ECONOMIC RESEARCH
FEDERAL RESERVE BANK OF ST. LOUIS

WORKING PAPER SERIES

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Authors	Gabriele Camera, and YiLi Chien
Working Paper Number	2012-058B
Revision Date	May 2013
Citable Link	https://doi.org/10.20955/wp.2012.058
Suggested Citation	Camera, G., Chien, Y., 2013; Understanding The Distributional Impact of Long-Run Inflation, Federal Reserve Bank of St. Louis Working Paper 2012-058. URL https://doi.org/10.20955/wp.2012.058

Published In	Journal of Money, Credit, and Banking
Publisher Link	https://doi.org/10.1111/jmcb.12136

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Understanding the Distributional Impact of Long-Run Inflation[†]

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April 26, 2013

Abstract

The impact of fully anticipated inflation is systematically studied in heterogeneous agent economies with an endogenous labor supply and portfolio choices. In stationary equilibrium, inflation non-linearly alters the endogenous distributions of income, wealth, and consumption. Small departures from zero inflation have the strongest impact. Three features determine how inflation impacts distributions and welfare: financial structure, shock persistence, and labor supply elasticity. When agents can self-insure only with money, inflation reduces wealth inequality but may raise consumption inequality. Otherwise, inflation reduces consumption inequality but may raise wealth inequality. Given persistent shocks and an inelastic labor supply, inflation may raise average welfare. The results hold when the model is extended to account for capital formation.

Keywords: Money; Heterogeneity; Wealth Inequality; Consumption Inequality.

JEL codes: E4, E5

[†] We thank two anonymous referee for helpful comments, and seminar participants at Michigan State, Simon Fraser, Fudan, Academia Sinica, Siena, Bologna, the Board of Governors of the Federal Reserve System, the Money Workshop at the Federal Reserve Bank of Chicago, Econometrics Society Summer Meetings 2011 and SAET meetings 2011. The views expressed are those of the individual authors and do not necessarily reflect the official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors. Camera: Faculty of Business and Economics, Department of Macroeconomics, University of Basel, Peter Merian-Weg 6, CH - 4002 Basel, Switzerland; e-mail: gabriele.camera@unibas.ch. Corresponding author: Chien: Research Division, Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO 63166-0442; E-mail: yilichien@gmail.com.

1 Introduction

Ongoing massive liquidity injections by the U.S. central bank are raising fears of significant long-run inflation and, with it, questions about the likely economic impact. Studies based on representative-agent models of the U.S. economy typically point to a negative, monotonic association between long-run inflation and social welfare. Most studies find that a representative household would give up some consumption to live in a zero-inflation economy, though the deadweight loss from moderate inflation quantitatively amounts to a fraction of 1% of consumption. Hence, from the vantage point of a representative agent the optimal policy prescription is non-inflationary, but departures from this policy are not very costly.[1]

But field economies are not populated by representative agents, so distributional issues must be taken into account. For instance, survey evidence suggests that low-income households are more concerned about inflation than wealthier households [9], and there is some empirical support for the view that inflation is negatively associated to wealth inequality in the U.S. (e.g., [17]). Unfortunately, only a handful of studies have focused on the impact of inflation in heterogeneous-agent monetary economies, and the results often differ on basic matters such as the impact on the distribution of wealth —which is the key aggregate state variable —whether it is optimal to target low inflation and, if so, what are the welfare gains.[2]

We report results from a study of the impact of long-run inflation in economies where inequality arises endogenously. We measure wealth and consumption inequality in the economy by means of Gini coefficients, by reporting some descriptive statistics (quartiles, top and bottom percentiles) and by plotting Lorenz curves, i.e., graphic displays reporting the cumulative share of a variable against the cumulative share of the population. The benchmark model is a production economy where labor supply and portfolio choices are endogenous. Capital is considered in an extension. Ex-ante homogeneous households hold money to trade on spot markets *and* to self-insure against idiosyncratic productivity shocks. Precautionary saving needs can also be satisfied by holding riskless debt securities that are illiquid. There

[1] To be precise, the optimal policy prescription is to run the so-called Friedman rule, i.e., to deflate at the common rate of time preference, but this policy is not implemented in practice. Notice also that higher welfare costs are found in some OECD economies; see [4].

[2] For examples, in [1] 10% inflation maximizes social welfare; in [16] average welfare increases in inflation if inflation is low. In contrast, [6] and [20] report that 10% inflation is worth, respectively, 0.6% and 8% of average consumption in the U.S.

is no aggregate risk. In stationary equilibrium productivity shocks induce heterogeneity in earnings, income, and wealth. Due to incomplete markets, consumption is heterogeneously distributed and the allocation is inefficient. Long-run inflation affects the (in)efficiency of the allocation by altering the equilibrium distributions of income, wealth, and consumption. Inflation results from money supply expansions achieved by fully anticipated lump-sum injections. In equilibrium, a 1% increase in the rate of monetary expansion raises inflation exactly by 1%. Stationary equilibrium is studied by computing and then comparing steady states for different inflation rates in a model calibrated to the U.S. economy.

The study makes several contributions. First, it identifies three features of an economy that determine the distributional impact of monetary policy: financial structure, elasticity of labor supply, and the process underlying earnings shocks. Disparities in earlier results can be traced back to different assumptions about one or more of these features. Second, it provides evidence of a non-linear impact of inflation on the distributions of endogenous variables. Small departures from zero inflation have the greatest consequences because they strongly alter the incentives to self-insure, inducing a significant drop in the size and the liquidity of savings portfolios. Third, the study reveals that although a faster rate of monetary expansion may reduce wealth concentration, this can *magnify* consumption inequality (and vice versa). It follows that inflation-induced reductions in the concentration of wealth do not necessarily result in improvements in average welfare. These results hold when the model is extended to incorporate capital formation.

There is clear intuition for these findings. When labor supply and portfolio choices are endogenous, a trade-off exists between inflation-induced consumption *redistribution* and output *decline*. A faster rate of monetary expansion has the potential to reduce consumption inequality by altering the distributions of wealth and income; however, it surely causes a permanent output decline due to endogenous labor choices. This mean variance trade-off depends in meaningful ways on the assets available to self-insure against shocks (money, bonds), the persistence of earnings shocks, and the response of the labor supply to changes in the real wage. The first two elements control the extent of inflation-induced reallocation of consumption and the third affects the overall consumption loss. We report that departures from a zero-inflation policy may raise average welfare when earnings shocks are persistent and the labor supply is inelastic. This helps us reconcile varying findings about the welfare-impact of monetary expansion in models where money is the *only* asset (e.g., as in [16], and [6]).

The financial structure matters for two reasons. First, inflation-induced wealth redistribution is tied to the composition of savings portfolios: When money is the only asset, a faster rate of monetary expansion acts as a progressive tax that lowers wealth inequality (as in [3]); When bonds can be traded, wealth inequality is less affected by inflation because the rich hold more illiquid portfolios than the poor. Second, the financial structure affects the ability to self-insure and to relax spending constraints. Higher inflation sharply lowers consumption of those who have tight spending constraints and cannot borrow. Shock persistence directly affects the extent of inequality, while the labor supply elasticity affects the inflation-induced output decline. An inelastic labor supply brings the model closer to an endowment economy where money injections simply induce mean-preserving consumption redistributions (e.g., as in [1] which does not have a labor supply response).

The paper proceeds with a literature review in Section 2. Section 3 presents the model, and Section 4 presents the results from the computational analysis. Section 5 concludes.

2 Related Literature

Only a handful of studies focus on the impact of fully anticipated inflation in economies with heterogeneity. Of these, only a very few perform a systematic investigation (i.e., very few studies consider a wide range of inflation rates). Most studies simply compute equilibria for two or three inflation values, usually 0%, 5% and 10%, and for a given financial structure. The study in [13], for instance, considers a pure exchange economy with a cash in advance constraint where inflation is generated through lump-sum money creation. There are two types of agents who can either have a high or a low endowment for the period, the endowment state follows a Markov process and the aggregate endowment is fixed. Hence, there is equilibrium heterogeneity in money holdings, which vary with inflation, but aggregate income and aggregate consumption are fixed and independent of inflation. The paper studies the average welfare cost of 5% and 10% inflation rates relative to an economy with no inflation (see the transactions cost model in [10], or the inventory-theoretic model in [8], for more recent examples). In contrast, our study develops a systematic investigation of the distributional impact of inflation across different financial structures.

The existing studies with more systematic investigations share some common elements. They all consider frameworks in which money is valued, at least partly, because it allows agents to self-insure against some idiosyncratic shock (e.g., [16], and [6]). They also typically adopt a computational methodology —and so do we. Such existing studies differ in their motivation for money (cash-in-advance restrictions, market timing frictions, trading constraints, and so on), and in the (un)availability of assets other than money. More importantly, the reported findings often differ in several key dimensions but such differences cannot be readily ascribed to the type of model adopted or the role played by money in the model. Dissimilar results are reported in matching models of money, cash-in-advance models, and models where money has only a precautionary role.

The article [1] studies optimal risk-sharing in a pure exchange economy where bonds and money are held only for precautionary purposes. A market-timing friction ensures agents with high endowments can only smooth consumption by holding money. Therefore, in equilibrium only high-income agents hold money and the demand for money vanishes as inflation gets out of hand. Positive inflation in this model ensures maximum risk-sharing, redistributing surplus to low-income agents, because at zero nominal interest rates bonds—which allow lending *and* borrowing, unlike money—are underutilized. At a zero nominal interest rate bonds and money pay the same return, hence the allocation is as in a money-only economy. But bonds allow borrowing, so positive inflation is optimal because it induces a bond demand (improves risk-sharing) and redistributes income from high-income to low income agents. The model, however, lacks a labor supply response to inflation. The central finding is that there is a positive inflation rate that maximizes welfare: inflation as high as 10% is *necessary* to maximize social welfare.

The precautionary money demand model in [20] delivers an antithetic result: 10% inflation is worth at least 8% of per capita consumption. This is a production economy where agents can hold precautionary money balances, as well as capital. The model is calibrated to match the distribution of money holdings in U.S. data by imposing random redistribution of net wealth. In the model inflation destroys the self-insurance value of money and raises the volatility of consumption for low-income households.

In a random matching model, [16] shows that some inflation can improve social welfare because higher inflation can reduce wealth and price dispersion, but only if inflation is low. The opposite holds if shocks are not persistent.[3] In this model agents experience iid shocks that constrain them to be buyer or seller in a period, but not both. Sellers and buyers are randomly and bilaterally matched in every period, and bargain to exchange goods for money. Random trading give rise to a nondegenerate distribution of money holdings and, by virtue of the bargaining protocol assumed, prices are an increasing function of the seller's wealth. [6] find that the welfare-improving effect of inflation vanishes when the search model in [16] is augmented with a market for money; inflation lowers average welfare, though the welfare cost is small. In this model after seller-buyer matches break, agents can enter another market by suffering fixed disutility. In that market, agents can buy or sell money. This allows for some market-based redistribution of money to take place in each period. The positive redistributive impact of inflation on welfare in Molico (2006) is thus substantially lessened. A small welfare cost of inflation also emerges from the matching model in [3], which shows how the financial structure matters a great deal for how this cost is distributed in the economy. The model assumes two markets open in sequence and quasilinear utility, which generates tractable equilibrium dispersion in wealth and earnings. They report that inflation does not generate large losses in societal welfare, but the distributional impact can be severe, and it depends on the financial sophistication of the economy. If money is the only asset, then inflation hurts mostly the wealthier and more productive agents, while the converse holds when agents can insure against consumption risk with assets other than money. In addition, they demonstrate that if money is *not* the only asset for self-insurance, then inflation can benefit the wealthier and harm the poorer households. Our study attempts to reconcile all these disparities.

[3] Moving away from zero inflation may also lead to a concentration in the distribution of wealth in the transaction cost model of [10]. Their study focuses on computationally studying how different transaction cost structures alter the distributional impact of inflation.

3 The model

Time is discrete, the horizon is infinite, and there is a continuum of ex-ante homogeneous infinitely lived households of measure one. Each household is a single economic decision unit composed of a shopper-worker pair. Thinking of each date as being divided into two subperiods (beginning and end); it is assumed that the shopper and the worker from each household are together only at the end of a period and otherwise are apart and undertake separate economic activities. Households consume a single perishable good and geometrically discount future consumption at rate $\beta \in (0, 1)$. At the beginning of each period, households operate on anonymous goods and labor markets, while they consume and operate on financial markets only at the end of each period.

On every date $t = 1, 2, ...$ a perishable good can be produced by a representative profit-maximizing firm. Labor is the only factor of production. The firm is owned by households in equal non-marketable shares and is defined by the production technology $Y : \mathbb{R}_+ \to \mathbb{R}_+$, is strictly increasing and concave, and satisfies the Inada conditions. Though Y is not homogeneous of degree one, such a feature can be recovered by adding an "entrepreneurial"factor in fixed supply to redistribute profits as factor payments (see [15]). With this in mind, the firm can be considered representative. It is assumed that the firm can pay the wage bill after selling its output. However, since the shopper and the worker are apart, workers demand monetary compensation and shoppers carry money balances.

At the start of each date, workers draw productivity shocks determining how many efficiency units of labor they can supply to the firm. For a worker from household n, let $h_{n,t} \in \{h_L, h_H\}$ denote the amount of "effective"labor she can supply on date t per unit of time worked, —that is, the worker in the household can have either high or low productivity with $0 \leq h_L < h_H < \infty$. It is assumed that for each household n and each date t the shock process follows a first-order Markov chain with transition probabilities $\Pr[h_{n,t+1} = h_j | h_{n,t} = h_j] = q$ for $j = L, H$ and $\Pr[h_{n,t+1} \neq h_j | h_{n,t} = h_j] = 1 - q$. The parameter $q \in (0, 1)$ affects the persistence of the shock, which is measured by the correlation coefficient $2q - 1$. Labor shocks introduce ex-post heterogeneity across households. The long-run distribution of labor productivity is invariant, with half of households having low productivity and the other half high productivity. Denoting by $\ell_{n,t}$ the labor supply of household n on date t, the per capita supply of effective labor is $L_t = \int_n \ell_{n,t} h_{n,t} dn$ so that the per capita output supply is $Y(L_t)$.[4]

Preferences are as follows. If household n consumes $c_{n,t} \geq 0$ goods and supplies $\ell_{n,t} \geq 0$ labor on any date t, then the household's utility is $u(c_{n,t}) - g(\ell_{n,t})$, where the function u is twice continuously differentiable, strictly increasing and concave, and g is convex with $g(0) = 0$. A government exists that is the sole supplier of fiat currency, of which there is an initial nominal stock $\bar{M} > 0$ evolving deterministically at gross rate π thanks to lump-sum transfers to households at the end of each period t. A bond market opens only at the end of each period. On t households sell or buy one-period nominal bonds (in zero net supply) that mature on $t+1$ and pay gross interest $i_{t+1} \geq 0$. Firms' dividends are distributed at the end of each period. It is assumed that all parties can commit to fulfill their financial obligations and that all economic agents are price takers.

3.1 Stationary Monetary Allocations

To set the stage for the analysis, consider the allocation selected by a planner who treats agents identically. We call it the *efficient allocation*. The planner maximizes the expected lifetime utility of a representative agent subject to the physical and technological constraints. The optimal plan, which solves a dynamic problem, has the following characteristics (See Supporting Materials) It is stationary, unique, and implies constant individual consumption and individual state-contingent labor supply. Put simply, the efficient allocation perfectly insures each household, a result that motivates our focus on stationary allocations of the monetary economy.

Definition 1. An allocation for a monetary economy is stationary if the distribution of consumption is time invariant and the real money stock is positive and stationary.

The efficient allocation can be decentralized by introducing a full set of contingent claims to be initially traded. However, the economy cannot achieve the efficient allocation because markets are assumed incomplete: Households can self-insure only with money and bonds.[5]

[4]In Section 4.3 we extend the analysis to the case of capital formation by considering $Y(L_t, K_t)$ as being generated by a CRS production technology with labor and capital.

[5]The presumption here is that frictions in financial markets prevent households from issuing a full set of claims that are contingent on their realized productivities. The solution to the problem with complete contingent claims is in the Supplementary Materials.

Notice that in a stationary monetary allocation, money market clearing implies that inflation is pinned down by the money growth process. Let $\pi > 0$ denote the stationary growth rate of nominal prices.

3.2 The Household's Problem

Households maximize expected lifetime utility, and since their problem is recursive we formalize it with a functional equation. Let V be the value function of a household at the start of a date *after* shocks are realized. Let $m \geq 0$ and $b \geq \underline{b} > -\infty$ denote the start-of-period household's portfolio of money and bonds, defined in real terms. Denote with a prime variables in the following period.

Consider a stationary distribution of wealth. Because households cannot trade state-contingent assets and can trade goods only on spot markets, then the relevant state of a household includes current productivity and portfolio of assets. In particular, the history of labor shocks s is relevant only if it affects the current labor shock. Hence, let (m, b, h) denote the current state of the household.

Given (m, b, h), in an economy where the nominal price sequence grows at constant rate π, the household's problem is to choose $(c, \ell, m') \geq 0$ and $b' \geq \underline{b}$ to maximize expected lifetime utility. The problem has a recursive representation:

$$\begin{aligned} V(m,b,h) = \quad & \max\{u(c) - g(\ell) + \beta EV(m',b',h')\} \\ & \text{s.t.} \quad c + \pi(m' + b') \leq w\ell h + m + bi + \xi + \tau, \\ & \qquad\; c \leq m, \end{aligned} \tag{1}$$

where nominal variables have been normalized by the contemporaneous nominal price of goods. The household faces two constraints. The budget constraint accounts for uses of funds, —that is, consumption $c \geq 0$ and real savings in the form of $b' \geq \underline{b}$ bonds and $m' \geq 0$ money balances. The latter are adjusted by gross inflation π because both assets are nominal. Sources of funds include $w\ell h$ income from supplying ℓh efficiency units of labor to the market, bond interest payments bi, m real balances, a lump-sum real balance transfer τ, and a dividend payment ξ.[6] There is also a cash-in-advance constraint because the buyer

[6] ξ can be considered compensation to a second productive input called "entrepreneurial capital". This asset is in positive and identical net supply across households and is not tradable.

must pay with money. Thus, disposable *income* includes $b(i-1)+\xi+\tau$ plus *earnings* $w\ell h$, while net *wealth* is $m+b$.

Conjecture that the function V exists and is differentiable. Let $V_x := \frac{\partial V}{\partial x}$ for $x = m, b$. Let $\mu \geq 0$ and $\lambda \geq 0$ be the multipliers on the first and second constraints:

$$\begin{aligned}
&u'(c) - \mu - \lambda = 0 \\
&g'(\ell) - \mu w h = 0 \\
&-\pi\mu + \beta E V_{m'} \leq 0 \\
&-\pi\mu + \beta E V_{b'} \leq 0
\end{aligned} \tag{2}$$

The envelope theorem implies

$$V_m = \mu + \lambda \quad \text{and} \quad V_b = \mu i, \tag{3}$$

that is, the marginal value of assets reflects the marginal utility of income μ, and the marginal utility of liquidity, λ.

The first-order conditions reveal that heterogeneity in income and consumption depends on three elements. A household's labor supply depends not only on its own productivity but also on wealth through the marginal value of income μ. Consumption depends on wealth as well as differences in the *liquidity premium* of money for that specific household, $u'(c)-\mu$. Liquidity premia will generally differ across households depending on the level *and* composition of savings.

From Equation (2) and Equation (3) we have

$$\begin{aligned}
&-\pi[u'(c)-\lambda] + \beta E[u'(c')] \leq 0 && (\text{with} = \text{if } m' > 0) \\
&-\pi[u'(c)-\lambda] + \beta i E[u'(c') - \lambda'] \leq 0 && (\text{with} = \text{if } b' > \underline{b}).
\end{aligned} \tag{4}$$

Bonds, unlike money, cannot be immediately traded for consumption, so they are illiquid. The expressions in Equation (4) indicate that, due to their illiquidity, bonds are held only if they dominate money in rate of return (i.e., if they pay a positive nominal interest rate). If money is held, then

$$u'(c) = \frac{\beta}{\pi} E[u'(c')] + \lambda \tag{5}$$

and if $b' > \underline{b}$, then

$$i = \frac{E[u'(c')]}{E[u'(c') - \lambda']}. \tag{6}$$

Instead, if $b' = \underline{b}$, then $i < \frac{E[u'(c')]}{E[u'(c')-\lambda']}$. Given uninsurable income shocks, the economy will generally exhibit heterogeneity in consumption, income, and wealth. Households will differ also in the composition of their portfolios. While an analytical characterization of stationary monetary outcome is beyond the scope of this paper, the following Lemma provides some useful results.

Lemma 1. In a stationary monetary outcome, the following must hold: (i) $m' > 0$ so that Equation (5) always holds; (ii) $\lambda > 0$ for at least some household; (iii) if $i \leq 1$, then $b' = \underline{b}$; and (iv) if $b' > \underline{b}$, then $E[\lambda'] > 0$.

Proof. See the Appendix. □

In a stationary monetary economy, every household holds a positive fraction of their savings in cash. Though not everyone may hold bonds, which are illiquid, those who do hold bonds will never hold "enough"money to satisfy any desired consumption level. In fact, those who trade on the bond market will optimally choose money balances that leave them liquidity-constrained in at least some possible future state. Put differently, when nominal interest rates are positive, no household will fully self-insure against all their possible future liquidity needs; if this were not the case, then lenders should optimally consume more today and accumulate less wealth, while borrowers should borrow even more (see equation (6)).

To understand these results, consider that in this economy savings fulfill a precautionary need. Households are subject to uninsurable income shocks, which expose them to income risk. Though income cannot be spent on contemporaneous consumption, it can be saved for future purchases. A low income shock may constrain future consumption levels because it restricts current monetary savings. Therefore, households in this economy will generally want to hold extra savings as a precaution against low income shocks. Since households want to maximize the return from savings, wealthier households in general will hold only a fraction of their savings in cash. In particular, no household will optimally hoard enough cash to be unconstrained in their consumption in every possible state.

Availability of a bond market improves the efficiency of the allocation for two reasons. First, the option to *buy* bonds reduces the opportunity cost of holding precautionary savings.

This especially matters to wealthy households who would otherwise be affected by a large inflation tax. Second, the option to *sell* bonds reduces the need to hold precautionary savings. This especially matters to poor households, which are more likely to be constrained. In the event of a low income shock, these households can spend all their current cash savings on consumption, and borrow money for future consumption by selling bonds. As a result, opening a bond market will increase the velocity of money because a smaller portion of the money supply is kept idle, and it will also mitigate the impact of low-productivity shocks on spending patterns. In turn, this should improve consumption smoothing and reduce consumption inequality.

Optimality of the firm's labor demand choice implies that in a stationary outcome the firm demands labor L that satisfies $w = F'(L)$. The firm distributes revenues in excess of labor compensation as dividends $\xi = Y(L) - wL$.

3.3 The Distribution of Savings and Stationary Equilibrium

Let the state of a household be denoted $\omega := (m, b, h) \in \Omega := M \times B \times H$, with $M = [0, \infty)$, $B = [\underline{b}, \infty)$, and $H = \{h_l, h_h\}$. Let $\mathcal{P}(H)$ denote the power set of H, $\mathcal{B}(M)$ and $\mathcal{B}(B)$ denote the Borel $\sigma-$algebra of M and B, respectively. Let $\mathcal{B}(\Omega) := \mathcal{B}(M) \times \mathcal{B}(B) \times \mathcal{P}(H)$ and define the subset of possible states $\mathbb{B}(\Omega) := (\mathbb{M}, \mathbb{B}, \mathbb{H}) \subseteq \mathcal{B}(\Omega)$. Finally, let $\{\Omega, \mathcal{B}(\Omega), \Phi\}$ define the probability space, where Φ is a probability measure.

Given current productivity, $h \in H$, $p(h'|h)$ denotes the conditional probability of reaching $h' \in H$ next period. The evolution of the distribution of the state ω can be characterized using a transition function $Q : \Omega \times \mathcal{B}(\Omega) \to [0, 1]$ defined by

$$Q(\omega, \mathbb{B}(\Omega)) = \begin{cases} \sum\limits_{h' \in \mathbb{H}} p(h'|h) & \text{if } (m'(\omega), b'(\omega)) \in \mathbb{M} \times \mathbb{B}, \\ 0 & \text{otherwise} \end{cases}$$

for all $\omega \in \Omega$ and all $\mathbb{B}(\Omega) \subseteq \mathcal{B}(\Omega)$. Put simply, the function Q allows us to calculate the probability of realizing any level of productivity $h' \in \mathbb{H}$ tomorrow, given that (i) today's state is $\omega \in \Omega$ and given that (ii) tomorrow's portfolios are restricted to be an element of $(\mathbb{M}, \mathbb{B})$. In particular, if portfolios are not in $(\mathbb{M}, \mathbb{B})$, then the function Q assigns probability zero to reaching the set of productivity $\mathbb{H}$. Let ϕ denote the joint probability density associated with the probability space $\mathcal{B}(\Omega)$. It is a mixed density, with a discrete random variable h and two continuous random variables, m and b. Given the transition function defined in Q, the law of motion for the probability measure Φ is given by

$$\Phi'(\mathbb{B}(\Omega)) = \sum_h \int_m \int_b Q(\omega, \mathbb{B}(\Omega))\phi(\omega) dm db.$$

A stationary monetary equilibrium is a time-invariant distribution of consumption, labor supplies, real money balances, and real bond holdings across the population of households, such that on each date the optimal plan of a household in state $\omega = (m, b, h) \in \Omega$ involves $c(\omega)$ consumption, $\ell(\omega)$ labor, $m'(\omega)$ and $b'(\omega)$ monetary and bonds savings, respectively, that solve the household problem (1), given that wages maximize the firm's profit, that all markets clear (goods, money, bonds, and labor), and given that the distribution of states (wealth and productivity) and the real value of the money supply are stationary.

Definition 2 (Recursive Equilibrium)**.** A stationary monetary recursive competitive equilibrium is a constant real value of liquidity $\bar{M}$, an inflation rate π, a wage rate w, an interest rate i, a set of policy functions $m' : \Omega \to \mathbb{R}_+, b' : \Omega \to [\underline{b}, \infty)$ and $c : \Omega \to \mathbb{R}_+$, $\ell : \Omega \to \mathbb{R}_+$, and an invariant probability measure Φ such that

1. Given π, i, and w, the policy functions $m'(\omega), b'(\omega), c(\omega)$ and $\ell(\omega)$ for $\omega \in \Omega$ solve the household problem.

2. Given w, the firm demands labor L that satisfies $w = F'(L)$ and distributes revenues in excess of labor compensation as dividends $\xi = Y(L) - wL$.

3. Markets clear:

$$\sum_{h \in H} \int_{m \in M} \int_{b \in B} \ell(\omega) h \phi(\omega) dm db = L \qquad \text{(money market)}$$

$$\sum_{h \in H} \int_{m \in M} \int_{b \in B} \pi m'(\omega) \phi(\omega) dm db = \bar{M} \qquad \text{(labor market)}$$

$$\sum_{h \in H} \int_{m \in M} \int_{b \in B} c(\omega) \phi(\omega) dm \, db = Y(L) \qquad \text{(goods market)}$$

$$\sum_{h \in H} \int_{m \in M} \int_{b \in B} b'(\omega) \phi(\omega) dm db = 0. \qquad \text{(bonds market)}$$

4. For all subsets $\mathbb{B}(\Omega) \subseteq \mathcal{B}(\Omega)$, the cdf $\Phi'(\mathbb{B}(\Omega))$ satisfies

$$\Phi'(\mathbb{B}(\Omega)) = \sum_h \int_m \int_b Q(\omega, \mathbb{B}(\Omega))\phi(\omega) dm db$$

   and $\Phi'(\mathbb{B}(\Omega)) = \Phi(\mathbb{B}(\Omega))$.

Clearly, stationary equilibrium, if it exists, is characterized by wealth and consumption inequality because markets are incomplete (Lemma 1). In equilibrium we denote average consumption by $\bar{c} = Y(L)$ and we let $\bar{m}$ denote average real money balances held at the start of a date, with

$$\bar{m} := \sum_{h \in H} \int_{m \in M} \int_{b \in B} m\phi(m, b, h) dmdb.$$

Characterization of equilibrium is an analytically intractable task because the distribution of money and bonds, which is the key aggregate state variable, is analytically intractable. Therefore, analysis is conducted using a computational methodology.[7]

4 Main Findings

This section reports findings from a computational analysis of steady states in a yearly model parameterized to match data for the U.S. for the sample period 1950-2006. Over that period, the average annual CPI inflation rate was 3.9% with a maximum of 14% and a minimum of -0.7%. Parameters are selected according to the procedure discussed below.

The production function is taken to have the form $Y(L) = L^{\alpha}$, where α is set to 0.7 as in the standard real business cycle model. To calibrate the idiosyncratic productivity shocks process we follow the procedure in [2]. They calibrate the shock process using estimates of the standard deviation and correlation of productivity shocks in the U.S. reported in [18]. Because there is no aggregate shock in our model, we adapt the procedure in [2] to our model by eliminating the counter-cyclical variation of labor income risk. In our model, the Markov process for the log of the productivity shocks must satisfy the estimated standard deviation $std(\ln h) = 0.71$ and correlation $\rho(\ln h) = 0.87$. This implies values for the idiosyncratic productivity shock $(h_L, h_H) = (0.1974, 0.8053)$ and a two-state transition matrix:

$$\begin{bmatrix} 0.935 & 0.065 \\ 0.065 & 0.935 \end{bmatrix}.$$

We also calibrate a separate model for the case of iid shock (i.e., where $\rho(\ln h) = 0$).

We adopt the functions $u(c) := \frac{c^{1-\gamma}}{1-\gamma}$ for $\gamma \neq 1$ with $u(c) := \ln c$ for $\gamma = 1$, and $g(\ell) := \frac{\ell^{\delta}}{\delta}$. To pin down δ, note that in competitive equilibrium the wage satisfies $wh = \ell^{\delta-1}$, where h

[7] Details of the numerical procedure, which is burdensome, are described in the Supporting Materials.

is the realization of the productivity shocks; hence the equilibrium labor supply is $\ell(w) = (wh)^{\frac{1}{\delta-1}}$. The elasticity of the labor supply with respect to the real wage is $\frac{d\ln\ell(w)}{d\ln w} = \frac{1}{\delta-1}$. Different studies use different measures; for instance, the literature on Social Security often uses 1 as a target for labor elasticity (e.g., [14]). However, estimates of the elasticity of labor supply vary from study to study and also according to the group considered. Following the literature on Social Security and estimates in the recent study by [11], in the benchmark calibration we target an elasticity of labor supply with respect to the own wage of 1, which implies $\delta = 2$. We also do a sensitivity analysis for two other measures of the elasticity of labor supply with respect to the own wage: 2 and 0.5, which correspond to $\delta = 1.5$ and 3, respectively. Given that we work with yearly data, we set $\beta = 0.97$.

Finally, to calibrate the intertemporal elasticity of substitution parameter γ, we match the variance of inverse velocity $\frac{M}{PY}$ in the data to that in the model. The nominal price level P is the GDP deflator, aggregate nominal output PY is nominal GDP, and the nominal money supply M is M1.[8] The calibrated value of γ depends on the value δ. When we select $\delta = 2$, we obtain $\gamma = 1.3$, which is in line with values calibrated in the literature. [9]

We first present results for economies where agents exclusively self-insure with money. This facilitates comparisons with the welfare cost of the inflation literature, which is based largely on models where money is the *only* asset. It also helps to clarify the role played by bond/credit markets, which we introduce subsequently. For expositional clarity, findings are reported as separate "Results", and reference to stationary equilibrium and to the baseline case are omitted when understood.

4.1 Money is the Only Available Asset

This section reports findings regarding stationary equilibrium allocations when the bond market is shut down. This means households exclusively self-insure with money and money

[8] M1 is in billions of dollars, December of each year, not seasonally adjusted. For the years before 1958, M1 data is from [12](p. 708-718, col. 7). For 1959-06, M1 is from the St. Louis Fed FRED database. Nominal GDP is from The National Income and Product Accounts of the United States.

[9] Ideally, we would parameterize the model by matching the inverse velocity of money (e.g., Chiu and Molico, 2010). However, ours is a "pure" cash in advance model, and so money circulates less than once per period, unlike in the data. Hence, we match the variance of inverse velocity. Although this is non-standard, it generates γ values that are in line with the literature, which gives us confidence in the calibration. To assess the sensitivity of the results to our choice of target, we ran the model also with other reasonable values of γ and do not find strong differences with the specification selected (see Supporting Materials).

balances represent the totality of savings (i.e., $b = 0$).

Result 1. Equilibrium exhibits endogenous inequality in income, wealth and consumption. Wealth and consumption inequality increase with the persistence of shocks.

Table 1 and Figures 1 and 2 provide supporting evidence.[10] Equilibrium inequality originates from idiosyncratic shocks to productivity. In particular, consumption inequality is tied to market incompleteness and the stochastic process responsible for productivity shocks, which are persistent in the baseline calibration. To understand the impact of persistence, we computed a version of the model in which we retain the same unconditional (long-run) mean productivity of the baseline calibration but assume iid shocks. We find that greater persistence reduces mobility across classes of wealth because it makes sudden earnings variations unlikely; wealth accumulation slows down for poor households and speeds up for rich households. More persistent shocks also generate a stronger desire to hold precautionary savings, which in turn affects consumption patterns. Wealthy households deplete their savings more slowly when they incur a negative shock that is more persistent; poor households more slowly accumulate wealth when they face a positive shock that is more persistent.[11] Result 1 thus suggests that the redistributive impact of monetary policy crucially depends on the process of earnings shocks.

Quantitatively, the money-only model—which is calibrated to match the persistence of earning shocks in the U.S. data—can account for roughly half of the income inequality in the U.S. data but only for one-third of the wealth inequality. In addition, the money-only model cannot fully account for the feature that wealth is substantially more concentrated than income. [7] report Gini coefficients for income and non-housing wealth —respectively, 0.548 and 0.861 in 1998, and 0.575 and 0.881 in 2007.

At 2% inflation, which roughly corresponds to the experience in those years, the model generates Gini coefficients of 0.249 and 0.313 for income and wealth.

[10] Wealth and consumption inequality are reported across inflation rates by means of Gini coefficients and, for 2% inflation, using Lorenz curves. A Lorenz curve represents inequality in the distribution of wealth by reporting the cumulative share of wealth against the cumulative share of the population. A curve coinciding with the 45-degree line corresponds to no inequality.

[11] Income inequality decreases with persistence (Table 1). Labor supply decisions are more extreme when shocks are iid; highly productive households supply more labor to take advantage of their temporarily high productivity; the opposite holds for low productivity agents. As a result, the variance of effective labor hours is higher under iid than persistent shocks. Overall, the responses of the two types of workers do not offset each other, so the per-capita supply of labor is higher and the wage rate lower under iid than persistent shocks.

Result 2. A faster rate of monetary expansion lowers income inequality and per-capita output.

Table 1 provides supporting evidence. Income inequality primarily falls because the monetary expansion is accomplished with lump-sum injections. Households whose wealth m is below per capita (or average) wealth $\bar{m}$ receive a net transfer $(\pi - 1)(\bar{m} - m)$, while all others are taxed. A faster rate of monetary expansion induces a permanent output decline because the rate of return on money falls with inflation, which raises the opportunity cost of savings. As the incentive to save declines, so does the incentive to supply labor. Hence, given a labor demand that is independent of inflation, equilibrium output falls; the severity of such a decline is a function of the labor supply elasticity. By market clearing, per capita output equals per capita consumption, so an increment in inflation induces a *permanent* decline in per-capita consumption. This decline is the key feature of representative-agent models with production, which explains why the representative household is typically found to be willing to give up some consumption to avoid *any* inflation. The message of Result 2 is that with heterogeneity, inflation-induced output declines do not necessarily render inflation socially undesirable because inflation also redistributes income and, as reported in the next two results, wealth and consumption.

[Table 1 about here.]

[Figure 1 about here.]

[Figure 2 about here.]

Result 3. Per capita wealth and wealth inequality decline nonlinearly with inflation. Small departures from zero inflation generate the steepest declines.

The message here is that expansionary monetary policy can be a tool for redistributing wealth. Table 1 and Figure 3 and 4 provide the evidence for this. We report that per capita savings rapidly fall as inflation grows above zero, and then slowly decline as inflation grows above 5% in the baseline calibration. A wealth decline occurs because the opportunity cost of precautionary savings grows with inflation. On the one hand, inflation lowers the self-insurance value of money, which reduces the desire to hold savings that exceed transactions needs (precautionary savings). Incentives to self-insure with money against earnings shocks are maximized when the opportunity cost of money is minimized (i.e., at zero inflation). In addition, lump-sum injections provide some insurance. Consequently, per capita wealth monotonically falls with inflation until it equals per capita expenditure. The pattern is nonlinear because the decline in precautionary savings is concentrated mostly among wealthy households who, unlike poor households, have significant precautionary savings. This also explains why wealth concentration declines, and why this decline is significant at low inflation rates; that is, when inequality is greater (Table 1). Figure 3 illustrates this phenomenon.

[Figure 3 about here.]

[Figure 4 about here.]

Wealth inequality *also* declines because lump-sum money injections directly redistribute income (Result 2). To appreciate this phenomenon, consider the trajectory of wealth by quartiles, across inflation rates (Figure 4 and Table 1). At low inflation, we identify three classes of households: those with a long history of similar income shocks, those who occupy the tails of the wealth distribution, and those households "in transition" (=the middle class). As inflation increases, precautionary savings decline; hence wealth levels increasingly reflect the household's most recent productivity shocks, so the middle class shrinks. From this point on, lump-sum money creation becomes the driving force behind inflation-induced wealth redistribution. If inflation is sufficiently high, then precautionary savings are virtually zero (Table 1). At that point, the wealth distribution becomes bimodal because only current earnings shocks matter, hence income and wealth inequality coincide.

In summary, two lessons have emerged so far. First, if money is the *only* source of self-insurance, then increments in fully anticipated inflation not only reduce income and wealth concentration, but also lower per capita income, output, and wealth. Second, the impact of inflation is nonlinear: Small departures from zero inflation have the strongest redistributive impact because the incentives to hold precautionary savings quickly vanish. Beyond a low inflation threshold, lump-sum money creation becomes the engine of inflation-induced redistribution. This threshold is very low in the baseline calibration; per capita savings exceed consumption by 270% at zero inflation and by only 6% at 5% inflation.

Results 2 and 3 might suggest that inflation necessarily reduces consumption inequality. In fact, this is not so.

Result 4. A faster rate of monetary expansion may elevate consumption inequality.

Tables 1 and Figures 5, 6 and 8 provide supporting evidence. We show that when households can self-insure only with money, consumption inequality grows with inflation if inflation is sufficiently low. At low rates, inflation has the potential to redistribute consumption shares *away* from the middle class toward those at the wealth distribution's tails (Figure 5). At high rates, consumption shares are redistributed top to bottom. This non-monotone association between inflation and consumption inequality is observed also when inequality is lower —that is, when earnings shocks are not very persistent (Result 1 and Table 1). The message from Result 4 is thus very simple: Inflating to decrease wealth concentration does not necessarily imply a decrease in the concentration of consumption.

[Figure 5 about here.]

[Figure 6 about here.]

Result 4 emerges because increments in inflation generate heterogeneous wealth and substitution effects. Lump-sum money transfers do induce strong wealth effects for the poorest households. However, inflation also unequally alters spending constraints, which are heterogeneously tight as a result of wealth inequality. As a result, an increase in inflation significantly raises the marginal value of money for households with tight liquidity constraints (poor and lower middle-class), so their consumption falls. Figure 7 illustrates this substitution effect when shocks are iid. The marginal value of money (V_m) nonlinearly declines in wealth because it reflects both the level of wealth and the severity of the household's liquidity constraints (the multipliers μ and λ). Moving from 0% to 10% inflation substantially increases the marginal value of money for poor households, while it minimally changes, or slightly lowers it for everyone else. A similar pattern emerges if shocks are persistent.

[Figure 7 about here.]

In summary, if money is the only available asset, then increments in fully anticipated inflation are sure to reduce per capita consumption and may also increase consumption inequality. The efficiency consequences are reported next.

Result 5. Per capita welfare is nonlinearly associated with inflation. The association is non-monotone when shocks are persistent.

Table 2 reports the (average) welfare cost of $x\%$ inflation, as opposed to no inflation, for different specifications of the model. Figure 8 reports welfare costs for the baseline case and inflation up to 40%.[12] The main point here is that qualitative and quantitative differences emerge based on the persistence of shocks, and the elasticity of labor supply. In a nutshell, labor elasticity and shocks affect the consumption mean variance trade-off associated to long-run inflation.

[Table 2 about here.]

[Figure 8 about here.]

With iid shocks, the welfare cost (of inflation) is positive and it monotonically increases with inflation (Panel B in Table 2). This is qualitatively in line with findings from representative-agent models and some heterogeneous-agent models. Quantitatively, the welfare cost is several times that found in representative agent models. With persistent shocks the average welfare cost remains positive but it becomes non-monotone in inflation (Panel A in Table 2); it initially increases as inflation rises above zero, then falls, and finally the average welfare cost rises again.

This non-monotonicity suggests caution should be taken in making qualitative and quantitative policy assessments. First, assessing the welfare impact monetary policy based on measurements for a few inflation targets, as is often done in the literature, can lead to troubling conclusions.[13] Second, with heterogeneity, the welfare impact of inflation hinges on the money injection mechanism in meaningful ways. In a sense, assuming lump-sum injections maximizes the possible welfare-enhancing impact of inflation, as opposed to assuming injection mechanisms that need not redistribute wealth in a socially desirable manner (e.g., open market operations, asymmetric transfers).

[12] Clearly, the average welfare cost obtained by comparing the monetary allocation with the efficient allocation is always positive and large because markets are incomplete. For example, at 2% inflation with persistent shocks, the average households would give up almost 15% consumption to be at the efficient allocation. See the Appendix for the definitions of welfare costs.

[13] For instance, [13] finds that (i) there is a welfare cost of inflation, (ii) it is larger than in representative agent models, and (iii) it is higher at 10% than at 5% inflation. However, infinite inflation is optimal in that setting. Large inflations push the economy closer to the planner's allocation without affecting mean consumption (the aggregate endowment is assumed fixed).

The non-monotonicity is even more prominent when output is less responsive to inflation because in this case some inflation may increase welfare. For example, consider $\delta = 3$ in the Panel A of Table 2, which corresponds to the case where the elasticity of labor supply is halved relative to baseline. To understand the role of the elasticity of the labor supply, consider that a planner would be willing to dissipate some output to reduce consumption inequality. Shock persistence and labor supply elasticity alter this trade-off between inflation-induced output decline and redistribution. Persistent shocks magnify inequality; hence the desirability of redistribution (Result 1): An inelastic labor supply brings the model closer to an endowment economy where output is unresponsive to inflation (as in [1] and [13]). Hence, a sufficiently inelastic labor supply allows moderate inflation to improve average welfare by redistributing consumption "top to bottom" without generating excessive output declines (Result 2). This inflation rate is generally bounded away from zero due to the initial increase in consumption inequality (Result 4).

Table 2 also reports welfare costs by wealth quartiles. In the baseline calibration (Panel A in Table 2, $\delta = 2$), every household dislikes inflation except for those in the bottom quartile. Interestingly, the welfare costs can be non-monotone across wealth levels. For instance, households in the second quartile would give up 9% consumption to avoid 5% inflation, which is twice what households in the next quartile would give up. This is tied to Result 4: Middle-class households lose consumption shares as inflation rises above zero. Panel B in Table 2 shows that every household dislikes inflation when shocks are iid, because there is less inequality than under persistent shocks.

4.2 Introducing a Credit Market

This section reports findings when we add the possibility to borrow and lend by trading risk-free bonds. One could think of this as introducing a financial innovation.[14] For comparison purposes, the economies are calibrated to the same parameter values used before; Tables 3-5

[14] This is not equivalent to assuming that agents can buy some goods with cash and the rest with credit; all consumption goods must still be purchased with cash because bonds are illiquid. However, now agents can relax their liquidity constraints through borrowing; in this sense, there is a similarity with a "cash & credit goods" model, where liquidity constrains are less stringent. In the computation the borrowing limit is set to $\underline{b} = -6\xi$, i.e., agents can borrow up to six times the dividend received in a period. The government borrows 100% of Y so the net bond supply is $\overline{B} = \int_n b_n dn = Y$. This is helpful for computational reasons; it allows us to bound the model away from $i = 0$.

and Figure 8 report the results. Start by observing that the introduction of a bond market alters per capita output; with no inflation, per capita output in the bond economy is 0.9% higher than in the money-only economy. Output is higher because demand is stronger due to the fact that agents can now borrow to smooth their consumption. Inflation has still a negative impact on per capita wealth and output. However, the output decline associated with expansionary monetary policy is slower relative to the money-only economy; at 40% inflation, output is almost 3% higher than in the money-only economy. In other words, with the introduction of bonds the price of inflation—in terms of lost output—declines.

Equilibrium is still characterized by endogenous wealth and consumption inequality because the introduction of debt securities is not sufficient to complete the market. Importantly, introducing a bond market alters both the *composition* and the *distribution* of portfolios in a meaningful way.

Result 6. When money is not the only asset, the liquidity of portfolios declines with inflation and the household's wealth.

Table 3 provides evidence on how the financial innovation we have considered affects the composition of portfolios. When households can self-insure with money and bonds, they reduce their exposure to the inflation tax by minimizing their money balances and holding bonds, hence the illiquid share of savings increases with inflation.

[Table 3 about here.]

Illiquid bonds form the bulk of precautionary savings because bonds dominate money in rate of return (Lemma 1). Consequently, money has primarily a transactions role. Since wealthy agents hold the bulk of precautionary savings, the monetary share of portfolios declines in the household's wealth. In the benchmark calibration, for instance, households in the bottom wealth quartile choose for their entire holdings only money to use for transactions purposes. This finding is qualitatively in line with U.S. data from the Survey of Consumer Finances, which reports that poor households hold essentially cash. We next report how these features alter the distribution of endogenous variables and, consequently, the distributional impact of inflation, relative to the model without money.

Result 7. Consumption inequality is lower and wealth inequality is greater when households can access a credit market, as opposed to when they cannot.

Table 4 provides supporting evidence.

[Table 4 about here.]

Introducing the option to borrow and lend improves risk-sharing, which in equilibrium lowers consumption inequality, therefore raising average welfare. Wealth inequality primarily increases relative to the money-only economy because poor households now can borrow. Because precautionary savings are now mostly composed of bonds (Result 6), per capita money balances more closely track per capita consumption, unlike the money-only economy.[15] The point here is that observing an increase in wealth concentration should not lead us to conclude that consumption has also become more skewed.

Introducing a bond market is a step in the right direction in terms of obtaining concentration measures closer to U.S. data. At 2% inflation, we obtain Gini coefficients 0.280 and 0.732 for income and wealth. The first measure still falls short in matching the income concentration in the data, but wealth concentration now accounts for about 85% of that observed in the data for 1998. As a result, the ratio of concentrations in wealth and income exceeds that in the data.[16]

The possibility to self-insure with interest-bearing assets has important implications for inflation-induced redistributions and the welfare cost of inflation.

Result 8. When money is not the only asset, a faster rate of monetary expansion reduces consumption inequality but does not decrease wealth inequality.

Financial innovation weakens the redistributive impact of inflation. Table 4 provides supporting evidence. When households can trade bonds, money plays a minor role in consumption smoothing, which makes inflation less effective in redistributing wealth. An increment in the money growth rate may increase wealth inequality because households can borrow to relax increasingly tight liquidity constraints. In addition, because monetary shares decline

[15] Consumption is more dispersed than money holdings because money is partially used for precautionary purposes. Most individuals hold small money balances. However, the liquidity of their asset portfolios differs. Individuals who are not very wealthy hold mostly liquid assets and may not spend their entire money balances, holding some for precautionary purposes. As inflation increases, money is never held for precautionary purposes; hence the Gini coefficients of consumption and money holdings become identical.

[16] In order to match the data better, households in the model should be given the possibility to over accumulate wealth, or experience substantial wealth losses. An alternative, is to consider financial market access constraints as in [5].

in wealth, a monetary expansion amounts to imposing a regressive tax on wealth. Consumption inequality monotonically falls with a faster rate of money growth for two primary reasons. First, money is injected via lump sum, which is a very effective way to reduce income inequality (Result 2). Second, the availability of a bond market allows households to relax increasingly binding spending constraints through borrowing. Clearly, when shocks are not persistent, inequality is already small so inflation-induced consumption redistribution is minimal. The availability of a credit market also has implications for the impact of inflation on average welfare, which we summarize as follows.

Result 9. When shocks are persistent and the labor supply is inelastic, expansionary monetary policy may raise average welfare.

Table 5 provides supporting evidence. Introducing a bond market generally lowers the average welfare cost of inflation because households can better self-insure and can also partially shield their precautionary savings from the inflation tax. As a consequence, the positive redistributive effect of lump-sum money injections is heightened. Whether inflation improves average welfare depends on the specifics of the economy in terms of shocks and labor supply.

With iid shocks, the welfare cost of inflation is positive and it monotonically increases with inflation (Panel B in Table 5). In this case, the associated inflation-welfare cost is qualitatively comparable to that observed in the economy where money in the only asset (Panel B in Table 2), but the welfare cost is now quantitatively smaller. For instance, moving from 10% to no inflation was worth 3.67% consumption in the money-only economy, while it is worth only 0.07% in the economy with bonds. The message here is that the economy with money, bonds, and iid shocks is qualitatively and quantitatively in line with representative-agent model findings.

[Table 5 about here.]

With persistent shocks, the average welfare cost is non-monotone; this is consistent with Result 5. However, now some inflation may improve average welfare, and this may occur when the concentration of wealth also increases (Panel A in Table 5). For example, the average household in the money-only economy would *give up* 5.3% consumption to avoid 10% inflation, but would *request* 1.11% additional consumption in the bond economy, even if wealth is more concentrated than at zero inflation. The welfare cost is U-shaped because inflation reduces both per capita output and income inequality. The first (negative) effect is dominated by the second (positive) at low inflation rates if the labor supply is sufficiently inelastic and shocks are persistent. In this case, income redistribution benefits a majority of households.

To see this, note that now the wealthiest households may also prefer some inflation (Panel A in Table 5, $\delta = 2$). The reason is simple: Unlike the money-only economy, wealthy households can now avoid the inflation tax by holding illiquid portfolios. In addition, lump-sum money injections provide some insurance against earnings risk, which can be valuable enough to overcome any decrease in current utility from lower consumption and less leisure. The value of this "additional" insurance increases with the persistence in earning shocks.[17]

Qualitatively, these findings hold when households can trade bonds—hence can borrow—after productivity shocks are realized and before the good market opens. In this case, constraints no longer bind for anyone, unless money and bonds are equivalent assets or households can borrow insufficient amounts (see Supporting Materials). Clearly, since households can fully relax their liquidity constraints through borrowing, we expect less inequality, hence somewhat weaker results. Indeed, when we calibrate the model to the same parameters used before, we find that consumption inequality is lower, and dispersion in money holdings simply reflects income disparities; no differences in qualitative results emerge.

4.3 The model with capital

In this section we extend the model to include physical capital to understand whether and how long-run adjustment in the capital/labor ratio—which are brought about by changes in long-run inflation—alter the distributional impact of inflation.

[17] There are variations in current utility and continuation utility as inflation increases. Inflation distorts labor supply decisions. With higher inflation all households supply less labor. Lump sum injections of money also decrease income inequality. As a result, current utility increases for the poor, because they consume more and work less. The welfare gain for the poor is significant because of their high marginal utility of consumption. Their continuation utility also increases. On the other hand, the rich suffer a current utility loss because they consume less but not very much. This utility loss is limited because the rich have a low marginal utility of consumption and a high marginal utility of leisure. Moreover, the rich's continuation utility may increase because the continuation utility from going to a low productivity state is now higher. Hence, the improvement in continuation utility can outweigh the loss in current utility, in which case the rich can also benefit from higher inflation.

Let aggregate output be produced by a representative firm with the CRS technology $Y(K_t, L_t) = K_t^{\frac{1}{3}} L_t^{\frac{2}{3}}$. The aggregate capital stock K_t is endogenously determined. Assuming a constant depreciation rate δ_d (set to 10% in the computation) and denoting by I_t aggregate investment on date t, the law of motion for capital is $K_{t+1} = (1-\delta_d)K_t + I_t$. In each period t the firm choose (L_t, K_t) to maximize profit $Y(K_t, L_t) - w_t L_t - r_t K_t$, where r_t denotes the rental rate. Clearly, $\frac{\partial Y(K_t,L_t)}{\partial L_t} = w_t$ and $\frac{\partial Y(K_t,L_t)}{\partial K_t} = r_t$. Hence, in equilibrium firms make zero profits and so households receive no lump-sum transfer ξ from the firm.

Now the household can save with money and also with two illiquid assets—bonds and capital. If we let $m, k \geq 0$ and $b \geq \underline{b} > -\infty$ denote the household's holdings of money, capital and bonds (all defined in real terms) at the start of a period, then the household's budget constraints is

$$c + \pi(m' + b') + k' \leq w\ell h + (r + 1 - \delta_d)k + m + bi + \tau,$$

where nominal variables have been normalized by the contemporaneous nominal price of goods.[18] Since there are no aggregate shocks, capital and bonds must have identical real return in equilibrium, i.e.,

$$r + 1 - \delta_d = \frac{i}{\pi}.$$

As a result, in stationary equilibrium households must be indifferent between holding capital or bonds because these two assets have the same return and are identically illiquid. Hence, if we let $\omega := \frac{k}{\pi} + b$ denote the household's non-monetary wealth, the budget constraint can be rewritten as

$$c + \pi(m' + \omega') \leq w\ell h + m + \omega i + \tau.$$

Given (m, ω, h) and a fully anticipated inflation rate π, the household's problem is to choose $(c, \ell, m', \omega') \geq 0$ and $\omega' \geq \underline{b} > -\infty$ to maximize expected lifetime utility. The problem has

[18]In the computation we set $\underline{b} = -6rK$, i.e., the household can borrow up to six times the share of capital income, which is in line with the assumption made in the bond economy (one can think the bond economy as one in which the capital stock is exogenously fixed at unity, hence the share of capital income is simply the dividend). The government still borrows 100% of Y.

the usual recursive representation

$$\begin{aligned} V(m,\omega,h) = \quad & \max\{u(c) - g(\ell) + \beta EV(m',\omega',h')\} \\ & \text{s.t.} \quad c + \pi(m' + \omega') \leq w\ell h + m + \omega i + \tau, \\ & \qquad\ \ c \leq m, \end{aligned}$$

This problem is identical to the one studied in Section 3.2. The only difference is a variable redefinition; we have ω instead of b. Hence, expressions (2)-(6) and Lemma 1 still hold, with the appropriate variable change.

Market clearing conditions now include the capital market, i.e.,

$$K_t = \int_n k_{n,t} dn = \pi \int_n (\omega_{n,t} - b_{n,t}) dn.$$

In this model inflation affects the degree of heterogeneity in the economy through the capital formation process, in addition to the other channels. Capital formation matters because it affects the marginal product of labor, hence the incentives to work. The open question is whether this additional channel significantly alters the results that emerged when households can only access a bond market.

To answer this question we calibrate the model with capital to the same parameter values used before; Tables 6-7 and Figure 8 report the results. The analysis reveals that the results are broadly in line with those reported in the previous section, with some differences, which we report next.

Income inequality is greater now that households can access a capital market, as opposed to when they could not (Table 6). Introducing a capital market is a step in the right direction to get income concentration closer to the recent U.S. experience. At 2% inflation, we obtain Gini coefficients 0.415 for income, under persistent shocks (roughly 75% of that observed in 1998). The ratio of wealth to income concentration is also much closer to the data than in the bond economy (1.57 in 1998). Wealth inequality, however, is not high enough—a common problem in the literature.

[Table 6 about here.]

The reason for the increase in income inequality is due to a marked change in the composition of portfolios and, therefore, in the distribution of capital income. Now that households can access a market for capital, portfolios of all but the poorest households are significantly more illiquid than in the bond economy (Table 7). The shift to illiquid assets is especially pronounced for households in the middle of the wealth distribution; the poorest, still hold no precautionary savings and simply hold cash. This also helps us to understand why wealth inequality is less pronounced than in the bond economy: a greater portion of households (those in the middle of the distribution) accumulates wealth.

[Table 7 about here.]

There are a few other features that are noteworthy. First, the equilibrium real interest rate is close to but slightly higher than in the model without capital, for inflation rates consistent with the U.S. experience post WWII. Second, the model does not generate a Tobin effect, i.e., greater steady state inflation is not associated to greater capital accumulation. This is interesting because one could imagine that households would more aggressively seek safety in capital assets as inflation increases. But this does not happen in the model, so that the impact of inflation on consumption and on average welfare remains in line with what we reported in Result 9 (Figure 8). In particular, we still find that when shocks are persistent and the labor supply is inelastic, expansionary monetary policy may still raise average welfare even if it lowers per capita consumption and even if it reduces welfare of the wealthier segment of society (see also Tables in Supplementary Materials).

5 Final Comments

This study has shown that whether inflation can be used as an instrument to improve social welfare crucially depends on the persistence of shocks to earnings, financial structure, and the elasticity of labor supply. Persistent shocks magnify inequality, hence the desirability of redistributive policies from an average welfare perspective. The financial structure determines the extent to which households with different wealth use money for self-insurance purposes, which has implications for how effectively inflation can redistribute wealth. The elasticity of labor supply affects the inflation-induced output loss.

In the model, long-run inflation results from fully anticipated lump sum money transfers. When households can self-insure only with money, increments in inflation have been shown to reduce wealth disparities. However, this does not necessarily make inflation socially desirable. In fact, we report that inflation can *increase* consumption inequality. When agents can access a credit market and can choose the composition of their portfolios–bonds and money, or also capital—inflation becomes ineffective at redistributing wealth. Overall, we report that—when we compare steady states of an annual model calibrated to U.S. data— a faster rate of fully anticipated monetary expansion may increase average welfare if shocks are persistent and the labor supply is inelastic.

Additional intuition could be gathered by studying the model when money creation is the result of open market operations and credit markets are not equally accessible to every household.

References

[1] Akyol, Ahmet. Optimal Monetary Policy in an Economy with Incomplete Markets and Idiosyncratic Risk. *Journal of Monetary Economics*, 51(6), 1245-1269, 2004

[2] Alvarez, F. and U. Jermann. Quantitative Asset Pricing Implications of Endogenous Solvency Constraints. *Review of Financial Studies*, 14, 1117-1152, 2001

[3] Boel, Paola and Gabriele Camera. The Welfare Cost Of Inflation In OECD Countries. *Macroeconomic Dynamics*, 15, 217-251, 2011

[4] Boel, Paola and Gabriele Camera. Financial Sophistication and the Distribution of the Welfare Cost of Inflation. *Journal of Monetary Economics*, 56(7), 968-978, 2011

[5] Chien, YiLi and Harold Cole and Hanno Lustig. A Multiplier Approach to Understanding the Macro Implications of Household Finance. *Review of Economic Studies*, 78(1), 199-234, 2011

[6] Chiu, Jonathan and Miguel Molico. Liquidity, redistribution, and the welfare cost of inflation. *Journal of Monetary Economics*, 57(4), 428-438,2010

[7] Díaz-Giménez, Javier and Andrew Glover and José-Víctor Ríos-Rull. Facts on the distributions of earnings, income, and wealth in the United States: 2007 update. *Quarterly Review*, 34(1), 2-31, 2011

[8] Dressler, Scott. Money Holdings, Inflation, And Welfare In A Competitive Market. *International Economic Review*, 52(2), 407-423, 2011

[9] Easterly, William and Fischer, Stanley. Inflation and the Poor. *Journal of Money, Credit and Banking*, 33(2), 160-178, 2001

[10] Erosa, Andres and Ventura, Gustavo. On Inflation as a Regressive Consumption Tax. *Journal of Monetary Economics*, 49(4), 761-795, 2002

[11] Fiorito, Riccardo and Giulio Zanella. The Anatomy of the Aggregate Labor Supply Elasticity. *Review of Economic Dynamics*, 15(2), 171-187, 2012

[12] Friedman, Milton. Inflation: Causes and Consequences. Proquest/Csa Journal Division, 1963

[13] Imrohoroglu, Ayse. The welfare cost of inflation under imperfect insurance. *Journal of Economic Dynamics and Control*, 16(1), 79-91, 1992

[14] Imrohoroglu, Selahattin and Kitao, Sagiri. Labor supply elasticity and social security reform. *Journal of Public Economics*, 93, 867-878, 2009

[15] McKenzie, Lionel. On the Existence of General Equilibrium for a Competitive Market. *Econometrica*, 27(1), 54-71, 1959

[16] Molico, Miguel. The Distribution Of Money And Prices In Search Equilibrium. *International Economic Review*, 47(3), 701-722, 2006

[17] Romer, Christina D. and David H. Romer. Monetary Policy and the Well-Being of the Poor. *Economic Review*, Q(I), 21-49, 1999

[18] Storesletten, Kjetil and Chris Telmer and Amir Yaron. Persistent Idiosyncratic Shocks and Incomplete Markets. Carnegie Mellon University, Tepper School of Business, GSIA Working Papers, 1997

[19] Storesletten, Kjetil and Chris Telmer and Amir Yaron. Cyclical Dynamics in Idiosyncratic Labor Market Risk. *Journal of Political Economy*, 112(3), 695-717, 2004

[20] Wen, Yi. Liquidity Demand and Welfare in a Heterogeneous Agent Economy. Federal Reserve Bank of St. Louis, Working Papers, 2010

Appendix

Proof of Lemma 1. Start by noticing that $u'(c) - \lambda = \mu > 0$, hence $u'(c) > \lambda \geq 0$. This follows from local non-satiation of preferences, so the budget constraint holds with equality and $\mu > 0$. To prove that $m' > 0$ for everyone we derive a contradiction. Suppose $m' = 0$ for someone. From

$$\begin{array}{ll} -\pi[u'(c) - \lambda] + \beta E[u'(c')] \leq 0 & \text{(with = if } m' > 0) \\ -\pi[u'(c) - \lambda] + \beta E[u'(c') - \lambda']i \leq 0 & \text{(with = if } b' > \underline{b}). \end{array} \tag{1}$$

optimality requires

$$-\pi[u'(c) - \lambda] + \beta E[u'(0)] \leq 0,$$

which is violated because $u'(c) - \lambda > 0$ and $u'(0) = \infty$.

To prove that $\lambda > 0$ for at least someone use a proof by contradiction. Suppose $\lambda = 0$ for everyone. Then

$$u'(c) = \frac{\beta}{\pi} E[u'(c')] + \lambda \tag{2}$$

implies $u'(c) = \frac{\beta}{\pi} E[u'(c')]$ for all households. Hence, current consumption is never constrained by monetary savings. This means that every household on average accumulates wealth, at least in some state. But this is suboptimal. They could spend some of it and improve their utility. This gives us the desired contradiction. Hence, $\lambda > 0$ for at least some agents, those with low money holdings.

To prove that if $i \leq 1$, then $b' = \underline{b}$ we also use a contradiction. Let $m' > 0$ (money is saved). Suppose $b' > \underline{b}$ and $i < 1$. Then from

$$i = \frac{E[u'(c')]}{E[u'(c') - \lambda']}. \tag{3}$$

we have $\frac{E[u'(c')]}{E[u'(c') - \lambda']} = i < 1$, which is impossible because $\lambda' \geq 0$. If $i = 1$, then money and bonds have the same return. Hence, if $\lambda' > 0$ in some state, then once again we derive a contradiction from Equation (3). and if $\lambda' = 0$ then money and bonds are identical assets, since the cash-in-advance constraint is not binding. Hence $b' = \underline{b}$ is optimal.

Finally let $i > 1$ and $m' > 0$. Suppose $b' > \underline{b}$. From Equation (3) we need $E[\lambda'] > 0$. □

Quantifying the Average Welfare Cost of Inflation

Fixing π, let $c_\pi(\omega)$, $\ell_\pi(\omega)$, $m'_\pi(\omega)$ and $b'_\pi(\omega)$ denote the optimal choices of a household in state $\omega := (m, b, h)$. Let $\omega' := (m'_\pi, b'_\pi, h')$ denote the state resulting from the current savings choices $m'_\pi(\omega)$ and $b'_\pi(\omega)$. Equilibrium *welfare* for this household is defined by $V_\pi(\omega)$ with

$$V_\pi(\omega) := u(c_\pi(\omega)) - g(\ell_\pi(\omega)) + \beta E V_\pi(\omega')$$

so that average welfare is W_π with

$$W_\pi := \sum_h \int_m \int_b V_\pi(\omega)\phi(\omega) dmdb \tag{4}$$

Inflation π affects average welfare W_π in two ways. Inflation redistributes monetary wealth thanks to inequalities in the *inflation tax* $(\pi - 1)(\bar{m} - m)$. There is no redistribution if $\pi = 1$ (no inflation). For all $\pi > 1$, monetary wealth is redistributed from those who have balances above average to those who have below-average balances. Redistribution under a deflation goes in the opposite direction. Second, inflation distorts savings decisions m'_π and b'_π and spending decisions c_π. In particular, inflation affects average consumption because aggregate output $Y(L)$ depends on individual labor supply decisions $\ell_\pi(\omega)$, which are a function of real wages as well as the individual state ω.

The welfare cost of $\pi > \beta$ inflation for a household with wealth ω is a standard compensating variation measure. It is the percentage adjustment in consumption that leaves the household indifferent, ex-ante, between inflation π and a lower rate $z \geq \beta$. Given that consumption is adjusted by the proportion $\bar{\Delta}_z$ in each period, and hours worked are unaltered, define the "compensated" value function $\hat{V}_z(\omega)$ by

$$\hat{V}_z(\omega) := u(\hat{\Delta}_z c_z(\omega)) - g(\ell_z(\omega)) + \beta E \hat{V}_z(\omega')$$

For a household in state ω, the welfare cost of z inflation, as opposed to π, is the value $\Delta_z = 1 - \hat{\Delta}_z$ that satisfies $V_\pi(\omega) = \hat{V}_z(\omega)$. If $\Delta_z > 0$, then this household is indifferent between π inflation or z inflation with consumption reduced by Δ_z percent.

In the numerical analysis we calculate average welfare cost **ex-ante**. It is the compensating variation from an ex-ante perspective, i.e., defining

$$\hat{W}_z := \sum_h \int_m \int_b \hat{V}_z(\omega)\phi(\omega) dmdb,$$

for a household at time zero the welfare cost of z inflation, as opposed to π, is the value $\Delta_z = 1 - \hat{\Delta}_z$ that satisfies $W_\pi = \hat{W}_z$. If $\Delta_z > 0$, then every household is ex-ante indifferent between π inflation or z inflation with consumption reduced by Δ_z percent. One can think of this as the percentage of consumption that the household would commit to give up, at the start of the economy, in order to be at π inflation instead of z.

The **ex-post** average welfare cost of inflation is

$$\bar{\Delta}_z := \sum_h \int_m \int_b \Delta_z(\omega)\phi(\omega) dmdb,$$

where the notation $\Delta_z(\omega)$ makes explicit that welfare costs of inflation are state-dependent, hence differ across households.

Table 1: Money-Only Economy

	Persistent Shocks					IID Shocks				
$\pi - 1$	$\overline{m}$	$\overline{c}$	$Gini_c$	$Gini_m$	$Gini_I$	$\overline{m}$	$\overline{c}$	$Gini_c$	$Gini_m$	$Gini_I$
0%	2.507	0.897	0.202	0.376	0.274	1.960	0.954	0.091	0.222	0.317
1%	1.843	0.890	0.210	0.347	0.259	1.767	0.947	0.100	0.218	0.309
2%	1.454	0.884	0.215	0.313	0.249	1.633	0.941	0.108	0.215	0.302
3%	1.201	0.880	0.220	0.277	0.242	1.533	0.936	0.115	0.212	0.296
4%	1.028	0.877	0.224	0.241	0.236	1.455	0.931	0.121	0.210	0.290
5%	0.928	0.874	0.226	0.214	0.232	1.394	0.926	0.126	0.208	0.285
10%	0.891	0.860	0.219	0.208	0.223	1.205	0.904	0.143	0.197	0.264
15%	0.870	0.848	0.212	0.206	0.215	1.105	0.886	0.151	0.190	0.247
20%	0.853	0.836	0.205	0.202	0.208	1.026	0.870	0.160	0.181	0.233
25%	0.840	0.826	0.199	0.196	0.202	0.967	0.856	0.167	0.174	0.221
30%	0.828	0.816	0.194	0.191	0.195	0.932	0.843	0.167	0.171	0.211
35%	0.817	0.807	0.188	0.186	0.190	0.904	0.830	0.165	0.170	0.203
40%	0.807	0.799	0.183	0.181	0.184	0.883	0.819	0.162	0.168	0.195

Note: $\gamma = 1.3$ and $\delta = 2$; $\pi - 1$ is the net inflation rate; $\overline{x}$ is the mean value, and $Gini_x$ is the Gini coefficient associated to the equilibrium random variable x. We define $m =$ money balances (in real terms), $c =$ consumption, $I =$ income net of taxes/transfers, $b =$ bonds balances (in real terms), wealth is $w = m + b$

Table 2: Distribution of Welfare Costs in Money-Only Economy

Panel A: Persistent Shocks							
	$\delta = 1.5$	$\delta = 2$					$\delta = 3$
$\pi - 1$	Δ_π	Δ_π	Q_1	Q_2	Q_3	Q_4	Δ_π
0%	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1%	2.213	1.674	−0.786	2.219	2.061	3.328	1.302
2%	4.155	3.143	−0.866	4.103	3.266	6.126	2.455
3%	5.866	4.438	−0.582	6.029	3.802	8.355	3.461
4%	7.389	5.569	−0.168	8.373	3.533	10.113	4.346
5%	8.739	6.300	0.096	9.020	4.281	11.251	4.336
10%	8.392	5.348	−0.918	7.856	3.827	10.888	3.373
15%	7.496	4.451	−1.821	6.739	3.479	10.388	2.499
20%	6.670	3.705	−2.636	5.833	3.184	10.053	1.774
25%	6.028	3.090	−3.284	5.000	3.014	9.776	1.185
30%	5.542	2.615	−3.783	4.289	2.912	9.631	0.711
35%	5.183	2.241	−4.219	3.725	2.884	9.547	0.307
40%	4.889	1.945	−4.571	3.230	2.888	9.533	−0.035

Panel B: IID Shocks							
	$\delta = 1.5$	$\delta = 2$					$\delta = 3$
$\pi - 1$	Δ_π	Δ_π	Q_1	Q_2	Q_3	Q_4	Δ_π
0%	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1%	0.641	0.522	0.183	0.417	0.601	0.882	0.419
2%	1.220	0.998	0.423	0.793	1.164	1.603	0.794
3%	1.759	1.432	0.668	1.119	1.722	2.200	1.129
4%	2.266	1.833	0.928	1.510	2.078	2.790	1.427
5%	2.750	2.196	1.203	1.855	2.441	3.250	1.688
10%	4.710	3.668	2.358	3.374	3.768	5.097	2.753
15%	6.169	4.748	3.252	4.472	4.755	6.411	3.560
20%	7.753	5.989	4.302	5.733	5.924	7.851	4.538
25%	9.455	7.103	5.255	6.875	6.991	9.104	5.181
30%	10.536	7.763	5.813	7.573	7.651	9.813	5.430
35%	11.394	8.322	6.292	8.167	8.227	10.383	5.664
40%	12.034	8.764	6.655	8.637	8.664	10.874	5.902

Note: $\gamma = 1.3$ and $\delta = 2$ unless otherwise noted; $\pi - 1$ is the net inflation rate. The welfare cost Δ_π is reported as the percent of current consumption the average household would give up to be at zero inflation (two-digit approximation; a negative number indicates a welfare gain). Q_i denotes the i-th quartile of the wealth distribution.

Table 3: Money/Total Asset Ratios by Wealth Quartiles

$\pi - 1$	Q_1	Q_2	Q_3	Q_4
0%	1.000	0.919	0.403	0.250
1%	1.000	0.907	0.393	0.250
2%	1.000	0.900	0.392	0.251
3%	1.000	0.902	0.393	0.251
4%	1.000	0.902	0.393	0.251
5%	1.000	0.901	0.393	0.250
10%	1.000	0.902	0.395	0.248
15%	1.000	0.905	0.397	0.246
20%	1.000	0.907	0.397	0.244
25%	1.000	0.907	0.397	0.242
30%	1.000	0.906	0.398	0.241
35%	1.000	0.905	0.399	0.240
40%	1.000	0.901	0.400	0.240

Note: The table is drawn for the benchmark calibration of persistent shocks with $\gamma = 1.3$ and $\delta = 2$; $\pi - 1$ is the net inflation rate. Columns 2-5 report the average real money balances divided by the average total assets held by households in each of the four wealth quartiles; this ratio varies from 0 to 1.

Table 4: Money and Bonds Economy

	Persistent Shocks						IID Shocks					
$\pi - 1$	$\overline{m}$	$\overline{c}$	$Gini_c$	$Gini_w$	$Gini_m$	$Gini_I$	$\overline{m}$	$\overline{c}$	$Gini_c$	$Gini_w$	$Gini_m$	$Gini_I$
0%	0.980	0.906	0.191	0.724	0.158	0.287	0.988	0.968	0.055	0.446	0.042	0.332
1%	0.939	0.902	0.190	0.732	0.173	0.283	0.979	0.965	0.055	0.448	0.044	0.329
2%	0.917	0.899	0.189	0.736	0.183	0.281	0.972	0.962	0.054	0.448	0.045	0.326
3%	0.908	0.896	0.188	0.739	0.185	0.278	0.966	0.959	0.054	0.448	0.047	0.323
4%	0.903	0.894	0.186	0.741	0.185	0.276	0.962	0.956	0.053	0.447	0.048	0.320
5%	0.900	0.891	0.185	0.740	0.184	0.274	0.957	0.953	0.053	0.446	0.048	0.317
10%	0.885	0.879	0.179	0.738	0.179	0.264	0.941	0.939	0.051	0.440	0.049	0.302
15%	0.873	0.868	0.173	0.737	0.173	0.255	0.927	0.926	0.049	0.437	0.049	0.289
20%	0.861	0.858	0.168	0.736	0.168	0.247	0.915	0.915	0.047	0.435	0.047	0.277
25%	0.850	0.849	0.163	0.734	0.163	0.239	0.903	0.903	0.046	0.436	0.046	0.266
30%	0.841	0.840	0.158	0.731	0.158	0.231	0.893	0.893	0.044	0.436	0.044	0.256
35%	0.832	0.831	0.154	0.729	0.154	0.224	0.883	0.883	0.043	0.437	0.043	0.247
40%	0.824	0.823	0.150	0.724	0.150	0.217	0.873	0.873	0.042	0.439	0.042	0.238

Note: $\gamma = 1.3$ and $\delta = 2$; $\pi - 1$ is the net inflation rate; $\overline{x}$ is the mean value, and $Gini_x$ is the Gini coefficient associated to the equilibrium random variable x. We define m = money balances (in real terms), c = consumption, I = income net of taxes/transfers, b = bonds balances (in real terms), wealth is $w = m + b$

Table 5: Distribution of Welfare Costs in Bond Economy

	Panel A: Persistent Shocks						
	$\delta = 1.5$			$\delta = 2$			$\delta = 3$
$\pi - 1$	Δ_π	Δ_π	Q_1	Q_2	Q_3	Q_4	Δ_π
0%	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1%	0.029	−0.039	−0.206	0.083	−0.045	0.097	−0.049
2%	0.027	−0.078	−0.331	0.109	−0.046	0.117	−0.122
3%	−0.058	−0.171	−0.495	0.099	−0.080	0.049	−0.242
4%	−0.147	−0.306	−0.671	−0.017	−0.150	−0.018	−0.376
5%	−0.331	−0.471	−0.891	−0.157	−0.278	−0.058	−0.527
10%	−1.029	−1.109	−1.741	−0.847	−0.462	−0.334	−1.097
15%	−1.553	−1.588	−2.491	−1.352	−0.515	−0.494	−1.583
20%	−1.951	−1.949	−3.031	−1.832	−0.491	−0.578	−1.942
25%	−2.221	−2.196	−3.446	−2.128	−0.427	−0.634	−2.222
30%	−2.392	−2.390	−3.763	−2.392	−0.493	−0.522	−2.433
35%	−2.481	−2.494	−4.036	−2.567	−0.421	−0.371	−2.599
40%	−2.515	−2.571	−4.269	−2.668	−0.324	−0.278	−2.691
	Panel B: IID Shocks						
	$\delta = 1.5$			$\delta = 2$			$\delta = 3$
$\pi - 1$	Δ_π	Δ_π	Q_1	Q_2	Q_3	Q_4	Δ_π
0%	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1%	0.012	0.021	0.029	0.028	0.007	0.022	0.004
2%	0.041	0.026	0.052	0.026	0.002	0.033	0.008
3%	0.060	0.042	0.089	0.028	0.029	0.034	0.010
4%	0.076	0.049	0.071	0.027	0.037	0.079	0.017
5%	0.101	0.069	0.086	0.042	0.057	0.113	0.019
10%	0.238	0.152	0.103	0.103	0.172	0.266	0.075
15%	0.422	0.278	0.193	0.237	0.363	0.368	0.123
20%	0.657	0.437	0.352	0.415	0.543	0.496	0.245
25%	0.931	0.648	0.574	0.637	0.804	0.638	0.377
30%	1.254	0.890	0.838	0.904	1.093	0.785	0.531
35%	1.607	1.158	1.111	1.196	1.409	0.976	0.691
40%	1.992	1.464	1.418	1.559	1.733	1.203	0.918

Note: $\gamma = 1.3$ and $\delta = 2$ unless otherwise noted; $\pi - 1$ is the net inflation rate. The welfare cost Δ_π is reported as the percent of current consumption the average household would give up to be at zero inflation (two-digit approximation; a negative number indicates a welfare gain). Q_i denotes the i-th quartile of the wealth distribution.

Table 6: Gini Coefficients in the economy with Capital

	Persistent Shocks				IID Shocks			
$\pi - 1$	$Gini_c$	$Gini_w$	$Gini_m$	$Gini_I$	$Gini_c$	$Gini_w$	$Gini_m$	$Gini_I$
0%	0.206	0.528	0.157	0.425	0.054	0.340	0.047	0.489
1%	0.205	0.530	0.179	0.420	0.054	0.340	0.049	0.484
2%	0.205	0.531	0.195	0.415	0.054	0.340	0.050	0.479
3%	0.204	0.532	0.202	0.411	0.054	0.339	0.051	0.473
4%	0.202	0.533	0.202	0.408	0.054	0.340	0.051	0.468
5%	0.201	0.534	0.201	0.404	0.053	0.340	0.051	0.463
10%	0.196	0.538	0.195	0.388	0.052	0.342	0.052	0.440
15%	0.191	0.543	0.191	0.372	0.052	0.346	0.051	0.419
20%	0.186	0.547	0.186	0.358	0.051	0.350	0.050	0.400
25%	0.182	0.551	0.181	0.345	0.050	0.355	0.049	0.383
30%	0.177	0.553	0.177	0.333	0.049	0.359	0.049	0.367
35%	0.173	0.555	0.173	0.321	0.048	0.363	0.048	0.352
40%	0.169	0.557	0.169	0.311	0.047	0.367	0.047	0.339

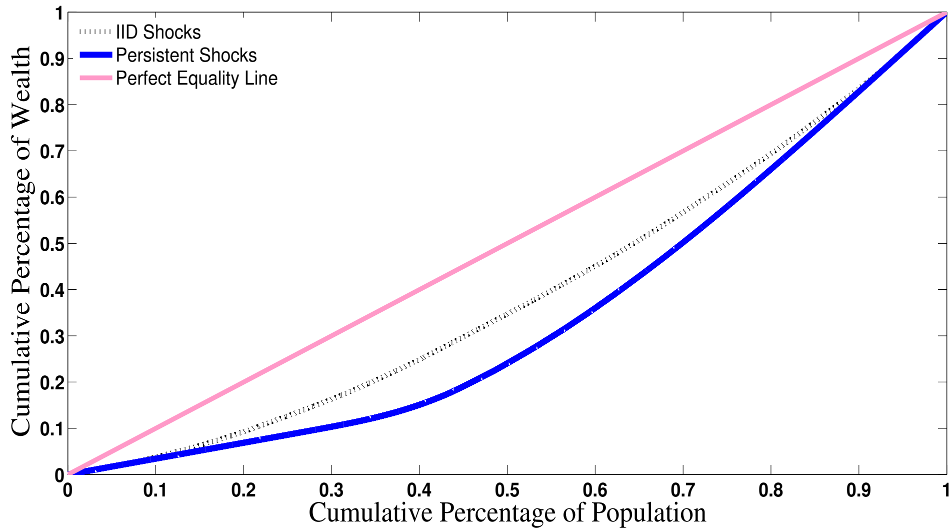
Note: $\gamma = 1.3$ and $\delta = 2$; $\pi - 1$ is the net inflation rate; $\overline{x}$ is the mean value, and $Gini_x$ is the Gini coefficient associated to the equilibrium random variable x. We define m = money balances (in real terms), c = consumption, I = income net of taxes/transfers, wealth is $w = m + \omega$

Table 7: Money/Total Asset Ratios by Wealth Quartiles in the economy with Capital

$\pi - 1$	Q_1	Q_2	Q_3	Q_4
0%	1.000	0.283	0.145	0.106
1%	1.000	0.270	0.142	0.107
2%	1.000	0.267	0.144	0.108
3%	1.000	0.270	0.145	0.108
4%	1.000	0.272	0.145	0.109
5%	1.000	0.275	0.146	0.109
10%	1.000	0.284	0.151	0.111
15%	1.000	0.296	0.155	0.113
20%	1.000	0.308	0.160	0.114
25%	1.000	0.319	0.164	0.116
30%	1.000	0.328	0.168	0.118
35%	1.000	0.338	0.172	0.120
40%	1.000	0.346	0.176	0.122

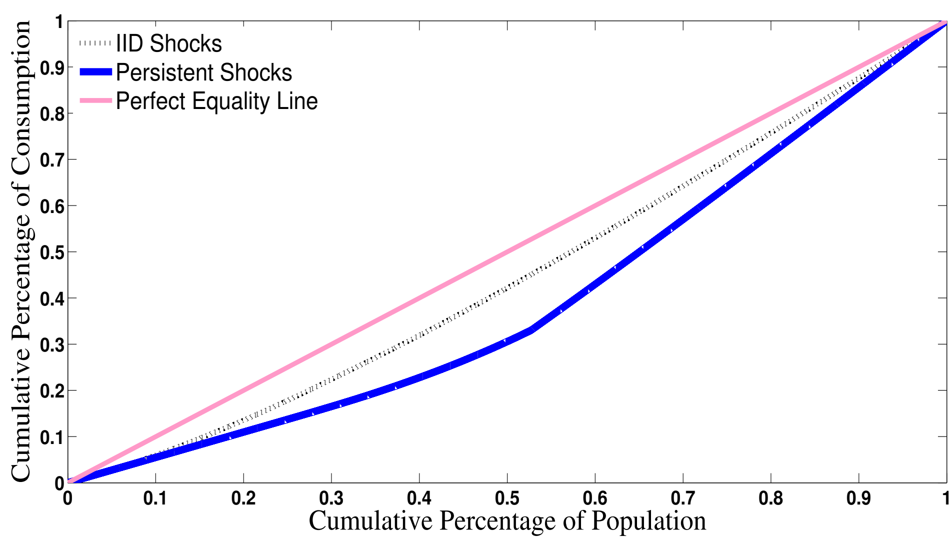
Note: The table is drawn for the benchmark calibration of persistent shocks with $\gamma = 1.3$ and $\delta = 2$; $\pi - 1$ is the net inflation rate. Columns 2-5 report the average real money balances divided by the average total assets held by households in each of the four wealth quartiles; this ratio varies from 0 to 1.

Figure 1: Lorenz Curve for Wealth



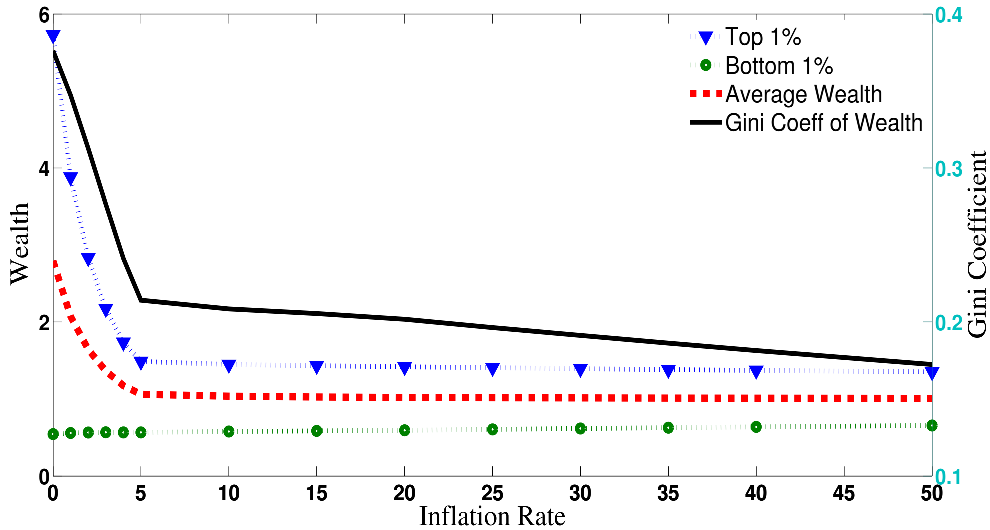
Notes: Lorenz curves are reported for stationary equilibrium at 2% inflation, baseline case (money only economy, persistent shocks, $\delta = 2$). The associated Gini coefficients are .215 and .313 for the case of iid and persistent shocks, respectively—Table 1.

Figure 2: Lorenz Curve for Consumption



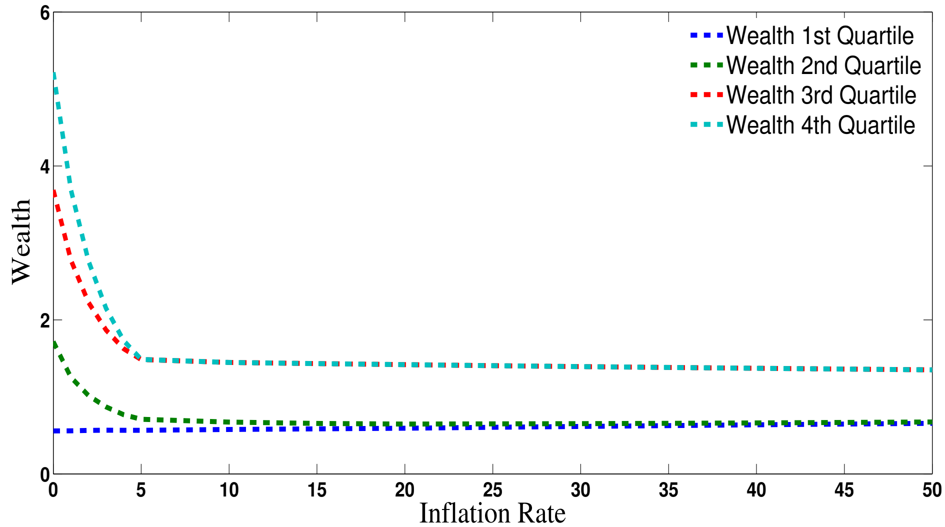
Notes: Lorenz curves are reported for stationary equilibrium at 2% inflation, baseline case (money only economy, persistent shocks, $\delta = 2$). The associated Gini coefficients are .108 and .215 for the case of iid and persistent shocks, respectively—Table 1.

Figure 3: Distribution of Wealth in Money Only Economy



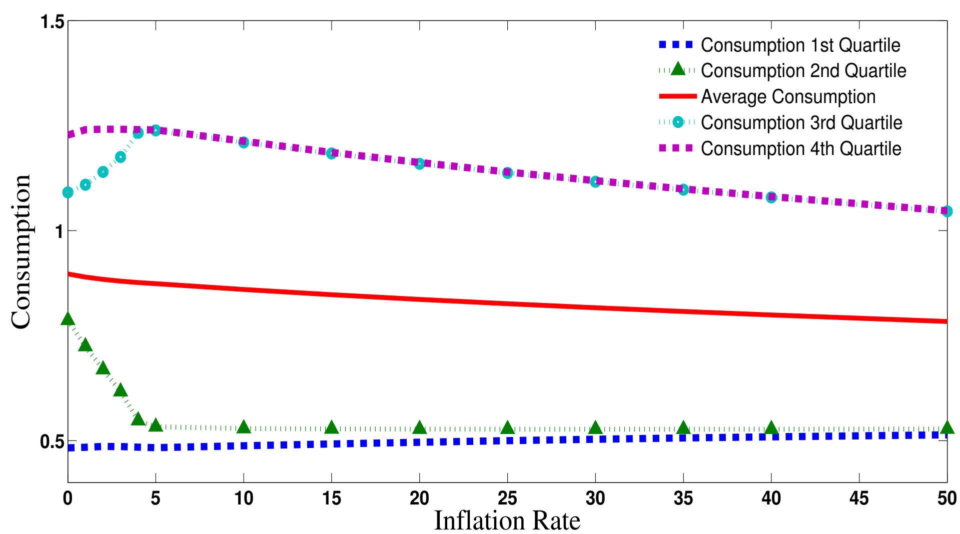
Notes: The figure reports statistics regarding the distribution of money for inflation rates ranging from 0% to 50%, in single-digit increments, for the baseline case (money only economy, persistent shocks, $\delta = 2$). Statistics reported are: mean balances, average balances held by households in the top and in the bottom percentile of the wealth distribution, and Gini coefficient (scale on right vertical axis). Non-housing wealth Gini coefficients for the U.S. in 1998 and 2007 were 0.861 and 0.881, respectively—see [7]
.

Figure 4: Quartiles of Wealth in Money Only Economy



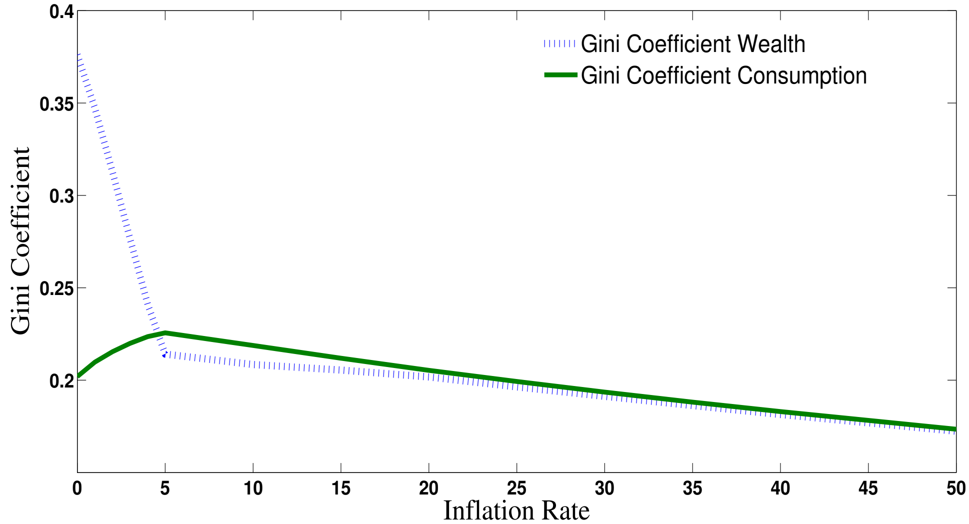
Notes: The figure reports the equilibrium wealth level by quartiles of wealth , for inflation rates ranging from 0% to 50%, in single-digit increments, for the baseline case (money only economy, persistent shocks, $\delta = 2$). The figure provides an illustration of how inequality develops with inflation. As inflation reaches 5%, the wealth distribution becomes bimodal: agents with below-average wealth hold similar balances, and those with more-than-average wealth hold more than twice as many balances. The figure also illustrates the nonlinear association between inflation and wealth inequality.

Figure 5: Consumption by Quartile of Wealth in Money Only Economy



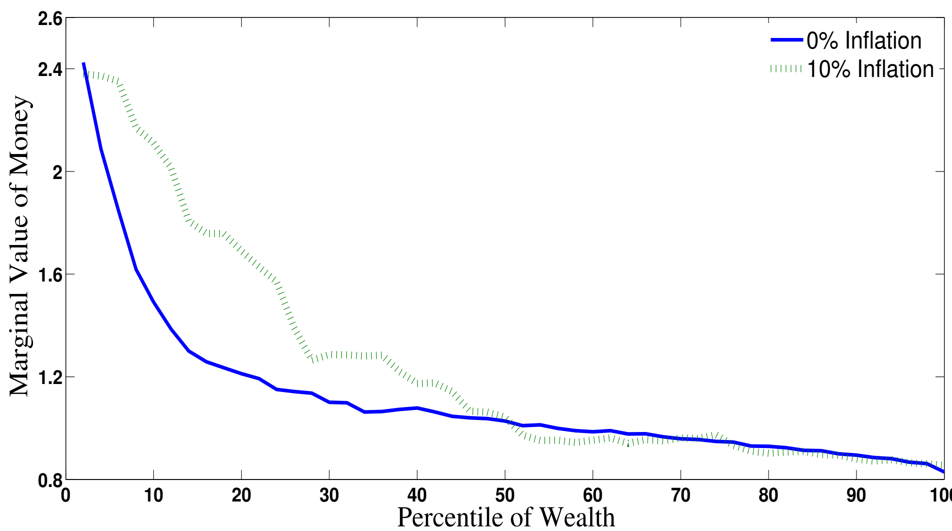
Notes: The figure reports the equilibrium consumption level by quartiles of wealth , for inflation rates ranging from 0% to 50%, in single-digit increments, for the baseline case (money only economy, persistent shocks, $\delta = 2$).

Figure 6: Gini Coefficients of Consumption and Wealth



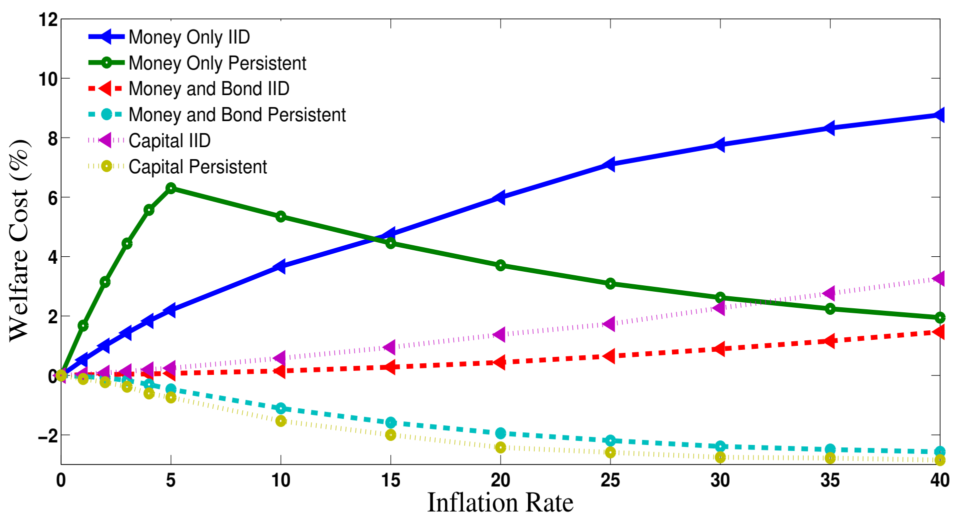
Notes: The figure reports the Gini coefficients of consumption and wealth for the baseline case (money only economy, persistent shocks, $\delta = 2$). Consumption and wealth inequality do not necessarily move in the same direction as inflation increases.

Figure 7: Marginal Value of Money



Notes: The figure reports the equilibrium marginal value of money by percentile of wealth, for inflation rates of 0% (solid line) and 10% (dashed line), for the baseline case (money only economy, persistent shocks, $\delta = 2$). The values reported are averages for two consecutive percentiles of wealth.

Figure 8: Welfare Costs of Inflation



Notes: The figure reports the average welfare cost of x% inflation as opposed to no inflation for inflation rates ranging from 0% to 40%. In the legend "Money Only" refers to the economy with only money, "Money and Bond" refers to the economy with bonds, and "Capital" refers to the economy where capital is also added. IID and Persistent differentiate between the two types of shocks considered.