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Low Real Interest Rates, Collateral Misrepresentation, and Monetary Policy

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Abstract

A model is constructed in which households and banks have incentives to fake the quality of collateral. These incentive problems matter when collateral is scarce in the aggregate – when real interest rates are low. Conventional monetary easing can exacerbate these problems, in that the mispresentation of collateral becomes more profitable, thus increasing haircuts and interest rate differentials. Central bank purchases of private mortgages may not be feasible, due to misrepresentation of asset quality. If feasible, central bank asset purchase programs work by circumventing suboptimal fiscal policy, not by mitigating incentive problems in asset markets. (Previously circulated under the title,"Central Bank Purchases of Private Assets.")

1 Introduction

This paper is primarily concerned with incentive problems in markets for collateral, and the implications of these incentive problems for conventional and unconventional monetary policy. In the model, households and banks may find it profitable to misprepresent the quality of collateral. To mitigate this problem, loan contracts incorporate haircuts – borrowing against a given quantity of collateral is limited. But this in turn limits the efficacy of collateral, in that collateral subject to misrepresentation cannot support as much lending as collateral not subject to this problem. In the model, the manifestation of this incentive problem is intimately related to limited commitment in credit markets, which is why collateral is useful in the first place. A scarcity of collateral in the aggregate tends to make real interest rates low, which then makes misrepresentation of collateral more profitable.

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Also contributing to scarce collateral and low real interest rates is conventional monetary policy. With binding collateral constraints, a low nominal interest rate tightens collateral constraints and makes the real interest rate low. So accommodative policy contributes to incentive problems. But unconventional monetary policy, in the form of central bank purchases of private assets, can improve welfare. These central bank asset purchases work, not by mitigating incentive problems, but by offsetting the collateral scarcity created by suboptimal fiscal policy.

A concern sometimes voiced is that low interest rates, engineered by the central bank, can cause a "reach for yield" or other incentive problems in financial markets. For example, prior to the financial crisis, Rajan (2005) presciently argued that financial markets had evolved in the direction of superior efficiency, but possibly at the expense of stability. One of Rajan's concerns was that financial institutions and their managers might be facing more severe incentive problems, stemming from moral hazard and herd behavior, for example. Rajan argued that accommodative monetary policy, in the form of low interest rates, could exacerbate these problems.

The financial incentive problem modeled here is certainly a relative of the ideas in Rajan (2005) and similar work, but the details are quite different, and we work out the broader implications for monetary policy. The foundation for the incentive problem comes from work by Li, Rocheteau, and Weill (2012), who are interested in how asset markets work when market participants can create counterfeit assets. The key to applying this idea in our context is that "counterfeiting" will involve the potential misrepresentation of collateral quality in credit contracts.

The model in this paper builds on the structure of Lagos and Wright (2005) and Rocheteau and Wright (2005). Some details of the structure of banks and fiscal policy in the model are in Williamson (2012, 2016a), but the issues addressed are different, as will be detailed in what follows. The basic assets in the model are currency, central bank reserves, government debt, and housing. These assets serve three purposes. First, assets are useful in exchange, for reasons that are standard in the monetary theory literature. Second, assets are useful as collateral in credit contracts, due to limited commitment. Third, housing is intrinsically useful, in that it yields a service flow to households. In the model, banks are useful in intermediating assets, as they solve a type of Diamond-Dybvig (1983) insurance problem. That is, the ability of some sellers in goods markets to evaluate collateral is limited, which means that buyers in goods markets face uncertainty about the means of payment that a seller will accept. Banks serve to insure against this uncertainty, by offering deposit contracts subject to withdrawal in currency.

In the model, there is an interesting, and novel, role for mortgage lending. In serving its insurance role, the banking sector needs collateral. Like the other economic agents in the model, the bank needs collateral, in the form of the assets in its portfolio. But, though housing is a collateralizable asset, it is efficient for households to hold housing rather than banks, as households receive the service flow from housing, while banks do not. Thus, an efficient financial arrangement is for the households to own the houses, and to borrow from banks in the form of mortgages, with houses serving as collateral on mortgage loans. Mortgage loans in turn serve as collateral backing the bank's deposit liabilities. In the model, the household does not borrow from the bank to finance the purchase of the house – it borrows because this is profitable, and profitability arises from the bank's need for collateral.

But, mortgage lending is subject to two types of incentive problems. First, a household can, at a cost, misrepresent the quality of the underlying house. Second, a bank can misrepresent, again at a cost, the quality of mortgages in its portfolio. The first type of misrepresentation can be interpreted as cheating on a mortgage appraisal – the type of misrepresentation that provisions in the Dodd-Frank Act of 2010 were written to help prevent. The second type of misrepresentation is typical of the array of moral hazard frictions associated with portfolio choice by banks.

Our key concern in the paper is understanding the key causes and consequences of these two types of misrepresentation. Misrepresentation tends to occur when real interest rates are low, but low real interest rates are in turn caused by a shortage of collateral. In the model, this is in part a fiscal policy problem – the fiscal authority could solve it by issuing more government debt, driving down the price of government debt, and increasing real interest rates. But our primary interest is in monetary policy, treating suboptimal fiscal policy as given. More accommodative monetary policy, in the form of lower nominal interest rates, will induce lower *real* interest rates, which exacerbate the collateral incentive problems, if incentive constraints bind.

Collateral misrepresentation can act through channels that result in large effects from monetary policy actions due to amplification and multiple equilibria. Amplification results from a complementarity in the behavior of banks and households. When collateral is scarce in the aggregate, an increase in the nominal interest rate, engineered by the central bank, relaxes the incentive constraints of banks, which reduces haircuts, and thus increases the effective stock of collateral. Higher interest rates on government debt increase mortgage rates, which reduces haircuts on housing collateral. This acts to increase the supply of mortgages. We might expect that higher interest rates would reduce the supply of collateral in the aggregate, because the demand for mortgages would naturally fall. But, if incentive constraints bind for banks and households, then the effect goes the other way. Higher interest rates can increase rather than decrease the supply of private collateral forthcoming – an amplification effect of monetary policy.

If incentive constraints bind for banks and for households in equilibrium, then there can exist other Pareto-ranked equilibria. That is, given monetary policy, as summarized by the nominal interest rate, the market for collateral can clear at a low real interest rate, with incentive constraints binding for banks and households. But then a higher real interest rate induces both higher demand and higher supply of collateral, and the market can also clear for a higher real interest rate. And, in our model, economic welfare will be higher in the equilibrium with the higher real interest rate. Thus, the complementarity that leads to amplification of the effects of monetary policy can also produce multiple equilibria.

The model is sufficiently rich to allow us to study how monetary policy works when there are no reserves outstanding, and the central bank supports a given nominal interest rate policy through open market operations, or when the central bank has a large balance sheet – strictly positive reserves outstanding – and pegs the nominal interest rate by setting the interest rate on reserves. The former and latter regimes correspond to what existed in the United States before and after the financial crisis, respectively. But in the model, the implications for monetary policy are the same in either case.

What does the model have to say about optimal conventional monetary policy? If collateral is not scarce in our model, the results are standard. That is, if collateral constraints do not bind when the nominal interest rate is zero, then the equilibrium allocation of resources is efficient. This is just a conventional Friedman rule result. But, what happens when collateral constraints bind? If collateral constraints bind and only one set of incentive constraints bind – either those for households or those for banks, then a zero nominal interest rate can be shown to be locally optimal. But, if incentive constraints bind for both households and banks, then there must be more than one equilibrium and, in the equilibrium with the lowest real interest rate, increasing the nominal interest rate always increases economic welfare. This is just the amplification effect at work. Further, given multiple equilibria, which requires that incentive constraints bind for both banks and households in at least one equilibrium, an increase in the nominal interest rate can eliminate low-welfare equilibria. Thus, in a model that otherwise tells us that a zero nominal interest rate is optimal, the central bank can increase welfare by increasing the nominal interest rate, if we include incentives to misrepresent collateral. This is because a higher nominal interest rate can increase the supply of collateral and relax collateral constraints.

The New Keynesian (NK) literature views the zero lower bound and the role of conventional monetary policy in a different way. In this literature, a low real interest rate can imply that the zero lower bound on the nominal interest rate is optimal. For example, Werning (2011) models a low-real-interest-rate period as arising from a high discount factor, and this implies that a zero nominal interest rate is optimal. Also, in Werning's model the nominal interest rate should stay at zero after the discount factor has fallen to its normal level, provided the central bank can commit to this policy. In NK models, the zero lower bound may also be a steady state equilibrium in which the central bank perpetually undershoots its inflation target. Indeed, Benhabib et al. (2001) have shown that aggressive Taylor rules give rise to multiple equilibria that converge to this zero-lower-bound steady state. Schmitt-Grohe and Uribe (2014) and Cochrane (2016) have suggested that escape from this liquidity trap is possible by simply raising the nominal interest rate, and either reverting to the standard Taylor rule (as in Schmitt-Grohe and Uribe 2014), or simply pegging the nominal interest rate at the appropriate higher level (as in Cochrane 2016).

During and after the financial crisis of 2008-2009, central banks have contemplated – and implemented – various unconventional asset purchase programs. The Bank of Japan, the Bank of England, the European Central Bank, and the Swiss National Bank, among other central banks, currently have large balance sheets, reflecting substantial unconventional asset purchases. In practice, large central bank asset purchases have been in the form of long-maturity government debt, and private assets. In particular, central bank purchases of private assets have included purchases of asset-backed securities, corporate debt, and exchange-traded funds. In the United States, the Federal Reserve System currently has a balance sheet of about \$4.5 trillion. The Fed's assets include no Treasury bills, and no repurchase agreements – assets that played an important role in conventional monetary policy prior to the financial crisis. But, the Fed is currently holding \$2.5 trillion in long-maturity Treasury debt, and \$1.8 million in mortgage-backed securities.[1] The latter assets were issued by government-sponsored enterprises – the Federal National Mortgage Association and the Federal Home Loan Mortgage Corporation – which have been under government conservatorship since September 2008. But the assets backing these securities are private mortgages, so effectively the Fed has a large portfolio of private assets.

In our model, the central bank can set up a purchase program for private assets. We assume that the central bank cannot lend directly to households, so central bank purchases of private assets – interpreted in the model as mortgages – are made from banks. In making asset purchases, the central bank has to be wary of the same incentive problem faced by private banks: If the central bank offers to purchase assets at too high a price, it will be on the receiving end of bad assets. So, feasibility of the asset purchase program puts bounds on its parameters.

Two possibilities are considered. First, the central bank could be a participant on the demand side of the mortgage market along with private banks. In this instance, the central bank chooses the size of its mortgage portfolio, but purchases assets at the market price. Second, at an extreme the central bank could purchase all mortgages forthcoming at a price that it dictates – with this price being at least as high as the price at which private lenders would purchase mortgages. In the first case, the central bank's asset purchase program is neutral. The central bank swaps reserves for mortgages, but these two assets are perfect substitutes if the asset purchase program is feasible. In the second case, the central bank's asset purchase program can work, but by relaxing collateral constraints, not incentive constraints. If incentive constraints bind, the central bank can purchase all mortgages loans forthcoming at the market interest rate, but this will have no effect on equilibrium quantities and prices. But, if incentive constraints do not bind, the central bank can purchase mortgages at a high price, expand the stock of mortgages outstanding, and thus expand the aggregate stock of collateral and relax collateral constraints. This is welfare-improving.

Thus, the incentive problem in the model does not create a role for uncon-

[1] See Board of Governors 2016.

ventional asset purchases. However, asset purchase programs by the central bank can work as a means for expanding the aggregate stock of collateral when collateral is scarce. But collateral is scarce in the model because of suboptimal fiscal policy. So unconventional central bank purchases can act to mitigate the harmful effects of suboptimal fiscal policy.

The role for collateral in this model is related to work by Kiyotaki and Moore (1997, 2012) and Venkateswaran and Wright (2013), for example.[2] The limited commitment problem of banks has something in common with models constructed by Gertler and Kiyotaki (2010) and Gertler and Karadi (2013), though in those models it is bank capital that is scarce, whereas our model features a scarcity of collateral in the aggregate, as in Andolfatto and Williamson (2015) and Williamson (2016a,b). Caballero and Farhi (2016) also explore some related implications of safe asset shortages.

Since some of the features of the model in Williamson (2016a) are similar to the those of the model here, it is useful to point out what the value-added of the current paper is. The key innovation here is exploring how incentives for misrepresenting collateral matter for interest rate spreads, haircuts, the aggregate supply of collateral, and conventional and unconventional monetary policy. Williamson (2016a) was concerned with a related but distinct issue: the consequences of quantitative easing in the form of swaps of short-maturity for long-maturity government debt.

The remainder of the paper is organized as follows. In the second section the model is constructed, and the third section contains the details of asset exchange and banking in the model. Then, an equilibrium is constructed and characterized in Section 4. Sections 5 and 6 contain an analysis of conventional and unconventional monetary policy, respectively, and the final section is a conclusion.

2 Model

The basic structure of the model is related to Lagos and Wright (2005) and Rocheteau and Wright (2005). Time is indexed by $t = 0, 1, 2, ...$, and in each period there are two sub-periods – the centralized market (CM) followed by the decentralized market (DM). There is a continuum of buyers and a continuum of sellers, each with unit mass. An individual buyer has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + F_t + u(x_t)],$$

where H_t is labor supply in the CM, F_t is consumption of housing services in the CM, x_t is consumption in the DM, and $0 < \beta < 1$. Assume that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with

[2] Although in contrast to Kiyotaki and Moore (1997, 2012), the collateral constraint is endogenous – credit must be secured given explicit limited commitment – and, in contrast to Venkateswaran and Wright (2013), haircuts are endogenous.

$u'(0) = \infty$, $u'(\infty) = 0$, and $-x\frac{u''(x)}{u'(x)} < 1$. Each seller has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t (X_t - h_t),$$

where X_t is consumption in the CM, and h_t is labor supply in the DM. Buyers can produce in the CM, but not in the DM, and sellers can produce in the DM, but not in the CM. One unit of labor input produces one unit of the perishable consumption good, in either the CM or the DM.

As well, there exists a continuum of banks. Each bank is an agent that maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t (X_t - H_t),$$

where X_t and H_t are consumption and labor supply, respectively, in the CM.

In the DM, there are random matches between buyers and sellers, and each buyer is matched with a seller. All DM matches have the property that there is no memory or recordkeeping, so that a matched buyer and seller have no knowledge of each others' histories. A key assumption is limited commitment – no one can be forced to work – and so lack of memory implies that there can be no unsecured credit. If any seller were to extend an unsecured loan to a buyer, the buyer would default.

Following Williamson (2012, 2016a), assume limitations on the information technology that imply that currency will be the means of payment in some DM transactions, and some form of credit (here it will be financial intermediary credit) will be used in other DM transactions. Suppose that, in a fraction ρ of DM transactions – denoted *currency transactions* – there is no means for verifying that the buyer possesses any assets other than currency. Thus, in these meetings, the seller can only verify the buyer's currency holdings, and so means of payment other than currency are not accepted in exchange. However, in a fraction $1 - \rho$ of DM meetings – denoted *non-currency transactions* – the seller can verify the entire portfolio held by the buyer. Assume that, in any DM meeting, the buyer makes a take-it-or-leave-it offer to the seller. At the beginning of the CM, buyers do not know what type of match they will have in the subsequent DM, but they learn this at the end of the CM, after consumption and production have taken place. A buyer's type (i.e. whether he or she will need currency to trade in the DM or not) is private information, and at the end of the CM, a buyer can meet at most one bank of his or her choice.[3]

In addition to currency, there are three other assets in the model: nominal government bonds, reserves and housing. A government bond sells for z_t units of money in the CM of period t, and pays off one unit of money in the CM of period $t + 1$. One unit of reserves can be acquired in exchange for z_t units of money in the CM in period t, and pays off one unit of money in the CM

[3] Type is private information, and trading opportunities are limited at the end of the CM so as to prevent the unwinding of bank deposit contracts. See Jacklin (1987) and Wallace (1988).

of period $t+1$. In principle, the prices of government bonds and reserves could be different in the CM, but if both assets are held in equilibrium, their prices are identical. Though government bonds and reserves are identical assets in equilibrium, it is important to include them separately in the analysis, as this will permit central bank asset purchase programs that may be infeasible without reserves. Housing is in fixed supply, with a perfectly divisible stock of one unit of housing in existence forever. If a buyer holds a_t units of housing at the beginning of the CM of period t, then that buyer receives $a_t y$ units of housing services, where $y > 0$. Only buyers receive utility from consuming housing services, and only the owner of a house can consume the services. Houses sell in the CM at the price ψ_t. Assume that there exists no rental market in housing.[4] Further, to guarantee that, in equilibrium, banks will never hold houses directly, assume that if a bank acquires a house in period t, that it immediately depreciates by 100%.[5]

3 Asset Exchange and Banking

In the spirit of Diamond-Dybvig (1983), banks play an insurance role. To illustrate this, suppose that banking is prohibited in this environment. Then, in the CM, a buyer would acquire a portfolio of currency, government bonds, reserves, and housing in the CM, anticipating that he or she may or may not need currency in the subsequent DM. In the DM, on the one hand, if the seller accepts only currency, then the buyer would exchange currency for goods. The government bonds, reserves, and housing in the buyer's portfolio would then be of no use in exchange, and the buyer would have to hold these assets until the next CM. On the other hand, if the buyer met a seller in the DM who could verify the existence of all assets in the buyer's portfolio, then government bonds, reserves, and housing could be used as collateral to obtain a loan from the seller. Currency could also be traded in this circumstance, but ex post the buyer would have been better off by acquiring higher-yielding assets rather than currency in the preceding CM. A bank, as we will show, is able to insure buyers against the need for different types of liquid assets in different types of exchange. The bank's deposit contract will allow the depositor to withdraw currency as needed, and to trade bank deposits backed by assets when that is feasible in the DM.

[4] We could model the reasons for the lack of a rental market in the model, for example arising from a moral hazard problem - a renter has private information about items needing repairs, but may have no incentive to make the repairs. But modeling these reasons for the missing rental market need not add anything useful to the analysis.

[5] If we did not make this assumption, then when the real interest rate is zero or lower, and counterfeiting costs are sufficiently low, banks may hold houses directly in equilibrium. Allowing for this does not appear to admit any important insights, and only makes the analysis more complicated.

3.1 Buyer's problem

Quasi-linear preferences for the buyer allows us to separate the buyer's contracting problem vis-a-vis the bank from his or her decisions about the remaining portfolio. In the CM, the buyer acquires housing a_t, at a price ψ_t, and holds this quantity of housing until the next CM, when the buyer receives the payoff $a_t(\psi_{t+1}+y)$ (asset quantity multiplied by market value plus the payoff in terms of housing services). As well, the buyer can borrow in the form of a mortgage from a bank. A mortgage, which is a promise to pay l_t^h units of consumption goods in the CM of period $t+1$, sells at the price q_t, in units of the CM good in period t. As well, a mortgage loan must be secured with housing assets, otherwise the borrower would abscond. But the buyer is able to produce "counterfeit housing," i.e. a buyer can produce assets that are indistinguishable to the bank from actual housing, at a cost of γ^h per unit of counterfeit housing. An interpretation is that this is a real-estate appraisal incentive problem. Anecdotes about cheating on real estate appraisals were common during the financial crisis, and the Dodd-Frank Act of 2010 contained provisions with respect to real estate appraisals similar to previous standards outlined in the Home Valuation Code of Conduct in 2009 (see Ding and Nakamura 2016). The intention of the Dodd-Frank provisions was to mitigate the problem we are building into our model.

In equilibrium, the buyer will not produce counterfeit housing (e.g. see Li, Rocheteau, and Weill 2012). For the buyer to keep himself or herself honest may require that he or she not borrow up to the full value of the collateral, so let $\theta_t^h \in [0,1]$ denote the fraction of the housing assets of the buyer that lenders can seize if the buyer defaults.

The buyer's collateral constraint is

$$l_t^h \leq (\psi_{t+1}+y)a_t\theta_t^h, \tag{1}$$

i.e. the payoff required in the CM of period $t+1$ on the buyer's mortgage loan cannot exceed the payoff on the housing collateral, discounted by θ_t^h, where we can interpret $1-\theta_t^h$ as the "haircut" on the housing collateral. For now, we will assume that (1) binds, and we will later determine conditions that guarantee this. Then, given (1) with equality, the buyer solves

$$\max_{a_t,\theta_t^h} a_t \left[-\psi_t + \beta(\psi_{t+1}+y) + (q_t-\beta)(\psi_{t+1}+y)\theta_t^h\right] \tag{2}$$

subject to

$$-\gamma^h + q_t(\psi_{t+1}+y)\theta_t^h \leq 0. \tag{3}$$

Here, (2) is the objective function for the buyer. The net payoff on one unit of housing assets, in the square parentheses in the objective function in (2), is minus the price of housing ψ_t plus the discounted direct payoff to the buyer from housing, plus the net discounted indirect payoff from using the housing as collateral to take out a mortgage. The constraint (3) is the incentive constraint for the buyer, which states that the net payoff to faking a house and borrowing against the fake house on the mortgage market must not be strictly positive. Off equilibrium, if the buyer were to fake a unit of housing and use the fake housing as collateral to borrow on the mortgage market, then he or she would default on the loan.

3.2 Bank's Problem

In the CM, when a bank writes deposit contracts with buyers, a buyer does not know his or her type, i.e. whether or not he or she will need currency to trade in the subsequent DM. Once the buyer learns his or her type, at the end of the CM, type remains private information to the buyer. The bank contract specifies that the buyer will deposit k_t units of goods with the bank in the CM, and gives the depositor one of two options. First, at the end of the current CM, the depositor can visit the bank and withdraw c_t in currency, in units of CM consumption goods, and have no other claims on the bank. Alternatively, if the depositor does not withdraw currency, he or she can have a claim to d_t units of consumption goods in the CM of period $t+1$, and these claims can be traded in the intervening DM. In equilibrium, a bank maximizes the expected utility of its representative depositor, subject to: (i) the bank earns nonnegative net payoff on the contract and (ii) the bank satisfies a collateral constraint and an incentive constraint. If the bank did not solve this problem in equilibrium, then another bank could enter the market, make depositors better off, and still earn a nonnegative expected payoff. As with a buyer, a bank must collateralize its deposit liabilities, though we assume that the bank can commit (say, by putting cash in the ATM) to meeting its promises to satisfy cash withdrawals. For the bank, collateral consists of mortgage loans, government bonds, and reserves. Further, the bank can create counterfeit loans in its asset portfolio, and in equilibrium the bank must have the incentive not to do that. This feature is intended to capture a moral hazard problem that exists in the context of limited commitment. The bank must back its deposit liabilities with collateral, but then the bank has an incentive, if there is a cost advantage, to compromise the quality of the collateral, unbeknownst to the bank's liability holders.

A depositing buyer receives expected utility from the bank's deposit contract,

$$EU = -k_t + \rho u\left(\beta \frac{\phi_{t+1}}{\phi_t} c_t\right) + (1-\rho)u(\beta d_t), \tag{4}$$

i.e. the buyer deposits k_t with the bank in the CM, and with probability ρ exchanges currency worth $\frac{\phi_{t+1}}{\phi_t} c_t$ in the CM of period $t+1$ with a seller, as the result of a take-it-or-leave-it offer by the buyer. With probability $1-\rho$ the buyer meets a seller who will accept claims on the bank, and the buyer makes a take-it-or-leave it offer which nets βd_t in DM consumption goods from the seller. The bank's net payoff, given the deposit contract and the bank's asset portfolio, must be nonnegative in equilibrium, or

$$k_t - z_t\left(m_t + b_t\right) - \rho c_t - q_t l_t - \beta(1-\rho)d_t + \beta \frac{\phi_{t+1}}{\phi_t}\left(m_t + b_t\right) + \beta l_t \geq 0, \tag{5}$$

where $k_t - z_t(m_t + b_t) - \rho c_t - q_t l_t$ denotes the payoff in the CM of period t from acquiring deposits and purchasing reserves, government bonds, currency, and loans. The quantity $-\beta(1-\rho)d_t + \beta\frac{\phi_{t+1}}{\phi_t}(m_t + b_t) + \beta l_t$ is the discounted net payoff to the bank in the CM in period $t+1$, from making good on deposit claims and collecting the payoffs on reserves, government bonds and loans. As well, the bank is subject to limited commitment, just as other agents in the model are. The bank's asset portfolio serves as collateral that backs its deposit liabilities. Thus, the bank faces a collateral constraint

$$-(1-\rho)d_t + \frac{\phi_{t+1}}{\phi_t}(m_t + b_t) + \theta_t l_t \geq 0, \tag{6}$$

which states that the bank's remaining deposit liabilities in the CM of period $t+1$ cannot exceed the value to the bank of the assets pledged as collateral against deposits. Here, $\theta_t \in [0,1]$ is the fraction of mortgage loans pledged as collateral, which is a choice variable for the bank that serves the same purpose as θ_t^h for a buyer.

In equilibrium, the bank must choose the banking contract, subject to its constraints, to maximize the depositor's expected utility (4). As well, in equilibrium, the bank's net payoff must be zero, i.e. (5) holds with equality. Finally, we will assume that the bank's collateral constraint (6) binds in equilibrium, and we will later determine conditions that guarantee this. Essentially, we assume that collateral is scarce in the aggregate in equilibrium, in a well-defined sense. Banks, similar to buyers, face incentive constraints, but for banks this is due to the fact that banks can fake mortgages. Letting γ denote the cost of faking one unit of mortgage loans, the net payoff to faking a mortgage must be non-positive, or

$$-\gamma + \theta_t \beta u'(\beta d_t) \leq 0. \tag{7}$$

3.3 Government

We will make explicit assumptions about the powers of the monetary and fiscal authorities, and the policy rules they follow, but what is important in determining an equilibrium are the consolidated government budget constraints. The consolidated government issues currency, reserves, and nominal bonds, denoted by, respectively, C_t, M_t, and B_t, in nominal terms, and issues liabilities and redeems them only in the CM. As well, the government makes a lump-sum transfer τ_t (in units of CM goods) to each buyer in the CM in period t.

Thus, the consolidated government budget constraints are given by

$$\phi_0 [C_0 + z_0 (M_0 + B_0)] - \tau_0 = 0 \tag{8}$$

$$\phi_t [C_t - C_{t-1} + z_t (M_t + B_t) - (M_{t-1} + B_{t-1})] - \tau_t = 0, \ t = 1, 2, 3, ... \tag{9}$$

4 Equilibrium

To solve for an equilibrium, we will first characterize the solutions to the buyer's and bank's problems. Then, we will make some assumptions about policy rules, and solve for a stationary equilibrium.

From the buyer's problem, (2) subject to (3), the haircut on housing collateral is determined by

$$\theta_t^h = \min\left[1, \frac{\gamma^h}{q_t(\psi_{t+1}+y)}\right], \tag{10}$$

where $1-\theta_t^h$ is the haircut, and asset prices must solve

$$-\psi_t + \beta(\psi_{t+1}+y) + \min\left[(q_t-\beta)(\psi_{t+1}+y), \gamma^h\left(1-\frac{\beta}{q_t}\right)\right] = 0. \tag{11}$$

Equation (10) states that, if the cost of faking a house is sufficiently small, the price of a mortgage is sufficiently high, and the price of housing and the flow of housing services are sufficiently high, then the buyer will not borrow fully against his or her housing collateral. The buyer does this in order to demonstrate to the bank that it is not posting fake collateral. Equation (11) states that the net payoff to the buyer from acquiring one unit of housing is zero in equilibrium.

Recall that, in equilibrium, a bank chooses the bank's deposit contract (k_t, c_t, d_t), its portfolio (m_t, b_t, l_t), and θ_t (which determines the haircut on the bank's mortgage portfolio, $1-\theta_t$), to maximize the expected utility of depositors, subject to a zero net payoff constraint (5), the binding collateral constraint (6), and the incentive constraint (7). Then, the following must hold in equilibrium:

$$-z_t + \beta\frac{\phi_{t+1}}{\phi_t}u'(\beta d_t) = 0 \tag{12}$$

$$-q_t + \beta[\theta_t u'(\beta d_t) + 1 - \theta_t] = 0 \tag{13}$$

$$\beta\frac{\phi_{t+1}}{\phi_t}u'\left(\beta\frac{\phi_{t+1}}{\phi_t}c_t\right) - 1 = 0 \tag{14}$$

$$-(1-\rho)d_t + \frac{\phi_{t+1}}{\phi_t}(m_t+b_t) + \theta_t l_t = 0 \tag{15}$$

$$\theta_t = \min\left(1, \frac{\gamma}{\beta u'(\beta d_t)}\right). \tag{16}$$

Note that the quantities c_t, d_t, m_t, b_t, and l_t in (14)-(15) denote, respectively, the quantities of currency and deposits promised to the representative depositor in the equilibrium banking contract, and the quantities of reserves, government bonds, and mortgage loans acquired by the representative bank in equilibrium.

In equilibrium, asset markets clear in the CM, so the representative bank's demands for currency, government bonds, and reserves are equal to the respective supplies coming from the government, i.e.

$$\rho c_t = \phi_t C_t, \tag{17}$$

$$b_t = \phi_t B_t, \tag{18}$$

$$m_t = \phi_t M_t. \tag{19}$$

As well, the demand for loans from banks equals the quantity supplied by buyers,

$$l_t = l_t^h, \tag{20}$$

and buyers' demand for housing is equal to the supply,

$$a_t = 1. \tag{21}$$

We will construct stationary equilibria, in which real quantities are constant forever, and all nominal quantities grow at the constant gross rate μ forever, so that the gross rate of return on money, $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ for all t (with μ endogenous). Then, from the government's budget constraints (8) and (9), and (17)-(19),

$$\rho c + z\left(m + b\right) = \tau_0 \tag{22}$$

$$\rho c\left(1 - \frac{1}{\mu}\right) + \left(z - \frac{1}{\mu}\right)\left(m + b\right) - \tau = 0,\ t = 1, 2, 3, ..., \tag{23}$$

where $\tau_t = \tau$ for $t = 1, 2, 3, ...,$ i.e. the transfer to buyers from the government may differ in period 0 from the transfer in each succeeding period. We will assume that the fiscal authority fixes the real value of the transfer in period 0, $\tau_0 = V$, i.e. V is exogenous. Then, from (23), we obtain

$$V\left(1 - \frac{1}{\mu}\right) + \left(\frac{m}{\mu} + \frac{b}{\mu}\right)(z - 1) = \tau, \tag{24}$$

where the transfer to buyers τ in each period $t = 1, 2, 3, ...,$ is endogenous. The fiscal policy rule is thus fixed in this sense, and the job of the central bank is to optimize treating the fiscal policy rule as given. So, in determining an equilibrium, all we need to take into account is equation (22) with $\tau_0 = V$, or

$$\rho c + z\left(m + b\right) = V. \tag{25}$$

In solving for a stationary equilibrium, it will prove convenient to express the equilibrium conditions in terms of the consumption allocation in the DM. This is helpful in part because, in this class of models, we can express aggregate welfare in terms of the DM consumption allocation. Let x_1 and x_2 denote, respectively, consumption in currency transactions and non-currency transactions in the DM, where $x_1 = \frac{\beta c}{\mu}$ and $x_2 = \beta d$. Then, from (12)-(16) and (25), we obtain:

$$z = \frac{u'(x_2)}{u'(x_1)}, \tag{26}$$

$$q = \min\left[\beta u'(x_2), \frac{u'(x_2)(\gamma + \beta) - \gamma}{u'(x_2)}\right], \tag{27}$$

$$-(1-\rho)x_2u'(x_2) - \rho x_1 u'(x_1) + V + l\min\left(\beta u'(x_2), \gamma\right) = 0, \tag{28}$$

$$\mu = \beta u'(x_1). \tag{29}$$

Similarly, from (11), (1) with equality, (20), and (21),

$$-\psi + \beta(\psi + y) + \min\left[(q-\beta)(\psi+y), \gamma^h\left(1 - \frac{\beta}{q}\right)\right] = 0, \tag{30}$$

$$l = \min\left[(\psi + y), \frac{\gamma^h}{q}\right]. \tag{31}$$

We will assume that conventional monetary policy consists of the choice of the price of short-term nominal government debt, z, which is then supported with the appropriate central bank balance sheet. Then, given V and z, equations (26)-(31) can be used to solve for x_1, x_2, q, μ, ψ, and l.

4.1 Scarce Collateral

Our focus in this paper will be on the behavior of the model economy when collateral is sufficiently scarce. Collateral is not scarce in this economy if the value of collateral is large enough that an efficient allocation can be supported in equilibrium. Confining attention to stationary allocations, efficiency is attained if surplus is maximized in all DM exchanges, i.e. if $x_1 = x_2 = x^*$, where $u'(x^*) = 1$.

Then, from (26), a necessary condition for efficiency is $z = 1$, i.e. the nominal interest rate on government debt must be zero, so conventional monetary policy must conform to the Friedman rule. Also, in an efficient equilibrium, from (27), $q = \beta$, and from (11), $\psi = \frac{\beta y}{1-\beta}$, so mortgages and houses are priced at their fundamental values, i.e. the sum of discounted payoffs on the respective assets. Of primary importance is that, for an efficient allocation to be feasible, the bank's collateral constraint (6) must be satisfied. From (28) and (31), the bank's collateral constraint holds if and only if

$$\underbrace{V}_{\text{public collateral}} + \underbrace{\min\left(\beta, \gamma\right)\min\left(\frac{y}{1-\beta}, \frac{\gamma^h}{\beta}\right)}_{\text{private collateral}} \geq x^*. \tag{32}$$

Inequality (32) states that the quantity of public collateral, given by V (the real value of the consolidated government debt), plus private collateral (the value of the stock of housing), must exceed the efficient quantity of consumption in the DM. If (32) holds, then an efficient allocation can be supported with conventional monetary policy, and the credit frictions – limited commitment and potential misrepresentation – in the model are irrelevant.

We will assume that (32) does not hold for any (γ, γ^h), i.e.

$$V + \frac{\beta y}{1-\beta} < x^*. \tag{33}$$

Inequality (33) states that the value of consolidated government debt plus the value of housing wealth to buyers is insufficient to support efficient exchange. Thus, (33) defines collateral scarcity. Note that, given the fiscal instruments available, the scarcity of collateral could be eliminated by fiscal policy, i.e. if the fiscal authority were to make V sufficiently large. Thus, a critical maintained assumption is that fiscal policy is suboptimal.

A stronger assumption, which will make life simpler in what follows, is

$$V + \frac{\beta y}{1-\beta} < (1-\rho)x^*. \tag{34}$$

Inequality (34) states that, even if z is very small so that, from (26), consumption purchased with currency in the DM is very small, there is not enough collateral to support efficient exchange in non-currency transactions.

4.2 Mortgage Market

Conventionally, mortgages are viewed as a means for financing the purchase of a house. But, in this model, a mortgage is actually an efficient means for reallocating collateral to its most productive use. To see how this works, recall that in the equilibria we want to study in this model, collateral is a scarce object, which is useful in securing the liabilities of banks. Those bank liabilities, in turn, are used in transactions in the DM. Houses are collateral, but it would be inefficient for banks to hold houses directly, as buyers receive a service flow from living in the house, while the bank does not. Thus, an efficient arrangement is for the buyer who owns the house to borrow from a bank in the form of a mortgage, using the house as collateral, so that the mortgage can serve as collateral backing the bank's liabilities.

To understand how the mortgage market works, and how the incentive to fake housing collateral matters, first consider the demand side of the mortgage market. It will be convenient to let r^h, r^m, and r, respectively, denote the gross real rates of return on housing (from the point of view of the buyer who owns the house), mortgages, and government debt, where $r^h = \frac{\psi+y}{\psi}$, $r^m = \frac{1}{q}$, and $r = \frac{1}{\beta u'(x_2)}$.

From the solution to the buyer's problem, summarized in (30) and (31), the buyer's incentive constraint binds if

$$r^m < \frac{y+\gamma^h}{\gamma^h}$$

and does not bind if

$$r^m \geq \frac{y+\gamma^h}{\gamma^h}.$$

Thus a lower mortgage rate makes cheating potentially more profitable – the buyer stands to gain more by faking collateral, borrowing against this fake collateral at a low interest rate, and defaulting on the loan. Recall that, if the buyer's incentive constraint binds, the buyer demonstrates that he or she has good collateral by not borrowing to the full amount that he or she could feasibly repay – there is a haircut on the housing collateral. Then, when the buyer's incentive constraint binds, this will restrict loan demand and will limit arbitrage between the mortgage market and the housing market. That is, the rates of return on mortgages and housing will not be equated when the buyer's incentive constraint binds.

To be more specific, from (30) and (31), the quantity of mortgage loans demanded is given by

$$\begin{aligned}\frac{l}{r^m} &= \gamma^h, \text{ if } r^m < \frac{y+\gamma^h}{\gamma^h}, \\ \frac{l}{r^m} &= \frac{y}{r^m-1}, \text{ if } r^m \geq \frac{y+\gamma^h}{\gamma^h}.\end{aligned} \tag{35}$$

Figure 1 shows the function described in (35). When the mortgage rate is low, demand is inelastic, determined by the cost of faking housing collateral, and when the incentive constraint does not bind the quantity demanded is decreasing in the mortgage rate, as intuition would tell us. Thus, the incentive problem in the mortgage market tends to limit demand. Note that, if the cost of faking housing collateral is sufficiently low, then the buyer's incentive constraint always binds, as $r^m \leq \frac{1}{\beta}$. That is, the buyer's incentive constraint always binds if

$$\gamma^h < \frac{\beta y}{1-\beta}. \tag{36}$$

So, Figure 1 is constructed for the case where (36) does not hold.

As well, from (30), we can determine the rate of return on housing (to the buyer) as a function of the mortgage rate:

$$\begin{aligned}r^h &= \frac{\gamma^h(1-\beta r^m)+y}{\gamma^h(1-\beta r^m)+\beta y}, \text{ if } r^m < \frac{y+\gamma^h}{\gamma^h}, \\ r^h &= r^m, \text{ if } r^m \geq \frac{y+\gamma^h}{\gamma^h}.\end{aligned} \tag{37}$$

Figure 2 depicts the relationship (37). For low mortgage rates, $r^h > r^m$, i.e. the private information friction acts to inhibit financial arbitrage, and this interest rate margin disappears with high mortgage rates, when the buyer's incentive constraint does not bind. But, if (36) holds, then the incentive constraint binds and a rate of return differential exists between housing and mortgages for all levels of the mortgage rate.

The supply of mortgage loans is determined by the behavior of banks, which face a gross market real interest rate on government debt r. From (27), mortgage loan supply is perfectly elastic at a mortgage rate determined by r. That is,

$$\begin{aligned}r^m &= \frac{1}{\gamma+\beta-\gamma\beta r}, \text{ if } r < \frac{1}{\gamma} \\ r^m &= r, \text{ if } r \geq \frac{1}{\gamma}.\end{aligned} \tag{38}$$

Figure 3 depicts the relationship between the mortgage rate and the interest rate on government debt, from (38), with the bank's incentive constraint binding when $r < \frac{1}{\gamma}$, i.e. when the real interest rate on government debt is sufficiently low. A lower real interest rate on government debt is reflected in a tighter collateral constraint for the bank, which increases the bank's incentive to fake collateral. Note that, if

$$\gamma < \beta, \tag{39}$$

then the bank's incentive constraint binds for any interest rate on government debt, since $r \leq \frac{1}{\beta}$ in equilibrium. Figure 3 is constructed for the case in which (39) does not hold. Thus, when the bank's incentive constraint binds, this induces a positive interest rate margin between the mortgage rate and the interest rate on government debt – the private information friction acts to inhibit arbitrage between government debt and mortgages.

The quantity of mortgages is determined by demand and supply in the mortgage market, and we have determined that the real interest rate r is important in determining the equilibrium quantity of mortgages. But the effective quantity of mortgages – mortgages that are used as collateral by banks – depends on the haircut applied to these mortgages by banks. From (16), the fraction of mortgages against which banks borrow is

$$\theta = \min(1, \gamma r). \tag{40}$$

That is, the bank's incentive constraint binds if $r < \frac{1}{\gamma}$, so when the real interest rate is low, an increase in r will increase the effective quantity of collateral, given the quantity of mortgages.

4.3 General Equilibrium: The Market For Collateral

We can construct a general equilibrium in terms of demand and supply in the market for collateral, given exogenous fiscal and monetary policy. This is useful, as it allows us to explore the implications of the model in an intuitive fashion. To see how this works, note that (26) and (28) imply an equilibrium relationship for the collateral market. That is, write (28) as

$$F(x_1, x_2) = G(x_2), \tag{41}$$

where

$$F(x_1, x_2) = (1-\rho)x_2 u'(x_2) + \rho x_1 u'(x_1) \tag{42}$$

is the demand for collateral, given the quantities of consumption x_1 and x_2, in the DM. The first term on the right-hand side of (42) is the demand for collateral in the form of assets backing bank deposits, and the second term is the demand for "collateral" in the form of currency (currency is of course traded on the spot, with immediate settlement). Further, because our assumptions on preferences (essentially implying asset demands are increasing in the rate of return on the asset) imply that $xu'(x)$ is strictly increasing in x, $F(\cdot,\cdot)$ is strictly increasing in both arguments. As well, in (41),

$$G(x_2) = V + l \min\left(\beta u'(x_2), \gamma\right), \tag{43}$$

which is the supply of collateral in the form of consolidated government debt and private assets, respectively, on the right-hand side of (43). Letting $R = \frac{1}{z}$ denote the gross nominal interest rate, from (26) we have

$$R = \frac{u'(x_1)}{u'(x_2)}, \tag{44}$$

which implies that, in equilibrium x_1 is an increasing function of x_2 and a decreasing function of R. Further, as $r = \frac{1}{\beta u'(x_2)}$, the real interest rate on government debt is increasing in x_2. Therefore, we can write equation (41) as

$$D(r, R) = S(r), \tag{45}$$

where the left-hand side of (45) is the demand for collateral as a function of the real interest rate on government debt and the exogenous nominal interest rate on government debt, and the right-hand side is the supply of collateral as a function of the real interest rate. From our analysis above, $D(r, R)$ is strictly increasing in r for $r \in \left[0, \frac{1}{\beta}\right]$, and strictly decreasing in R for $R \geq 1$, and we can find the equilibrium real interest rate r by solving (45).

The demand for collateral is increasing in the real rate of return on government debt, as the return on safe collateral (where "safe" means that it cannot be faked) is what is relevant for the demand for collateral in the aggregate. The nominal interest rate matters, essentially because of the Fisher effect. In a stationary equilibrium, the gross rate of inflation is equal to R/r, and the rate of inflation determines the rate of return on currency. In the model, the demand for currency, in real terms, increases with the rate of return on currency, i.e. it is decreasing in R/r. Thus, the aggregate demand for collateral $D(r, R)$ (which includes the demand for currency) is decreasing in R, given r.

Let IC denote an incentive constraint. From (27), (30), and (31), we can determine that

$$\begin{aligned}
S(r) &= V + \frac{y}{r-1}, \text{ if neither the buyer's nor the bank's } IC \text{ binds.} \qquad (46)\\
S(r) &= V + \gamma^h, \text{ if the buyer's } IC \text{ binds, and the bank's does not.}\\
S(r) &= V + \frac{y\gamma}{1-\gamma-\beta+\gamma\beta r}, \text{ if buyer's } IC \text{ not binding, and bank's is binding.}\\
S(r) &= V + \frac{\gamma\gamma^h}{\gamma+\beta-\gamma\beta r}, \text{ if both } ICs \text{ bind.}
\end{aligned}$$

In each case in (46) the first term in the supply function is the supply of public collateral, while the second is the supply of private collateral. Note that the supply of collateral is decreasing in the real interest rate in the cases where the buyer's IC does not bind, but if the buyer's IC binds, then the supply of collateral is either inelastic or *increasing* in the real interest rate. Why is the supply of collateral increasing with the real interest rate when both incentive constraints bind? First, in Figures 1 and 3, the equilibrium quantity of mortgages is inelastic with respect to r when r is low, due to the fact that the buyer borrows more as r increases and his or her incentive constraint is relaxed. Second, as r increases, with the bank's incentive constraint binding, from (40) this relaxes the incentive constraint for banks and increases the effective supply of collateral, as the bank's haircut on mortgage collateral falls. Thus, when incentive constraints bind for buyers and banks, an increase in r means that the bank can borrow more against an inelastic quantity of mortgage loans, so the effective supply of collateral increases in the aggregate. Once r is high enough that the buyer's incentive constraint does not bind, then buyers borrow less as r and r^m increase.

From (46) and our previous analysis, we can piece together a supply function for collateral. But what this function looks like depends on parameters. From (35)-(40), and (46),

1. If $\gamma < \beta$ and $\gamma^h < \frac{\beta y}{1-\beta}$, then both ICs bind for all $r \in \left[0, \frac{1}{\beta}\right]$.

2. If $\gamma \geq \beta$ and $\gamma^h < \frac{\beta y}{1-\beta}$, then both ICs bind for $r \in [0, \frac{1}{\gamma})$; bank's IC does not bind and the buyer's IC binds for $r \in \left[\frac{1}{\gamma}, \frac{1}{\beta}\right]$.

3. If $\gamma < \beta$, $\gamma^h \geq \frac{\beta y}{1-\beta}$, and $\frac{1}{\gamma+\beta} \geq \frac{y+\gamma^h}{\gamma^h}$, then the buyer's IC does not bind and the bank's does, for all $r \in \left[0, \frac{1}{\beta}\right]$.

4. If $\gamma < \beta$, $\gamma^h \geq \frac{\beta y}{1-\beta}$, and $\frac{1}{\gamma+\beta} < \frac{y+\gamma^h}{\gamma^h}$, then both ICs bind for $r \in [0, \frac{1}{\gamma} + \frac{1}{\beta} - \frac{\gamma^h}{\gamma\beta(y+\gamma^h)})$, IC does not bind for buyer, but binds for the bank, if $r \in \left[\frac{1}{\gamma} + \frac{1}{\beta} - \frac{\gamma^h}{\gamma\beta(y+\gamma^h)}, \frac{1}{\beta}\right]$.

5. If $\gamma \geq \beta$, $\gamma^h \geq \frac{\beta y}{1-\beta}$, and $\frac{1}{\gamma+\beta} \geq \frac{y+\gamma^h}{\gamma^h}$, then the buyer's IC does not bind and the bank's IC binds for $r \in [0, \frac{1}{\gamma})$; neither IC binds for $r \in [\frac{1}{\gamma}, \frac{1}{\beta}]$.

6. If $\gamma \geq \beta$, $\gamma^h \geq \frac{\beta y}{1-\beta}$, $\frac{1}{\gamma+\beta} \geq \frac{y+\gamma^h}{\gamma^h}$, and $\frac{1}{\gamma} < \frac{y+\gamma^h}{\gamma^h}$, then both ICs bind for $r \in [0, \frac{1}{\gamma})$, the buyer's IC binds and the bank's does not for $r \in [\frac{1}{\gamma}, \frac{y+\gamma^h}{\gamma^h})$, and neither constraint binds for $r \in \left[\frac{y+\gamma^h}{\gamma^h}, \frac{1}{\beta}\right]$.

7. If $\gamma \geq \beta$, $\gamma^h \geq \frac{\beta y}{1-\beta}$, $\frac{1}{\gamma+\beta} \geq \frac{y+\gamma^h}{\gamma^h}$, and $\frac{1}{\gamma} \geq \frac{y+\gamma^h}{\gamma^h}$, then both ICs bind for $r \in [0, \frac{1}{\gamma} + \frac{1}{\beta} - \frac{\gamma^h}{\gamma\beta(y+\gamma^h)})$, the buyer's IC does not bind and the bank's IC does for $r \in [\frac{1}{\gamma} + \frac{1}{\beta} - \frac{\gamma^h}{\gamma\beta(y+\gamma^h)}, \frac{1}{\gamma})$, and neither incentive constraint binds for $r \in \left[\frac{1}{\gamma}, \frac{1}{\beta}\right]$.

There are two key conclusions from (1)-(7) above. First, except for cases (3) and (5) both ICs bind and $S(r)$ is strictly increasing and strictly convex for low values of r. Second, unless $\gamma \geq \beta$ and $\gamma^h \geq \frac{\beta y}{1-\beta}$, as in cases (5), (6), and (7), some incentive constraint binds for all $r \in \left[0, \frac{1}{\beta}\right]$.

In cases (3) and (5), $S(r)$ is a decreasing function of r on $\left[0, \frac{1}{\beta}\right]$. Therefore, since $D(r, R)$ is strictly increasing in r, given R, the equilibrium is unique in those cases. Further, by continuity, the equilibrium exists in these cases, as $D(0, R) = 0 < S(0)$, and $D\left(\frac{1}{\beta}, R\right) > S\left(\frac{1}{\beta}\right)$ (from (34)). But in the other cases – other than (3) and (5) that is – there is the possibility of multiple equilibria.

For example, consider case (2), in which both ICs bind for low values of r, and only the buyer's IC binds for high values of r. In this case, as depicted in Figure 4, the supply of collateral function is strictly convex for $r \in [0, \frac{1}{\gamma})$, and is constant for $r \in \left[\frac{1}{\gamma}, \frac{1}{\beta}\right]$. Then, there exist demand-for-collateral functions $D(r, R)$ that can yield multiple equilibria. For example, in Figure 4, there are three equilibria, denoted by points A, B, and D. For equilibria A and B, both IC constraints bind, and for equilibrium D, the buyer's IC binds but the bank's does not.

To show that the configuration in Figure 4 is a possibility, consider the case where $u(x) = \frac{x^{1-\alpha}}{1-\alpha}$ and, given our assumptions, $\alpha < 1$, which implies that asset demand is strictly increasing in the asset's real rate of return. Then, from (42), (44), and $r = \frac{1}{\beta u'(x_2)}$, we get

$$D(r, R) = (r\beta)^{\frac{1}{\alpha}-1}\left[1 - \rho + \rho R^{-\frac{1}{\alpha}+1}\right] \tag{47}$$

From (47), note that $D(\frac{1}{\beta}, 1) = 1$, so if $\gamma^h < 1$, by continuity there must be at least one equilibrium in the set $(0, \frac{1}{\beta})$. But as long as γ^h is sufficiently close to 1, and $\frac{1}{2} < \alpha < 1$ (so $D(r, R)$ is strictly concave), there must be three equilibria as in Figure 4.

If there are multiple equilibria as in Figure 4, it is then straightforward to rank equilibria according to welfare. To see this, if we add utilities across agents in a stationary equilibrium to determine total welfare, this is just the weighted sum of surpluses in DM exchange, or

$$W(x_1, x_2) = \rho[u(x_1) - x_1] + (1 - \rho)[u(x_2) - x_2], \tag{48}$$

The welfare function $W(x_1, x_2)$ is strictly increasing in both arguments in any equilibrium, due to the inefficiency caused by scarce collateral, which implies $u'(x_i) > 1$, for $i = 1, 2$. Further, from (44) and $r = \frac{1}{\beta u'(x_2)}$, we have

$$u'(x_1) = \frac{R}{r\beta}, \tag{49}$$

$$u'(x_2) = \frac{1}{r\beta}. \tag{50}$$

Therefore, in Figure 4, for example, R is held constant across the three equilibria, but A has the lowest equilibrium real interest rate, the real interest rate is higher in equilibrium B, and it is highest in equilibrium D. Therefore, inefficiencies in both types of DM exchanges (currency and non-currency trades) decline from equilibrium A to B to D. Thus, welfare is lowest in equilibrium A, higher in equilibrium B, and highest in equilibrium D.

This makes sense, as we know from our analysis above that a higher real interest rate tends to relax the bank's incentive constraint, and also tends to increase the mortgage rate, which tends to relax the buyer's incentive constraint. Thus, higher real interest rates in general will tend to increase the effective supply of collateral, since effective collateral increases as incentive constraints are relaxed.

5 Conventional Monetary Policy

Conventional monetary policy is defined here as the interest rate policy of the central bank, which is implemented through swaps of reserves for government debt, or vice-versa. We have assumed that central bank reserves and government debt have exactly the same properties – each asset has the same payoffs, there are no restrictions on who can hold these assets, and each is equally useful in backing deposit claims that are used in exchange. In practice, there are some subtleties associated with the fact that reserves can be held only by a select set of financial institutions, and that government debt can be used in different ways than reserves – for example in repurchase agreements (see for example Williamson 2016b). But, for our purposes, assuming that these assets are in most respects the same – except that the first is issued by the monetary authority and the second by the fiscal authority – is a good approximation to reality.

But, if government debt and reserves are perfect substitutes, why will swaps by the central bank of one asset for the other matter? The key difference between the two assets is that reserves can be converted one-for-one into currency, with the currency/reserves composition of outside money determined by the terms on which buyers and banks wish to hold each asset in equilibrium. For example, we could have a regime in which the central bank targets a price for government debt z, and then purchases just enough government debt by issuing outside money that the total quantity of outside money is equal to the quantity of currency held in equilibrium, with zero reserves outstanding. This is a typical "corridor system," in which the quantity of reserves is zero and z determines the required open market policy of the central bank.

Alternatively, the central bank could set the price of reserves at z, and purchase a quantity of government debt larger than the quantity of currency that will be outstanding in equilibrium given z. Then, there will be a positive quantity of reserves in equilibrium, and the nominal interest rate on reserves will determine the nominal interest rate on government debt, as arbitrage dictates that the two interest rates are equal in equilibrium. This is a "floor system" in which monetary policy is conducted by a central bank with a large balance sheet. In this model, conventional monetary policy under a channel system and a floor system are identical. That is, given the nominal interest rate, increasing the quantity of reserves from zero to the entire quantity of government debt issued by the fiscal authority will have no effect – a type of liquidity trap.

Thus, conventional monetary policy can be summarized by the price of short-term government debt z, which is treated as an exogenous policy instrument. Or equivalently, the gross nominal interest rate $R = \frac{1}{z}$ is the monetary policy instrument. In this section we want to explore how conventional monetary policy affects economic welfare, as defined by the welfare function (48). First, we will determine conditions under which $R = 1$ will and will not be optimal, at least locally. Then, we will examine some potential consequences of changes in R for existence of equilibria, and for welfare.

5.1 When is the Zero Lower Bound Optimal?

In this model, if collateral were not scarce then it would be clear that the central bank should set $R = z = 1$, i.e. a zero nominal interest rate would be optimal. This is just a version of the Friedman rule. That is, if the central bank sets $z = 1$, and if, in equation (28), the right hand side is greater than x^* (the efficient quantity of DM consumption) in equilibrium, then $z = 1$ maximizes (48). A sufficient condition for optimality is $V \geq x^*$, so that incentive problems affecting the private supply of collateral are irrelevant. In this case the government has supplied sufficient collateral for efficiency.

But what if collateral is scarce in equilibrium, so that $r < \frac{1}{\beta}$, and the equilibrium allocation is suboptimal? What is optimal conventional monetary policy, i.e. what is optimal R, given suboptimal fiscal policy?

As in the example in the previous section, consider the case where $u(x) = \frac{x^{1-\alpha}}{1-\alpha}$, with $0 < \alpha < 1$. Then, from (46) and (47), we can solve for equilibrium r given R, from

$$(r\beta)^{\frac{1}{\alpha}-1}\left[1-\rho+\rho R^{-\frac{1}{\alpha}+1}\right] = S(r). \tag{51}$$

Then, from (48), (49), and (50), we can determine the derivative of aggregate welfare with respect to R, evaluated for $R = 1$:

$$\frac{dW}{dR} = \rho\left[\frac{\left(\frac{1}{\alpha}-1\right)\beta^{\frac{1}{\alpha}-1}r^{\frac{1}{\alpha}-2}}{\left(\frac{1}{\alpha}-1\right)\beta^{\frac{1}{\alpha}-1}r^{\frac{1}{\alpha}-2}-S'(r)}-1\right]. \tag{52}$$

Therefore, from (46), $S'(r) \leq 0$ if one IC binds, or if neither binds. Then, from (52), in those cases the zero lower bound is locally optimal. That is $\frac{dW}{dR} \leq 0$ for $R = 1$. In spite of the inefficiency that arises because of scarce collateral and binding collateral constraints, welfare will not go up if the central bank increases the nominal interest rate from zero. In the cases in which either IC or both ICs are non-binding, conventional monetary policy should operate as it would in this model in the absence of scarce collateral – the zero lower bound is optimal, at least locally.

But, if both ICs bind then, from (46), $S'(r) > 0$. Then, in an equilibrium such as A in Figure 4 (supposing Figure 4 is constructed for $R = 1$), the denominator in the first term in square parentheses in (52) is positive, so $\frac{dW}{dR} > 0$, and the zero lower bound is not optimal. However, for an equilibrium such as B in Figure 4 (again, constructing the equilibrium for $R = 1$), the denominator in the first term in square parentheses in (52) is negative, so $\frac{dW}{dR} < 0$ and the zero lower bound is locally optimal.

Therefore, if both ICs bind, an increase in the nominal interest rate from zero, which leads to an increase in the real interest rate, can relax incentive constraints sufficiently that welfare must increase, at least in one equilibrium. Indeed, we could have circumstances giving rise to Figure 5, in which A is the equilibrium with a zero nominal interest rate, and the equilibrium is unique, with both ICs binding. Then, it must be the case that welfare increases if R increases to $R_1 > 1$, provided R_1 is sufficiently close to 1, with the new equilibrium at B in the figure.

5.2 Multiple Equilibria

Given the existence of multiple equilibria, it can be difficult to make meaningful statements about the effect of policy. However, there are some cases in which there are clear-cut policy conclusions. In Figure 6, suppose we have a case of multiple equilibria with $R = 1$, much like in Figure 4. That is, in Figure 6, when $R = 1$, the equilibria are A (two ICs bind), B (two ICs bind), and D (one IC binds). Recall that welfare is lowest in equilibrium A, higher in equilibrium B, and highest in D. Suppose that the central bank increases R to R_1, in which case there is only one equilibrium, which is F, in which one IC binds. Then, if the change in R is sufficiently small, since $S'(r) = 0$ in equilibrium D, from (52) the change in welfare from D to F is essentially zero, if the change in R is sufficiently small. Thus, the change in policy in this case eliminates two inferior equilibria, with no change in welfare in the superior equilibrium. In this case the policy change – moving off the zero lower bound – is unambiguously welfare-improving.

5.3 Discussion

In terms of comparative statics for particular equilibria, and in multiple equilibrium contexts, we have shown that increasing the nominal interest rate from zero, in circumstances in which incentive problems are severe – both ICs bind – can increase welfare. This is because, given collateral scarcity, binding collateral constraints, and low real interest rates, increasing the nominal interest rate relaxes collateral constraints, increases the real rate, and in turn relaxes incentive constraints. This is an amplification effect. The open market operation that supports the increase in the nominal interest rate (or the substitution between currency and reserves resulting from an increase in the interest rate on reserves, if there are reserves outstanding) increases the stock of collateral in non-currency transactions, the real interest rate rises, and then the relaxation in incentive constraints further increases the effective stock of collateral.

It is possible to relate the key incentive problem in our model to financial crises. Typically, theories of crises require large effects from small shocks. This can either arise from multiple equilibria, for example in the Diamond-Dybvig (1983) model, or through some amplification effect, for example in financial frictions models like the financial accelerator model (Bernanke et al. 1999). Our model has both multiple equilibria and amplification effects. For a given nominal interest rate there can be several equilibria that we can rank in terms of welfare, and an increase in the central bank's nominal interest rate target can kill off undesirable equilibria. As well, an increase in the nominal interest rate, through supporting open market operations, can increase the stock of effective collateral, which in turn relaxes incentive constraints, further increasing the stock of effective collateral.

6 Unconventional Monetary Policy: Private Asset Purchases by the Central Bank

In recent years, the world's central banks have engaged in various types of unconventional monetary policies, including large-scale purchases of private assets. In the United States, the Fed currently has large holdings of mortgage-backed securities, which were issued by government-sponsored enterprises, and are backed by private mortgages. As well, the Bank of Japan has extensive holdings of shares in exchange-traded funds, and the European Central Bank has purchased private corporate debt, along with other private assets.

So far, we have shown the consequences of conventional monetary policy for incentives in private asset markets, and for quantities, prices, and economic welfare. But what if the central bank in our model chose to purchase and hold private mortgages? What consequences would this have? For example, could this help to mitigate incentive problems in the mortgage market? We will answer these questions in this section.

We will assume that mortgages must be purchased from banks – the central bank in our model cannot lend directly to buyers. Private banks will make zero profits in lending to buyers and passing mortgages on to the central bank, so the gross real mortgage interest rate the government receives on its mortgage portfolio in a stationary equilibrium is $r^m = \frac{1}{q}$. Letting l^g denote the quantity of mortgage loans held by the central bank, we need to modify the government's budget constraint in a stationary equilibrium from (25) to

$$\rho c + z\left(m + b\right) = V + \frac{l^g}{r^m}. \tag{53}$$

In equilibrium, a private bank must weakly prefer selling a mortgage to the central bank to holding the mortgage on its balance sheet. Therefore, from (13) and (16), and given $r = \frac{1}{\beta u'(x_2)}$ in equilibrium,

$$r^m \leq \max\left[r, \frac{1}{\beta + \gamma - \gamma\beta r}\right]. \tag{54}$$

As well, so that banks do not have the incentive to sell fake mortgages to the central bank,

$$r^m \geq \frac{1}{\gamma}. \tag{55}$$

Inequalities (54) and (55) imply that a necessary condition for feasibility of the central bank asset purchase program is

$$\gamma \geq \beta, \tag{56}$$

As well, from (54) and (55), feasibility of the central bank's asset purchase program implies

$$r \geq \frac{1}{\gamma}. \tag{57}$$

That is, binding incentive constraints for banks are incompatible with a feasible central bank asset purchase program.

We need to consider two cases in turn. In the first, the central bank does not purchase the entire quantity of mortgage loans in the market each period, while in the second case the central bank holds the entire aggregate stock of mortgage debt, and therefore sets the market price.

6.1 Central Bank Asset Purchases Coexist with Private Mortgage Lending

Given (54)-(57), if there are mortgages held by both the central bank and private banks, and the central bank's asset purchase program is feasible, then banks' incentive constraints do not bind in equilibrium and $r = r^m$. Then, from (30) and (31), if l denotes the total quantity of loans (held by the central bank and private banks), then

$$l = \min\left[\frac{ry}{r-1}, r\gamma^h\right] \tag{58}$$

We need to modify (28), using the government budget constraint (53) to account for the central bank's mortgage purchases. That is, in equation (28), replace V with $V + \frac{l^g}{r}$, to account for the mortgage debt held by the central bank, and replace l with $l - l^g$, to net out mortgages purchased by the central bank from private mortgage loan supply. Then, in a manner similar to (45), the demand for collateral equals supply in equilibrium, but from (58) total supply is given by

$$S(r) = V + \min\left[\frac{y}{r-1}, \gamma^h\right]. \tag{59}$$

So, if an equilibrium exists in which central bank asset purchases are feasible, and the central bank does not buy the entire stock of mortgages outstanding, equilibrium r must solve (45), with r satisfying (57), and (56) holding. But note that the supply of collateral, $S(r)$, in (59) does not depend on l^g, the quantity of mortgage loans purchased by the central bank. That is, the central bank purchases mortgage loans by issuing more reserves, and these reserves are held by banks on the same terms as the mortgage loans that they replace in banks' portfolios. Therefore, if the central bank does not purchase the entire stock of mortgage loans, a feasible central bank asset purchase program is neutral.

6.2 The Central Bank Intermediates All Private Mortgages

Since central bank asset purchases that compete with private mortgage lending were shown to be irrelevant in the last subsection, the only possibility of purchases having any effect exists when the central bank purchases the entire stock of mortgage loans. As in the previous subsection, (53)-(57) must hold for any feasible central bank asset purchase program. But here, since the central bank purchases all mortgages, from (30) and (31),

$$l^g = l = \min\left(r^m\gamma^h, \frac{r^m y}{r^m - 1}\right), \tag{60}$$

and then we can characterize the program in terms of a setting for the mortgage loan rate r^m by the central bank – the interest rate implied by the price at the which the central bank takes mortgages off the hands of private banks. Then, given (53)-(57), we can write the aggregate supply of collateral under the program, using (60), as

$$S(r, r^m) = V + \min\left(\gamma^h, \frac{y}{r^m - 1}\right) \tag{61}$$

In an equilibrium in which the central bank's asset purchase program is feasible, an optimal purchase program will maximize the supply of collateral in the aggregate, which from (61) implies minimizing the mortgage loan interest rate. However, note that reducing this rate below the point at which the buyer's incentive constraint binds has no effect. That is, if $r^m \leq \frac{\gamma^h + y}{\gamma^h}$, then reducing r^m results in no change in the aggregate supply of collateral.

In the absence of a central bank asset purchase program under which all mortgages are purchased by the central bank, and given feasibility of the program, the supply of collateral is given by (59), rather than (61) with the program. Essentially, feasibility of the asset purchase program requires that the bank's incentive constraint not bind, otherwise the central bank will receive only fake mortgages from banks. As well, the asset purchase program will only increase the supply of collateral in the aggregate if the buyer's incentive constraint does not bind. Therefore, we can conclude, from (59) and (61), that the asset purchase program will increase equilibrium r if and only if, in the absence of the asset purchase program,

$$r > \frac{\gamma^h + y}{\gamma^h}.$$

Further, note that welfare will increase as a result of the program if and only if r increases.

Figure 7 shows a case in which a central bank asset purchase program can increase welfare. The demand for collateral in the figure is given by $D(R, r)$, and in the absence of an asset purchase program the supply of collateral is $S(r)$. The equilibrium in this case is $r = r_1$, with neither incentive constraint binding. But with an optimal central bank asset purchase program in place, the supply of collateral is given by $S'(r)$ in Figure 7, and the equilibrium real interest rate is $r_2 > r_1$, with conventional monetary policy held constant, i.e. fixing R. In this case, the asset purchase purchase program is welfare improving.

Thus, the asset purchase program, if it works, does so by increasing the stock of collateral and relaxing collateral constraints, which acts to reduce the liquidity premium on government debt and increase the real interest rate r. But the program does not work by affecting the incentive to cheat on collateral, as the program is not feasible if the bank's incentive constraint binds, and is neutral if the buyer's incentive constraint binds. Basically, if the program is effective, it acts to sidestep fiscal policy, which has limited the supply of safe collateral. The central bank can buy mortgages at a low interest rate, which increases the demand for such mortgages, and this expands the aggregate stock of collateral, so long as the mortgage interest rate is not so low as to induce binding collateral constraints for mortgage borrowers.

Thus, the model tells us that central bank purchases of private assets work only if the central bank purchases good collateral – that is collateral not subject to incentive problems. In the model, the central bank does not have any special advantage in mitigating incentive problems in the market for collateral.

These results seem to be consistent with Krishnamurthy and Vissing-Jorgensen (2013), who argue that the Fed's large-scale purchases of mortgage-backed securities were more effective than purchases of long-maturity Treasury debt. In our model, private asset purchases can relax collateral constraints, given fiscal policy and conventional monetary policy, while purchases of government debt do not. But consistency of our model with Krishnamurthy and Vissing-Jorgensen (2013) depends on the existence of market segmentation in practice. That is, it would have had to be the case that, when the Fed purchased mortgage-backed securities, it was at times purchasing all available mortgages in some segment of the market.

7 Conclusion

We have built a model where there are incentive problems in the mortgage market – banks can fake the quality of mortgage debt, and consumers can fake the quality of housing posted as collateral. These incentive problems matter when real interest rates are low, that is when collateral is scarce in the aggregate. And conventional monetary policy easing – a lower nominal interest rate – lowers real rates when collateral constraints bind, which tends to exacerbate incentive problems.

Inventive problems with respect to collateral act to produce amplification effects of monetary policy, and multiple equilibria, due to the fact that higher interest rates can increase the stock of private collateral. This then matters for optimal monetary policy, in that raising nominal interest rates can increase welfare, and can eliminate suboptimal equilibria.

If the central bank purchases private mortgages which are subject to incentive problems, such purchases are neutral unless the central bank purchases the entire stock of mortgages outstanding, and does not cause households to misrepresent housing collateral. But the beneficial effects of these purchases come from relaxing collateral constraints and sidestepping suboptimal fiscal policy, not from mitigating incentive problems.

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Figure 1

Demand for Mortgages

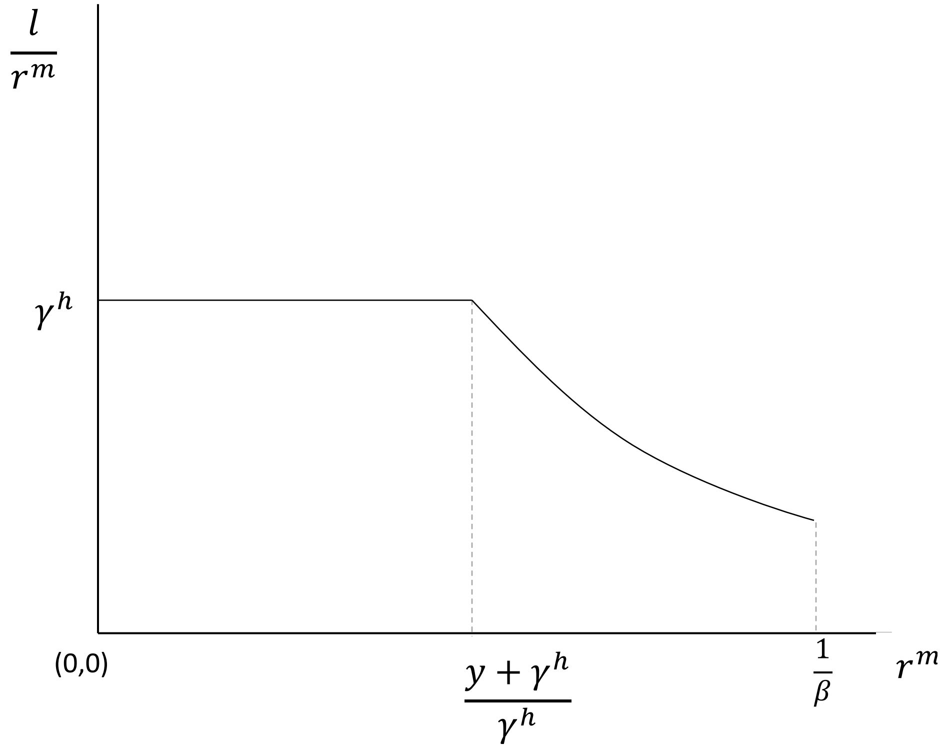

Figure 2

Rate of Return on Housing vs. Mortgage Rate

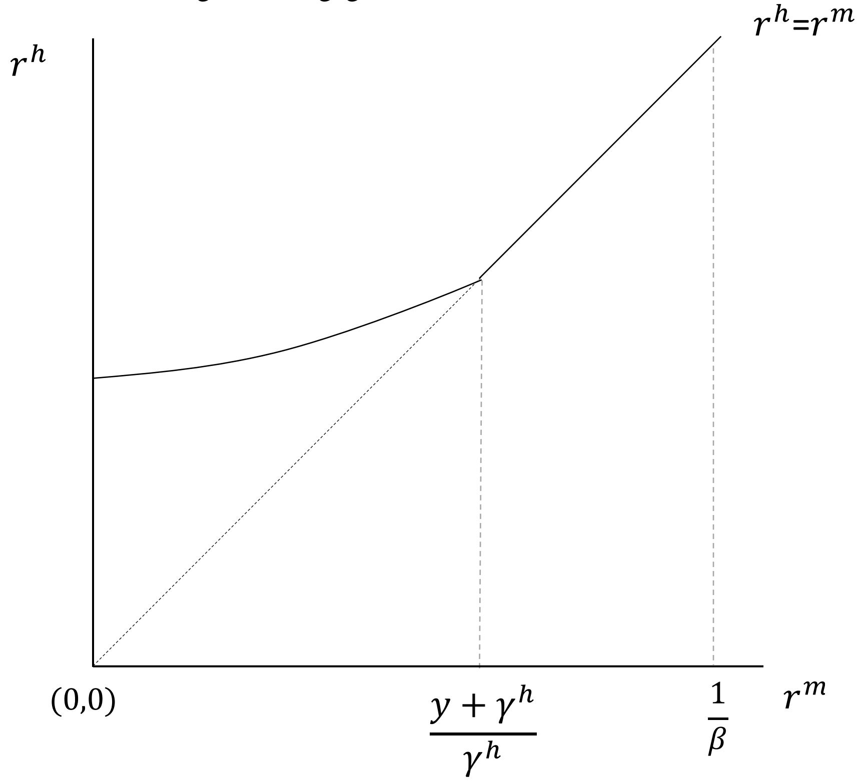

Figure 3

Mortgage Rate vs. Interest Rate on Government Debt

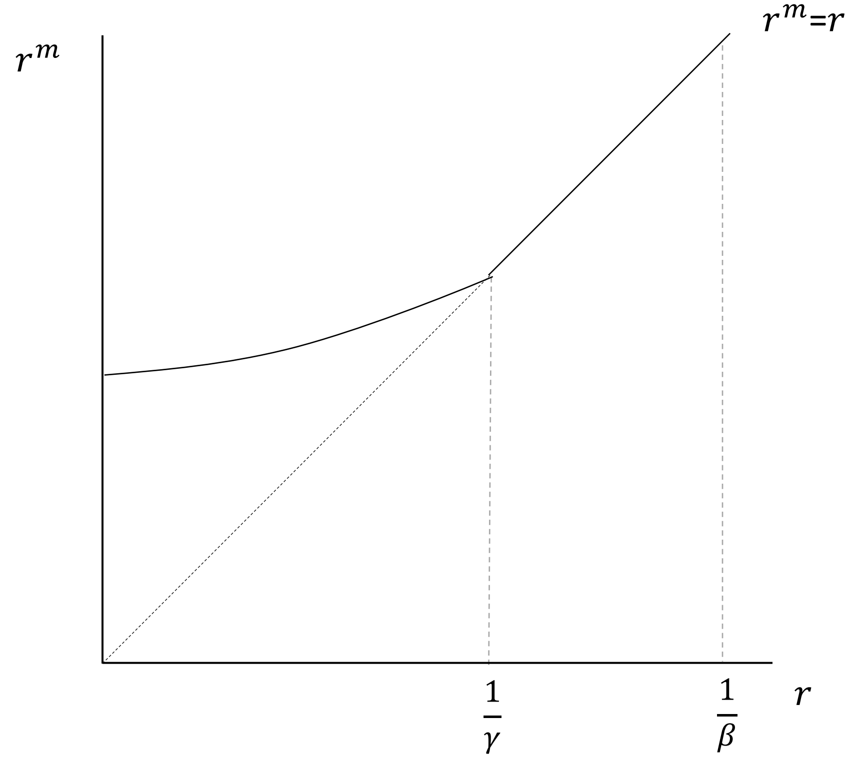

Figure 4

Multiple Equilibria

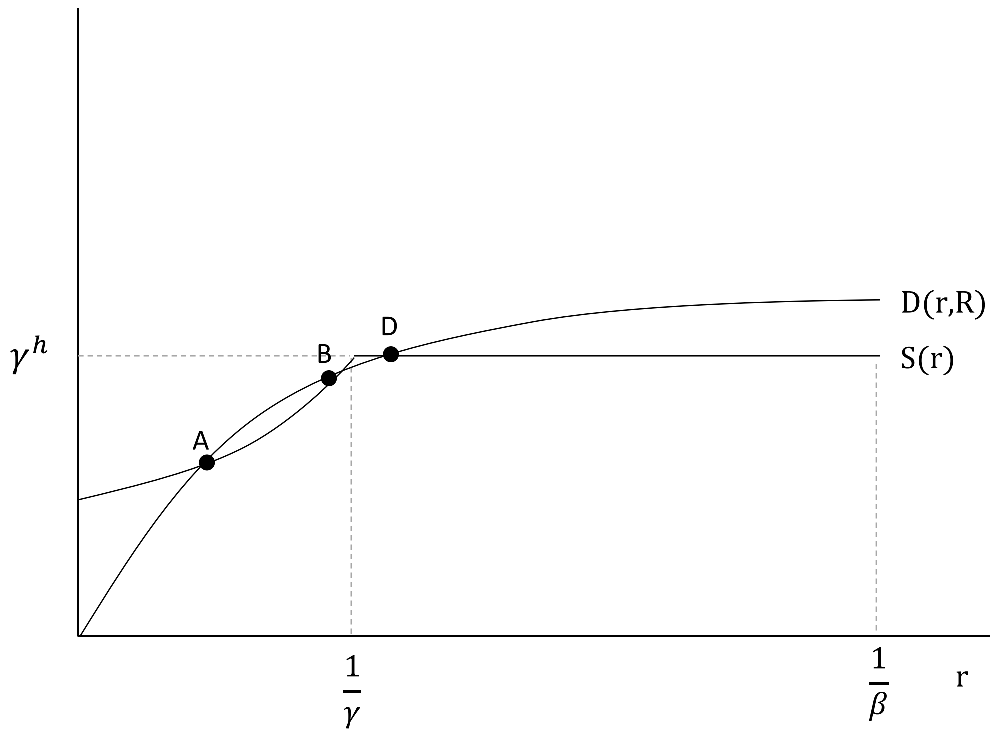

Figure 5

Increase in R Increases Welfare

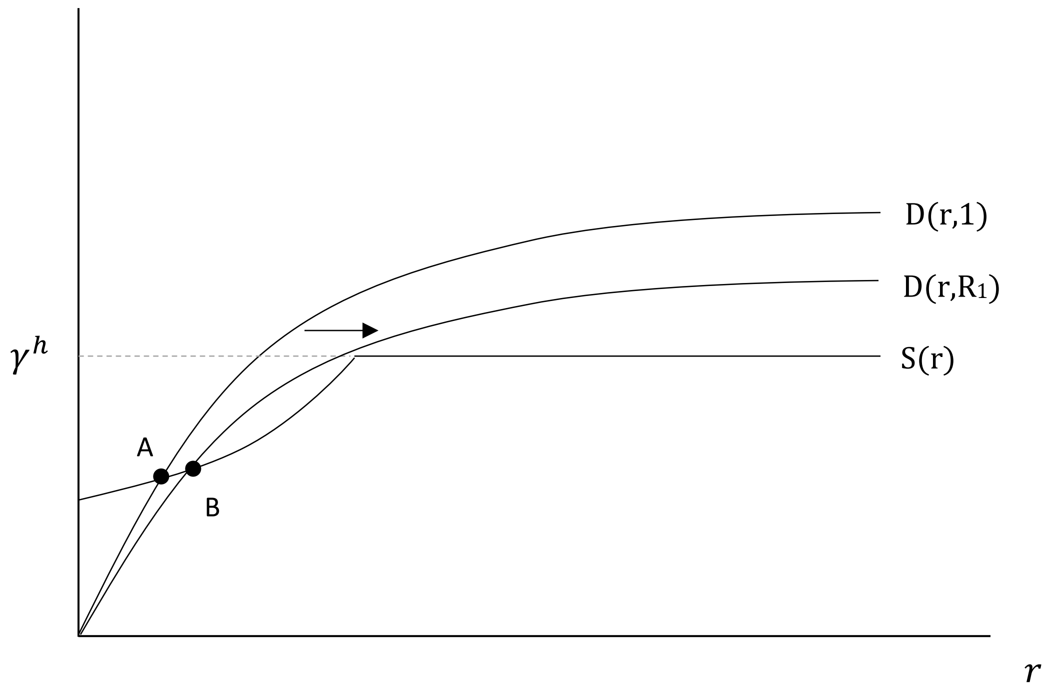

Figure 6

Increase in R Eliminates Suboptimal Equilibria

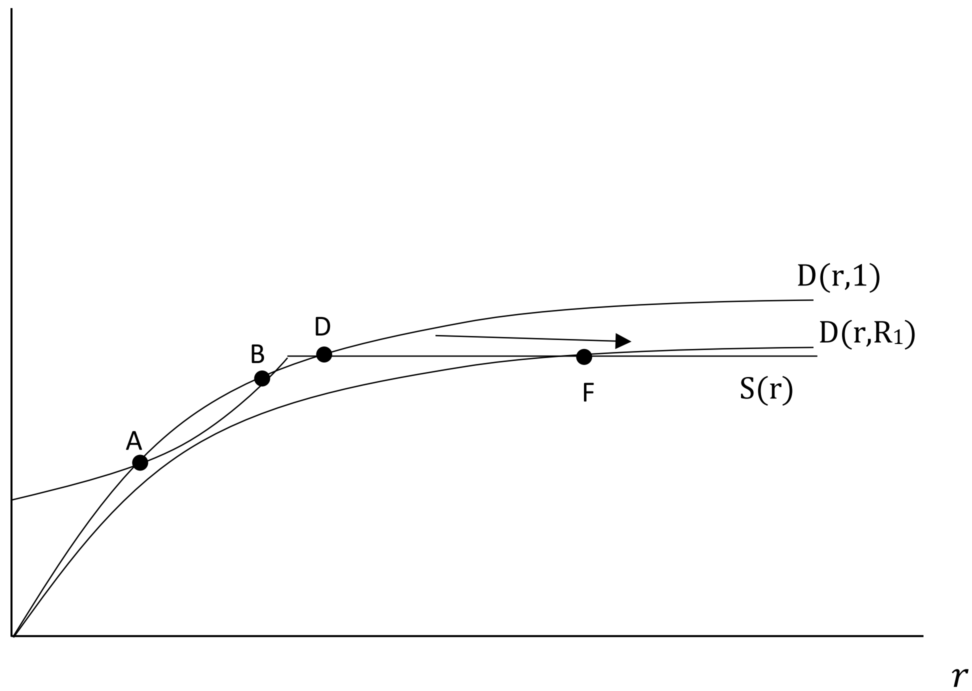

Figure 7

Central Bank Asset Purchases