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Working Paper Number	2014-031D
Revision Date	July 2017
Citable Link	https://doi.org/10.20955/wp.2014.031
Suggested Citation	Sanchez, J.M., Sapriza, H., Yurdagul, E., 2017; Sovereign Default and Maturity Choice, Federal Reserve Bank of St. Louis Working Paper 2014-031. URL https://doi.org/10.20955/wp.2014.031

Published In	Journal of Monetary Economics
Publisher Link	https://doi.org/10.1016/j.jmoneco.2018.01.001

Sovereign Default and Maturity Choice*

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September 24, 2017

Abstract

This study develops a novel model of endogenous sovereign debt maturity choice that rationalizes various stylized facts about debt maturity and the yield spread curve: first, sovereign debt duration and maturity generally exceed one year, and co-move positively with the business cycle. Second, sovereign yield spread curves are usually non-linear and upward-sloped, and may become non-monotonic and inverted during a period of high credit market stress, such as a default episode. Finally, output volatility, sudden stops, impatience and risk aversion are key determinants of maturity, both in our model and in the data.

JEL Classification: F34, F41, G15
Keywords: Crises, Default, Yield Curve, Spreads, Bond Duration.

*Corresponding author: Juan Sánchez, juan.m.sanchez78@gmail.com. The views expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of St. Louis or the Federal Reserve System. The authors thank Javier Bianchi, Huberto Ennis, Andres Erosa, Emilio Espino, Juan C. Hatchondo, Leo Martinez, Rody Manuelli, and Ricardo Reis for insightful comments. The feedback received at the ASU, the St. Louis Fed, the Federal Reserve Board, the Midwest Macro Meetings, the SED, the Bank of France, the RIDGE Workshop on Financial Crisis, and the Madrid Macro Workshop, is also appreciated. Yurdagul gratefully acknowledges the support from the Ministerio Economìa y Competitividad (Spain), Marìa de Maeztu grant (MDM 2014-0431), and from Comunidad de Madrid, MadEco-CM (S2015/HUM-3444).

1 Introduction

Our paper proposes a new approach to studying the maturity of sovereign debt and the term structure of interest rate spreads. The new framework helps rationalize the maturity choice for sovereign debt and the pricing of this debt at each maturity. Our model shows that when economic growth weakens (bad times), sovereign debt maturity and duration shorten, default risk and interest rate spreads over risk-free debt increase, and the term spreads decline, often resulting in a humped or in a negatively sloped yield spread curve for the borrowing country.

The model also helps identify the key factors determining a country's debt maturity. Our results show that sovereign debt maturity and duration are explained by the trade-off between the benefits of long-term debt from the hedging of changes in the rollover costs of the debt, and its costs from higher spreads due to debt dilution. Changes in economic conditions or country characteristics that increase hedging incentives, such as increasing the risk aversion and the probability of sudden stops, imply a longer equilibrium debt maturity. Changes that tend to lower sovereign borrowing costs, such as increasing the discount factor, lead to higher debt maturity in equilibrium, as it becomes relatively less expensive for the economy to borrow longer term. In contrast, changes that tend to increase sovereign borrowing costs, such as higher income volatility, lead to a reduction in debt maturity to partially offset the increase in yield spreads. Our calibrated model accounts, qualitatively and quantitatively, for differences in debt maturity across several economies over the last 20 years.

The behavior of debt maturity and yield spreads over the business cycle using data for several economies is summarized in Table 1. The first column of the table shows that the maturity of sovereign debt tends to be procyclical, i.e., bad times are linked to the shortening of the average debt maturity. The table also highlights that even during bad times, countries tend to sustain debt maturities that significantly exceed one year.[1] Finally, the last two columns of the table show that 1-year and 10-year sovereign bond yield spreads are countercyclical.

Our new modeling approach identifies four economic and financial market features as key in determining sovereign debt maturity. The first factor is the volatility of GDP growth. We find that a country with a more volatile process for income growth seeks to mitigate the higher yield spreads from higher default risk by both deleveraging and lowering its debt maturity. We document this type of debt management equilibrium outcome on the level and time profile of debt, in both our model and our empirical analysis.

The other factor is the possibility of a sudden stop. The presence of sudden stops as a long-standing feature of international credit markets has been well-documented in the literature (see for instance Edwards, 2007). Sudden stops may significantly shape the maturity profile of sovereign debt, as the possibility of a sudden withdrawal of funding that makes the sovereign unable to repay its immediate debt obligations creates a strong incentive for the sovereign to borrow at longer maturities. Other things equal, economies with less open capital accounts are less exposed to sudden stops. We show, using data for several economies, that countries with more open capital accounts have a longer maturity of their sovereign debt.

Another economic feature that can significantly affect the choice of sovereign debt maturity is the degree of risk aversion of the borrower. The higher the risk aversion, the higher

[1]For some countries in the sample we do not have data on debt duration. To maximize our sample size, we omit the duration data, but replicating the same exercise with duration delivers very similar results.

the incentive of the sovereign to insure itself against movements in market interest rates or other shocks that may deteriorate the country's borrowing conditions, by fixing the terms of the debt contract. Interestingly, a higher risk aversion of the sovereign borrower may also be interpreted as an economy with a higher level of after-tax income inequality among heterogeneous households, as argued by Ferriere (2015), who also associates inequality to stronger incentives to default. Thus, our model results suggest that economies with more pronounced income inequality tend to borrow at longer maturities, a result that we also find in the data.

Finally, a fourth economic characteristic with key implications for the choice of debt maturity by the sovereign is the degree of patience in the economy, which may be proxied in the data by the fraction of youngsters in the overall population of the country. Our model predicts that a country with a higher level of patience should exhibit lower outstanding debt and yield spreads, but higher maturity and duration, a testable hypotheses that we confirm in the cross-country dataset. Intuitively, a more patient country can partially trade the lower average yield spreads from a lower outstanding debt balance by extending its maturity profile to improve its rollover risk profile.

Our sovereign debt setup also helps explain debt pricing behavior often observed during debt crises that has not been addressed in the literature. In general, as an economy approaches a period of relatively high financial and economic stress, the sovereign yield spread curve shifts up, flattens and may develop a hump or even invert. Figure 1 illustrates these yield spread curve dynamics prior to periods of substantial credit market distress for some of the countries in our sample. The left plot shows the significant upward shift and inversion of the Argentinian curve from 2000 to 2001, as the economy headed to its 2002 sovereign debt crisis. At the onset of the global financial crisis, the Colombian curve, on the right panel, did not quite invert, but it also shifted up markedly and developed a noticeable hump. The literature on macroeconomics and the term structure of interest rates has long recognized both the relevance of non-linearities, i.e., curvature and humps, in characterizing the yield curve (see, for instance, Ang and Piazzesi, 2003; Neely, 2012) and the importance of considering the whole term structure of interest rates for aggregate variables (Gurkaynak and Wright, 2012). Interestingly, previous papers on quantitative models of sovereign default have discussed term yield spreads considering only two rates, one derived from a one-period bond and another derived from a perpetuity bond with a counterfactual decay in coupon payments.

Our analysis borrows from different strands of the literature on sovereign debt, default, and debt maturity. Following the seminal work on international sovereign debt by Eaton and Gersovitz (1981), a large portion of the literature on quantitative models of sovereign debt default has used only one-period debt (Arellano, 2008; Aguiar and Gopinath, 2006; D'Erasmo, 2008; Yue, 2010; Mendoza and Yue, 2012, among others). Models of long debt duration, such as Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009), feature exogenous maturity. In contrast, our quantitative model features endogenous sovereign debt maturity and repayment.

The work of Arellano and Ramanarayanan (2012) includes the choice of maturity, where the debt instruments available to the country are a one-period bond and a perpetuity bond with an exogenous, exponentially decaying coupon payment structure. There are several important differences between their approach to model debt maturity and our framework[2]:

[2]See also Hatchondo and Martinez (2013), and Hatchondo et al. (2014).

(i) by construction, those models only capture duration, since the maturities of their bonds are equal to one or infinite periods, while our model captures any maturity between 1 and N periods; (ii) the equilibrium duration in these models depends heavily on an exogenous coupon payment decay parameter of the long-term bond, suggesting that duration is not fully endogenous[3]; (iii) as pointed out in Bai et al. (2014), the assumption that payments are front-loaded, which is required to keep the discounted present value of repayments bounded, is at odds with the data[4]; (iv) the 10-1 years term spread computed in these models is not directly comparable with the data because in these models the 10-year spread is derived for a coupon bond, but their empirical analyses of yield curves consider the price of zero-coupon bonds of different maturity. In contrast, in our setup it is extremely easy to generate the entire yield curve; and (v) these models have two continuous variables to capture debt duration (the coupons for each bond) while our setup has one continuous variable and one discrete variable (which can be exactly represented by the number of maturities allowed in the model, say 10 or 15). This is a relevant aspect of these models, as their computation involves discretizing those continuous variables (see the discussion in Chatterjee and Eyigungor, 2012) and the number of points in the discretization must be very large to obtain accurate solutions.

Cole and Kehoe (2000) study self-fulfilling debt crisis equilibria considering a different timing from Eaton and Gersovitz (1981), where the government issues new bonds before it decides to default within the period. Cole and Kehoe (2000) characterize crisis zones, i.e.,

[3]Hatchondo et al. (2016) calibrate the decay parameter of the long-term bond targeting the average debt duration of the economy - the 2014 working paper version states that the parameter is set at 3.4% to obtain an average duration of 4.2 years in their model simulations.

[4]While Bai et al. (2014) tackle the problem of a front-loaded repayment schedule, that framework does not consider debt dilution, i.e., that additional borrowing decreases the price of outstanding debt, a feature considered by the literature to be important to understand maturity choice and the term yield spread (see Hatchondo et al. (2014).

the values and maturity structure of sovereign debt under which a financial crisis can arise due to a loss of confidence in the government. We model sudden stops as exogenous shocks and focus on the country's choice of maturity. In our framework, countries choose longer maturities partly to prevent the cost of sudden stops.

Aguiar and Amador (2013) also point to the optimality of strategies involving only short-term debt in the context of sovereign default risk models. Their study finds that during periods of deleveraging it is optimal for sovereigns to engage in strategies that use only one-period bonds. Aguiar and Amador (2013) argue that active shifts in the maturity structure by the sovereign may affect deleveraging incentives, and hence change the equilibrium price of long-term bonds. The price movements of long-term bonds will shrink the budget set of the sovereign, creating the incentive to use only one-period bonds during a period of deleveraging. Similarly, our model generates shorter maturity at times of high default risk.

The remainder of the paper proceeds as follows. In Section 2, we present the economic environment and the benchmark model, and we define the equilibrium. In Section 3, we calibrate the benchmark model and discuss the fit, and in Section 4 we compare the quantitative implications of the model for the dynamics of debt and prices at different maturities with the patterns found in the data. In Section 5, we study quantitatively the role of key country-specific and international credit market features on the choice of maturity. In Section 6, we conclude.

2 Model

We consider a stochastic small open economy with households, a benevolent government, and foreign lenders. The economy receives a stochastic endowment and may experience a sudden stop. The government trades in bonds of different maturities with risk-neutral foreign lenders. Debt contracts are not enforceable, as the government has the option to default on them. Default is costly for the country, and the foreign lenders charge a premium to account for the probability of not being paid back by the government.

We characterize the country's debt structure by a constant stream of payments, b, and the number of years that those payments are promised for, i.e., the debt maturity, denoted by m. This flat payment schedule, however, does not imply that the *actual* payments will be constant, as b is adjusted every year. Thus, for instance, a country's debt can be a promise to pay 1 unit of resources per year, for the next 5 years. The country has the option to change the debt maturity and coupon payment every year, so the following year the country may choose to have debt that promises to pay 1.2 units per year over the next 8 years.

As the country decides to default or repay its entire debt, what matters is not if this debt consists of only one constant coupon bond or many zero coupon bonds, but the yearly stream of future payments promised by the country and the number of years those payments were promised for. Therefore, b and m, together with the country's income and the sudden stop shock, are the state variables in the economy.

2.1 Preferences and endowments

Time is discrete and denoted by $t \in \{0, 1, 2, ...\}$. Each period, the small open economy has a stochastic endowment y_t that follows a finite-state first-order Markov chain with state space $Y \subset \mathbb{R}_{++}$ and transition probability $\Pr\{y_{t+1} = y' \mid y_t = y\}$.

The benevolent government in the country maximizes the expected utility over consumption sequences of the representative household. The discount factor is $\beta \in (0, 1)$, and the risk aversion coefficient is $\gamma \geq 1$. The momentary utility function is

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}. \tag{1}$$

2.2 Decision problem

A country with an outstanding amount of assets, b (debt if $b < 0$), has two actions to choose from. The first option is to make the payment (G, for good credit status). The second option for the country is to default (D, for default). The country's choice to either pay or default is represented by

$$V(y, a, b, m) = \max\left[V^G(y, a, b, m), V^D(y)\right], \tag{2}$$

where the policy function $D(y, a, b, m)$ takes the value of 1 if default is preferred and 0 otherwise. The variable a takes the value of 1 if the country does not receive a sudden stop shock and 0 otherwise.

If the country does not receive a sudden stop shock ($a = 1$), and decides not to default, it selects the maturity of the new portfolio, m', and the new level of promised annual payment,

b'. The value of this scenario for the country is:

$$V^G(y,1,b,m) = \max_{b',m'} \frac{c^{1-\gamma}}{1-\gamma} + \beta E_{y',a'|y} V(y',a',b',m') \tag{3}$$

subject to

$$c = y + b + \underbrace{[q(y,b',m';m-1)b - q(y,b',m';m')b']}_{\text{net resources obtained from international markets}}$$
$$b' \in \mathbb{R}_-, m' \in \mathbf{M}(m).$$

The constraint implies that consumption is equal to income, y, net of debt payments, b, plus a new term shown in brackets that captures the net resources that are obtained from or paid to international markets. The first summand in the brackets, $q(y,b',m';m-1)b$, is the market value of outstanding obligations. The price, $q(y,b',m';m-1)$, takes into account not just the current income, y, but also the obligations that the country will have from the beginning of the next period, (b',m'). These three variables determine the risk of default. This price also depends on $m-1$, which is the remaining number of years of payments left in the outstanding debt after the current year's payment. As $q(y,b',m';m-1)$ captures the price per unit of resources promised per year, it is multiplied by b to capture the market value of the total outstanding obligations at the beginning of the current period. With b negative, this term represents the gross resources leaving the country. Similarly, the term $q(y,b',m';m')b'$ represents the value of the outstanding debt at the end of the current period. Thus, this term represents the gross resources obtained from international markets, and the combination of both terms captures the net resources obtained from international markets.

The country's resource constraint can be rearranged such that this equation can be reinterpreted without relying on the country repurchasing the entire amount of outstanding debt every period. In particular, it can be written as

$$c = y + b - q(y, b', m'; m-1)(b' - b) - [q(y, b', m'; m') - q(y, b', m'; m-1)]\, b'. \tag{4}$$

To understand this equation, consider the case in which the country does not change the maturity date; this means that $m' = m - 1$. Then, Equation (4) reduces to

$$c = y + b - q(y, b', m'; m-1)(b' - b), \tag{5}$$

an expression that is very similar to the resource constraint with exogenous maturity, because the only change is in the amount of yearly payments, from b to b'.

The remaining term of the resource constraint, $[q(y, b', m'; m') - q(y, b', m'; m-1)]\, b'$, captures the change in maturity. Consider now that the country wants to extend the date of maturity by one year, such that the debt maturity remains constant, $m' = m$. In such case, the expression $[q(y, b', m; m) - q(y, b', m; m-1)]$ is the price of a zero coupon bond that promises to pay 1 unit in m years.

A country that receives a sudden stop shock and therefore has no access to credit markets ($a = 0$), may pay but cannot issue debt:

$$V^G(y, 0, b, m) = \frac{(y+b)^{1-\gamma}}{1-\gamma} + \beta E_{y',a'|y} V(y', a', b, m-1). \tag{6}$$

The policy functions for the amount of debt and the maturity are $B(y, a, b, m)$ and $M(y, a, b, m)$. When a country makes only its debt payment, the policy functions take the values $B(y, a, b, m) =$

b and $M(y,a,b,m) = m-1$. Then, it follows that if $a=0$, we must have $B(y,0,b,m) = b$ and $M(y,0,b,m) = m-1$.

Default brings immediate financial autarky for a stochastic number of periods and a direct output loss to the defaulting country. Formally, the value of default is:

$$V^D(y) = \frac{(y-\Phi(y))^{1-\gamma}}{1-\gamma} + \beta E_{y',a'|y}\left[(1-\lambda)V^D(y') + \lambda V(y',a',0,1)\right]. \tag{7}$$

Here, maturity is set to 1 as a normalization when the country return from a default with no debt.

2.3 Equilibrium

The price of the country's debt must be consistent with zero expected discounted profits, given the world interest rate r. The price of a bond with maturity $n>0$ of a country with income y, new debt $-b'$, and maturity $m'>0$ can be represented by

$$q(y,b',m';n) = \frac{E_{y',a'|y}}{1+r}\{(1-D(y',a',b',m'))[1+q(y',B(y',a',b',m'),M(y',a',b',m');n-1)]\}.$$

As the debt being priced matures in n periods, after the country repays 1 unit, in the next period there will be $n-1$ periods left to repay the remaining units. Because the state of the economy may change going forward, the price of those future promises incorporates the country's new income, y', and new debt profile represented by the policy functions B and M. In contrast, in the existing literature of quantitative models of sovereign debt and default, the price accounts for the optimal choice of future debt B, but not of maturity, M.

Therefore, our representation of maturity choice allows us to define the price function that captures debt dilution in a parsimonious way.

3 Calibration

In this section, we describe the calibration strategy and we compare the results with the data. We solve the model numerically, calibrating the parameters based on the literature and on the available data.

The model period is one year. Each year, the country can change the maturity by at most one year: i.e., $\mathbf{M}(m) = \{m-1, m, m+1\}$.[5] We set the maximum possible debt maturity to 10 years, significantly larger than the maturity observed for most emerging markets.[6]

We use the standard function for the income loss in case of default:

$$\Phi(y) = \begin{cases} 0, & \text{if } y < \phi \\ \phi, & \text{otherwise.} \end{cases}$$

We follow Arellano and Ramanarayanan (2012) in setting the annual risk-free interest rate to 0.032, and the probability of redemption if the country is excluded from the financial markets to 0.17. We assume a risk-aversion of 2 as is standard in the literature.

In specifying the rest of the model parameters, we use data from Colombia, an emerging market economy with good data availability and mean debt statistics close to those of the overall sample of countries. The standard deviation of the income shock is set to 0.019, and

[5]This is consistent with observed annual variations in maturity and saves computing time, and our results are robust to considering $\mathbf{M}(m) = \{m-2, m-1, m, m+1, m+2\}$.

[6]Our results are robust to allowing for longer maximum maturities. In particular, the moments shown in this section do not change up to two decimals points when we set the maximum maturity at 11 years.

the persistence is set to 0.86 to replicate the detrended log income process of Colombia.[7] We set the sudden stop probability at 14.3 percent, the frequency of sudden stops for Colombia according to Comelli (2015).

The remaining parameters are calibrated to match specific targets. In particular, we calibrate the discount factor, β, and the threshold of income in the default cost function, ϕ, to match two important moments in the data: the debt-to-output ratio and the volatility of consumption relative to that of output for Colombia. The model matches the targeted moments closely, generating a consumption volatility relative to output volatility of 1.16, and a debt-to-output ratio of 0.31.[8] The model parameters are summarized in Table 2.

3.1 Model fit of non-targeted moments

Our model can closely match several key empirical stylized facts. As illustrated in Table 3, in addition to capturing the moments usually discussed in the literature, such as the correlation of consumption and the trade balance with output, our model statistics mimic closely the median sovereign debt maturity and duration from the data, as well as their cyclical behavior.[9] Accordingly, the model generates the lower debt maturity and duration found in the data during bad times. Overall, the model delivers a maturity of 6.93 years and a duration of 3.75 years. During bad times, both debt maturity and duration in the model are about 10 percent lower than their averages, with maturity declining to 6.22 years (7.34

[7] We use GDP per capita to measure the income in the data, consistently with the rest of the paper.

[8] The debt-to-output ratio is the ratio of the face value of the debt to output, nb/y. This ratio is decreasing in the default cost parameter ϕ and in the discount factor β, while the relative volatility of consumption is decreasing in the discount factor β. Accordingly, for a given default cost, we set the discount factor to match the relative volatility of consumption, and we adjust the default cost to match the debt-to-output ratio.

[9] Consistent with the data, we use the Macaulay definition to derive the duration of sovereign debt. See the Appendix for definitions of debt duration and yield spreads.

years in the data) and duration declining to 3.41 years (4.26 years in the data), so the model captures slightly more than 80 percent of the observed maturity and duration levels.

As we focus on sovereign default risk, we consider sovereign bond yield spreads over risk-free debt instruments.[10] Our framework underpredicts the level of the spreads, which is to be expected in a model where bond prices are derived assuming risk-neutral lending. Our spreads are lower than those in models with exogenous maturity because, as it will be shown below, especially for periods of high spreads, the country chooses a debt maturity that is shorter than the fixed maturity set in the standard calibration of those models. Nevertheless, our model captures the dynamics of yield spreads for different bond maturities over the business cycle. As summarized in Table 3, yield spreads for 1, 2 and 10-year instruments are countercyclical, and spreads for short term bonds are lower than those for longer-term instruments.

4 Quantitative analysis

This section discusses the implications of our model for the dynamics of the macroeconomic variables of interest, and contrasts these model implications with the patterns observed in the data. We first use our analytical framework to rationalize the joint dynamics of debt and other macro variables in the years prior to a debt crisis episode. We then analyze the non-linearity and non-monotonicity of the spread curve, a relevant empirical pattern that the model helps explain, and we close the section comparing our model implications with

[10] The spread at each maturity is the difference between the yield on a zero-coupon bond with default risk, and the yield on a bond with the same characteristics but with no default risk. We present the details of the model computations in the Appendix.

those from a fixed maturity framework to highlight our contribution.

4.1 Default episodes

In this subsection, we look at the evolution of the main debt and economic activity indicators as the economy approaches a debt crisis episode. Figure 2 summarizes the behavior of several key variables according to the model in the ten periods leading to a default, with the values for the variables measuring deviations from the average values at default. As the top left panel on output shows, in the years before a crisis, economic activity deteriorates at an increasing pace, especially in the three years preceding the event. The decline in output contributes to an increase in the value of the debt-to-GDP ratio, as illustrated in the top right panel. As the debt burden weighs further, the incentives for debt dilution become stronger and, starting about three years before the crisis, the country increasingly reduces its debt duration (bottom left panel), which helps to fuel the observed dramatic rise in short-term yield spreads, especially during the last three years preceding the default, as the bottom right panel shows.

4.2 Non-linearities and non-monotonicities of the spread curve

The panels from Figure 1 illustrated episodes of spread curve reversals and non-monotonicity in Argentina and Colombia. As already pointed out by Levine (1975), market prices imply curves of considerable variation, e.g., multiple humped curves. Work on the term structure of interest rates that generate humped term structure curves include studies as early as Vasicek (1977). The non-linearity and non-monotonicity of the curves suggest that analyzing only the 1-year vs 10-year yield term spread, the focus of the current models of sovereign debt maturity and default, can leave out pricing information that is relevant to understand the dynamics of debt maturity, and may even be misleading. For instance, focusing only on the 1-year and 10-year spreads in the case of the somewhat humped curve shown in Figure 1 for Colombia in 2008, would incorrectly imply a flatter pattern that understates the true funding cost faced by the borrower at intermediate maturities, as it would miss credit market risk information from the higher intermediate-maturity spreads. Interestingly, the sovereign default literature has recognized the non-linear dynamics of sovereign debt pricing, but it has not addressed either the non-linearity or the non-monotonicity of the yield spread curve. Our paper helps to parsimoniously fill this void.

In line with the data, the yield spread curves generated by the model are typically upward sloped and concave, as illustrated in the top panel of Figure 3. The shape of the curve for a country can also change significantly according to the data, which is consistent with the country experiencing time-varying debt repayment probabilities at different maturities. Despite the bond pricing limitations arising from our risk-neutral lending setup, our model economy captures well the changes in the yield spread curve found in the data as countries approach default, as illustrated in the other panels of Figure 3. The middle panel shows that a few years before a default, the yield spread curve becomes non-monotone and concave, peaking at the horizon in which default is more likely, and the bottom panel shows that yields are high and decreasing with maturity in two years prior to a default. The concavity and non-monotonicity of the yield spread curve in the presence of sovereign default risk are stylized facts not addressed before in the literature.

To understand why default risk may explain changes in the yield spread curve, notice that the yield of a zero-coupon bond that pays in j periods is,

$$i_t^0(j) = \frac{1+r}{(\prod_{i=1}^{j} P_{t+i})^{1/j}}, \tag{8}$$

where $P_{t+1}, P_{t+2}, ..., P_{t+j}$ denote the repayment probabilities in $1, 2, ..., j$ periods ahead and r is the risk-free interest rate.[11] As a result, depending on the future repayment probabilities P_{t+i}, the yield curve may be increasing, decreasing, or non-monotone, following the different possible patterns observed in the data, as discussed in the literature. In our model, these probabilities are endogenous, and depend on the country's income, as well as on the level, the maturity and the duration of its debt.

4.3 Role of endogeneous maturity

As the main contribution of the paper is to endogenize debt maturity, we compare our benchmark model implications with those from a model with the same parameterization but fixed maturity. We first consider a fixed maturity of 7 years, the level implied by our benchmark model, and then of 10 years, roughly the level observed for Colombia in our sample.[12]

The results are given in Table 4. We find that the implied moments of the model are quite sensitive to the fixed maturity level. Importantly, the default rate is higher under fixed maturity. In line with this, observed spreads become larger, the correlation between output and consumption increases slightly, and the relative standard deviation of consumption to income increases, which indicates that the fixed maturity makes debt a poorer insurance

[11] The derivation of this formula is presented in the Appendix.

[12] Maturity is fixed except when a sudden stop occurs, in which case maturity decreases by one year.

instrument against shocks.

The sensitivity of the model-implied moments to the fixed maturity level makes the imposed maturity level very important. If we were to fix the maturity at the level observed in the data for Colombia, which is roughly 10 years, the model would imply a default rate that is clearly larger than our benchmark, with comparable debt-to-output ratio, higher spreads and higher correlation of consumption and output. Surely, the model with fixed maturity set at 7 years, the equilibrium level of the benchmark model, delivers outcomes closer to our main exercise, but we would not have a clear idea of what is that level without solving our benchmark model.

5 The cross-section of maturity choices

In this section, we study the cross-section of sovereign debt maturity. We highlight four important factors in the data affecting the debt maturity choices of countries, and we show that our model offers an explanation for these empirical findings.

We collect data on several countries' sovereign debt maturity and macroeconomic characteristics that can be mapped to different parameters in our model. The data contains 92 annual observations for the period from 1998 to 2014 (with gaps) for 10 countries: Argentina, Brazil, Chile, Colombia, Hungary, Mexico, Peru, Poland, Slovenia and Turkey.[13] We provide details on the data source in the Appendix. Table 5 shows the descriptive statistics of the key variables under study. Interestingly, there is large variation both in the maturity of

[13]The data that we use in Section 3 for Colombia is a subset of the data we use in this cross-section analysis. In particular, the data on maturity, sudden stop probability, and GDP per capita are the same data we use in our calibration and in the model-data comparison in Section 3. Similarly, we use the same data for the maturity and output variables given in Table 1 of Section 1 as we do in this section.

sovereign debt and in the explanatory macroeconomic variables.

The mappings of the volatility of income growth, the probability of a sudden stop, and the business cycle, between the model and the data are quite straightforward. To study variations in debt maturity and in other macroeconomic variables in our model as we change income growth volatility, we use the country-specific standard deviation of detrended (Hodrick-Prescott (HP)-filtered) real log GDP per capita as the counterpart in the data. As the empirical counterpart of the model's sudden stop probability, we consider the number of sudden stop episodes documented in the country over the years in the sample. To this end, we follow the list of sudden stop events in Comelli (2015). GDP per capita cycle is the deviation of log GDP per capita from its long run trend. In addition, we consider a measure of inequality -the Gini coefficient- as a proxy for the level of risk aversion in the model. Ferriere (2015) argues that higher risk aversion of the sovereign borrower may be interpreted as a higher level of income inequality in an economy populated by heterogeneous hand-to-mouth consumers. Finally, we use the share of the population with age 0-14 years old as a proxy for the model's discount factor, β, as a younger population will place a larger weight in the future than an older population. Thus, a larger proportion of young individuals in the economy will be associated with a larger value of β.

Table 6 illustrates the relationship between maturity and the key macroeconomic variables described above. Regressions were performed using pooled OLS and year fixed effects, which highlights the importance of controlling by common factors in a given year.[14] We also added GDP per capita, population, and a dummy for Argentina because the country was

[14] A fixed effect at the country level is not possible because two of the variables of interest, i.e., volatility and sudden stop probability, vary only across countries.

technically in default during the available sample period.

In the regressions with and without year fixed effects, all the macroeconomic factors of interest are statistically significant. Moreover, almost all the regression coefficients exhibit significance levels at the 1 percent level. To accurately gauge how much these factors affect sovereign debt maturity, we consider the impact that a one-standard-deviation change in each variable would have on maturity. As shown in the last two columns of Table 6, such change in volatility would decrease maturity by 0.95-1.02 years. A one-standard-deviation increase in the probability of a sudden stop would increase maturity by 1.02-1.08 years. Similarly, such change in inequality would increase debt maturity by 0.95-1.01 years, and a one-standard-deviation increase in the share of the population 0-14 years old would increase maturity by 2-2.07 years. Implications of variations over the cyclical component of the GDP per capita are similar in magnitude: a one-standard-deviation increase in the business cycle component of GDP per capita would increase maturity by 0.79-1.21 years.

Next, as illustrated in Table 7, we analyze variations of the four macroeconomic factors in the context of our model. First, we show the results with lower and higher variance of the endowment shock. Keeping the level and maturity of debt constant, an increase in the volatility of income would imply an increase in the probability of default and thus on the cost of debt, i.e., the yield spreads. In equilibrium, the country reduces the amount of debt and its maturity to mitigate the increase in the probability of default, and thus of the borrowing costs.

Second, we show the effect of changing the probability of a sudden stop event. To show that the sudden stop probability plays a very important role in obtaining long maturity debt in equilibrium, we computed a case in which that probability is zero. In such economy, the optimal maturity is equal to 1 year in the majority of the periods, and as a consequence, median duration and maturity are 1.[15] Interestingly, the effect of the sudden stop probability on maturity is much larger than on several other macro variables. For instance, when the sudden stop probability changes from 0 to 30 percent, the mean maturity of sovereign debt increases from 1 to 9.92 years, while the debt-to-output ratio only varies from 0.27 to 0.32.

The likelihood of experiencing a sudden stop episode in an environment of increased sovereign default risk is relevant to explain the recent debt dynamics of peripheral euro-area economies. During this period, the higher risk of a credit event in Greece increased the likelihood of a sharp fall in short-term funding to Ireland. Ireland exited its 2010 international bailout at the end of 2013 without any European Union or International Monetary Fund precautionary credit line as an insurance policy. Tight budget conditions and the country's sole reliance on market funding without a safety net provided the Irish government with strong incentives to borrow short term. However, as predicted by our model, policymakers opted to issue long-term debt. Issuing more long maturity debt helped Ireland to reduce its rollover risk.

Third, Table 7 also shows the role of risk aversion. As mentioned earlier, an increase in risk aversion can also be interpreted as a rise in inequality in an economy populated by heterogeneous hand-to-mouth consumers (Ferriere, 2015). If the country is more risk averse, it chooses a significantly longer maturity to take advantage of the hedging properties of long debt maturity (Arellano and Ramanarayanan, 2012).

Finally, the last two columns of Table 7 show the effect of changing the discount factor.

[15] This result resembles the welfare comparison of the economies with short vs. long term bonds in Chatterjee and Eyigungor (2012).

As expected, more patient countries have less outstanding debt. Without any changes in debt maturity, this would imply a very drastic decline in yield spreads. The country must trade off some of the decline in the average yield spread by choosing a longer maturity that reduces the risk of default due to a sudden stop. This is reflected in the sharp decline in spreads of all maturities and in all periods.

Overall, the results show that sovereign debt maturity and duration are determined by the trade-off between the benefits of long-term debt from the hedging of changes in the rollover costs of the debt, and its costs from higher spreads due to debt dilution. Changes in economic conditions or country characteristics that increase the incentives to hedge, such as increasing the risk aversion and the probability of sudden stops, imply a longer equilibrium debt maturity. Changes that tend to reduce sovereign borrowing costs, such as increasing the discount factor, lead to an increase in equilibrium debt maturity, as it becomes relatively cheaper for the economy to borrow longer term. In contrast, changes that tend to increase sovereign borrowing costs, such as higher income volatility, lead to a reduction in debt maturity to partially offset the increase in yield spreads.

Are the magnitudes of the changes similar in the regressions and in the model? Notice that for the discount factor and for the risk aversion we only have proxies in the data, so only a qualitative comparison can be performed. In the case of volatility and sudden stop probability, however, a quantitative comparison can be provided. The magnitudes in the model and in the data are quite similar. For instance, consider the regressions for the level (not the log) of maturity. In the model, as the sudden stop probabilities change from 0 to 0.3, maturity increases from 1 to 9.9 years; i.e. a change of about 8.9 years. In the data, a change of 0.3 in the probability of sudden stop implies an increase of maturity of about

5.5-5.8 years. The measure of volatility is not the same in the model as in the regression, but they are proportional, so we consider a 10 percent increase in volatility, the same change we computed in the model. There, such a change implies a reduction of 0.4 years, which is about a 5.8 percent reduction in maturity. In the regressions, an increase of magnitude around 10 percent of the average volatility leads to 0.29-0.31 years of reduction in the maturity, which is about 3.6-3.8 percent of the average maturity in the data. If instead we use the regression in logs, the same change in volatility generates about 4.4-4.6 percent change in maturity.

6 Conclusions

In this paper, we develop a novel approach to endogenize sovereign debt maturity in a tractable manner in the presence of default risk. In our framework, the sovereign chooses the maturity structure instead of being constrained to a combination of a one-period bond and a perpetuity bond with an exogenous coupon payment decay, as done in the literature, thus providing a more realistic and flexible portrayal of debt maturity choice. Our study also rationalizes and quantitatively mimics key properties of sovereign debt maturity, duration and the yield spread curve found in the data: first, the maturity and duration of sovereign debt generally exceed one year and comove positively with the borrowing country's business cycle. Second, the yield spread curve is generally upward-sloped and non-linear, and may turn non-monotonic and even invert when the economy approaches a severe debt stress event.

Crucially, our new framework also helps assess the role of country-specific and international financial market conditions in driving stylized facts on debt maturity identified in the analysis. Both in the model and in the data, we find that the presence of sudden stops, a country's risk aversion, income volatility and the degree of impatience, have a large impact on a country's maturity choice. Moreover, the quantitative results on the importance of these different macroeconomic conditions obtained in the calibrated model match very closely those from the data.

Finally, our model provides an excellent framework to explore other aspects of sovereign debt that are not addressed in this paper. For instance, one could quantitatively assess alternative debt rescheduling mechanisms by extending the baseline model to incorporate debt default restructurings. We leave this type of extension for future research.

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A Appendix (For online publication)

A.1 Computation

Approximating the solution using additional shocks

It is well-known that numerical solutions of defaultable sovereign debt models are prone to convergence issues. In this paper, we overcome such problems by introducing two idiosyncratic shocks with small variances. First, there is an i.i.d. income shock that adds to the persistent component, as highlighted in Chatterjee and Eyigungor (2012). This component of income, x_t, has a truncated normal distribution that is i.i.d. with mean 0 and variance σ_x and the bounds are given by $\underline{x}$ and $\overline{x}$. In addition to the transitory income shocks, there is a stochastic value shifter, χ_t, that affects the value of increasing the maturity of the portfolio. This variable is also i.i.d., normally distributed with mean 0 and variance σ_χ.

Decision problem and timing

At the beginning of each period, the persistent component of the endowment, y, the sudden stop shock and the value shifter, χ, are realized. If the country did not receive a sudden stop shock, it selects maturity of the new portfolio, m', that will be in effect only if the country ends up not defaulting by the end of the period. The value at that point is:

$$V(y,1,b,m,\chi) = \max_{m' \in \{\max\{m-1,1\},m,\min\{m+1,N\}\}} \{\hat{V}(y,b,m,m') + \mathbb{I}(m'=m)\chi + \mathbb{I}(m'=m+1)2\chi\}.$$

where the value of committing (in case of not defaulting) to $m' \in \{\min\{m+1,N\}, m, \max\{m-1,1\}\}$ is given by

$$\hat{V}(y,b,m,m') = E_x\left[\max\left(V^G(y,b,m,m',x), V^D(y)\right)\right].$$

After the decision on the maturity, the idiosyncratic component of the endowment, x, is realized. Then the country decides on whether or not to default on the existing debt, and in case of not defaulting, on the debt level, b'. Given the decision on the maturity of the portfolio, and the transitory income shock, the value is:

$$V^G(y,b,m,m',x) = \max_{b'} \tfrac{c^{1-\gamma}}{1-\gamma} + \beta E_{y',\chi',a'|y} V(y',a',b',m',\chi')$$

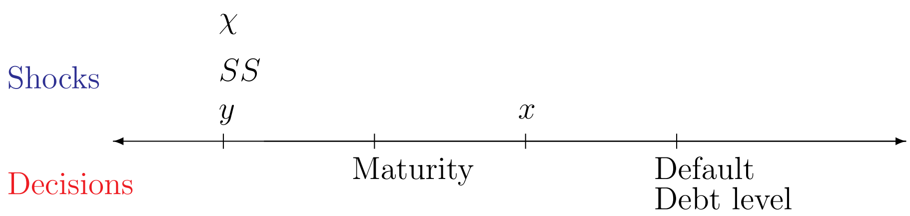
subject to

$$c = y + x + b - q(y,b',m';m')b' + q(y,b',m';m-1)b$$
$$b' \in \mathbb{R}_-.$$

A default brings financial autarky for a stochastic number of periods and a direct output loss to the defaulting country. In case of default, the i.i.d. income shock realization is assumed to be zero. Formally, the value of default is:

$$V^D(y) = \tfrac{(y-\Phi(y))^{1-\gamma}}{1-\gamma} + \beta E_{y',\chi',a'|y}\left[(1-\lambda)V^D(y') + \lambda V(y',a',0,1,\chi')\right].$$

Figure A1: Timing of shocks and decisions within a period



If the country receives the sudden stop shock in the beginning of the period, it has no decisions to make until x is realized. Then it has two options. First is to pay the debt promises and continue with the existing portfolio, and the second option is to default.

$$V(y, 0, b, m, \chi) = E_x \left[\max\left(V^P(y, b, m, x), V^D(y)\right)\right].$$

$$V^P(y, b, m, x) = \tfrac{(y+x+b)^{1-\gamma}}{1-\gamma} + \beta E_{y',\chi',a'|y} V(y', a', b, m-1, \chi')$$

if $m > 1$ and,

$$V^P(y, b, 1, x) = \tfrac{(y+b)^{1-\gamma}}{1-\gamma} + \beta E_{y',\chi',a'|y} V(y', a', 0, 1, \chi').$$

Figure A1 illustrates the timing of shocks and decisions in a given period. Notice that if the two additional shocks are degenerate, i.e. with zero variance, then the model we are actually solving coincides with the model described in Section 2.

Equilibrium

Given the world interest rate r, the price of the country's debt must be consistent with zero expected discounted profits. The price of a bond of maturity $n > 0$ in a country with income y, new debt $-b'$, and maturity $m' > 0$ can be represented by

$$q(y, b', m'; n) = \frac{E_{y',a',\chi',x'|y}}{1+r}\{(1-D)[1 + q(y', B, M; n-1)]\},$$

where we denote $M(y', b', m', \chi', a')$ by M, $B(y', b', m', M, a', x')$ by B, and $D(y', b', m', M, a', x')$ by D for simplicity.

Solution algorithm

We start an iteration by solving the optimal debt levels, given the guesses for (i) the price function and (ii) the expected values for next period value function (for both good and bad financial standing). For every component j on the persistent income shock grid $\mathbf{y}$, k on the transitory income shock grid $\mathbf{x}$, i on the debt grid $\mathbf{b}$, and for every maturity level m, we solve the optimal debt level of next period, for each possible maturity choice, m'. To find the optimal debt level, we use a discrete grid search method, and we apply the divide-and-conquer algorithm highlighted by Gordon and Qiu (2015) to limit the boundaries of the debt selection.

Chatterjee and Eyigungor (2012) have shown that the optimal debt level is increasing with respect to the transitory income shock in their framework, and it is also the case in ours. We use this monotonicity and the optimal debt decisions for each $(\mathbf{y}(j), \mathbf{b}(i), m, m', \mathbf{x}(k))$ to get the threshold levels of the transitory income shock that correspond to each different choice of the debt level. Formally, if the points $\{\tilde{b}_1, .., \tilde{b}_h\}$ on the debt grid are chosen for $(\mathbf{y}(j), \mathbf{b}(i), m, m')$, we find the $h-1$ threshold levels of the transitory shock, $\{\bar{x}_1, .., \bar{x}_{h-1}\}$ such that $\tilde{b}_1$ is optimal for all $x \leq \bar{x}_1$, $\tilde{b}_2$ is optimal for all $x \in (\bar{x}_1, x_2]$, ... , and $\tilde{b}_h$ is optimal for all $x > \bar{x}_{h-1}$. In addition to the optimal debt level, we also find the threshold level for default $\hat{x}_{def}$, using the fact that $V^G(y, b, m, m', x)$ is increasing in x and $V^D(y)$ does not depend on it. Next, we obtain the value before the realization of the transitory income shock, $\hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m')$ using these thresholds. Using these values, we then get the relevant thresholds for the maturity choice:[16]

$$\begin{aligned}
\bar{\chi}_+ &\equiv \max\{\hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m) - \hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m+1), \\
&\quad (\hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m-1) - \hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m+1))/2\} \\
\bar{\chi}_{-,1} &\equiv \hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m) - \hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m+1) \\
\bar{\chi}_{-,2} &\equiv \hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m-1) - \hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m).
\end{aligned}$$

The probabilities of the sovereign increasing the maturity by one, keeping it constant, and decreasing it by one, are $P_+ \equiv 1 - \Phi(\bar{\chi}_+)$, $P_0 \equiv \max\{0, \Phi(\bar{\chi}_{-,1}) - \Phi(\bar{\chi}_{-,2})\}$, and $P_- \equiv 1 - P_+ - P_-$, respectively. Hence, the value of the country that has not received a sudden stop shock is given by:

$$\begin{aligned}
E_\chi\left(V(y, 1, b, m, \chi)\right) &= P_+ \times \left[\hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m+1) + 2 \times E\left(\chi | \chi > \bar{\chi}_+\right)\right] \\
&\quad + P_0 \times \left[\hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m) + E\left(\chi | \bar{\chi}_{-,1} > \chi > \bar{\chi}_{-,2}\right)\right] \\
&\quad + P_- \times \hat{V}(\mathbf{y}(j), \mathbf{b}(i), m, m-1).
\end{aligned}$$

We follow the same approach to find the default policy and the expected value under the event that the country receives a sudden stop, except that the only threshold level to find in that case is the default threshold for x.

Then, using the thresholds of χ, above which the country chooses to increase its debt maturity, and the threshold levels of x that determine the optimal debt levels and the default policies of the country, we derive the price matrix q.

The algorithm described so far solves one iteration of the model. We start the iterations with a guess $W_0(y, b, m)$ for the expected value of being included in the financial markets; a guess $W_0^d(y)$ for the value of being excluded; and a guess $Q_0(y, b, m; n)$ for the price function, i.e. $q(y, b, m; n)$. Using these guesses, we solve the policy, value and price functions in the first iterations, which gives us W_1, W_1^d, and Q_1. We follow the same step for 15000 iterations and take the iteration delivering the "closest" price function to that of the previous iteration. Our convergence criteria considers the maximum relative distance:

$$\delta_h^Q \equiv \max_{j,k,m,n} \frac{|Q_h(j,k,m,n) - Q_{h+1}(j,k,m,n)|}{\frac{Q_h(j,k,m,n) + Q_{h+1}(j,k,m,n) + 0.001}{2}}.$$

[16] For simplicity, suppose here that $1 < m < N$.

Basics

We solve the model numerically using value function iteration on a discrete state space for debt and income shocks. For the debt level, we use maturity-specific grids of evenly-spread 201 points. We solve the policy and value functions for all points on these grids, and we do the discrete search to find the optimal debt policy also over these. We solve the price function for 41 equally-spaced points on this grid and lineary interpolate the implied function in the other parts of the algorithm. A maturity-specific grid is used to avoid points on the grid that are not chosen in equilibrium. Specifically, the maximum debt level, which is equal to 0.4 (40 percent of median income) for maturity 1, is decreased to 0.24 at maturity 10. We use a grid of 101 points for the persistent income shock. We equally spread 90 points below the median income, and 10 points above. We follow this approach since defaults only arise below the median income level, hence the price function is steeper in that region. We approximate the process of persistent income component using the Tauchen method. Table A1 shows the robustness of the results to reducing the number of points on each grid.[17]

After solving for the policy and value functions, we simulate 1500 countries (paths) for 500 years and we exclude the first 100 periods to calculate the statistical moments. The model counterparts to the empirical correlation and standard deviation statistics are averages across samples. For the first-order moments, we take country-specific medians before averaging across countries. For the data, we follow the same approach when generating statistics.

Role of the two additional idiosyncratic shocks

The non-convex nature of the value functions often makes it difficult to obtain convergence in defaultable sovereign debt models. In particular, the objective function of the sovereign, in the case of not defaulting, might have multiple local maxima, associated with potentially similar implied values. Hence, little differences in the guessed price functions can cause jumps in the optimal debt level, even if the maturity decision does not change. This is a problem common to most sovereign debt models—unless taken care of with a technique specifically designed to mitigate this issue. As our model also involves another layer of discrete choice, which is that of the maturity, non-convexity issues are magnified in our baseline.

[17]For the computation with 51 points on the income grid, we proportionally reduce the number of points above and below the median by a half, i.e. to 45 points below and 5 points above the median.

Table A1: Robustness to alternative sizes for debt and income grids

	BM	Alternative grids		
		Debt		Income
Grids for b	201	121	201	201
Grids for b (coarse)	41	41	26	41
Grids for y	101	101	101	51
Maturity	6.93	6.82	7.00	6.95
Maturity (bad times)	6.22	6.10	6.44	6.27
Duration	3.75	3.67	3.82	3.76
Duration (bad times)	3.41	3.38	3.45	3.40
$\rho(n, log(y))$	0.57	0.56	0.56	0.56
$\rho(dur, log(y))$	0.64	0.62	0.63	0.63
1-year spread	0.03	0.03	0.03	0.03
1-year spread (bad times)	0.37	0.35	0.34	0.35
2-year spread	0.17	0.17	0.17	0.19
2-year spread (bad times)	0.69	0.67	0.68	0.67
10-year spread	0.64	0.63	0.65	0.66
10-year spread (bad times)	0.88	0.86	0.88	0.88
$\rho(1YS, log(y))$	-0.43	-0.43	-0.43	-0.43
$\rho(2YS, log(y))$	-0.51	-0.51	-0.51	-0.50
$\rho(10YS, log(y))$	-0.64	-0.65	-0.65	-0.66
Default (%)	0.63	0.62	0.61	0.64
$\rho(log(c), log(y))$	0.90	0.90	0.90	0.91
$\rho(TB/y, log(y))$	-0.08	-0.08	-0.08	-0.08
$\sigma(log(TB/y))/\sigma(log(y))$	0.50	0.51	0.49	0.47
Debt/Income	0.31	0.31	0.30	0.30
$\sigma(log(c))/\sigma(log(y))$	1.16	1.16	1.15	1.14

Note: For all columns, the same parameters are used as in the benchmark (see Table 2). "Grids for b" gives the number of points on the main debt grid. "Grids for b (coarse)" stands for the number of points on the coarse debt grid, the one for which we solve the price function. "Grids for y" gives the number of points on our income grid. $\rho(x, y)$ represents the correlations between x and y, and $\sigma(x)$ gives the standard deviation of x. The bad times are the years when the log-output is less than the median. Spreads are given as percentages. See the Appendix for details about the simulations.

The additional shocks we introduce for numerical purposes help for convergence because they randomize the decision about the portfolio and about default, hence convexifying the value and price functions. Intuitively, when default decisions, optimal debt level, and optimal maturity depend on a continuous random variable, small changes in the price function do not lead to discrete changes in the default decision or choosing one debt level-maturity combination over another. Instead, these small changes in the price function lead to continuous and potentially small changes in the probability of a particular decision being made. Hence, over two iterations, smaller changes in the price function lead to smaller changes in the optimal policies, and in turn, to smaller changes in the price function in the next iteration.

Chatterjee and Eyigungor (2012) document that the presence of the income shock x helps significantly to obtain convergence in solving sovereign debt models by randomizing the decision on default and the optimal debt level. However, the presence of such a shock in our model would not be sufficient for two reasons related to the maturity choice structure: First, maturity choice makes the optimal debt level non-monotone with respect to the i.i.d. income shock. This is the reason why we assume that these shocks are realized after the maturity choice being made, as given the maturity choice, the debt level is monotone with respect to x. Second, the discrete choice of maturity calls for additional randomization by itself, as changes in the price function could affect the choice of m'. This is why we introduce the maturity shocks.

For the results shown in sections 3 to 5, we use a standard deviation for the i.i.d. income shocks, σ_x, equal to 0.05 percent of the mean income. The standard deviation for the i.i.d. maturity shocks, σ_χ, are set to 0.002 percent of the absolute flow utility of consuming the mean income, $\bar{y}$. In Table A2 we show the moments implied by alternative specifications for the sizes of the additional shocks, around the values we use for the benchmark. Moreover, we include convergence outcomes for each of the models. In particular, in the bottom panel of the table, (i) the first row corresponds to the measure of our criteria, δ^Q, the maximum relative distance in the price function between two iterations, (ii) the second row considers the maximum absolute distance in the price function, and (iii) the third row shows the average absolute distance in the price function over all the points. In the final three rows, we present the distance measures using the solution for the value function, W, instead of the solution for the price function, Q. Halving or doubling the variance of each shock relative to the benchmark calibration do not significantly change the moments implied by the model.[18]

[18]To economize space, Table A2 does not include the other settings that we use in this paper. The convergence measure δ^Q for the settings of Section 5 with σ_y equal to 0.018 and 0.020 are 6.44×10^{-10} and 5.08×10^{-10}; they are 9.69×10^{-12} and 1.53×10^{-9} for the models with the sudden stop probability p_s equal to 0 and 30; 1.06×10^{-11} and 3.00×10^{-10} for the settings with the risk aversion γ equal to 1 and 3; and they are 3.87×10^{-9} and 5.18×10^{-10} for the settings with the discount factor β equal to 0.939 and 0.953.

Table A2: Analyzing the role of the additional shocks

	Benchmark	Alternative variances for IID shocks on			
		Income		Maturity	
$\sigma_x/\bar{y}$	0.0005	0.00025	0.001	0.0005	0.0005
$\sigma_\chi/\lvert u(\bar{y})\rvert$	0.00002	0.00002	0.00002	0.00001	0.00004
Maturity	6.93	6.96	6.89	6.92	6.96
Maturity (bad times)	6.22	6.31	6.17	6.21	6.28
Duration	3.75	3.78	3.72	3.75	3.78
Duration (bad times)	3.41	3.43	3.40	3.41	3.42
$\rho(n, log(y))$	0.57	0.56	0.58	0.57	0.55
$\rho(dur, log(y))$	0.64	0.63	0.64	0.64	0.62
1-year spread	0.03	0.03	0.03	0.03	0.03
1-year spread (bad times)	0.37	0.37	0.36	0.36	0.36
2-year spread	0.17	0.17	0.17	0.17	0.17
2-year spread (bad times)	0.69	0.70	0.69	0.69	0.69
10-year spread	0.64	0.64	0.64	0.64	0.64
10-year spread (bad times)	0.88	0.88	0.87	0.88	0.88
$\rho(1YS, log(y))$	-0.43	-0.43	-0.43	-0.44	-0.43
$\rho(2YS, log(y))$	-0.51	-0.51	-0.51	-0.51	-0.50
$\rho(10YS, log(y))$	-0.64	-0.64	-0.64	-0.65	-0.64
Default (%)	0.63	0.63	0.63	0.63	0.63
$\rho(log(c), log(y))$	0.90	0.90	0.90	0.90	0.90
$\rho(TB/y, log(y))$	-0.08	-0.08	-0.08	-0.08	-0.08
$\sigma(log(TB/y))/\sigma(log(y))$	0.50	0.50	0.50	0.50	0.50
Debt/Income	0.31	0.31	0.31	0.31	0.31
$\sigma(log(c))/\sigma(log(y))$	1.16	1.15	1.16	1.16	1.16
Distance in price, relative	8.0×10^{-11}	9.5×10^{-10}	4.1×10^{-11}	1.6×10^{-10}	1.9×10^{-10}
Distance in price, max	3.0×10^{-10}	1.6×10^{-9}	2.0×10^{-10}	5.4×10^{-10}	5.6×10^{-10}
Distance in price, per state	6.0×10^{-13}	3.8×10^{-12}	3.8×10^{-13}	6.8×10^{-13}	1.3×10^{-12}
Distance in value, relative	9.5×10^{-15}	3.3×10^{-14}	5.8×10^{-13}	8.4×10^{-15}	1.4×10^{-14}
Distance in value, max	1.6×10^{-13}	5.9×10^{-13}	1.0×10^{-11}	1.5×10^{-13}	2.5×10^{-13}
Distance in value, per state	2.6×10^{-14}	2.8×10^{-14}	2.5×10^{-14}	2.6×10^{-14}	2.6×10^{-14}

Note: For all the columns, the same parameters are used as in the benchmark (see Table 2). $\rho(x, y)$ represents the correlations between x and y, and $\sigma(x)$ gives the standard deviation of x. The bad times are the years when the log-output is less than the median. Spreads are given as percentages. Details on simulations are given in the Appendix.

A.2 Notes on variable definitions

Duration. For the duration of a bond, we use the Macaulay definition (as in Hatchondo and Martinez, 2009) which is a weighted sum of future coupon payments:

$$\frac{q(y,b',m';1)+2\times(q(y,b',m';2)-q(y,b',m';1))+...+n\times(q(y,b',m';n)-q(y,b',m';n-1))}{q(y,b',m';n)}.$$

Yield to maturity. The yield for maturity n for an observation with borrowing level b', income y and maturity of the held bond m, is calculated as follows:

$$YTM(y,b',m';n)\equiv\left(\frac{1}{q(y,b',m';n)-q(y,b',m';n-1)}\right)^{\frac{1}{n}}-1.$$

Then the spread for maturity m is $YTM(y,b',m';n)-r$.

A.3 Yield curve and repayment probability

Consider bonds that make constant unitary coupon payments for m periods. In this section we explain how the unit bond price depends on the default probabilities. For simplicity, we consider the repayment probabilities in periods $1,2,...,j$ denoted by $P_{t+1},P_{t+2},...,P_{t+j}$. Then, the price of a bond that pays one unit the next period is

$$q_t(1)=\frac{P_{t+1}}{1+r},$$

where r is the risk free rate. Similarly, the price for bonds that pay one unit for two periods is

$$q_t(2)=\frac{P_{t+1}}{1+r}+\frac{P_{t+1}P_{t+2}}{(1+r)^2},$$

and more generally, the price for a bonds that pays for j periods is

$$q_t(j)=\frac{P_{t+1}}{1+r}+\frac{P_{t+1}P_{t+2}}{(1+r)^2}+...+\frac{\prod_{i=1}^{j}P_{t+i}}{(1+r)^j}.$$

With these prices at hand, we can write prices of zero-coupon bonds that pay in $1,2,...,j$ periods

$$\begin{aligned}
q_t^0(1) &= q_t(1)=\frac{P_{t+1}}{1+r},\\
q_t^0(2) &= q_t(2)-q_t(1)=\frac{P_{t+1}P_{t+2}}{(1+r)^2},\\
&\cdots\\
q_t^0(j) &= q_t(j)-q_t(j-1)=\frac{\prod_{i=1}^{j}P_{t+i}}{(1+r)^j}.
\end{aligned}$$

Then, the yield of a zero-coupon bond that pays in j period is $i_t^0(j)$,

$$
\begin{aligned}
i_t^0(1) &= \frac{1+r}{P_{t+1}}, \\
i_t^0(2) &= \frac{1+r}{(P_{t+1}P_{t+2})^{1/2}}, \\
&\cdots \\
i_t^0(j) &= \frac{1+r}{(\prod_{i=1}^{j} P_{t+i})^{1/j}}.
\end{aligned}
$$

A.4 Empirical analysis

The data that we use in the cross-country analysis are for Argentina, Brazil, Chile, Colombia, Hungary, Mexico, Peru, Poland, Slovenia and Turkey. For the cross-country relationships between maturity and country-level characteristics, we use the following variables:

1. Maturity: The information on maturity is for external debt, except Argentina which is for gross debt. For Colombia (2001-2014) and Brazil (2005-2015) we use data provided by the HAVER database. For Chile (1990-2010), Hungary (1999-2010), Mexico (2007-2010), Poland (1998-2010), Slovenia (2003-2010) and Turkey (2005-2010), we use the data provided by the OECD database. For Peru, we get the maturity data from the Ministerio de Economia y Finanzas (2001-2013). The corresponding statistics for Argentina are provided by Ministerio de Economia y Finanzas Publicas (2000-2014). When data is given in higher frequency than a year, we take the median within a year.

2. Sudden stop probability: Our data source is the list of sudden stop episodes in Comelli (2015). For each country, we take the mean of the dummy variable indicating a sudden stop over the years for which we have data on maturity.

3. GDP per capita, cycle: We get the GDP per capita data from the World Development Indicators (WDI) provided by the World Bank (constant 2005 US$). For each country, we detrend the log-GDP per capita of all the available data with HP filtering using a smoothing parameter of 1600 as in Hatchondo and Martinez (2009), and we use the cyclical component.

4. GDP per capita, volatility: It is the standard deviation of the cyclical component described above for the entire period.

5. Population under 14 years old: Percentage of population under 14 years of age, from the WDI.

6. Gini coefficient: Provided by the WDI.

7. Population: Provided by the WDI.

For consistency, we use the same Colombian dataset to assess the implications of our model, to construct our calibration targets, and for the cross-country analysis. Below we describe the construction and sources for the rest of the variables that we use in the paper.

1. Duration: The data on duration for Colombia come from the HAVER database and span years 2001 to 2014, like the country's maturity data. This datasource follows the Macaulay definition when constructing the variable, as is standard in the literature, and it is consistent with our computation of duration in the model.[19]

2. Debt to output: External debt stocks (% of GNI) provided by the WDI for the entire period.

3. Consumption: Households' final consumption expenditure per capita (constant 2005 US$), provided by the WDI. For the volatility and correlations, we follow the same approach as for the GDP per capita, by taking HP filtering the log consumption per capita for the entire period.

4. Trade balance over output: We substract the aforementioned consumption per capita from the GDP per capita, and divide by the output per capita for the entire period.

5. Spreads: The yields are US dollar sovereign yields obtained from Bloomberg. We get the yield spreads by substracting US yields from the same data source.

In Section 1 Table 1, we use the same data as described above for each country to get the information on maturity and the cyclical component of log GDP per capita. The yields are the US dollar sovereign yields from Bloomberg for Brazil, Colombia, Mexico and Peru; and from HAVER for Chile, Slovenia and Turkey, the countries in the cross-country analysis that have available data for both 1 and 10-years yields for sufficiently many years. We get the yield spreads by substracting US yields from the same data source. Spreads given in Figure 1 are also from the same data. We exclude the spreads data for Argentina in the table because we only have 2 years with available data. For Hungary and Poland, we do not have any years with both 1 and 10 year spreads.

In Table 1, we take averages across country-specific correlations and medians. We classify a year as a bad time if the detrended log-output of the country is below 0. This treatment of the data is consistent with our approach to the model simulations and to our treatment of the data for Colombia in the remainder of the paper.

[19]An alternative measure of duration for Colombia computed with our data on maturity and the country's official average interest on new external debt commitments (International Debt Statistics provided by the World Bank) produces very similar numbers. In particular, the median then is 5.01 instead of the 5.42 value from the HAVER database.

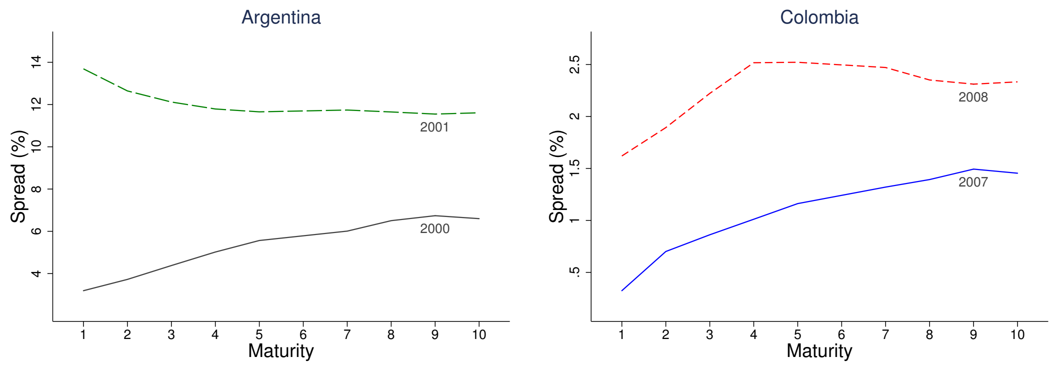
Tables and figures

Table 1: Maturity and yield spreads over the business cycle

	Maturity	1-year spread	10-year spread
Overall	7.70	1.23	1.79
Bad times	7.12	1.58	2.93
Corr. with log(output)	0.46	-0.21	-0.43

Note: The maturity data are from 117 observations, from Argentina, Brazil, Chile, Colombia, Hungary, Mexico, Peru, Poland, Slovenia and Turkey. The yield spreads are from Brazil, Chile, Colombia, Mexico, Peru, Slovenia and Turkey and include 72 observations with both 1 and 10-year maturity spreads information for more than 6 years. For correlations, we show the averages across country-specific correlations. For the rest, we show the averages across country-specific medians. Bad times are the years when the detrended (HP) log output is below 0. In the data, we compute output as GDP per capita. Spreads are given as percentages. See the Appendix for additional description of the variables.

Figure 1: Sovereign yield spread curves, selected years and countries



Note: The figures plot US dollar sovereign bond yield spreads over the US yields. The source is Bloomberg. See the Appendix for additional description of the variable definitions.

Table 2: Calibrated parameters

Parameter	Value	Basis
Interest rate, r	0.032	Arellano and Ramanarayanan (2012)
Redemption prob., λ	0.17	”
Risk aversion, γ	2	Standard
Income shock std, σ_y	0.019	Colombia income process, WDI
Income autocorrelation, ρ_y	0.86	”
Sudden stop prob., p_s	0.143	Colombia SS frequency, Comelli (2015)
Discount factor, β	0.946	Colombia Debt-to-output ratio = 31%, WDI
Cost of default, ϕ	0.905	Colombia $\sigma(log(c))/\sigma(log(y)) = 1.15$, WDI

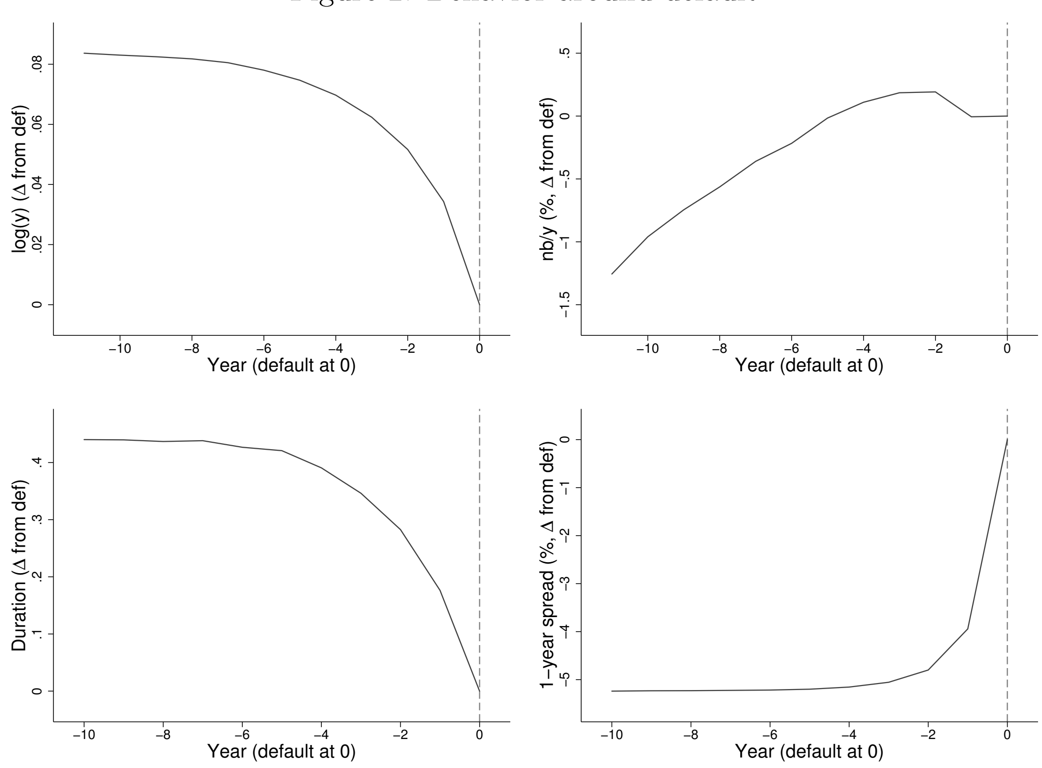
Note: We use the detrended (HP) log GDP per capita from the World Development Indicators (WDI) to get the income process paramaters ρ_y and σ_y. See the Appendix for the definitions of the rest of the variables.

Table 3: Fit of non-targeted moments

	Data	Model
Maturity	9.69	6.93
Maturity (bad times)	7.34	6.22
Duration	5.42	3.75
Duration (bad times)	4.26	3.41
$\rho(n, log(y))$	0.92	0.57
$\rho(dur, log(y))$	0.93	0.64
1-year spr.	0.96	0.03
1-year spr. (bad times)	1.42	0.37
2-year spread	1.28	0.17
2-year spread (bad times)	1.89	0.69
10-year spr.	2.33	0.64
10-year spr. (bad times)	4.32	0.88
$\rho(1YS, log(y))$	-0.61	-0.43
$\rho(2YS, log(y))$	-0.68	-0.51
$\rho(10YS, log(y))$	-0.89	-0.64
$\rho(log(c), log(y))$	0.90	0.90
$\rho(TB/y, log(y))$	-0.08	-0.08
$\sigma(log(TB/y))/\sigma(log(y))$	1.36	0.50

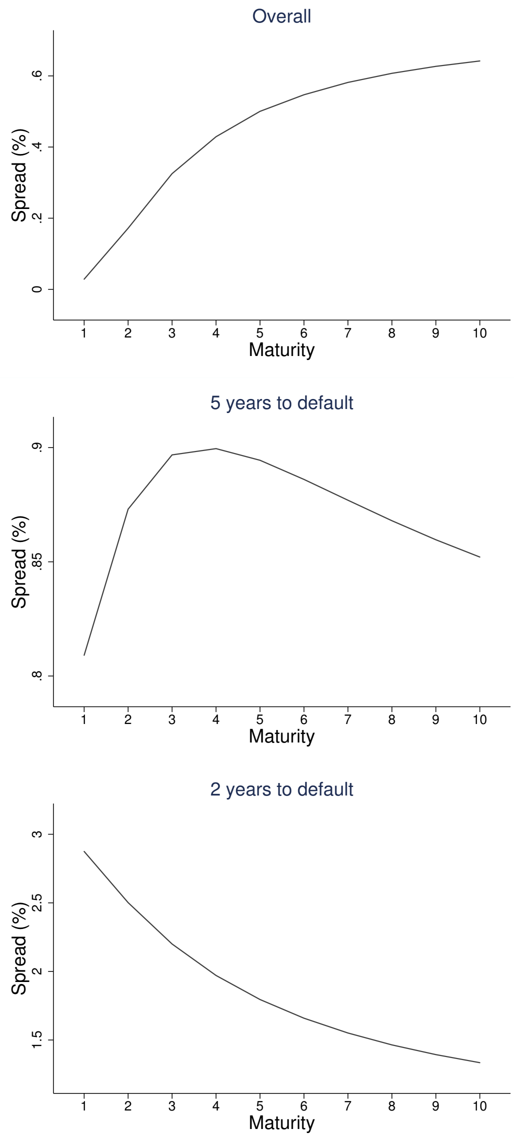
Note: The data are for Colombia. Duration is derived following the Macaulay definition. $\rho(x, y)$ represents the correlations between x and y. $\sigma(x)$ stands for the standard deviation of x. Bad times are the years when the log-output is less than the median. For the data, we use detrended log output and consumption per capita. Spreads are in percentages. See the Appendix for details about the data and simulations.

Figure 2: Behavior around default



Note: For each simulated path, we take the median of each statistic over observations $j = \{0, ..., 10\}$ years before a default. We then take averages across these simulations for every j, and we plot the deviations from the average at default, i.e. $j = 0$.

Figure 3: Sovereign yield spread curves



Note: Figures titled "j year(s) to default", $j = \{2, 5\}$, use observations that precede a default by j years. For each simulated path, we take the median spreads over observations j years before a default, and then we take averages across these simulations to form the yield spread curves. For the figure titled "Overall", we do the same using every observation for each simulation, except observations in default or financial exclusion.

Table 4: The implications of fixing maturity in the model

	Benchmark	Fixed maturity	
		7	10
Duration	3.75	3.85	5.17
Duration (bad times)	3.41	3.83	5.11
$\rho(dur, log(y))$	0.64	0.13	0.26
1-year spread	0.03	0.03	0.04
1-year spread (bad times)	0.37	0.38	0.55
2-year spread	0.17	0.16	0.22
2-year spread (bad times)	0.69	0.73	1.04
10-year spread	0.64	0.66	0.92
10-year spread (bad times)	0.88	0.98	1.39
$\rho(1YS, log(y))$	-0.43	-0.44	-0.43
$\rho(2YS, log(y))$	-0.51	-0.53	-0.50
$\rho(10YS, log(y))$	-0.64	-0.70	-0.67
Default (%)	0.63	0.67	0.89
$\rho(log(c), log(y))$	0.90	0.90	0.93
$\rho(TB/y, log(y))$	-0.08	-0.06	0.00
$\sigma(log(TB/y))/\sigma(log(y))$	0.50	0.48	0.39
Debt/Income	0.31	0.30	0.34
$\sigma(log(c))/\sigma(log(y))$	1.16	1.14	1.07

Note: For each alternative model with fixed maturity $N = \{7, 10\}$, we repeat the same simulation strategy as in the benchmark calibration, keeping the model parameters unchanged. $\rho(x, y)$ represents the correlations between x and y. $\sigma(x)$ stands for the standard deviation of x. Bad times are the years when the log-output is less than the median. For the data, we use detrended log output and consumption per capita. Spreads are given as percentages. See the Appendix for details about the simulations.

Table 5: Key macroeconomic variables

Variable	Mean	Std. Dev.	Min	Max
Maturity	8.15	2.50	2.61	15.42
Volatility	0.055	0.018	0.033	0.082
Sudden stop pr.	0.183	0.056	0.083	0.286
Gini coeff.	44.93	9.57	26.84	58.25
Share pop. under 14 (%)	25.00	5.83	14.64	35.05
GDP per capita cycle	0.003	0.052	-0.221	0.094
GDP per capita (2005 US$)	6,478	2,461	2,329	11,750
Population (millions)	57.1	53.0	10.0	206.1

Note: Descriptive statistics of the variables of interest at an annual frequency from 1998 to 2014 for Argentina, Brazil, Chile, Colombia, Hungary, Mexico, Peru, Poland, Slovenia and Turkey. We only use the cases with non-missing information on all of the variables mentioned in the table, which gives 92 observations. "Maturity" is that of sovereign external debt, "Volatility" is the country-specific standard deviation of detrended (HP-filtered) real log GDP per capita. "Sudden stop pr." is the country-specific annual probability of a sudden stop, "Gini coeff." is a measure of inequality that proxies for risk aversion in the model, "Share pop. under 14" is the fraction of the total population of a country that is below 14 years old, and it is used as a proxy for the discount factor in the model. "GDP per capita cycle" is the deviation of real log output per capita from its HP trend. See the Appendix for details on the data.

Table 6: Maturity and key macroeconomic variables, regressions

	log(Maturity)		Maturity	
Variables	Pooled OLS	Year FE	Pooled OLS	Year FE
Volatility	-8.362***	-8.089***	-56.81***	-52.77***
Sudden stop pr.	2.822***	2.919***	18.30***	19.29***
Gini coef.	0.0130**	0.0123*	0.0994**	0.106**
Share pop. under 14	0.0436***	0.0463***	0.343***	0.355***
GDP cycle	3.087***	2.113***	23.20***	15.24***
R-squared	0.684	0.682	0.643	0.652

Note: The regressions were performed using annual data from 1998 to 2014 for Argentina, Brazil, Chile, Colombia, Hungary, Mexico, Peru, Poland, Slovenia and Turkey. There are 92 observations for each regression, which is the number of country-year level observations with data on all variables used in these estimations. The regressors also include GDP per capita, population, and a dummy for Argentina because the country was technically in default during the available sample period. "Maturity" is the maturity of sovereign external debt, "Volatility" is the country-specific standard deviation of detrended (HP-filtered) real log GDP per capita. "Sudden stop pr." is the country-specific annual probability of a sudden stop, "Gini coeff." is a measure of inequality that proxies for risk aversion in the model, "Share pop. under 14" is the fraction of the total population of a country that is below 14 years old, and it is used as a proxy for the discount factor in the model. "GDP per capita cycle" is the deviation of real log output per capita from its HP trend. *, ** and *** indicate significance at 10, 5 and 1 percent level of the coefficient, respectively. See the Appendix for details on the data.

Table 7: The role of macroeconomic factors

	Benchmark	Income shock sd		Sudden stop pr.		Risk aversion		Discount factor	
σ_y, std. dev. y	0.019	0.018	0.020	0.019	0.019	0.019	0.019	0.019	0.019
p_s, sudden stop pr. (%)	14.3	14.3	14.3	0	30	14.3	14.3	14.3	14.3
γ, risk aversion	2	2	2	2	2	1	3	2	2
β, discount factor	0.946	0.946	0.946	0.946	0.946	0.946	0.946	0.939	0.953
Maturity	6.93	7.01	6.60	1.02	9.92	5.01	8.23	6.04	7.88
Duration	3.75	3.81	3.57	1.01	4.79	2.93	4.36	3.39	4.19
1-year spread	0.03	0.03	0.03	0.05	0.01	0.08	0.01	0.06	0.01
2-year spread	0.17	0.17	0.17	0.12	0.13	0.27	0.11	0.27	0.07
10-year spread	0.64	0.61	0.66	0.22	0.73	0.58	0.66	0.81	0.43
Default (%)	0.63	0.61	0.64	0.22	0.70	0.55	0.69	0.77	0.45
$\rho(log(c), log(y))$	0.90	0.89	0.91	0.93	0.88	0.83	0.93	0.87	0.93
Debt/Income	0.31	0.32	0.29	0.27	0.32	0.31	0.30	0.32	0.28
$\sigma(log(c))/\sigma(log(y))$	1.16	1.17	1.14	1.22	1.14	1.33	1.07	1.20	1.09

Note: The table shows moments of the model for different values of the standard deviation of the income shock, the probability of a sudden stop, risk aversion, and the discount factor. For all columns, the same parameters are used as in the benchmark except the one that corresponds to the particular exercise (see Table 2). For the volatility of the income shock, we consider values 5 percent below and above the benchmark value of 0.019. $\rho(x, y)$ represents the correlations between x and y, and $\sigma(x)$ stands for the standard deviation of x. For the data, we use detrended log-output and log-consumption. Spreads are given as percentages. The details about the simulations are discussed in the Appendix.