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The Cost of Business Cycles with Heterogeneous Trading Technologies *

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Abstract

This paper investigates the welfare cost of business cycles in an economy where households have heterogeneous trading technologies. In an economy with aggregate risk, the different portfolio choices induced by heterogeneous trading technologies lead to a larger consumption inequality in equilibrium, while this source of inequality vanishes in an economy without business cycles. Put simply, the heterogeneity in trading technologies amplifies the effect of aggregate output fluctuation on consumption inequality. The welfare cost of business cycles is, therefore, larger in such an economy. In the benchmark economy with a reasonable low risk aversion rate, the business cycle costs 6.49% per period consumption for an average household when I calibrate this model to match the risk premium.

*The views expressed are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors.

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1 Introduction

In a calibrated representative agent model, Lucas (1987) shows a very insignificant welfare gain from the elimination of business cycles. His work suggests that the benefits of stabilizing the cyclical fluctuations in an economy are very limited. Hence, studying the business cycle might not be the top priority in macroeconomics. More recently, Lucas (2003) has argued that most macroeconomics models still fail to generate a sizeable welfare cost of business cycles.

This paper investigates the welfare cost of business cycles in an economy where households have heterogeneous trading technologies. In contrast to most papers in the incomplete market literature, the menu of assets available in this economy is quite rich. However, households in this model have heterogeneous abilities to access the menu of assets from the market. This paper distinguishes between passive traders, who hold fixed portfolios of stocks and bonds, and active traders, who frequently adjust their portfolios in response to a change in the investment opportunity set.

In the equilibrium of the calibrated economy, heterogeneous trading technologies result in a clear difference between active traders and passive traders with respect to their portfolio choices. In response to the high risk premium, households with more sophisticated trading technologies take greater aggregate risks by holding a large fraction of equity in their portfolios. They also optimally adjust their portfolios in response to the frequent changes in the investment opportunity set. On the other hand, households with less sophisticated trading technology take a more cautious approach. On average, they bear less aggregate risk by holding less equity share in their portfolios and do not actively respond to the changes in the investment opportunity set. The active traders ultimately earn a high rate of return on their portfolios, accumulating more wealth and enjoying a high level of consumption, while the passive investors earn a low return on their portfolios, acquiring relatively low levels of wealth and consuming less. Hence, heterogeneous trading technologies induce more consumption inequality in this economy.

Clearly, the source of consumption inequality depends heavily on the level of risk premium as well as the variation of market price of risk. Both of them are linked tightly to the business cycle fluctuation. A reduction of aggregate output volatility helps to reduce not only the size but also the time variation of risk premium. This reduction downplays the role of portfolio choice, and hence, improves the consumption inequality and welfare in the economy. Without aggregate risk, the inequality in consumption caused by the heterogeneous trading technologies disappears since the composition and timing of portfolio choice no longer matter in the rate of return. In short, all assets are risk free and offer exactly the same rate of return. A sophisticated trading technology does not offer any advantage in an environment without aggregate risk.

I conjecture that heterogeneous trading technologies may contribute to the welfare cost of business cycles. In an economy with aggregate risk, the different portfolio choices across households lead to a larger consumption inequality in equilibrium, while this source of inequality vanishes in an economy without business cycles. In short, the consumption inequality due to the aggregate output fluctuation is amplified by the heterogeneous trading technologies. The welfare cost of business cycles is, therefore, larger in such an economy.

This paper uses the modified macroeconomics model developed by Chien, Cole, and Lustig (2011) to evaluate the conjecture quantitatively. Their model incorporates heterogeneous trading technologies into an otherwise standard macroeconomics model. In my use of the model, the heterogeneity in trading technologies is calibrated to match the high risk premiums shown in the US historical data. The welfare cost is measured by the percentage of consumption compensation to households in an economy without business cycles, so that they are indifferent to an environment with aggregate fluctuations. I find that the welfare gain from the elimination of business cycles is large, with a reasonably low risk aversion coefficient. In my benchmark case in which the risk aversion coefficient is 4, the business cycle costs 6.49% per period consumption for an average household in my economy. The welfare cost is even larger, 9.37% per period consumption, when the risk aversion rate is lowered to 3 and this economy is recalibrated to match the risk premiums. These welfare cost numbers are definitely much larger than in Lucas's calculation.

Also, I compute the case in which all the households are active traders and endowed with the same sophisticated trading technologies. Given the parameter values in my benchmark calibration, the results show a low risk premium and a much smaller cost of business cycles. The importance of this computational exercise is twofold. First, it shows how an inferior investment technology among some of the investors influences the patterns of return in asset markets. If all households make no investment mistakes, the good asset pricing result is dampened compared to that in my benchmark economy. In addition, it demonstrates a large welfare loss in regards to poor investment strategies. This exercise shows that the welfare cost of business cycles is much smaller if no household makes investment errors. The heterogeneous trading technologies contribute significantly and mostly to the welfare cost number in my benchmark economy.

The assumption of heterogeneous trading technologies is critical to my results. The question thus arises: How realistic is the assumption of heterogeneous trading technologies? The answer can be found in empirical studies and data that have shown a high amount of heterogeneity in household portfolio choices. Different households behave as if they had access to a different menu of tradable assets. In the United States, a majority of households do not invest directly in equity in spite of the sizeable historical equity premiums. Even for those who participate in the equity market, most do not frequently adjust the composition of their portfolios, regardless of the large countercyclical variation of Sharpe ratios in the equity market. Put simply, they miss the market timing. However, there is a small fraction of households that hold a large share of stock and constantly change their equity position in response to the high, variable risk premiums. Therefore, these households end up richer but have more exposure to aggregate risk. Parker and Vissing-Jorgensen (2009) show that the consumption of the richest 10% of US households is five times more exposed to aggregate risk than that of average households.

This paper is closely related to the body of literature in which the distribution effects on consumption inequality might justify a large welfare cost of business cycles. Krusell and Smith (1999) propose the idea that business cycles might worsen the consumption inequality across the population while the impact on average households is insignificant. The higher cost of business cycles is due to the distributional impact of consumption among the rich and poor. Evidently, the distributional impact is missing in a representative agent economy. Krusell, Mukoyama, Sahin, and Smith (2009) employ an incomplete market model calibrated to the wealth distribution in the United States, in order to evaluate the welfare effect from the elimination of business cycles. Using the same parameter for risk aversion as in Lucas (1987), they find the welfare cost to be approximately 0.1% of household consumption. Although the welfare cost number is already one magnitude larger than that from Lucas's calculation, it is still negligible in an economic sense. Storesletten, Telmer, and Yaron (2001) consider the welfare cost of business cycles in an environment with the countercyclical variations in idiosyncratic shock. A more volatile idiosyncratic income risk during the recession can amplify the cost of aggregate risk in individual consumption and leads to a higher distributional impact. Although the welfare cost of business cycles is still insignificant, the cost increases rapidly as the risk aversion coefficient increases. Krebs (2007) extends the concept of idiosyncratic labor income shock by adding a permanent job displacement risk. The risk of job displacement is assumed to be closely associated with the business cycle. His paper finds a sizeable cost of business cycles due to the important role played by the displacement risks.

The central idea of this corpus of literature is to translate a small scale of aggregate risk into a large consumption inequality. I follow this concept, but the large consumption inequality in my model is caused by a novel feature: heterogeneity in trading technologies. Most articles in this literature operate under incomplete market models in which all households can only trade a very limited menu of tradable assets. However, the actual menu of assets that households can trade is quite rich. Instead of assuming a limited set of tradable assets, I introduce the heterogeneous ability in accessing the menu of assets. This idea is motivated by the empirical evidence of heterogeneity in portfolio choices. With heterogeneous trading technologies, households' total incomes not only differ because of their idiosyncratic risk in labor income but also because of the variations in their investment returns resulting from the heterogeneity in trading technologies. In addition, heterogeneous trading technologies affect the return of portfolio choices only in an economy with aggregate risk. Without business cycles, the cost of consumption inequality from different trading technologies disappears. Therefore, the heterogeneity in trading technologies only enlarges the consumption inequality in an economy with aggregate risk and, hence, amplifies the cost of business cycles.

Alvarez and Jermann (2004) demonstrate a close link between the cost of business cycles and the risk premium. They offer an alternative way to measure the cost of business cycles by asset pricing data. Their work illustrates that, under a representative agent economy, the welfare cost of business cycles is limited by an upper bound, which can be approximated by the risk premium between an aggregate consumption claim and a risk free asset, regardless of the assumption of utility function. One of the contributions from Alvarez and Jermann (2004) is the following observation: If a model can generate a reasonable asset pricing result, then it might produce a large welfare cost of business cycles as well. The calibrated model does produce a reasonable asset pricing result; however, the large welfare cost of business cycles results mainly from the consumption inequality induced by heterogeneous trading technologies, not directly from the variations in aggregate consumption.

This paper also relates to the fast growing body of literature on household finance. Campbell (2006) points out that some households might make various mistakes when facing complicated financial decisions. This paper evaluates the welfare cost of some of these mistakes. In the model economy, passive traders make two types of mistakes. Households that do not participate in the equity market forgo the large equity premium. Also, those equity investors who do not frequently change their portfolio choices miss the market timing. By comparing the results of two model economies, with heterogeneous trading technologies and without heterogeneous trading technologies, I demonstrate that these investment mistakes not only affect the patterns of risk premium but also cause a large welfare cost. If all households comprise active traders who do not make any investment mistakes, then the risk premium is low and stable in the calibrated economy. Moreover, the welfare cost of business cycles is almost negligible and similar to the result found by Lucas (1987). This finding emphasizes the importance of the study of household finance because preventing investment mistakes can considerably improve welfare.

The outline of the paper is as follows: Section 2 describes the environment and the trading technologies. Section 3 discusses the calibration of the model. Section 4 displays the results and sensitivity analyses of my model. Finally, section 5 offers the conclusion.

2 Model

The model setup follows closely to the one in Chien, Cole, and Lustig (2011). The novel feature of their model is the imposition of restrictions on the menu of assets that households are able to trade, which defines the trading technology a household owns. These restrictions are imposed exogenously. The goal of these restrictions is to capture the observed portfolio behavior of most households.

I will refer to households as being *passive traders* if they take their portfolio composition as given and simply choose how much to save or dissave in each period. Other households constantly manage their portfolios in response to changes in the investment opportunity set. I refer to these traders as *active traders* since they optimally adjust the composition of their portfolios every period. Note that the passive traders are completely rational expect their portfolio choice decisions. They fully acknowledge the rate of return on their portfolios and adjust their consumption and saving decisions optimally. Hence, the results are clearly driven by the only additional novel assumption, heterogeneous trading technologies, in contrast to most papers in the incomplete market literature.

2.1 Environment

This endowment economy consists of a continuum of heterogeneous households that are subject to idiosyncratic income shocks as well as aggregate output shocks. The total measure of households is normalized to be one. The heterogeneity across households arises from two assumptions. In the planning period 0, households are received a one time permanent shock in their trading technologies, while all other characteristics of the households are identical. Starting at period 1, these households also differ in terms of the realization of idiosyncratic income shock at all subsequent periods. Initially, all households start with the same initial wealth and face an identical stochastic process of idiosyncratic income shocks.

In the model, time is discrete, infinite, and indexed by $t = 0, 1, 2, ...$ The first period, $t = 0$, is a planning period in which financial contracting takes place. I use $z_t \in Z$ to denote the aggregate shock in period t and $\eta_t \in N$ to denote the idiosyncratic shock in period t. z^t denotes the history of aggregate shocks, and similarly, η^t denotes the history of idiosyncratic shocks for a household. The idiosyncratic events η are i.i.d. across households with the mean normalized to be one. I use $\pi(z^t, \eta^t)$ to denote the unconditional probability of state (z^t, η^t) being realized. The events are first-order Markov, and I assume that

$$\pi(z^{t+1}, \eta^{t+1}|z^t, \eta^t) = \pi(z_{t+1}|z_t)\pi(\eta_{t+1}|\eta_t). \tag{1}$$

Note that the probability of idiosyncratic events does not depend on the realization of aggregate shocks. As I will show later, this paper does not consider the counter-cyclical variation of idiosyncratic risk. Since I can appeal to the law of large numbers, $\pi(z^t, \eta^t)/\pi(z^t)$ also denotes the fraction of agents in state z^t that have drawn a history η^t. I use $\pi(\eta^t|z^t)$ to denote that fraction. I introduce some additional notation: $z^{t+1} \succ z^t$ or $y^{t+1} \succ y^t$ means that the left hand side node is a successor node to the right hand side node. I denote by $\{z^\tau \succ z^t\}$ the set of successor aggregate histories for z^t including the many periods in the future.

There is a single non-durable goods available for consumption in each period, and its aggregate supply is given by $Y_t(z^t)$, which evolves according to

$$Y_t(z^t) = \exp\{z_t\}Y(z^{t-1}), \tag{2}$$

with $Y(z^0) = 1$. This endowment goods comes in two forms. The first part is non-diversifiable income that is subject to idiosyncratic risk and it is given by $\gamma Y(z^t)\eta_t$; hence γ is the share of income that is non-diversifiable. Non-diversifiable income cannot be traded in the future market and may be considered as labor income. The second part is diversifiable income, which is not subject to idiosyncratic shocks, and is given by $(1-\gamma)Y_t(z^t)$.

All households are infinitely lived and rank stochastic consumption streams according to the following utility function

$$U(\{c\}) = \sum_{t\geq 1,(z^t,\eta^t)}^{\infty} \beta^t \pi(z^t,\eta^t) \frac{c_t(z^t,\eta^t)^{1-\alpha}}{1-\alpha}, \tag{3}$$

where $\alpha > 0$ denotes the coefficient of relative risk aversion, and $c_t(z^t,\eta^t)$ denotes the household's consumption in state (z^t,η^t).

2.2 Assets Traded

There are three assets available in this economy: equity, bond, and contingent claims on aggregate shocks. All of these assets are claims to diversifiable income. The actual menu of assets that a household can trade depends on the trading technology. However, this is still an incomplete market economy since there is no state contingent asset on idiosyncratic shocks.

Following Abel (1999), I simply consider equity as a leveraged claim to aggregate diversifiable income ($(1-\gamma)Y_t(z^t)$). The leverage ratio is assumed to be constant over time and denoted by ψ. Let $B_t(z^t)$ denotes for the supply of one period risk free bond in period t and $R^f_{t,t-1}(z^{t-1})$ denotes the risk free rate between period $t-1$ and t given the aggregate history z^{t-1}.With a constant leverage ratio, the total supply of $B_t(z^t)$ has to be adjusted such that

$$B_t(z^t) = \psi\left[\omega_t(z^t) - B_t(z^t)\right], \tag{4}$$

where $\omega_t(z^t)$ is denoted for the price of a claim to aggregate diversifiable income. Because the aggregate diversified income can be decomposed into the interest payment to bond holders and dividend payment to shareholders, the dividend payment, $d_t(z^t)$, is given by

$$D_t(z^t) = (1-\gamma)Y_t(z^t) - R^f_{t,t-1}(z^{t-1})B_{t-1}(z^{t-1}) + B_t(z^t) \tag{5}$$

Traders who invest a fraction $\psi/(1+\psi)$ of their wealth in bonds and the rest in equity hold the market portfolio. I denote the price of the equity (a claim to dividends payment $D_t(z^t)$) by $V_t(z^t)$.

The third available asset is the aggregate state contingent claims. I denote the price of a unit claim to the final good in aggregate state z^{t+1} acquired in aggregate state z^t by $Q_t(z_{t+1}, z^t)$.

I consider a household entering the period with a net financial wealth $\hat{a}_t(z^t, \eta^t)$. This household buys securities in financial markets (state contingent claims $a_t(z^{t+1}, \eta^{t+1})$, risk free bonds $b_t(z^t, \eta^t)$, and equity shares $s^D_t(z^t, \eta^t)$) and consumption $c_t(z^t, \eta^t)$ in the goods markets subject to this one-period budget constraint:

$$\sum_{z^{t+1}\succ z^t, \eta^{t+1}\succ \eta^t} Q_t(z_{t+1}, z^t)a_t(z^{t+1}, \eta^{t+1})\pi(\eta_{t+1}|z_{t+1}, \eta_t) + s^D_t(z^t, \eta^t)V_t(z^t) + b_t(z^t, \eta^t) + c_t(z^t, \eta^t) \tag{6}$$

$$\leq \hat{a}_t(z^t, \eta^t) + \gamma Y_t(z^t)\eta_t, \forall z^t, \eta^t, \tag{7}$$

where $\hat{a}_t(z^t, \eta^t)$, the agent's net financial wealth in state (z^t, η^t), is given by the payoffs of his state-contingent claim acquired last period, the payoffs from his equity position, and the risk free bond payoffs:

$$\hat{a}_t(z^t, \eta^t) = a_{t-1}(z^t, \eta^t) + s^D_t(z^{t-1}, \eta^{t-1})\left[D_t(z^t) + V_t(z^t)\right] + R^f_{t,t-1}(z^{t-1})b_{t-1}(z^{t-1}). \tag{8}$$

2.3 Trading Technology

There are two main classes of traders: active traders and passive traders. Active traders are able to trade state contingent claims on aggregate shock. They change their portfolio composition in equity of bonds optimally every period in response to the variations of state contingent prices. These active traders make no mistakes in their investment choices. In contrast, passive traders cannot trade state contingent claims and their portfolio choice is limited by an exogenously assigned and fixed target ϖ for the equity share. I refer to these traders as passive precisely because of their inelastic response to the changes in investment opportunity. These passive traders potentially make two kinds of investment mistakes. First, they miss the market timing if the volatility of market price of risk is not constant in the equilibrium. Second, for those passive traders who hold small or zero fractions of equity in their portfolios, they relinquish the risk premiums. The welfare cost of their mistakes may be large in the equilibrium, exhibiting a large risk premium and a volatile Sharpe Ratio in equity.

In addition, households face exogenous limits on their net asset positions, or solvency constraints,

$$\hat{a}_t(z^t, \eta^t) \geq 0. \tag{9}$$

Equation (9) reflects the fact that traders cannot borrow against their future non-diversifiable income.

2.4 Measurability Restrictions

To capture these portfolio restrictions implied by the different trading technologies, I use measurability constraints (see Chien, Cole, and Lustig (2011) for a detailed discussion) on net wealth. These restrictions allow us to solve for equilibrium allocations and prices without searching for all the equilibrium prices that clear each security market.

Active Trader Since idiosyncratic shocks are not spanned for the active traders, their net wealth needs to satisfy:

$$\hat{a}_t\left(z^t, \left[\eta_t, \eta^{t-1}\right]\right) = \hat{a}_t\left(z^t, \left[\tilde{\eta}_t, \eta^{t-1}\right]\right), \tag{10}$$

for all t and η_t, $\tilde{\eta}_t \in N$.

Passive Trader Passive traders who hold a fixed fraction ϖ in levered equity and $1 - \varpi$ in non-contingent bonds in their portfolio earn a portfolio return:

$$R_t^p(\varpi, z^t) = \varpi R_{t,t-1}^d(z^t) + (1 - \varpi) R_{t,t-1}^f(z^{t-1}) \tag{11}$$

where $R_{t,t-1}^d(z^t)$ denotes for the equity return between period t and $t-1$ given the realization of history state z^t. Hence, their net financial wealth satisfies this measurability restriction:

$$\frac{\hat{a}_t\left([z_t, z^{t-1}], [\eta_t, \eta^{t-1}]\right)}{R_t^p(\varpi, [z_t, z^{t-1}])} = \frac{\hat{a}_t\left([\tilde{z}_t, z^{t-1}], [\tilde{\eta}_t, \eta^{t-1}]\right)}{R_t^p(\varpi, [\tilde{z}_t, z^{t-1}])}, \tag{12}$$

for all t, z_t, $\tilde{z}_t \in Z$, and η_t, $\tilde{\eta}_t \in N$. If $\varpi = 1/(1+\psi)$, then this trader holds the market in each period and earns the return on a claim to aggregate diversifiable income. There is a special type of passive trader who does not participate in the equity market and only holds risk free assets. I call them non-participants, who can be thought of as those passive traders with zero equity target share, $\varpi = 0$.

2.5 Competitive Equilibrium

A competitive equilibrium for this economy is defined in a standard way. It consists of a list of bond, equity, and state contingent claims holdings; a consumption allocation; and a list of bond, equity, and state contingent prices such that: (i) given these prices, a trader's asset and consumption choices maximize her expected utility subject to the budget constraints, the solvency constraints, and the measurability constraints, and (ii) all asset markets clear.

3 Calibration

This section discusses the calibration of the parameters, the endowment processes, and the composition of trader pools. Section 4 uses a calibrated version of the model to evaluate the welfare effect of eliminating business cycles.

To compute the equilibrium of this economy, I follow the algorithm described by Chien, Cole, and Lustig (2011), who use truncated aggregate histories as state variables. I keep track of the lagged aggregate histories up to 7 periods.

3.1 Preferences and Endowments

Lucas (2003) suggested that a reasonable risk aversion coefficient should lie between 1 and 4. My benchmark calibration set the coefficient of relative risk aversion α to four. In order to check the robustness of my results with respect to the choice of risk aversion rate, I conduct a sensitivity analysis in subsection 4.3. The model is calibrated to annual data. The time discount factor β is set to .95. To match the collateralizable wealth to income ratio in the data, the diversifiable or collateralizable income share $1-\gamma$ is set to 10%, as discussed below. The average ratio of household wealth to aggregate income in the US is 4.30 between 1950 and 2005. The wealth measure is the total net wealth of households and non-profit organizations (Flow of Funds Tables). With a 10% diversifiable income share, the implied ratio of wealth to consumption is 5.40 in my benchmark calibration. [1]

The process of aggregate output is calibrated to match the aggregate consumption growth moments from Alvarez and Jermann (2001) and Mehra and Prescott (1985). The average consumption growth rate is 1.83% and the standard deviation is 3.15%. The auto-correlation of consumption growth is -0.14. Expansions are more frequent than recessions: 73% of realizations are high aggregate consumption growth states. I calibrate the labor income process as in Storesletten, Telmer, and Yaron (2004) and Storesletten, Telmer, and Yaron (2007), except that I eliminate the counter-cyclical variation of labor income risk. The variance of labor income risk is constant in this model. An invariant labor income risk setup highlights the role of the new feature (heterogeneous trading technologies) considered in this paper. The main driving force of my result comes from the heterogeneity in trading technology, not from the counter-cyclical variation of labor income risk. The Markov process for $\log \eta(y, z)$ has a standard deviation of 0.71, and the autocorrelation is 0.89. I use a 4-state discretization for both aggregate and idiosyncratic risk. The elements of the process for $\log \eta$ are 0.38 and 1.61 for low and high shocks, respectively.

The equity in my model is simply a leveraged claim to diversifiable income. In the Flow of Funds, the ratio of corporate debt-to-net worth is around 0.65, suggesting a leverage parameter ψ of 2. However, Cecchetti, Lam, and Mark (1990) report that the standard deviation for the growth rate of dividends is at least 3.6 times that of aggregate consumption, suggesting that the appropriate leverage level is over 3. Following Abel (1999) and Bansal and Yaron (2004), I choose to set the leverage parameter ψ to 3.

[1] As is standard in this literature, I compare the ratio of total outside wealth to aggregate non-durable consumption in the endowment economy to the ratio of total tradable wealth to aggregate income in the data. Aggregate income exceeds aggregate non-durable consumption because of durable consumption and investment.

3.2 Composition of Trader Pools and Equity Target Share

I set the fraction of non-participants at 50% based on the fact that 51.1% of households reported owning stocks directly or indirectly on the recent Survey of Consumer Finances. In order to match the large equity premium (7.53%) measured in the post-war US data, a relatively small fraction of active traders needs to bear the large amount of residual aggregate risks created by non-participants. Hence, I set the share of active traders at 10% and the passive equity traders at 40%.

Among those households that hold equity, I am not able to distinguish in the data between active traders and passive equity traders. It is hard to calibrate the equity target share of passive equity traders, since I do not know who they are. However, empirical studies have shown that rich households tend to be more sophisticated traders. Therefore, I consider the richest 10% of households to be active traders and the poorest 50% of households to be non-participants. The equity target share of passive equity traders is therefore calibrated to match the average fraction of equality share among those households that possess a percentile of wealth between 50% and 90%. According to the data from the Survey of Consumer Finances, the average equity share among these middle wealthy households is 24.2%. I, therefore, set the equity target share of the passive equity traders at 24%. This calibration also reflects the observation that the rich tend to hold a higher fraction of equity than the poor.

[Table 1 about here.]

4 Quantitative Results

I consider two cases in the quantitative exercise. The first is the benchmark economy, where the parameters are calibrated as described above. I refer to this case as the BM (benchmark) economy. I also consider another economy in which there is no heterogeneity in trading technologies. All households are able to access all assets available on the market without any restriction. I label this economy as the NHT (no heterogeneous trading) economy. Table I reports moments of asset prices in both of the economies I am considering. These results are generated by the simulating data from a model with 12,000 agents for 10,000 periods. Panels I and II report results for the cases of the BM economy and the NHT economy, respectively.

4.1 Asset Prices

In the upper part of table I, I report the maximum unconditional Sharpe ratio (SR) or market price of risk ($\frac{\sigma(m)}{E(m)}$), the standard deviation of the maximum SR ($Std(\frac{\sigma_t(m)}{E_t(m)})$), the equity risk premium $E\left(R^D_{t+1,t} - R^f_{t+1,t}\right)$, the standard deviation of excess returns $\sigma\left(R^D_{t+1,t} - R^f_{t+1,t}\right)$, the Sharpe ratio on equity, the mean risk-free rate $E\left(R^f_{t+1,t}\right)$, and the standard deviation of the risk-free rate $\sigma\left(R^f_{t+1,t}\right)$.

Benchmark Economy In the benchmark economy, the maximum SR is 0.37 and the standard deviation of the maximum SR is 4.04%. The equity premium is 7.53% and the Sharpe ratio on equity is .37. The average risk-free rate is 1.91% and its volatility is 2.27%. Clearly, the benchmark economy generates several key features of asset pricing observed in the data, such as high equity premiums; a low and stable risk free interest rate, and a relatively volatile Sharpe Ratio [2].

The large fraction of non-participant traders is critical for the results for high risk premiums. Those households that only hold risk free assets do not take any aggregate risk since their portfolio return is independent of the realization of aggregate shocks. Additionally, passive equity traders only take a limited amount of aggregate risks because of their relatively low and constant target equity share. Therefore, a large amount of aggregate risks has to be absorbed by a small fraction of active traders. In equilibrium, a high risk premium is necessary so that active traders are willing to bear these extra aggregate risks. The key mechanism is to concentrate the aggregate risk in a small fraction of the population.

No Heterogeneous Trading Economy In an economy where all the households are active traders, the asset pricing results are dampened. Compared to the benchmark case, the maximum SR is only 0.15 and the standard deviation of the maximum SR is down to 1.01%. The equity premium reduces to 3.01% and the Sharpe ratio on equity is only 0.15. The average risk-free rate increases to 3.09% and its volatility remains roughly the same, 2.27%. The heterogeneity in trading technologies considerably affects the patterns of asset pricing results. The reason for the low equity premium is clear: The aggregate risk is equally borne by all households and there is no

[2]The Sharpe ratio estimated from the data is enormous and highly counter-cyclical. My model still falls short to match data quantitatively. However, Chien, Cole, and Lustig (2010) extend a similar version of this model by introducing inertia investment behavior among some of the households. Their work shows that the inertia investment behavior helps significantly to explain the large counter-cyclical variation in Sharpe Ratio.

concentration of risk in a small fraction of households as in the benchmark economy.

Approximation In general, the prices of state contingent claims depend on the entire aggregate history. However, I am unable to keep track of the entire aggregate history of shocks in the state space. Following Chien, Cole, and Lustig (2011), I use truncated aggregate histories as state variables to forecast state contingent prices. In order to show the accuracy of my approximation, I report the implied R^2 in a linear regression of the actual realization of state contingent prices on the predicted state contingent prices which based on the truncated aggregate histories. This measure of precision is closer to the one by Krusell and Smith (1998). As shown in table I, the R^2 for this regression is higher than 0.9995 in the benchmark case and higher than 0.9999 in the case without heterogeneous trading technologies. This result shows that the approximation is accurate and comparable to others reported in the literature for models with heterogeneous agents and incomplete markets.

4.2 Welfare Costs of Business Cycle

The welfare cost of eliminating business cycles is defined as the average welfare difference between two economies: one with aggregate shocks and the other without. Given the fact that households are heterogeneous in terms of their wealth, income shocks, and trading technologies in the long run equilibrium, the average welfare of one economy is computed by taking the expectation across all idiosyncratic features of the population. In addition, I measure the average welfare gap between the two economies by the percentage of per period consumption. Therefore, the welfare cost is defined as the expected percentage of consumption compensation to a household in an economy without business cycles, so that this household is indifferent to join the benchmark economy. The welfare cost is reported at the bottom of table I.

Benchmark Economy In the benchmark economy, the welfare cost of business cycles is 6.49%. This number means that the average household in the benchmark economy is willing to relinquish up to 6.49% of his per period consumption in order to be in the other economy without aggregate uncertainty, all else being equal. The welfare cost is much larger than most of the findings in the body of literature. This result demonstrates that heterogeneous trading technologies play an important role not only in the patterns of asset pricing but also in the distributional effects of consumption. In the benchmark economy, those households with better trading technologies earn a higher return on their wealth, while those households with less sophisticated trading technologies earn a lower return. This phenomenon generates a distributional impact on consumption and eventually widens the welfare gap across households. However, the welfare inequality caused by heterogeneous trading technologies vanishes in an economy without business cycles. The reason for this is quite simple: Since all assets are risk free, the portfolio choice between equity and risk free bonds does not affect the return on the portfolios. There is no investment advantage for a household that owns an advanced trading technology. The returns on wealth between active traders and passive traders are identical in an environment without aggregate risk.

No Heterogeneous Trading Economy In my second exercise in which all the households are active traders, the welfare cost is only 1.46%. This low welfare cost is consistent with the findings in the cost of business cycles literature. This result suggests that the welfare cost of business cycles is less significant in an environment where all agents have sophisticated trading technologies and make no investment mistakes. This outcome can easily be understood. Because all households have the same trading technologies, there is no heterogeneity in portfolio choice. The income and consumption inequality are greatly reduced in this case. The aggregate risk does not amplify the distributional impact on consumption any more, so the welfare cost of business decreases considerably.

The amount of reduction in the welfare cost of business cycles can be seen as the average welfare gain from preventing the investment mistakes made by passive traders in my model. Clearly, the results show that the average welfare loss due to these investment errors is large, 5.03% of per period consumption (the welfare cost difference between the BM economy and the NHT economy). This number implies that the welfare cost of having inferior trading technologies is sizeable. Also, my findings shed a light on the importance of understanding the investment mistakes made by passive traders, since avoiding them can improve the average welfare of the society.

[Table 2 about here.]

4.3 Sensitivity Analysis

Risk Aversion Coefficient The benchmark calibration set the risk aversion coefficient to 4. Although my choice of risk aversion is in the range considered in many macroeconomics models, it is different from the choice made by Lucas (1987), who uses a log utility. More importantly, the welfare cost of business cycles might be sensitive to the risk aversion rate. Here, I investigate the sensitivity of the results to the change in risk aversion coefficient. I conduct two sensitivity analyses with respect to the change in risk aversion rate. In each analysis, I vary the risk aversion coefficient from 3 to 1.

The first analysis only considers the change in risk aversion rate while keeping all other parameters unchanged. Table II reports the results of my first analysis. The decrease in risk aversion rate lowers the risk premium as well as the welfare cost. The risk premium drops substantially, from 5.18% with risk aversion coefficient 3, to 1.28% in the log utility case. In addition, the welfare cost of eliminating business cycles decreases in a non-linear pattern. With a risk aversion rate of 3 or 2, the welfare costs are still very significant, 5.27% and 4.22%, respectively. However, it reduces sharply to 0.6% when I consider the case of log utility. This analysis demonstrates a close relationship between the risk premium and welfare cost of business cycles. This is not surprising, because the welfare cost of business cycles in my paper depends critically on the magnitude of consumption dispersion, which is based on the return difference between equity and risk free bonds. As the risk premium decreases, the heterogeneity in wealth returns reduces along with the welfare cost.

The first analysis indicates that when households become less and less risk averse, the model misses the calibration target, equity premium, more and more. Therefore, I conduct a second sensitivity analysis. For each risk aversion rate considered above, I adjust the composition between active traders and passive equity traders in order to match the historical risk premium as much as possible, while keeping all other parameters fixed. The results of the second analysis are shown in table III.

The first panel reports the results of the case in which the risk aversion coefficient is 3. In order to match the high historical risk premium, the fractions of active traders and passive equity traders are adjusted to be 3% and 47%, respectively. The asset pricing results are similar to those in my benchmark economy. The risk premium is high, 7.38%, and volatile, 19.20% in standard deviation, while the risk free rate is low, 2.25%, and stable, 1.66% in standard deviation. Most importantly, the welfare cost of inflation increases to 9.37%. The higher welfare cost result can be understood as follows. First, active traders are those who respond to the change in state contingent prices and are those who bear extra aggregate risk. Put simply, they are marginal traders who price the risk premium. Second, if these active traders still bear the same amount of aggregate risk as in the benchmark case, then the risk premium will drop since their risk aversion rate is lower now. In order to maintain the same high level of risk premium while giving a lower risk aversion rate, a larger amount of aggregate risk has to be concentrated and borne by a smaller fraction of active traders. As the fraction of active traders is adjusted from 10% to 3%, each active trader bears more aggregate risk, but is able to enjoy a even higher level of consumption in terms of compensation. The reduction in the fraction of active traders worsens the consumption inequality and, consequently, increases the welfare cost of business cycles.

Panels 2 and 3 report the results of the case with $\alpha = 2$ and 1, respectively. In both cases, I am unable to match the high risk premium shown in the data even when the fraction of active traders is set to be only 1% of the total population. The risk premiums of both cases are significantly smaller: 5.78% for the case with $\alpha = 2$ and only 3.04% for the case with log utility. Nevertheless, the welfare cost of business cycles is even higher, 9.56%, in the case where the risk aversion coefficient is 2. The reason for this is simply because of the higher inequality in consumption. Although the lower risk premium reduces the inequality of consumption by decreasing the heterogeneity of wealth return across the population, the smaller fraction of active traders amplifies the consumption inequality even more. The second effect on consumption inequality due to the diminishing size of active traders dominates the first effect resulting from the lower risk premium. Consequently, the welfare cost increases slightly. The last panel reports the results in a log utility case. The welfare cost drops substantially from 9.56% to 3.81% when the risk aversion coefficient changes from 2 to 1. This result is not surprising given the same composition of traders in both panels II and III.

The second sensitivity analysis demonstrates that the welfare cost of business cycles is even larger with a lower risk aversion coefficient whenever the historical, high risk premium can be matched in my calibration economy. Additionally, in the log utility case, the welfare cost of business cycles is still significant even if my calibration fails to match the risk premium. The welfare cost number is 3.81% in the log utility case when active traders comprise 1% of the total population.

[Table 3 about here.]

5 Conclusion

This paper demonstrates that heterogeneous trading technologies can play an important role not only in the patterns of asset pricing but also in the welfare cost of business cycles. In my calibrated model, a large amount of aggregate risk is borne by a small fraction of households, while a large fraction of households bear little or even no aggregate risk. The concentration of risk in a limited set of households drives the large risk premium in my model. As a result, sophisticated investors who hold a large fraction of equity are compensated with a much higher return on wealth while less sophisticated investors earn a lower return on their wealth. The wealth return difference worsens the income and consumption inequality. In addition, the new feature of my model, heterogeneous trading technologies, has no distributional effect on consumption in an economy without aggregate shocks, because the return difference between stocks and bonds vanishes. A ceasing of aggregate shocks can greatly improve the consumption inequality caused by the heterogeneity in investment behavior. Therefore, the welfare cost of business cycles is more pronounced in an economy with heterogeneous trading technologies.

In the case with homogeneous trading technologies, the results show an insignificant welfare cost of business cycles. This result implies a large welfare difference between economies with and those without heterogeneous trading technologies, which can be thought of as the welfare cost of investment mistakes made by passive traders. These mistakes include relinquishing high risk premiums and missing the market timing. The significant welfare cost of investment errors highlights the importance of the study of household finance. If I can find a way to avoid these investment mistakes, the average welfare of the society can be improved considerably. Additionally, the results indicate that the welfare improvement from avoiding these investment errors is comparable to that of eliminating business cycles. Therefore, if the elimination of aggregate output volatility is infeasible or extremely expensive, then putting a lot of resources into preventing investment mistakes made by households may be reasonable.

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Table I: Results of BM and NHT economy

	Panel I: BM Economy	Panel II: NHT Economies
Active Traders	10%	100%
Passive Equity Traders	40%	0%
Non-participants	50%	0%
	Asset Pricing	
$\frac{\sigma(M)}{E(M)}$	0.3739	0.1528
$Std\left[\frac{\sigma_t(M)}{E_t(M)}\right]$	4.0440	1.0106
$E\left(R^D_{t+1,t} - R^f_{t+1,t}\right)$	7.5368	3.0077
$\sigma\left(R^D_{t+1,t} - R^f_{t+1,t}\right)$	20.3867	19.7216
Sharpe Ratio	0.3697	0.1525
$E\left(R^f_{t+1,t}\right)$	1.9141	3.0900
$\sigma\left(R^f_{t+1,t}\right)$	2.2729	2.2539
	Approximation	
R^2	0.9995	0.9999
	Welfare Cost	
Welfare Cost of Business Cycle(%)	6.49	1.45

BM refers to the benchmark economy and NHT refers to no heterogeneous trading economy. Storesletten, Telmer, and Yaron (2007) calibration of idiosyncratic shocks without counter cyclical variation risk; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\alpha = 4$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $12,000$ agents and $10,000$ periods.

Table II: Results of Sensitivity Analysis I

	Panel I	*Panel II*	*Panel III*
risk aversion rate (α)	3	2	1
Active Traders	10%	10%	10%
Passive Equity Traders	40%	40%	40%
Non-participants	50%	50%	50%
	Asset Pricing		
$\frac{\sigma(M)}{E(M)}$	0.2856	0.1868	0.0872
$Std\left[\frac{\sigma_t(M)}{E_t(M)}\right]$	3.2615	2.1882	1.0307
$E\left(R^D_{t+1,t} - R^f_{t+1,t}\right)$	5.1849	3.0434	1.2799
$\sigma\left(R^D_{t+1,t} - R^f_{t+1,t}\right)$	18.3498	16.4103	14.6313
Sharpe Ratio	0.2826	0.1855	0.0875
$E\left(R^f_{t+1,t}\right)$	2.8186	3.8275	4.8322
$\sigma\left(R^f_{t+1,t}\right)$	1.6744	1.0946	0.5360
	Approximation		
R^2	0.9997	0.9997	0.9998
	Welfare Cost		
Welfare Cost of Business Cycle(%)	5.27	4.22	0.6

Storesletten, Telmer, and Yaron (2007) calibration of idiosyncratic shocks without counter cyclical variation risk; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $12,000$ agents and $10,000$ periods.

Table III: Results of Sensitivity Analysis II

	Panel I	*Panel II*	*Panel III*
risk aversion rate (α)	3	2	1
Active Traders	3%	1%	1%
Passive Equity Traders	47%	49%	49%
Non-participants	50%	50%	50%
	Asset Pricing		
$\frac{\sigma(M)}{E(M)}$	0.3957	0.3461	0.2238
$Std\left[\frac{\sigma_t(M)}{E_t(M)}\right]$	7.5114	9.6565	9.5149
$E\left(R^D_{t+1,t} - R^f_{t+1,t}\right)$	7.3828	5.7938	3.0422
$\sigma\left(R^D_{t+1,t} - R^f_{t+1,t}\right)$	19.2098	17.7495	15.6695
Sharpe Ratio	0.3843	0.3264	0.1941
$E\left(R^f_{t+1,t}\right)$	2.2503	3.0917	4.3834
$\sigma\left(R^f_{t+1,t}\right)$	1.6619	1.0823	0.5280
	Approximation		
R^2	0.9995	0.9995	0.9997
	Welfare Cost		
Welfare Cost of Business Cycle(%)	9.37	9.56	3.81

Storesletten, Telmer, and Yaron (2007) calibration of idiosyncratic shocks without counter cyclical variation risk; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $12,000$ agents and $10,000$ periods.