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WORKING PAPER

PREVENTING BANK RUNS

BY DAVID ANDOLFATTO, ED NOSAL AND BRUNO SULTANUM*

JULY, 2014

Diamond and Dybvig (1983) is commonly understood as providing a formal rationale for the existence of bank-run equilibria. It has never been clear, however, whether bank-run equilibria in this framework are a natural byproduct of the economic environment or an artifact of suboptimal contractual arrangements. In the class of direct mechanisms, Peck and Shell (2003) demonstrate that bank-run equilibria can exist under an optimal contractual arrangement. The difficulty of preventing runs within this class of mechanism is that banks cannot identify whether withdrawals are being driven by psychology or by fundamentals. Our solution to this problem is an *indirect* mechanism with the following two properties. First, it provides depositors an incentive to communicate whether they believe a run is on or not. Second, the mechanism threatens a suspension of convertibility conditional on what is revealed in these communications. Together, these two properties can eliminate the prospect of bank-run equilibria in the Diamond-Dybvig environment.

KEYWORDS: bank runs, optimal deposit contract, financial fragility.
JEL CLASSIFICATION: D82, E58, G21.

1. INTRODUCTION

BANKING is the business of transforming long-maturity illiquid assets into short-maturity liquid liabilities. The demandable debt issued by commerical banks constitutes the quintessential example of this type of credit arrangement. The use of short-maturity debt to finance long-maturity asset holdings is also prevalent in the shadow-banking sector.[1] Demandable debt or short-maturity debt in general has long been viewed by economists and regulators as an inherently fragile financial structure—a credit arrangement that is susceptible to runs or roll-over risk. The argument is a familiar one. Suppose that depositors expect a run—a wave of early redemptions driven by fear, rather than by liquidity needs. By the definition of illiquidity, the value of what can be recouped in a fire-sale of assets must fall short of existing obligations.[2] Because the bank cannot honor its promises in this event, it becomes insolvent. In this manner, the fear of run can become a self-fulfilling prophecy.

If demandable debt is run prone, then why not tax it, or better yet, legislate it out of existence?[3] Bryant (1980) suggests that the American put option embedded in bank liabilities is a way to insure against unobservable liquidity risk. In short, banking is an efficient risk-sharing arrangement when assets are illiquid, depositors are risk averse, and liquidity preference is private information. But if this is the case, then the solution to this one problem seems to open the door to another. Indeed, the seminal paper by Diamond and Dybvig (1983) on bank runs demonstrates precisely this possibility: Demandable debt as an efficient risk-sharing arrangement is also a source of indeterminacy and financial instability.

Diamond and Dybvig (1983) is most often viewed as a theory of bank runs, but it also offers a prescription for how to *prevent* bank runs for the case in which aggregate risk is absent. The prescription entails embedding bank liabilities with a suspension clause that is triggered when redemptions exceed a specified threshold. This simple fix prevents bank runs.

As Diamond and Dybvig (1983) point out, a full suspension of convertibility—conditional on a threshold level of redemption activity being breached—is not likely to be optimal in the presence of aggregate risk.[4] In the absence of aggregate uncertainty, redemptions exceeding the appropriate threshold constitutes a signal that a run is occurring. With aggregate uncertainty, the optimal redemption schedule

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[1]This sector includes, but is not limited to, structured investment vehicles (SIVs), asset-backed commercial paper (ABCP) conduits, money market funds (MMFs), and markets for repurchase agreements (repos).

[2]If the underlying assets are not illiquid, the demand for maturity transformation would be absent.

[3]This is essentially the recommendation recently put forth by Cochrane (2014).

[4]Diamond and Dybvig (1983) do not actually characterize the optimal contract for the case in which aggregate risk is present.

is state contingent.[5] As a consequence, it is not possible to confirm whether heavy redemptions are driven by fundamentals or by psychology. Threatening full suspension is desirable in the latter case, but not the former.

Our proposed solution to the bank-run problem under aggregate uncertainty is to exploit the idea that while the bank may not know whether a run is on, there are agents in the economy that do. That is, in equilibrium, the beliefs of agents in the economy are consistent with the reality unfolding around them. Can the bank somehow elicit this information in an incentive-compatible manner? If it can, then might the threat of suspensions conditional on such information—and not on withdrawals—serve to eliminate run equilibria?

We provide a positive answer for both these questions, and by so doing depart from the direct mechanism approach usual in the literature. In a direct mechanism, a depositor in the sequential service queue simply requests to withdraw or not. That is, the depositor communicates only his type; impatient if he withdraws or patient if he does not. Our indirect mechanism expands the message space to accommodate additional communications. In this way, we permit a depositor to communicate his belief that a run is on. We can show that the threat of suspension conditional on this communication eliminates the possibility of a run equilibrium.

In practice, such information could be gleaned by introducing a separate financial instrument, the choice of which implicitly reveals what the depositor believes.[6] Our mechanism rewards the depositor for delivering such a message when a run is on. The reward is such that his payoff is higher compared to the payoff associated with concealing his belief that a run is on and making an early withdrawal—that is, misrepresenting his type and running with the other agents. Upon receiving such a message, the mechanism fully suspends all further redemptions. The design of our mechanism ensures that a patient agent never has an incentive to either run when a run is on or announce that he believes a run is on when it is not. At the end of the day, we are able to construct an indirect mechanism that implements the constrained-efficient allocation in iterated elimination of strictly dominated strategies.

Literature Review

A number of papers have studied bank fragility under optimal arrangements in the Diamond and Dybvig (1983) setting. Green and Lin (2003) were the first to characterize an optimal bank contract under private information, sequential service, and aggregate uncertainty. In their version of the Diamond-Dybvig model, the first-best allocation is implementable as a *unique* Bayes-Nash equilibrium of a direct revelation game.

The allocation rule in Green and Lin (2003) allows early withdrawal payments in the sequential service queue to depend on the history of announcements—"I want to withdraw" or "I do not want to withdraw"—and payments to that point. The maximum withdrawal amount faced by an agent in the service queue is lower the larger is the number of preceding withdrawals. This partial suspension scheme is in stark contrast to Diamond and Dybvig (1983), who restrict the maximum withdrawal amount to be insensitive to realized withdrawal demand, so that resources are necessarily exhausted in the event of a run.[7]

Peck and Shell (2003) modify the Green and Lin (2003) environment in at least two important ways. First, they alter the preferences so that incentive-compatibility constraints bind at the optimum. This implies, among other things, that the first-best allocation cannot be implemented. Second, they assume

[5] This property was suggested by Wallace (1988) and later confirmed by Green and Lin (2003).

[6] We elaborate on this in section 8.

[7] Wallace (1990) reports that partial suspensions were prevalent in the banking panic of 1907, and that in one form or another must have been a feature of other suspension episodes as well.

that depositors do not know (or are not told) their position in the service queue. If depositors do not know their queue position, then it is not possible to use backward induction argument of Green and Lin (2003) to eliminate a bank-run equilibrium. It also turns out—and this was not recognized at the time—not revealing queue positions to depositors is part of an optimal mechanism when incentive-compatibility constraints bind.[8] Peck and Shell (2003) use a direct revelation mechanism and demonstrate by example that the optimal direct mechanism can have a bank-run equilibrium.

Ennis and Keister (2009b) modify the Green-Lin environment by assuming the distribution of depositors types is correlated; Green and Lin (2003) assume independence. Using a direct revelation mechanism, Ennis and Keister (2009b) demonstrate that a bank-run equilibrium can exist. But, it is no longer obvious, that a direct revelation mechanism is the "best" mechanism since it does not deliver a uniqueness result. Indeed, Cavalcanti and Monteiro (2011) examine indirect mechanisms in the Ennis and Keister (2009b) environment and demonstrate that the best allocation can be uniquely implemented in dominant strategies. Unfortunately, the backward induction argument implicitly embedded in their mechanism—which is key to their uniqueness proof—will not work in the more general Peck and Shell (2003) environment since depositors do not know their positions in the queue.

There is a mechanism design literature that studies how indirect mechanisms can help to implement optimal outcomes uniquely. Demski and Sappington (1984) examine a principal-two-agent setting where agents separately make production decisions and their costs are private and correlated. The optimal *direct* mechanism has two equilibria: A truth-telling equilibrium and a "cheating" equilibrium, where the cheating equilibrium leaves both agents better off and the principal worse off compared to the truth-telling equilibrium. Ma et al. (1988) shows how an indirect mechanism can prevent agents from misrepresenting their types—or stop agents from cheating—in the Demski and Sappington (1984) model.[9] Mookherjee and Reichelstein (1990) generalizes this approach. Unfortunately, these results cannot be directly applied to the banking problem because sequential service, which is absent in the mechanism design models, complicates the analysis.

The paper is organized as follows. The next section describes the economic environment. Section 3 characterizes the best weakly implementable allocation. In Section 4 we provide a stripped down version of model to illustrative the key features of our mechanism. In Section 5 we construct an indirect mechanism and provide sufficient conditions for unique implementation of the best weakly implementable allocation. In Section 6, we examine examples for which the sufficient conditions are not valid and Section 7 examines an alternative indirect mechanism that addresses these examples. Some policy remarks are offered in the final section.

2. ENVIRONMENT

There are three dates: 0, 1 and 2. The economy is endowed with $Y > 0$ units of date-1 goods. A constant returns to scale investment technology transforms y units of date-1 goods into $yR > y$ units of date-2 goods. There are N *ex-ante* identical agents who turn out to be one of two types: $t \in T = \{1,2\}$. We label a type $t = 1$ agent "impatient" and a type $t = 2$ agent "patient". The number of patient agents in the economy is drawn from the distribution $\pi = (\pi_0, \ldots, \pi_N)$, where $\pi_n > 0$, $n \in \mathbb{N} \equiv \{0,1,\ldots,N\}$, is the probability that there are n patient agents.[10] A *queue* is a vector $t^N = (t_1, \ldots, t_N) \in T^N$, where $t_k \in T$ is the

[8]By not revealing or knowing queue positions, multiple incentive compatibility constraints can be replaced by a single incentive compatibility constraint. As a result, the set of implementable incentive compatible allocations expands.

[9]Postlewaite and Schmeidler (1986) also produced an example where an indirect mechanism has a unique equilibrium yielding the optimal outcome while the corresponding direct mechanisms possess multiple equilibria.

[10]The full support assumption is not crucial to any result. It is imposed only for simplicity.

type of the agent that occupies the k^{th} position/coordinate in the queue. Let $P_n = \{t^N \in T^N | \; n_{t^N} = n\}$ and $Q_n(t^N) = \{j \mid t_j = 2 \text{ for } t^N \in P_n\}$, where n_{t^N} denotes the number of patient agents in the queue t^N. P_n is the set of queues with n patient agents and $Q_n(t^N)$ is the set of queue positions of the n patient agents in queue $t^N \in P_n$.[11] The probability of a queue $t^N \in P_n$ is $\pi_n / \binom{N}{n}$, where the binomial coefficient, $\binom{N}{n}$, is the number of queues $t^N \in P_n$. In other words, all queues with n patient agents are equally likely. Agents are randomly assigned to a queue position, where the unconditional probability that an agent is assigned to position k is $1/N$. Label an agent assigned to position k agent k. The queue realization, t^N, is observed by no one; not by any of the agents nor the planner. Agent k does not observe his queue position, k, but does privately observe his type $t \in T$. The utility function for an impatient agent is $U(c^1, c^2; 1) = u\left(c^1\right)$ and the utility function of a patient agent is $U(c^1, c^2; 2) = \rho u\left(c^1 + c^2\right)$, where c^1 is date-1 consumption and c^2 is date-2 consumption. The function u is increasing, strictly concave and twice continuously differentiable, and $\rho > 0$ is a parameter.[12] Agents maximize expected utility.

The timing of events and actions are as follows. At date 0, the planner constructs a mechanism that determines how date-1 and date-2 consumption are allocated among the N agents. A mechanism consists of a set of announcements, M, and an allocation rule, $c = (c^1, c^2)$, where $c^1 = (c^1_1, \ldots c^1_N)$ and $c^2 = (c^2_1, \ldots c^2_N)$. The planner can commit to the mechanism.[13] The queue t^N is realized at the beginning of date 1. Then agents meet the planner sequentially, starting with agent 1. Each agent k makes an announcement $m_k \in M$.[14] Only agent k and the planner can directly observe m_k. There is a sequential service constraint at date 1, which means the planner allocates date-1 consumption to agent k based on the announcements of agents $j \le k$, (m^{k-1}, m_k), where $m^{k-1} = (m_1, \ldots, m_{k-1})$, and each agent k consumes $c^1_k(m^{k-1}, m_k)$ at his date-1 meeting with the planner. Date 1 ends after all agents meet the planner. In between dates 1 and 2 the planner's resources are augmented by a factor of R. At date 2, the planner allocates the date-2 consumption good to each agent based on the date-1 announcements, i.e., agent k receives $c^2_k(m^N)$, where $m^N = (m_1, \ldots, m_N) \in M^N$. Figure 1 depicts the sequence of actions.

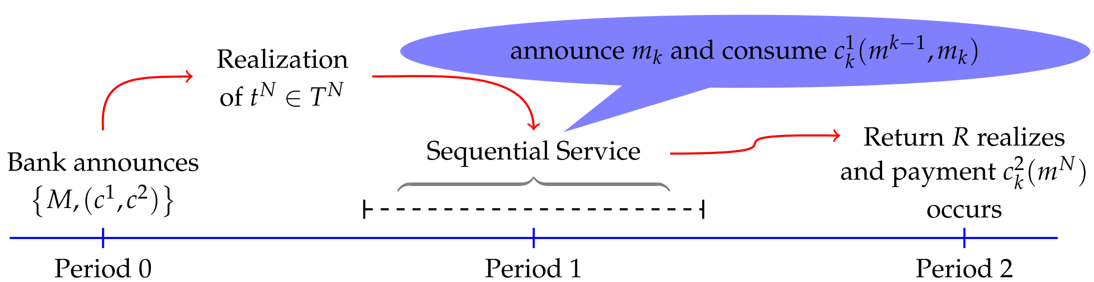

Figure 1: Sequence of Actions.

3. THE BEST WEAKLY IMPLEMENTABLE OUTCOME

An allocation is weakly implementable if it is an equilibrium outcome of a mechanism; it is strongly or uniquely implementable if it is the unique equilibrium outcome of a mechanism. Among the set of weakly implementable allocations, the best weakly implementable allocation provides agents with the

[11] We omit the argument of $Q_n(t^N)$ throughout the paper to keep the notation short.

[12] These preferences are identical to the ones in Diamond and Dybvig (1983). In addition, they assume that $\rho R > 1$ and $\rho \le 1$.

[13] For a discussion of bank fragility in a setting without commitment, see Ennis and Keister (2009a).

[14] One could imagine that the planner makes announcement a_k to agent k before k makes his announcement. For example, the planner could tell agent k his queue position, as in Green and Lin (2003), or the set of all messages sent in the previous $k-1$ planner-agent meetings, as in Andolfatto et al. (2007), or "nothing", $a_k = \emptyset$, as in Peck and Shell (2003). The optimal mechanism, however, will have the planner announcing nothing. To reduce notation, and without loss of generality, we assume that the planner cannot make announcements to agents, unless otherwise specified. See footnote 16 for a discussion.

highest expected utility. To characterize the best weakly implementable allocation, it is without loss of generality to restrict the planner to use a direct revelation mechanism, where agents only announce $m_k = t_k \in T = \{1,2\}$. The welfare—which we measure as the expected utility of an agent before he learns his type—associated with allocation rule c when agents use strategies $m_k \in T$ is

$$(1)\qquad \sum_{n=0}^{N} \frac{\pi_n}{\binom{N}{n}} \sum_{t^N \in P_n} \sum_{k=1}^{N} U\left[c_k^1\left(m^{k-1}, m_k\right), c_k^2\left(m_1^N\right); t_k\right].$$

The allocation rule $c = (c_1, c_2)$ is *feasible* if for all $m^N \in T^N$

$$(2)\qquad \sum_{k=1}^{N} \left[Rc_k^1\left(m^{k-1}, m_k\right) + c_k^2\left(m^M\right)\right] \leq RY.$$

The best weakly implementable allocation has all agents k announcing truthfully, i.e., $m_k = t_k$. Allocation rule c must be incentive compatible in the sense that agent k has no reason to announce $m_k \neq t_k$. Since impatient agents k only value date-1 consumption, they always announce $m_k = 1$.[15] Patient agent k has no incentive to defect from the strategy $m_k = 2$, assuming that all other agents announce truthfully, if

$$(3)\qquad \sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho \left\{ u\left[c_k^1\left(t^{k-1}, 2\right) + c_k^2\left(t^N\right)\right] - u\left[c_k^1\left(t^{k-1}, 1\right) + c_k^2\left(t^{k-1}, 1, t_{k+1}^N\right)\right]\right\} \geq \delta,$$

where, for any vector $x^N = (x_1, \ldots, x_N)$, x_i^j denotes $(x_i, \ldots, x_j)$, $\delta > 0$ is a parameter, and

$$\hat{\pi}_n = \frac{\pi_n / \binom{N}{n}}{\sum_{n=1}^{N} \pi_n / \binom{N}{n}}$$

is the conditional probability that agent k is in a specific queue with n patient agents.[16] The $1/n$ term that appears in (3) reflects that a patient agent has a $1/n$ chance of occupying each of the patient queue positions in Q_n.

The best weakly implementable allocation is given by the solution to

$$(4)\qquad \max\ (1) \text{ subject to } (2) \text{ and } (3),$$

where $m_k = t_k$ for all $k \in \mathbb{N}$. We restrict $\delta > 0$ to those values that admit a solution to problem (4). Let $c^*(\delta) = \left(c^{1*}(\delta), c^{2*}(\delta)\right)$ be a solution to problem (4) and let $W^*(\delta)$ denotes its maximum. We consider $\delta > 0$ to guarantee that the incentive compatibility holds in an open neighbourhood of c^*. The existence of such neighbourhood is necessary for our uniqueness result but $\delta > 0$ can be made arbitrarily small. Therefore, we can apply Berge's maximum theorem, which says that $W^*(0)$ is approximated by $W^*(\delta)$ when δ is close to zero. The allocation rule $c^*(\delta)$ has the following features: (i) an agent k who announces $m_k = 1$ consumes only at date 1, that is, $c_k^{2*}(m_1, \ldots, m_{k-1}, 1, m_{k+1}, \ldots, m_N) = 0$ for all $k \in \mathbb{N}$; (ii) an agent k who announces $m_k = 2$ consumes only at date 2, that is, $c_k^{1*}(m_1, \ldots, m_{k-1}, 2) = 0$ for all $k \in \mathbb{N}$; and (iii) all

[15] This anticipates the result that the best weakly implementable allocation provides zero date-1 consumption to agents who announce that they are patient, which implies that the incentive compatibility constraint for impatient agents is always slack.

[16] To characterize the best weakly implementable allocation, one wants to choose from the largest possible set of incentive compatible allocations. This implies the planner should not make any announcements, as noted in footnote 14. In particular, if the planner does not make any announcements, then there is only one incentive compatibility constraint for all patient agents, (3). If, however, the planner did make an announcement a_k to agent k, there will be additional incentive constraints for the agent who received the announcement. For example, suppose that $a_k = k$ for all k, i.e., the planner announces to each agent his place in the queue. Then there would be N incentive compatibility constraints for patient agents, one for each queue position. Since an appropriately weighted average of these distinct incentive constraints implies (3), the set of incentive compatible allocations when the planner makes announcements is a subset of the set of incentive compatible allocations when he does not. By not making any announcements, the planner is able to choose from a larger set of incentive feasible allocations.

agents j and k announcing $m_j = m_k = 2$ consume identical amounts at date 2, that is, $c_j^{2*}(m^N) = c_k^2(m^N)$ for all $m_j = m_k = 2$. The best-weakly implementable allocation is $c^*(0)$, which corresponds to the allocation rule derived in Peck and Shell's (2003) Appendix B.

Define a bank run as a non-truthtelling equilibrium for the mechanism $\{M, c\}$, where some and possibly all $k \in Q_n(t^N)$ announce $m_k = 1$. Both Peck and Shell (2003) and Ennis and Keister (2009b) demonstrate, by example, that the direct mechanism $\{T, c^*(0)\}$ can have two equilibria: one where agents play truth-telling strategies, $m_k = t_k$ for all k, and another where all patient agents k play bank-run strategies, $m_k = 1$.[17] We claim that bank-run equilibria arise in these examples because the *direct* revelation mechanism they use, $\{T, c^*(0)\}$, is not optimal in the sense that there exists a different mechanism which strongly implements the best weakly implementable allocation. Before we demonstrate this result, we provide a simple example that illustrates the basic intuition underling our optimal mechanism.

4. A SIMPLE EXAMPLE

Consider a stripped-down version of a Diamond-Dybvig model where there are only 2 agents—column and row—and both agents are patient. Agents simultaneously announce that they are either patient, $m = 2$, or impatient, $m = 1$. The payoffs to agents for this game are given by

	$m = 1$	$m = 2$
$m = 1$	1 , 1	2 , 0
$m = 2$	0 , 2	3 , 3

This simple normal form game captures two important insights of the Diamond-Dybvig model. First, there are multiple equilibria: one where both agents announce the truth, $m = 2$, one where both agents announce they are impatient, $m = 1$, and another where both agents randomize between each strategy with probability half. And second, the truth-telling equilibrium generates the higher payoffs for agents than a bank-run equilibrium.

Consider now a normal form game that simply augments the announcement space of the original game from $\{1,2\}$ to $\{1,2,g\}$, with associated payoffs

	$m = 1$	$m = 2$	$m = g$
$m = 1$	1 , 1	2 , 0	$0 , 1 + \epsilon$
$m = 2$	0 , 2	3 , 3	$3 , 2 + \epsilon$
$m = g$	$1 + \epsilon , 0$	$2 + \epsilon , 3$	ϵ , ϵ

There are three features of the augmented game that we would like to highlight. First, when agents restrict their announcements to $\{1,2\}$, the payoffs they receive are identical to the original game. Second, announcement $m = g$ strictly dominates announcement $m = 1$. And finally, the payoff to an agent who announces $m = 2$ is the same regardless if his opponent announces $m = 2$ or $m = g$.

[17] The Ennis and Keister (2009b) bank-run example is in section 4.2 of their paper. There, agents do not know their position in the queue, as in Peck and Shell (2003), and the utility functions of patient and impatient agents are the same, $\rho = 1$, as in Green and Lin (2003).

Since agents never play $m=1$ in the augmented game—it is strictly dominated by playing $m=g$—the relevant augmented game that agents play is

	$m=2$	$m=g$
$m=2$	$3\,,\,3$	$2\,,\,0$
$m=g$	$2+\epsilon\,,\,3$	$\epsilon\,,\,\epsilon$

But in this relevant augmented game, announcement $m=g$ is strictly dominated by announcement $m=2$. Therefore, the unique iterated strict dominant equilibrium to the augmented game is one of truthtelling, $m=2$. Hence, by modifying the game that agents play, we get rid of the "bad" bank-run equilibria that existed in the original game.

The best weakly implementable allocation described in Section 3, $c^*(\delta)$, is somewhat more complicated than the payoff structure in the stripped-down example. Nevertheless, our approach to eliminate the bad equilibria is the same: We construct an indirect mechanism $\{\hat{M},\hat{c}\}$ with the properties: (i) $\hat{M}=\{1,2,g\}$; (ii) announcing $\hat{m}_k=1$ is strictly dominated by announcing $\hat{m}_k=g$ for patient agents; and (iii) after announcement $\hat{m}_k=1$ is eliminated for patient agents announcing $\hat{m}_k=2$ strictly dominates announcing $m_k=g$. The uniqueness result is a bit more tricky to prove because we need enough resources to construct an allocation rule $\hat{c}$ that provides sufficiently high payoffs to patient agents so that announcing truthfully is the unique rational strategy. In the subsequent section, we characterize an indirect mechanism and provide sufficient conditions under which this mechanism uniquely implements the best weakly implementable allocation using dominance arguments similar to the simple example.

5. AN INDIRECT MECHANISM

Consider an indirect mechanism $\{\hat{M},\hat{c}\}$, where $\hat{M}=\{1,2,g\}$ and $\hat{c}$ is described below. The basic construction of the allocation rule $\hat{c}$ uses $c^*(\delta)$. If agent j announces $\hat{m}_k=1$, then

$$(5)\qquad \hat{c}_k^1\left(\hat{m}^{k-1},1\right)=\begin{cases} c_k^{1*}(\delta)\left(\hat{m}^{k-1},1\right) & \text{if } \ \hat{m}_j\in\{1,2\} \text{ for all } j<k \\ 0 & \text{if } \ \hat{m}_j=g \text{ for some } j<k \end{cases} \quad \text{and} \quad \hat{c}_k^2\left(\hat{m}^{k-1},1,\hat{m}_{k+1}^N\right)=0.$$

An agent k announcing $\hat{m}_k=1$ receives the date-1 consumption payoff under the direct revelation mechanism $\{T,c^*(\delta)\}$ *only if all earlier agents $j<k$ announce either $\hat{m}_j=1$ or $\hat{m}_j=2$*; otherwise he receives zero. That is, there is a suspension of first period payments after an agent $j<k$ announces $\hat{m}_j=g$. The date-2 consumption payoff associated with the announcement $\hat{m}_k=1$ is zero, as in the direct revelation mechanism $\{T,c^*(\delta)\}$. If agent k announces $\hat{m}_k=g$, then

$$(6)\qquad \hat{c}_k^1\left(\hat{m}^{k-1},g\right)=0 \quad \text{and} \quad \hat{c}_k^2\left(\hat{m}^{k-1},g,\hat{m}_{k+1}^N\right)=\hat{c}_k^1\left(\hat{m}^{k-1},1\right)+\epsilon,$$

where $\epsilon>0$ is an arbitrarily small number. To keep the presentation simple, we assume throughout the paper that ϵ is taken small enough so all results hold. If agent k announces $\hat{m}_k=g$, then he receives a zero payoff at date 1. At date 2, he receives a payoff that is slightly bigger than the date-1 payoff he would receive by announcing $\hat{m}_k=1$, see (5), which implies that $\hat{c}_k^2(\hat{m}^{k-1},g,\hat{m}_{k+1}^N)=\hat{c}_k^1(\hat{m}^{k-1},1)+\epsilon$. Hence, announcing $\hat{m}_k=g$ strictly dominates announcing $\hat{m}_k=1$ for any patient agent k. Finally, if agent k

announces $\hat{m}_k = 2$, then

$$\text{(7)} \quad \hat{c}_k^1\left(\hat{m}^{k-1},2\right) = 0 \quad \text{and} \quad \hat{c}_k^2\left(\hat{m}^{k-1},2,\hat{m}_{k+1}^N\right) = \frac{R\left[Y - \sum_{j=1}^N \hat{c}_j^1(\hat{m}^j)\right] - \sum_{j=1}^N \hat{c}_j^2(\hat{m}^N)\mathbb{1}_{\hat{m}_j=g}}{n_{\hat{m}^N}}$$

where $n_{\hat{m}^N}$ represents the number of agents who announced $\hat{m} = 2$ in the announcement vector $\hat{m}^N$ and $\mathbb{1}_{\hat{m}_j=g}$ is an indicator function, where $\mathbb{1}_{\hat{m}_j=g} = 1$ if $m_j = 1$ and 0 otherwise. If agent k announces $\hat{m}_k = 2$, then he receives a $1/n_{\hat{m}^N}$ share of the total date-2 output that remains after payments to agents j who announced either $m_j = 1$ or $m_j = g$ are made. Since the allocation rule $\hat{c}$, given by (5)-(7), depends on δ and ϵ, we will denote it as $\hat{c}\,(\delta,\epsilon)$.

Generally speaking, a patient agent j who announces $m_j = 1$ adversely affects the payoffs of truthfully announcing patient agents in two ways. First, the payments to an agent who announces $m_j = 1$ are made in period 1 which implies that these resources cannot benefit from the investment opportunity, R, available between dates 1 and date 2. Second, if impatient agents have a relatively high marginal utility of consumption compared to patient agents, *i.e.*, ρ is small, then, due to risk-sharing considerations, payments to agents who announce $m_j = 1$ can be quite high, leading to less resources available to the patient agents. Interestingly, the story is a bit different when patient agent j announces $\hat{m}_j = g$ and impatient agents have a relatively low marginal utility of consumption compared to patient agents. Following a g announcement there is a suspension of date 1 payments and agents who announce g receive their payments at date 2. Hence, all suspended payments benefit from the investment opportunity that is available between dates 1 and 2, and patient agents who announced truthfully will receive a fraction of the investment return, R. In addition, if ρ is relatively large, then the date-2 payment to agent j will be relatively low, which benefits truth-telling patient agents.

Patient agent k who announces truthfully will benefit from a $m_j = g$ if allocation rule $\hat{c}(\delta,\epsilon)$ has the following property

$$\text{(P1)} \quad \hat{c}_k^2(\delta,\epsilon)(\hat{m}^{k-1},2,\hat{m}_{k+1}^N) \geq \hat{c}_k^2(\delta,\epsilon)(\hat{t}^{k-1},2,\hat{t}_{k+1}^N) = c_k^{2*}(\delta)(\hat{t}^{k-1},2,\hat{t}_{k+1}^N),$$

where $\hat{t}^i \in T^i$ ($\hat{t}_i^N \in T_i^N$) is a vector of length i ($T - i$) such that for each $j \leq i$ ($i \leq j \leq N$), $\hat{t}_j = 1$ if $\hat{m}_j = 1$ and $\hat{t}_j = 2$ if either $\hat{m}_j = 2$ or $\hat{m}_j = g$. In words, vector $\hat{t}^i$ ($\hat{t}_i^N$) is constructed from the message vector $\hat{m}^i$ ($\hat{m}_i^N$) by replacing all of the g's with 2's. The first term in (P1) is the payoff to a truthfully announcing patient agent when some (patient) agents announce g. The second term is the payoff to patient players when those g announcements are replaced by 2, which, by construction, also equals the payment from the best implementable allocation. If the contract $\hat{c}(\delta,\epsilon)$ is characterized by property (P1), then, clearly, a truthfully announcing patient agent benefits if some other (patient) agent announces g. In fact, his payoff will exceed that associated with the best weakly implementable allocation, $c^*(\delta)$.

Under what circumstances will the allocation rule $\hat{c}(\delta,\epsilon)$ have property (P1)? The above discussion suggests that truthfully announcing patient agents benefit from a $m_j = g$ announcement the larger is R and/or the larger is ρ. (Recall that the higher is ρ, the smaller will be the payments to impatient agents.) Our first proposition verifies this intuition.

PROPOSITION 1: *If $\rho R > 1$, then property (P1) holds.*

PROOF: See Appendix. □

Property (P1) seems to imply that, since more resources are available to patient players who announce truthfully and less to patient players who announce g, it is rational for patient players to announce truthfully. Our main proposition demonstrates that this intuition is, in fact, correct.

PROPOSITION 2: *If property (P1) holds, then the indirect mechanism $\{\hat{M},\hat{c}(\delta,\epsilon)\}$ strongly implements allocation $c^*(\delta)$ in rationalizable strategies.*

PROOF: The mechanism $\{\hat{M},\hat{c}(\delta,\epsilon)\}$ induces a symmetric Bayesian game $\Gamma = \{T,S\}$ where, $T = \{1,2\}$ is the set of types, $s_t \in \hat{M}$ is the player's message contingent on his type $t \in T$ and $S = \{(s_1,s_2) \in \hat{M}^2\}$ is the set of pure strategies. We solve the game by iterated elimination of strictly dominated strategies in two rounds.

Round 1 **-** Any strategy $(s_1,s_2) \in S$, with $s_1 \neq 1$, is strictly dominated by $(1,s_2)$ since, contingent on being impatient, an agent only derives utility from period 1 consumption. Additionally, any strategy $(s_1,1)$ is strictly dominated by (s_1,g) since, contingent on being patient, agents are indifferent between period 1 or period 2 consumption and announcing g always gives a total payment that is ϵ higher than announcing 1. Let $S^1 = \{(1,2),(1,g)\}$ denote the set of strategies that survive the first round of elimination of strictly dominated strategies.

Round 2 **-** When strategies are restricted to S^1, impatient agents announce 1 and patient agents announce either 2 or g. From property (P1), the lower bound on the expected payoff to a patient player who announces 2 is

$$\sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u \left(c_k^{2*}(\delta)(t^{k-1},2,t_{k+1}^N) \right).$$

Since the payment to agent k who announces $m_k = g$ is either $c_k^{1*}\left(t^{k-1},1\right) + \epsilon$ or ϵ, the expected payoff to a patient player who announces g is bounded above by

$$\sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u \left(c_k^{1*}(\delta)\left(t^{k-1},1\right) + \epsilon \right).$$

Since u is continuous, there exists an $\epsilon > 0$ sufficiently small so that

$$\sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \left\{ \rho u \left(c_k^{1*}(\delta)\left(t^{k-1},1\right) + \epsilon \right) - \rho u \left(c_k^{1*}(\delta)\left(t^{k-1},1\right) \right) \right\} < \delta.$$

The incentive compatibility condition (3) can be rewritten as

$$\sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u \left(c_k^{2*}(\delta)\left(t^{k-1},2,t_{k+1}^N\right) \right) \geq \sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u \left(c_k^{1*}(\delta)\left(t^{k-1},1\right) \right) + \delta.$$

Combining the above two inequalities, we get

$$\sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u \left(c_k^{2*}(\delta)\left(t^{k-1},2,t_{k+1}^N\right) \right) > \sum_{n=1}^{N} \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} \rho u \left(c_k^{1*}(\delta)\left(t^{k-1},1\right) + \epsilon \right). \tag{8}$$

Therefore, the strategy $(1,g)$ is strictly dominated by the strategy $(1,2)$ in S^1. Let S^2 be the set of strategies that survive the second round of elimination of strictly dominated strategies. Since $S^2 = \{(1,2)\}$ is a singleton, the game is iterated strict dominance solvable. The unique equilibrium strategy is the truth-telling $s = (1,2)$, which implies the same outcome as the truth-telling equilibrium of the direct mechanism $\{T,c^*(\delta)\}$. □

If allocation $\hat{c}(\delta,\epsilon)$ has property (P1), then, just as in the stripped-down example from Section 4, mechanism $\{\hat{M},\hat{c}(\delta,\epsilon)\}$ admits only one equilibrium characterized by truthtelling for all agents. Hence, mechanism $\{\hat{M},\hat{c}\left(\delta,\epsilon\right)\}$ does not allow bank runs. In addition, the allocation delivered by the mechanism, $\hat{c}\left(\delta,\epsilon\right)$, can be made arbitrarily close to the best weakly implementable allocation $c^*\left(0\right)$ by choosing δ

arbitrarily close to zero.

Together, Propositions 1 and 2 imply that a sufficient condition for unique implementation is $\rho R > 1$. This is quite interesting and, perhaps, even remarkable. Diamond and Dybvig (1983) construct a model where fractional reserve banks endogenously arise and use the model to help us understand the notion that banks are inherently *unstable*. Their 1983 article requires that $\rho R > 1$. Propositions 1 and 2 in this article, however, indicates that for this parametrization banks are always *stable*.

We want to emphasize that conditions stated in Propositions 1 and 2 are only sufficient conditions. Regarding Proposition 1, one can see from the proof that if incentive compatibility condition (3) does not bind, then condition the $\rho R > 1$ is *not* necessary. This means that contract $\hat{c}(\delta,\epsilon)$ can be consistent with property (P1) even if $\rho R < 1$. In the subsequent section, we provide an example of this (even when the incentive compatibility condition (3) binds). Regarding Proposition 2, property (P1) allows us to derive a lower bound on the expected payoff of a patient agent announcing $m = 2$ and, therefore, to use dominance arguments to demonstrate uniqueness. But neither, such lower bound or dominance arguments, are necessary for uniqueness. In the subsequent section we provide an example where contract allocation $\hat{c}(\delta,\epsilon)$ does not have property (P1) but the indirect mechanism $\{\hat{M},\hat{c}(\delta,\epsilon)\}$ uniquely implements $\hat{c}(\delta,\epsilon)$.

6. SOME EXAMPLES

In this section we provide some examples that show the sufficient conditions described in Propositions 1 and 2 are not necessary for unique implementation of the allocation rule $c^*(\delta)$. The first example shows that property (P1) can hold when $\rho R < 1$. A second example shows that allocation rule $c^*(\delta)$ can be uniquely implemented when property (P1) is violated.

Common to all examples are: (i) $R = 1.05$; (ii) $Y = 6$; (iii) $\rho R < 1$; (iv) $\delta = 10^{-10}$; and (v) the general structure of preferences is given by [18]

$$u(x) = \frac{(x+1)^{1-\gamma} - 1}{1-\gamma}, \ \gamma > 1. \tag{9}$$

In the first example, $N = 2$, $\rho = 0.9$, $\gamma = 1.01$ and $(\pi_0, \pi_1, \pi_2) = (0.005, 0.4975, 0.4975)$. Notice that $\rho R < 1$. The best weakly implementable allocation, $c^*(0)$, which is obtained by solving (4), has $c_1^{*1}(1) = 3.1487$ and $c_2^{*1}(2,1) = 3.1481$. The other payments can be derived from the resource constraint (2) holding at equality. It is straight forward to show that the direct mechanism $\{T, c^*(0)\}$ admits a bank-run equilibrium for this example. For ϵ arbitrarily small, property (P1) holds, even though $\rho R < 1$. Therefore, although $\rho R > 1$ is a sufficient condition for property (P1), it is not a necessary one. Since property (P1) is satisfied in this example, Proposition 2 implies that $\{\hat{M},\hat{c}(\delta,\epsilon)\}$ uniquely implements allocation $c^*(\delta)$ for δ and ϵ small. In this example, constraint (3) binds. This implies that incentive constraints in the Green and Lin (2003) environment—where agents know their queue positions—will also bind and that the best implementable allocation from that environment is not equal to $c^*(0)$.[19] This implies that the Green and Lin (2003) mechanism is unable to even weakly implement the allocation $c^*(\delta)$, where δ is arbitrarily small.

The second example replicates the Peck and Shell (2003) example in Appendix B. The only difference between the examples is the specification of preferences. Peck and Shell (2003) assume that $u(x) = c^{1-\gamma}/(1-\gamma)$, which implies that $u(0) = -\infty$. For these preferences, our mechanism trivially uniquely implements allocation $c^*(\delta)$, since patient agent k will never announce $m_k = g$ if there is a probability,

[18] Notice that $u(0) = 0$.

[19] Our environment can be turned into the Green and Lin (2003) environment by allowing the planner to tell agent k his queue position, k, before agent k makes sends his message.

however small, that some other agent j will announce $m_j = g$. The parameters for our second example are $N = 2$, $\rho = 0.1$, $\gamma = 2$ and $(\pi_0, \pi_1, \pi_2) = (0.25, 0.5, 0.25)$. Notice that $\rho R < 1$. The best weakly implementable allocation, $c^*(0)$, is characterized by $c_1^{*1}(1) = 3.0951$ and $c_2^{*1}(2,1) = 3.1994$. Allocation $c^*(0)$ features bank runs and a binding incentive constraint (3). (This implies that a Green and Lin (2003) mechanism cannot weakly implement $c^*(0)$.) It is straightforward to demonstrate that the mechanism $\{\hat{M}, \hat{c}(\delta, \epsilon)\}$ uniquely implements allocation $c^*(\delta)$, for δ and ϵ arbitrarily close to zero. For this example $c_1^{*1}(2,1) + c^{*2}(2,2) > RY$, which implies that (P1) is not satisfied for all $\hat{m}^N \in \hat{M}^N$. Hence, property (P1) is not necessary for unique implementation. We are not aware of any mechanism in the literature that can implement the best weakly implementable allocations from these two examples. We have experimented with many combinations of model parameters. We are unable to find a set of parameters for which the indirect mechanism $\{\hat{M}, \hat{c}(\delta, \epsilon)\}$ *cannot* uniquely implement an allocation that is arbitrarily close to the best weakly implementable allocation. Our search, however, was restricted to $N \in \{2, 3\}$. It is, of course, possible that the indirect mechanism $\{\hat{M}, \hat{c}(\delta, \epsilon)\}$ does not uniquely implement the best weakly implementable allocation for some set of parameters—that we were unable to recover—when $\rho R \leq 1$. In the next section, we propose an alternative indirect mechanism to deal with this case.

7. AN ALTERNATIVE MECHANISM

The indirect mechanism $\{\hat{M}, \hat{c}(\delta, \epsilon)\}$ uniquely implements allocation $c^*(\delta)$ for the $N \in \{2, 3\}$ examples we considered, but there may exist primitives for which it does *not*. To address this issue, we construct an alternative mechanism that uniquely implements $c^*(\delta)$ in pure and symmetric strategies. The mechanism, however, does rule out the existence of mixed or asymmetric Nash equilibria.[20]

The alternative indirect mechanism is denoted by $\{\tilde{M}, \tilde{c}\}$, where $\tilde{M} = \{1, 2, g\}$ and $\tilde{c}$ is described below. For a given $\tilde{m}^{k-1} \in \tilde{M}^{k-1}$, define $\tilde{t}^{k-1} \in T^{k-1}$ as a vector of length $k-1$, where for each $j \leq k-1$, $\tilde{t}_j = 1$ if either $\tilde{m}_j = 1$ or $\tilde{m} = g$; and $\tilde{t}_j = 2$ if $\tilde{m}_j = 2$. It is important to emphasize that the relationship between $\tilde{m}_j$ and $\tilde{t}_j$ is different from that of $\hat{m}_j$ and $\hat{t}_j$. Specifically, the vector $\tilde{t}^{k-1}$ is constructed from $\tilde{m}^{k-1}$ by replacing any g's with 1's, while vector $\hat{t}^{k-1}$ is constructed from $\hat{m}^{k-1}$ by replacing any g's with 2's.

The construction of the allocation rule $\tilde{c}$ uses the best weakly implementable allocation rule, $c^*(0)$. If agent k announces $\tilde{m}_k = 1$, then

$$(10) \quad \tilde{c}_k^1(\tilde{m}^{k-1}, 1) = c_k^{1*}(0)\left(\tilde{t}^{j-1}, 1\right) \quad \text{and} \quad \tilde{c}_j^2(\tilde{m}^{k-1}, 1, \tilde{m}_{k+1}^N) = 0.$$

When agent k announces $\tilde{m}_k = 1$ he receives the consumption associated with announcing $m_k = 1$ in the direct revelation mechanism $\{T, c^*(0)\}$, where announcement $\tilde{m}_j = g$ in the indirect mechanism is treated as if it is $\tilde{m}_j = 1$. If agent k announces $\tilde{m}_k = g$, then

$$(11) \quad \tilde{c}_k^1(\tilde{m}^{k-1}, g) = 0 \quad \text{and} \quad \tilde{c}_k^2(\tilde{m}^{k-1}, g, \tilde{m}_{k+1}^N) = \begin{cases} c_k^{1*}(0)\left(\tilde{t}^{k-1}, 1\right) + \epsilon & \text{if} \quad \tilde{m}_j = 1 \text{ for all } j \neq k \\ 0 & \text{otherwise} \end{cases},$$

where $\epsilon > 0$ is arbitrarily small. If agent k announces $\tilde{m}_k = g$, he receives a zero date 1 payoff. His date 2 payoff is slightly bigger than what he would receive by announcing $\tilde{m}_k = 1$ but only if *all* other agents j announce $m_j = 1$; otherwise, he receives a payoff of zero. Finally, if agent k announces $\tilde{m}_k = 2$, then

$$(12) \quad \tilde{c}_k^1\left(\tilde{m}^{k-1}, 2\right) = 0 \quad \text{and} \quad \tilde{c}_k^2\left(\tilde{m}^{k-1}, 2, \tilde{m}_{k+1}^N\right) = \frac{R\left[Y - \sum_{j=1}^N \tilde{c}_j^1(\tilde{m}^k)\right] - \sum_{j=1}^N \tilde{c}_j^2(\tilde{m}^N)\mathbb{1}_{\tilde{m}_j = g}}{n_{\tilde{m}^N}},$$

[20] Mechanism $\{\hat{M}, \hat{c}(\delta)\}$ *does* rule out these equilibria when $\rho R > 1$. It is interesting to note, however, that in the literature virtually all of the analyses of the Diamond-Dybvig model focus on pure and symmetric equilibria.

where $n_{\tilde{m}^N}$ represents the number of agents j who announced $\tilde{m}_j = 2$. If agent k announces $\tilde{m}_k = 2$, then he receives an equal share of date-2 output net of any payments made to agent j who announce $\tilde{m}_j = g$. Since the allocation rule $\tilde{c}$ given by (10)–(12) depends on δ, we will denote it as $\tilde{c}(\varepsilon)$.

When considering only pure and symmetric equilibria, the indirect mechanism $\{\tilde{M}, \tilde{c}(\varepsilon)\}$ is quite powerful. Specifically,

PROPOSITION 3: *The indirect mechanism $\{\tilde{M}, \tilde{c}(\varepsilon)\}$ uniquely implements the best weakly implementable allocation $c^*(0)$ in pure and symmetric Nash equilibrium.*

PROOF: All impatient agents k announce truthfully since announcing $\tilde{m}_k = 1$ results in a strictly positive date-1 payoff and announcing $\tilde{m}_k \neq 1$ results in a date-1 payoff equal to zero.

First, there cannot exist an equilibrium where all patient agents k announce $\tilde{m}_k = 1$. Suppose such an equilibrium exists. Then some patient agent j can defect from proposed equilibrium and announce $\tilde{m}_j = g$. Agent j's payoff is strictly greater than the payoff associated with announcing $\tilde{m}_j = 1$ by the amount $\epsilon > 0$; a contradiction.

Second, there cannot be an equilibrium where all patient players k announce $\tilde{m}_k = g$. To see this, note that if agent k announces $\tilde{m}_k = g$, then his payoff will be zero if there are other patient agents in the economy. The (proposed) equilibrium payoff, therefore, is

$$\frac{\hat{\pi}_1}{N} \sum_{k=1}^{N} \rho u \left[c_k^{1*}(0) \left(1^{k-1}, 1 \right) + \varepsilon \right] + (1 - \hat{\pi}_1) \rho u(0). \tag{13}$$

If instead, agent k defects from proposed play announces $\tilde{m}_k = 1$, his payoff will be

$$\frac{1}{N} \sum_{k=1}^{N} \rho u \left[c_k^{1*}(0) \left(1^{k-1}, 1 \right) \right]. \tag{14}$$

Since $\hat{\pi}_1 < 1$, for $\epsilon > 0$ sufficiently small (14) exceeds (13); a contradiction.

Third, there is an equilibrium where all patient agents k announce $\tilde{m}_k = 2$. By construction, patient agent j has no incentive to announce $\tilde{m}_j = 1$ when all other agents announce truthfully, i.e., allocation rule $c^*(0)$ is incentive compatible for patient agents when $\tilde{m}_j$ is restricted to the set $\{1, 2\}$. Suppose, instead, that patient agent j defects from equilibrium play and announces $\tilde{m}_j = g$. In this case, his payoff will be only slightly greater than the payoff associated with announcing $\tilde{m}_j = 1$ *if and only if he is the only patient agent in the economy*—an event that occurs with probability $\hat{\pi}_1$. With probably $1 - \hat{\pi}_1$, there are other patient agents k who announce $\tilde{m}_k = 2$, which implies that patient agent j receives a zero payoff. For any $\hat{\pi}_1 < 1$, there exists an $\varepsilon > 0$ sufficiently small so that the expected payoff associated with announcing $\tilde{m}_j = g$ is strictly less than that associated with announcing $\tilde{m}_j = 1$, when all other agents announce truthfully.

The unique symmetric and pure equilibrium strategy for mechanism $\{\tilde{M}, \tilde{c}\}$ is characterized by truth-telling, i.e., $\tilde{m}_k = t_k$ for all k. By construction, these strategies implement the best weakly implementable allocation in $c^*(0)$. □

There is an interesting tradeoff between the two indirect mechanisms that have been studied. Mechanism $\{\hat{M}, \hat{c}\}$ has a very weak equilibrium concept, rationalizability. However, unique implementation is guaranteed only if the restriction $\rho R > 1$ is satisfied. Unique implementation is possible when $\rho R \leq 1$, as our examples demonstrate, but it has to be verified on a case-by-case basis. Mechanism $\{\tilde{M}, \tilde{c}\}$ has a very strong equilibrium concept, pure and symmetric Nash equilibria. However, no restriction are required on model parameters to guarantee unique implementation. Unique implementation is possible for when mix strategies are allowed, but it has to be verified on a case-by-case basis.

Finally, the indirect mechanism $(\hat{M}, \hat{c})$ relies on both punishments and suspension for unique implementation. Since strategies are restricted to be pure and symmetric, indirect mechanism $(\tilde{M}, \tilde{c})$ only relies on punishments for unique implementation.

8. POLICY DISCUSSION

The most common prescription for enhancing the stability of demandable debt is to modify the contract to include a partial suspension clause. For example, Cochrane (2014), suggests that if securities are designed so debtors have the right to delay payment, suspend convertibility, or pay in part, then it is much harder for a run to develop. Santos and Neftci (2003) recommend the use of extendable debt—which is a suspension in payments—in the sovereign debt market to help mitigate the frequent debt crises that have afflicted emerging economies and, recently, more advanced economies as well. In June 2013, the Securities and Exchange Commission (SEC) announced a set of proposals to help stabilize money market funds (MMFs). One of the key proposals recommends that the MMF board of directors have the discretion to impose of penalty redemption fees and redemption gates—or suspsension of payments—in times of heavy redemption activity.

The effect of such proposals is to render demandable debt more state-contingent. In this sense, the proposals above are consistent with the properties of the optimal debt contracts described in Diamond and Dybvig (1983), Green and Lin (2003), and Peck and Shell (2003). But given that bank-run equilibria remain a possibility in the latter model, one is led to question whether the the use of such measures constitute only necessary, and not sufficient conditions, for stability.

The key question concerns the issue of precisely *what* information is be used to condition the suspension/extension clause. In the Diamond and Dybvig (1983) model without aggregate risk, suspension is triggered when "reserves" are reach a well-specified critical level. Evidently, this conditioning factor is sufficient to prevent runs in that environment. Similarly, the partial suspension schedules described in Green and Lin (2003) and Peck and Shell (2003) are triggered by measures of reserve depletion (more precisely, the history of reported types). In reality, the volatility of redemption rates varies across different classes of MMFs. Schmidt et al. (2013), for example, report that MMFs with volatile flow rates prior to the financial crisis of 2008 were more likely to experience runs during the crisis. How are directors of these funds to ascertain whether a spike in redemptions is attributable to fear rather than fundamentals? Our indirect mechanism suggests that information beyond some measure of redemption activity or resource availability is needed to prevent the possibility of a bank-run. We need to know *why* depositors are exercising their redemption option. For better or worse, this information is private and must therefore be elicited directly—as in our model—or inferred indirectly—through some other means. Of course, information revelation must be incentive compatible.

Just how realistic is this idea? There is, in fact, historical precedence for the practice of soliciting additional information in periods of heavy redemption activity. For example, banks would sometimes permit limited redemptions to occur for depositors that could demonstrate evidence of impatience, e.g., a need to meet payroll. Gorton (1985, fn 7) reports that 19th century clearinghouses would regularly investigate rumors pertaining to the financial health of member banks.

As a practical matter, the spirit of our mechanism could be implemented in several different ways. One way would be to permit depositors to pay a small fee for the right to have their funds diverted to a segregated, priority account.[21] Such an action could be interpreted as a communication of an impending run. The priority debt differs from other debt only in the event of failure and the ratio of priority to

[21] This is effectively what happens in our mechanism when a depositor reports $m = g$.

non-priority debt outstanding informs the issuer on the degree to which depositors expect the bank to fail. In principle, the suspension clause could be made conditional on this ratio hitting some specified threshold. It does not need to be official as long as there is a mutual understanding that it will be used. And along the lines suggested by our mechanism, if one knows that the bank will suspend before any rumor-induced trouble affects their balance sheet, then depositors know that there will be no reason, in equilibrium, to actually exercise the option of converting their claims to priority debt.

To summarize, current policy proposals designed to prevent, or at least mitigate, bank runs in demandable debt structures focus on enhancing state-contingency, with contingencies dictated by some measure of redemption activity or resource depletion. Our analysis suggests that while state contingency is necessary, it may not be sufficient to prevent bank runs. Suspension clauses should be conditioned on information relating to depositor beliefs about what they perceive to be happening around them. The desired information could be elicited in an incentive compatible manner through an appropriate modification of the deposit contract—an example of which we described above. If we are wrong in our present assessment, the inclusion of such a clause would be inconsequential. But if we are correct, then the inclusion of such a clause may help to prevent bank runs in debt structures that are presently run prone.

APPENDIX: PROOF OF PROPOSITION 1

In order to prove proposition 1 we first establish the following result.

LEMMA 1: *If* $\rho R > 1$ *then* $c_k^{1*}(\delta)(\bar{t}^{k-1}, 1) < c^{2*}(\delta)(\bar{t}^{k-1}, \mathbf{2}^{N-k+1})$ *for all* $k \in \mathbb{N}$ *and* $\bar{t}^{k-1} \in T^{k-1}$. *Where* $\mathbf{2}^n$ *denotes the* $n-$*dimensional vector of twos.*

PROOF: Since $c^*(\delta)$ solves problem (4), it satisfies the implied first-order conditions.[22] Let λ_{t^N} denote the Lagrange multiplier of the feasibility constraint (2) for each $t^N \in T^N$ and μ denotes the Lagrange multiplier of the incentive compatibility (3). By simplicity, λ_{t^N} is normalized by $\pi_{n_{t^N}}/\binom{N}{n_{t^N}}$, where n_{t^N} denotes the number of type 2 players in queue t^N. And μ is normalized by $\bar{\pi} = \sum_{n=1}^{N} \pi_n/\binom{N}{n}$. Since $u'(0) = \infty$ the constraint $c^1 \geq 0$ and $c^2 \geq 0$ are not binding and the respective Lagrange multipliers can be ignored. The first order conditions of the problem are given below.

$$(15) \quad \left[c_k^1(\bar{t}^k)\right]: \quad \sum_{n=0}^{N} \frac{\pi_n}{\binom{N}{n}} \sum_{\substack{t^N \in P_n \\ t^k = \bar{t}^k}} \left\{ u'\left[c_k^1(\bar{t}^k)\right] - \lambda_{t^N} R \right\} - \sum_{n=1}^{N} \frac{\pi_n}{\binom{N}{n}} \sum_{\substack{t^N \in P_n \\ t^k = (\bar{t}^{k-1}, 2)}} \frac{\mu\rho}{n_{t^N}} u'\left[c_k^1(\bar{t}^k)\right] = 0$$

for all $k \in \mathbb{N}$ and $\bar{t}^{k-1} \in T^{k-1}$ such that $\bar{t}_k = 1$; and

$$(16) \quad \left[c^2(t^N)\right]: \quad \frac{\pi_n}{\binom{N}{n_{t^N}}} \left\{ \rho u'\left[c^2(t^N)\right] - \lambda_{t^N} + \frac{\mu\rho}{n_{t^N}} u'\left[c^2(t^N)\right] \right\} = 0$$

for all $t^N \in T^N$ such that $n_{t^N} > 0$. We can solve the above equations for λ_{t^N} and obtain

$$\lambda_{t^N} = \begin{cases} \rho\left(1 + \frac{\mu}{n_{t^N}}\right) u'\left[c^2(t^N)\right] & \text{if} \quad n_{\bar{t}^N} > 0 \\ \frac{1}{R} u'\left[c_N^1(t^N)\right] & \text{if} \quad n_{\bar{t}^N} = 0 \end{cases}.$$

Note that $c^2(t^N)$ is not defined if $t^N = \mathbf{1}^N = (1, 1, \ldots, 1)$—there is no second period payments when every depositor announces to be of type impatient in the first period. In order to keep the notation short, let us

[22] From now on we will denote $c^*(\delta)$ just by c in order to keep the notation short.

define $u'\left[c_N^1\left(\mathbf{1}^N\right)\right]=\rho R u'\left[c^2\left(\mathbf{1}^N\right)\right]$ and $1/n_{\mathbf{1}^N}=0$. Then, λ_{t^N} is given by

$$\lambda_{t^N}=\rho\left(1+\frac{\mu}{n_{t^N}}\right)u'\left[c^2\left(t^N\right)\right]. \tag{17}$$

After replace equation (17) in equation (15) we obtain that for all $k\in\mathbb{N}$ and $\bar{t}^k=(\bar{t}^{k-1},1)\in T^{k-1}$:

$$\sum_{n=0}^{N}\frac{\pi_n}{\binom{N}{n}}\sum_{\substack{t^N\in P_n\\ t^k=\bar{t}^k}}u'\left[c_k^1(\bar{t}^k)\right]-\sum_{n=1}^{N}\frac{\pi_n}{\binom{N}{n}}\sum_{\substack{t^N\in P_n\\ t^k=(\bar{t}^{k-1},2)}}\frac{\mu\rho}{n_{t^N}}u'\left[c_k^1(\bar{t}^k)\right]=\sum_{n=0}^{N}\frac{\pi_n}{\binom{N}{n}}\sum_{\substack{t^N\in P_n\\ t^k=(\bar{t}^{k-1},1)}}R\rho\left(1+\frac{\mu}{n_{t^N}}\right)u'\left[c^2\left(t^N\right)\right]$$

which is equivalent to

$$\left\{\mathbb{P}\left[t^k=(\bar{t}^{k-1},1)\right]-\sum_{n=1}^{N}\frac{\pi_n}{\binom{N}{n}}\sum_{\substack{t^N\in P_n\\ t^k=(\bar{t}^{k-1},2)}}\frac{\mu\rho}{n_{t^N}}\right\}u'\left[c_k^1(\bar{t}^k)\right]=\sum_{n=0}^{N}\frac{\pi_n}{\binom{N}{n}}\sum_{\substack{t^N\in P_n\\ t^k=(\bar{t}^{k-1},1)}}R\rho\left(1+\frac{\mu}{n_{t^N}}\right)u'\left[c^2\left(t^N\right)\right].$$

We can also write the equation in expectations, which yields to the formula

$$\left[1-\gamma\left(\bar{t}^{k-1}\right)\right]u'\left[c_k^1\left(\bar{t}^k\right)\right]=\mathbb{E}_{t^N|t^k=(\bar{t}^{k-1},1)}\left\{R\rho\left(1+\frac{\mu}{n_{t^N}}\right)u'\left[c^2\left(t^N\right)\right]\right\} \tag{18}$$

where $\gamma\left(\bar{t}^{k-1}\right)=\mathbb{P}\left[t^k=(\bar{t}^{k-1},2)\right]\mathbb{E}_{t^N|t^k=(\bar{t}^{k-1},2)}\left[\mu\rho/n_{t^N}\right]/\mathbb{P}\left[t^k=(\bar{t}^{k-1},1)\right]$.

The result will be derived from equation (18). Let us use induction on $k\in\mathbb{N}$ starting from $k=N$ and going down until $k=1$.

Proof for $k=N$**:** Fix any $\bar{t}^N=(\bar{t}^{N-1},1)$. From equation (18) we have that

$$\left[1-\frac{\mathbb{P}\left[t^N=(\bar{t}^{N-1},2)\right]}{\mathbb{P}\left[t^N=(\bar{t}^{N-1},1)\right]}\times\frac{\mu\rho}{n_{(\bar{t}^{N-1},2)}}\right]u'\left[c_N^1\left(\bar{t}^{N-1},1\right)\right]=R\rho\left(1+\frac{\mu}{n_{(\bar{t}^{N-1},1)}}\right)u'\left[c^2\left(\bar{t}^{N-1},1\right)\right].$$

which implies that $u'\left[(c_N^1\left(\bar{t}^{N-1},1\right)\right]>u'\left[c^2\left(\bar{t}^{N-1},1\right)\right]$. Thus, $c_N^1\left(\bar{t}^{N-1},1\right)<c^2\left(\bar{t}^{N-1},1\right)$. We know that the resources constraints holds at equality because u is strictly increasing. Therefore,

$$n_{(\bar{t}^{N-1},2)}c^2\left(\bar{t}^{N-1},2\right)=\left[n_{\bar{t}^N}+1\right]c^2\left(\bar{t}^{N-1},2\right)=n_{\bar{t}^N}c^2\left(\bar{t}^{N-1},1\right)+Rc_N^1\left(\bar{t}^{N-1},1\right)$$

And after reorganize the equation above we have that

$$c^2(\bar{t}^{N-1},2)=\frac{n(\bar{t}^{N-1},1)}{n(\bar{t}^{N-1},1)+1}c^2(\bar{t}^{N-1},1)+\frac{1}{n(\bar{t}^{N-1},1)+1}Rc_N^1(\bar{t}^{N-1},1)>c_N^1(\bar{t}^{N-1},1).$$

Hence, for the case $k=N$, we can conclude that $c_k^1\left(\bar{t}^{k-1},1\right)<c^2\left(\bar{t}^{k-1},\mathbf{2}^{N-k+1}\right)$.

Proof for $k<N$**:** Assume the result holds for all $j>k$ and $\bar{t}^j=(\bar{t}^{j-1},1)\in T^j$. That is, for all $j>k$ we have $c_j^1\left(\bar{t}^{j-1},1\right)<c^2\left(\bar{t}^{j-1},\mathbf{2}^{N-j}\right)$. Let us show it also holds for k. Fix some $\bar{t}^k=(\bar{t}^{k-1},1)\in T^{k-1}$, then equation (18) is given by

$$u'\left[c_k^1\left(\bar{t}^k\right)\right]=\frac{1}{1-\gamma\left(\bar{t}^{k-1}\right)}\mathbb{E}_{t^N|t^k=(\bar{t}^{k-1},1)}\left\{R\rho\left(1+\frac{\mu}{n_{t^N}}\right)u'\left[c^2\left(t^N\right)\right]\right\}.$$

Note that, for any function $X:T^N\to\mathbb{R}$, the conditional expectation can be decomposed as

$$\mathbb{E}_{t^N\left|t^k=\bar{t}^k\right.}\left\{X\left(t^N\right)\right\}=\sum_{j=k+1}^{N}\mathbb{P}\left[t^j=\left(\bar{t}^k,\mathbf{2}^{j-k-1},1\right)\middle|t^k=\bar{t}^k\right]\mathbb{E}_{t^N\left|t^j=\left(\bar{t}^k,\mathbf{2}^{j-k-1},1\right)\right.}\left\{X\left(t^N\right)\right\}+$$
$$\mathbb{P}\left[t^N=\left(\bar{t}^k,\mathbf{2}^{N-k}\right)\middle|t^k=\bar{t}^k\right]X\left(\bar{t}^k,\mathbf{2}^{N-k}\right).$$

Applying this decomposition to equation (18) we obtain

$$u'\left[c_k^1\left(\bar{t}^k\right)\right] = \left\{ \sum_{j=k+1}^{N} \mathbb{P}\left[t^j = \left(\bar{t}^k, \mathbf{2}^{j-k-1}, 1\right) \Big| t^k = \bar{t}^k\right] \mathbb{E}_{t^N \left| t^j = \left(\bar{t}^k, \mathbf{2}^{j-k-1}, 1\right)\right.} \left\{ R\rho\left(1 + \frac{\mu}{n_{t^N}}\right) u'\left[c^2\left(t^N\right)\right]\right\} + \mathbb{P}\left[t^N = \left(\bar{t}^k, \mathbf{2}^{N-k}\right) \Big| t^k = \bar{t}^k\right] R\rho\left(1 + \frac{\mu}{n_{\left(\bar{t}^k, \mathbf{2}^{N-k}\right)}}\right) u'\left[c^2\left(\bar{t}^k, \mathbf{2}^{N-k}\right)\right] \right\} \frac{1}{1-\gamma\left(\bar{t}^{k-1}\right)}.$$

By equation (18) we know that

$$\left[1-\gamma\left(\bar{t}^k, \mathbf{2}^{j-k-1}\right)\right] u'\left[c_j^1\left(\bar{t}^k, \mathbf{2}^{j-k-1}, 1\right)\right] = \mathbb{E}_{t^N \left| t^j = \left(\bar{t}^k, \mathbf{2}^{j-k-1}, 1\right)\right.} \left\{ R\rho\left(1 + \frac{\mu}{n_{t^N}}\right) u'\left[c^2\left(t^N\right)\right]\right\}$$

for $j = k+1, \ldots, N$. Hence,

$$u'\left[c_k^1\left(\bar{t}^k\right)\right] = \left\{ \sum_{j=k+1}^{N} \mathbb{P}\left[t^j = \left(\bar{t}^k, \mathbf{2}^{j-k-1}, 1\right) \Big| t^k = \bar{t}^k\right] \left[1-\gamma\left(\bar{t}^k, \mathbf{2}^{j-k-1}\right)\right] u'\left[c_j^1\left(\bar{t}^k, \mathbf{2}^{j-k-1}, 1\right)\right] + \mathbb{P}\left[t^N = \left(\bar{t}^k, \mathbf{2}^{N-k}\right) \Big| t^k = \bar{t}^k\right] R\rho\left(1 + \frac{\mu}{n_{\left(\bar{t}^k, \mathbf{2}^{N-k}\right)}}\right) u'\left[c^2\left(\bar{t}^k, \mathbf{2}^{N-k}\right)\right] \right\} \frac{1}{1-\gamma\left(\bar{t}^{k-1}\right)}.$$

By the inductive hypothesis we know that $c_j^1\left(\bar{t}^k, \mathbf{2}^{j-k-1}, 1\right) < c^2\left(\bar{t}^k, \mathbf{2}^{N-k}\right)$, which implies that

$$\begin{aligned}
u'\left[c_k^1\left(\bar{t}^k\right)\right] &> \frac{1}{1-\gamma\left(\bar{t}^{k-1}\right)} \left\{ \sum_{j=k+1}^{N} \mathbb{P}\left[t^j = \left(\bar{t}^k, \mathbf{2}^{j-k-1}, 1\right) \Big| t^k = \bar{t}^k\right] \left[1-\gamma\left(\bar{t}^k, \mathbf{2}^{j-k-1}\right)\right] u'\left[c^2\left(\bar{t}^k, \mathbf{2}^{N-k}\right)\right] \right. \\
&\quad \left. + \mathbb{P}\left[t^N = \left(\bar{t}^k, \mathbf{2}^{N-k}\right) \Big| t^k = \bar{t}^k\right] R\rho\left(1 + \frac{\mu}{n_{\left(\bar{t}^k, \mathbf{2}^{N-k}\right)}}\right) u'\left[c^2\left(\bar{t}^k, \mathbf{2}^{N-k}\right)\right] \right\} \\
&= \frac{1}{1-\gamma\left(\bar{t}^{k-1}\right)} \left\{ \sum_{j=k+1}^{N} \mathbb{P}\left[t^j = \left(\bar{t}^k, \mathbf{2}^{j-k-1}, 1\right) \Big| t^k = \bar{t}^k\right] \left[1-\gamma\left(\bar{t}^k, \mathbf{2}^{j-k-1}\right)\right] \right. \\
&\quad \left. + \mathbb{P}\left[t^N = \left(\bar{t}^k, \mathbf{2}^{N-k}\right) \Big| t^k = \bar{t}^k\right] R\rho\left(1 + \frac{\mu}{n_{\left(\bar{t}^k, \mathbf{2}^{N-k}\right)}}\right) \right\} u'\left[c^2\left(\bar{t}^k, \mathbf{2}^{N-k}\right)\right] \\
&= \frac{1}{1-\gamma\left(\bar{t}^{k-1}\right)} \left\{ \sum_{j=k+1}^{N} \mathbb{P}\left[t^j = \left(\bar{t}^k, \mathbf{2}^{j-k-1}, 1\right) \Big| t^k = \bar{t}^k\right] \right. \\
&\quad - \sum_{j=k+1}^{N} \mathbb{P}\left[t^j = \left(\bar{t}^k, \mathbf{2}^{j-k-1}, 1\right) \Big| t^k = \bar{t}^k\right] \frac{\mathbb{P}\left[t^j = \left(\bar{t}^k, \mathbf{2}^{j-k}\right)\right]}{\mathbb{P}\left[t^j = \left(\bar{t}^k, \mathbf{2}^{j-k-1}, 1\right)\right]} \mathbb{E}_{t^N | t^j = \left(\bar{t}^k, \mathbf{2}^{j-k}\right)} \left[\frac{\mu\rho}{n_{t^N}}\right] \\
&\quad \left. + \mathbb{P}\left[t^N = \left(\bar{t}^k, \mathbf{2}^{N-k}\right) \Big| t^k = \bar{t}^k\right] R\rho\left(1 + \frac{\mu}{n_{\left(\bar{t}^k, \mathbf{2}^{N-k}\right)}}\right) \right\} u'\left[c^2\left(\bar{t}^k, \mathbf{2}^{N-k}\right)\right]
\end{aligned}$$

After simplify the above equation we obtain

$$\begin{aligned}
u'\left[c_k^1\left(\bar{t}^k\right)\right] &> \frac{1}{1-\gamma\left(\bar{t}^{k-1}\right)} \left\{ 1 - \mathbb{P}\left[t^N = \left(\bar{t}^k, \mathbf{2}^{N-k}\right) \Big| t^k = \bar{t}^k\right] \right. \\
&\quad - \sum_{j=k+1}^{N} \frac{\mathbb{P}\left[t^j = \left(\bar{t}^k, \mathbf{2}^{j-k}\right)\right]}{\mathbb{P}\left[t^k = \bar{t}^k\right]} \mathbb{E}_{t^N | t^j = \left(\bar{t}^k, \mathbf{2}^{j-k}\right)} \left[\frac{\mu\rho}{n_{t^N}}\right] \\
&\quad \left. + \mathbb{P}\left[t^N = \left(\bar{t}^k, \mathbf{2}^{N-k}\right) \Big| t^k = \bar{t}^k\right] R\rho\left(1 + \frac{\mu}{n_{\left(\bar{t}^k, \mathbf{2}^{N-k}\right)}}\right) \right\} u'\left[c^2\left(\bar{t}^k, \mathbf{2}^{N-k}\right)\right].
\end{aligned} \tag{19}$$

The fact that the queue position is withdrawn uniformly implies that

$$\mathbb{P}\left[t^j = (\bar{t}^k, \mathbf{2}^{j-k})\right] = \mathbb{P}\left[t^j = (\bar{t}^{k-1}, 1, \mathbf{2}^{j-k})\right] = \mathbb{P}\left[t^j = (\bar{t}^{k-1}, \mathbf{2}^{j-k}, 1)\right]$$

and

$$\mathbb{E}_{t^N|t^j=(\bar{t}^k,\mathbf{2}^{j-k})}\left[\frac{\mu\rho}{n_{t^N}}\right] = \mathbb{E}_{t^N|t^j=(\bar{t}^{k-1},1,\mathbf{2}^{j-k})}\left[\frac{\mu\rho}{n_{t^N}}\right] = \mathbb{E}_{t^N|t^j=(\bar{t}^{k-1},\mathbf{2}^{j-k},1)}\left[\frac{\mu\rho}{n_{t^N}}\right].$$

This implies that

$$\begin{aligned}
(20)\quad \sum_{j=k+1}^{N}\frac{\mathbb{P}\left[t^j = (\bar{t}^k, \mathbf{2}^{j-k})\right]}{\mathbb{P}\left[t^k = \bar{t}^k\right]}\mathbb{E}_{t^N|t^j=(\bar{t}^k,\mathbf{2}^{j-k})}\left[\frac{\mu\rho}{n_{t^N}}\right] &= \sum_{j=k+1}^{N}\frac{\mathbb{P}\left[t^j = (\bar{t}^{k-1}, 1, \mathbf{2}^{j-k})\right]}{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, 1)\right]}\mathbb{E}_{t^N|t^j=(\bar{t}^{k-1},1,\mathbf{2}^{j-k})}\left[\frac{\mu\rho}{n_{t^N}}\right] \\
&= \sum_{j=k+1}^{N}\frac{\mathbb{P}\left[t^j = (\bar{t}^{k-1}, \mathbf{2}^{j-k}, 1)\right]}{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, 1)\right]}\mathbb{E}_{t^N|t^j=(\bar{t}^{k-1},\mathbf{2}^{j-k},1)}\left[\frac{\mu\rho}{n_{t^N}}\right] \\
&= \sum_{j=k+1}^{N}\frac{\mathbb{P}\left[t^j = (\bar{t}^{k-1}, \mathbf{2}^{j-k}, 1)\right]}{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, 1)\right]}\mathbb{E}_{t^N|t^j=(\bar{t}^{k-1},\mathbf{2}^{j-k},1)}\left[\frac{\mu\rho}{n_{t^N}}\right] \\
&\quad + \frac{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, \mathbf{2}^{N-k})\right]}{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, 1)\right]}\frac{\mu\rho}{n_{(\bar{t}^{k-1},\mathbf{2}^{N-k})}} \\
&\quad - \frac{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, \mathbf{2}^{N-k})\right]}{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, 1)\right]}\frac{\mu\rho}{n_{(\bar{t}^{k-1},\mathbf{2}^{N-k})}} \\
&= \frac{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, 2)\right]}{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, 1)\right]}\mathbb{E}_{t^N|t^k=(\bar{t}^{k-1},2)}\left[\frac{\mu\rho}{n_{t^N}}\right] \\
&\quad - \frac{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, \mathbf{2}^{N-k})\right]}{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, 1)\right]}\frac{\mu\rho}{n_{(\bar{t}^{k-1},\mathbf{2}^{N-k})}} \\
&= \gamma(\bar{t}^{k-1}) - \frac{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, \mathbf{2}^{N-k})\right]}{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, 1)\right]}\frac{\mu\rho}{n_{(\bar{t}^{k-1},\mathbf{2}^{N-k})}}.
\end{aligned}$$

Replacing equation (20) in inequality (19) and reorganising the terms in the inequality, we obtain

$$\begin{aligned}
(21)\quad u'\left[c_k^1(\bar{t}^k)\right] > \frac{1}{1-\gamma(\bar{t}^{k-1})}\Bigg\{1 - \gamma(\bar{t}^{k-1}) + \frac{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, \mathbf{2}^{N-k})\right]}{\mathbb{P}\left[t^k = (\bar{t}^{k-1}, 1)\right]}\frac{\mu\rho}{n_{(\bar{t}^{k-1},\mathbf{2}^{N-k})}} \\
+ \mathbb{P}\left[t^N = (\bar{t}^k, \mathbf{2}^{N-k})\middle| t^k = \bar{t}^k\right] R\rho\left(1 + \frac{\mu}{n_{(\bar{t}^k,\mathbf{2}^{N-k})}} - \frac{1}{R\rho}\right)\Bigg\} u'\left[c^2(\bar{t}^k, \mathbf{2}^{N-k})\right].
\end{aligned}$$

Because $R\rho > 1$, the inequality (21) implies that

$$u'\left[c_k^1(\bar{t}^{k-1}, 1)\right] = u'\left[c_k^1(\bar{t}^k)\right] > u'\left[c^2(\bar{t}^k, \mathbf{2}^{N-k})\right] = u'\left[c^2(\bar{t}^{k-1}, 1, \mathbf{2}^{N-k})\right].$$

And since u is concave, it implies that $c_k^1(\bar{t}^{k-1}, 1) < c^2(\bar{t}^{k-1}, 1, \mathbf{2}^{N-k})$. The resources constraint implies that

$$\left[n_{(\bar{t}^{k-1},1,\mathbf{2}^{N-k})} + 1\right] c^2(\bar{t}^{k-1}, \mathbf{2}^{N-k+1}) = n_{(\bar{t}^{k-1},1,\mathbf{2}^{N-k})} c^2(\bar{t}^{k-1}, 1, \mathbf{2}^{N-k}) + Rc_k^1(\bar{t}^{k-1}, 1).$$

And finally we can conclude that

$$c^2(\bar{t}^{k-1}, \mathbf{2}^{N-k+1}) = \frac{n_{(\bar{t}^{k-1},1,\mathbf{2}^{N-k})}}{n_{(\bar{t}^{k-1},1,\mathbf{2}^{N-k})} + 1} c^2(\bar{t}^{k-1}, 1, \mathbf{2}^{N-k}) + \frac{1}{n_{(\bar{t}^{k-1},1,\mathbf{2}^{N-k})} + 1} Rc_k^1(\bar{t}^{k-1}, 1) > c_k^1(\bar{t}^{k-1}, 1).$$

We have shown that the result holds for $k = N$ and that if it holds for all $j \in \{k+1, \ldots, N\}$ it holds for k. Therefore, by induction, we can conclude that the result holds for all $k \in \mathbb{N}$. □

Proposition 1

PROOF: We know that for any vector of announcements $\hat{m}^N \in \hat{M}^N$, if either $\hat{m}^N \in T^N$ or $\hat{m}_k \neq 2$ for all k, the result is trivial. Consider a realized vector of announcements $\hat{m}^N \in \hat{M}^N$, with $\hat{m}^N \notin T^N$, $\hat{m}_k = 2$, and let j be the queue position of the first agent to announce g. As before, $\hat{t}^N \in T^N$ denotes the vector $\hat{m}^N$ we replace all g's with 2's. When agent j announced g the in the first period payments were suspended, hence, the total resources in the beginning of period 2 is

$$R\left[Y - \sum_{i=1}^{N} \hat{c}_i^1(\hat{m}^i)\right] = R\left[Y - \sum_{i=1}^{j} \hat{c}_i^{*1}(\hat{t}^i)\right] = n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})} c_k^{*2}(\hat{t}^{j-1},\mathbf{2}^{N-j+1}).$$

Where $n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})}$ is the number of 2's in the vector $(\hat{t}^{j-1},\mathbf{2}^{N-j+1})$. Let $d_{\hat{m}^N}$ denote the number of agents who have announced g and $n_{\hat{m}^N}$ the number of agents who announced 2. The total payments in the second period to agents who announced g is given by

$$\sum_{k=1}^{N} \hat{c}_k^2(\hat{m}^N)\mathbb{1}_{\hat{m}_k=g} = c_j^{*1}(\hat{t}^{j-1},1) + d_{\hat{m}^N}\epsilon.$$

Hence, payment to agent k is

$$\begin{aligned}
\hat{c}_k^2\left(\hat{m}^{k-1},2,\hat{m}_{k+1}^N\right) &= \frac{R\left[Y - \sum_{k=1}^{N} \hat{c}_k^1(\hat{m}^k)\right] - \sum_{k=1}^{N} \hat{c}_k^2(\hat{m}^N)\mathbb{1}_{\hat{m}_k=g}}{n_{\hat{m}^N}} \\
&= \frac{n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})} c_k^{*2}(\hat{t}^{j-1},\mathbf{2}^{N-j+1}) - c_j^{*1}(\hat{t}^{j-1},1) - d_{\hat{m}^N}\epsilon}{n_{\hat{m}^N}}.
\end{aligned}$$

By lemma (1) we know that $c_j^{*1}\left(\bar{t}^{j-1},1\right) < c_j^{*2}(\hat{t}^{j-1},\mathbf{2}^{N-j+1})$. Thus, by taking $\epsilon > 0$ small enough, we have that,

$$\hat{c}_k^2\left(\hat{m}^{k-1},2,\hat{m}_{k+1}^N\right) \geq \frac{[n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})} - 1]c_k^{*2}(\hat{t}^{j-1},\mathbf{2}^{N-j+1})}{n_{\hat{m}^N}}.$$

By construction we have that

$$\begin{aligned}
\hat{c}_k^2\left(\hat{t}^{k-1},2,\hat{t}_{k+1}^N\right) &= c_k^{*2}\left(\hat{t}^{k-1},2,\hat{t}_{k+1}^N\right) \\
&= \frac{R\left[Y - \sum_{i=1}^{N} \hat{c}_i^{*1}(\hat{t}^i)\right]}{n_{\hat{m}^N} + d_{\hat{m}^N}} \leq \frac{R\left[Y - \sum_{i=1}^{j} \hat{c}_i^{*1}(\hat{t}^i)\right]}{n_{\hat{m}^N} + 1} = \frac{n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})} c_k^{*2}(\hat{t}^{j-1},\mathbf{2}^{N-j+1})}{n_{\hat{m}^N} + 1}.
\end{aligned}$$

Note that,

$$\begin{aligned}
&\frac{n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})} - 1}{n_{\hat{m}^N}} \geq \frac{n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})}}{n_{\hat{m}^N} + 1} \iff \\
&n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})} n_{\hat{m}^N} + n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})} - n_{\hat{m}^N} - 1 \geq n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})} n_{\hat{m}^N} \iff n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})} \geq n_{\hat{m}^N} + 1.
\end{aligned}$$

The last inequality holds because $n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})} \geq n_{\hat{t}^N} = n_{\hat{m}^N} + d_{\hat{m}^N}$. Hence,

$$\begin{aligned}
\hat{c}_k^2\left(\hat{m}^{k-1},2,\hat{m}_{k+1}^N\right) &\geq \frac{[n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})} - 1]c_k^{*2}(\hat{t}^{j-1},\mathbf{2}^{N-j+1})}{n_{\hat{m}^N}} \\
&\geq \frac{n_{(\hat{t}^{j-1},\mathbf{2}^{N-j+1})} c_k^{*2}(\hat{t}^{j-1},\mathbf{2}^{N-j+1})}{n_{\hat{m}^N} + 1} \geq \hat{c}_k^2\left(\hat{t}^{k-1},2,\hat{t}_{k+1}^N\right).
\end{aligned}$$

Which concludes the proof. $\square$

References

Andolfatto, D., Nosal, E., and Wallace, N. (2007). The role of independence in the green–lin diamond–dybvig model. *Journal of Economic Theory*, 137(1):709–715.

Bryant, J. (1980). A model of reserves, bank runs, and deposit insurance. *Journal of Banking & Finance*, 4(4):335–344.

Cavalcanti, R. and Monteiro, P. K. (2011). Enriching information to prevent bank runs.

Cochrane, J. H. (2014). Toward a run-free financial system. *Available at SSRN 2425883*.

Demski, J. S. and Sappington, D. (1984). Optimal incentive contracts with multiple agents. *Journal of Economic Theory*, 33(1):152–171.

Diamond, D. and Dybvig, P. (1983). Bank runs, deposit insurance, and liquidity. *The Journal of Political Economy*, pages 401–419.

Ennis, H. M. and Keister, T. (2009a). Bank runs and institutions: The perils of intervention. *The American Economic Review*, pages 1588–1607.

Ennis, H. M. and Keister, T. (2009b). Run equilibria in the green–lin model of financial intermediation. *Journal of Economic Theory*, 144(5):1996–2020.

Green, E. and Lin, P. (2003). Implementing efficient allocations in a model of financial intermediation. *Journal of Economic Theory*, 109(1):1–23.

Ma, C.-T., Moore, J., and Turnbull, S. (1988). Stopping agents from "cheating". *Journal of Economic Theory*, 46(2):355–372.

Mookherjee, D. and Reichelstein, S. (1990). Implementation via augmented revelation mechanisms. *The Review of Economic Studies*, 57(3):453–475.

Peck, J. and Shell, K. (2003). Equilibrium bank runs. *Journal of Political Economy*, pages 103–123.

Postlewaite, A. and Schmeidler, D. (1986). Implementation in differential information economies. *Journal of Economic Theory*, 39(1):14–33.

Santos, A. O. and Neftci, S. N. (2003). Puttable and extendible bonds: Developing interest rate derivatives for emerging markets.

Schmidt, L., Timmermann, A., and Wermers, R. (2013). Runs on money market mutual funds.

Wallace, N. (1990). A banking model in which partial suspension is best. *Federal Reserve Bank of Minneapolis Quarterly Review*, 14(4):11–23.