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Endogenous Borrowing Constraints and Stagnation in Latin America

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Abstract

The Latin American debt crisis of the 1980's had a major and long-lasting effect on per-capita consumption: its level in 2005 was not that different from that in 1980. This paper studies the long stagnation in per-capita consumption that followed the crisis, and its relationship with recessions and sovereign risk, using a small open economy real business cycle model with complete markets, endogenous borrowing limits (limited commitment), endogenous capital accumulation, and domestic productivity and international interest rate shocks. I find that the model does an excellent job at explaining the observed behavior of per-capita consumption and that both the productivity and international interest rate shocks are important. Furthermore, I show that the participation constraint in this kind of representative agent model can bind not only in good times but also in prolonged bad times.

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1 Introduction

The Latin American debt crisis of the 1980's had a major and long-lasting effect on per-capita consumption: its level in 2005 was not that different from that in 1980 (see Figure 1). This paper analyzes this debt crisis through the lens of a limited commitment model to gain insight on the long stagnation in per-capita consumption that followed the crisis and its connection with recessions and sovereign risk.

A limited commitment model is a natural setup to study the relationship and feedback mechanisms between debt crises, recessions, and consumption because it is a model with a notion of sovereign default where capital flows continuously. In standard models of sovereign debt, there are no capital flows during a default period because they assume that in this state a country is in financial autarky; while in the data capital still flows during default episodes, and countries do not go into financial autarky.

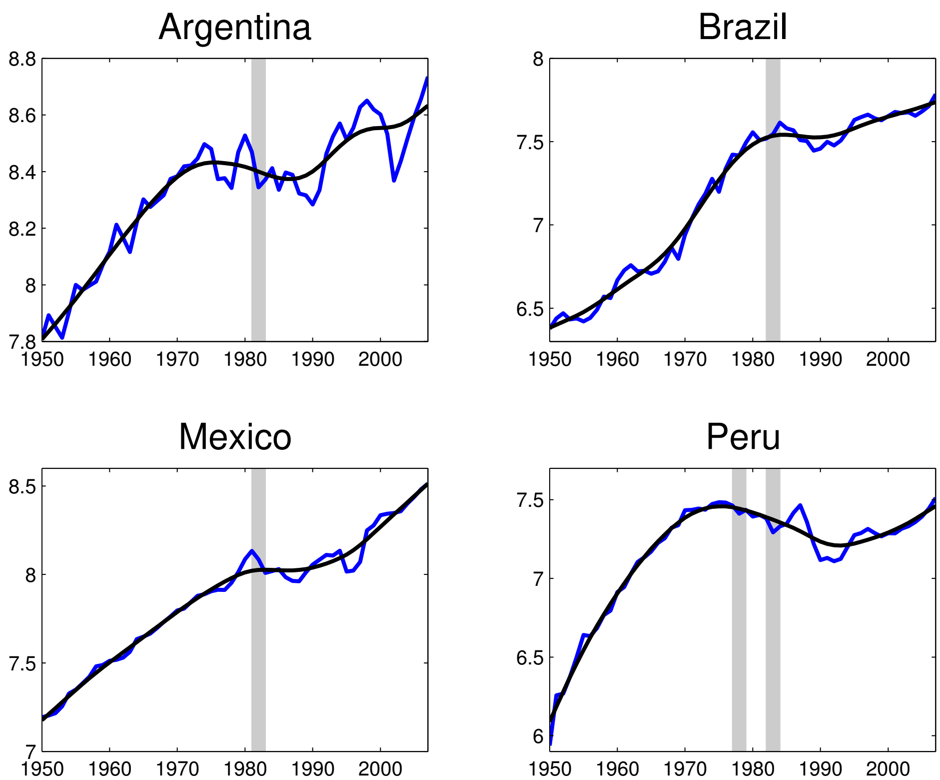

Figure 1: Log of per-capita consumption in Latin America

Note: The level is represented by the blue line and the black line is the Hodrick and Presscott trend. The shaded gray bars represent the debt crises of the 1980's.

In a limited commitment model there are complete markets, but international loans are feasible only to the extent to which they can be enforced by the threat of exclusion from future inter-temporal and inter-state trade.[1] In other words, there is an endogenous borrowing limit that comes from a participation constraint that states that the discounted present value of being in a contract with a lender has to be greater or equal to the discounted present value of being in autarky. So although in this type of model there is no default in equilibrium—because the participation constraint will always prevent the borrower from going into autarky—there is a notion of default because when the participation constraint binds it means that there is a renegotiation of terms such that the value of staying in the contract is at least as good as the value of autarky. In the data, most default episodes are renegotiations, countries do not go into financial autarky, and capital keeps flowing.[2] As such, I interpret a binding budget constraint as a default episode: it is a renegotiation of terms, the same as in the data.

The model consists of a small open economy (SOE) that interacts with a risk-neutral lender. The SOE produces a single good using labor and capital as inputs, and its production is subject to a total factor productivity shock, which grows at a stochastic rate. The SOE cannot commit to repaying its debt due to borrowing limits that arise endogenously as in Kehoe and Levine (1993), Alvarez and Jermann (2000), or Kehoe and Perri (2002), among others, and there are shocks to the international interest rate that are modeled as a shock to the discount factor of the lender. Incorporating the productivity and international interest rate shocks allows me to study the impact of domestic and external factors that are often cited in analyzing fluctuations in emerging economies[3] and evaluate the effect that these shocks have on the medium-to-long-term economic performance when there are limits to international borrowing.

Note that apart from the shocks to productivity and the international interest rate, the model is an off-the-shelf limited commitment model. As such, the goal of this paper is to take this type of

[1] I see having complete markets as an advantage because in this setting borrowing constraints are endogenous and one does not have to take a stand on portfolio restrictions.

[2] In the data, the official definition of default by Standard and Poor's is the following:

Sovereign default is the failure to meet a principal or interest payment on the due date (or within the specified grace period) contained in the original terms of the debt issue. In particular, each issuer's debt is considered in default in any of the following circumstances: (1) for local and foreign currency bonds, notes and bills, when either scheduled debt service is not paid on the due date, *or an exchange offer of new debt contains terms less favorable than the original issue*; (2) for central bank currency, when notes are converted into new currency of less than equivalent face value; and (3) for bank loans, when either scheduled debt service is not paid on the due date, *or a rescheduling of principal and/or interest is agreed to by creditors at less favorable terms then the original loan. Such rescheduling agreements covering short- and long-term debt are considered defaults even where, for legal or regulatory reasons, creditors deem forced rollover of principal to be voluntary.*

[3] See Aguiar and Gopinath (2007) and Neumeyer and Perri (2005), for example.

model to the data in the context of a specific episode in history—for which it seems particularly well suited—and study in which dimensions it can help us rationalize the path taken by fundamentals and in which dimensions it fails to shed any light.

Unlike what is standard in the literature, I concentrate on understanding the medium-to-low frequencies of the data instead of focusing on the moments at business cycle frequencies. As such, I want to test the ability of the model to rationalize the consumption slowdown that takes place after 1980 and the subsequent pickup that is observed after 1990.

To take the model to the data, I calibrate it to Argentina. For the productivity shock I use the Solow residual, and for the interest rate shock I use interest rate data directly. In the non-stationary model, productivity grows at a stochastic rate, while the stationary equilibrium is a function of the growth rate of productivity. I solve for the policy functions of the stationary model and feed the productivity growth and interest rate shocks back in to recover the equilibrium path followed by fundamentals. I compare the trend of the equilibrium path generated by the model with that of the observed data to assess how endogenous variations in borrowing capacity have affected consumption, output, investment, and capital flows in Argentina and how these variations in borrowing capacity are affected by the productivity *growth* shocks and interest rate shocks.

The pattern of the binding constraint depends on how the value of being in autarky, given the actual capital stock and productivity, compares with the value of being in the contract. Whenever the value of autarky would exceed the value of being in the contract, the borrowing constraint binds. At this point the lender has to raise the value of the contract by increasing consumption/capital assigned to the SOE, so that the participation constraint holds with equality. The binding pattern of the constraint determines the behavior of consumption in equilibrium.

In the model when the constraint is non-binding, the agent front-loads consumption due to impatience (which is stochastic because of the shocks to the discount factor of the lender). This front loading of consumption implies that consumption is being run down until the insurance provided by the contract goes to zero (consumption is declining and asset position is depreciating), triggering the participation constraint. The rate at which consumption is run down is determined by the interest rate/discount factor shock. A lower interest rate means that consumption is being run down at a faster pace (more front-loading), which will trigger the constraint sooner. In other words, the discount factor affects the speed at which the constraint is approached, and hence it affects the level of consumption.

I find that the model does an excellent job matching the observed behavior of per-capita consumption. More specifically, from the first panel of Figure 1, it can be seen that per-capita consumption in Argentina grew fast prior to 1980, it had a slightly negative growth rate in the 1980's, and it picked up again at a slower pace from the 1990's to 2005. The model is able to generate the decade-long slowdown and the pickup after the 1990's. Furthermore, the interaction of both productivity and interest rate shocks with the borrowing limit matters.

From a theoretical standpoint, there are two mechanisms that are important for this quantitative result. First, in this type of model the participation constraint tends to bind a large fraction of the time (which might prevent consumption from falling). However, in this setup with shocks to the stochastic discount factor of the lender, the constraint will bind less often when the risk free rate is high. A high interest rate—which implies a lower discount factor for the lender relative to the SOE—slows the rate at which consumption is front-loaded, which delays the binding of the constraint, allowing for longer periods of declining consumption. In the period at hand—due to the oil shocks—interest rates increased toward the end of the 1970's and remained high until 1987 helping the model explain the decline in consumption of the 1980's. Second, the participation constraint not only binds in good times–as is known of limited commitment models–but can also be triggered in prolonged bad times. This feature helps explain the pickup in consumption that is observed in the 1990's. This feature that the constraint can bind in prolonged bad times is not unique to this setup. I believe that it would be true in any representative agent limited commitment model; it is just that previous literature had only looked at certain moments of the data and not at the full equilibrium path, which is the context in which this feature becomes apparent.

The periods where the constraint binds mostly coincide with the periods where productivity *growth* is high. In these states, autarky is an attractive option because the borrower is expected to repay in the future. However, in Argentina, productivity declined during the decade of the 1980's and here is where the participation constraint starts to bind. Because the shocks to productivity are to its growth rate and not its level, the model can reconcile the fact that the constraint binds during bad times (declining productivity) because this is also a period in which there are positive spikes in the growth rate of productivity.

The model does a relatively good job at rationalizing the data for output and the level of hours worked (although they are much more volatile in the model). However, its ability to generate investment (capital) and net exports is limited. The model does generate the capital flows reversal that we observe at the beginning of the 1980's, although it is unable to capture the prolonged debt crisis.

There are two dimensions that explain why the model has difficulty generating observed net exports and investment. On the one hand, net exports are determined by the pattern of the binding constraint: they are positive when the constraint is binding and negative otherwise. We know that the constraint does not bind for many consecutive periods, and hence the model cannot generate the sustained capital outflows that we observe in the data during the 1980's. Because we know how output, consumption, and net exports are determined in equilibrium, investment is a residual; and because we know that the model does not do well at generating net exports, then it is not surprising that it does not do well at explaining investment. Note that this intuition speaks about the cyclical behavior of investment and not its level.

On the other hand, from the Euler equation we know that expectations of a binding constraint generate a friction that is isomorphic to a tax on the return to investment. This means that if the planner expects that the constraint will bind tomorrow, they will deter the SEO from accumulating capital in order to make autarky less attractive. This will make the level of investment lower under this circumstance. As such we observe that capital in the model is higher than in the data at the beginning of the period—because the constraint is not binding—and then it is lower in the period where the constraint is binding on and off.

Related Literature

As mentioned above, I follow the literature on international debt that relies on the willingness to pay as well as the literature on debt-constrained asset markets. This literature studies the theoretical implications of limited commitment constraints, but mostly in pure-exchange, closed-economy setups. I follow the international debt literature—in particular, Eaton and Gersovitz (1981), where a debtor who defaults faces permanent exclusion from international capital markets, and Atkeson (1991), who considers an environment where the participation constraint interacts with a moral hazard problem.

Kehoe and Perri (2002) go a step further and extend the work of Kehoe and Levine (1993) and Kocherlakota (1996) to a full-blown international business cycle model with production. They study business cycle co-movements across industrial countries; their paper is one of the few quantitative applications of this type of model. My paper follows Kehoe and Perri (2002) in the sense that it is an open economy setup and it has endogenous capital. However, there are three main differences between their work and mine. First, their model is a two-country model while I focus on a SOE that interacts with a risk-neutral lender; second, they focus on explaining the data moments and do not look at the full equilibrium path like I do; and third, they consider shocks to productivity while I consider shocks to the growth rate of productivity and the international interest rate.

Tsyrennikov (2013) analyzes a SOE model with limited commitment and moral hazard and finds that introducing moral hazard improves the performance of the model in several dimensions when trying to explain the moments of the Argentinian business cycle. He argues that introducing moral hazard can reverse the result that, in limited commitment models, the participation constraint binds only in good times; however, it is unclear whether this can occur under a reasonable model parameterization. Once more, one important difference between Tsyrennikov's work and mine is that he looks only at data moments and not the full equilibrium path of the data. Dovis (2019) introduces information frictions into an otherwise limited commitment model and shows that key aspects of sovereign debt crises can be rationalized as part of the efficient risk-sharing arrangement between a sovereign borrower and foreign lenders.

Bai and Zhang (2010) explore the impact of limited enforcement constraints on the Feldstein-Horioka puzzle and point out that it is necessary to couple the limited enforcement with limited spanning in order to generate the volume of capital flows and a savings-investment correlation like those in the data.

Tomz and Wright (2010) show that in practice defaults occur both in good and bad times, which is in line with the results found here. Bai and Zhang (2012) show that sovereign debt renegotiations take an average of five years for bank loans but only one year for bonds, which also is in line with the short binding spells in the model.

From a theoretical standpoint, the problem with introducing capitalistic production into limited commitment models is that capital introduces a non-convexity to the outside option. This generates complications when it comes to solving the model because the solution can be infeasible and/or suboptimal. This issue is pointed out in Messner and Pavoni (2016). Feasibility can be verified, but dealing with the sub-optimality is more involved. I suggest a quantitative solution to this problem in Section 2.

Aguiar, Amador, and Gopinath (2009) consider a SOE where the government cannot commit to policy and seeks to insure a risk-averse domestic constituency. Their setup and mine coincide in that the limited commitment is one-sided in the context of a SOE. My model differs in that the SOE is subject to international interest rate shocks, labor supply is elastic, and markets are complete.

Finally, this paper also relates to the international real business cycle literature. This literature has shown that the excess volatility of consumption in Latin America can be explained by shocks to the growth rate of productivity (see Aguiar and Gopinath (2007)) and by shocks to the international interest rate (see Neumeyer and Perri (2005)). In this paper I show the medium- and long-term importance of these two types of shocks when they affect borrowing constraints.

The rest of the paper proceeds as follows: Section 2 presents the limited commitment model and describes the methodology. Section 3 explains the model dynamics. Section 4 explains the calibration and solution method. Section 5 shows the results. And Section 6 states some final remarks.

2 Theoretical framework

The model consists of a small open economy (SOE) that interacts with a risk-neutral lender, and there is a dynamic contracting problem between these two parties. The SOE produces a good using domestic labor and capital, and production is subject to a country-specific productivity shock that grows at a stochastic rate. Output is used for domestic consumption and domestic investment and to make a transfer to the lender. This transfer can be positive or negative (depending on whether the SOE is a net lender or borrower), but the SOE cannot commit to repaying its debt: the SOE has limited commitment.

The lender has linear utility on the transfers received from the SOE and faces a stochastic discount factor. The discount factor is the inverse of the risk-free interest rate. Note that in the model, there are two stochastic elements driving business cycle fluctuations: the productivity shock for the SOE and the shock to the discount factor of the lender (or international interest rate shock).

Time is discrete and runs to infinity. In each period t, the state of the world s_t, determined by the productivity and the stochastic discount factor shocks, is realized. I denote by $s^t = (s_0, ..., s_t)$ the history of events up to and including period t. The probability of any particular history s^t as of period 0 is given by $\pi\left(s^t\right)$, and the initial realization s_0 is such that $\pi\left(s_0\right) = 1$. In period t, the SOE produces a tradable good using capital $K(s^{t-1})$ and per-capita hours worked $h(s^t)$, and production is affected by a productivity shock $A(s^t)$, which has a stochastic trend.

Output at history s^t is given by

$$Y\left(s^t\right) = A\left(s^t\right) F\left(K\left(s^{t-1}\right), h\left(s^t\right)\right), \tag{1}$$

where F is a standard constant returns to scale production function.

Consumers in the SOE have preferences

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)u\left(C\left(s^t\right),h\left(s^t\right)\right), \tag{2}$$

where $C\left(s^t\right)$ denotes consumption at s^t and β denotes the discount factor of the SOE. The resource constraint of the SOE is given by

$$C\left(s^t\right)+X\left(s^t\right)+T\left(s^t\right)\leq A\left(s^t\right)F\left(K\left(s^{t-1}\right),h\left(s^t\right)\right), \tag{3}$$

where $T\left(s^t\right)$ denotes the transfers received/made from/to the lender at s^t and $X\left(s^t\right)$ denotes investment at s^t. Note that the transfers $T\left(s^t\right)$ can also be interpreted as the net exports of the SOE, and investment is determined by the standard capital-accumulation equation: $X\left(s^t\right)=K\left(s^t\right)-(1-\delta)K\left(s^{t-1}\right)$.

The lender faces the following utility function:

$$\sum_{t=0}^{\infty}\sum_{s^t}Q\left(s^t\right)\pi\left(s^t\right)T\left(s^t\right), \tag{4}$$

where $Q\left(s^t\right)$ denotes the stochastic discount factor, $Q\left(s^t\right)=q\left(s^0\right)q\left(s^1\right)...q\left(s^t\right)$, and $q\left(s^t\right)$ is the realization of the stochastic discount factor shock at s^t. It is assumed that the SOE is more impatient than the lender, $Q\left(s^t\right)>\beta^t$. This is a common assumption in the literature to prevent the SOE from infinitely accumulating assets over time. Alternatively, from a political economy standpoint, it can be interpreted as a reflection of the fact that in developing countries political uncertainty is higher and politicians are short sighted.

If we solve the model in a centralized manner, in the absence of limited commitment the planner would maximize a weighted sum of the expected discounted utilities of the lender and the SOE

$$\max_{\{K(s^t),C(s^t),h(s^t)\}}\left[\sum_{t=0}^{\infty}\sum_{s^t}Q\left(s^t\right)\pi\left(s^t\right)T\left(s^t\right)+\mu\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)u\left(C\left(s^t\right),h\left(s^t\right)\right)\right] \tag{5}$$

subject to the resource constraint (3) and the capital accumulation equation for the SOE.

Now consider a situation where the SOE cannot commit to repaying its debt: it has limited commitment. In this case, apart from the resource and capital accumulation constraints, the SOE faces a participation constraint. By enforcing this constraint the planner guarantees that the SOE will stay in the contract with the lender and repay its debt. The participation constraint states that at every point in time and every state of the world, the SOE has to weakly prefer the allocation it receives by being in a contract with the lender (and ship $T(s^t)$ units of output to the lender) to the allocation it could attain if it were in autarky from then on. The participation constraint is of the form

$$\sum_{r=t}^{\infty}\sum_{s^t}\beta^{r-t}\pi\left(s^r|s^t\right)u\left(C\left(s^r\right),h\left(s^r\right)\right)\geq V^A\left(K\left(s^{t-1}\right),s^t\right)\ \forall r,\ s^r, \tag{6}$$

where $\pi(s^r|s^t)$ denotes the conditional probability of s^r given s^t, $\pi(s^t|s^t)=1$, and $V^A(K(s^{t-1}),s^t)$ denotes the value of autarky from s^t onward. The value of autarky corresponds to the utility delivered by the following problem:

$$V^A\left(K\left(s^{t-1}\right),s^t\right)=\max_{\{K(s^r),C(s^r),h(s^r)\}}\sum_{r=t}^{\infty}\sum_{s^r}\beta^{r-t}\pi\left(s^r|s^t\right)u\left(C\left(s^r\right),h\left(s^r\right)\right)$$

subject to

$$C\left(s^r\right)+K\left(s^r\right)\leq A\left(s^r\right)F\left(K\left(s^{r-1}\right),h\left(s^r\right)\right)+\left(1-\delta\right)K\left(s^{r-1}\right)\ \forall r,\ s^r,$$

where $r\geq t$, and $K(s^{r-1})$ is given.

In the context of limited commitment, the planner maximizes the weighted sum of the expected discounted utilities of the big and small country (5), subject to the participation constraint (6), the resource constraint (3), and the capital accumulation constraint of the SOE

$$\max_{\{K(s^t),C(s^t),h(s^t)\}}\left[\sum_{t=0}^{\infty}\sum_{s^t}Q\left(s^t\right)\pi\left(s^t\right)T\left(s^t\right)+\mu\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)u\left(C\left(s^t\right),h\left(s^t\right)\right)\right] \tag{7}$$

subject to

$$\sum_{r=t}^{\infty}\sum_{s^t}\beta^{r-t}\pi\left(s^r|s^t\right)u\left(C\left(s^r\right),h\left(s^r\right)\right)\geq V^A\left(K\left(s^{t-1}\right),s^t\right)\ \forall r,\ s^r,$$

$$C\left(s^t\right)+K\left(s^t\right)-\left(1-\delta\right)K\left(s^{t-1}\right)+T\left(s^t\right)\leq A\left(s^t\right)F\left(K\left(s^{t-1}\right),h\left(s^t\right)\right).$$

Notice that in the model with limited commitment, although markets are complete, there are endogenous borrowing constraints because there is a limit to the amount of contingent claims of a particular type that can be sold. The limit is determined by the amount the SOE is willing to repay according to the participation constraint.

Solution Method

Productivity has a stochastic trend $\frac{A_t}{A_{t-1}} = a_t$ that follows an autorregressive process of order one:

$$\ln a_t = \rho_a \ln a_{ss} + (1 - \rho_a) \ln a_{t-1} + \sigma_a \varepsilon_t; \tag{8}$$

as such, the model has a trend equal to $Z_t = A_t^{\frac{1}{1-\alpha}}$. We re-define the model in effective units by detrending all variables by Z_{t-1}, where $z_t = \frac{Z_t}{Z_{t-1}}$, and $n_t = \frac{N_t}{Z_{t-1}}$ stands for any detrended variable n.

We can write the stationary version of the problem in the following way

$$\max_{\{k(s^t), c(s^t), h(s^t)\}} \left[\sum_{t=0}^{\infty} \sum_{s^t} Q\left(s^t\right) \pi\left(s^t\right) t\left(s^t\right) + \mu \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) u\left(c\left(s^t\right), h\left(s^t\right)\right) \right]$$

subject to

$$\sum_{r=t}^{\infty} \sum_{s^t} \beta^{r-t} \pi\left(s^r | s^t\right) u\left(c\left(s^r\right), h\left(s^r\right)\right) \geq V^A\left(k\left(s^{t-1}\right), s^t\right) \ \forall r, \ s^r,$$

$$c\left(s^t\right) + z\left(s^t\right) k\left(s^t\right) - (1 - \delta) k\left(s^{t-1}\right) + t\left(s^t\right) \leq a\left(s^t\right) F\left(k\left(s^{t-1}\right), h\left(s^t\right)\right).$$

Solving this problem is difficult because it has an infinite number of participation/enforcement constraints, which can have complicated binding patterns. Furthermore, given that consumption and leisure enter the current enforcement constraint, the standard dynamic programming approach cannot be used.

Kydland and Prescott (1980) show that when this feature is present, the state space can be expanded to include an extra state variable; in this way the problem has a solution that is stationary in the new expanded state space. Marcet and Marimon (2019) follow Kydland and Prescott (1980) and extend their approach to different applications.

To solve the limited commitment model, I extend the recursive contract approach of Marcet and Marimon (2019) in a similar way to Kehoe and Perri (2002). The added state variable is the current relative weight of the small country in the planning problem. Adding this state variable, and assuming that the shocks to productivity and the stochastic discount factor are Markovian, allows me to write a recursive problem.

If we denote by $\beta^t \pi\left(s^t\right) \lambda\left(s^t\right)$ the multipliers on the participation constraint, $M\left(s^t\right) = M\left(s^{t-1}\right) + \lambda\left(s^t\right)$ and $M\left(s_{-1}\right) = \mu$, where $M\left(s^t\right)$ is defined as the original planner weight for the small country (μ) plus the sum of the past multipliers on the enforcement constraint along history s^t, then the optimality conditions for consumption, hours worked, and capital are given by

$$u_{c(s^t)}\left(c\left(s^t\right),h\left(s^t\right)\right)=\frac{Q\left(s^t\right)}{\beta^t\left(M\left(s^{t-1}\right)+\lambda\left(s^t\right)\right)}, \tag{9}$$

$$-u_{h(s^t)}\left(c\left(s^t\right),h\left(s^t\right)\right)=u_{c(s^t)}\left(c\left(s^t\right),h\left(s^t\right)\right)a\left(s^t\right)F_{h(s^t)}, \tag{10}$$

$$z\left(s^t\right)u_{c(s^t)}=\beta\sum_{s^{t+1}}\pi\left(s^{t+1}|s^t\right)\left[u_{c(s^{t+1})}\frac{M\left(s^{t+1}\right)}{M\left(s^t\right)}\left(a\left(s^{t+1}\right)F_{k(s^t)}+1-\delta\right)-\frac{\lambda\left(s^{t+1}\right)}{M\left(s^t\right)}V^A_{k(s^t)}\right], \tag{11}$$

and the complementary slackness condition.[4]

To preserve stationarity and following Kehoe and Perri (2002), I normalize the multiplier by defining $\upsilon\left(s^t\right)=\frac{\lambda\left(s^t\right)}{M\left(s^t\right)}$. I also denote the right-hand side of equation (9) as

$$\frac{Q\left(s^t\right)}{\beta^t\left(M\left(s^{t-1}\right)+\lambda\left(s^t\right)\right)}=\frac{Q\left(s^t\right)}{\beta^t M\left(s^t\right)}=\hat{M}\left(s^t\right), \tag{12}$$

such that a transition law for $\hat{M}\left(s^t\right)$ is enough to determine its evolution. This transition equation is given by

$$\hat{M}\left(s^t\right)=\frac{q\left(s^t\right)}{\beta}\left(1-\upsilon\left(s^t\right)\right)\hat{M}\left(s^{t-1}\right). \tag{13}$$

Using Equation (12), the first-order conditions can be re-written and summarized by Equation (10),

$$\hat{M}\left(s^t\right)=u_{c(s^t)}\left(c\left(s^t\right),h\left(s^t\right)\right) \tag{14}$$

instead of Equation (9),

$$z\left(s^t\right)u_{c(s^t)}=\beta\sum_{s^{t+1}}\pi\left(s^{t+1}|s^t\right)\left[\frac{u_{c(s^{t+1})}}{1-\upsilon\left(s^{t+1}\right)}\left(a\left(s^{t+1}\right)F_{k(s^t)}(.)+1-\delta\right)-\frac{\upsilon\left(s^{t+1}\right)}{1-\upsilon\left(s^{t+1}\right)}V^A_{k(s^t)}\right] \tag{15}$$

instead of Equation (11), Equation (12), and the complementary slackness condition.

Note that Equation (13) can be written in terms of consumption by using (14):

$$u_{c(s^t)}\left(c\left(s^t\right),h\left(s^t\right)\right)=\frac{q\left(s^t\right)}{\beta}\left(1-\upsilon\left(s^t\right)\right)u_{c(s^{t-1})}\left(c\left(s^{t-1}\right),h\left(s^{t-1}\right)\right). \tag{16}$$

[4] See Appendix for the derivation.

This substitution changes the nature of consumption within the model and transforms it from a control variable to a state variable. Hence the state space of the problem is given by $x_t = (c(s^{t-1}), k(s^{t-1}), s_t)$, where $s = (A(s^t), q(s^t))$.

I assume that the underlying shocks for productivity growth $a(s^t)$ and the stochastic discount factor $q(s^t)$ are Markov. This assumption implies that the conditional probability $\pi(s^t|s^{t-1})$ can be written as $\pi\left(s_t|s_{t-1}\right)$, and the solution to the programming problem in (7) can be characterized recursively by policy rules for $k(s^t)$, $c(s^t)$, $h(s^t)$, and $\upsilon(s^t)$, where the state is x_t.

The policy rules satisfy the first-order conditions (10), (15), (16), the participation constraint (6) expressed in effective units, and the complementary slackness condition on the multiplier.

To calculate the policy functions I use a version of policy function iteration and modify it to handle enforcement constraints in a similar way to Kehoe and Perri (2002). Specifically, I define a grid X on the state space. I restrict the search to functions that take arbitrary values for every $x \in X$ and are completely characterized over the state space when their value for every $x \in X$ is identified.

I define a value function for each party: $W(x)$ for the SOE and $P(x)$ for the lender. These value functions satisfy the first-order conditions (10), (15), (16), the participation constraint (6), and the complementary slackness condition on the multiplier and are of the form

$$W(x) = u(c(x), h(x)) + \beta \sum_{s'} \pi\left(s'|s\right) W\left(x'\right), \tag{17}$$

$$P(x) = T(x) + \sum_{s'} \pi\left(s'|s\right) q\left(x'\right) P\left(x'\right). \tag{18}$$

I start with the solution to the planner's problem when there is no limited commitment. This guarantees that the initial value functions $W^0(x)$ and $P^0(x)$ are uniformly greater than or equal to the value of the true solution. This condition is needed for the algorithm to converge to the right solution.

Given the first-order conditions and the initial guess for labor, the normalized multiplier, and the value functions $(h^0(x), \upsilon^0(x), W^0(x), P^0(x))$, I find a new set of policy functions

$$\left(k^1(x), c^1(x), h^1(x), \upsilon^1(x), W^1(x), P^1(x)\right).$$

To do so, I first assume that the participation constraint does not bind and find a set of allocations $\left(h, c', k'\right)$. When the participation constraint does not bind $\upsilon = 0$, and the set of allocations has to satisfy (10),

$$u_{c(s^t)} = \frac{q\left(s^t\right)}{\beta} u_{c(s^{t-1})}, \tag{19}$$

and

$$u_{c(s^t)} = \beta \sum_{s^{t+1}} \pi\left(s^{t+1}|s^t\right) u_{c(s^{t+1})} \left(a\left(s^{t+1}\right) F_{k(s^t)} + 1 - \delta\right). \tag{20}$$

After finding this set of allocations, I check if they satisfy the participation constraint

$$u\left(c,h\right) + \beta \sum_{s'} \pi\left(s'|s\right) W^0\left(x'\right) \geq V^A\left(k,s\right); \tag{21}$$

if they do, then I define them to be the new set of allocations for x, and $\upsilon^1\left(x\right) = 0$. If they don't satisfy the participation constraint (21), then I solve for a set of allocations $\left(h, \upsilon, c', k'\right)$ that satisfy (10), (15), (16), and the participation constraint (21). This new set of allocations then becomes $\left(k^1\left(x\right), c^1\left(x\right), h^1\left(x\right), \upsilon^1\left(x\right), W^1\left(x\right), P^1\left(x\right)\right)$.

Note that because there is endogenous capital accumulation that affects the value of autarky, there is a non-convexity in the problem. As pointed out by Messner and Pavoni (2016) this makes it such that applying Marcet and Marimon (2019) to solve the model does not guarantee feasibility or optimality of the solution. Feasibility can be verified, and I do, but guaranteeing that the solution is a global optimum is more complicated.

To address the sub-optimality issue I introduce a homotopy when doing the updating of the policy functions to guarantee that I do not jump over the global optimum. Because the model is solved on a grid, to implement a homotopy, I convexify between the old and new policy function at the grid-point level such that the steps between the two are small enough to avoid jumping over the global optimum solution and risk reaching a local optimum.

3 Model dynamics

The model dynamics depend on whether the participation constraint is binding or not, and the binding patterns of the constraint depend on the productivity and stochastic discount factor shock.

When the participation constraint is not binding, $\upsilon = 0$. Assuming that the utility function is logarithmic in consumption, and from (16), then

$$c\left(s^t\right) = \frac{\beta}{q\left(s^t\right)} c\left(s^{t-1}\right). \tag{22}$$

Given that the SOE is more impatient than the lender $\frac{\beta}{q(s^t)} < 1$, the relative impatience of the SOE leads to declining consumption. Furthermore, due to the discount factor shocks, consumption declines at a stochastic rate. High discount factor shocks (which correspond to a decrease in the international interest rate) make consumption decline at a faster rate—as is better to front-load consumption—and low discount factor shocks make consumption decline more slowly.

When the participation constraint is binding, $\upsilon \neq 0$. From (16) and keeping the assumption that the utility function is logarithmic in consumption, we know that

$$c\left(s^t\right) = \underbrace{\frac{\beta}{q\left(s^t\right)}}_{A} \underbrace{\frac{1}{1-\upsilon\left(s^t\right)}}_{B} c\left(s^{t-1}\right). \tag{23}$$

Equation (23) states that there are two competing forces leading the long-run properties of consumption. Part A represents the relative impatience of the SOE and drives consumption down. On the other hand, part B drives consumption up. As a result, when the constraint is binding, consumption might increase, decrease, or remain constant, depending on which one of the two forces dominates.

When the participation constraint is expected to bind tomorrow, the Euler equation is given by

$$z\left(s^t\right) u_{c(s^t)} = \beta \sum_{s^{t+1}} \pi\left(s^{t+1}|s^t\right) \left[\frac{u_{c(s^{t+1})}}{1-\upsilon\left(s^{t+1}\right)} \left(a\left(s^{t+1}\right) F_{k(s^t)} + 1 - \delta\right) - \underbrace{\frac{\upsilon\left(s^{t+1}\right)}{1-\upsilon\left(s^{t+1}\right)} V^A_{k(s^t)}}_{C}\right]. \tag{24}$$

Note that, in this case, the Euler equation depends on the marginal product of capital tomorrow (as usual), but it has an extra term that depends on the expected impact of capital on next period's participation constraint. In other words the return to capital tomorrow is now affected by the change in the value of autarky when there is an extra unit of capital. If having one more unit of capital tomorrow increases the value of autarky, then it is more likely that the SOE will want to walk away from the contract tomorrow. As a result, the optimal strategy for the planner is to discourage capital accumulation by "taxing" the return to capital by c in Equation (24). This term is analogous to an investment "wedge"; see Chari, Kehoe, and McGrattan (2007) and Ohanian, Restrepo-Echavarria, and Wright (2018); and it is isomorphic to a tax on investment. Note that this feature is not exclusive to the model presented here, but all limited commitment models endogenously generate a "tax" on investment.

Summarizing, consumption shrinks if the participation constraint does not bind. If the participation constraint binds, the dynamics of consumption depend on the tightness of the constraint (i.e., whether A or B dominates in Equation (23)). History matters through $c(s^t)$ and $k(s^t)$.[5] When consumption and capital are low, the participation constraint is more likely to bind and generate a higher investment tax. When consumption and capital are high, the participation constraint is less likely to bind and the investment tax is lower.

Up to now I have not discussed the effects that the productivity and stochastic discount factor shocks have on the constraints. Productivity shocks have a different effect in the short and long run. In the short run, a high productivity shock tightens the constraint because it increases the value of autarky. When the value of autarky is higher, the need for the insurance provided by the contract goes down and the SOE is more likely to want to walk away from the contract. In the long run, the situation can change. It is still true that a long sequence of high productivity shocks will keep the constraint tight, but it is also the case that a long sequence of low productivity shocks will trigger the constraint. This is true because when productivity goes down, so does the value of autarky. When the value of autarky goes down, the constraint loosens and the planner assigns less consumption and capital to the SOE in that period. If this happens repeatedly, the SOE will depreciate its asset position until the insurance provided by the contract is no longer better than being in autarky; at this point the participation constraint will be triggered again.

On the other hand, shocks to the stochastic discount factor have the same effect in the short and long run. A higher international interest rate, which is the same as a lower stochastic discount factor, decreases the return to domestic capital through the effect that it has on the growth rate of consumption (which enters the Euler equation). A lower return decreases capital accumulation, and a lower capital stock means that the value of autarky is lower. This loosens the constraint.

Finally, hours worked are determined by (10), (26), and (27)

$$h\left(s^t\right) = \left(\frac{1}{\psi c\left(s^t\right)} a\left(s^t\right)(1-\alpha)\, k\left(s^{t-1}\right)^{\alpha}\right)^{\frac{1}{\gamma+\alpha}}, \tag{25}$$

independent of whether the constraint is binding or not.

[5] Unlike the endowment economy where history doesn't matter.

4 Parametrization, functional forms, and application

From among the Latin American countries, I chose to calibrate the model to Argentina, as is common in the SOE literature. To be able to compare the equilibrium path of the model-generated data with that of the observed data, I parameterize the model to compute the policy functions. I also pin down the observed productivity and discount factor shocks to feed into the model and use data on consumption, investment, output, hours worked, and net exports.

The model has four structural parameters. The structural parameters define the preferences, the production function, and the capital accumulation equation.

I assume that the preferences for the SOE are given by

$$U(c,h) = \log c_t - \left(\frac{\psi}{1+\gamma} (h_t)^{1+\gamma} \right) \tag{26}$$

and output is determined by a standard constant-returns-to-scale Cobb-Douglas production function

$$F(k,h) = k^{\alpha} h^{1-\alpha}. \tag{27}$$

I set the capital share in the production function to $\alpha = 0.36$, as is standard in the literature. The preference for leisure ψ is set such that the level of hours worked in the model matches the data, and $\gamma = 1.5$, which implies a Frisch elasticity of labor supply of two-thirds. This is within the range typically estimated using micro data on the labor supply intensive margin, a little higher than estimates using micro data on the extensive margin, but smaller than estimates typically found using macro data (see the surveys by Pencavel (1987), Keane (2011), and Reichling and Whalen (2012)). The depreciation rate is set to $\delta = 0.07$.

The discount factor of the SOE (β) is set to 0.85. The level of this parameter on its own is not that important. What really matters for the dynamics of the model is the relative impatience of the SOE to the lender, the difference between β and $\bar{q}$. Where $\bar{q}$ is the mean of the stochastic discount factor of the lender.

The path for q is the inverse of the international interest rate. For the international interest rate I use the nominal lending rate adjusted for inflation as measured by the GDP deflator for the United States. The lending rate is the base rate charged by banks on short-term business loans. The monthly rate is the average of rates of all calendar days and is posted by a majority of the top 25 insured chartered commercial banks; $\bar{q}$ is 0.96 and an average discount factor of 0.96 implies an annual international interest rate of 4%, which is close to the average rate of return on capital

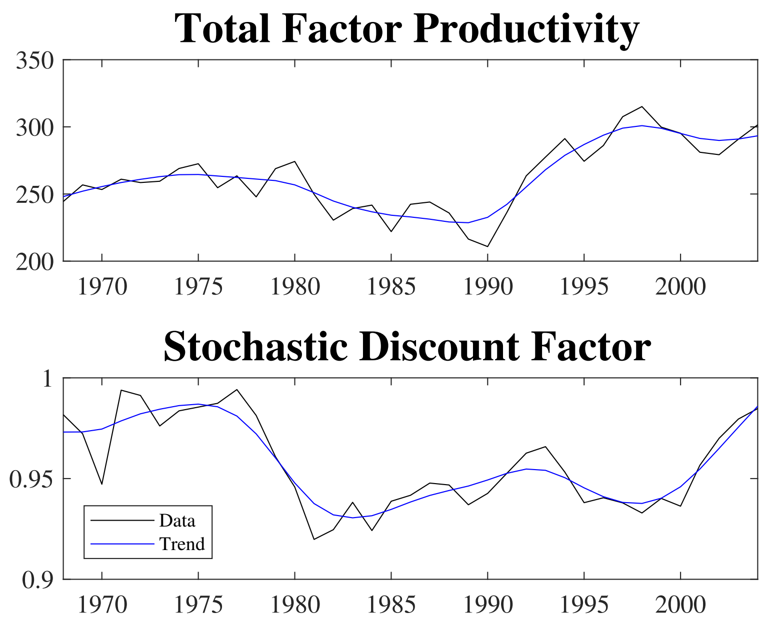

Figure 2: Time series for exogenous processes

over the past hundred years.

The lower panel in Figure (2) shows the time path followed by the stochastic discount factor (in black) and its Hodrick and Prescott (HP) trend (in blue). Interest rates were low prior to the 80's. At the beginning of the 1980's, after the oil shocks, interest rates go up and remain high until around the year 2000, when they decrease but do not return to the level they were prior to the 80's.

I assume that the logarithm of the stochastic discount factor follows a standard autorregressive process of order one:

$$\log(q_{t+1}) = (1-\rho_q)\log(\bar{q}) + \rho_q \log(q_t) + \sigma_q \varepsilon_{qt}. \tag{28}$$

The estimated parameters for the discount factor process are the following: $\rho_q = 0.81$ and $\sigma_q = 0.0131$.

To recover the time series for the productivity shock, I calculate the Solow residual by using the production function (27) and aggregate data on output, hours worked, and investment for Argentina. To recover capital I use the perpetual inventory method using as initial value the number calculated by Nehru, Vikram, and Ashok Dhareshwar (1993) for 1950. The upper panel of Figure (2) shows the recovered time series, its level (in black), and its HP trend (in blue). As can be seen, prior to 1980 productivity was relatively stable and at a moderate level; after 1980, productivity goes down for ten years and then recovers to a value above its pre-80's level.

Recall that it is assumed that productivity has a stochastic trend and that the growth rate of productivity is stationary and follows a standard autorregressive process of order one:

$$\log(a_{t+1}) = (1-\rho_a)\log(a_{ss}) + \rho_a \log(a_t) + \sigma_a \varepsilon_{at}. \tag{29}$$

The upper panel of Figure 3 shows the stochastic process for the growth rate of productivity (the Solow residual) in black. The estimated parameters for Equation 29 are the following: $\rho_a = 0.03$ and $\sigma_A = 0.05$.

In order to solve the model, I have to discretize the stochastic discount factor and productivity shocks. To do so, I follow the methodology suggested by Tauchen and Hussey (1991) to transform the stochastic processes into Markov chains. I discretize a to 5 states and q to 3 states, for a total of 15 exogenous states. Figure 3 shows the Markov processes in black and the resulting approximated Markov chains in blue.

I solve the model using dynamic programing, where capital and consumption (or the cumulative multiplier) are the endogenous states and are on a grid, and productivity and the stochastic discount factor are the exogenous states in a grid comprising fifteen states.

5 Results

This section compares the model-generated data with the observed data for Argentina. To produce the model-generated data, I feed in the time series for productivity growth and stochastic discount factor to the policy functions of the stationary model (blue lines in Figure 3).

Because I am interested in the ability of the model to explain the low-to-medium frequencies of the data, I compare the trends of the observed data and the model-generated data. Figure 4 shows the HP trend for the observed per-capita consumption data (black line) and the HP trend for the model-generated consumption (blue line).[6] The shaded areas in the figure represent the periods in which the constraint was binding. One can see that the model does a very good job at explaining the path of the data, and it does particularly well at explaining the drop in per-capita consumption observed between the 80's and the 90's, the fast recovery between 1990 and 1998,

[6]Note that in Figure 4 what is plotted is the HP trend of C instead of c (not consumption in efficiency units but aggregate consumption to make it comparable with the data).

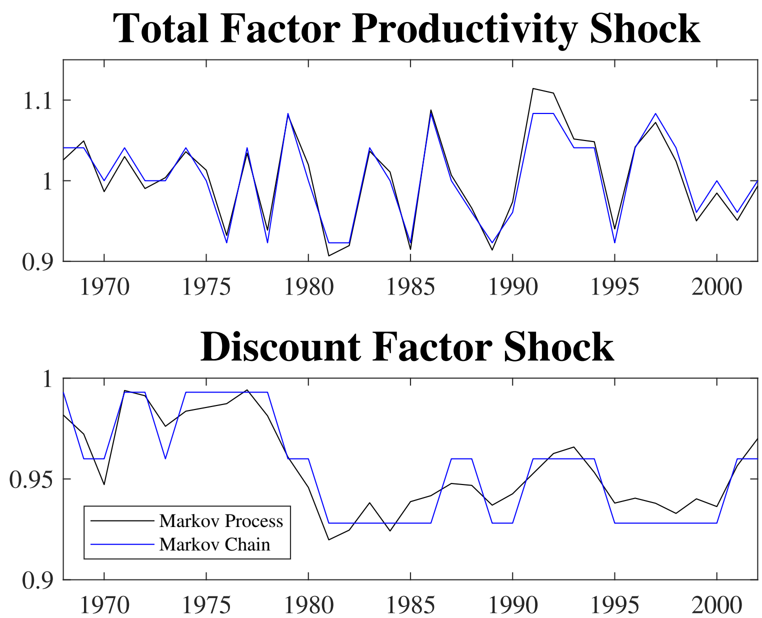

Figure 3: Exogenous Shocks

and the slowdown that follows.

The shaded regions in the graph show the periods for which the participation constraint was binding. During this period, Argentina defaulted on its external debt in 1982—the Latin American debt crisis—and in January 2002. There was another default in 1988-89, but it was on domestic debt. As we can see from the plot, the model does generate a binding constraint in 1982 and another one in 2000, among others. This reflects the ability of the model to replicate Argentina's inability to repay, which—in the model—is reflected in a tightening of the participation constraint.

To better understand what triggers the participation constraint, Figure 5 is the same as Figure 4, but now the time series paths for productivity and discount factor are superimposed on the per-capita consumption graph. The purpose is to compare the binding patterns of the participation constraint with the changes in productivity growth and the discount factor. Note that the shocks are not to scale in the graph, and the left axis does not reflect their values; this superimposed plot is just to convey the qualitative behavior of the shocks over time.

From Figure 5 we can see that the periods where the constraint binds mostly coincide with the periods where productivity *growth* is high. This is expected because high productivity growth reflects a higher value of autarky; and when state-contingent insurance is not needed, there is a

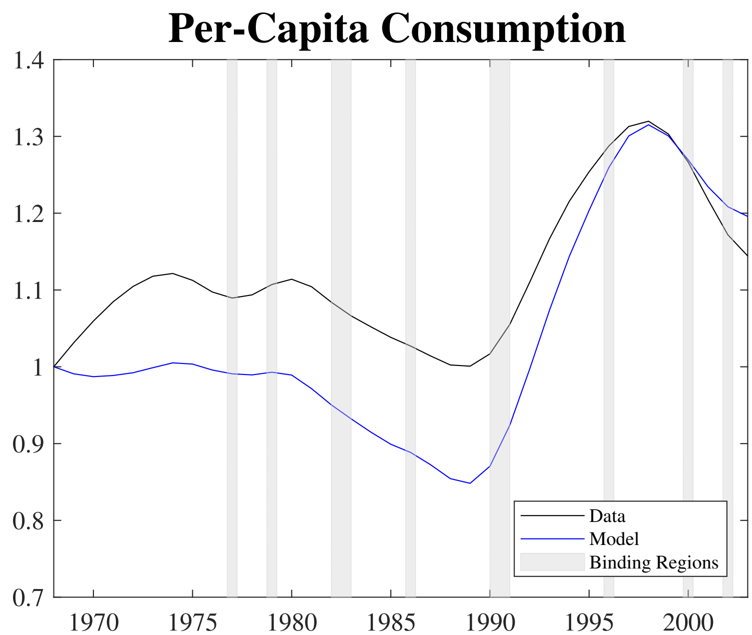

Figure 4: Observed and model-generated per-capita consumption and binding regions

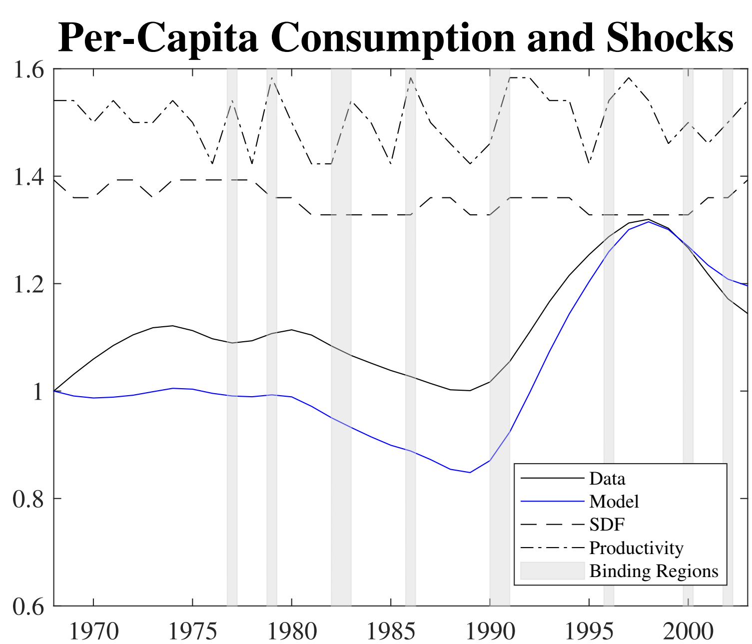

Figure 5: Per-capita consumption and observed stochastic shocks

threat of default to increase the value of being in the contract and the participation constraint binds. However, note as well from the upper panel of Figure 2 that productivity is declining during the decade of the 80's, and here is where we mostly see that the participation constraint starts to bind. This is a period where, although there are spikes in the growth rate of productivity, in terms of the level, it is a prolonged bad time. It is also the time period during which Argentina experienced the lowest productivity growth rates and we know that when state contingent insurance is needed the constraint is non-binding and the agent front-loads consumption due to impatience. This front loading of consumption implies that consumption is being run down until the insurance provided by the contract goes to zero (consumption is declining and asset position is depreciating), triggering the participation constraint. So this is a case where the participation constraint is triggered because of a high productivity *growth* shock or because of prolonged bad times.

Recall that the rate at which consumption goes down—when the agent is unconstrained—is determined by the stochastic discount factor shock. A higher discount factor (lower interest rate) means that consumption is being run down at a faster pace, which will trigger the constraint sooner. In other words, the discount factor affects the speed at which the constraint is approached and hence it affects the level of consumption. This mechanism is not obvious from the pattern of the binding constraint and the behavior of the stochastic discount factor in Figure 5, but this point is illustrated in Figure 6.

In Figure 6 I plot a counterfactual exercise where I feed in the path for the growth rate of productivity, but set the discount factor at a constant level. The black line shows the case where the discount factor is set at its mean, the dashed blue line is the case where the discount factor is at the lowest level, and the dotted blue line shows the case where the discount factor takes the highest value in the grid. This plot also shows us the relative importance of the productivity and interest rate shocks. As can be seen, the pattern followed by the data is solely dictated by the productivity *growth* shock but its level is affected by the discount factor shock. A low discount factor—high interest rate—decreases the speed at which consumption is run down and hence increases the level of consumption; likewise, when consumption falls faster due to a high discount factor, its level is lower.

Note that modeling the growth rate of productivity (and not just the level) allows us to reconcile the fact that the participation constraint binds in "good times"—which in this case means that it binds when there is a high productivity *growth* rate shock—as well as in prolonged bad times—in the decade of the 1980's when the *level* of productivity was low.

It is also important to note that the equilibrium path generated by the model spends a good

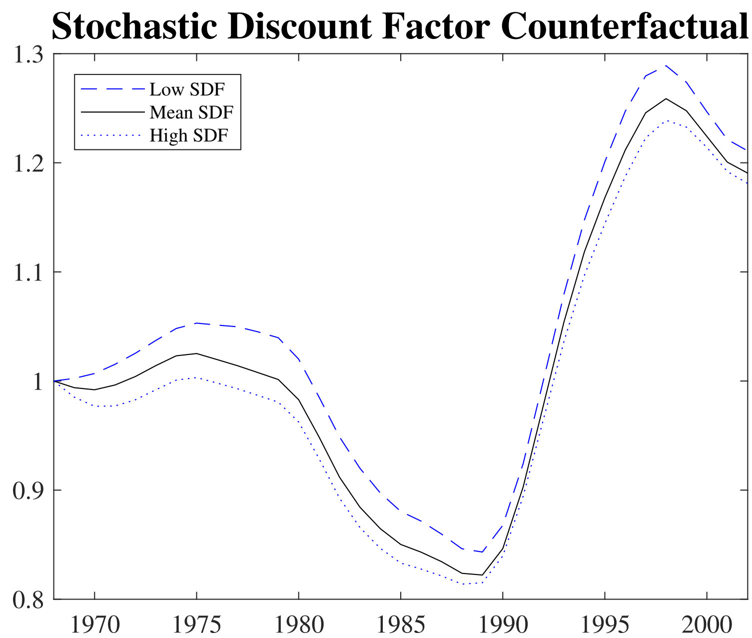

Figure 6: Counterfactual per-capita consumption for different values of the stochastic discount factor

amount of time outside the constraint. In other words, the solution is considerably different from that of the closed economy (the constraint is not constantly binding). This is important because a common concern about this type of model is that the constraint binds too much (see Kehoe and Perri (2002)). The existing literature avoids this problem by penalizing the outside option: they fix the outside option to the value of autarky minus some percentage of output, which would be lost in the case of walking away from the contract. In this model this is not necessary due to the discount factor shocks. The discount factor of the risk-neutral lender determines the speed at which the constraint is approached. High interest rate shocks (low stochastic discount factor) imply that consumption goes down at a slower rate and, hence, the equilibrium path is unconstrained longer because consumption is higher.

Finally, Figure 7 shows the model-generated and observed data for output, capital, hours, and net exports. The black line represents the observed data and the blue line represents the model-generated data. We can see that the model does a relatively good job generating the data for output and capital is not too far off. The output boom of the first decade is generated by capital, while the output boom of the last ten years is generated by a combination of an increase in hours worked, high productivity growth and an increasing capital stock. From the Euler equation we know that

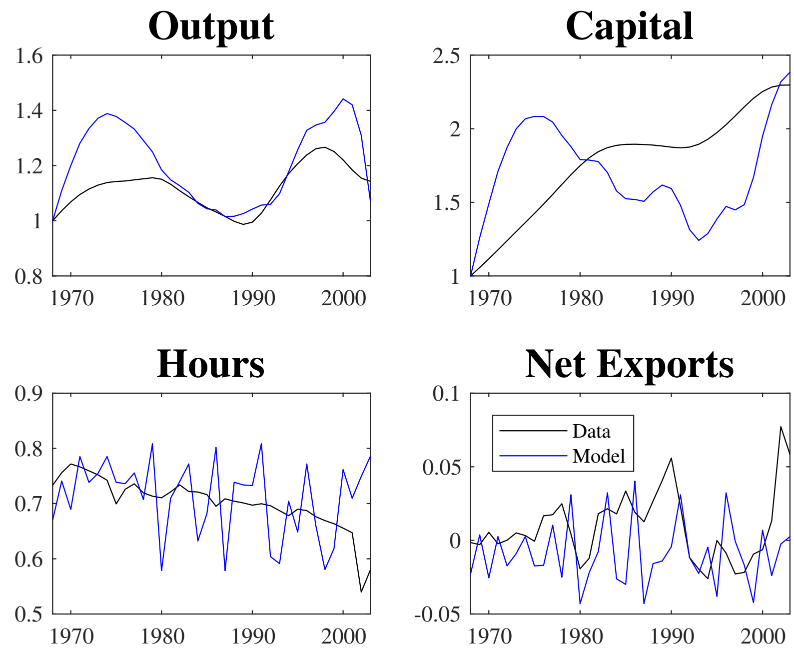

Figure 7: Other Macroeconomic Variables

expectations of a binding constraint generate a friction that is isomorphic to a tax on the return to investment. This means that if the planner expects that the constraint will bind tomorrow, they will deter the SOE from accumulating capital in order to make autarky less attractive. This will make the level of investment lower under this circumstance. As such we observe that capital in the model is higher than in the data at the beginning of the period— because the constraint is not binding—and then it is lower in the period where the constraint is binding on and off. However, as mentioned earlier, capital starts increasing around 1993, contributing to the output boom observed in the model after the same period.

Hours worked and net exports are much more volatile in the model than in the data. Nevertheless the model does generate the capital flows reversal that we observe at the beginning of the 80's. This is because the participation constraint binds exactly in 1982 and 1983, and we know that in a limited commitment model, net exports will be positive only when the participation constraint is binding and otherwise they will be negative.

However, although the model is able to generate the sharp reversal of capital flows around 1982, it is not able to capture the prolonged debt crisis. This is expected because in the model the intent to default is corrected immediately (there is no default in equilibrium), while in reality we know that the debt crisis took around a decade to overcome.

6 Conclusion

In this paper I show that in Latin America, following the recession and debt crises of the early 1980's, per-capita consumption declined for about ten years and per-capita consumption in 2005 was not very different from that in 1980. Motivated by the fact that this decline in per-capita consumption coincides with the start of the Latin American debt crisis, this paper examines the ability of a real business cycle model with limited commitment, capital in the production function, and domestic productivity *growth* shocks and international interest rate shocks to explain the path followed by macroeconomic variables in Argentina.

I find that a model with limited commitment and productivity *growth* and international interest rate shocks can account for the behavior of per-capita consumption in Argentina and does relatively well at explaining output and the level of hours worked. Furthermore, I find that the productivity *growth* shock governs the pattern of the constraint and that the international interest rate shock governs the tightness of the constraint. On the negative side I find that such a model is not well suited to explain the behavior of net exports or investment.

These results have implications for theory. The interest rate shock determines the speed at which the constraint is approached (unlike Kehoe and Perri (2002) and Aguiar, Amador, and Gopinath (2009))—the lower relative impatience generated by high international interest rate shocks buys more time outside the constraint—in this way the outside option does not have to be penalized with loss of output to prevent the equilibrium from being mostly on the constraint (or the same as the closed economy solution). Also, when exogenous stochastic growth of total factor productivity is added to the model, the constraint can bind in both good times—as is known about limited commitment models—and in prolonged bad times. This last feature was not known (in a representative agent model) because previous work does not study medium- and long-run properties of limited commitment models.

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Appendix

I can write the Lagrangian for problem (7) as follows:

$$\max_{\{k(s^t),h(s^t),c(s^t)\}} \sum_{t=0}^{\infty}\sum_{s^t} Q\left(s^t\right)\pi\left(s^t\right)\left\{a\left(s^t\right)F\left(k\left(s^{t-1}\right),h\left(s^t\right)\right)-c\left(s^t\right)-z\left(s^t\right)k\left(s^t\right)+(1-\delta)k\left(s^{t-1}\right)\right\}$$

$$+\mu\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)u\left(c\left(s^t\right),h\left(s^t\right)\right)$$

$$+\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\lambda\left(s^t\right)\left\{\sum_{r=t}^{\infty}\sum_{s^r}\beta^{r-t}\pi\left(s^r|s^t\right)u\left(c\left(s^r\right),h\left(s^r\right)\right)-V^A\left(k\left(s^{t-1}\right),s^t\right)\right\},$$

where $\beta^t\pi\left(s^t\right)\lambda\left(s^t\right)$ denotes the multipliers on the participation constraint. Note that the set up of this Lagrangian is standard; I only substituted transfers (T) in the lender's utility by their definition from the resource constraint.

Marcet and Marimon (1994) point out that, given we know $\pi\left(s^r\right)=\pi\left(s^r|s^t\right)\pi\left(s^t\right)$, we can define

$$\sum_{t=0}^{\infty}\beta^t\lambda\left(s^t\right)\sum_{r=t}^{\infty}\beta^{r-t}u\left(c_r\right)=\sum_{t=0}^{\infty}\beta^t M\left(s^t\right)u\left(c_t\right),$$

where $M\left(s^t\right)=M\left(s^{t-1}\right)+\lambda\left(s^t\right)$, $M\left(s_{-1}\right)=0$, and $\lambda\left(s^t\right)$ is the Lagrange multiplier associated with the participation constraint.

Given this, the participation constraint can be written as

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\left\{M\left(s^t\right)u\left(c\left(s^t\right),h\left(s^t\right)\right)-\lambda\left(s^t\right)V^A\left(k\left(s^{t-1}\right),s^t\right)\right\},$$

where $M\left(s^t\right)=M\left(s^{t-1}\right)+\lambda\left(s^t\right)$ and $M\left(s_{-1}\right)=\mu$. Hence, the Lagrangian can be re-written as

$$\max_{\{k(s^t),h(s^t),c(s^t)\}} \sum_{t=0}^{\infty}\sum_{s^t} Q\left(s^t\right)\pi\left(s^t\right)\left\{A\left(s^t\right)F\left(k\left(s^{t-1}\right),h\left(s^t\right)\right)-c\left(s^t\right)-z\left(s^t\right)k\left(s^t\right)+(1-\delta)k\left(s^{t-1}\right)\right\}$$

$$+\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\left\{M\left(s^{t-1}\right)u\left(c\left(s^t\right),h\left(s^t\right)\right)+\lambda\left(s^t\right)\left(u\left(c\left(s^t\right),h\left(s^t\right)\right)-V^A\left(k\left(s^{t-1}\right),s^t\right)\right)\right\}, \tag{30}$$

where $M\left(s^t\right)$ is defined as the original planner weight for the small country μ, plus the sum of the past multipliers on the enforcement constraint along history s^t.

The optimality conditions for consumption, hours worked, and capital from (30) are given by

$$u_{c(s^t)}\left(c\left(s^t\right),h\left(s^t\right)\right)=\frac{Q\left(s^t\right)}{\beta^t\left(M\left(s^{t-1}\right)+\lambda\left(s^t\right)\right)}, \tag{31}$$

$$-u_{h(s^t)}\left(c\left(s^t\right),h\left(s^t\right)\right)=u_{c(s^t)}\left(c\left(s^t\right),h\left(s^t\right)\right)a\left(s^t\right)F_{h(s^t)}, \tag{32}$$

$$z\left(s^{t}\right)u_{c(s^{t})}=\beta\sum_{s^{t+1}}\pi\left(s^{t+1}|s^{t}\right)\left[u_{c(s^{t+1})}\frac{M\left(s^{t+1}\right)}{M\left(s^{t}\right)}\left(a\left(s^{t+1}\right)F_{k(s^{t})}+1-\delta\right)-\frac{\lambda\left(s^{t+1}\right)}{M\left(s^{t}\right)}V^{A}_{k(s^{t})}\right], \tag{33}$$

and the complementary slackness condition.