Research Division
Federal Reserve Bank of St. Louis
Working Paper Series

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Working Paper 2013-038D
http://research.stlouisfed.org/wp/2013/2013-038.pdf

December 2013
Revised February 2015

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The Limitations of Forward Guidance*

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February 7, 2015

Abstract

This article examines forward guidance via news shocks to the monetary policy rule in a nonlinear New Keynesian model with an occasionally binding zero lower bound (ZLB) constraint on the policy rate. Three findings emerge: (1) The stimulative effect of forward guidance is small when the economy is in a deep recession or households expect a slow recovery; (2) News shocks are more stimulative than unanticipated shocks at and away from the ZLB; (3) Forward guidance can have longer and larger cumulative effects on output without increasing the variance of the news shocks by redistributing the news over longer horizons.

Keywords: Forward Guidance; News Shocks; Zero Lower Bound; Nonlinear

JEL Classifications: E43; E58; E61

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1 INTRODUCTION

The global economic slowdown in 2008 led many central banks to sharply reduce their policy rates. When rates could not be reduced further, some central banks resorted to unconventional policies, such as forward guidance. This article examines the stimulative effect of forward guidance at and away from the zero lower bound (ZLB) on the policy rate. Forward guidance refers to central bank communication about future policy, which takes many forms including announcements about objectives, contingencies, policy actions, and speeches. We focus on communication about the future path of the policy rate, which operates through the real interest rate to affect current demand.

Several papers examine the effects of forward guidance in an economy with a binding ZLB constraint through the perspective of optimal monetary policy under commitment (i.e., a central bank promises to implement a specific policy regardless of any changes in economic conditions). Eggertsson and Woodford (2003) and Jung et al. (2005) analyze forward guidance by solving for the optimal commitment policy assuming the nominal interest rate initially equals zero and cannot return to its ZLB once it increases. They find the optimal commitment policy is to maintain a policy rate equal to zero even after the natural real interest rate rises. Such a policy generates higher future inflation and lowers the real interest rate, which moderates the sharp declines in output and inflation that occur at the ZLB.[1] Levin et al. (2010) then show the optimal policy can fully stabilize the economy after small shocks but not after large and persistent shocks. In that situation, they argue a central bank must employ other unconventional policies, such as large-scale asset purchases, to stabilize the economy. Adam and Billi (2006) relax the assumption that the nominal rate is initially at zero by allowing the ZLB to occasionally bind. They find the optimal policy under commitment is to react more aggressively to shocks that cause both output and inflation to decline.[2]

In keeping with the optimal commitment policy literature, we examine the economic effects of forward guidance in a nonlinear New Keynesian model where the policy rate is constrained by the ZLB and the central bank has perfect credibility. This paper, however, models forward guidance as anticipated news shocks similar to Laséen and Svensson (2011), so news about future policy rates affects expectations in much the same way as news about future technology.[3] The central bank implements forward guidance by communicating its intended policy rate path over a particular forecast horizon. The news is then the difference between the central bank's announced policy rate path and the policy rate path that would have occurred in the absence of forward guidance. This modeling approach has two main benefits. One, the central bank can provide forward guidance in any state of the economy. Two, the effects of providing additional news can be isolated from the effects caused by a longer forward guidance horizon. A promise to delay an increase in the future policy rate generates higher future inflation, lowers real interest rates, and offsets the declines in current inflation and output—the same mechanism driving the optimal policies under commitment. That stimulative effect, however, is more limited when the economy is in a deep recession.

To our knowledge, this paper is the first to study forward guidance with news shocks using a

[1] Krugman (1998) uses the same intuition to suggest several policy proposals. Reifschneider and Williams (2000) apply this intuition by augmenting a standard Taylor rule in a dynamic model to neutralize the effects of the ZLB.

[2] Adam and Billi (2007) find discretionary monetary policy is unable to generate the future inflation that is necessary to offset shocks that are amplified by the ZLB constraint. Werning (2011) builds on the optimal commitment policy literature, which is usually modeled in discrete time, by using a continuous-time model with a ZLB constraint. He finds interest rates jump discretely under the optimal commitment policy despite fundamentals that vary continuously.

[3] Milani and Treadwell (2012) estimate a New Keynesian model (without a ZLB) with news shocks to the monetary policy rule. They find that news shocks have larger and more persistent effects on output than unanticipated shocks.

nonlinear solution method that endogenizes the occurrence of ZLB events.[4] Demand shocks occasionally push the nominal interest rate to its ZLB. The size of those shocks and whether news shocks occur determine how long the nominal interest rate remains at zero. Households use information about existing economic conditions and the entire distribution of news shocks to form expectations about whether the ZLB will bind in future periods.[5] As demand falls, the ZLB constraint further limits the stimulative effect of forward guidance by preventing current and future policy rates from falling below zero. We examine the economic effects of forward guidance across the entire distribution of news shocks, while imposing a ZLB constraint on current and future policy rates and holding the variance of the news shocks constant across forward guidance horizons.

Three key findings emerge from our analysis of forward guidance:

1. The stimulative effect of forward guidance is small when the economy is in a deep recession or households expect a slow recovery.
2. News shocks are more stimulative than unanticipated shocks at and away from the ZLB.
3. Forward guidance can have longer and larger cumulative effects on output without increasing the variance of the news shocks by redistributing the news over longer horizons.

We compare our model's predictions to survey data collected immediately before and after recent episodes of forward guidance by the Fed. Our theoretical results reveal that if households simultaneously receive forward guidance and information that the recovery is weaker than expected, then the observed stimulative effect of forward guidance will be smaller and possibly negative. Data indicate that some of the Fed's recent forward guidance policies have been successful at flattening the yield curve, although the yield curve was fairly flat prior to those announcements. Moreover, the announcements were usually followed by a decline in consensus forecasts of output and inflation. Those outcomes suggest that recent forward guidance has had a limited effect on the economy.

Campbell et al. (2012) introduce two terms to differentiate the types of forward guidance: Delphic and Odyssean. Delphic forward guidance is a central bank's forecast of its own policy, which is based on its projections for inflation and real GDP growth as well as an established policy rule. When combined with economic projections, Delphic forward guidance can help clarify the central bank's policy strategy.[6] Odyssean forward guidance is a commitment to deviate from the established policy rule at some point in the future by promising to set the policy rate lower than the policy rule recommends. News shocks are one way to model Odyssean forward guidance.

In recent years, central banks have used both date-based and threshold-based forward guidance. Date-based forward guidance provides information on the intended path of policy over a fixed period of time and is frequently modeled using an interest rate peg. To a modeler, an interest rate

[4]There are other papers that use nonlinear methods to examine the effects of a ZLB constraint. See, for example, Aruoba and Schorfheide (2013), Basu and Bundick (2012), Fernández-Villaverde et al. (2012), Gavin et al. (2014), Gust et al. (2013), Mertens and Ravn (2014), Nakata (2012), Plante et al. (2014), and Wolman (2005).

[5]Several papers that solve for optimal commitment policy in models with a ZLB also use global solution methods, specifically collocation methods that involve discretizing the model's state space. Nakov (2008) parameterizes the expectation functions, which is not a contraction mapping technique and may fail to find a solution. Adam and Billi (2006) and Kato and Nishiyama (2005) iterate on the value function approximated with cubic splines. We also use a collocation method but approximate the policy functions with linear splines, which are better suited for approximating the kink in the policy functions at the ZLB than either cubic splines or other smooth approximating functions.

[6]See the Bank of England (2013) for a discussion of how forward guidance helps the public form more accurate expectations about future central bank policy. See den Haan (2013) for a collection of essays about forward guidance and the International Monetary Fund (2013) for a detailed account of recent unconventional monetary policies.

peg represents a promise by the central bank to fix the policy rate for a set number of periods after the policy rate would have otherwise risen. Many researchers who use an interest rate peg to model forward guidance, such as Carlstrom et al. (2012), assume the policy rate initially equals zero and stays there for a fixed period of time.[7] Once that period ends, the policy rate starts to rise and never returns to zero. Instead of restricting the model to one ZLB event of predetermined length, we assume a set of stochastic shocks determines whether the ZLB binds. Another type of forward guidance is threshold-based where the central bank agrees to keep a certain policy in place until a particular event occurs. For example, the central bank might announce that it intends to keep its policy rate at zero until the unemployment rate falls below some value. That style of forward guidance recognizes that economic shocks or other factors can change when the policy rate adjusts.

Our news shock approach is similar to threshold-based forward guidance because it enables the policy rate to endogenously respond to changes in economic conditions unlike an interest rate peg which fixes the policy rate regardless of economic conditions. Specifically, expansionary news shocks push down future policy rates, but those rates can still rise above zero if the endogenous response of the policy rate to output and inflation is strong enough to compensate for the news shocks. Another advantage of using news shocks is that households' expectations incorporate the possibility that the central bank alters its previous forward guidance policy. For example, suppose the central bank announces a plan to keep its policy rate lower than its policy rule suggests for the next q quarters. Households account for the possibility that the central bank might change its policy in the intervening periods. A strict interest rate peg, in contrast, does not allow for the possibility that future economic conditions may cause an unanticipated shift in the policy rate.[8]

The paper is organized as follows. Section 2 provides a post-financial crisis account of Federal Open Market Committee (FOMC) forward guidance in its policy statements. Section 3 describes the nonlinear model, including the specification of forward guidance, its calibration, and its solution method. Section 4 quantifies the stimulative effect of forward guidance at and away from the ZLB. Specifically, we describe the decision rules and the impulse responses to news shocks across forward guidance horizons. Section 5 examines how our model's predictions compare with the data using case studies of specific FOMC forward guidance announcements. Section 6 concludes.

2 FORWARD GUIDANCE AT THE FEDERAL RESERVE

The FOMC uses two primary methods to communicate information about the path of future policy rates. One, it releases the individual forecasts of its members four times per year, but that information can be diverse and reveal differences of opinions. Two, it provides forward guidance about the future federal funds rate in its policy statements and has consistently done so since 2008.[9]

At the December 16, 2008 meeting, the FOMC cut the federal funds rate to 0.25% and announced it would remain at that unusually low level for an *extended* period. The FOMC continued to use that vague language until its August 9, 2011 statement which said that low range was likely

[7] Blake (2012) examines alternate ways to peg the policy rate in a model with an endogenous monetary policy rule.

[8] Walsh (2009) contends inflation-targeting central banks that promise expansionary future policies may lack the credibility to fulfill that promise. In our setup, households account for this time inconsistency problem.

[9] Various forward guidance policies have also been used by the Bank of Canada, Bank of England, European Central Bank, Bank of Japan, Reserve Bank of New Zealand, Norges Bank, and the Riksbank. See Andersson and Hofmann (2010), Filardo and Hofmann (2014), Kool and Thornton (2012), Moessner and Nelson (2008), Svensson (2011), and Swanson and Williams (2014) for an overview of the policies and econometric analysis of their economic impacts.

warranted "…*at least through mid-2013*." The announcement was the FOMC's first use of date-based forward guidance, and it had a major effect on current and expected future interest rates.

The next change in forward guidance occurred in the statement release following the January 25, 2012 FOMC meeting. This policy statement was different in two ways. One, the time that the federal funds rate was expected to remain at zero was updated to read "…*at least through late 2014*," which was an increase of six quarters. Two, the FOMC expressed a more pessimistic economic outlook and indicated the projected path for the federal funds rate was conditional on that outlook. Subsequent speeches by policymakers provided support for that assessment.

The forward guidance provided in the January 25, 2012 statement was likely viewed as Delphic for two reasons. One, the statement expressed more pessimism about the economy, which suggests the FOMC's policy rule was already projecting a later date for raising the federal funds rate. Two, the FOMC never stated the new projected interest rate path was different from the path implied by its policy rule. By late summer 2012, the economy continued to disappoint policymakers and the statement issued following the September 13, 2012 meeting was amended to read:

> To support continued progress toward maximum employment and price stability, …*a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens*. …the Committee also decided today to keep the target range for the federal funds rate at 0 to $1/4$ percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted *at least through mid-2015*.

That statement included a 2-quarter extension to the time the FOMC promised to keep its policy rate at zero and a new pledge to add $85 billion to the Fed's balance sheet every month until the labor market significantly improved. The language "…for a considerable time after the economic recovery strengthens" conveys Odyssean forward guidance. Without that language, it suggests the FOMC would raise its policy rate as the recovery strengthens. On the other hand, the FOMC statement included information about business spending that led to lower real GDP growth forecasts.

On December 12, 2012, the FOMC switched its forward guidance from the date-based language "at least through mid-2015" to threshold-based language. The policy statement read:

> …this exceptionally low range for the federal funds rate will be appropriate *at least as long as the unemployment rate remains above* 6-$1/2$ *percent*, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee's 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored.

FOMC participants' forecasts indicated the unemployment rate would likely hit 6.5% in mid-2015. Therefore, the statement was not intended to change expectations about when the policy rate would rise, but rather to emphasize that the timing of the FOMC's decision to increase its policy rate is conditional on inflation expectations and labor market conditions. The phrase "at least as long as the unemployment rate remains above 6-$1/2$ percent" emphasizes that the unemployment threshold was not a trigger for when the FOMC would automatically raise its policy rate.

Over the next year, the labor market continued to improve and it was evident the unemployment rate might cross the 6.5% threshold. On December 18, 2013, the FOMC decided to begin tapering their monthly asset purchases and reformulated their forward guidance communication to explain how they intended to react to future economic developments. Specifically, the statement read:

> …it likely will be appropriate to maintain the current target range for the federal funds rate *well past the time that the unemployment rate declines below* 6-$1/2$ *percent*, especially if projected inflation continues to run below the Committee's 2 percent longer-run goal.

The change in language from "at least as long as" to "well past" may have been viewed as Odyssean in nature because the FOMC implied that the federal funds rate would likely remain near zero even though stronger economic conditions would normally cause the FOMC to raise its policy rate.

In 2014, the FOMC continued to reduce its asset purchases and focused on communicating state-contingent forward guidance. For example, the July 18, 2014 statement read:

> ...it likely will be appropriate to maintain the current target range for the federal funds rate for a *considerable time after the asset purchase program ends...*

FOMC participant forecasts of output and inflation for 2015 were near their long-run levels, but their federal funds rate forecasts remained about 2 percentage points below its long-run level. We believe that statement also conveys Odyssean forward guidance since the FOMC communicated that they expected to maintain a low policy rate well after concluding their asset purchase program.

3 Economic Model, Calibration, and Solution Method

This section develops a textbook New Keynesian model in which the ZLB constraint on the short-term nominal interest rate occasionally binds due to persistent discount factor (demand) shocks. Forward guidance enters our model through anticipated shocks to the monetary policy rule.

3.1 Households A unit measure of households choose $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility, $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t [c_t^{1-\gamma}/(1-\gamma) - \chi n_t^{1+\eta}/(1+\eta)]$, where c_t is consumption, n_t is labor hours, b_t is the real value of a 1-quarter nominal bond, γ is the coefficient of relative risk aversion, $1/\eta$ is the Frisch elasticity of labor supply, $\tilde{\beta}_0 \equiv 1$, and $\tilde{\beta}_t = \prod_{i=1}^{t} \beta_i$ for $t > 0$. Following Eggertsson and Woodford (2003), β_t is a time-varying discount factor that follows $\beta_t = \bar{\beta}(\beta_{t-1}/\bar{\beta})^{\rho_\beta} \exp(\upsilon_t)$, where $\bar{\beta}$ is the steady-state discount factor, $0 \leq \rho_\beta < 1$, and $\upsilon_t \sim \mathbb{N}(0, \sigma_\upsilon^2)$.

Each household's choices are constrained by $c_t + b_t = w_t n_t + i_{t-1} b_{t-1}/\pi_t + d_t$, where π_t is the gross inflation rate, w_t is the real wage rate, i_t is the gross nominal interest rate, and d_t are profits from intermediate firms. The optimality conditions to each household's problem imply

$$w_t = \chi n_t^\eta c_t^\gamma, \tag{1}$$

$$1 = i_t E_t[\beta_{t+1}(c_t/c_{t+1})^\gamma/\pi_{t+1}]. \tag{2}$$

3.2 Firms The production sector consists of monopolistically competitive intermediate goods firms and a final goods firm. Intermediate firm $f \in [0, 1]$ produces a differentiated good, $y_t(f)$, according to $y_t(f) = n_t(f)$, where $n_t(f)$ is the labor used by firm f. Each intermediate firm chooses its labor input to minimize operating costs, $w_t n_t(f)$, subject to its production function. The final goods firm purchases $y_t(f)$ units from each intermediate goods firm to produce the final good, $y_t \equiv [\int_0^1 y_t(f)^{(\theta-1)/\theta} df]^{\theta/(\theta-1)}$ according to a Dixit and Stiglitz (1977) aggregator, where $\theta > 1$ measures the elasticity of substitution between the intermediate goods. The optimality condition to the firm's profit maximization problem then yields the demand function for intermediate inputs given by $y_t(f) = (p_t(f)/p_t)^{-\theta} y_t$, where $p_t = [\int_0^1 p_t(f)^{1-\theta} df]^{1/(1-\theta)}$ is the price of the final good.

Following Rotemberg (1982), each firm faces a price adjustment cost, $adj_t(f)$, which emphasizes the negative effect that price changes can have on customer-firm relationships. Using the function in Ireland (1997), $adj_t(f) = \varphi[p_t(f)/(\bar{\pi} p_{t-1}(f)) - 1]^2 y_t/2$, where $\varphi \geq 0$ scales the size of the adjustment costs and $\bar{\pi}$ is the steady-state gross inflation rate. Real profits are then given by $d_t(f) = (p_t(f)/p_t)y_t(f) - w_t n_t(f) - adj_t(f)$. Firm f chooses its price, $p_t(f)$, to maximize the expected discounted present value of real profits $E_t \sum_{k=t}^{\infty} \lambda_{t,k} d_k(f)$, where $\lambda_{t,t} \equiv 1$, $\lambda_{t,t+1} = \beta_{t+1}(c_t/c_{t+1})$ is the pricing kernel, and $\lambda_{t,k} \equiv \prod_{j=t+1}^{k} \lambda_{j-1,j}$. In a symmetric equilibrium, all firms make identical decisions and the optimality condition implies

$$\varphi\left(\frac{\pi_t}{\bar{\pi}} - 1\right)\frac{\pi_t}{\bar{\pi}} = (1-\theta) + \theta w_t + \varphi E_t\left[\lambda_{t,t+1}\left(\frac{\pi_{t+1}}{\bar{\pi}} - 1\right)\frac{\pi_{t+1}}{\bar{\pi}}\frac{y_{t+1}}{y_t}\right]. \tag{3}$$

Without price adjustment costs (i.e., $\varphi = 0$), the real marginal cost of producing a unit of output (w_t) equals $(\theta-1)/\theta$, which is the inverse of a firm's markup of price over marginal cost.

3.3 MONETARY POLICY Households receive forward guidance (news) about future monetary policy through anticipated monetary policy shocks. Specifically, the central bank sets the gross nominal interest rate according to the following Taylor rule, subject to the ZLB constraint:

$$\begin{gathered} i_t = \max\{1, \bar{\imath}(\pi_t/\pi^*)^{\phi_\pi}(y_t/\bar{y})^{\phi_y}\exp(x_t)\}, \\ x_t \equiv \sum_{j=0}^{q} \alpha_j \varepsilon_{t-j}, \quad \sum_{j=0}^{q} \alpha_j^2 = 1, \end{gathered} \tag{4}$$

where π^* is the inflation rate target, ϕ_π and ϕ_y are the policy responses to inflation and output, $\varepsilon_t \sim \mathbb{N}(0, \sigma_\varepsilon^2)$ is a monetary policy shock, α_j is the intensity of the news j periods in the future, and $q \geq 0$ is the forward guidance horizon. For example, when $(\alpha_0, \alpha_1, \ldots, \alpha_q) = (1, 0, \ldots, 0)$, the shock is unanticipated (no forward guidance) and when $(\alpha_0, \alpha_1, \ldots, \alpha_q) = (0, 0, \ldots, 1)$, households anticipate the shock q periods before it occurs (q-period forward guidance). The restriction on α_j guarantees the variance of the MA(q) process, x_t, is the same as ε_t. In other words, the distribution of the news does not affect the variance of the monetary policy shock process. That restriction is particularly important because it isolates the effect of lengthening the forward guidance horizon.[10]

3.4 EQUILIBRIUM, CALIBRATION, AND SOLUTION METHOD The resource constraint is given by $c_t = y_t - adj_t \equiv y_t^{gdp}$, where y_t^{gdp} includes the value added by intermediate firms, which is their output minus quadratic price adjustment costs. Thus, y_t^{gdp} represents real GDP in the model. A competitive equilibrium consists of sequences of quantities $\{c_t, n_t, y_t, b_t\}_{t=0}^{\infty}$, prices $\{w_t, i_t, \pi_t\}_{t=0}^{\infty}$, and discount factors $\{\beta_t\}_{t=0}^{\infty}$ that satisfy each household's and each firm's optimality conditions, (1)-(3), the monetary policy rule, (4), the production function, $y_t = n_t$, the bond market clearing condition, $b_t = 0$, the discount factor process, and the resource constraint.

We calibrate the model at a quarterly frequency using common values in the monetary policy literature. The risk-free real interest rate is set to 4 percent annually, which implies a steady-state quarterly discount factor, $\bar{\beta}$, equal to 0.99. Both the constant of relative risk aversion, γ, and the Frisch elasticity of labor supply, $1/\eta$, are set to 1. The leisure preference parameter, χ, is set so that steady-state labor equals $1/3$ of the available time. The elasticity of substitution between goods, θ, is calibrated to 6, which corresponds to a 20 percent average markup of price over the wage rate. The costly price adjustment parameter, φ, is set to 58.25, which is similar to a Calvo (1983) price-setting specification in which prices change on average once every four quarters. The steady-state

[10] Appendix B describes how to model an exogenous interest rate peg in a stochastic model with an occasionally binding ZLB constraint. It then solves that model and highlights the comparative advantages of using news shocks.

inflation rate, $\bar{\pi}$, is calibrated to 1.005, so the annual target, π^*, is 2%. The monetary response to changes in inflation, ϕ_π, is set to 1.5 and the response to changes in output, ϕ_y, is set to 0.1.

Richter and Throckmorton (2015) show that as the persistence of a shock process increases, the standard deviation of that shock must decline; otherwise, our algorithm will not converge to a minimum state variable solution. The failure to converge occurs because the economy either remains at the ZLB too long when the shocks are very persistent or falls to the ZLB too frequently when the processes are highly volatile. We set the persistence of the discount factor, ρ_β, to 0.8 and the standard deviation of the shock, σ_υ, to 0.0025. We chose those values so that the average duration of each ZLB event in our model is consistent with the length of time forecasters expected the policy rate to remain at its ZLB in 2009. They are also the same values used in Fernández-Villaverde et al. (2012). The standard deviation of the monetary policy shock, σ_ε, is set to 0.0025.

We solve the model using the policy function iteration algorithm described in Richter et al. (2014), which is based on the theoretical work on monotone operators in Coleman (1991). This solution method discretizes the state space and uses time iteration to solve for the updated policy functions until the tolerance criterion is met. We use linear interpolation to approximate future variables, since this method accurately captures the kink in the policy functions, and Gauss-Hermite quadrature to numerically integrate. See Appendix C for a formal description of the algorithm.[11]

4 FORWARD GUIDANCE AND THE STATE OF THE ECONOMY

This section presents our theoretical results. We first show the largest possible stimulative effect that could occur at the ZLB under 1-quarter forward guidance and then discuss two main reasons why the stimulative effect is more likely to be much smaller. Next, we examine how the stimulative effect changes when households receive news over forward guidance horizons up to 10 quarters.

4.1 ONE-QUARTER HORIZON ($q = 1$) We begin by showing the stimulative effect of forward guidance across the entire distribution of news shocks. Figure 1 plots the decision rules for real GDP, the inflation rate, and the current and expected future nominal interest rates as a function of the monetary policy shock, $\hat{\varepsilon}_t$, which ranges from -1% to 1%.[12] The subscript on the shock represents the period in which households learn about the shock and not necessarily the period the shock impacts the economy. If, for example, the central bank provides no forward guidance, then $\hat{\varepsilon}_t$ represents an unanticipated monetary policy shock, which is observed and impacts the economy in period t. When the central bank provides 1-quarter forward guidance, $\hat{\varepsilon}_t$ represents a news shock that households learn about in period t but does not impact the economy until period $t+1$. The news shocks in the policy rule create an innovation in the expected value of future interest rates. One way to map the size of those shocks to actual forward guidance policies is to look at the difference between mean forecasts of interest rates before and after central bank communication.

We focus on a cross section of the decision rules where the initial notional interest rate ($\tilde{i}_0^*$)—the nominal interest rate a central bank would set if there were no ZLB constraint—equals zero. When the initial notional rate is zero, the discount factor is 1.2% above its steady state. The high discount factor depresses real GDP by about 1.2%. Households expect the discount factor will follow its law

[11] Benhabib et al. (2001) show that models with a ZLB constraint have two steady-state equilibria. See Gavin et al. (2014) for a discussion of the equilibrium that our algorithm converges to in both a deterministic and stochastic model.

[12] In our results, a hat denotes percent deviation from the deterministic steady state (i.e., for some generic variable x in levels, $\hat{x}_t \equiv 100(x_t - \bar{x})/\bar{x}$) and a tilde denotes a net rate (i.e., for some gross rate x, $\tilde{x}_t = 100(x_t - 1)$).

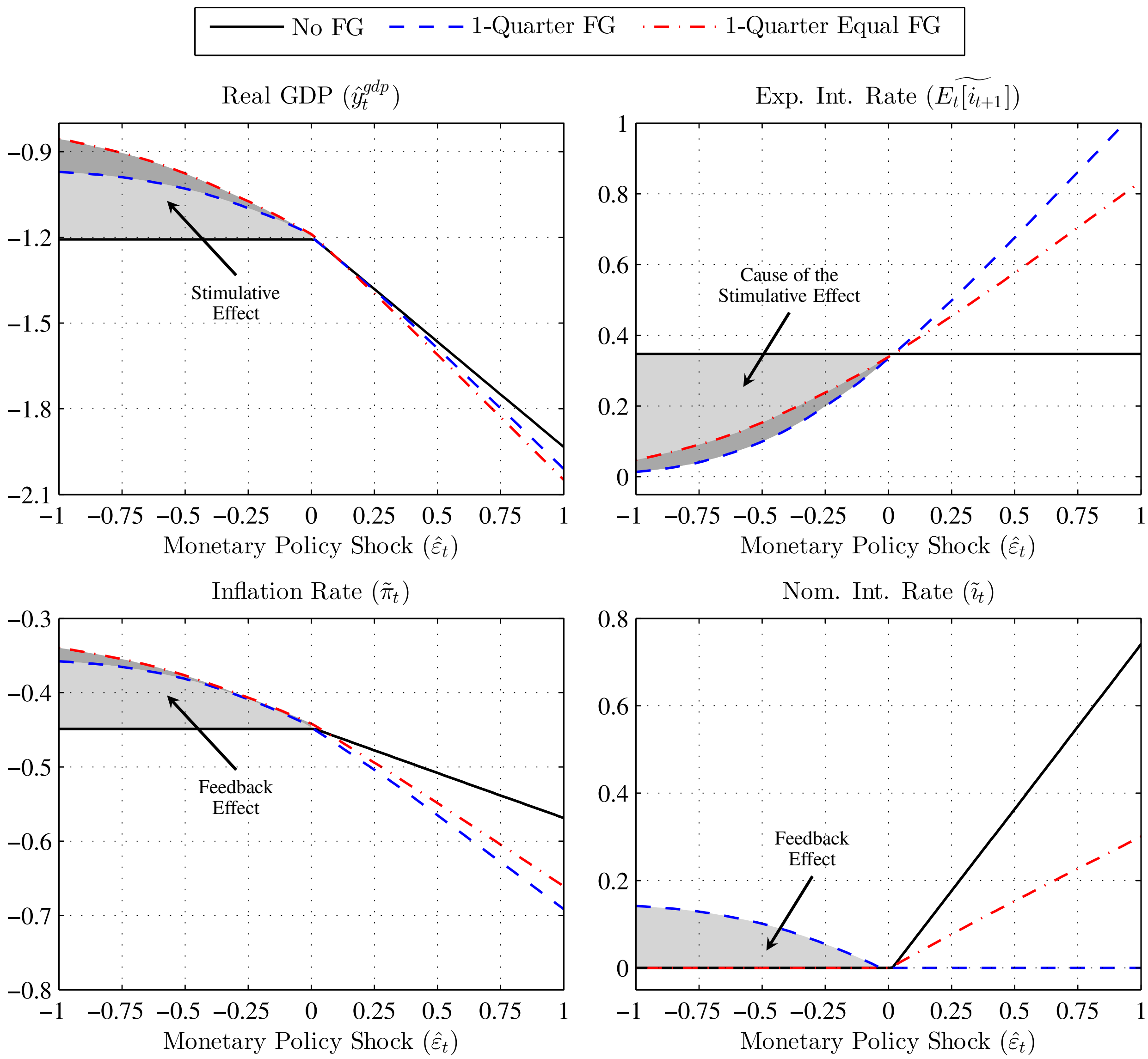

Figure 1: Decision rules as a function of the monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$ (solid line), 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line), and 1-quarter equal forward guidance, $(\alpha_0, \alpha_1) = (\sqrt{1/2}, \sqrt{1/2})$ (dash-dotted line). The values are based on a cross section where the initial notional rate equals zero.

of motion and revert to its steady state over time. If the central bank does not intervene, that belief raises the expected nominal interest rate in all future periods above zero. In this cross section, next period's expected nominal interest rate is equal to 0.35%. We chose this state of the economy ($\hat{\beta}_{-1} = 1.2$) because it produces the largest possible stimulative effect of forward guidance at the ZLB. Later in the paper, we discuss reasons why the stimulative effect is typically smaller.

When $(\alpha_0, \alpha_1) = (1, 0)$ (solid line), the central bank provides no forward guidance, so $\hat{\varepsilon}_t$ represents an unanticipated policy shock. If $\hat{\varepsilon}_t > 0$, then the shock contracts economic activity by raising the current nominal interest rate and lowering inflation and real GDP. The expected nominal interest rate is unaffected since the shock is serially uncorrelated. If, on the other hand, $\hat{\varepsilon}_t < 0$, then monetary policy has no impact on the current nominal interest rate since it is already at its ZLB.

Thus, the decision rules are flat when $\hat{\varepsilon}_t < 0$, meaning conventional monetary policy is ineffective.

When $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line), the central bank provides households with 1-quarter forward guidance. A central bank announcement this period about future monetary policy shocks can affect the current economy even though the shock has not yet happened. For example, suppose households receive information in period t that an expansionary monetary policy shock, $\hat{\varepsilon}_t < 0$, will occur in period $t+1$. If the discount factor is *not* expected to revert to its mean (i.e., the economy is expected to stagnate), then forward guidance will have hardly any stimulative effect since households expect that next period's nominal interest rate will remain near its ZLB. In this figure, the initial discount factor is above its mean and households expect it to decline in the future. That belief increases expected future nominal interest rates. When households are informed this period about an expansionary monetary policy shock next period, they expect next period's nominal interest rate to increase less than if they received no forward guidance. That expectational effect stimulates real GDP, which pushes up inflation and the current nominal interest rate—what we call *feedback effects*—even though the discount factor remains 1.2% above its steady state.[13]

Another way to understand how forward guidance stimulates demand is through an intertemporal consumption smoothing motive. Households know an expansionary monetary policy shock will occur in period $t+1$ and expect higher future consumption. Consequently, households raise current consumption to smooth their consumption across time. The increase in demand pushes up real GDP and inflation, which feeds into the Taylor rule and drives up the current nominal interest rate. The light-shaded regions in figure 1 represent the effects of 1-quarter forward guidance.

The increase in the current nominal interest rate dampens the stimulative effect of 1-quarter forward guidance. The central bank could provide *additional* stimulus in the form of an unanticipated shock in the current period that would mitigate the feedback effect (e.g., set $(\alpha_0, \alpha_1) = (1, 1)$), but it could also *redistribute* the news so the variance of the news process remains constant while still enhancing the stimulative effect. An example of that policy is $(\alpha_0, \alpha_1) = (\sqrt{1/2}, \sqrt{1/2})$ (dash-dotted line), which we call 1-quarter equal forward guidance. That specification equally shocks the policy rule in periods t and $t+1$ but keeps the variance of the news process constant.

The dark-shaded regions in figure 1 represent the marginal effects on the decision rules of using 1-quarter equal forward guidance instead of 1-quarter forward guidance. The effects of 1-quarter equal forward guidance differ in two ways. One, the current expansionary policy shock further stimulates real GDP because it eliminates the feedback effect that causes the current nominal interest rate to rise with 1-quarter forward guidance. Two, the smaller expansionary policy shock next period with 1-quarter equal forward guidance moderates the decline in the expected nominal interest rate. Overall, the stimulative effect of a lower current nominal interest rate dominates the negative effects of a smaller drop in the expected nominal interest rate. Thus, real GDP rises more with 1-quarter equal forward guidance at the cost of only a small increase in the inflation rate.

A larger expansionary shock under both 1-quarter and 1-quarter equal forward guidance have a diminishing positive impact on real GDP. For example, a -0.5% shock under 1-quarter forward guidance increases real GDP by 0.18 percentage points (from -1.21% to -1.03% below steady state), whereas a -1% shock increases real GDP by 0.23 percentage points (from -1.21% to -0.98% below steady state). Thus, doubling the strength of the news only leads to a small additional increase in real GDP (0.05 percentage points). That result stems from the ZLB constraint on the possible realizations of the nominal interest rate next period. A larger shock causes a greater

[13] Appendix A shows that the ZLB constraint significantly reduces the stimulative effect of forward guidance.

fraction of the future nominal interest rate distribution to be constrained by the ZLB. That fact is evident from the decision rule for the expected (net) nominal interest rate, which approaches 0 as the strength of the news increases. The result is important because it means that monetary policy shocks have nonlinear effects on the economy when households anticipate that the ZLB may bind in the future, which is in sharp contrast with linear models that do not include a ZLB constraint.

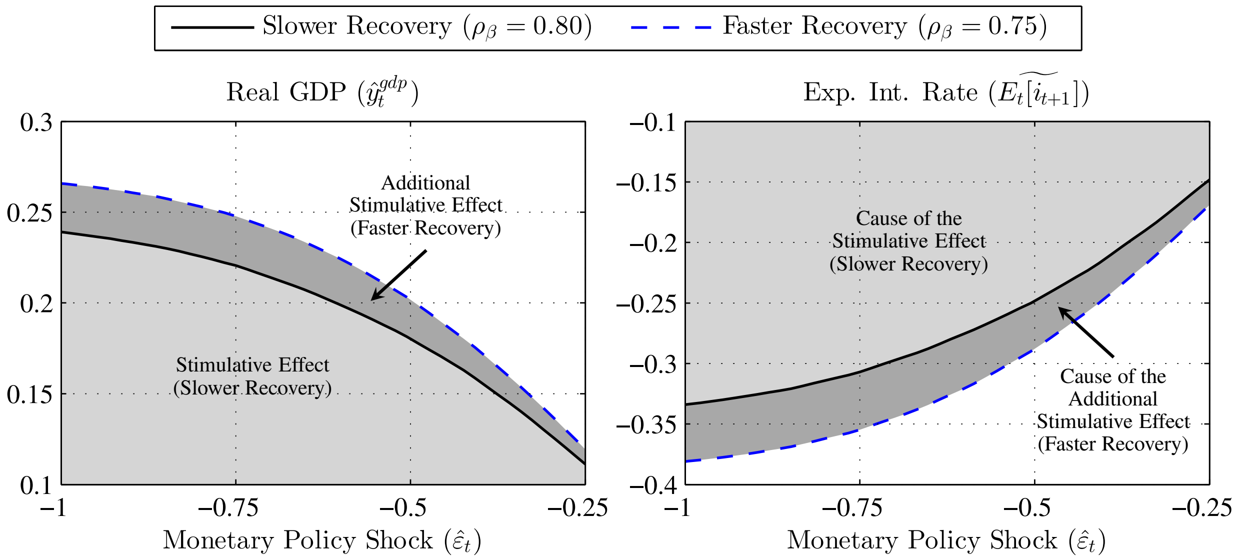

Figure 2: The stimulative effect of forward guidance given a slower recovery (solid line) and a faster recovery (dashed line). The vertical axis is the difference between 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$, and no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$. The values are based on a cross section where the initial notional rate equals zero.

Figure 2 shows the stimulative effect of forward guidance is stronger when households expect a faster economic recovery.[14] The left panel displays the decision rules for real GDP while the right panel shows the decision rules for the expected nominal interest rate. The vertical axis on the left (right) panel illustrates the percentage (percentage point) difference in the decision rules from the model with no forward guidance. The light-shaded region is the same as the region shown in figure 1. The dark-shaded region shows how the stimulative effect of forward guidance changes when households expect a faster recovery. Just like in figure 1, the initial discount factor is elevated in period t, which depresses current real GDP and inflation. A lower ρ_β is a proxy for a quicker recovery because the discount rate is expected to return to its steady state faster. A more rapid decline in the discount factor raises households' expected consumption growth rate, which further drives up next period's expected nominal interest rate. The larger jump in the expected nominal rate implies that a promise by the central bank to maintain a low policy rate in the future will have a greater effect on real GDP because a smaller percentage of next period's nominal interest rate distribution will be constrained. For example, an announcement this period by the central bank to change its policy rate by -0.5% next period causes real GDP to rise by 0.18% when $\rho_\beta = 0.80$ and by 0.20% when $\rho_\beta = 0.75$. Those results suggest that if households' expectations about future economic conditions change it will alter the stimulative effect of forward guidance policy.

[14] Levin et al. (2010) make a similar point in their study of optimal commitment policy. They assume a real rate shock hits the economy in period 0 and decays at a constant rate for 4 periods. In period 5, households are uncertain about how quickly the economy will recover, which is captured by a Markov switching process on the shock persistence.

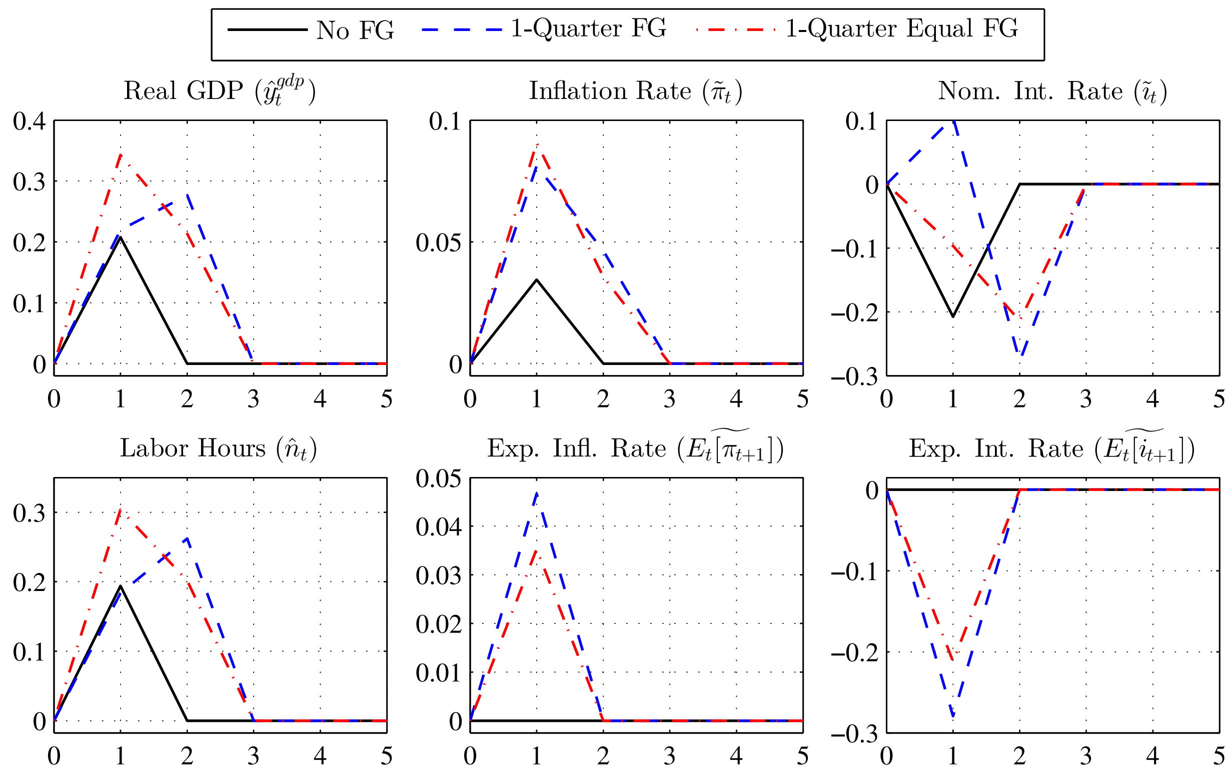

Figure 3: Generalized impulse responses to a -0.5% monetary policy shock. Three types of monetary policy are examined: No forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$ (solid line); 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line); and 1-quarter equal forward guidance, $(\alpha_0, \alpha_1) = (\sqrt{1/2}, \sqrt{1/2})$ (dash-dotted line). Each simulation is initialized at the average discount factor conditional on the ZLB binding in a 500,000 quarter simulation.

We examine the dynamic effects of forward guidance using generalized impulse response functions (GIRFs). GIRFs are based on an average of model simulations where the realization of shocks is consistent with households' expectations over time. Figure 3 plots the GIRFs to a -0.5% policy shock at the ZLB with no forward guidance (solid line), 1-quarter forward guidance (dashed line), and 1-quarter equal forward guidance (dash-dotted line).[15] To compute the GIRFs, we calculate the mean of 10,000 simulations conditional on a random shock in the first quarter. We then calculate a second mean from another set of 10,000 simulations, but this time the random shock in the first quarter of each simulation is replaced with the -0.5% policy shock.[16] The GIRFs are the percentage change (or the difference in rates) between the two means. Each simulation is initialized at the discount factor conditional on the ZLB binding in a 500,000 quarter simulation of the model. The notional interest rate equals -0.24% at that mean value. Households expect the discount factor to revert to its mean, so the nominal interest rate eventually exits the ZLB in every simulation.

In each GIRF, households learn about the monetary policy shock in period 1. With no forward guidance, the shock is unanticipated and occurs in period 1. With 1-quarter forward guidance, households are informed in period 1 about the policy shock that hits in period 2. An unanticipated expansionary shock $[(\alpha_0, \alpha_1) = (1, 0)$, solid line$]$ is stimulative on average because 27% of the

[15] Appendix A conducts sensitivity analysis using GIRFs. A higher degree of risk aversion decreases the stimulative effect of forward guidance. Changes in the monetary policy parameters, however, only have a small effect.

[16] The general procedure for computing GIRFs is outlined in Koop et al. (1996). See Appendix E for details.

simulations exit the ZLB despite the cut in the nominal interest rate. The expected future nominal interest rate is unchanged since the shock is transitory. A 1-quarter news shock [$(\alpha_0, \alpha_1) = (0, 1)$, dashed line], in contrast, lowers next period's expected nominal interest rate, which raises current consumption, the inflation rate, real GDP, and labor hours. It also causes the current nominal rate to rise due to the feedback effect. Specifically, news about a -0.5% monetary policy shock raises expected consumption and lowers the expected nominal interest rate for period 2. That change causes households to increase their current consumption and reduce their labor supply. Firms respond to the higher demand by raising prices, real GDP, and labor demand. The additional labor demand dominates the decline in labor supply so equilibrium hours and the real wage rate both increase. Thus, 1-quarter forward guidance stimulates the economy over the forward guidance horizon by raising expected inflation, just like it does in the optimal commitment policy literature.

The stimulative effect of 1-quarter forward guidance increases the current nominal interest rate by an average of 0.10% due to the feedback effect. To offset that effect, the central bank could provide an unanticipated expansionary monetary policy shock in period 1. For example, with 1-quarter equal forward guidance [$(\alpha_0, \alpha_1) = (\sqrt{1/2}, \sqrt{1/2})$, dash-dotted line], households learn that identical policy shocks will impact the economy in periods 1 and 2. The unanticipated shock in period 1 reduces the feedback effect so the current nominal interest rate falls. As a result, real GDP rises 56% more on impact than with 1-quarter forward guidance. Moreover, the boost in real GDP in period 2 is only slightly smaller, even though the intensity of the news is much weaker.

The stimulative effect of forward guidance also depends on the initial state of the economy. Figure 4 displays GIRFs to two different types of -0.5% monetary policy shocks: an unanticipated shock (left column) and a 1-quarter equal forward guidance shock (right column). For each shock, we examine its impact on real GDP given four alternative initial notional interest rates: (1) $\tilde{\imath}_0^* = 1.5$ (solid line) represents an economy at its steady state; (2) $\tilde{\imath}_0^* = 0.25$ (dashed line) corresponds to a low but positive policy rate, such as the 1% federal funds rate in 2004; (3) $\tilde{\imath}_0^* = 0$ (circle markers) denotes an economy that is just weak enough so that the ZLB binds, and is the same value used in earlier figures; and (4) $\tilde{\imath}_0^* = -0.5$ (triangle markers) represents an economy in a severe recession where the central bank is constrained by the ZLB, which is based on its estimated value during the Great Recession.[17] The initial notional rate is inversely related to the discount factor state, which is a proxy for the current level of demand. When the discount factor is high, households prefer to consume less and save more today, which depresses demand.

Two key results emerge from our simulations. One, monetary policy, regardless of whether it is unanticipated or provided with forward guidance, is less stimulative when the initial notional interest rate is near or below zero. Consider the case in which the economy is at its steady state, $\tilde{\imath}_0^* = 1.5$, so the current nominal interest rate is far enough from its ZLB that no plausible monetary policy shock will push it to zero. In that situation, a -0.5% policy shock (either unanticipated or anticipated) generates the largest decline in the nominal interest rate and has the greatest stimulative effect on real GDP because the policy rate rarely falls by enough to hit its ZLB. As the initial notional interest rate declines, the effects of a policy shock become more limited. For example, a -0.5% policy shock has a smaller effect on real GDP when $\tilde{\imath}_0^* = 0.25$ since the nominal interest rate hits its ZLB in a greater percentage of the simulations. Those effects are further dampened when $\tilde{\imath}_0^*$ equals 0% and -0.5%. This behavior also occurs with forward guidance because there is

[17] Bauer and Rudebusch (2014), Gust et al. (2013), Krippner (2013), and Wu and Xia (2014) estimate the notional federal funds rate when the ZLB binds and find it was well below zero during the Great Recession.

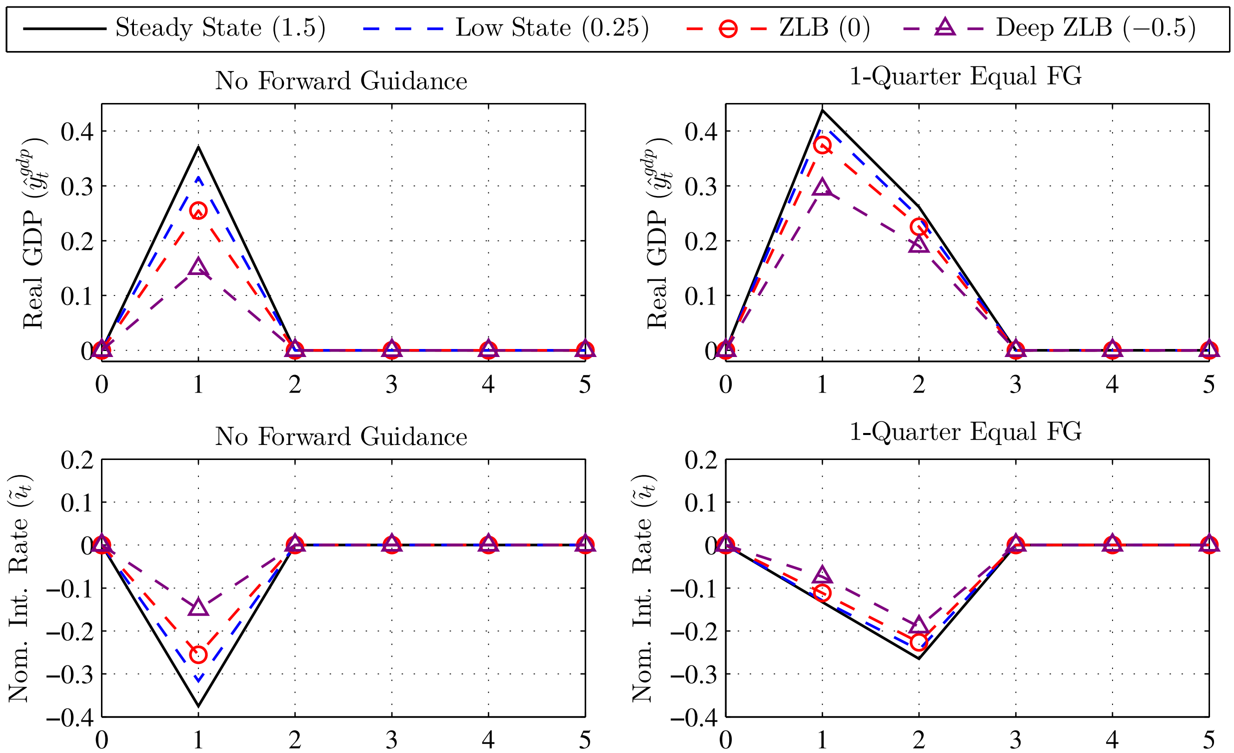

Figure 4: Generalized impulse responses to a -0.5% monetary policy shock. Two types of monetary policy are examined: No forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$ (left panels); and 1-quarter equal forward guidance, $(\alpha_0, \alpha_1) = (\sqrt{1/2}, \sqrt{1/2})$ (right panels). Each line type represents a simulation initialized at a specific notional interest rate.

an increasingly smaller margin for the news to operate as the fraction of the simulations where the nominal interest rate rises above zero declines.[18] Two, a news shock has a larger economic effect on real GDP than an identical unanticipated monetary policy shock in any state of the economy. For example, when $\tilde{\imath}_0^* = 1.5$, a -0.5% unanticipated shock increases real GDP by 0.37%, whereas a 1-quarter equal forward guidance shock increases real GDP by 0.44%. In states of the economy where the notional interest rate is lower, the marginal benefit of news shocks over unanticipated shocks becomes even larger. A key implication of those results is that forward guidance becomes an increasingly important policy tool as the nominal interest rate approaches its ZLB.

4.2 Two-Quarter Horizon ($q = 2$)

Forward guidance beyond 1-quarter will also affect the current economy since households are forward looking. This section analyzes the effects of 2-quarter forward guidance when the initial notional interest rate is zero. Figure 5 plots the 2-quarter forward guidance decision rules [$(\alpha_0, \alpha_1, \alpha_2) = (0, 0, 1)$, dashed line] as a function of the current monetary policy shock, $\hat{\varepsilon}_t$. As a reference, we also show the decision rules without forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (1, 0, 0)$, solid line]. With 2-quarter forward guidance, households receive news about a future monetary policy shock two periods before that shock impacts the economy.

If households receive news in period t that an expansionary monetary policy shock will occur in period $t + 2$, then the impact on real GDP is similar to the impact with 1-quarter forward guidance shown in figure 1. Given households prefer a smooth consumption path, the expectation

[18] Appendix A shows the distribution of future nominal interest rates initialized at different states of the economy.

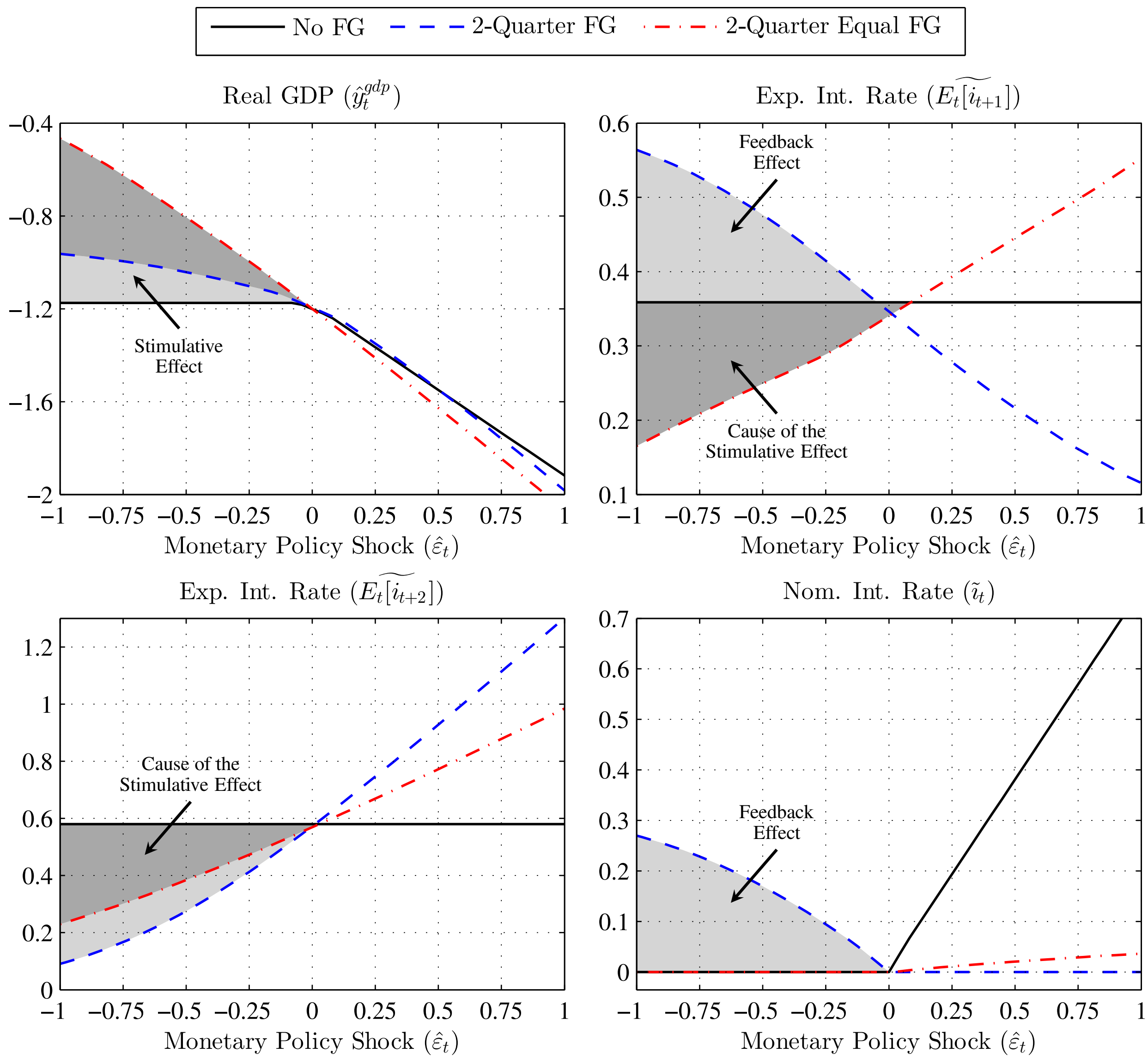

Figure 5: Decision rules as a function of the monetary policy shock with no forward guidance (solid line); 2-quarter forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (0, 0, 1)$ (dashed line); and 2-quarter equal forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = \sqrt{1/3}(1, 1, 1)$ (dash-dotted line). The values are based on a cross section where the initial notional rate equals zero.

of monetary stimulus in period $t + 2$ encourages households to raise their consumption not only in period $t + 2$, but also in periods t and $t + 1$. The higher consumption in those intervening periods increases real GDP, the inflation rate, and the current and expected future nominal interest rates.

Central banks, in practice, offset increases in current and expected future nominal interest rates by promising to keep the nominal rate at zero over the entire forward guidance horizon. Figure 5 also shows the decision rules when households receive 2-quarter equal forward guidance $[(\alpha_0, \alpha_1, \alpha_2) = (\sqrt{1/3}, \sqrt{1/3}, \sqrt{1/3})$, dash-dotted line$]$. Substantial differences exist between the two types of 2-quarter forward guidance. With 2-quarter equal forward guidance, the central bank announces in period t that an expansionary monetary policy shock will occur in periods t, $t+1$, and $t+2$. The shocks in periods t and $t+1$, which are not present with 2-quarter forward guidance, hold the current nominal interest rate at zero and lower next period's nominal interest rate. Thus, the period t and $t+1$ policy shocks eliminate the feedback effects from 2-quarter forward guidance. Those two additional policy shocks more than compensate for the smaller weight on the period $t+2$ news shock, so that 2-quarter equal forward guidance produces a larger stimulative effect than the more intense news shock that occurs in period $t+2$. For example, a -0.5% (-1%) shock announced in period t increases current output by 0.24 (0.50) percentage points *more* with 2-quarter equal forward guidance than with 2-quarter forward guidance. Those results are due to the fact that the anticipated shock in the intervening period lowers the expected nominal interest rate by 23 (40) basis points, even though the expected nominal interest rate 2 quarters ahead is 11 (14) basis points higher than it is with 2-quarter forward guidance. Regardless of how the news is distributed, forward guidance is less stimulative as the notional interest rate becomes more negative. Therefore, the stimulative effect of forward guidance is smaller than figures 1 and 5 indicate when the news coincides with information about a weaker state of the economy.

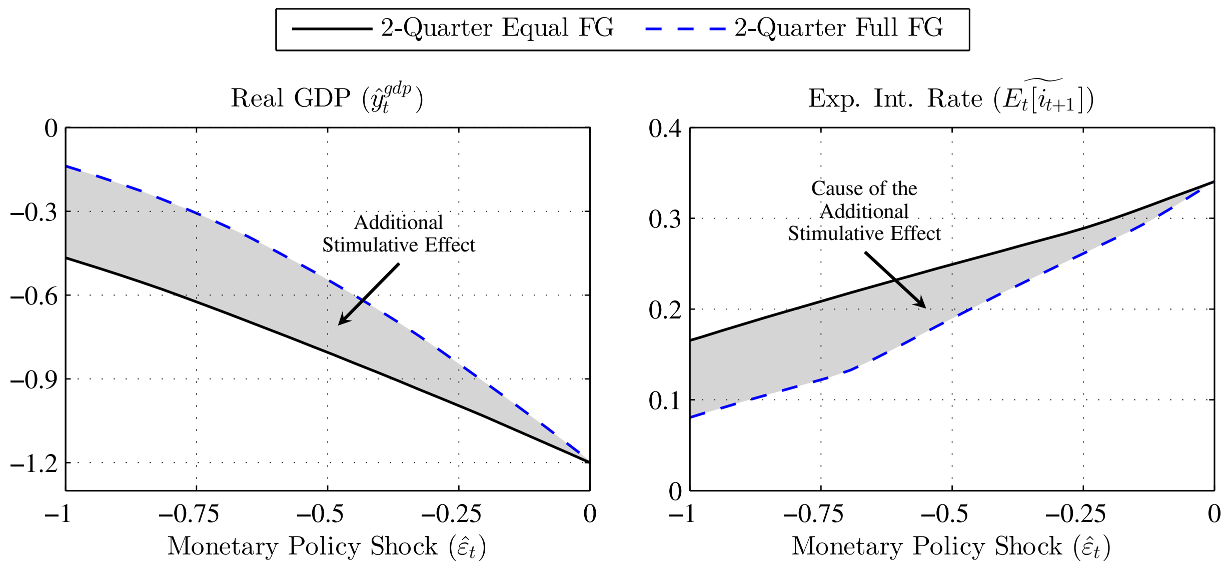

Figure 6: Decision rules as a function of the monetary policy shock with 2-quarter equal forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (\sqrt{1/3}, \sqrt{1/3}, \sqrt{1/3})$ (solid line), and 2-quarter full forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (1, 1, 1)$ (dashed line). The values are based on a cross section where the initial notional rate equals zero.

Extending the forward guidance horizon from 1 to 2 quarters does not simply double the size of its stimulative effect, unless the monetary policy shock exceeds 1%. For example, a -0.5% (-1%) policy shock increases current output by 0.23 (0.35) percentage points with 1-quarter equal forward guidance and by 0.37 (0.71) percentage points with 2-quarter equal forward guidance. Thus, the extra quarter of forward guidance raises output by an additional 0.14 (0.36) percentage points. Those results stem from the fact that the variance of the news process is held constant across alternative forward guidance horizons. Without that restriction, extending the horizon by an additional quarter increases the intensity of the news. Figure 6 contrasts key decision rules with 2-quarter equal forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (\sqrt{1/3}, \sqrt{1/3}, \sqrt{1/3})$, solid line] to 2-quarter *full* forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (1, 1, 1)$, dashed line]. Not surprisingly, a higher intensity of news with 2-quarter full forward guidance further stimulates real GDP. For example, a -0.5% (-1%) monetary policy shock with 2-quarter full forward guidance increases real GDP by 0.26

(0.33) percentage points *more* than 2-quarter equal forward guidance because the 1-quarter ahead expected nominal interest rate falls by 6 (8.5) basis points and the 2-quarter ahead expected rate declines even more. Thus, a large fraction of the additional stimulus is due to the higher intensity of the news and not the longer forward guidance horizon when we do not restrict the news process.

4.3 LONGER HORIZONS ($q > 2$) This section compares the stimulative effect of forward guidance policies over horizons beyond 2 quarters. Our results in sections 4.1 and 4.2 rely on Gauss-Hermite quadrature to evaluate expectations. That approach enables us to obtain an accurate approximation of the decision rules and to quantify the stimulative effect of forward guidance for a *continuous range* of news shocks. The solution method, however, is numerically infeasible with longer forward guidance horizons because of the size of the state space. Therefore, we reduce the dimensionality of the problem when analyzing forward guidance horizons beyond 2 quarters by discretizing the continuous distribution of the shock process using the method described in Tauchen (1986). Specifically, we assign three values for each monetary policy shock, $(-0.5, 0, 0.5)$, and calculate the probabilities of each transitional event. Tauchen's (1986) method is particularly useful for calculating GIRFs because it enables us to analyze the effects of *specific* shocks to the news process without solving the model across the complete distribution of shocks.[19]

Figure 7 shows the GIRFs to a -0.5% monetary policy shock that is equally distributed over a 1-quarter (solid line), 4-quarter (dashed line), 8-quarter (circle markers), and 10-quarter (triangle markers) forward guidance horizon.[20] We set $\alpha_i = \sqrt{1/(q+1)}$ for $i \in \{0, \ldots, q\}$ to keep the variance of the news process constant for each value of q. The simulations are initialized at the average discount factor conditional on the ZLB binding in a $500{,}000$ quarter simulation, which corresponds to an average notional interest rate of -0.24%. In every case, households know in period 1 about the current and future policy shocks, but their effects on the economy depend on the forward guidance horizon. An unanticipated expansionary policy shock has, on average, a small stimulative effect due to the discount factor slowly falling to its steady state. When the central bank provides 1-quarter equal forward guidance, households expect a lower nominal interest rate in period 2, which stimulates the economy in period 1. Over that 1-quarter horizon, the stimulative effect of equal forward guidance is considerable and the unexpected policy shock in period 1 is usually big enough to reverse the feedback effect on the current nominal interest rate.

Additional quarters of forward guidance generate more persistent increases in labor hours, the inflation rate, and real GDP because households expect a lower future nominal interest rate and higher future consumption in every period until the last shock hits the economy in period q. At longer horizons, the feedback effect from forward guidance dominates the smaller intensity of the news shocks, so the current and expected nominal interest rates on average increase. Those feedback effects, however, can be eliminated by either increasing the size of the shocks over the entire forward guidance horizon or redistributing the intensity of the news toward the present. Beyond quarter q, forward guidance has no effect on the economy. Our findings reveal that forward guidance for up to 10 quarters does not cause the stimulative effect to explode, but rather spreads the effect across the entire horizon when the variance of the news process is held constant.

The stimulative effects in figure 7 stem from the reduction in expected future policy rates.

[19] See Appendix D for more details on how this solution procedure differs from the method used in earlier sections.

[20] Del Negro et al. (2012) show that when the forward guidance horizon is extended, it leads to predictions that overstate the actual increase in output and inflation, which they call the "forward guidance puzzle." Their experiment assumes the central bank uses news shocks to peg the policy rate at 25 basis points for three years.

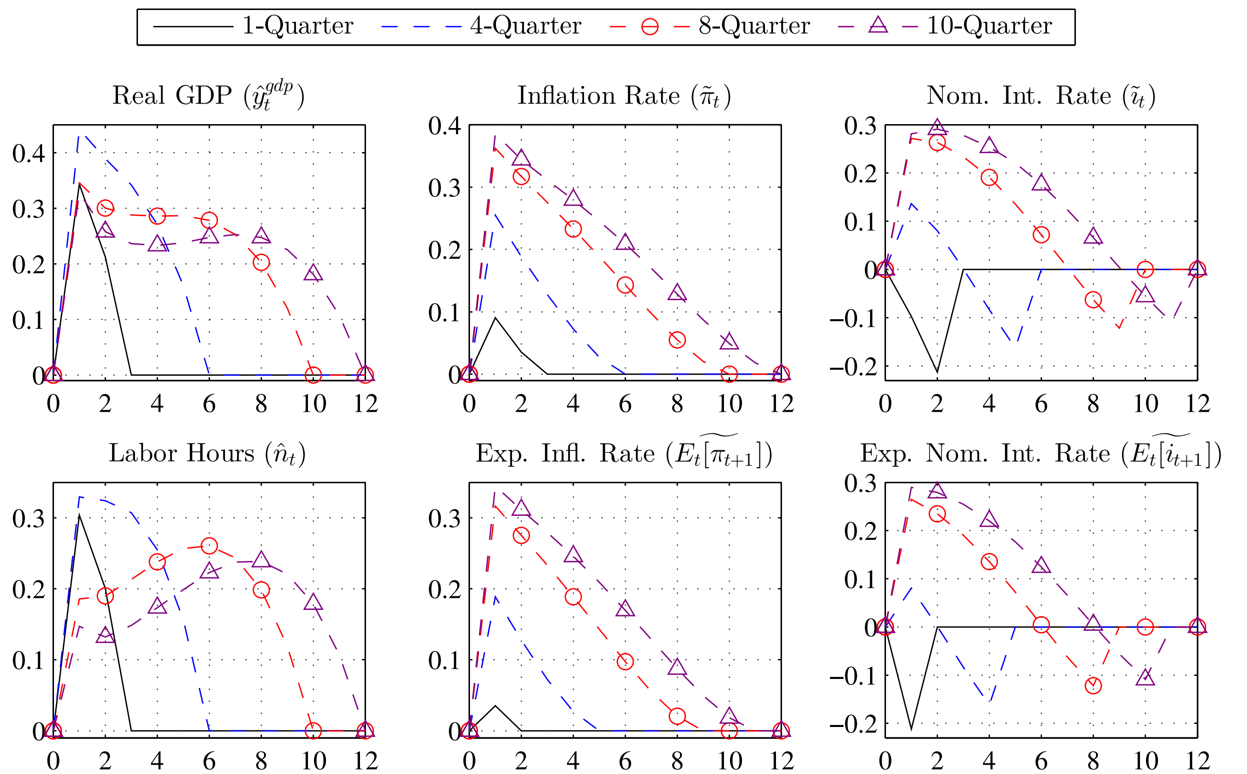

Figure 7: Generalized impulse responses to a -0.5% monetary policy shock with 1-quarter (solid line), 4-quarter (dashed line), 8-quarter (circle markers), and 10-quarter (triangle markers) equal forward guidance. Each simulation is initialized at the average discount factor conditional on the ZLB binding in a 500,000 quarter simulation.

Figure 8 compares the paths of the expected future nominal interest rate (left panel) and the yield curve (right panel) at various forward guidance horizons when the initial notional interest rate is zero.[21] In this example, the central bank announces in period 0 that it will cut the nominal interest rate by -0.5% with no forward guidance (solid line), or with 1-quarter (dashed line), 2-quarter (circle markers), or 4-quarter (triangle markers) equal forward guidance. The intensity parameters with q-quarter equal forward guidance are set to $\alpha_i = \sqrt{1/(q+1)}$ for $q \in \{0, 1, 2, 4\}$.

The left panel of figure 8 shows the impact of forward guidance on the expected nominal interest rate. In most cases, forward guidance lowers the expected nominal rate during the forward guidance horizon, but in all cases, it returns to its expected value without forward guidance in quarters beyond that horizon. The expected increase in the nominal interest rate without forward guidance reflects households' expectation that the discount factor will revert to its mean. The 1- and 2-quarter equal forward guidance cases lower the expected nominal rates during the forecast horizon but revert to their no forward guidance expected values beginning in periods 2 and 3, respectively. The 4-quarter equal forward guidance horizon behaves like the 1- and 2-quarter cases, except the policy shock is not strong enough in period 0 to eliminate the feedback effect.

The right panel of figure 8 shows the yield curve for maturities from 0 to 5 quarters. The yield at each maturity, m, is calculated according to the expectations hypothesis as $(\Pi_{j=0}^{m} E_0[i_j])^{1/(m+1)}$. The stimulative effect of forward guidance is evident as the yield curve flattens with longer horizons. For example, 4-quarter equal forward guidance reduces the expected increase in the 1-year

[21] Swanson and Williams (2014) show how the ZLB affects intermediate- and longer-term yields in the data.

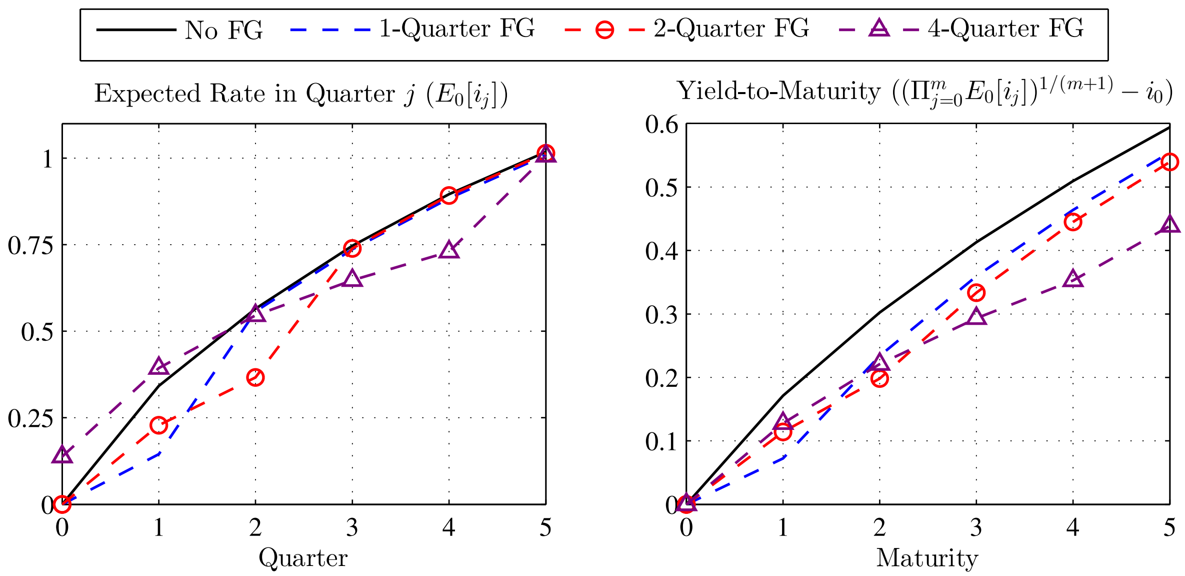

Figure 8: Comparison of expected rates and yields with no forward guidance (solid line) and 1-quarter (dashed line), 2-quarter (circle markers), and 4-quarter (triangle markers) equal forward guidance. The left panel shows the expected increase in the net nominal interest rate j periods in the future. The right panel shows the yield curves as a percentage point difference from the initial net nominal interest rate. In period 0, households receive news of a -0.5% monetary policy shock. The values in both panels are based on a cross section where the initial notional rate equals zero.

yield (i.e., $m = 3$) by 13 basis points. Beyond the forward guidance horizon, the slopes of the yield curves with q-quarter forward guidance converges to the yield curve with no forward guidance.

Figure 9 compares the effect of forward guidance on the yield curve for two different initial notional interest rates: $\tilde{\imath}_0^* = 0$ (left panel) and $\tilde{\imath}_0^* = -0.5$ (right panel). The solid line is the yield curve when the central bank provides no forward guidance [$(\alpha_0, \alpha_1) = (1, 0)$], while the dashed line is the yield curve with 4-quarter equal forward guidance [$\alpha_i = \sqrt{1/5}$ for $i \in \{0, \ldots, 4\}$]. In the left panel, the economy is in a recession that causes the ZLB to bind, but households place a high probability on exiting the ZLB next period. If the central bank provides 4-quarter equal forward guidance, then the expected future nominal interest rates over the forward guidance horizon fall, which flattens the yield curve. In the right panel, the economy is in a deep recession and households do not expect a recovery in the near term. An announcement of 4-quarter equal forward guidance has little stimulative effect because households expect the nominal interest rate to remain near zero over most of the forward guidance horizon. In that situation, the yield curve with 4-quarter equal forward guidance is much closer to the yield curve with no forward guidance.

Our results show why the initial state of the economy changes the effect of forward guidance. In particular, they show that models can easily overpredict the stimulative effect of forward guidance by assuming a counterfactually high state of the economy when the news is announced. Suppose the central bank provides a -0.5% news shock that is equally distributed over 4 quarters. The response of the yield curve depends on how the nominal interest rate is expected to respond over the next 4 quarters. When there is a strong expectation that the nominal interest rate will exit the ZLB within the forward guidance horizon [$\tilde{\imath}_0^* = 0$], the 4-quarter equal forward guidance shock pushes down the annual yield on the 4-quarter bond by about 54 basis points. That response declines to 8 basis points when the economy is weak enough that households place a high probability on the

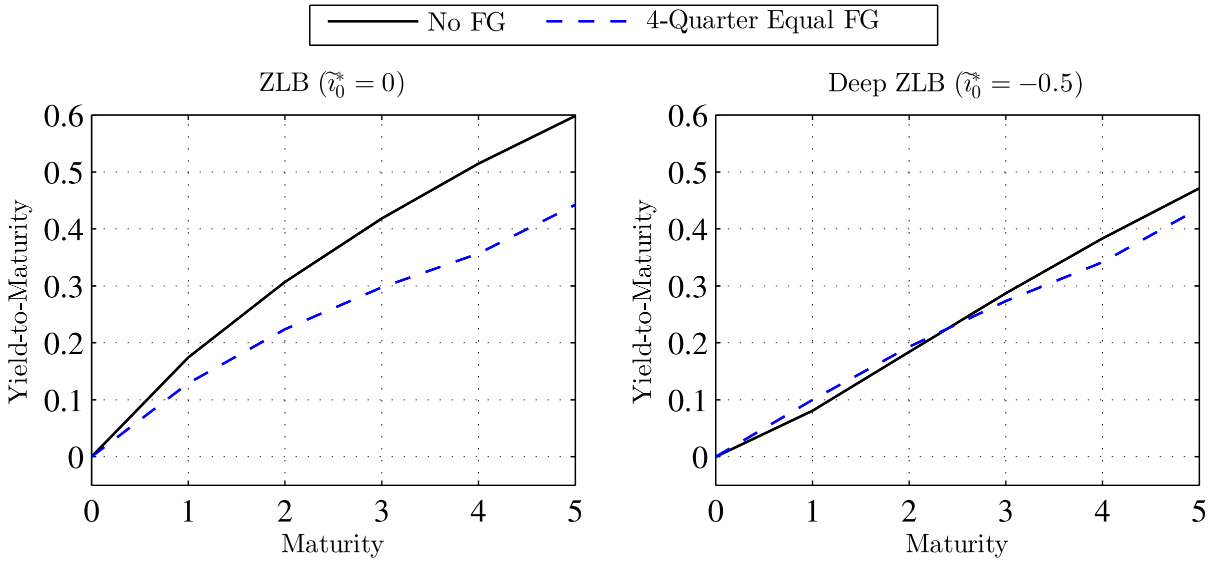

Figure 9: Comparison of yield curves with no forward guidance (solid line) and 4-quarter equal forward guidance (dashed line) at two initial notional rates: $\tilde{\imath}_0^* = 0$ (left panel) and $\tilde{\imath}_0^* = -0.5$ (right panel). In period 0, households receive news of a -0.5% monetary policy shock. The values are shown as percentage point differences from $\tilde{\imath}_0$.

nominal rate remaining at or near zero [$\tilde{\imath}_0^* = -0.5$]. Therefore, any information about weaker current and future economic conditions will dampen the stimulative effect of forward guidance.

Figure 10 compares the separate and combined effects from a negative demand shock and 4-quarter equal forward guidance. To assess their combined effects, we compute GIRFs to a simultaneous 1 standard deviation positive discount factor shock (i.e., lower demand) and a -0.5% monetary policy shock with 4-quarter equal forward guidance (solid line). Those responses are then compared to the GIRFs with only the monetary policy shock (dashed line) and the GIRFs with only the discount factor shock (dash-dotted line). The simulations are initialized in the same way as our previous GIRFs. The distance between the dashed line and the solid line measures the effect of the negative demand shock, whereas the distance between the dash-dotted line and the solid line quantifies the marginal benefit of implementing 4-quarter equal forward guidance.

A central bank announcement that it intends to keep its policy rate low for the next 4 quarters reduces expected nominal interest rates and raises real GDP over the forward guidance horizon, regardless of whether there is a downward revision in economic conditions. If, however, the central bank does not provide forward guidance after a negative demand shock, then the fall in real GDP is larger but the yield curve still flattens.[22] Those findings illustrate two key points. One, it is difficult to identify the source of changes in the yield curve observed in the data because households often simultaneously receive forward guidance from the Fed and information about the current and future state of the economy. Two, forward guidance by itself is stimulative, but its effect on the economy will be smaller or even negative if other information about the economy reduces demand.

[22]Christiano et al. (2014) find that if the Fed had not provided any forward guidance after 2011, then the policy rate would have started to rise in 2014 and output would have been 2% lower. Bernanke (2014) reiterated this viewpoint by saying, "Skeptics have pointed out that the pace of recovery has been disappointingly slow...[h]owever...economic growth might well have been considerably weaker, or even negative, without substantial monetary policy support."

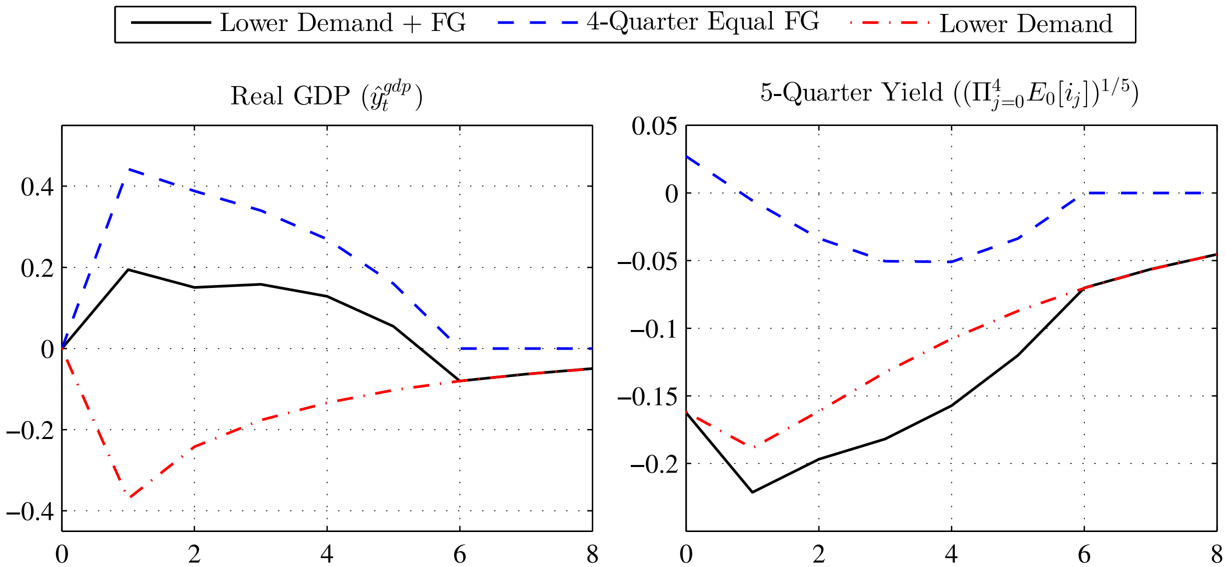

Figure 10: Generalized impulse responses to a 1 standard deviation positive discount factor shock and a -0.5% monetary policy shock with 4-quarter equal forward guidance (solid line). We compare those responses to cases with only the monetary policy shock (dashed line) and only the discount factor shock (dash-dotted line). Each simulation is initialized at the average discount factor conditional on the ZLB binding in a 500,000 quarter simulation.

5 Case Studies of Federal Reserve Forward Guidance

Given our theoretical results, this section examines how well our model's predictions correspond to the effects of recent Fed policy statements that communicated forward guidance.

5.1 2011 Policy Statement On August 9, 2011, the FOMC announced it "anticipates that economic conditions...are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013," which was a more specific statement than in the past. It also said, "The Committee now expects a somewhat slower pace of recovery over coming quarters." That forward guidance could be described as Delphic in nature since the FOMC communicated it with a more pessimistic economic outlook. If, however, the forward guidance horizon exceeds the time the economy is expected to remain weak, then the statement conveys Odyssean forward guidance.[23]

Crump et al. (2013) find the August 9th FOMC statement changed Blue Chip forecasts of interest rates, GDP, and inflation. Estimating the effect of that announcement on economic forecasts is complicated by a downward revision of GDP just 11 days before the statement was released. To separate the impact of the two events, Crump et al. (2013) compare the consensus forecasts of the federal funds rate from the Blue Chip Financial Forecasts (BCFF) survey to the consensus forecasts of the 3-month Treasury bill rate from the Blue Chip Economic Indicators (BCEI) survey.

Table 1 shows the term structure implied by the BCFF and BCEI forecasts. The BCFF forecasts were made on July 20-21 before the GDP revisions were released on July 29th, while the BCEI forecasts were made on August 4-5 just prior to the August 9th FOMC announcement. The difference between the late-July and early-August forecasts is an implicit measure of the impact

[23] Woodford (2012) argues that information about the central bank's policy intentions is more likely to impact market expectations than knowledge about its economic forecasts because the central bank has undisclosed information about its own policy intentions, whereas it does not enjoy the same advantage when forecasting economic conditions.

the GDP revisions had on the economic forecasts. The next BCFF survey forecasts were made on August 24-25. The difference between the BCEI's August 4-5 forecasts and the BCFF's August 24-25 forecasts is an indirect measure of the effect of forward guidance on those forecasts.

	$1Q$	$2Q$	$3Q$	$4Q$	$5Q$	$6Q$
BCFF (7/20-21)	0.140	0.150	0.187	0.237	0.336	0.459
BCEI 3-mo. T-bill (8/4-5)	0.090	0.110	0.137	0.175	0.240	0.328
BCFF (8/24-25)	0.120	0.125	0.130	0.135	0.138	0.145
Total Change	−0.020	−0.025	−0.057	−0.102	−0.197	−0.313
Change following GDP	−0.050	−0.040	−0.050	−0.062	−0.096	−0.131
Change following FOMC	0.030	0.015	−0.007	−0.040	−0.102	−0.183

Table 1: Term structure implied by Blue Chip consensus forecasts. All values are annualized net interest rates.

Our calculations indicate the July 29th GDP revisions led to a 6 basis point decline in the 4-quarter rate and a 13 basis point decline in the 6-quarter rate. The flatter yield curve reflects a weaker economic outlook. The August 9th FOMC announcement also pushed down the yield curve, with the 4-quarter and 6-quarter rates falling by an additional 4 and 18 basis points. Furthermore, the 6-quarter rate was only 2.5 basis points higher than the 1-quarter rate, which suggests forecasters believed the policy rate would not change over the next year and a half. That change in expectations likely can be attributed to both the Fed's forward guidance and a weaker economy.

Using a distribution of expected future Eurodollar rates, Raskin (2013) estimates the August 9th statement generated a significant decline in 1- to 5-year ahead future Eurodollar rates and a substantial decline in the dispersion of those expected interest rates. For example, the 2-year ahead Eurodollar rate fell by about 25 basis points after the statement was released. Moessner (2013) also finds the 1-year ahead U.S. Treasury bill rate at 3- to 8-year horizons fell by a similar amount.

	Pre-GDP	Post-GDP	Post-FOMC
2012 Real GDP growth rate	2.8	2.5	2.2
2012 CPI growth rate	2.3	2.2	2.2

Table 2: Blue Chip consensus forecasts reproduced from Crump et al. (2013). All values are annualized rates.

Table 2 shows the 2012 consensus forecasts of GDP and inflation as summarized by Crump et al. (2013). The forecast for 2012 GDP growth dropped 0.3 percentage points after the downward revisions to GDP on July 29th and another 0.3 percentage points following the release of the FOMC's policy statement on August 9th. The inflation forecast, on the other hand, dropped only 0.1 percentage points following the GDP revisions and remained unchanged after the policy statement release. Those forecasts suggest the economic pessimism expressed in the policy statement affected the forecasts more than the stimulative effect from the change in forward guidance.[24]

[24] Walsh (2009) cautions that aggressively reducing the policy rate in response to adverse shocks may cause a downward revision in households' economic outlook when their information set differs from the central bank. Bullard (2012) and Woodford (2012) contend that date-based forward guidance may cause people to expect worse economic conditions over its horizon. Threshold-based forward guidance alleviates that problem by linking policy rate changes to economic conditions. Yellen (2013, 2014) refers to this type of communication as an "automatic stabilizer."

Our model predicts forward guidance will reduce longer-term interest rates and push up output given the right economic conditions. An analysis of the data following the August 9th FOMC announcement indicates that longer-term interest rates and output declined. We contend those outcomes occurred for two main reasons. One, the implied yield curve from the 8/4-5 BCEI survey was fairly flat. Specifically, the yield curve observed prior to the FOMC statement was flatter than the yield curve in the right panel of figure 9, even though it is based on a notional interest rate equal to -0.5%. That result suggests households placed a low probability on exiting the ZLB before the August 9th forward guidance announcement, which decreased its effectiveness. Two, there was a downward revision in forecasts of future economic conditions. Figure 10 helps explain these empirical results. It shows that a negative demand shock can limit the observed stimulative effect of forward guidance by simultaneously flattening the yield curve and reducing real GDP.

5.2 2012 Policy Statements

Two other FOMC statements (January 25 and September 13) lengthened the forward guidance horizon for the federal funds rate. The January statement extended the horizon by 6 quarters (from mid-2013 to late-2014) similar to the August 2011 statement, but it was announced 5 quarters before the end of the August 2011 forward guidance horizon. The September statement then extended that horizon by an additional 2 quarters (from late-2014 to mid-2015), 6 quarters before the January forward guidance ended. That statement also contained Odyssean-like language that the FOMC "expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens," whereas the January statement only included language related to the forward guidance horizon.

Announcement	Expected Change in 1-Year Rate j-Years Ahead						
Date	0	1	2	3	4	6	8
08/09/2011	-0.10	-0.20	-0.27	-0.30	-0.31	-0.27	-0.21
01/25/2012	-0.04	-0.10	-0.13	-0.14	-0.13	-0.08	-0.02
09/13/2012	-0.01	-0.03	-0.05	-0.07	-0.07	-0.04	0.02

Table 3: Expected changes in future 1-year rates on the date of the announcement. Values are annualized net rates.

Table 3 shows the expected changes in future 1-year interest rates following three FOMC statements. The rates are constructed from daily term structure data by Gürkaynak et al. (2007) and regularly updated by the Board of Governors. Following the January 2012 statement, the decline in the 1-year rate 0 to 3 years ahead was about half the decline that occurred after the August 2011 statement. At longer horizons, the response is smaller and at 8 years ahead it is near zero. The September 2012 statement had an even smaller effect. Both of those findings are consistent with Raskin (2013). When analyzing the data, he finds the 6-quarter extension of forward guidance only marginally reduced expected interest rates and was not statistically significant. He argues that the difference in the market's reaction to the August 2011 and the January 2012 statements is attributed to market participants' beliefs. Specifically, the market was surprised by the first forward guidance announcement but not by the second statement of a longer forward guidance horizon. He also suggests that the greater uncertainty about the longer horizon may have made the later announcement less relevant for the market's expectations of future interest rates.

Table 4 includes Blue Chip consensus forecasts of real GDP growth and CPI inflation. We focus on the 2012 (2013) forecasts immediately before and after the January 2012 (September 2012)

	2012 Forecasts (Jan. 2012)		2013 Forecasts (Sept. 2012)	
	Pre-FOMC	Post-FOMC	Pre-FOMC	Post-FOMC
Real GDP growth rate	2.2	2.2	2.2	2.0
CPI inflation	2.1	2.1	2.0	2.0

Table 4: Blue Chip consensus forecasts. All values are annualized rates.

FOMC statement. The only meaningful revision in these forecasts was a reduction in 2013 real GDP growth by 0.2 percentage points following the September announcement. As in the case of the August 2011 statement, both 2012 forward guidance announcements were communicated with forecasts of a weaker economy in the months ahead. Specifically, both statements said, "Strains in global financial markets continue to pose significant downside risks to economic growth." The FOMC, however, was generally less pessimistic in its 2012 statements than in its August 2011 statement, which may have contributed to the smaller revisions in the forecasts of real GDP.

The evidence is inconclusive on whether the 2012 forward guidance announcements flattened the yield curve, but concurrent information about a weak economy likely lowered yields and slowed the recovery. In fact, a headline in the *New York Times* on the day of the January FOMC statement release read, "Fed Signals That a Full Recovery Is Years Away." That reaction to the policy statement demonstrates the challenge central banks face in communicating future policy rates. The findings are also consistent with those in figure 10. Although the data does not separate out the effects of the forward guidance and the downward revision in GDP, our results suggest forward guidance would have been more effective if people had expected a quicker recovery.

6 CONCLUSION

This paper examines the effects of forward guidance at and away from the ZLB. The central bank conducts forward guidance by promising to keep future nominal interest rates lower than its policy rule suggests. That policy can stimulate economic activity if households believe the economy will recover and exit the ZLB. If, on the other hand, households do not expect any meaningful recovery, then future nominal interest rates will remain at or near zero and forward guidance will have little effect on the economy. Therefore, the ability of forward guidance to stimulate demand is limited when the economy is in a deep recession or households expect a slow recovery. News shocks, however, are more stimulative than unanticipated shocks and longer forward guidance horizons can help stimulate the economy for several periods, even if the amount of news is held constant.

Empirical estimates indicate that the Fed's recent forward guidance has likely reduced long-term interest rates [Campbell et al. (2012), Gürkaynak et al. (2005), Moessner (2013), and Swanson and Williams (2014)]. What is unclear, however, is how much of that decline was due to the Fed's communication and how much was because of simultaneous changes in economic conditions. Our results suggest that if there is a decline in the current economic outlook when communicating forward guidance, then its observed stimulative effect will be much more limited.

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A SUPPLEMENTAL RESULTS

A.1 EFFECT OF THE ZLB CONSTRAINT The ZLB constraint truncates the distribution for next period’s nominal interest rate at zero so that any negative realizations are automatically set to zero. Thus, the constraint affects the expected nominal interest rate even when most of the distribution is positive. To demonstrate how the ZLB constraint impacts households’ expectations, figure 11 compares the effects of 1-quarter forward guidance with (solid line) and without (dashed line) a ZLB constraint under the assumption that the the initial notional interest rate equals zero due to an elevated discount factor. We use 1-quarter forward guidance rather than 1-quarter equal forward guidance so the stimulative effect is due only to changes in the expected nominal interest rate. Without a ZLB constraint, the expected nominal interest rate can be negative, which leads to an overstatement of the stimulative effect of forward guidance because a significant portion of next period’s nominal interest rate distribution is well below zero. For example, a -0.5% (-1%) news shock in the constrained model reduces the expected nominal interest rate to 10 (1) basis points and increases real GDP to 1.03% (0.97%) below its steady state. The same shock in the unconstrained model pushes down the expected nominal rate to -4 (-40) basis points and raises real GDP to 0.89% (0.64%) below its steady state. A negative expected nominal interest rate occurs whenever

$\hat{\varepsilon}_t < -0.44$. Those results show that any analysis that does not constrain both current and expected future nominal interest rates will significantly overstate the stimulative effect of forward guidance. That overstatement becomes even more severe as the forward guidance horizon lengthens.

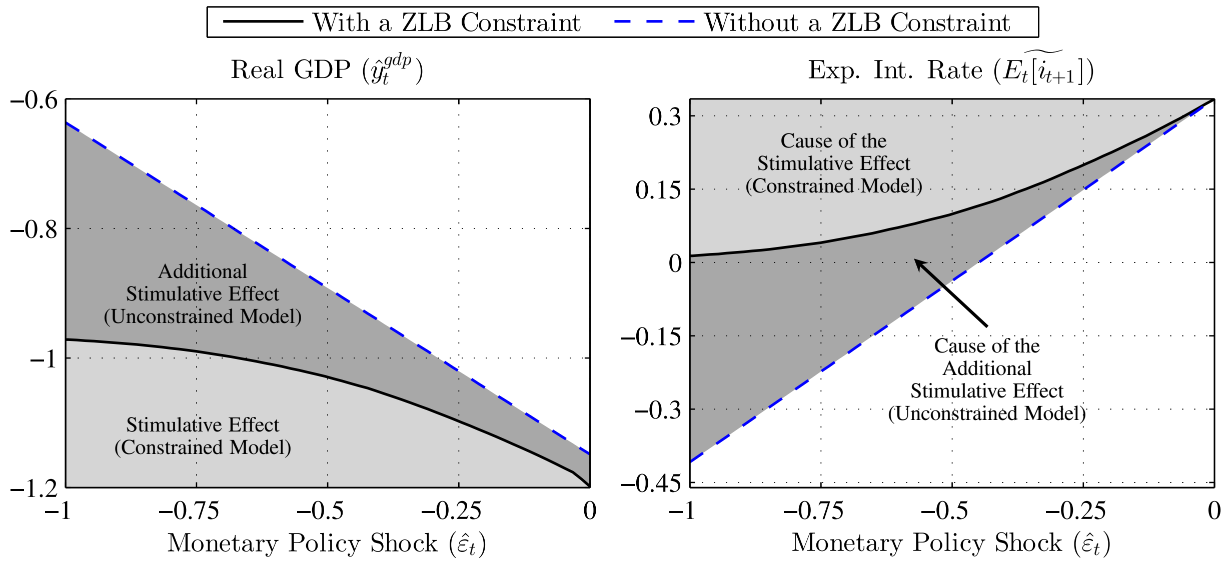

Figure 11: Comparison of decision rules with (solid line) and without (dashed line) a ZLB constraint given 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$. The initial notional rate equals zero in these decision rules.

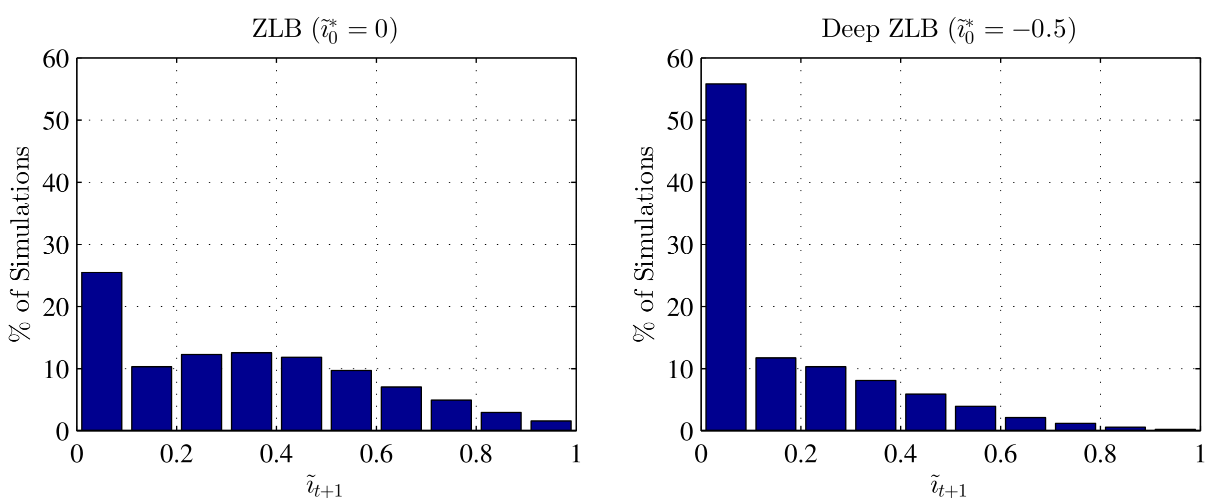

Figure 12: Histograms of the simulated values of next quarter's nominal interest rate without forward guidance. The simulations are initialized at two alternative notional interest rates: $\tilde{i}_0^* = 0$ (left panel) and $\tilde{i}_0^* = -0.5$ (right panel).

To better understand how economic conditions impact the distribution of future nominal interest rates, figure 12 plots histograms of the simulated values of next quarter's nominal interest rate without forward guidance. The simulations are initialized at two alternative notional interest rates: $\tilde{i}_0^* = 0$ (left panel) and $\tilde{i}_0^* = -0.5$ (right panel). Since the discount factor is above its steady state, households expect it to mean revert and the nominal interest rate to rise over time. These histograms reveal how much the initial notional interest rate skews the distribution for the expected nominal rate. When $\tilde{\imath}_0^* = 0$, $\tilde{\imath}_{t+1}$ is between 0% and 0.1% in 26% of the simulations, but that percentage rises to 56% when the economy is in a deep recession ($\tilde{\imath}_0^* = -0.5$). Therefore, forward guidance has a smaller stimulative effect at lower initial notional interest rates because the expected nominal rate is closer to zero and cannot decline as much. For example, figure 4 shows that a -0.5% news shock under 1-quarter equal forward guidance generates a real GDP increase of only 0.15% on average when $\tilde{\imath}_0^* = -0.5$, but that increase rises to 0.25% when $\tilde{\imath}_0^* = 0$.

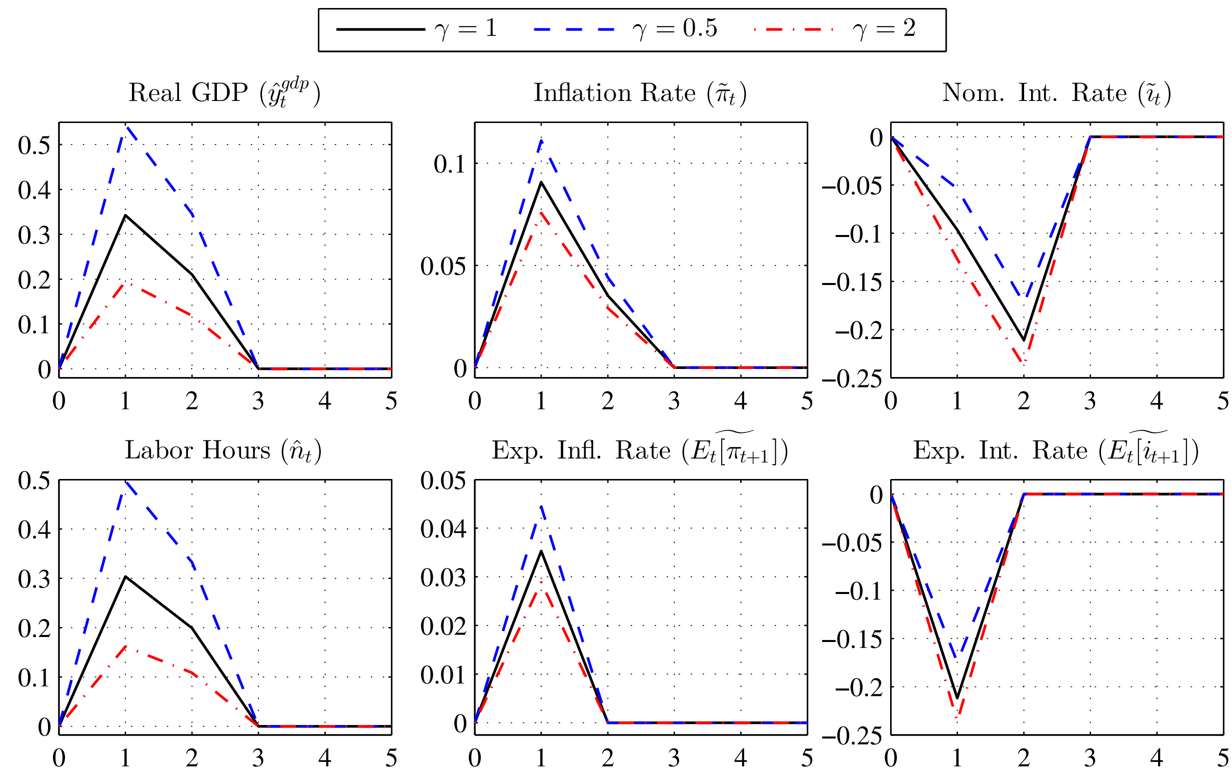

Figure 13: Generalized impulse responses to a -0.5% monetary policy shock. Three degrees of risk aversion are examined: $\gamma = 1$ (solid line), $\gamma = 0.5$ (dashed line), and $\gamma = 2$ (dash-dotted line). Each simulation is initialized at the average discount factor conditional on the ZLB binding in a 500,000 quarter simulation.

A.2 COEFFICIENT OF RELATIVE RISK AVERSION Figure 13 compares the stimulative effect of 1-quarter equal forward guidance across different degrees of risk aversion by plotting GIRFs to a -0.5% monetary policy shock. The simulations are initialized at the ZLB in the same way as in figure 3. When households are more risk averse (i.e., a higher value of γ), they are less willing to substitute consumption between periods. Therefore, as γ increases the central bank's promise to reduce next period's policy rate leads to a smaller increase in expected consumption, expected inflation, and current real GDP, even though the news generates a larger decline in the expected nominal interest rate. That is, an increase in γ from 0.5 to 2 causes the expected nominal rate to decrease by an additional 7 basis points (from 17 basis points to 24 basis points), while the rise in real GDP declines by 0.34 percentage points (from 0.54% to 0.20%). Our results are consistent with Levin et al. (2010). They show that a larger value of γ leads to a smaller increase in expected inflation. Given that some estimates of γ are well above 1, those results provide another plausible explanation for why the observed effect of forward guidance has been fairly small since 2008.

B MODELING FORWARD GUIDANCE WITH AN INTEREST RATE PEG

An alternative to using anticipated monetary policy shocks to model forward guidance is an exogenous interest rate peg. This section describes one way to introduce an interest rate peg into a stochastic model with an endogenous ZLB constraint. Suppose the central bank sets the gross nominal interest rate conditional on the realization of a discrete state variable, s_t, with the rule:

$$(i_t, i_{t+1}) = \begin{cases} \left(\max\{1, i_t^*\}, \max\{1, i_{t+1}^*\}\right) & \text{for } s_t = 1 \\ (\max\{1, i_t^*\}, 1) & \text{for } s_t = 2 \\ \left(1, \max\{1, i_{t+1}^*\}\right) & \text{for } s_t = 3 \\ (1, 1) & \text{for } s_t = 4 \end{cases},$$

where $i_t^* \equiv \bar{\imath}(\pi_t/\pi^*)^{\phi_\pi}(y_t/\bar{y})^{\phi_y}$ and s_t follows a Markov process with transition matrix,

$$P = \begin{bmatrix} p & 1-p & 0 & 0 \\ 0 & 0 & p & 1-p \\ p & 1-p & 0 & 0 \\ 0 & 0 & p & 1-p \end{bmatrix}.$$

When $s_t = 1$, the ZLB constraint is endogenous this period and next period. The probability that policy stays in state 1 is p while there is a probability $1-p$ that policy will enter state 2. When $s_t = 2$, the current nominal interest rate, i_t, is still set endogenously and constrained by the ZLB, but the central bank credibly announces next quarter's nominal interest rate, i_{t+1}, will be pegged to 1 regardless of economic conditions. That promise places a dogmatic distribution on the future nominal interest rate unlike news shocks, which always allow for the possibility that $i_{t+1} > 1$. The economy transitions from $s_t = 2$ to a state where the current nominal interest rate is pegged to 1, but the future nominal interest rate is either set endogenously ($s_t = 3$ with probability p) or the central bank lengthens the interest rate peg by one quarter ($s_t = 4$ with probability $1-p$).

Using an exogenous interest rate peg to model forward guidance is less flexible than news shocks for several reasons. One, an exogenous interest rate peg cannot respond to changes in current economic conditions, which is inconsistent with the threshold-based nature of recent forward guidance. Two, a 1-quarter interest rate peg is a special case of the news shock approach where a large anticipated shock causes the expected policy rate to equal zero. Three, the central bank cannot reverse previously announced forward guidance policies. Four, the effects from additional news cannot be separated from a longer forward guidance horizon because extending the horizon with an interest rate peg is analogous to providing increasingly large news shocks over that horizon.

Figure 14 shows impulse responses to a 1.5% increase in the discount factor given different interest rate pegs. We specify a discount factor shock that is large enough to push the nominal interest rate to its ZLB in the first quarter but small enough that its mean reverting process will push the expected nominal rate above its ZLB in the second quarter without forward guidance. With a 1-quarter peg, the economy enters state 2 in the first quarter, where the central bank promises to keep next quarter's nominal interest rate at zero. The economy then transitions to state 3 in the second quarter, where the central bank holds the current nominal interest rate at its ZLB but makes no promises for the third quarter. Without additional forward guidance, households expect the nominal interest rate to rise as the economy recovers. With a 2-quarter peg, the economy transitions to state 4 in the second quarter. The additional quarter of forward guidance provides

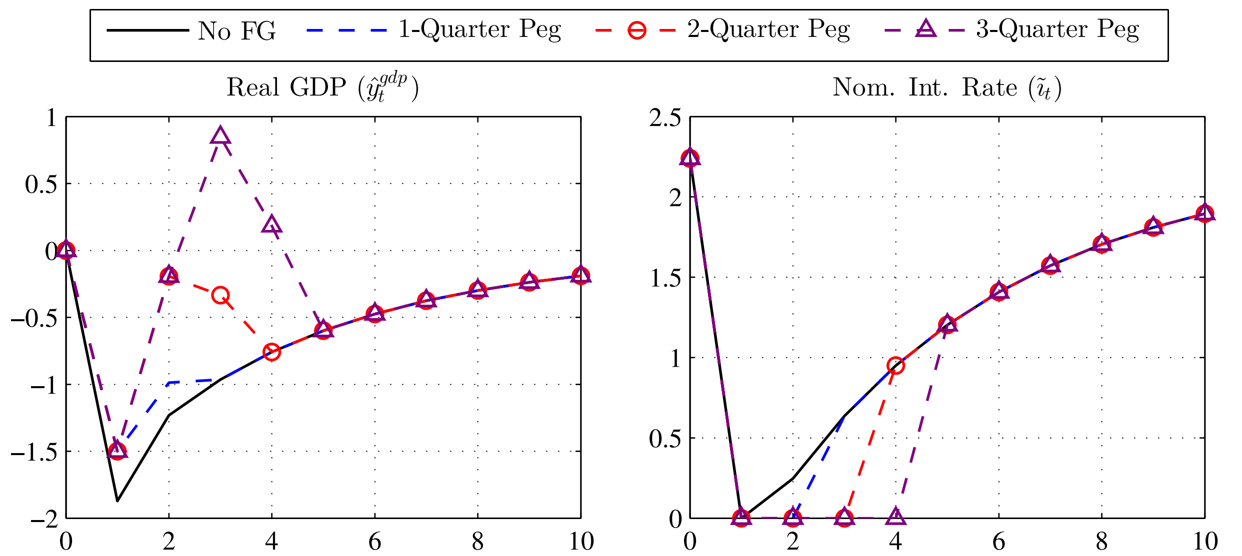

Figure 14: Impulse responses to a 1.5% increase in the discount factor with different interest rate pegs.

a larger boost to real GDP since the expected nominal interest rate in the absence of the peg is further above zero. If the economy stays in state 4, the peg lasts for 3 quarters and is stimulative enough to push real GDP above its steady state in the third quarter. Each additional quarter in the interest rate peg is analogous to increasingly larger news shocks that cause the expected nominal interest rate to equal zero. Thus, an interest rate peg generates much larger increases in real GDP.

C NUMERICAL ALGORITHM

A formal description of the numerical algorithm begins by writing the model compactly as

$$\mathbb{E}[f(\mathbf{z}_{t+1}, \mathbf{w}_{t+1}, \mathbf{z}_t, \mathbf{w}_t)|\Omega_t] = 0,$$

where f is a vector-valued function that contains the equilibrium system, $\mathbf{z}$ is a vector of exogenous variables, $\mathbf{w}$ is a vector of endogenous variables, and $\Omega_t = \{M, P, \mathbf{z}_t\}$ is households' information set in period t, which contains the structural model, M, its parameters, P, and the state vector, $\mathbf{z}$. With 1-quarter forward guidance, $\mathbf{z}_t = (\varepsilon_{t-1}, \varepsilon_t, \beta_t)$ and with 2-quarter forward guidance, $\mathbf{z}_t = (\varepsilon_{t-2}, \varepsilon_{t-1}, \varepsilon_t, \beta_t)$. For both forward guidance horizons $\mathbf{w} = (c, \pi, y, n, w, r)$.

Policy function iteration approximates the vector of decision rules, Φ, as a function of the state vector, $\mathbf{z}$. The decision rules for the model are

$$\underbrace{\Phi(\mathbf{z}_t)}_{\text{True RE Solution}} \approx \underbrace{\hat{\Phi}(\mathbf{z}_t)}_{\substack{\text{Approximating} \\ \text{Function}}}.$$

We choose to iterate on $\Phi = (c, \pi)$ so that we can easily solve for future variables that enter the households' expectations using f. Each state variable in $\mathbf{z}$ is discretized into N^d points, where $d \in \{1, \ldots, D\}$ and D is the dimension of the state space. Thus, the discretized state space contains $N = \Pi_{d=1}^{D} N^d$ nodes. We set the bounds of stochastic state variables to encompass 99.999 percent of the probability mass of the distribution. We specify 61 grid points for each continuous state variable and use 15 Gauss-Hermite nodes for each continuous shock (with 1-quarter forward guidance $N = 226{,}981$ and with 2-quarter forward guidance $N = 13{,}845{,}841$). Those techniques minimize extrapolation and ensure that the location of the kink in the decision rules is accurate.

The following outline summarizes the policy function algorithm we employ. Let $i \in \{0, \ldots, I\}$ index the iterations of the algorithm and $n \in \{1, \ldots, N\}$ index the nodes.

1. Obtain initial conjectures for the approximating functions, $\hat{c}_0$ and $\hat{\pi}_0$, on each node, from the log-linear model without the ZLB imposed. We use `gensys.m` to obtain those conjectures.
2. For $i \in \{1, ..., I\}$, implement the following steps:
   (a) On each node, solve for $\{i_t, w_t\}$ given $\hat{c}_{i-1}(\mathbf{z}_t^n)$ and $\hat{\pi}_{i-1}(\mathbf{z}_t^n)$ with the ZLB imposed.
   (b) Linearly interpolate $\{c_{t+1}, \pi_{t+1}\}$ given the state, $\{\varepsilon_t, \varepsilon_{t+1}^m, \beta_{t+1}^m\}_{m=1}^M$ (1-quarter forward guidance) and $\{\varepsilon_{t-1}, \varepsilon_t, \varepsilon_{t+1}^m, \beta_{t+1}^m\}_{m=1}^M$ (2-quarter forward guidance). Each of the M pairs of $\{\varepsilon_{t+1}^m, \beta_{t+1}^m\}$ are Gauss-Hermite quadrature nodes. We use Gauss-Hermite quadrature to numerically integrate, since it is very accurate for normally distributed shocks. We use piecewise linear interpolation to approximate future variables that show up in expectation, since this approach more accurately captures the kink in the decision rules than continuous functions such as cubic splines or Chebyshev polynomials.
   (c) On each node, solve for time $t+1$ variables, $\{y_{t+1}^m, c_{t+1}^m\}_{m=1}^M$ that enter the expectation operators. Then, numerically integrate to approximate the expectation operators,

$$\mathbb{E}\left[f(\mathbf{x}_{t+1}^m, \mathbf{x}_t^n)|\Omega_t\right] \approx \frac{1}{\sqrt{\pi}} \sum_{m=1}^{M} f(\mathbf{x}_{t+1}^m, \mathbf{x}_t^n)\phi(\varepsilon_{t+1}^m, \beta_{t+1}^m),$$

   where $\mathbf{x} \equiv (\mathbf{z}, \mathbf{w})$, and ϕ are the respective Gauss-Hermite weights. The superscripts on $\mathbf{x}$ indicate which realizations of the state variables are used to compute expectations. Finally, use the nonlinear solver, `csolve.m`, to minimize the Euler equation errors.
3. Define $\text{maxdist}_i \equiv \max\{|\hat{c}_i - \hat{c}_{i-1}|, |\hat{\pi}_i - \hat{\pi}_{i-1}|\}$. Repeat step 2 until $\text{maxdist}_i < 10^{-13}$ for all n and for 10 consecutive iterations. At that point, the algorithm converged to a solution.

Richter et al. (2014) demonstrate the accuracy of our algorithm in a model with a ZLB constraint.

D COMPUTING LONGER HORIZONS

To make our numerical algorithm tractable with longer forward guidance horizons (i.e., $q > 2$), we discretize each monetary policy shock with 3 points. Although this simplification prevents us from examining the entire distribution of each shock, it still allows us to compute GIRFs of a shock. We use the method outlined in Tauchen (1986) to obtain the 3 points and the corresponding weights for numerical integration. The state vector, $\mathbf{z}_t = (\beta_t, s_{0,t}, s_{1,t})$, is independent of the forward guidance horizon and determines the realization of the monetary policy shock, ε_t, according to

$$\varepsilon_t = \begin{cases} -0.005 & \text{for} \quad s_{0,t} = 1 \\ 0 & \text{for} \quad s_{0,t} = 2 \\ 0.005 & \text{for} \quad s_{0,t} = 3 \end{cases}.$$

When $s_{0,t} = 1$, there is a -0.5% monetary policy shock, which is the same size shock used to generate the GIRFs in the paper. $s_1 \in \{1, 2, \ldots, 3^q - 1, 3^q\}$ determines the realization of the lagged shocks in the news process, where q is the forward guidance horizon. For example, if $q = 1$, then $\varepsilon_{t-1} \in \{-0.005, 0, 0.005\}$, and if $q = 2$, then $(\varepsilon_{t-1}, \varepsilon_{t-2}) \in$

$$\{(-0.005, -0.005), (0, -0.005), (0.005, -0.005), \ldots, (0, 0.005), (0.005, 0.005)\}.$$

The transition matrix for $s_{0,t}$ is ergodic and is characterized by a single vector of probabilities,

$$P = (\lambda_1, \lambda_2, \lambda_3) = (0.1587, 0.6827, 0.1587),$$

where $\lambda_k = \Pr(s_{0,t+1} = k)$. We discretize β_t with 151 points so the state space contains $N = 151 \times 3 \times 3^q$ nodes. Expectations formation is given by

$$\mathbb{E}\left[f(\mathbf{x}^k_{t+1}, \mathbf{x}^n_t)|\Omega_t\right] \approx \sum_{k=1}^{3} \lambda_k \frac{1}{\pi} \sum_{m=1}^{M} f(\mathbf{x}^k_{t+1}, \mathbf{x}^n_t)\phi(\beta^m_{t+1}),$$

where $\mathbf{x} \equiv (\mathbf{z}, \mathbf{w})$, and ϕ are the Gauss-Hermite weights to numerically integrate across realizations of β_{t+1}. The superscript k on $\mathbf{x}$ indicates the realization of $s_{0,t+1}$. The policy function iteration algorithm is otherwise the same as outlined in Appendix C.

E GENERALIZED IMPULSE RESPONSE FUNCTIONS

The GIRFs are based on the average of 10,000 Monte Carlo simulations of the model. The advantage of this approach is that the realization of shocks are consistent with households' expectations that the stochastic processes will mean revert when the GIRF is initialized away from the model's stochastic steady state. The general procedure for calculating GIRFs is laid out in Koop et al. (1996). We apply the following steps to our models:

1. Find the state vector at which to initialize each Monte Carlo simulation by simulating the model for 500,000 quarters using random draws of discount factor shocks. The initial state vector is the average state vector conditional on the ZLB binding, $\mathbf{z}_0^{zlb}$.
2. Draw random monetary policy and discount factor shocks, $\{\varepsilon_t, \upsilon_t\}_{t=0}^N$, from their distributions. Simulate the model for R different draws of the sequence of shocks beginning at the initial state vector, $\mathbf{z}_0^{zlb}$. This yields R equilibrium paths, $\{\mathbf{x}_t^j(\mathbf{z}_0^{zlb})\}_{t=0}^N$, where $j \in \{1, 2, \ldots, R\}$. N corresponds to the number of quarters to plot and we set $R = 10{,}000$.
3. Using the same R draws of shocks from step 2, replace the policy rate shock in period one with a -0.5% shock (i.e., set $\varepsilon_1 = -0.5$ for all $j \in \{1, 2, \ldots, R\}$). Simulate the model with these alternate sequences of shocks. This yields R equilibrium paths, $\{\mathbf{x}_t^j(\mathbf{z}_0^{zlb}, \varepsilon_{z,1})\}_{t=0}^N$.
4. Average across the R simulations from step 2 and step 3 to obtain average paths given by

$$\bar{\mathbf{x}}_t(\mathbf{z}_0^{zlb}) = \frac{1}{R}\sum_{j=1}^{R} \mathbf{x}_t^j(\mathbf{z}_0^{zlb}), \qquad \bar{\mathbf{x}}_t(\mathbf{z}_0^{zlb}, \varepsilon_{z,1}) = \frac{1}{R}\sum_{j=1}^{R} \mathbf{x}_t^j(\mathbf{z}_0^{zlb}, \varepsilon_{z,1}).$$

5. The difference between the two paths is a GIRF. In our figures, a variable with a hat equals $100(\bar{\mathbf{x}}_t(\mathbf{z}_0^{zlb}, \varepsilon_{z,1})/\bar{\mathbf{x}}_t(\mathbf{z}_0^{s}) - 1)$, and a variable with a tilde equals $100(\bar{\mathbf{x}}_t(\mathbf{z}_0^{zlb}, \varepsilon_{z,1}) - \bar{\mathbf{x}}_t(\mathbf{z}_0^{zlb}))$.