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Information Disclosure and Exchange Media

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Abstract

When commitment is lacking, intertemporal trade is facilitated with the use of exchange media—interpreted broadly to include monetary and collateral assets. We study the properties of a model commonly used to motivate monetary exchange, extended to include a physical asset whose expected short-run return is subject to a news shock, but whose expected long-run return is stable. The nondisclosure of news enhances the asset’s property as an exchange medium, and generally improves social welfare. When a nondisclosure policy is infeasible, the framework admits a role for government debt, including fiat money. When lump-sum taxation is not permitted, fiat money may still improve welfare—but only if its circulation is supported by a cash-in-advance constraint.

Keywords: Money, Collateral, News, Nondisclosure

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1 Introduction

It is common sense that more information should be preferred to less; or, at least, weakly so. But since at least Hirshleifer (1971), economists have known that this eminently reasonable proposition need not hold fast in all social settings.

There is by now a large literature that studies optimal information revelation in a variety of contexts. Much of this work resides in the agency literature; see, for example, Prat (2005).\footnote{A few macroeconomic applications include Citanna and Villanacci (2000), Morris and Shin (2002), and Kaplan (2006).} The purpose of our paper is to examine the role information disclosure in financial markets that rely on exchange media to support intertemporal trade.

We should be clear what we mean by the term “exchange media.” Exchange media are assets that are used to facilitate intertemporal exchange. This includes all monetary assets, since money represents, in one way or another, a claim to future resources. It also includes assets that are used as collateral to support short-term credit arrangements, as exemplified by the type of exchanges that occur in the overnight repo market. We adopt this broad definition of exchange media because our theory tells us to.\footnote{Exchange media in the form of circulating physical objects is required only if other forms of record-keeping are absent; see Ostroy (1973), Townsend (1987), and Kocherlakota (1998).}

Our theoretical framework is based on the monetary model introduced by Lagos and Wright (2005). There is nothing particularly special about this framework, apart from its tractability—a property we intend to exploit below. Because private exchange media play such an important role in payments, and because such objects predate government fiat money in history, we begin by studying a private-money economy. To this end, we introduce an asset in the form of a Lucas (1978) tree. Because agents lack commitment, the asset can be used to support intertemporal exchange, either as a collateral object, or in the form of direct claims against the asset. Our setup here is very similar to Geromichalos, Licari, and Suárez-Lledó (2007), except that the return to the asset in our model is stochastic. This simple (and natural) extension turns out to have some interesting implications.

To be more specific, we assume that the asset’s expected return over short
horizons is stochastic—its expected return over long horizons is constant. This structure is meant to capture the idea that while the long-run fundamentals underlying an asset may be stable, its expected return in the short-run may be subject to variation. Conditional expectations over future asset returns fluctuate with the arrival of information we call news. Because the asset is in fixed supply, news plays no allocative role—as in Hirshleifer (1971), it has no social value. Consequently, news plays no role in optima, even though it is priced in equilibria. Consistent with what others have found, we demonstrate that if the asset is sufficiently scarce (in a well-defined sense), then equilibria are generally inefficient. In addition, we find that asset return uncertainty exacerbates the asset shortage for a given supply of assets.

Evidently, the usual inefficiency is exacerbated by “bad news” events that lead to temporarily depressed asset prices. For assets that do not serve as exchange media, these events are innocuous as far as the operation of the payments system is concerned. But if the assets in question are used to support intertemporal trade, then even a temporary asset price collapse can result in binding debt constraints and depressed economic activity. The economy in this case shows many of the symptoms commonly associated with a financial crisis or “credit crunch.” There is a sense in which the price of exchange media are “excessively sensitive” to certain types of information events in the competitive equilibria of economies with limited commitment.

Of course, the private sector goes to considerable lengths to supply high quality payment instruments. The tranching of assets, with only the most senior tranches serving as exchange media is a common practice. Think, for example, of demand deposit liabilities used every day in retail payments, and the AAA rated tranches of asset-backed securities that (up until recently) served as collateral in the repo market. In our model, the tranching of claims against the Lucas tree is possible, but ultimately ineffecual because there is simply not enough high quality tranches available.

Private sector attempts to enhance asset quality may take another, more surprising, form; namely, the nondisclosure of information. As alluded to above, it is theoretically possible for the equilibrium price of exchange media to fluctuate excessively in response to news events. One way to enhance asset “quality” (in terms of the asset’s usefulness as a payment instrument) is to suppress any information unrelated to the asset’s long-term fundamentals (or, at least, to disclose such information with a lag). This may be one
reason why banks typically prefer to report asset valuations that are based on internal “mark-to-model” methods, rather than potentially more volatile “mark-to-market” methods. A similar motivation may explain why money market mutual funds can avoid “breaking the buck” at the discretion of their board members.\textsuperscript{3} It is interesting to note that similar practices are evident among central banks and financial regulators. The Federal Reserve Bank of the United States, for example, does not disclose the identity of agencies that make use of its discount window facility. Nor do federal regulators make public their internal assessments of the financial soundness of private banks under federal supervision.\textsuperscript{4} These nondisclosure practices are typically justified as promoting a more efficient payments system—a theme consistent with our own results.

When the private sector is limited in its ability to supply quality payment instruments, the introduction of government liabilities may constitute a desirable innovation. As usual, a lot depends on what one assumes in the way of available tax instruments and government objectives. Not surprisingly, some version of the Friedman rule implements the first-best allocation (a solution that requires lump-sum taxation to finance a real return on money).

If lump-sum taxation is not permitted, then welfare may be improved even with a constant supply of fiat money—but only if a cash-in-advance constraint is imposed. The unbacked nature of fiat money is frequently viewed as a defect because its value in exchange must be supported by a (possibly fragile) self-fulfilling expectation. But if the short-run value of private assets fluctuates excessively owing to news, then the unbacked nature of fiat money turns out to be an advantage. In particular, news concerning the nature of what is backing fiat money is irrelevant—everyone already knows that no backing exists. This relative insensitivity to news is a property that confers an advantage to fiat money over private money instruments. But whether this advantage implies a welfare-improving role for fiat money turns out to

\textsuperscript{3}Rule 2a-7 of the Investment Company Act of 1940 stipulates that “The board of directors of the money market fund shall determine, in good faith, that it is in the best interests of the fund and its shareholders to maintain a stable net asset value per share or stable price per share, by virtue of either the Amortized Cost Method or the Penny-Rounding Method, and that the money market fund will continue to use such method only so long as the board of directors believes that it fairly reflects the market-based net asset value per share.”

\textsuperscript{4}These are the so-called CAMELS ratings, performed by the Federal Reserve Bank, the Office of the Comptroller of the Currency, and the Federal Deposit Insurance Corporation.
depend on parameters.

2 The environment

There is a unit measure of infinitely-lived individuals, distributed uniformly on $[0, 1]$. Time is discrete; with each time-period $t = 0, 1, ..., \infty$ divided into two subperiods, labeled *day* and *night*. Agents gather at centralized locations in both the day and night.$^5$

Output is produced in the day and the night. Let $x_t(i) \in \mathbb{R}$ denote consumption in the day by individual $i \in [0, 1]$ at date $t$; where $x_t(i) < 0$ is interpreted as production. Utility is linear in $x_t(i)$.

At the beginning of the night, agents experience an idiosyncratic shock that determines their type: *consumer* or *producer*. Consumption at night is denoted $c_t(i) \in \mathbb{R}^+$ and generates (for a consumer) the utility flow $u(c_t(i)) \in \mathbb{R}$; where $u'' < 0 < u'$ and $u(0) = 0$, $u'(0) = \infty$. Production at night is denoted $y_t(i) \in \mathbb{R}^+$ and generates (for a producer) the utility flow $-h(y_t(i)) \in \mathbb{R}$; where $h(0) = h'(0) = 0$, $h' > 0$ for $y > 0$ and $h'' \geq 0$.

For each individual, the stochastic process generating types is i.i.d. across time. Assume that the population at night is at all times divided equally between the two types. Preferences for individual $i$ at the beginning of time are represented by

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ x_t(i) + 0.5u(c_t(i)) - 0.5h(y_t(i)) \right]
\]

where $0 < \beta < 1$.

There is a durable asset that generates an exogenous and stochastic output flow $z_t \in [z, \bar{z}]$ at the beginning of each day; $0 \leq z \leq \bar{z} < \infty$. This aggregate shock follows a Markov process, $\Pr[z_{t+1} \leq z^+ | \eta_t = \eta] = F(z^+ | \eta)$; where $F$ is a cumulative distribution function, conditional on information $\eta_t$.

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$^5$We choose centralized locations for simplicity and because our main results do not hinge on search frictions. It may be worth mentioning that a common misconception is that a search friction is necessary to rationalize monetary exchange. In fact, all that is needed is anonymity (lack of commitment and record-keeping). Centralized trade in the Lagos-Wright model is studied in Rocheteau and Wright (2005).
(news) received at the beginning of the night. Assume, for simplicity, that news is either bad or good; \( \eta_t \in \{b, g\} \) and that \( \pi \equiv \Pr[\eta_t = b] \).\(^6\) Define

\[
    z(\eta) \equiv \int z^+ dF(z^+ | \eta)
\]  

(2)

where \( z(b) \leq z(g) \). That is, \( z(\eta) \) is a “short-term” conditional forecast made at night over the dividend payment that is to be realized the next day. In contrast, the “long-term” forecast (horizons extending from one day to the next and beyond) is invariant to news; i.e.,

\[
    z^e \equiv \pi z(b) + (1 - \pi) z(g)
\]  

(3)

As all output is nonstorable, there are two resource constraints

\[
    z_t \geq \int x_t(i) di \\
    \int y_t(i) di \geq \int c_t(i) di
\]  

(4)

(5)

The first-best allocation maximizes (1) for an ex ante representative individual, subject to the resource constraints (4), (5); and assuming that expectations are consistent with (2). The first-best allocation may, without loss, assign \( x_t(i) = z_t \); so that each agent receives (in expectation) \( z^e \) units of output in the day.\(^7\)

Strict concavity of \( u \) implies \( c_t(i) = c_t \). If \( h \) is strictly convex then \( y_t(i) = y_t \); if \( h \) is linear, then we focus on a symmetric allocation, so that again, \( y_t(i) = y_t \). An equal population of types at night implies \( c_t = y_t \); by virtue of (5) holding with equality. Optimality requires \( y_t = y^* \); with \( 0 < y^* < \infty \) satisfying

\[
    u'(y^*) = h'(y^*)
\]  

(6)

\(^6\)We could easily model a continuum of news states; see, Andolfatto and Martin (2009). Doing so would not affect our results in any substantive manner.

\(^7\)Note that owing to the quasilinear property of preferences, the presence of risk in the day (whether aggregate or idiosyncratic) has no effect on ex ante welfare. The first-best allocation here is also consistent with any lottery over \( \{x_t(i)\} \) that generates expected utility \( z^e \) for the agent.
The first-best allocation delivers \textit{ex ante} utility

\[ W^* = (1 - \beta)^{-1} [z^\epsilon + 0.5u(y^*) - 0.5h(y^*)] \]

**Proposition 1** The first-best allocation is independent of news.

**Proof.** The first-best allocation is characterized by (6) and \( x_t(i) = z_t \) for all \( i, t \); restrictions that are independent of \( \eta_t \). \( \blacksquare \)

In more general settings, the first-best allocation may depend on some news events and not others; see Andolfatto and Martin (2009). The model studied here represents a special case in which news has zero social value. We choose to focus on this case because the scenario is less well understood than the alternative where news possesses social value. Moreover, as we shall demonstrate below, while optima may not depend on zero-value news, the same is not true of equilibria. And to the extent that equilibria do depend on zero-value news, we know by Proposition 1 that equilibria with this property must be inefficient—so that welfare-improving interventions are in principle possible.

### 3 Competitive equilibrium

In this section, we characterize a competitive equilibrium without government intervention. Individuals are anonymous, so that private credit secured by the promise of future labor is infeasible. Anonymity gives rise to a demand for exchange media. We assume that the exchange medium takes the form of a security representing a state-contingent claim against the economy’s asset. The security may be used as a means of payment, or as collateral securing a short-term consumption loan. Either interpretation is legitimate here.\(^8\)

Markets are competitive. Each individual is initially endowed with one unit (an ownership share) of the physical asset. Apart from the initial period, we anticipate that the equilibrium distribution of shares at the beginning of each day will fall on a two-point set \( \{s_c, s_p\} \); where \( s_j \geq 0 \) and \( j \) denotes the

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\(^8\)Ferraris and Watanabe (2008) consider the case where an asset is pledged as collateral. Lagos and Rocheteau (2008) consider the case where an asset is used directly in exchange. The mathematics are identical. The key is that the asset facilitates intertemporal exchange; which is why we label such assets \textit{exchange media}. 
individual’s type in the previous night (consumer or producer). Let \((\phi_1, \phi_2)\) denote the price of a share measured in units of output, in the day and night, respectively. In what follows, \(\phi_1\) denotes the ex-dividend price.

### 3.1 Decision making in the day

Let \(s \geq 0\) denote shares carried forward into the night. The day budget constraint is then given by

\[
x = (z + \phi_1) s_j - \phi_1 s
\]  

(7)

Let \(D(s_j, z)\) denote the value of entering the day with shares \(s_j\) and with realized dividend income \(z\). Let \(N(s, \eta)\) denote the \textit{ex ante} (before type is known) value of entering the night-market with share-holdings \(s\) when the news is \(\eta\). The value functions \(D\) and \(N\) must satisfy the following recursion

\[
D(s_j, z) \equiv \max_{s \geq 0} \left\{ (z + \phi_1) s_j - \phi_1 s + E_\eta[N(s, \eta)] \right\}
\]  

(8)

where here, we have substituted in the budget constraint (7).

Assume that the value function \(N\) is increasing and at least weakly concave in \(s\); i.e., \(N_{11} \leq 0 < N_1\). In fact, these are properties that will hold in equilibrium. If \(N_{11} < 0\), which occurs whenever an agent is not satiated in exchange media, then each individual leaves the day-market with identical share-holdings \(s\) characterized by

\[
\phi_1 = E_\eta[N_1(s, \eta)]
\]  

(9)

As in Lagos and Wright (2005), the distribution of wealth at the end of the day is degenerate. If \(N_{11} = 0\), then desired individual share-holdings are indeterminate; at least, beyond some strictly positive lower bound. Even in this case, however, condition (9) will continue to hold in any equilibrium.\(^9\)

By the envelope theorem, \(D_1(s_j, z) = z + \phi_1\); so that \(D_1(s_j^+, z^+) = z^+ + \phi_1^+\). Given that the stochastic dividend flow is an \textit{i.i.d.} process from one day to the next, and given quasi-linearity, the ex-dividend price of equity in the day will remain constant over time; i.e. \(\phi_1 = \phi_1^+\). In this case,

\[
\int D_1(s_j^+, z^+) dF(z^+ | \eta) = z(\eta) + \phi_1
\]  

(10)

\(^9\)If it did not hold, then the demand for shares would either be zero or infinity.
3.2 Decision making at night

Let $P(s, \eta)$ denote the value of being a producer at night, with money $s$ and when news is $\eta$. Using $y = \phi_2(\eta)(s_p^+ - s)$, the choice problem may be stated as

$$P(s, \eta) \equiv \max_{s_p^+ \geq 0} \left\{ -h(\phi_2(\eta)(s_p^+ - s)) + \beta \int D(s_p^+, z^+)dF(z^+ | \eta) \right\}$$

(11)

Note that as a producer has no desire to consume, his debt-constraint is necessarily slack. Utilizing (10), desired production is characterized by

$$\phi_2(\eta)h'(y(\eta)) = \beta [z(\eta) + \phi_1]$$

(12)

Let $C(s, \eta)$ denote the value of being a consumer at night, with shares $s$ and when news is $\eta$. Using $c = \phi_2(\eta)(s - s_c^+)$, the choice problem may be stated as

$$C(s, \eta) \equiv \max_{s_c^+ \geq 0} \left\{ u(\phi_2(\eta)(s - s_c^+)) + \beta \int D(s_c^+, z^+)dF(z^+ | \eta) \right\}$$

(13)

The consumer’s debt-constraint $s_c^+ \geq 0$ plays an important role in what follows. Utilizing (10), desired consumption is characterized by

$$\phi_2(\eta)u'(c(\eta)) = \beta [z(\eta) + \phi_1] \quad \text{if} \quad \phi_2(\eta)s \geq c(\eta)$$

$$c(\eta) = \frac{\phi_2(\eta)s}{\phi(\eta)} \quad \text{otherwise}$$

(14)

3.3 Equilibrium restrictions

The market-clearing conditions are given by $s = 1$ and $c(\eta) = y(\eta)$.

The object of interest here is the equilibrium allocation at night $y(\eta)$, together with the corresponding price system $\phi_1$ and $\phi_2(\eta)$.\(^\text{10}\) To begin, consider (9). Note that $N_1(s, \eta) \equiv 0.5C_1(s, \eta) + 0.5P_1(s, \eta)$. Applying the envelope theorem to (11) and (13), $N_1(s, \eta) \equiv 0.5\phi_2(\eta)u'(y(\eta)) + 0.5\phi_2(\eta)h'(y(\eta))$. Condition (9) may therefore be expressed as

$$\phi_1 = \pi\phi_2(b)h'(y(b))A(y(b)) + (1 - \pi)\phi_2(g)h'(y(g))A(y(g))$$

(15)

\(^{10}\)Once these objects are determined, the remaining variables can be deduced from budget constraints, etc.
where

\[ A(y) \equiv 0.5 \left[ \frac{u'(y)}{h'(y)} + 1 \right] \quad (16) \]

Note that \( A(y^*) = 1 \) and \( A'(y) < 0 \).

Next, observe that condition (12) implies the asset-price function

\[ \phi_2(\eta) = \beta \left[ \frac{z(\eta) + \phi_1}{h'(y(\eta))} \right] \quad (17) \]

Finally, conditions (12) and (14), together with market-clearing, imply

\[
y(\eta) = y^* \quad \text{if} \quad \phi_2(\eta) \geq y^* \\
\phi_2(\eta) = y(\eta) < y^* \quad \text{otherwise} \quad (18)
\]

Conditions (15), (17) and (18) constitute the key restrictions that characterize the general equilibrium allocation and price-system for this competitive economy.

### 3.3.1 A no-news economy

As a benchmark, it is useful to consider the case in which news is uninformative; i.e., \( z(b) = z(g) \). In this no-news economy, we have \( z(\eta) = z^e \) for \( \eta \in \{b, g\} \). It follows that \( \phi_2(\eta) = \phi_2 \) and \( y(\eta) = y \).

Combining (15) and (17), we obtain the following expression for the asset day-price

\[
\phi_1 = \left[ \frac{\beta A(y)}{1 - \beta A(y)} \right] z^e \quad (19)
\]

To begin, conjecture that the debt-constraint remains slack. Then (18) implies that \( y = y^* \) and given \( A(y^*) = 1 \), from (19) we find

\[
\phi_1 = \left( \frac{\beta}{1 - \beta} \right) z^e; \quad (20)
\]

which happens to be the standard asset-pricing formula that one typically derives for risk-neutral agents.
We need to confirm that the conjecture made with respect to (18) holds in equilibrium; i.e., that \( \phi_2 \geq y^* \). Using (17) and (20), this latter condition can be expressed as

\[
\left( \frac{\beta}{1 - \beta} \right) z^e \geq h'(y^*)y^*
\]

Whether this condition holds or not depends on parameters. Define the following object:

\[
\hat{\beta}(z^e) \equiv \left[ 1 + \frac{z^e}{h'(y^*)y^*} \right]^{-1}
\]

**Proposition 2** A competitive equilibrium implements the first-best allocation for any \( \beta \geq \hat{\beta}(z^e) \).

**Proof.** A competitive equilibrium price-system and allocation satisfy conditions (15), (17) and (18). \( \beta \geq \hat{\beta}(z^e) \) implies that condition (21) holds, which, in turn, implies that the debt-constraint remains slack in all states of the world.

Note that \( \hat{\beta}(z^e) \) is strictly decreasing in \( z^e \); so that a higher expected asset return expands the set of economies for which the first-best is implementable; see also Proposition 1 in Geromichalos, et. al. (2007). An analogous result holds for production economies; see Lagos and Rocheteau (2008). Indeed, a similar property holds in overlapping generations models, where the competitive equilibrium is known to be Pareto efficient if the equilibrium real rate of interest (the expected marginal product of capital) is sufficiently high.

The interesting case arises when \( \beta < \hat{\beta}(z^e) \). In this case, the economy can be said to experience an “asset shortage” in the sense of Caballero (2006). The fundamental object in short supply is commitment. The asset itself is just an instrument that helps overcome the limited commitment friction (the assumption here is that, unlike future labor, the asset’s dividend can be pledged as collateral). When the asset is sufficiently scarce (an extreme example is when it is absent entirely), then debt-constraint binds tightly; so that (18) implies \( \phi_2 = y < y^* \).

The asset price function (19) suggests that equity is “over-valued” in the debt-constrained equilibrium relative to its “fundamental” value.\(^{11}\) That is,

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\(^{11}\)Imagine that we have two assets in this economy that are identical in every way except
people would like to borrow (or short equity) at night, but cannot. In terms of the expected rate of return on equity (from one day to the next)

$$1 < \left[ \frac{z^e + \phi_1}{\phi_1} \right] < \frac{1}{\beta}$$

That is, the effect of the binding debt constraint—the consequence of an “asset shortage”—is to confer a “liquidity premium” on the price of equity (and all liquid assets); so that equity earns a lower expected rate of return.$^{12}$

3.3.2 A news economy

We now consider the case in which news is informative; i.e., $z(b) < z^e < z(g)$.

If the debt-constraint never binds, then by (18), the competitive equilibrium implements the efficient allocation $y(\eta) = y^*$. As a consequence, the equilibrium asset price in the day is given by (20). Condition (17) then delivers an expression for the price of equity at night. Clearly, the equilibrium share price at night responds to news in the way one would expect; i.e., $\phi_2(b) < \phi_2(g)$.

Thus, it is possible to have an asset serve as an efficient exchange medium, even if its price fluctuates in response to “short-run” news events. That is, while the price of the asset fluctuates randomly at night in response to information, this price volatility in no way inhibits \textit{ex ante} efficiency. This is true as long as asset price movements do not leave consumers debt-constrained in any state of the world; a possibility that we consider next.

\textbf{Proposition 3} If $z(b) < z^e < z(g)$ and $\beta = \hat{\beta}(z^e)$, then the consumer debt constraint binds tightly in the bad news state and remains slack in the good news state.

\textbf{Proof.} See appendix A. \hfill \blacksquare

that only one can be used as an exchange medium. Then the illiquid asset will be priced at its fundamental value (20) and the liquid asset will trade at a premium (19). See also Lagos (2010).

$^{12}$In a model with endogenous capital accumulation, the analogous result is an over-accumulation of capital; see Lagos and Rocheteau (2008).

12
Proposition 3 can be understood in the following way. What we do is fix a pair \((\beta, z^e)\) such that the competitive equilibrium just manages to implement the first-best allocation in the absence of news; see Proposition 2. In this case, the debt-constraint remains weakly slack. We then perform a mean-preserving-spread over the short-run conditional forecast of the future asset return. That is, we keep the unconditional expectation \(z^e\) fixed, and increase the variance of the short-run forecast around this mean. From (18), we know that \(\phi_2(\eta) \geq y^*\) must hold in all news states \(\eta\) if consumers are to avoid being debt-constrained at night. In the absence of news and for our parameters \((\beta, z^e)\), we know that \(\phi^2 = y^*\). The mean-preserving spread in conditional forecasts then implies \(\phi_2(g) > y^* > \phi_2(b)\). That is, good-news slackens a constraint that was binding only weakly; while bad news causes the constraint to bind tightly.14

Proposition 3 and condition (18) imply that \(\phi_1(y(b)) = y^*\) and

\[
\phi_1 = \pi \beta [z(b) + \phi_1] A(y(b)) + (1 - \pi) \beta [z(g) + \phi_1] A(y(g))
\]

Solving for the ex-dividend price of equity in the day

\[
\phi_1 = \beta A(y(b)) + (1 - \pi) \beta A(y(g))]
\]

Note that (23) reduces to (20) when \(y(b) = y^*\). Hence, as long as \(y(b) < y^*\), equity commands a “liquidity premium.”

As for the equilibrium price of equity at night, refer to condition (17)

\[
\phi_2(b) = \frac{\beta [z(b) + \phi_1]}{h'(y(b))}
\]

and

\[
\phi_2(g) = \frac{\beta [z(g) + \phi_1]}{h'(y^*)}
\]

13 The condition \(\beta = \beta(z^e)\) in Proposition 3 is sufficient but not necessary for the stated result. One can show that the result holds for a range \(\beta \geq \beta''\), where \(\beta'' < \beta(z^e)\).

14 It is interesting to ask whether creating low-risk tranches of the asset might be helpful here. If the asset return \(z\) is bounded below by some \(\underline{z}\), then one could, for example, create two securities, with one representing a claim to \(\underline{z}\) (senior claim) and the other representing a claim to the residual return \((z - \underline{z})\) (junior claim). Tranching assets in this manner, however, confers no benefit in our environment. However, tranching does play a vital role in Dang, Gorton and Holmström (2009).
It is curious to note that $\phi_2(b) > \phi_2(g)$ appears possible here (unless $h$ is linear). If this is so, then the debt constraint would bind in the good news state and remain slack in the bad news state; a possibility ruled out by Proposition 3. Hence, $\phi_2(b) < \phi_2(g)$; a result that is immediately apparent for the special case in which $h$ is linear.

4 Optimal disclosure policies

Consider two economies that are identical in every respect except one. The first economy has the property $z(b) = z^e = z(g)$; while in the other economy, $z(b) < z^e < z(g)$. (Keep in mind that the unconditional expected dividend flow $z^e$ is assumed to be the same across the two economies.)

The former case is what we have called the no-news economy, while the latter is a news economy. The two economies differ only in that a particular form of information is assumed to exist or not. Alternatively—and equivalently—we might instead imagine that the information exists in both economies, but that it is somehow withheld from public viewing in the no-news economy.

The thought experiment we have in mind is as follows. Imagine that there exists an “information switch” that can be turned on or off at the beginning of time. If the switch is turned on, news is made public information. If the switch is turned off, news is hidden from society. The switch is not under the control of any individual; instead, society must choose whether to engage the switch or not. Because there is a representative agent ex ante when the choice is made, there will be perfect agreement over this choice. The question is simply this: what will society choose? In particular, can a nondisclosure policy (switch off) be socially desirable?

While it is true that the information we consider here has no social value, it is not immediately obvious why making it public is not innocuous. In fact, there are cases in which this turns out to be true. But what we wish to emphasize here is that there are also cases in which nondisclosure is strictly preferred. This latter proposition can be seen very easily for the case in which $\beta = \hat{\beta}(z^e)$, so let us focus on this parameterization to drive the basic point home.

Recall from condition (21), that $\hat{\beta}(z^e)$ was chosen such that for a given
the first-best allocation was just implementable in the no-news economy. What this means is that choosing to keep the information switch off is consistent with maximum social welfare. However, it turns out that we can say more than this. In particular, by Proposition 3, we know that if the information switch is turned on, then the debt constraint binds in the bad news state. We conclude that social welfare is strictly higher when the switch is turned off—nondisclosure is an optimal policy.

Can it ever be welfare improving, from an ex ante perspective, to turn the information switch on? If the information to be released has zero social value, then the answer is always no. One case for which this is not so obvious is when the debt constraint is binding in both news states. Under a nondisclosure policy, the debt constraint binds a given amount, independent of news. Under a disclosure policy, the constraint is relaxed in the good news state, but binds even more tightly in the bad news state. Evidently, the strict concavity of $u$ implies that from an ex ante perspective, society would prefer a steady but “average” degree of tightness in the debt constraint, relative to fluctuating extremes of tightness.

4.1 Is nondisclosure time-consistent?

If the debt constraint binds under a nondisclosure policy, then there is an obvious temptation for the “information manager” to reveal good news when it transpires. The reason is clear—good news will relax the debt constraint and expand the volume of trade in the night-market. If the news is released prior to types becoming known, the expected flow utility at night is increased.

Of course, a policy of disclosing good news and hiding bad news is infeasible here. In particular, agents would be able to infer bad news events from the lack of public disclosure. Consequently, if the temptation to reveal good news at night is strong enough, then the only time-consistent (i.e.,

\footnote{15 Of course, if information has social value, as in Andolfatto and Martin (2009), then the answer can be yes.}

\footnote{16 While we have not formally proved this result, many numerical examples failed to provide a counterexample.}

\footnote{17 It follows that if the debt constraint does not bind under a nondisclosure policy, there is no incentive to make information public.}

\footnote{18 If the information is revealed after types become known, then the welfare effect is likely to differ across consumers and producers.}
sequentially rational) policy is one of full disclosure.

As it turns out, a nondisclosure policy is time-consistent only for patient economies—that is, for economies with sufficiently high $\beta$. The reason is straightforward. There is a one-shot gain to revealing hidden news when it is good. But there is a long run loss associated with full disclosure. Nondisclosure in this case can only be time-consistent if the long-run loss outweighs the short-run gain. This, in turn, is possible only for economies that are sufficiently patient.

5 Fiat money

Imagine that there is an asset shortage in the sense that $\beta \leq \hat{\beta}(z^e)$. Moreover, assume that $z(b) < z^e < z(g)$ and that the nondisclosure of news is not possible. It follows from Propositions 2 and 3 that the competitive equilibrium is inefficient. In this section, we ask whether the introduction of fiat money can improve social welfare.

We introduce fiat money in exactly the same way as Geromichalos, et. al. (2007).\(^{19}\) Let $M^+ = \mu M$, where $M$ is the stock of money at the beginning of the day and $\mu$ is the gross money growth rate. New money is introduced as a lump-sum transfer in the day (a tax, if the money supply is contracted). Let $v_1$ be the value of money during the day and $v_2$ the value of money at night; both these values are normalized by the aggregate money stock at the end of the day, i.e., by $M^+$. We anticipate that in a stationary equilibrium, the value of money is constant in the day and potentially depends on news at night. Let $\tau$ be the money transfer, expressed in terms of day goods. The government budget constraint is,

$$\tau = v_1 \left[ 1 - \frac{1}{\mu} \right] \quad \text{(24)}$$

5.1 Decision making in the day

An agent starts the day with money holdings $m_j$, $j = \{c, p\}$, which are normalized by the beginning-of-the-day aggregate money stock. Let $m$ be

\(^{19}\)Here, we restrict money to earn zero nominal interest. But the analysis generalizes to interest-bearing money, or interest-bearing government debt; see Andolfatto (2010).
the money holdings he chooses to take into the night market, which are normalized by the end-of-the-day money stock. The day budget constraint of an agent is now given by
\[ \mathbf{x} = (\mathbf{z} + \mathbf{\phi}_1)s_j - \mathbf{\phi}_1s + (v_1/\mu)m_j - v_1m + \tau. \]
The choice problem in the day (the analog to (8) is,
\[ \mathbf{D}(s_j, m_j, z) \equiv \max_{s \geq 0, m \geq 0} (z + \mathbf{\phi}_1)s_j - \mathbf{\phi}_1s + (v_1/\mu)m_j - v_1m + \tau + E_\eta[N(s, m, \eta)] \] (25)

At an interior solution, individual asset demands must satisfy,
\[ \mathbf{\phi}_1 = E_\eta[N_1(s, m, \eta)] \] (26)
\[ v_1 = E_\eta[N_2(s, m, \eta)] \] (27)
By the Envelope Theorem, \( D_1(s_j, m_j, z) = z + \mathbf{\phi}_1 \) and \( D_2(s_j, m_j, z) = v_1/\mu. \)
In a stationary equilibrium, \( \mathbf{\phi}_1 \) and \( v_1 \) are constant over time. Consequently,
\[ \int D_1(s_j^+, m_j^+, z^+)dF(z^+ | \eta) = z(\eta) + \mathbf{\phi}_1 \] (28)
\[ \int D_2(s_j^+, m_j^+, z^+)dF(z^+ | \eta) = v_1/\mu \] (29)

5.2 Decision making at night

Let \( P(s, m, \eta) \) denote the value of being a producer at night, with portfolio \( (s, m) \) when the news is \( \eta \). Using \( y(\eta) = \phi_2(\eta)(s_p^+ - s^+) + v_2(\eta)(m_p^+ - m^+) \), the choice problem may be stated as
\[ P(s, m, \eta) \equiv \max_{s_p^+ \geq 0, m_p^+ \geq 0} \left\{ -h(\phi_2(\eta)(s_p^+ - s^+) + v_2(\eta)(m_p^+ - m^+)) \right\} + \beta \int D(s_p^+, m_p^+, z^+)dF(z^+ | \eta) \]
As before, the debt constraints will not bind for the producer. There is the question, however, as to which asset is to be preferred as a payment instrument. We want to restrict attention to equilibria in which the two assets coexist. For this to be true, the following rate-of-return equality condition must hold:
\[ R(\eta) \equiv \left[ \frac{z(\eta) + \mathbf{\phi}_1}{\phi_2(\eta)} \right] = \left[ \frac{v_1/\mu}{v_2(\eta)} \right] \] (30)
That is, the expected rate of return on assets from the night to the next day (conditional on \( \eta \)) must be the same if both assets are to be accepted as payment. At the individual level then, portfolio composition is indeterminate
in equilibrium—all that matters is the common rate of return \( R(\eta) \). Consequently (making use of the envelope results in 28 and 29), the producer’s optimal behavior is characterized by

\[
h'(y(\eta)) = \beta R(\eta) \tag{31}
\]

Let \( C(s, m, \eta) \) denote the value of being a consumer at night, with portfolio \((s, m)\) when the news is \( \eta \). Using \( c = \phi_2(\eta)(s - s^+_c) + v_2(\eta)(m - m^+_c) \), the choice problem may be stated as,

\[
C(s, m, \eta) \equiv \max_{s^+_c \geq 0, m^+_c \geq 0} \left\{ u(\phi_2(\eta)(s - s^+_c) + v_2(\eta)(m - m^+_c)) + \beta \int D(s^+_c, m^+_c, z^+) dF(z^+ | \eta) \right\}
\]

It is easy to show that if \( s^+_c = 0 \), then \( m^+_c = 0 \); and vice versa. So the debt-constraint either binds or remains slack. Using (28) and (29), optimal behavior on the part of the consumer is characterized by (again, assuming that condition 30 holds):

\[
\begin{align*}
&u'(c(\eta)) = \beta R(\eta) & \text{if } \phi_2(\eta)s + v_2(\eta)m \geq c(\eta) \\
&c(\eta) = \phi_2(\eta)s + v_2(\eta)m & \text{otherwise}
\end{align*} \tag{32}
\]

### 5.3 A competitive monetary equilibrium

In a monetary equilibrium, market-clearing implies \( c(\eta) = y(\eta) \), and \( s = m = 1 \).

As before, the object of interest here is the equilibrium allocation at night \( y(\eta) \), together with the corresponding price system \( \{\phi_1, \phi_2(\eta), v_1, v_2(\eta)\} \). To begin, consider conditions (26) and (27). Employing the usual envelope results, the former condition can be shown to correspond exactly to (15). A similar calculation allows us to restate condition (27) as

\[
v_1 = \pi v_2(b)h'(y(b)) A(y(b)) + (1 - \pi) v_2(g)h'(y(g)) A(y(g)) \tag{33}
\]

Next, note that the equilibrium share price at night \( \phi_2(\eta) \) continues to be characterized by the earlier condition (17). We need an analogous condition for the price of money at night. From the restrictions characterizing optimal producer behavior, we can recover the expression

\[
v_2(\eta) = \frac{\beta v_1}{\mu h'(y(\eta))} \tag{34}
\]
Finally, note that (31) and (32), together with market-clearing, imply
\[
y(\eta) = y^* \quad \text{if} \quad \phi_2(\eta) + v_2(\eta) \geq y^* \\
\phi_2(\eta) + v_2(\eta) = y(\eta) < y^* \quad \text{otherwise}
\] (35)

Conditions (33), (34), and (35), together with conditions (15) and (17) derived earlier, constitute the key restrictions that characterize the general equilibrium allocation and price-system for an economy in which fiat money is valued.\(^{20}\) The following proposition reports an expected result.

**Proposition 4** The Friedman rule \((\mu = \beta)\) implements the first-best allocation.

**Proof.** Combine (33) and (34) to derive \(\mu = \beta[\pi A(y(b)) + (1 - \pi)A(y(g))]\). Since \(A(y^*) = 1\), this condition is satisfied at the Friedman rule. All that remains to be shown is that the consumer debt constraint remains slack; i.e., \(\phi_2(\eta) + v_2(\eta) \geq y^*\). Using (17) and (34), the debt constraint may be rewritten as \(\beta [z(\eta) + \phi_1] + v_1 \geq h'(y^*)y^*\). We can use (20) to derive an equilibrium value for \(\phi_1 > 0\), so that all we need is
\[
v_1 \geq h'(y^*)y^* - \beta [z(b) + \phi_1]
\] (36)

Utilizing (33), we derive the familiar result that real money balances are indeterminate at the Friedman rule. Consequently, any value \(v_1 < \infty\) satisfying (36) is an equilibrium. \(\blacksquare\)

It is interesting to compare Proposition 4 with (say) Proposition 2. The latter proposition suggests that for private-money economies, the first-best allocation is attainable only for sufficiently large \(z^e\) and/or \(\beta\). In contrast, the former proposition states that the first-best allocation can be implemented independently of these parameters. The key to understanding this result is that the power to lump-sum tax circumvents the asset shortage. If one wants to think of the transferable utility term \(x\) as labor, for example, then the government effectively has the ability to create assets out of labor (e.g., by issuing interest-bearing debt that is effectively backed by labor power).

\(^{20}\) As before, the remaining endogenous variables can be recovered by appealing to the remaining restrictions. For example, with \(v_1\) determined and a given \(\mu\), the equilibrium transfer is given by (24).
Can fully unbacked money (money whose return is completely unsupported by taxation, or any other real asset) coexist with private money? If individuals are free to choose their most preferred payment instrument, then the answer to this question is no.

**Proposition 5** A stationary monetary equilibrium cannot exist for any \( \mu \geq 1 \).

**Proof.** Consider an allocation \( \{y(b), y(g)\} \). In a monetary equilibrium, this allocation must satisfy restrictions (33) and (34); which together can be combined to form
\[
\mu = \beta [\pi A(y(b)) + (1 - \pi) A(y(g))]
\]
At the same time, the equilibrium asset price \( 0 < \phi_1 < \infty \) must satisfy condition (23), which implies
\[
1 > \beta [\pi A(y(b)) + (1 - \pi) A(y(g))]
\]
It follows that if fiat money is to coexist, then we must necessarily have \( \mu < 1 \). □

The proposition above asserts that at least some deflation (interest on money financed by \( \tau < 0 \)) is necessary for coexistence. We already know from Proposition 4 that deflating at the Friedman rule ensures coexistence. For a given parameterization, it can be shown that there exists a \( \overline{\mu} < 1 \) such that coexistence is possible for money growth rates in the range \( \beta \leq \mu < \overline{\mu} \). The result here is standard: lower inflation is associated with greater output and higher welfare; see Geromichalos, et. al. (2007).

Proposition 5 is surprising in a way. The asset price is sensitive to news. If there is an asset shortage, this price sensitivity is undesirable. Introducing a second asset to this economy—an asset whose return is insensitive to news—might, one would think, be valued even if its return is low. A constant supply of fiat money should generate stable return for money, making it valuable as a payment instrument—at least, for some parameter values (e.g., a high degree of risk aversion). Proposition 5 states that this is not the case.

The result stated in Proposition 5 is, in part, an artifact of quasilinear preferences. In particular, the no-arbitrage-condition that must hold at night
(30) makes no reference to risk-aversion parameters. But there is another factor as well. That is, even for nonlinear preferences, currency competition implies that the short-run expected return on fiat money will be indirectly affected by news. The only way to render the short-run rate of return on fiat money insensitive to news is to restrict the use of private money. The desirability of such a policy is investigated next.

5.4 A cash-in-advance constraint

Assume now that only fiat money can be used to make payments at night. The problem faced by agents in the day remains unaffected. But their decisions at night are obviously affected by what is effectively a cash-in-advance constraint.

The supply of night output is characterized by condition (31),

\[ h'(y(\eta)) = \frac{\beta v_1}{\mu v_2(\eta)} \]

The no-arbitrage-condition (30) is obviously irrelevant here. The demand for night output is characterized by condition (32) with the added constraint \( s = 0 \); i.e.,

\[ u'(c(\eta)) = \beta R(\eta) \quad \text{if} \quad v_2(\eta) m \geq c(\eta) \]
\[ c(\eta) = v_2(\eta) m \quad \text{otherwise} \]

We anticipate that for a growing supply of money (\( \mu \geq 1 \), the cash-in-advance constraint will bind. Equilibrium implies \( m = 1 \) and \( c(\eta) = y(\eta) \), so that \( v_2(\eta) = y(\eta) \). Together, these restrictions imply \( h'(y(\eta)) = \beta v_1/(\mu y(\eta)) \). The implication here is that the equilibrium level of output at night is independent of news; i.e., \( y(\eta) = y \). Of course, this is exactly what we would expect, given that payments at night are now made solely with a risk-free asset. Consequently, we have the equilibrium restriction

\[ v_1 = \mu \beta^{-1} h'(y)y \] (37)

Combining (33) and (34), we have the equilibrium restriction

\[ \mu = \beta A(y) \] (38)

Conditions (37) and (38) characterize the equilibrium pair \((v_1, y)\).
Since $A(y^*) = 1$ and $A(y)$ is increasing in $y$, it follows from (38) that $y < y^*$ for any $\mu > \beta$. Moreover, the equilibrium level of $y$ is decreasing in $\mu$. Consequently, imposing a cash-in-advance constraint when cash earns a low rate of return ($\mu \geq 1$) constrains the level of night activity and implies a welfare cost. On the other hand, there is potentially a welfare gain to be had as well, since a bad news shock no longer depresses economic activity at night. Before confirming this possibility, we establish the following result.

**Proposition 6** Imposing a cash-in-advance constraint in a no-news economy with a constant supply of fiat money unambiguously reduces welfare.

**Proof.** Consider a competitive equilibrium in a no-news economy where the debt constraint binds. Combining conditions (17), (18), (19), this implies

$$\beta z^e = h'(y)y [1 - \beta A(y)]$$

The restriction above implies that $y$ is increasing in $z^e$ as long as the debt-constraint binds. Note that the equilibrium associated with the cash-in-advance economy corresponds to the equilibrium of the private-money economy when $z^e = 0$. Consequently, imposing a cash-advance-constraint in a no-news economy when $z^e > 0$ must necessarily restrict the volume of trade and lower social welfare.

Proposition 6 implies that if a cash-in-advance constraint is to improve social welfare, it must do so within the context of a news-economy. Unfortunately, it is not easy to show analytically how a cash-in-advance constraint can improve welfare.

If night-output in the bad news state is particularly low due to severely binding debt-constraints, then replacing private money with an informationally-insensitive asset, such as fiat money, increases welfare, as we note below. On the other hand, a numerical example should suffice to demonstrate the possibility.

**Claim 7** Imposing a cash-in-advance constraint in a news economy with a constant supply of fiat money has ambiguous welfare consequences.

Assume $u(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$ and $h(y) = y$, which implies $y^* = 1$. Let $\beta = 0.99$ and $\sigma = 10$. At this point, we can compute the night-allocation
in the cash-in-advance economy, since it does not depend on news or the rate of return on the asset. With these parameters, the night-allocation is: \( y = 0.9980 \). We now fix \( z(b) = 0, \pi = 0.25 \) and show the effects of varying \( z(g) \) on the night-allocation in the competitive equilibrium of the news economy. First, pick \( z(g) \) so that condition (21) is satisfied; i.e., \( z(g) \approx 0.0135 \). Then, the night-allocation in the news economy is: \( y(b) = 0.9999 \) and \( y(g) = 1 \). For this parameterization, welfare in the news economy is higher. Next, lower \( z(g) \) so that the debt-constraint in the good state is barely slack; we choose \( z(g) = 0.0051 \). The night-allocation in the the news economy is: \( y(b) = 0.9951 \) and \( y(g) = 1 \). Welfare now is higher in the cash-in-advance economy.

We are able to find several parameter combinations for which a cash-in-advance constraint improves welfare. The intuition seems straightforward enough. The constraint eliminates currency competition, so that the equilibrium rate of return on fiat money is no longer linked by an arbitrage condition to the value of private money instruments. While the average rate of return on fiat money is low, it is not terribly low. Moreover, fiat money retains its purchasing power when bad news causes a collapse in the price of private assets. The stability in the rate of return on fiat money is valued by society because it cushions the economy against bad news events that would otherwise disrupt the payments system.

6 Conclusion

The main premise of this paper is that commitment is limited in financial markets and that at least some types of information relating to expected asset returns is of limited social value.

A lack of commitment induces a demand for exchange media; that is, assets that are used to support intertemporal exchange. The ability of the private sector to supply such assets, however, may be limited. In our paper, we modeled this asset scarcity as a technological property—a low value for \( z^e \). But there is an institutional interpretation as well; i.e., with \( z^e \) reflecting the return on a set of assets for which property rights are well enforced. Indeed, the general lack of commitment, a factor that plays such a critical role in the analysis above, can be thought of in such terms.
While information may have private value, it need not have social value. Financial markets can be expected to capitalize all relevant information into asset prices. If equilibrium asset prices capitalize information of limited social value, the effect is innocuous if there is no asset shortage or if the asset in question does not play a supporting role in the payments system. But if these two conditions fail to hold, then equilibrium asset prices will move too much. In particular, capitalizing (socially irrelevant) bad news into the price of an exchange medium can result in an undesirable “credit crunch.”

One way to create “informationally insensitive” exchange media is to create high quality tranches out of the existing asset supply. This is of course problematic if there is an asset shortage to begin with.\(^2\) The suppliers of exchange media may employ alternative strategies for enhancing asset quality. We have identified the nondisclosure of a certain type of information related to short-run asset returns as one such strategy. This is perhaps one reason why banks should not be required to report asset valuations using “mark-to-market” methods at high frequency.\(^2\)

If the private sector is limited in its ability to create enough high-quality exchange media, then there is, in principle, a role for government intervention. It should come as no surprise that if the government has enough instruments, efficiency can be restored. The simplest and most direct intervention would entail a news-contingent tax/transfer policy.\(^3\) Alternatively, the introduction of interest-bearing government debt earning an appropriate constant rate of return will also restore efficiency. If government debt is constrained to earn zero nominal interest, then the first-best allocation can be supported

\(^2\)Gorton and Pennacchi (1990) develop a model that it relies on the presence of asymmetric information between “informed” and “uninformed” traders. In their environment, one solution to this problem is for a firm to split the cash flow of their asset portfolio between risky equity and risk-free debt. The debt instrument here is “informationally insensitive” in that its value is independent of any news received by informed traders. In this manner, uninformed agents can be induced to acquire and use debt for transaction purposes.

\(^3\)The Financial Accounting Standards Board Rule 157 (Fair Value Measurements) issued in September 2006 requires banks to report the value of their assets at market value. Some people, including former FDIC chair William Isaac, have blamed this legislation for exacerbating the negative consequences of the Great Recession; see, www.williamisaac.com/ published-works/ providing-relief-from-the-crisis/

\(^2\)In particular, efficiency is restored by applying a distortionary subsidy/tax on expected asset returns at night. Lump sum taxes/transfers can be used in the day to balance the budget.
via an appropriate deflation. Either way, the real rate of return on government debt must be financed with lump-sum taxes; at least, to implement the first-best allocation.

As it turns out, even a constant supply of fiat money can improve welfare under some circumstances. Evidently, the welfare benefit of fiat money (or government debt) stems not only from its ability to eliminate the liquidity premium on private assets (increasing the real rate of return), but also from it’s relative insensitivity to news of zero (or limited) social value. If the price of private monetary instruments fluctuate “excessively,” then it may be desirable to prohibit their use as exchange media.24 A cash-in-advance constraint insulates the return on fiat money from competing currencies, making it more desirable as a payment instrument. A solution of this sort of course presumes a willingness and ability on the part of the monetary authority to keep inflation low and stable.

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24 Or, alternatively, to provide some form of government insurance, for example, the way the FDIC insures demand deposits. It is interesting to speculate whether the collapse of the repo market in 2008 would have been avoided with government guarantees on AAA rated MBS.
7 References


Appendix A
Proof to Proposition 2

Proposition 2 asserts that if \( z(b) < z^e < z(g) \) and \( \beta = \hat{\beta}(z^e) \), then the consumer debt constraint will bind tightly in the bad news state and remain slack in the good news state. This can be demonstrated by way of the following lemmas.

**Lemma 1** The debt-constraint cannot remain slack in both news states.

**Proof.** Assume that the debt-constraint remains slack in both news states. Then \( y(b) = y(g) = y^* \), so that (15) implies

\[
\phi_1 = \beta(z^e + \phi_1)
\]

Moreover, conditions (17) and (18) imply

\[
\phi_2(\eta) = \beta \left[ \frac{z(\eta) + \phi_1}{h'(y^*)} \right] \geq y^* \text{ for } \eta \in \{b, g\}
\]

This latter condition implies \( \beta [z(b) + \phi_1] \geq y^* h'(y^*) \). Since \( z(b) < z^e < z(g) \), it follows that

\[
\phi_1 = \beta(z^e + \phi_1) > \beta [z(b) + \phi_1] \geq y^* h'(y^*) = \left( \frac{\beta}{1 - \beta} \right) z^e = \phi_1;
\]

which is a contradiction. ■

**Lemma 2** The debt-constraint cannot bind tightly in both news states.

**Proof.** Assume that the debt-constraint binds tightly in both news states. Then (15) and (17) imply

\[
\phi_1 = \pi \beta [z(b) + \phi_1] A(y(b)) + (1 - \pi) \beta [z(g) + \phi_1] A(y(g))
\]

or, by collecting terms,

\[
\phi_1 [1 - \pi \beta A(y(b)) - (1 - \pi) \beta A(y(g))] = \pi \beta z(b) A(y(b)) + (1 - \pi) \beta z(g) A(y(g))
\]

28
As both debt-constraints bind, (18) implies that \( y(\eta) < y^* \) for \( \eta \in \{b, g\} \); so that \( A(y(\eta)) > 1 \) for \( \eta \in \{b, g\} \). Combining this information with the equation above, we see that the asset commands a liquidity premium; i.e.,

\[
\phi_1 > \left( \frac{\beta}{1 - \beta} \right) z^e = y^* h'(y^*)
\]

The expression above implies

\[
\beta [z^e + \phi_1] > \beta [z^e + y^* h'(y^*)] > y^* h'(y^*)
\]  
(39)

Condition (18) implies \( \phi_2(\eta) = y(\eta) < y^* \) for \( \eta \in \{b, g\} \), so that by condition (17)

\[
y(b)h'(y(b)) = \beta [z(b) + \phi_1] \\
y(g)h'(y(g)) = \beta [z(g) + \phi_1]
\]

Since \( z^e = \pi z(b) + (1 - \pi) z(g) \), it follows from these latter two restriction that

\[
\pi y_2(b)h'(y(b)) + (1 - \pi)y_2(g)h'(y(g)) = \beta [z^e + \phi_1]
\]  
(40)

Conditions (39) and (40) imply

\[
\pi y_2(b)h'(y(b)) + (1 - \pi)y_2(g)h'(y(g)) > y^* h'(y^*)
\]  
(41)

But as \( yh'(y) \) is strictly increasing in \( y \), and as \( y(\eta) < y^* \), the inequality in (41) is impossible. \[\blacksquare\]

**Lemma 3** The debt-constraint cannot bind in the good-news state and remain slack in the bad-news state.

**Proof.** Assume that the debt-constraint binds in the good-news state and remains slack in the bad-news state. Then (18) implies \( \phi_2(b) > y^* \) and \( \phi_2(g) = y(g) < y^* \). Moreover, by condition (17)

\[
\phi_2(b)h'(y^*) = \beta [z(b) + \phi_1] \\
y(g)h'(y(g)) = \beta [z(g) + \phi_1]
\]

As \( z(g) > z(b) \), these latter equations imply

\[
y(g)h'(y(g)) > \phi_2(b)h'(y^*) > y^* h'(y^*)
\]
But this is impossible; as $y h'(y)$ is strictly increasing in $y$ and as $y(g) < y^*$. 

The three lemmas above rule out three out of the four possible configurations. The only remaining configuration is as characterized in the text; where the debt-constraint binds in the bad-news state and remains slack in the good-news state.