cohort, completed at age 35 for white people starting in 1876 until 1975. We HP-filtered the time series and used, for calibration purposes, cohorts from 1880 to 1915. These cohorts of people are of age 35 in years 1915 through 1950. As expected, the data revels a substantial increase in schooling years. A 35-year-old person in 1915 had completed about 8 years of schooling while the same-age person in 1950 had completed close to 10.5 years.[17] We explicitly avoid using data on years of schooling past 1950 in our calibration procedure. The reason is that there is a large literature emphasizing the role of skill-biased technological progress, which we abstract from in this paper, for the increase in education and wage inequality in the second half of the twentieth century.[18]

Market hours data come from various sources. For the period 1830 to 1880 we use data from

[16]See Goldin and Katz (2008, Figures 1.4–1.6).

[17]Since the cross-country data we use for average years of schooling is from Barro and Lee (2010), we verified that the Goldin and Katz data used for calibration is consistent with the Barro and Lee data for the United States for the overlapping time period.

[18]See for instance Restuccia and Vandenbroucke (2013b) for a quantitative assessment of the role of skill-biased technological change (SBTC) for education in the United States since 1940. It is quite plausible that SBTC plays a role in explaining schooling across countries since there are large differences in labor market institutions and lags in technology adoption that would derive in differences in the returns to skills across countries. We do not pursue this possibility because we found it difficult to obtain systematic cross-country data to measure SBTC in a satisfactory way, specially for the large number of countries we have in our sample. Moreover, the notion of skill/unskilled might be quite different in poor (low education) countries than in rich (high education) countries, raising issues of measurement and modelling of SBTC. We leave this non-trivial departure of our analysis for future work.

Whaples (1990, Table 2.1), for the period 1890 to 1940 we use data from Kendrick (1961, Table A-IX), and for the period 1950 to 2000 we use data reported by McGrattan and Rogerson (2004, Table 1).[19] We HP-filtered the data and linearly interpolate between census dates to build a time series of hours from 1830 through 2000. The trend shows a decline from close to 72 hours per week in 1830 to 40 hours in 2000. Importantly, the rate of this decline is non-constant. There is a moderate decline in hours from 1830 to about 1910, followed by a sharp decline until about 1980, and a substantial flattening after 1980. The fact that the workweek declined significantly in the United States has been recognized elsewhere. Rogerson (2006), for example, uses data from Whaples (1990) and proposes to rationalize the decline in the workweek using non-homothetic preferences similar to our specification. Maddison (1987, Table A-9) and Huberman and Minns (2007, Table 1) show patterns of hours over time for countries such as Belgium, Denmark, France, Germany, Ireland, Italy, Japan, the Netherlands, Spain, Sweden, Switzerland and the U.K. between 1870 and 1990 that are similar to the pattern in the United States.

For nonmarket time we use data from Aguiar and Hurst (2007, Table II). We use the beginning and end of their period of study, from 1965 to 2003, as targets. For 1965 they report 36 hours of total market work per person (for men and women together). Our data shows 42 hours per worker in 1965. We thus apply the ratio 42/36 to transform their measure of 22 hours of non-market work per person into 25.6 ($22 \times 42/36$) hours per worker. Similarly, they report 32 hours of total market work per person in 2005. Our data shows 40 hours per worker at this time. We use the ratio 40/32 to transform their measure of 18 hours of non-market work per person into 22.5 hours per worker.[20]

[19] The Whaples data are weekly hours worked collected from two surveys of manufacturing hours taken by the federal government in the context of the 1880 Census. The Kendrick data are average weekly hours in the private non-farm sector, finally the McGrattan and Rogerson data are average weekly hours worked for all workers.

[20] Hours data are available in calendar time while the model predicts hours by generation. We choose to associate the 1830 hours data with the 1795 generation from the model, i.e. the generation that is 35 years old in 1830. That is, when we compare the model's predictions to the U.S. data we compare the hours chosen by the 1795 generation in the model with the 1830 data on hours. We associate subsequent data points and generations in the same way.

To calibrate the model lifespan T we note that its empirical counterpart is not life expectancy per se but rather the sum of years spent in school and on the labor market for a generation. Hazan (2009) reports market years for cohorts born in $1840, 1850, \ldots, 1930$. We add this data to Goldin and Katz's figures for years of schooling achieved by these generations to obtain a measure of T_τ for cohorts born in $1840, 1850, \ldots, 1930$. We then estimate a linear time trend for T_τ and obtain $T_\tau = 0.1716 \times \tau - 279.38$, and use it to compute T_τ for all cohorts of the model.[21]

We now describe the details of the calibration procedure. We start by normalizing the productivity parameter $z^m_{1795} = 1$. We set the discount factor to 4 percent, i.e., $\rho = 0.04$ and, following Bils and Klenow (2000), we choose $\gamma = 0.1$ and $\psi = 0.3$.[22] McGrattan et al. (1997) report an estimated value of 0.4 for σ. We use this estimate. We also impose that nonmarket productivity is a function of market productivity, namely

$$z^n_\tau = \sum_{i=0}^{3} a_i \left(z^m_\tau\right)^i ,$$

where the parameters a_i are to be determined. The motivation for this approach is that we do not have data to discipline nonmarket productivity in poor countries over time. We return to this issue in detail in the next section.

The remaining parameters are

$$\omega = \left(\bar{c}, \phi, \mu, \theta, \alpha, \beta, g^m, a_0, a_1, a_2, a_3\right)' .$$

We let $\hat{s}_\tau(\omega)$, $\hat{n}_\tau(\omega)$, $\hat{\ell}_\tau(\omega)$ and $\hat{y}_\tau(\omega)$ denote the model's predictions for years of schooling, nonmarket time, leisure time and age-35 income for generation τ. We choose ω to solve the

[21] The t-statistics for the slope and intercept in this regression are 18 and 15, respectively. The regression's r^2 is 0.98.

[22] Bils and Klenow (2000) report estimates for ψ between 0 and 0.6. Our choice of 0.3 is the middle of that range. We present robustness checks with respect to ψ in the online appendix.

following minimization problem:

$$\min_{\omega} \sum_{\tau=1880}^{1915} \left(\frac{\hat{s}_{\tau}(\omega)}{\mathbf{s}_{\tau}} - 1 \right)^2 + \sum_{\tau=1795}^{1965} \left(\frac{1 - \hat{\ell}_{\tau}(\omega) - \hat{n}_{\tau}(\omega)}{\mathbf{n}_{\tau}/112} - 1 \right)^2 + M'(\omega)M(\omega),$$

where

$$M(\omega) = \begin{pmatrix} \frac{\hat{y}_{1965}(\omega)/\hat{y}_{1795}(\omega)}{e^{0.02\times170}} - 1 \\ \hat{n}_{1968}(\omega)/22.5 - 1 \\ \hat{n}_{1938}(\omega)/25.6 - 1 \end{pmatrix}.$$

This objective function implies that we choose the parameters of the model such that (i) predicted and actual years of schooling are close, in a least square sense, for generations up to the 1915 generation; (ii) predicted and actual market time are close up to the 1965 generation; (iii) the average growth rate of income per person is 2 percent per year; (iv) the nonmarket time of the the 1938 and 1968 generation match the nonmarket time data of Aguiar and Hurst (2007) and the beginning and end of their period of study.[23]

Table 2 reports the calibrated parameters. Figures 4 and 5 show the model's fit to the U.S. data on schooling and market hours. In particular, Figure 4 reports the time series of years of schooling in the model and the U.S. data. The dashed line separates the period of U.S. data used in the calibration (the period between 1915 and 1950) with the period that is not used for calibration (after 1950). As the figure shows, the model is able to capture the trend in schooling well for the calibrated period until 1950 and under predicts the increase in schooling during the second half of the twentieth century. As discussed earlier, this increase is best attributed to other forces not in the model such as skill-biased technical change and hence it is important that this increase in schooling is not attributed to parameters pertaining to the income effect in the model. We also note the fit of the model to the hours data. Figure 5 illustrates how the time series of hours implied by the model fits the changing pattern of the rate of change in actual hours.

[23] We assume that a person needs 8 hours for sleep and other necessities each day. There are then $(24-8)\times 7 = 112$ hours of discretionary time in a week.

We have computed the Mincer returns implied by our model and found that they are consistent with the patterns reported in the literature, e.g., Psacharopoulos and Patrinos (2004). In particular, Mincer returns are higher when education is low and lower when education is high, resulting in Mincer returns that are decreasing with time as a country gets richer. However, the calibrated value of θ is such that the level of Mincer returns are lower in our model than those reported in the literature. For instance, our calibrated values imply Mincer returns of between 2 and 3 percent in the benchmark economy whereas in the data the estimated returns are around 8 to 10 percent for the United States. We emphasize two points. First, our calibration procedure selects the value of θ (among other parameters) to target the time series of schooling s in the U.S. data. This implies that the model is calibrating the returns to schooling to best fit the schooling data. Second, while it is clear that the model should match the "right" returns to schooling, it is not clear that the model should match the Mincer returns estimated in the empirical literature. There are several reasons for that: 1) Heckman et al. (2006), among others, show that the conditions under which a Mincer regression coefficient corresponds to the rate of return on schooling are violated in our model. Specifically, the assumption that schooling entails a loss of working life and that there is a utility benefit of schooling imply that the Mincer regression coefficient is not the rate of of return on schooling and 2) as pointed out by Bils and Klenow (2000), the estimates of Mincer returns to schooling in the data may be biased upward when more able individuals obtain more schooling and when the returns to schooling are not constant across individuals as assumed in the Mincer specification.[24]

The growth rate of income in the calibrated model is 2.0% per year. The rate of growth of market productivity, g^m, needed to achieve a 2.0 percent annual growth rate of income, is close: 1.9 percent. This is because our framework abstracts from amplification effects such as those emphasized in Manuelli and Seshadri (2006) and Erosa et al. (2010). It is also because the

[24]We have computed the solution of our model with Mincer returns fixed at the level reported in the literature, 10 percent for the United States. We do this by taking θ out of our algorithm for calibration and selecting this value so that the model matches the level of estimated Mincer returns. We found that the model's fit to the U.S. schooling data is noticeably worse than when θ is selected to best fit the targeted data. But more importantly, what we find is that the cross-country results in Section 5 are quite similar and hence, the main results of the paper are not critically affected by the difference in the level of the Mincer returns to schooling.

downward trend in market hours magnifies the lack of amplification. Increases in productivity would give rise to larger increases in income if hours remained constant instead of decreasing. We emphasize, however, that abstracting from these amplification effects does not affect our quantitative results. For the purpose of calibrating the model, larger amplification effects would imply a smaller value of g^m and, in the cross-country experiments that follow, they would reduce the size of the productivity gaps needed to reproduce the calibrated income differences across countries leaving the impact of income on schooling the same. We also note that our calibration implies that the average rate of growth of nonmarket productivity, z^n, is 1.7 percent per year. Figure 6 shows the path of market, nonmarket and leisure time in the model. There is a decline in work time both in the home and the market, and an increase in leisure hours per worker. This feature of the model is consistent with the evidence described by Greenwood et al. (2005) and Aguiar and Hurst (2007).[25]

A key parameter of our model is $\bar{c}$ since it drives the strength of the income effect.[26] To assess its calibrated value, we use the model to compute the ratio of $\bar{c}$ to aggregate consumption in the time series: $\bar{c}/c$. We find that it declines from 63 percent to 2 percent between 1800 and 2000. In the context of our model, we argue that this is a sensible measure of the constraint imposed by $\bar{c}$ on economic behavior. Two alternative measures such as the ratio of subsistence to income, $\bar{c}/y$, and the ratio of subsistence to potential income $\bar{c}/[z^m H(s,x)]$ are less informative since they do not take into account the possibility that subsistence consumption can be provided by nonmarket work. Nevertheless, it is difficult to compare the 2% figure for 2000 with existing data. One approach is to relate it to the final expenditures on food relative to total expenditures.[27] For the United States, the expenditure share of food is 5.2% in 1996.

[25]The measurement of long-run trends in nonmarket hours is the subject of an ongoing debate. Ramey and Francis (2009) argue that hours of leisure per person have been essentially constant during the Twentieth century.

[26]We have performed sensitivity analysis with respect to the value of subsistence consumption and report the results in the on-line appendix. We reduced the calibrated value of $\bar{c}$ by 25 percent keeping all other parameters the same. The increase in schooling in the benchmark economy is 88 percent of that of the baseline whereas the reduction in hours is 81 percent that of the baseline. We find that even with this diminished role of productivity our model still accounts for a sizeable fraction of the variation in schooling across countries and over time.

[27]The data is for the 1996 Benchmark study of the International Comparison Program.

Another possibility is to compare the incomes of countries far back in time. Maddison (2009) reports that GDP per capita in Western Europe between 1 and 1500 was between 450 and 771 at constant 1990 dollars, representing a range of 2 to 4.5 percent of the 1970 GDP per capita. In the context of these numbers, we conclude that the 2 percent ratio of subsistence to aggregate consumption implied by our model for the year 2000 is reasonable.

5 Cross-Country Experiments

5.1 Baseline Experiment

We use the calibration of the benchmark economy and assume that countries are identical except in terms of productivity and life expectancy. In particular, we assume that countries differ in their initial level of market productivity z^m and its growth rate g^m as well as in the level and rate of change of life expectancy. We discipline our choice of life expectancy across countries by estimating two cross-sectional relationships between life expectancy and GDP per capita for 1950 and 2005. We then use our model to compute schooling, time allocation and income for a set of 10 economies representing the 10 deciles of Table 1. To do this, we search for 10 combinations of initial market productivity, z^m, and growth rate, g^m, to match the relative income gaps in 1950 and 2005, as described in Table 1, while imposing that life expectancy in 1950 and 2005 be as described by the estimated cross-sectional relationships. A detailed description of this procedure is in the online appendix. A key element in our strategy is to exploit the assumption that nonmarket productivity z_n is a function of market productivity. We note that in the absence of this assumption we would need cross-country and time-series data on nonmarket hours and/or nonmarket consumption to discipline z^n for each country. Since such data is not available, our approach is to assume that nonmarket productivity depends upon market productivity and use available data for market productivity to discipline nonmarket productivity.

Table 3 displays the results of our baseline experiments. There are two sets of results that we emphasize: the cross-sectional implications of the model relative to the data in 1950 and the time-series behavior across countries relative to the data. We start with the cross-sectional implications in 1950. The model implies that poor countries in 1950 attain very few years of schooling compared to rich countries. Thus, the strong positive association between schooling and per capita income in the data is captured by the model. To summarize our findings, we note that the model accounts for 90 percent of the difference in schooling between countries in the 1st decile and the United States. To understand how we obtain this number, note that for countries in the poorest decile of income in 1950, the model implies 2.2 years of schooling whereas the data is 1.28 years (see the schooling data in Table 1). In 1950, the United States has 10.2 years of schooling in our benchmark economy (10.3 in the data). Hence, the model accounts for (10.2-2.2)/(10.2-1.28)=90% of the difference. The "Cross Section" column of Table 3 shows a similar calculation for each decile. It transpires that the model accounts for a lower percentage of schooling differences for countries in higher deciles of income. For example, the model accounts for 67% of the difference in schooling with the U.S. for the 5th decile and 30% for the 10th decile. This tendency for the model to account for lower fractions of the schooling data as we consider richer countries results from the mechanisms emphasized in our theory: the quantitative importance of non-homotheticity in preferences tends to vanish at high levels of income to eventually play no role. For rich countries, factors other than income levels have first-order importance in the determination of schooling, e.g., skill-biased technical change, public policy towards education, labor market institutions that compress wages, among many others. In poor countries, however, increases in productivity and income allow individuals to move farther away from subsistence consumption having a first-order effect on the allocation of time in schooling.

We now turn to the time-series implications of the model for years of schooling across countries. Our first observation is that the model accounts for 64 percent of the increase in schooling in poor countries. We compute this statistic as follows. For the economy in the 1st decile, years of schooling increase from 2.2 in 1950 to 5.2 in 2005, a $\ln(5.2/2.2)/55 = 1.6\%$ annual rate of increase. This compares with a 2.5% annual rate of increase in the data. Thus, for this economy, the model accounts for 1.6/2.5=64% of the increase in years of schooling. The "Time Series" column of Table 3 shows a similar calculation for each decile. At the fifth and tenth decile the corresponding accounting numbers are 53 and 91%. Our second observation is that the model is consistent with the fact that schooling increased faster in poor countries relative to rich countries. We show this by comparing the differential annualized growth rate of years of schooling between the poorest countries in the first decile and the richest countries in the tenth decile, both in the model and the data. In the model, years of schooling increased 2.86 times faster in the poorest decile than in richest decile while in the data the difference is a factor 4.05. Thus, by this metric, the model accounts for 2.86/4.05=71% of the faster increase in schooling in poor relative to rich countries.[28] We also highlight the faster growth of schooling in poor countries generated by the model by computing the cross-sectional elasticity of schooling relative to income using model-generated data. We find this elasticity to be 0.52 (v. 0.76 in the data) in 1950 and 0.35 in 2005 (v. 0.28 in the data). Again, the decrease of this elasticity is evidence of the reduced dispersion in years of schooling across countries in 2005 and, therefore, of the faster increase in schooling in poorer countries.

We report additional statistics in Table 3. The column $\bar{c}/c$ indicates the size of subsistence consumption relative to aggregate consumption. It varies from 77% for the poorest countries in 1950 to 2% for the richest countries in 2005. The column $\hat{c}/c$ reports the compensating variation for reducing schooling by one year, measured in terms of aggregate consumption. Namely, $\hat{c}$ is the solution of

$$\int_0^{T_\tau} e^{-\rho t}\left[U(c_\tau) + \alpha V(\ell_\tau)\right] dt + \beta W\left(s_\tau\right) = \int_0^{T_\tau} e^{-\rho t}\left[U\left(\hat{c}_\tau\right) + \alpha V(\ell_\tau)\right] dt + \beta W\left(s_\tau - 1\right),$$

that is, it is the consumption needed for an individual to be indifferent between his current

[28]Note that this metric is simply the ratio of the time series accounting in each decile relative to the top decile in Table 3.

situation and one where schooling would be one less year, holding all else constant. Our calculations reveal that the consumption value of schooling is relatively small, i.e. less than one percent of aggregate consumption. Thus, even though the utility benefit (or cost) of schooling is necessary for productivity to matter in schooling decisions as we showed in Section 3, the actual utility benefit of schooling implied by our calibration are not large.

In terms of hours there is limited data that can be brought to bear on the implications of the model. Nevertheless, we use the available market hours data from the Conference Board (2010). They report yearly hours per worker and we plot the hours data for 1950 and 2005 against GDP per capita in Figure 3. We note that not only hours of work decline with income in 1950 and 2005, but also hours decrease as income rises for each country and hours fall faster for the poor than the rich countries. In the data in 1950, hours of work in poor countries relative to rich is about 1.4, while the same ratio drops to 1.2 in 2005. In the model, the ratio of hours in the poorest economy relative to the benchmark is 1.5 in 1950 and drops to 1.2 in 2005. While this comparison is crude since hours data is missing for the poorest countries, its suggests that the hours implications of the model are broadly in line with the data in terms of both the magnitude of hours differences in 1950 and the faster decline in hours over time in poor countries. Table 3 also reports leisure and nonmarket hours. Although we do not have data to compare to these predictions, we note an important mechanism at work in our model: as countries get poorer, they work more both in and out of the market in order to meet the subsistence level of consumption.

5.2 Country by Country Implications

We use our model to compute implications for individual countries in our sample. That is, for each of the 84 countries in our sample we seek for, as in Section 5.1, a combination of market productivity level and growth such that the model generates the relative income for this country in 1950 and 2005 observed in the data. A detailed list of countries with the implications for years of schooling in the model compared to data is available in the online appendix. We summarize these implications in Figure 7. The first two quadrants of this Figure report the years of schooling for the model and data in 1950 and 2005. Consistent with our findings for the average of each decile, the model broadly captures the pattern of years of schooling in the data. In particular, in 1950 as expected the model captures for most countries only a fraction of their schooling level (most countries above the 45 degree line). Some outliers where the model implies a much higher years of schooling than in the data stand out such as Venezuela, Austria, France, and Luxembourg. It is interesting to note that these countries are middle to high income countries in 1950 and hence the discrepancy between the model and the data for these countries is unlikely to be related to the income effect emphasized in the paper. To put it differently, in relatively high income countries the income effect is likely to be small and hence differences in schooling must be related to policy and other institutional features that are not the focus of our analysis. There are also some outliers where the model implies a much lower years of schooling than in the data such as Albania and New Zealand. Again, these departures are most likely related to policy and country-specific institutional features rather than an assessment of the income effect.

In the third and fourth quadrant of Figure 7, we report the growth rate of schooling between 1950 and 2005 for each country in both the model (third quadrant) and the data (fourth quadrant). In both cases, we order countries by the relative income per capita in 1950. We make a few observations. First, as expected our model implies lower schooling growth than in the data, that is the model accounts for a fraction of the schooling growth observed in the data. This result also transpired from the "Time Series" column of Table 3. Second, the model implies a decreasing relationship between per-capita income and the growth rate of schooling across all countries, consistent with the data. Third, some noticeable outliers are worth emphasizing. Many countries in Africa and the Middle East that were around 20 percent of the income per capita in the US in 1950 observed changes in schooling much larger than implied by the model. On the top end of the income distribution in 1950, Venezuela experienced an annualized growth rate in years of schooling of about 2 percent despite its relatively large GDP per capita in 1950, whereas our model implies a growth rate of schooling of less than 0.5 percent per year for Venezuela.

5.3 Equal Growth Rates across Countries

To illustrate the importance of differences in productivity growth and changes in life expectancy across countries, we conduct two additional experiments. First, we conduct an experiment similar to the baseline except that we assume that the rate of growth of market productivity, g^m, is the same across countries and given by the rate of the benchmark economy. In a second experiment we assume, in addition, that the change in life expectancy over time is also the same across countries and given by the change in the benchmark economy. We show with these experiments that the implications of the model for the cross-country differences in schooling in 1950 are not substantially affected by the differential growth components. The critical factors determining the time allocation in 1950 are the levels of income per capita and life expectancy. We also show that even when abstracting from cross-country differences in productivity growth and changes in life expectancy the model still accounts for a substantial portion of the changes in schooling over time across countries.

In the first experiment, we assume equal market productivity growth g^m across countries. Thus, for each decile we find an initial level of market productivity so that the model reproduces the 1950 distribution of relative income across deciles. We then compute, as in the baseline, the model's implications for hours and schooling across time. We report the results of this experiment in Table 4. In the 1950 cross-section the model accounts for 90 percent of the difference in schooling between countries in the 1st decile and the United States. This is the same as the 90 percent we found in the baseline experiment. At the 5th and 10th deciles the numbers are 72 and 33 percent (v. 67 and 30 in the baseline). These figures are reported in the "Cross Section" column of Table 4 for all deciles, under "Experiment 1." Turning to the time-series implications, this experiment account for 65% of the growth in schooling in the 1st decile (64% in the baseline). At the 5th and 10th deciles the numbers are 48 and 89% (53 and 91% in the baseline). These figures are reported in the "Time Series" column of Table 4. Just as in the baseline, the model with equal g^m predicts a narrowing of the schooling gap relative to income as observed in the data: the income elasticity of schooling across countries, estimated on the model-generated data, is 0.51 in 1950 and 0.29 in 2005.

We emphasize that these results do not imply that productivity growth is not important for schooling decisions as reported in the related literature, e.g. Bils and Klenow (2000). Instead, these results arise for two main reasons. First, the productivity growth differences among the deciles in our sample are small since there are no substantial departures from relative per-capita income between 1950 and 2000 (see column "Rel Inc" in Table 3). Second, productivity growth reduces market hours which mitigates the impact of growth on the returns to schooling and this effect is stronger for poorer countries. Abstracting from labor supply exaggerates the impact of growth on schooling, specially so for the very poor countries. Nevertheless, we verify that productivity growth can have an important impact on schooling in our model. For instance, an increase in productivity growth in 1 percentage point in the United States would increase schooling by more than 25 percent and a larger increase when hours of work are relatively flat (an increase of 2 years in 1915 and 3 years in 2000). This corroborates that growth can have a substantial impact on schooling levels in our model but this happens when the change in productivity growth is substantial and in economies for which the decline in hours of work is relatively flat.

In the second experiment we assume, in addition to a constant g^m, that the change in life expectancy for each country is the same as in the benchmark economy. That is, relative to the previous experiment, life expectancy in 1950 is unchanged but in 2005 it is 9.3 years longer for all countries. This figure is the increase in life expectancy in the benchmark economy between 1950 and 2005. The results of this experiment are reported in Table 4, under "Experiment 2."

We first note that the cross-sectional implications in 1950 of the model in this experiment and the previous experiment are identical. This is due to the fact that, by design, the experiment implies that the level of life expectancy and income are the same in 1950. The only difference between the two experiments is the level of life expectancy in 2005. In terms of the time series, abstracting from differences in rate of change of T reduces the ability of the model to account for the increase in schooling. Nevertheless, even when abstracting from differences in productivity growth and changes in life expectancy, the model accounts for 51% of the growth rate of schooling in the 1st decile. At the 5th and 10th deciles the corresponding figure are 36 and 85%. These figures are reported in the "Time Series" column of Table 4 for all deciles. The model also implies a faster growth in schooling in poor than in rich countries: the income elasticity of schooling measured across countries falls from 0.51 in 1950 to 0.34 in 2005.

6 Conclusion

We developed a model of human capital accumulation to quantitatively assess the importance of productivity and life expectancy in explaining differences in educational attainment across countries and over time. We calibrated a benchmark economy to reproduce the historical evolution of schooling and hours in the United States. We found that the model accounts for 90 percent of the difference in schooling between rich and poor countries in 1950. The model accounts for 64 percent of the increase in schooling levels over time in poor countries. The model generates a faster increase in schooling levels in poor than in rich countries. Hence, it explains the convergence in cross-country schooling levels observed in the data. Our results emphasize the importance of productivity (and life expectancy) in explaining the bulk of differences in educational attainment across countries and their evolution over time. As such, our results support an alternative view of the determinants of low education in developing countries that is based on low productivity. Nevertheless, we think that extending our framework to incorporate other complementary factors such as credit market frictions and the role of public education can yield additional insights with important implications for educational policy. We leave these relevant explorations for future work.

References

Acemoglu, Daron, "Technical Change, Inequality, and the Labor Market," *Journal of Economic Literature*, 2002, *40* (1), 7–72.

— **and Simon Johnson**, "Disease and Development: The Effect of Life Expectancy on Economic Growth," *Journal of Political Economy*, 2007, *115* (6), 925–985.

Aguiar, Mark and Erik Hurst, "Measuring Trends in Leisure: The Allocation of Time Over Five Decades," *The Quarterly Journal of Economics*, August 2007, *122* (3), 969–1006.

Atkeson, Andrew and Masao Ogaki, "Wealth-Varying Inter-temporal Elasticities of Substitution: Evidence from Panel and Aggregate Data," *Journal of Political Economy*, 1996, *38* (3), 507–534.

Barro, Robert J., "Economic Growth in a Cross Section of Countries," *The Quarterly Journal of Economics*, 1991, *106* (2), 407–443.

— **and Jong-Wha Lee**, "A New Data Set of Educational Attainment in the World, 1950-2010," 2010. NBER Working Paper No 15902.

Becker, Gary S., "Investment in Human Capital: A Theoretical Analysis," *Journal of Political Economy*, 1962, *70* (5), 9–49.

Ben-Porath, Yoram, "The Production of Human Capital and the Life Cycle of Earnings," *Journal of Political Economy*, 1967, *75* (4), 352–365.

Benhabib, Jess and Mark M. Spiegel, "The role of human capital in economic development evidence from aggregate cross-country data," *Journal of Monetary Economics*, 1994, *34* (2), 143–173.

Bils, Mark and Peter J. Klenow, "Does Schooling Cause Growth?," *The American Economic Review*, 2000, *90* (5), 1160–1183.

Blinder, Alan S. and Yoram Weiss, "Human Capital and Labor Supply: A Synthesis," *Journal of Political Economy*, 1976, *84* (3), 449–472.

Christiano, Lawrence J., "Understanding Japan's Saving Rate: The Reconstruction Hypothesis," *Federal Reserve Bank of Minneapolis, Quarterly Review*, 1989, (1322).

Conference Board, "The Conference Board Total Economy Database," 2010. http://www.conference-board.org/database.cfm.

Cordoba, Juan-Carlos and Marla Ripoll, "What Explains Schooling Differences across Countries?," *Journal of Monetary Economics*, March 2013, *60* (2), 184–202.

Duarte, Margarida and Diego Restuccia, "The Role of the Structural Transformation in Aggregate Productivity," *The Quarterly Journal of Economics*, 2010, *125* (1), 129–173.

Duflo, Esther, "Schooling and Labor Market Consequences of School Construction in Indonesia: Evidence from an Unusual Policy Experiment," *The American Economic Review*, 2001, *91* (4), 795–813.

Erosa, Andres, Tatyana Koreshkova, and Diego Restuccia, "How Important is Human Capital? A Quantitative Theory Assessment of World Income Inequality," *The Review of Economic Studies*, 2010, *77* (4), 1421–1449.

Fieler, Ana Cecília, "Non-Homotheticity and Bilateral Trade: Evidence and a Quantitative Explanation," *Econometrica*, 2011, *79* (4), 1069–1101.

Goldin, Claudia and Lawrence F. Katz, *The Race Between Education and Technology*, Cambridge, MA: The Belknap Press of Harvard University Press, 2008.

Gollin, Douglas, Stephen Parente, and Richard Rogerson, "The Role of Agriculture in Development," *The American Economic Review*, 2002, *92* (2), 160–164.

Greenwood, Jeremy, Ananth Seshadri, and Mehmet Yorukoglu, "Engines of liberation," *The Review of Economic Studies*, 2005, *72* (1), 109–133.

_ and _ , "Technological Progress and Economic Transformation," in Philippe Aghion and Steven N. Durlauf, eds., *Handbook of Economic Growth*, Amsterdam: Elsevier North-Holland, 2005, chapter 19, pp. 1225–1273.

Guvenen, Fatih and Burhanettin Kuruscu, "A Quantitative Analysis of the Evolution of the U.S. Wage Distribution, 1970-2000," *NBER Macroeconomics Annual*, 2010, *24* (1), 227–276.

Hazan, Moshe, "Longevity and Lifetime Labor Supply: Evidence and Implications," *Econometrica*, 2009, *77* (6), 1829–1863.

Heckman, James J., "Estimates of a Human Capital Production Function Embedded in a Life-Cycle Model of Labor Supply," in Nestor E. Terleckyj, ed., *Household Production and Consumption*, New York, NY: NBER Studies in Income and Wealth, 1976, chapter 4, pp. 225–264.

_ , Lance Lochner, and Christopher Taber, "Explaining Rising Wage Inequality: Explorations with a Dynamic General Equilibrium Model of Labor Earnings with Heterogeneous Agents," *Review of Economic Dynamics*, 1998, *1* (1), 1–58.

_ , _ , and Petra Todd, "Earnings Functions Rates of Return and Treatment Effects: The Mincer Equation and Beyond," in Eric A. Hanushek and Finis Welsh, eds., *Handbook of the Economics of Education*, Vol. 1, Elsevier North-Holland, 2006, chapter 7, pp. 307–458.

Hendricks, Lutz, "How Important Is Human Capital for Development? Evidence from Immigrant Earnings," *The American Economic Review*, 2002, *92* (1), 198–219.

Hodrick, Robert J. and Edward C. Prescott, "Postwar U.S. Business Cycles: An Empirical Investigation," *Journal of Money, Credit and Banking*, 1997, *29* (1), 1–16.

Huberman, Michael and Christopher Minns, "The Times They Are Not Changin': Days and Hours of Work in Old and New Worlds, 1870-2000," *Explorations in Economic History*, 2007, *44* (4), 538–567.

Jayachandran, Seema and Adriana Lleras-Muney, "Life Expectancy and Human Capital Investments: Evidence from Maternal Mortality Declines," *The Quarterly Journal of Economics*, 2009, *124* (1), 349–397.

Kendrick, John W., *Productivity Trends in the United State*, Princeton, NJ: Princeton University Press, 1961.

Kongsamut, Piyabha, Sergio Rebelo, and Danyang Xie, "Beyond Balanced Growth," *The Review of Economic Studies*, 2001, *68* (4), 869–882.

Laitner, John, "Structural Change and Economic Growth," *The Review of Economic Studies*, 2000, *67* (3), 545–561.

Maddison, Angus, "Growth and Slowdown in Advanced Capitalist Economies: Techniques of Quantitative Assessment," *Journal of Economic Literature*, 1987, *25* (2), 649–698.

_ , "Historical Statistics of the World Economy: 1-2008 AD," 2009. http://www.ggdc.net/maddison/.

Manuelli, Rodolfo and Ananth Seshadri, "Human Capital and the Wealth of Nations," 2006. Manuscript.

_ **and** _ , "Neoclassical Miracles," 2009. Manuscript.

Markusen, Jame R., "Putting per Capita Income Back into Trade Theory," 2010. NBER Working Paper No 15903.

McGrattan, Ellen R. and Richard Rogerson, "Changes in Hours Worked, 1950-2000," *Federal Reserve Bank of Minneapolis, Quarterly Review*, 2004, *28* (1).

_ **,** _ **, and Randall Wright**, "An equilibrium model of the business cycle with household production and fiscal policy," *International Economic Review*, May 1997, *38* (2), 267–290.

Mincer, Jacob, *Schooling, Experience, and Earnings*, New York, NY: NBER Press, 1974.

Ogaki, Masao, "Engel's Law and Cointegration," *Journal of Political Economy*, 1992, *100* (5), 1027–1046.

Psacharopoulos, George and Harry A. Patrinos, "Returns to investment in education: a further update," *Education Economics*, July 2004, *12* (2), 111–134.

Ramey, Valerie A. and Neville Francis, "A Century of Work and Leisure," *American Economic Journal: Macroeconomics*, July 2009, *1* (2), 189–224.

Restuccia, Diego and Guillaume Vandenbroucke, "A Century of Human Capital and Hours," *Economic Inquiry*, July 2013, *51* (3), 1849–1866.

_ **and** _ , "The Evolution of Education: A Macroeconomic Analysis," *International Economic Review*, August 2013, *54* (3), 915–936.

Rogerson, Richard, "Understanding differences in hours worked," *Review of Economic Dynamics*, 2006, *9* (3), 365–409.

_ , "Structural Transformation and the Deterioration of European Labor Market Outcomes," *Journal of Political Economy*, 2008, *116* (2), 235–259.

Schoellman, Todd, "Education Quality and Development Accounting," *The Review of Economic Studies*, 2012, *79* (1), 388–417.

Schultz, Theodore W., *The economic value of education*, New York, NY: Columbia University Press, 1963.

Soares, Rodrigo R., "Mortality Reductions, Educational Attainment, and Fertility Choice," *The American Economic Review*, 2005, *95* (3), 580–601.

Violante, Gianluca, "Skill-biased technical change," in Steven N. Durlauf and Lawrence E. Blume, eds., *The New Palgrave Dictionary of Economics*, Palgrave Macmillan, 2008.

Whaples, Robert, "The Shortening of the American Work Week: An Economic and Historical Analysis of its Context, Causes, and Consequences." PhD dissertation, University of Pennsylvania 1990.

Wolpin, Kenneth I. and Donghoon Lee, "Accounting for Wage and Employment Changes in the U.S. from 1968-2000: A Dynamic Model of Labor Market Equilibrium," *Journal of Econometrics*, 2010, *156* (1), 68–85.

Table 1: GDP per Capita and Schooling across Countries

	1950		2005		
Decile	y_{50}	s_{50}	y_{05}	s_{05}	s_{05}/s_{50}
1	0.05	1.28	0.06	5.01	3.91
2	0.07	1.50	0.05	6.85	4.57
3	0.09	3.18	0.21	8.42	2.65
4	0.12	2.04	0.10	7.88	3.87
5	0.17	2.43	0.22	9.41	3.87
6	0.21	3.91	0.31	9.96	2.55
7	0.24	4.06	0.34	9.95	2.45
8	0.38	5.83	0.61	11.25	1.93
9	0.58	6.70	0.71	11.75	1.75
10	0.81	7.96	0.77	11.15	1.40
$R_{10/1}$	17.56	6.22	13.95	2.23	-
$R_{9/1}$	12.51	5.23	12.85	2.35	-

Note: The source of data is Barro and Lee (2010) for schooling and the Conference Board (2010), Total Economy Database for GDP per capita. The symbol y refers to real GDP per capita relative to the United States, and s is average years of schooling of the 25-29 year old population. Numbers reported are the average of each decile. The countries in each decile are the same in each year and represent the 1950 distribution of GDP per capita.

Table 2: Calibration

Preferences	$\rho = 0.04$, $\bar{c} = 0.03$, $\phi = 0.10$
	$\sigma = 0.40$, $\alpha = 0.68$, $\beta = 0.71$, $\mu = 0.23$
Technology	$z^m_{1795} = 1.0$, $g^m = 0.019$
	$a_0 = 0.03410$, $a_1 = 0.00097$, $a_2 = 0.00463$, $a_3 = -0.00010$
	$\gamma = 0.1$, $\psi = 0.30$, $\theta = 0.06$

Table 3: Model's Implications – The Baseline Experiment

	Dec	Rel Inc	Life Exp	Leis Hrs	Home Hrs	Mkt Hrs	Schl Yrs	$\bar{c}/c$	$\hat{c}/c$	Accounting
										Cross-Section
	1	0.05	20.9	1.3	45.2	65.5	2.2	0.77	1.007	0.90
	2	0.07	24.1	4.8	39.1	68.0	2.6	0.66	1.008	0.87
	3	0.09	26.4	2.3	41.2	68.5	3.7	0.72	1.004	0.92
1	4	0.12	29.2	9.4	33.9	68.7	3.7	0.56	1.006	0.79
9	5	0.17	32.5	10.0	31.9	70.1	5.0	0.54	1.004	0.67
5	6	0.21	34.5	11.4	30.2	70.4	5.8	0.50	1.004	0.71
0	7	0.24	35.7	13.5	28.8	69.7	6.1	0.47	1.004	0.66
	8	0.38	40.1	18.5	25.2	68.3	8.0	0.37	1.003	0.50
	9	0.58	44.1	30.0	22.8	59.2	9.0	0.21	1.003	0.35
	10	0.81	47.3	41.8	23.1	47.1	9.5	0.09	1.003	0.30
	U.S.	1.00	49.3	44.9	23.3	43.8	10.2	0.07	1.003	-
										Time Series
	1	0.06	35.6	15.2	29.4	67.4	5.2	0.45	1.004	0.64
	2	0.05	38.5	16.6	29.7	65.6	5.0	0.44	1.005	0.42
	3	0.21	42.9	27.5	22.2	62.3	9.6	0.23	1.003	0.97
2	4	0.10	44.1	27.4	25.1	59.5	7.0	0.28	1.004	0.47
0	5	0.22	48.2	33.9	22.3	55.8	10.1	0.16	1.003	0.53
0	6	0.31	50.3	36.4	21.6	54.0	11.7	0.12	1.003	0.76
5	7	0.34	51.0	38.0	21.6	52.4	12.0	0.10	1.003	0.75
	8	0.61	54.0	41.7	21.3	49.0	14.4	0.05	1.002	0.89
	9	0.71	56.0	46.1	22.4	43.5	14.1	0.03	1.002	0.80
	10	0.77	57.4	49.4	23.4	39.2	13.0	0.02	1.002	0.91
	U.S.	1.00	58.7	48.9	23.1	40.0	13.0	0.02	1.002	-

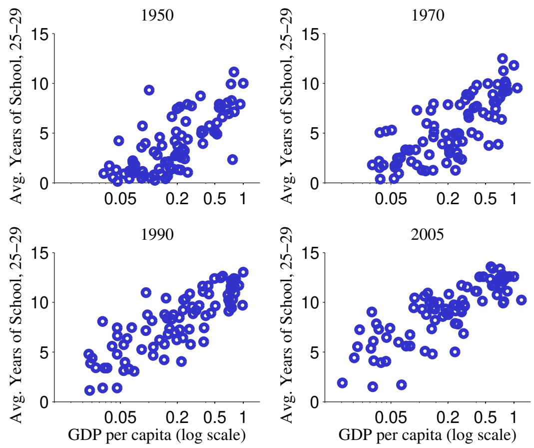
Note: The column "Life Exp" indicates the life expectancy of the generations reaching age 35 in 1950 and 2005, as implied by our calibration procedure. The column "$\bar{c}/c$" indicates the ratio of subsistence consumption to aggregate consumption. The column "$\hat{c}/c$" indicates the compensating variation, in terms of aggregate consumption, of reducing schooling by one year. The "Cross-Section" column indicates what fraction of the observed difference with U.S. schooling is accounted for by the model in 1950. The "Times Series" column indicates what fraction of the observed growth rate of school years is accounted for by the model.

Table 4: Model's Implications – Experiments

	Dec	Experiment 1 Rel Inc	Mkt Hrs	Schl Yrs	Accounting	Experiment 2 Rel Inc	Mkt Hrs	Schl Yrs	Accounting
					Cross Section				Cross Section
	1	0.05	65.4	2.2	0.90	0.05	65.4	2.2	0.90
	2	0.07	67.9	2.8	0.85	0.07	67.9	2.8	0.85
	3	0.09	69.0	3.3	0.99	0.09	69.0	3.3	0.99
1	4	0.12	69.4	3.9	0.78	0.12	69.4	3.9	0.78
9	5	0.17	68.2	4.6	0.72	0.17	68.2	4.6	0.72
5	6	0.21	66.6	5.2	0.80	0.21	66.6	5.2	0.80
0	7	0.24	65.1	5.5	0.76	0.24	65.1	5.5	0.76
	8	0.38	58.5	6.9	0.76	0.38	58.5	6.9	0.76
	9	0.58	51.4	8.3	0.55	0.58	51.4	8.3	0.55
	10	0.81	46.4	9.5	0.33	0.81	46.4	9.5	0.33
	U.S.	1.00	43.8	10.2	-	1.00	43.8	10.2	-
					Time Series				Time Series
	1	0.06	67.4	5.3	0.65	0.06	66.9	4.4	0.51
	2	0.08	65.0	6.1	0.51	0.08	64.5	5.1	0.40
	3	0.10	62.8	7.1	0.80	0.09	62.0	5.7	0.58
2	4	0.12	59.5	7.6	0.51	0.11	58.8	6.5	0.39
0	5	0.16	55.1	8.9	0.48	0.15	54.3	7.6	0.36
0	6	0.19	52.3	9.6	0.66	0.18	51.4	8.2	0.50
5	7	0.21	50.4	9.9	0.65	0.20	49.6	8.7	0.50
	8	0.33	44.4	11.3	0.75	0.31	43.7	10.2	0.61
	9	0.51	40.2	12.3	0.70	0.49	39.8	11.7	0.61
	10	0.75	38.9	12.8	0.89	0.74	38.8	12.6	0.85
	U.S.	1.00	40.0	13.0	-	1.00	40.0	12.9	-

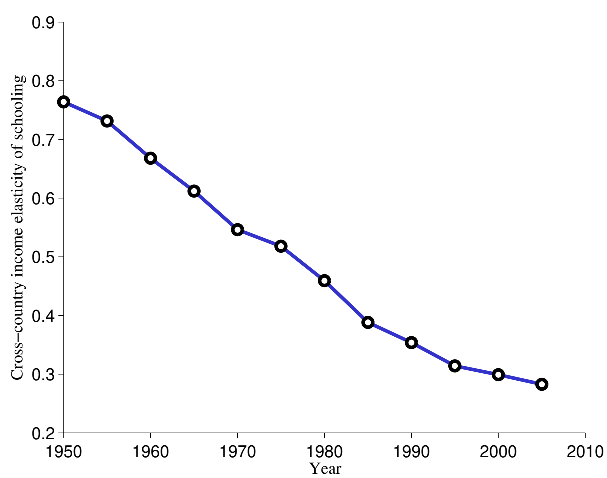
Note: In "Experiment 1" we impose that the growth rate of market productivity is the same across countries, and given by the growth of market productivity in the benchmark economy. In "Experiment 2" we impose in addition that the change in life expectancy is the same across countries between 1950 ans 2005, and given by the change in the benchmark economy. The "Cross-Section" columns indicate what fraction of the observed difference with U.S. schooling is accounted for by the model in 1950. The "Times Series" columns indicate what fraction of the observed growth rate of school years is accounted for by the model.

Figure 1: Average Years of Schooling Population 25 to 29 – Selected Years



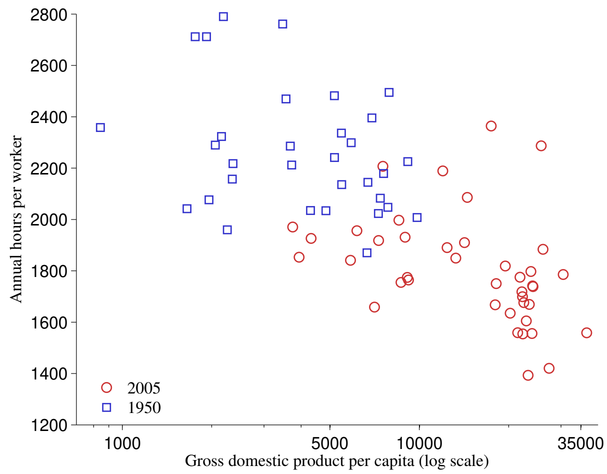
Note: The source of data is Barro and Lee (2010) for schooling and the Conference Board (2010), Total Economy Database for GDP per capita. The horizontal axis measures GDP per capita relative to the United States. The vertical axis measures average years of schooling for the 25-29 population.

Figure 2: Income Elasticity of Schooling across Countries



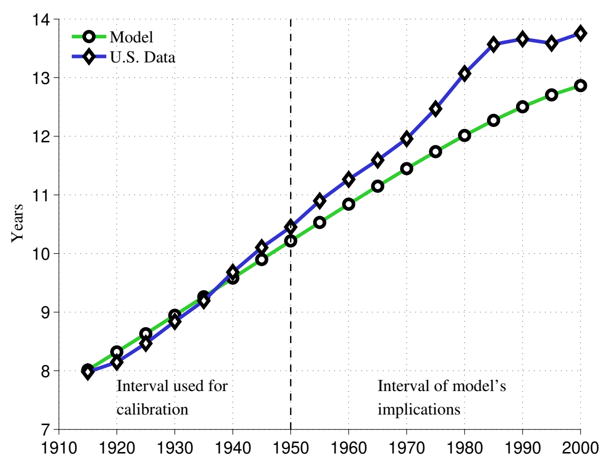
Note: For each year, we regress the (natural) logarithm of average years of schooling on a constant and log real GDP per capita across countries in our sample. The slope coefficient is plotted for each year.

Figure 3: Work Hours Across Countries



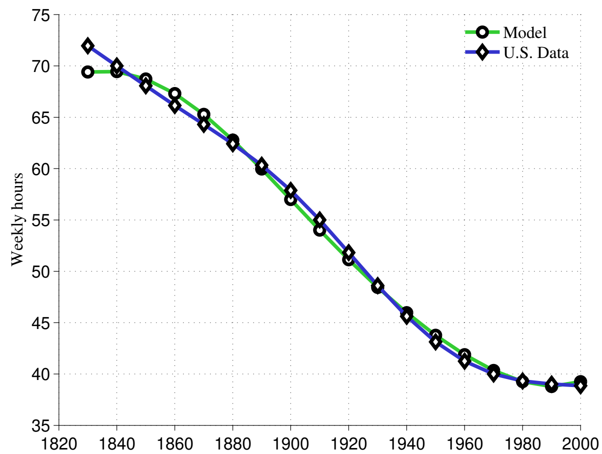
Note: The source of data is the Conference Board (2010), Total Economy Database.

Figure 4: Years of Schooling Completed at Age 35, Model and U.S. Data



Note: The source of data is Goldin and Katz (2008).

Figure 5: Work Hours, Model and U.S. Data



Note: The source of data is Kendrick (1961), Whaples (1990, Table 2.1) and McGrattan and Rogerson (2004). See text for details.

Figure 6: The Allocation of Time over the Time Series in the Benchmark Economy

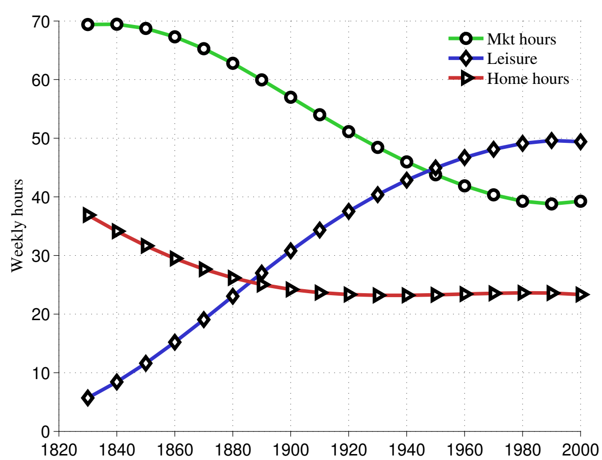
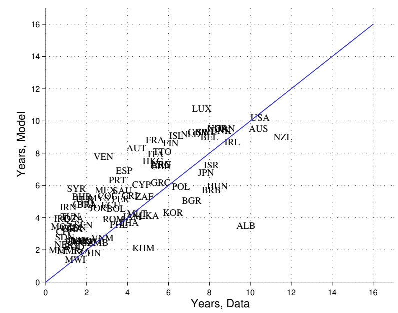
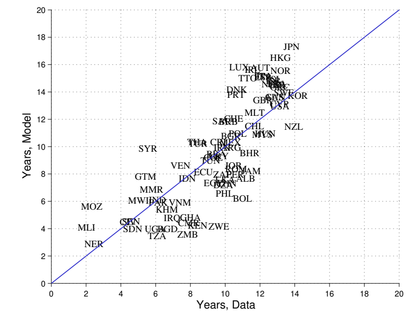


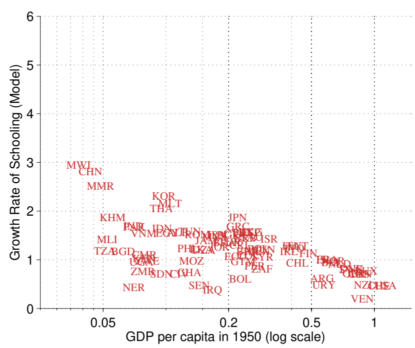
Figure 7: Country by Country Predictions for Years of Schooling



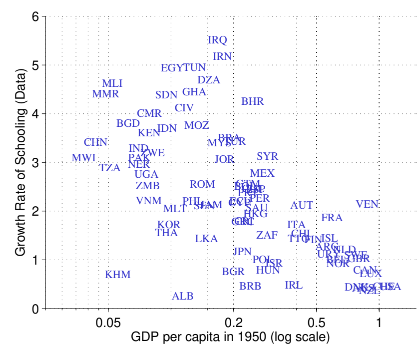
Years of School, Model v. Data
1950



Years of School, Model v. Data
2005



Growth Rate of Schooling
by 1950 Income, Model



Growth Rate of Schooling
by 1950 Income, Data

Note: The growth rate of schooling in the model and data is the annualized growth rate between 1950 and 2005.