Basel Accord and Financial Intermediation: The Impact of Policy

Martin Berka
and
Christian Zimmermann

Working Paper 2011-042A

December 2011

FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
P.O. Box 442
St. Louis, MO 63166

The views expressed are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors.

Federal Reserve Bank of St. Louis Working Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Federal Reserve Bank of St. Louis Working Papers (other than an acknowledgment that the writer has had access to unpublished material) should be cleared with the author or authors.
Basel Accord and Financial Intermediation:
The Impact of Policy

Martin Berka and Christian Zimmermann

December 23, 2011

1This work was initiated while both authors were visiting the Bank of Canada and benefited from initial stimulus from Mingwei Yuan and discussions with Scott Hendry, Césaire Meh, Kevin Moran, Vincenzo Quadrini, and seminar participants at the Universities of Alabama, Connecticut, Florida International, Montréal, Boston College, the Dallas and St. Louis Fed, the national Bank of Poland, the NBER Summer Institute, and the Midwest Macro Meetings at the Chicago Fed. The views expressed in this paper are those of the authors. No responsibility should be attributed to the Bank of Canada. The views expressed are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors. Any errors or omissions are those of the authors.

2Berka: School of Economics and Finance, Victoria University of Wellington, P.O. Box 600, Wellington, New Zealand; phone: ++64 4 463 5893; email: martin.berka@vuw.ac.nz
Zimmermann: Economic Research, Federal Reserve Bank of St. Louis, P.O. Box 442 St. Louis MO 63166-0442, phone (314) 444 8647; fax (314) 444 8731; email: zimmermann@stlouisfed.org
Abstract

This paper studies loan activity in a context where banks must follow Basel Accord-type rules and acquire financing from households. Loan activity typically decreases when entrepreneurs’ investment returns decline, and we study which type of policy could revitalize an economy in a trough. We find that active monetary policy increases loan volume even when the economy is in good shape; introducing active capital requirement policy can be effective as well if it implies tightening of regulation in bad times. This is performed with an heterogeneous agent economy with occupational choice, financial intermediation and aggregate shocks to the distribution of entrepreneurial returns.

Keywords: Bank Capital Channel, Capital Requirements, Basel Accord, Occupational Choice, Bankruptcy, Credit Crunch.

JEL Classification: E44, E22, G28, E58
1 Introduction

Traditionally, the literature on financial intermediation and credit channels, especially credit crunches, has emphasized the relation between banks and entrepreneurs who require credit and has neglected the funding of banks. With this paper, we want to be much more precise in this respect and study the impact of funding on credit. Indeed, the world wide capital adequacy requirements imposed by the Basel Accords limit the amount of bank loans by the level of bank equity. A crucial and yet under-explored element we focus on in the model is the dependance of the amount of equity banks can issue on the supply of equity by households (who also purchase deposits).

In our model economy, households have heterogenous asset holdings because of different labor histories and because only some of them qualify to apply for credit as entrepreneurs. (among those, the return on investment is stochastic.) Non-entrepreneur households invest in bank deposits and bank equity, and banks maximize profits while following regulations. A central bank conducts the monetary policy and regulates the banks.

Therefore, when banks reduce their loan portfolio, the displaced entrepreneurs also become new equity holders, thereby acting as “automatic stabilizers.” However, banks typically reduce their quantity of loans when their loan portfolio becomes too risky, and households may then want to hold less equity in banks that are now more risky. Whether or not banks have to severely tighten credit in this situation depends very much on the distribution of assets across households and their equity decisions.

We solve this very rich model using numerical methods, in particular for the transitional dynamics that may lead an economy into a possible credit crunch. We then look for policies that may help the economy out of a trough or prevent it altogether. We find that the endogenous distribution of assets has strong implications that should not be neglected in future research. Also, monetary policy can have positive real effects only if the central bank is able to commit to act in certain ways.

We find some evidence in our model that a credit crunch can arise in the presence of capital requirements, as documented in the data by Bernanke and Lown (1991). The numerical simulations show that the size of the crunch is relatively small. We then investigate the potential role of flexible capital requirements. One would first think that loosening those requirements in a trough would expand the loan mass. It appears that, on the contrary, tighter capital requirements increase the demand for equity, and thus facilitate the financing of banks sufficiently to offset the reduction of allowable loans for given equity. Again, this highlights the importance of the savings decisions for the supply of bank capital. This result is particularly important in the light of the second Basel Accord, whose more flexible requirements essentially tighten the equity requirements when the economy passes through a rough patch, as highlighted by Catarineu-Rabell, Jackson and Tsomocos (2005). This cyclicality of capital requirements was previously

---

1The welfare costs of bank capital requirements, notwithstanding the possible costs of a credit crunch, are studied by Van den Heuvel (2008).
thought to have a negative impact on credit. For example, Kashyap and Stein (2004) argue that Basel II exacerbates business cycle fluctuations by requiring banks to hold more capital during downturns. Higher costs of raising capital in downturns force banks to further contract lending, a credit crunch. However, such demand-side arguments ignores the negative impact of declining capital adequacy on the supply of bank capital. Rising funding costs in bad times are partly a result of a drop in the supply of bank’s capital because of the banks’ deteriorating capitalization. Our model articulates both of the channels. Under standard calibration, the supply-side channel in our model dominates the demand considerations highlighted in Kashyap and Stein (2004). Consequently, more stringent capital adequacy requirements help bank recapitalization and facilitate a smoother loan time-path. The conservative lending behavior implied by such a policy in the face of increased aggregate uncertainty has been observed in the data – for example by Baum, Caglayan, and Ozkan (2002).

We are not the first to highlight the real impact of monetary policy through lending. Bernanke and Gertler (1995) highlight two channels. In the balance sheet channel, Fed policy affects the financial position of borrowers and hence their ability to post collateral or self-finance. In the bank lending channel, Fed policy shifts the supply of bank credit, in particular loans. They argue the importance of the latter channel has declined with deregulation, as this channel relies on reserves. Van de Heuvel (2002) identifies another channel stemming specifically from Basel Accord-like rules. The “bank capital channel” arises from maturity transformation through banks: higher short-term interest rates depress profits and consequently equity and capital adequacy – all this in an environment with an imperfect market for bank equity, which forces banks to raise new equity by retaining their earnings. Van de Heuvel’s model has a very detailed banking structure, but neglects the problems of households and firms. Our model has a simpler banking structure but emphasizes the source of financing (households) and the demand for loans (entrepreneurs) by modeling occupational choice, savings, and bankruptcy.

Chami and Cosimano (2010) identify a “bank-balance sheet channel” using the concept of increasing marginal cost of external financing. Oligopolistic banks avoid expected profit declines caused by the anticipation of a binding capital adequacy constraint by holding capital above the regulatory level. Tighter monetary policy raises banks’ cost of funding and reduces loan supply. Persistent increases in deposit rates reduce the option value of bank capital, further propagating the loan contraction. As in Van den Heuvel (2002), these authors need market power in the banking industry to obtain the result. Our model has fully competitive banks. Furthermore, they summarize the demands for loans with a reduced form while we try to present a general equilibrium framework. Bolton and Freixas (2006) have a model where higher costliness of bank capital, due to asymmetric information about the quality of banks’ assets, motivates banks to hold the

---

2The empirical relevance of the standard bank lending channel has recently been questioned by den Haan et al. (2007), who document large loan portfolio changes following monetary contractions.

3The heterogeneity of firms we obtain is endogenous. Bernanke, Gertler, and Gilchrist (1999) also have heterogeneous firms, but they exogenously fix a share of firms to have easy access to credit.
minimum required amount of capital. Capital requirements are a cause of the credit crunch when spreads are low — one of model’s multiple equilibria. Monetary policy tightening operates by pricing riskier firms out of the loan market. Our model puts more emphasis on the financing side and does not impose the asymmetric information assumption. In a model where banks as well as entrepreneurs face moral hazard, Meh and Moran (2004) show that responses to monetary and technology shocks are dampened but more persistent. The bank capital/asset ratio in their model is countercyclical, as in the data, but the authors do not explore the behavior of credit.\footnote{Regulations in many countries limit or prevent banks from investing equity (capital) in their borrowers’ business, arguably to limit their risk exposure. However, contribution of the model in Meh and Moran (2004) is based on the assumption which implies that banks invest a share of their capital into borrowers’ business.} The discussion of cyclicity of capital requirements has intensified recently. Covas and Fujita (2010) extend Kato’s (2006) model of liquidity provision under asymmetric information with bank capital requirements. They find that Basel-II type cyclical capital requirements reduce household welfare by increasing output volatility. In the overlapping generations framework of Repullo and Suarez (2009), banks cannot access equity markets every period and consequently hold large capital buffers. Buffers are insufficient to prevent credit contractions in downturns. In a static partial equilibrium model, Heid (2007) shows that capital buffers held by banks in excess of the Basel II requirements strongly influence the degree of cyclical magnification of the business cycle.

Gertler and Karadi (2011) model agency problems at the bank level as the cause of difficulties in raising capital in a model with heterogeneity within a family in a representative agent framework. They highlight that a non-standard monetary policy, whereby the central bank intermediates in a crisis, reduces the credit volume downturns as a result of the elimination of the agency problem (but at a cost of lower efficiency). Gertler and Kiyotaki (2010) combine the aforementioned model with Kiyotaoki and Moore’s (2008) model of liquidity risk. A contraction limits the intermediary’s ability to obtain credit, driving a wedge between the loan and deposit rates that constrains the non-financial borrowers. In our model, counter-cyclical loan-deposit spreads and bank funding difficulties in the downturn arise without any informational asymmetries. Christensen et al. (2011) introduces the double hazard problem of Holmström and Tirole (1997) into a New-Keynesian environment and find that higher capital requirements lead to increased monitoring of borrowers. As a consequence, countercyclical capital requirements dampens fluctuations and improves welfare.

The major points of difference of our model are with the endogenous financing of bank and heterogeneity of agents. Loan applicants are endogenously divided into entrepreneurs and depositors according to their wealth. Not only are the loan supply and demand endogenous, changes in macroeconomic conditions generate distributional effects that feed back to the aggregate level in a general equilibrium setting. Because of these distributional considerations, Basel-II type requirements do not magnify the downturns, unlike in all other work discussed above. In fact, ours is the first model
in which tightening of the capital adequacy requirements in bad times facilitates bank funding, by lowering risk of bank equity and making it more attractive to savers. In fact, as households anticipate this, they provide more equity to banks even when the economy is doing well, leading to an improvement in steady-state as well. This is consistent with evidence from Guidara et al. (2010) that shows Canadian banks’ capital was more procyclical after the implementation of Basel II.

Credit constraints in our model lead to asymmetric effects of the monetary policy, with tightening having more significant real consequences than a monetary policy loosening (one of the key properties of monetary propagation mechanism in general, as highlighted by Kocherlakota, 2000). Furthermore, our model is the only one so far that can be used for addressing both the aggregate effects and the redistributional effects of credit market policies. The structure of this paper is as follows. Section 2 presents the model. Section 3 presents the calibration of the model. Section 4 analyzes bank lending and optimal monetary policy behavior following negative shocks. Section 5 concludes. Appendices give additional details about various aspects of the model and the solution strategy.

2 Model

2.1 Overview

There are three types of agents in the economy: households, banks, and a central bank. Households in a productive stage of their lives aim to become entrepreneurs, but a shortage of internal financing forces them to apply for external funds. Successful applicants become entrepreneurs while others become workers. Workers face an idiosyncratic shock of becoming unemployed while entrepreneurs face risky returns on their investment. All households in a productive stage of life (entrepreneurs, and employed and unemployed workers) face a risk of becoming permanently retired, and all retirees face a risk of dying. New households are born to replace the deceased ones.

When households make their consumption-saving decision, they decide optimally on allocation of their savings between bank deposits and bank equity. Banks collect deposits and equity, provide loans to entrepreneurs and purchase risk-free government bonds in order to maximize their profits. Banks screen loan applications and accept them according to the level of each household’s net worth. Banks also have to purchase deposit insurance and are subject to a capital adequacy requirement imposed by the central bank. The central bank controls the government bond rate.

We now go through the model in more detail. The economy is subject to aggregate shocks and can thus be represented by an aggregate state vector including the current shock and the current distribution of assets and occupations that we ignore to simplify

5Empirically, such earnings shocks are critical for the savings decisions, as outlined in the 2001 Survey of Consumer Finances (Chatterjee et al. (2007)).
2.2 Households

In the model economy, there is a continuum of measure one of households, each maximizing their expected discounted lifetime utility by choosing an optimal consumption-savings path. A household can either be productive or retired, and the probability of a productive household retiring \( \tau \) is exogenous.\(^6\)

Each productive household \( i \) is endowed with one investment project of size \( x^i \), which is always greater than the household’s net worth \( m^i \). We assume that the total investment is a fixed multiple of household’s net worth: \( x^i = \phi m^i \), where \( \phi > 1 \). The project is indivisible, and so \((\phi - 1)m^i\) has to be funded by the bank in order for a project to be undertaken.\(^7\) If a household receives a loan it becomes an entrepreneur and invests in a project, receiving a return \( r^i \) drawn from a trinomial distribution. The distribution of returns is such that households always prefer investing in projects and becoming entrepreneurs to becoming workers. We study an equilibrium in which this participation constraint is satisfied in all cases for households that receive loans. The returns are drawn independently across households and time. The lowest of the returns is sufficiently negative with a positive probability to lead to bankruptcy, in which case a household is guaranteed a minimal amount of consumption \( c_{\text{min}} \) and starts next period with no assets.\(^8\)

When the bank rejects a loan application, the household enters the work force and faces exogenous probabilities \( 1 - u \) of becoming employed and \( u \) of becoming unemployed. Workers inelastically supply their labor and receive an after tax wage income \( y \). Unemployed workers receive unemployment benefits \( \theta y \) where \( \theta \) is the replacement ratio.

Labor supply is inelastic at an individual level. At the aggregate level, labor supply is determined by moves between the pools of workers (employed and unemployed), entrepreneurs, and retirees. This further strengthens the role that asset accumulation plays in the economy. We use aggregate labor input data on the average hours per worker to calibrate the labor demand. Therefore, the labor market clears implicitly at the level of the utility function.

After retirement, the household earns income from its savings and pension (which equals the unemployment benefit payments). Retirees face a probability \( \delta \) of dying. They are then replaced by agents with no assets and any remaining assets are lost (no bequests).

\(^6\)Once retired, household cannot become productive again. This innocuous assumption simplifies the calibration of birth rates.

\(^7\)Kocherlakota (2000) shows how credit constraints are fundamental for the amplification of income shocks at a macroeconomic level when capital and labour shares in production function are substantial. Our net worth constraint works in a similar manner, but by rationing an endogenous mass of the agent distribution with positive assets below some \( m^* \).

\(^8\)Consumption insurance is used in a number of papers, recently in Gertler and Karadi (2011)
The households make their consumption-savings decision to maximize their expected lifetime utility. The contemporaneous utility function is a CRRA type:

\[ U(c, l_j) = \frac{\left( l_j^{\sigma} c^{1-\sigma} \right)^{1-\rho} - 1}{1 - \rho}, \]

where \( j \in \{W, U, E, R\} \), \( l \) denotes leisure, \( c \) consumption, and \( \rho \) is a risk-aversion parameter. As mentioned above, the labor supply is inelastic and the values \( l_j \) represent market-clearing values for leisure.

Let \( V_j \) denote the value functions and \( m^* \) be the minimum net worth necessary for external financing. A worker with a net worth \( m < m^* \) faces probability \((1 - u)\) of being employed, following which he receives labor income \( y = (1 - l_W)w \) and interest income \((1 - \tau)Rd \), pays a banking fee \( \xi \), consumes a desired level, and invests his remaining net worth \( m' \) in a bank. If unemployed, he receives unemployment benefit payment \( \theta y \) and makes a similar consumption-savings decision. In the next period, depending on the level of \( m' \), a worker may either become an entrepreneur (borrower) or remain a worker (depositor).

For an employed worker, the Bellman equation is

\[ V_W(m^i) = \max_{c^i, m'^i} \{ U_W(l_W, c^i) + \beta(1 - \tau)((1 - u)V_W(m'^i) + uV_U(m'^i) + E_r'V_E(m'^i, r'^i) + \tau V_R(m'^i)) \} \] (1)

such that

\[ c^i + m'^i = (1 + r^d)m^i + y - \xi, \]

\[ m'^i \geq 0. \]

For an unemployed worker,

\[ V_U(m^i) = \max_{c^i, m'^i} \{ U_U(l_U, c^i) + \beta(1 - \tau)((1 - u)V_W(m'^i) + uV_U(m'^i) + E_r'V_E(m'^i, r'^i) + \tau V_R(m'^i)) \} \] (2)

such that

\[ c^i + m'^i = (1 + r^d)m^i + \theta y - \xi, \]

\[ m'^i \geq 0. \]

An entrepreneur \( i \) invests in a project of size \( x^i \), earns a stochastic net return \( r^i \) and labor income \( y = (1 - l_E)w \), and pays the borrowing cost \( r^i(x^i - m^i) \), while making a consumption-savings decision to maximize his expected utility. Because the net wealth is constrained to be non-negative, significant project losses may drive the entrepreneur into bankruptcy. When bankrupt, an entrepreneur defaults on the portion of the debt.

\[ \xi \] We will justify in the calibration the use of \( \xi \).

\[ \theta \] A prime (') denotes variable values in the next period.
he cannot repay, less a minimal consumption allowance \( c_{\text{min}} \), which has to be granted by the bank. Upon default, the entrepreneur starts the next period as a household with no assets and no liabilities. The returns on project \( r_i \) are drawn independently across time and individuals and follow a trinomial distribution. The lowest of the returns is sufficiently negative to lead the entrepreneur to bankruptcy. The value function of an entrepreneur is as follows:

\[
V_E(m^i, r^i) = \max_{c^i, m^{i'}} \left\{ U_E(l_E, c^i) + \beta[(1 - \tau)](1 - u)V_W(m^{i'}) + uV_U(m^{i'}) + E_r V_E(m^{i'}, r^{i'}) + \tau V_R(m^{i'}) \right\}
\]

such that

\[
c^i = \max\{c_{\text{min}}, m^i + y + (1 + r^i)x^i - r^i(x^i - m^i) - \xi - m^{i'} \},
\]

\[
x^i = \phi m^i,
\]

\[m^{i'} \geq 0.\]

The assumption that project \( x^i \) is proportional to the entrepreneur’s asset holdings \( m^i \) can be justified by the collateral requirements typically observed in credit markets. The proportionality parameter \( \phi \) can easily be calibrated from the data. We assume that households \textit{ex ante} always prefer to apply for a loan. This implies a participation constraint for households in a productive stage of their lives that needs to be satisfied for all households that obtain a loan:

\[
E_r V_E(m, r) \geq (1 - u)V_W(m) + uV_U(m), \quad \forall m \geq m^*.
\]

Every household faces an exogenous probability of retirement \( \tau \). Once retired, the household collects retirement income \( y_R = \theta w \) and manages its assets subject to the risk of death; \( \delta \).

\[
V_R(m) = \max_{c^r, m^{i'}} \left\{ U_R(1, c^r) + \beta[(1 - \delta)V_R(m^{i'})] \right\}
\]

such that

\[
c^r + m^{i'} = (1 + r^d)m + y_R - \xi,
\]

\[m^{i'} \geq 0.\]

Risk-averse agents smooth their consumption over time. Heterogeneous risks of unemployment and retirement as well as the heterogeneity in project returns cause uncertain income streams and lead to a non-degenerate distribution of assets in the economy. Without these risks, there would be no reason to save other than to invest in a project, and the asset distribution would unrealistically collapse along \( m = 0 \) and \( m = m^* \). This would make financial intermediation impossible because of the lack of funds (no depositors). Furthermore, as pointed out by Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), there would be no bankruptcy without agent heterogeneity. All equilibria in
such bimodal distribution are very unstable because all entrepreneurs can drift to zero assets following a shock. The distribution of assets plays a crucial role in determining the dynamics of the aggregate variables.

The decision to allocate savings between bank equity and bank deposits is obtained by maximizing a risk-adjusted return on portfolio $r_i^{\text{port}}$ across agent types $i, i \in \{W, U, R\}$:

$$
\max_{\omega_i} r_i^{\text{port}} - \frac{1}{2} \lambda_i \sigma_{\text{port}}^2,
$$

where $r_i^{\text{port}} = r_e \frac{E_i}{M_i} + r_d \frac{D_i}{M_i} = \omega_i r_e + (1 - \omega_i) r_d$, $\omega_i \equiv E_i / M_i$ is a weight on the risky (equity) investment for agent type $i$, $\lambda_i$ is a risk-aversion parameter, and $\sigma_{\text{port}}^2$ is a variance of the portfolio return. Because bank deposits carry no risk ($\sigma_d^2 = 0$), the household maximizes:

$$
\max_{\omega_i} \omega_i r_e + (1 - \omega_i) r_d, -\frac{1}{2} \lambda_i \omega_i^2 \sigma_e^2
$$

which yields the optimal share of equity $\omega_{i}^* = \frac{r_e - r_d}{\lambda_i \sigma_e^2}$. This in turn defines the demand for equity (and implicitly for deposits) given savings $M_i$ for any agent group:

$$
\frac{E_i}{M_i} = \frac{r_e - r_d}{\lambda_i \sigma_e^2}.
$$

Note that we have separated this portfolio problem from the intertemporal utility maximization of the household. We do this for computational reasons: given that with aggregate shocks we need to include the entire asset distribution in the state space, we need to avoid having to track for each household two separate assets to keep the state space dimensionality within computationally efficient bounds. Appendix B shows the details of making savings decisions depend on asset levels.

### 2.3 Financial Sector

#### 2.3.1 Bank

The representative bank maximizes its expected profits, taking the asset distribution in the economy as given. Profits equal asset returns less the funding costs, deposit insurance payments, and the expected loan losses and liquidation costs. The bank’s choice variables are loans $L$, bonds $B$, equity $E$, and deposits $D$. Because the bank takes the distribution of assets as well as all returns as given, the choice of loan volume is identical to the choice of a threshold level of net worth $m^*$. Formally, the problem can be stated as

$$
\max_{L, B, D, E} r^L L + r^B B - r^d D - r^e E - \delta \left( \frac{D}{E} \right)^\gamma D - (1 + lc) \epsilon L + \xi
$$

8
subject to

\[ B + L = D + E \equiv M \]  
\[ \frac{E}{L} \geq \alpha \]  
\[ D + E \geq L, \]  

where \( M \) is the total amount of loanable funds that are exogenous from the point of view of the bank, \( \delta \) is a per-unit deposit insurance cost parameter, \( \epsilon \) is an expected share of loan losses, and \( lc \) is a liquidation cost parameter. Equation (8) is the usual balance sheet constraint, (9) is the regulatory requirement on capital adequacy, and (10) is a non-negativity constraint on bond holdings. The profit function (7) is non-linear because of the inclusion of deposit insurance costs, which are an increasing function of the deposit/equity ratio. Because profits increase with larger loans for any given asset distribution, one and only one of the constraints (9) and (10) will bind at any time.\(^{11}\) The solution of the profit maximization is described in the appendix.

2.4 Central bank

The central bank in this model determines the bond interest rate \( r^b \) and supplies government bonds at that rate. In addition, it determines the capital-to-asset ratio parameter \( \alpha \). Therefore \( \alpha \) and \( r^b \) are the only monetary policy instruments it has at hand. In the simulation in Section 4, we show how different monetary policy actions, as represented by mean-preserving changes in \( r^b \) across the aggregate states, influence the behavior of the different types of households and of the representative bank. We also do similar exercises with mean-preserving changes in the capital requirements.

2.5 Market clearing

On the financial side, markets for loans, bonds, equity, and deposits must clear. The bond market clears automatically because of an infinitely elastic supply of bonds.\(^{12}\) The remaining market clearing conditions are:

\[ D^S = D^D = \sum_{m^i < m^*} m^i (1 - \omega_r) \]  
\[ E^S = E^D = \sum_{m^i < m^*} m^i \omega_r \]  
\[ L = \sum_{m^i \geq m^*} (\phi - 1) m^i \]  

\(^{11}\)The chances that both of them bind at the same time can be dismissed as arbitrarily low.

\(^{12}\)One can think of banks depositing their non-loanable investments at the central bank, which also sets the deposit rate in this model.
\[
M = \sum_{m^i < m^*} m^i = D + E = B + L.
\] (14)

Also, expected losses of the bank must in equilibrium equal the realized loan losses:
\[
\epsilon = \sum_{m^i \geq m^*} \max\{0, (1 + \mu)[r^l(\phi - 1)m^i - \phi m^i(1 + r^i)]\} + c_{\min},
\]
where \(\mu\) are auditing costs.

Equity market clearing implicitly defines the return on equity \(r^e\) as a function of all other returns. In the case of an interior solution, equations (21) and (6) imply
\[
\frac{1}{\delta}(r^e - r^d)^3 - \left[\frac{1}{\alpha \delta}(r^l - r^d - (1 + l_c)\epsilon) + 1\right](r^e - r^d)^2 + 2\lambda \sigma_e^2(r^e - r^d) - \lambda^2 \sigma_e^4 = 0. \tag{15}
\]
In the case of a corner solution, equations (24) and (6) imply
\[
r^e^3 - r^e^2 \left[2r^d + r^l - (1 + l_c)\epsilon + 1\right] - r^e \left[r^d^2 + 2r^d(r^l - (1 + l_c)\epsilon + 1) + 2\lambda \sigma_e^2\right]
- \left[r^d^2(r^l - (1 + l_c)\epsilon + 1) + 2\lambda \sigma_e^2 r^d + \delta \lambda^2 \sigma_e^4\right] = 0. \tag{16}
\]

To illustrate the functioning of the equity market, it is useful to undergo the following thought experiment. Consider a case of an increase in the lending interest rate \(r^l\), possibly because of an increase in the demand for loans. If the ratio of expected losses as a proportion of loans \(\epsilon\) rises less than \(r^l\), the bank’s profit margin on the additional new loan goes up, which prompts the bank to lend more. To do so, the bank has to raise more equity (it starts without excess equity: \(E = \alpha L\)), which is why the equity supply equation (21) is increasing in the loan profit margin. Household demand for equity (6) is unaffected by the return on loans, and so to raise more equity, the bank’s offered \(r^e\) has to increase. \(r^e\) plays an important role in the bank’s liability management because the bond rate is exogenous and determines the deposit rate in an interior solution, and also because the bank cannot choose the size of its balance sheet \(M\). The increase of \(r^e\) leads to a rise in the total amount of equity raised and to a more-than-proportional increase in the \(E/D\) ratio for any size of the balance sheet \(M\).\textsuperscript{13}

It is therefore easy to see that when the bank increases the share of loans in its portfolio, it has to fund the higher equity holdings at an ever-increasing price. Eventually, the original profit margin disappears and a new optimal loan level is achieved. Two cases can occur. First, the total amount of new loans is less than the new balance sheet level, and loan market clearing conditions are satisfied and constitute a potential equilibrium. Second, the total amount of new loans may exceed the new balance sheet volume \(M\), which is what we defined earlier as a corner solution. In the latter case the loan market does not clear and the banks ration some of the eligible loan applicants. Because there is no asymmetric information problem in this model (hence no adverse selection), an increase in the price of loans does not affect its quality and a higher \(r^l\) is needed to clear the market. Therefore we have a choice of focusing on market-clearing

\textsuperscript{13}This follows from the fact that \(\frac{E}{D} = \frac{\epsilon}{\omega_r}\) and \(\omega_r\) increases in \(r^e\).
equilibria, which rule out corner solutions and equity “hoarding”, or allowing credit rationing when multiple equilibria may arise and excess equity is kept as a backup in case the total amount of loanable funds $M$ increases. For simplicity, we focus on the market-clearing equilibria, implying that equation (16) becomes irrelevant. This implies that we do not observe the bank holding excess equity in equilibrium, and so regulatory changes in capital adequacy ratio $\rho$ will have a direct effect on the loan volume.

Market-clearing condition (15) defines a return on equity as a function of all other returns and some parameters: $r^e = r^e(r^d, r^d, \sigma^2_e, \lambda, \alpha)$. The cubic equation can be solved analytically but does not determine the $r^e$ uniquely. Depending on the parameter values, two of the three roots may be complex. We disregard the complex roots.

The system is recursive: conditional on $M$, equation (21) determines the optimal level of equity $E$, equation (23) determines the optimal level of deposits $D$, equation (14) determines the optimal level of bonds $B$, and equation (22) determines the optimal level of loans $L$. We therefore have \{r^e, r^d, E, D, L, B\} as a function of \{r^d, M\} and exogenous variables.

2.6 Equilibrium

A recursive equilibrium in this model economy includes the four value functions $V_j(m, s)$, where $s$ represents the aggregate state (current shock, distribution of $m$), for $j \in \{E, W, U, R\}$, decision rules \{\(g^m_j(m, s), g^d_M(s), g^e_M(s), g^m_B(s), g^d_B(s)\}\}, government policies \{\(\alpha(s), r^b(s)\}\}, prices \{r^d(s), r^{port}(s), r^e(s)\}\}, aggregate asset levels \{L, D, B, E\}\, and a function $\Psi(\mu)$ such that

1. decision rules $g^m_j(m, s)$ solve each household’s problem with the associated value functions $V_j(m, s)$.
2. decision rules $g^d_M(s)$ and $g^e_M(s)$ solve portfolio problem of the household.
3. decision rules $g^m_B(s)$ and $g^d_B(s)$ solve the banks’ problems.
4. loan, equity, and deposit markets clear:

$$L(s) = \sum_{m \geq m^*} (\phi - 1)m\mu(m, s)$$  \hfill (17)

$$E(s) = \frac{r^e - r^d}{\gamma \sigma^2_e} \sum_{m < m^*} m\mu(m, s)$$  \hfill (18)

$$D(s) = \left(1 - \frac{r^e - r^d}{\gamma \sigma^2_e}\right) \sum_{m < m^*} m\mu(m, s).$$  \hfill (19)

5. the distribution of households is the fixed point of the law of motion $\Phi$:

$$\mu'(m, s) = \Psi(m, s).$$
3 Parametrization

To simulate the economy and obtain numerical results, we parametrize the model to the Canadian economy in the years of 1988 to 1992, in accordance with the available data on project return distributions. Indeed, these are the only years for which Statistics Canada published such data.

First we calibrate the household sector. Several parameters are set in accordance with the literature: $\rho = 2.5$, $\beta = 0.96$, and $\sigma = 0.67$. In accordance with the models that include explicit leisure specification, $l_E = l_W = l_U = 0.55$ while $l_R = 1$, as a result of which the labor input of entrepreneurs and workers, and the search effort of unemployed are set to 0.45. Wages are exogenous and while they completely characterize the labor income of entrepreneurs and workers, the incomes of unemployed and retired are determined by the ratio of unemployment insurance benefits to wages $\theta = 0.3$.\(^{14}\)

The probability of unemployment is set equal to the average Canadian unemployment rate for the considered period: $u = 0.0924$. The probability of retirement $\tau$ and the mortality rate $\delta$ are set at 0.05 and 0.1, respectively so that the number of expected periods as a worker is 20 and as a retiree is 10. Longer expected lifetime horizon generates a stronger wealth effect, and consequently stronger responses of savings over time than in the usual 2-period models (e.g., Williamson (1987) and Bernanke and Gertler (1989)).

Now we turn to the financial side. Following the calibration in Yuan and Zimmermann (2004), we set the real bond rate $r^b$ at 1%, such that the deposit rate $r^d$ is about 0.9%, which corresponds to an average of savings rates and guaranteed investment certificate rates. The parameter $\alpha$ of the capital adequacy constraint is taken to represent the tier-1 capital requirements imposed by the 1998 Basel Accord and set to $\alpha = 0.08$. The deposit insurance parameter $\delta$ is calibrated using the premium rates of the Canadian Deposit Insurance Corporation for banks in 2000/2001 (0.0417% of insured deposits). This per-unit rate corresponds to $\delta = 0.0000417$ for an average D/E ratio of 10. The loan administration cost $l_c$ is assumed to equal 0. The account flat fee $\xi$ is set at 0.0003 by trial and error in order to get the banks to break even. The parameters of the equity market that need to be calibrated are $\lambda$ and $\sigma_E^2$. The variance of returns on equity of the banks is calculated from the TSE (Toronto Stock Exchange) monthly series on financial enterprises’ returns on equity from September 1978 until December 2000, which are deflated by the CPI. Therefore, $\sigma_E^2 = 0.24$. The risk-aversion parameter of the portfolio optimization problem $\lambda$ is calibrated from the market-clearing condition (15) using the observed average real deposit, lending and ROE rates. This implies $\lambda = 16$.

The distribution of returns follows a two-state Markov process calibrated such that the high state occurs 75% of the time. Specifically, a high state has a 75% chance of reoccurring the next period, while a low state can repeat itself with a 25% chance. The distributions of project returns in both aggregate states are calibrated from firms’ return on equity data. Statistics Canada (1994) reports the distribution of return on equity by non-financial enterprises from the fourth quarter of 1988 until the fourth quarter of 1992.

\(^{14}\)This measure is based on the effective replacement rate of Hornstein and Yuan (1999).
Average returns in each quarter are reported for the top, middle, and bottom tertile. Assuming the underlying distribution is normal, we find the returns and associated probabilities for trinomial distributions such that a) average returns are replicated, and b) we have have two extreme returns, one implying bankruptcy. We compute two such distributions, one for the high aggregate state, corresponding to the average of the 75% best quarters in the sample period, and the other for the low state. The returns and the associated distributions are the following:

\[
\text{High: } \left( \begin{array}{ccc} -50\% & 5.2\% & 60\% \\ 0.71\% & 98.48\% & 0.81\% \end{array} \right), \quad \text{Low: } \left( \begin{array}{ccc} -50\% & 2.57\% & 60\% \\ 1.79\% & 97.42\% & 0.79\% \end{array} \right)
\]

The ratio of investment to net worth \((\phi - 1)\) is calibrated to equal the average debt-equity ratio during the reference period, and so \(\phi = 2.2\). With a minimum return on investment of -50%, we have occasional bankruptcies.

4 Capital requirements, bank lending, and monetary policy

In this section, we want to understand the behavior of the model economy. This is no easy task, as the model is quite complex. The rich aggregate state space implies that many different histories of shocks can be studied. We focus here on one particular history which we believe is empirically relevant from the business cycle perspective. The model economy has been hit by a long sequence of High aggregate shocks, thus the distribution of assets has converged to a high steady state. Given the way the shock process has been calibrated, the economy spends on average 50% of time in the initial state of this experiment, a state that we will sometimes refer to as “normal times.” Our experiment then starts with a succession of five low shocks and then five high shocks. Thus, the model economy wanders through a whole cycle, bottoming out in the middle. Note that this is a particular history of shocks among many others and that this history is not anticipated.

Figure 1 shows the behavior of various indicators in a benchmark economy, i.e., one with no policy intervention from the central bank on the interest rate for bonds or capital requirements. When the initial bad shock hits the economy, the lending rate jumps up, essentially to cover against higher expected loan losses, and because agents need time to adjust their asset holdings. As the bad news (negative shocks) accumulate, the lending rate decreases as \(m^*\) increases and the households adjust their asset levels. Banks ration more and more loans as bad shocks accumulate but revert to “normal” behavior as soon as good news comes in. From peak to trough, the amount of loans decreases by 3.0%, and 3.6% of all entrepreneurs are driven out. The consequence is that the size of an average loan increases by 0.6%, which corresponds to the empirically documented phenomenon that small businesses suffer more in a downturn.
Do the results of the benchmark calibration imply a credit crunch? Despite the fact that banks can increase the interest rate on loans to compensate for higher default rates, they have to decrease the total loan mass. The reason is the following. Facing increased risk, more entrepreneurs are forced to become workers because of a higher bankruptcy rate. With more agents that save, the volume of assets increases. However, a smaller share of those assets are channeled to bank equity because its return is too low given its risk. The banks are then squeezed by the capital requirement and have to ration credit and invest more into less profitable government bonds. Without the capital requirement, banks could make more loans, in principle, by charging even higher loan rates, and entrepreneurs would still be willing to pay these rates. Although all agents behave optimally, we have a situation that can be described as a credit crunch to the extent that the marginal returns and the marginal costs of loans are not equal.

Capital requirements imply that changes in the composition of banks’ liabilities affect the amount of credit in the economy. An adverse productivity shock increases the number of depositors and lowers the number of borrowers. Yet risk averse depositors shy away from the risky bank equity, which leads to a further credit decline (due to the binding capital requirements). Thus, our unique heterogeneous agents setup generates “procyclical” bank capital requirements even without $\alpha$ dependant on risk.\textsuperscript{15} However, the movements described above are relatively small.

4.1 Countercyclical monetary policy

The following experiments explain the consequences of various policy actions. The first policy experiment, described in Figure 2, involves a 25-basis-point reduction of the bond rate in the worst aggregate state (current shock Low, long history of Low shocks).\textsuperscript{16} Thus, the central bank reacts only after a prolonged decline in the economy. Note that the decisions of the banks are changed only in this specific state: $m^*$ and the lending rates are unaffected when the central bank does not move, but when it does banks reduce the lending rate by the same margin and, more importantly, significantly relax their loan threshold $m^*$. Thus the situation for entrepreneurs should improve noticeably: easier access to credit at better conditions. Loan activity is negatively affected, however, and equity is reduced compared with the benchmark. This is because workers decide to save less (interest rates are lower) and put a smaller fraction of their saving into equity (the return of equity drops more). Note that household decisions are affected even when the central bank has left the bond rate untouched, in anticipation of possible changes. Ultimately, the same number of entrepreneurs get loans and the average loan is now smaller.

A one-time drop in the interest rate therefore does not appear to be an effective policy, as it weakly magnifies the credit cycle rather than smoothing it. What now if

\textsuperscript{15}The recent literature described above discusses how procyclical bank capital requirements arise in the scope of Basel II. Yet in our model, banks effectively operate under Basel I.

\textsuperscript{16}Note that all experiments are designed such that the average $r^b$ or $m^*$ stay at the same level.
the interest rate is gradually reduced by 5 points after each bad shock and goes back to normal whenever a good shock comes by? This policy should better reflect the expectations households formulate and consequently yield a stronger wealth effect. In Figure 3, we see that the outcome is quite different from Figure 2. Equity-rich banks are more generous to entrepreneurs in bad times, both in terms of lower lending rates in bad times (but higher in good ones) and quite significantly in terms of $m^\ast$. In all aggregate states, there are more entrepreneurs, loans, deposits, and equity. While the average loan is larger in normal times compared with the benchmark, it is smaller in almost any other. This means that asset accumulation has increased for households: entrepreneurship is more interesting as monetary policy counterbalances the increased risk in bad times. This effect is facilitated by an increasing spread, which makes investment of households into bank equity more attractive. Indeed, while firms face lower average returns and higher bankruptcy rates, monetary policy forces banks to offer better conditions. This has an impact on asset accumulation even in good times, due to the households’ long planning horizon. We conclude that an active countercyclical monetary policy can have a significant positive impact. Note, however, that it cannot remove the cyclical nature of loans.

### 4.2 Procyclical monetary policy

If a policy of lower interest rates may have negative consequences under some circumstances, one may naturally ask whether an interest rate increase can do some good. Indeed, higher bond rates mean higher returns on savings and potentially more equity to satisfy the loan needs in the presence of capital requirements.

In Figure 4, we find that the model economy does not behave in a symmetric way, as compared with Figure 2. While the lending rate increases as expected, $m^\ast$ barely changes. Consequently, the loan activity does not change much as households barely change their decisions compared with the benchmark. The average loan size decreases. However, the change in the average loan size is smaller than in the countercyclical policy.

Figure 5 shows the model equilibrium when interest rates are increased gradually. The asymmetry of the monetary policy observed in Figure 3 is also present for the gradual policy. A gradual increase of the bond rate has a negative, but much smaller impact on the various assets. The asymmetry of monetary policy due to the existence of credit constraints has been highlighted by Kocherlakota (2000). In our model, the explanation for this asymmetry is as follows. Countercyclical monetary policy induces a drop in $m^\ast$, leading to an increase in the loan volume as more smaller agents can become entrepreneurs. Moreover, a lower $m^\ast$ induces workers to save more (consumption drops) at any given deposit rate because entrepreneurship is more likely to be attained (a move that is slightly offset by the distributional movement from workers to entrepreneurs). Because of this boom in banks’ liabilities, the asset side of bank’s balance sheets expands which reinforces the initial loan volume increase.

On the other hand, a gradual procyclical monetary policy induces a small rise in $m^\ast$. 

15
This is a strong saving disincentive for workers who want to become entrepreneurs and leads to a drop in the volume of deposits and equity. Such a drop is offset by a larger increase of the pool of depositors and a rise in the deposit interest rate. Because of these offsetting effects, the change in the volume and the composition of the banks’ balance sheets is relatively small.

The reason why the offsetting distributional effect is stronger when \( m^* \) rises is that the distribution of agents is skewed in the neighborhood of \( m^* \). Households have little reason to attain an asset level just below \( m^* \) because of a higher expected utility of an entrepreneur than of a worker. Thus, an increase in \( m^* \) has stronger consequences on loans than a decrease. This explains why \( m^* \) does not have to rise as much as it had to decline.

Finally, banks’ decisions to change \( m^* \) in an asymmetric way is just a reflection of the equilibrium nature of the problem. With a countercyclical monetary policy, in order for banks to give more loans, a rise in their equity funding is required (capital requirements bind). Yet the equity is more risky in bad states and households channel their savings away from equity and into deposits. Therefore, in order to expand their loans, banks must make the vision of entrepreneurship – a motivation for saving – more desirable. This is why \( m^* \) drops sharply. On the contrary, a procyclical monetary policy achieves a loan volume drop by an increase in \( m^* \). Such increase can be small because, for any amount of savings, risk-averse households prefer deposits in bad times anyway.

### 4.3 Countercyclical capital requirements

The interest rate is one of two instruments the central bank can use. The other is to modify the capital requirements, which in the benchmark economy are set at an 8% equity/loan ratio, as in the original Basle Accord. We now explore the policy of cyclical changes in capital requirements. In the first experiment (see Figure 6), equity/loan ratio is lowered to 7% in the worst aggregate state. This has two effects. First, it facilitates bank lending in the worst state. Second, households shift from equity to deposits because of the increased risk. The second effect is sufficiently large to counterbalance the first, resulting in a decrease of the loan volume. As in the case of a bond rate reduction, the average loan size decreases as the number of entrepreneurs barely changes compared with the benchmark economy.

The next experiment involves a gradual decline of the capital requirements during the bad shocks (see Figure 7). One would expect that the regulator allowing the banks to take more risks during a downturn may generate more loans. To the contrary, equity declines even more, resulting in a smaller loan mass under a binding regulatory constraint. Interestingly, loans are lower even when the regulator does not intervene and has in fact slightly more stringent capital requirements to maintain the same average as in the benchmark. The reasons are the same as previously: households shy away from banks when they take on more risk.
4.4 Procyclical capital requirements

If countercyclical capital requirements have adverse effects, maybe procyclical ones have a positive impact on lending ability. Figure 8 looks at the one-time policy, and Figure 9 at the gradual one. Both policies have positive effects: small local effects in the first case, and large global effects in the second case. Thus, tightening of the capital requirements is good for loan activity because it improves the financing of the banks. An increase in the equity-deposit spread facilitates the portfolio reallocation of households from deposits to equity. The lending rate increases in both the one-time and the gradual scenarios, to offset higher equity costs. In the gradual scenario, higher loan volume is facilitated by a drop in $m^*$. This brings in more entrepreneurs, both directly (those that were just below $m^*$ before its drop) and indirectly by increasing saving as entrepreneurship becomes more achievable to a wider spectrum of households (hence the increase in deposits as well as equity). These smaller new loans result in a lower average loan size under this policy.

Although this policy effectively mimics the Basel Accord II requirement of higher capital requirements in a downturn, its effects on the model economy contrast sharply with those described in the literature (see Section 1). In the literature, the negative effect of tightening requirements in a downturn is a result of a demand-side argument where higher capital raising costs further contract banks’ lending. However, such argument ignores the effect of more stringent capital requirements on the supply of bank capital. In our model, this latter effect dominates and stringent capital requirements improve bank funding. Well-capitalized banks are more attractive to savers. Furthermore, the loan volume increases as a result of lending to previously ineligible borrowers (those with net worth just below initial $m^*$), providing further incentive to save - a welcome news for banks in need of funding.

Note that we have no informational problem in the model economy that would actually require the imposition of capital requirements. One can easily imagine that if the model would include this feature, it would only reinforce the result: the presence of more entrepreneurial risk leads to a higher impact of asymmetric information and risk, thus furthering the need for regulation.

4.5 Credit crunch? What exactly happens in the model?

A negative aggregate shock lowers the expected project returns and increases their volatility. This affects the loan volume and the lending rate in four ways. First, both these effects decrease the expected value of risk-averse entrepreneurs ($E r V E$) while the value functions of non-entrepreneurial households do not see a first-order effect. There-fore the incentive to accumulate assets in order to be eligible for a loan declines. This lowers the quantity demanded for credit because fewer agents save enough to pass the $m^*$ cutoff. Second, the risk-neutral banks care only about the expected return of projects. The relative net payoff of bonds versus loans rises and induces a substitution from loans

\footnote{There is a second order effect coming from expectations to be an entrepreneur in the future.}
to bonds. The loan supply drops and the lending rate $r^L$ increases to compensate for higher loan losses. This is the credit supply effect (i.e., the “crunch”). Third, an increase in $r^L$ further discourages loan applicants because their net return on investment declines, and the equilibrium credit level drops further. Therefore the post-shock equilibrium exhibits a higher lending rate and a lower level of loans which further propagates the shock. Note that the decline in the market-clearing volume of credit is partly demand-driven and cannot be attributed to only the credit crunch behavior of the banks. Fourth, the household perceives more risk in the bank when entrepreneurial risk increases. It then shifts from equity to insured deposits, thus making it harder for banks to meet the capital requirements. They further reduce the supply of loans.

4.6 Does the equity market worsen or soften the credit decline?

The existence of an equity market can either amplify or reduce the impact of a negative shock on a volume of credit. Only the second and fourth of the above mentioned four effects is directly affected by the existence of an equity market. The equilibrium condition (15) shows that only changes in $r^L$ and $\epsilon$ affect credit behavior through the equity channel, and they do so in an offsetting manner. An increase in $\epsilon$ (higher loan losses) increases the return on equity $r^E$, while an increase in $r^L$ lowers it. We therefore distinguish two cases. (A) If $d(r^L - (1 + l_c)\epsilon) < 0$, then a rise in $r^E$ increases the cost of funds to the bank, which squeezes the profit margin further and leads to an additional substitution from loans to bonds ($L^S$ drops) as well as an increase in $r^L$. At the same time the portfolio return $r_{PORT}$ increases, making borrowing relatively less attractive (demand for credit drops). In this case, the presence of the equity market worsens the credit decline: a higher $r^E$ is compatible only with a lower amount of equity $E$ on the market, as households are risk averse while banks are risk neutral, which in turn requires an additional drop in loans due to a binding capital adequacy constraint (see equation (22)). Case (B) when $d(r^L - (1 + l_c)\epsilon) > 0$ has the opposite implication – it softens the effects of the financial accelerator. According to the simulations (comparing peak and trough states), $d(r^L - (1 + l_c)\epsilon) = 0.0002$ and we can conclude that the presence of the equity market softens the credit crunch.

5 Conclusion

We used a complex model to study the interaction of household saving decisions, project returns, Basel Accord type banking regulation, and credit activity. We find that the Basel Accord has a noticeable impact on loans when project returns decline through the cycle. Active monetary policy through interest rate reductions in bad times is able to increase the loan volume, but without removing its cyclical nature.

A relaxation of the Basel Accord capital requirements in bad times obtains negative results, as households shy away from the equity that banks need to make loans. As in models with informational problems, which do not explicitly exist in our model, a
tightening is in order. Therefore, this calls for regulatory policy to be active through the cycle, in the sense of the Basel Accord II, which forces banks to adopt a risk evaluation method à la Merton, as pointed out by Catarineu-Rabell, Jackson, and Tsomocos (2005).

Our results also emphasized that it is important to take into account the financing of banks. Given capital requirements, banks are limited in their lending by the bank equity that households are willing to hold. As this decision is influenced by interest rates, it gives rise to another channel of monetary policy. This channel has also been identified by Chami and Cosimano (2010) and van der Heuvel (2008). Unlike these papers, we do not require explicit asymmetric information, market power in the banking industry, or the increasing marginal cost of financing.

A Appendix: Solving the banks’ problem

Due to the inequality constraints, we have to use a Kuhn-Tucker approach and be careful about the corner solutions. The Lagrangean for this problem is

\[
\mathcal{L} = r^d L + r^b B - r^r D - r^e E - \delta \left( \frac{D}{E} \right)^\gamma D - (1 + l_c) \epsilon L + \lambda_1 (D + E - B - L) + \lambda_2 (E/L - \alpha) + \lambda_3 (D + E - L).
\]

Then the first-order conditions are

\[
\begin{align*}
    r^d - \lambda_1 - \lambda_2 E/L^2 - \lambda_3 - \epsilon (1 + l_c) &= 0 \\
    r^b - \lambda_1 &= 0 \\
    -r^d - \delta (\gamma - 1) \left( \frac{D}{E} \right)^\gamma + \lambda_1 + \lambda_3 &= 0 \\
    -r^e + \delta \gamma \left( \frac{D}{E} \right)^{\gamma+1} + \lambda_1 + \lambda_2/L + \lambda_3 &= 0.
\end{align*}
\]

As noted above, there are two possibilities: either constraint (9) or constraint (10) bind. In terms of the Lagrangean we therefore need to consider two cases. The one where \( \lambda_2 > 0 \) and \( \lambda_3 = 0 \) (i.e., (9) binds while (10) does not) will be referred to as an “interior solution” because not all loanable funds are invested into loans. The opposite case where \( \lambda_3 > 0 \) and \( \lambda_2 = 0 \) will be referred to as a “corner solution.” For simplicity, in what follows we assume \( \gamma = 1 \).

\textbf{Interior solution}

This is the case when bank holds just enough equity to satisfy the capital adequacy requirement \( (E/L = \alpha \) and therefore \( D + E > L \)). The above first-order conditions can be combined into

\[
\begin{align*}
    r^d &= r^b - 2\delta \frac{D}{E} \\
    \frac{M}{E} &= 1 + \left[ \frac{1}{\delta} (r^e - r^d) - \frac{1}{\alpha \delta} (r^d - r^d - (1 + l_c) \epsilon) \right]^{\frac{1}{2}}.
\end{align*}
\]

19
\[ L = \frac{1}{\alpha} E \]  
\[ D = M - E, \]  
where (21) is an equity (or implicitly deposit) supply equation. Conditional on particular values of M and all levels of prices, equations (20) to (23) form a recursive system which uniquely determines all quantities.

**Corner solution**

In a corner solution, bank holds more equity than required by the capital adequacy requirement \((D + E = L)\) and therefore \(E/L > \alpha\). Now, \(r^b > r^d\), and the above first-order conditions can be combined into

\[ \frac{M}{E} = 1 + \left[ \frac{r^e - r^l + (1 + l_c)\epsilon}{\delta} \right]^{\frac{1}{2}} \]  
\[ L = M \]  
\[ D = M - E \]  
\[ r^l - r^b - (1 + l_c)\epsilon = r^b - r^d. \]

where (24) is again an equity supply equation. Note that now loans and equity supply decisions are disconnected. Equation (27) shows a wedge between the bond and deposit rates. The bond “premium” on the right hand side equals the profit differential between net returns on loans and bonds that would equal zero in an interior solution.

**B Appendix: Portfolio optimization**

The Euler equation for equity is

\[ MU_t = \beta E_t[MU_{t+1}R^e_{t+1}] \]  
\[ MU_t = \beta E_t[MU_{t+1}]E_t[R^e_{t+1}] + \text{cov}_t(MU_{t+1}, R^e_{t+1}). \]

Because returns on equity are uncertain, the covariance between expected gross return and expected marginal utility at \(t + 1\) affects consumption decisions of households in equilibrium. The intuition is easy to see if we think of starting from certain returns on equity and then allow for uncertainty with \(\text{cov} > 0\) – returns on equity tend to be high when MU of consumption is high. Such change makes allocations under certainty suboptimal because \(MU_t < PV(E_tMU_{t+1})\). An inter-temporal re-allocation by shifting consumption from today to the future period increases \(MU_t\) and lowers \(MU_{t+1}\) which brings Euler equation into balance. So when comovement between \(MU\) and \(R^e\) is positive – an empirical regularity, see Duffee (2005) – households save more through equity than through deposits with the same return \((R^d\) is certain). A non-zero covariance justifies existence of two assets in equilibrium.

\[ ^{18}\text{A lower demand for bank’s financing by deposits (relative to equity) depresses their price.} \]
This implies differences in saving patterns across heterogeneous agents arising from differences in their expected income (and consumption) levels. On average, entrepreneurs earn most, workers less, and unemployed and retirees least of all. With increasing and concave utility $MU_{\text{worker}} < MU_{\text{retired}} < MU_{\text{unemployed}}$.\(^{19}\) If $\text{cov}(MU, R^e)$ is a constant wedge, the last Euler equation above implies that covariance between $R^e$ and $C$ will be least important for agents with highest average $MU$ (unemployed) and most important for workers because of their low $MU$. Then, workers will save most through the equity while unemployed will save least via equity, because the comovement of risky returns is relatively less important when $MU$ is high.

To implement this idea in the scope of our model, we introduce three mutual funds for each of the three types of depositors: workers, unemployed and retirees. Equation (6) show that choices of workers, unemployed and retirees of share of risky equity in their portfolio saving depends on their risk aversion parameter $\lambda$. Guided by the above discussion and standard micro theory we calibrate $\lambda$ as proportional to the distance $X_i$ between utility of mean consumption $U(C_i)$ and average of utilities $U(C_i - T)$ and $U(C_i + T)$ where $T$ is chosen to clear equity market ($E_W + E_U + E_R = E$). Specifically for workers (and similarly for unemployed and retired), $\lambda_W = \frac{M_W + M_U X_W X_U + M_R X_R}{M/\lambda}$ where $\lambda$ is the aggregate lambda implied by the overall market’s equity holdings (the homogeneous part of the model), $M_i$ is overall asset holding of agent type $i$.

### C Appendix: The solution procedure

Heterogeneous agents models with aggregate shocks are difficult to solve because the distribution of agents is not invariant and becomes a highly dimensional state variable. The two main strategies to solve this problem is to either find a good way to summarize the distribution with very few variables, as Krusell and Smith (1998) demonstrate, or to work with linearization, as Cooley and Quadrini (2006) do. Unfortunately, neither is possible here due to some highly non-linear phenomena that are crucial in our model. For example, decision rules change abruptly in the vicinity of $m^*$. Finally, second degree effects appear to be quite important, and they are likely to vanish with linearization.

Our strategy uses the realization that aggregate shocks in a two-state Markov process lead to transitional states somewhere between two steady-states corresponding to repeated identical shocks. We therefore choose a sufficient number of aggregate states to represent a large proportion of actual aggregate states.

The aggregate state space is assumed two-dimensional: one dimension is the current shock, high or low, the other is a counter of how far from the the high steady-state the economy is. Specifically, this counter is incremented by one each time a low shocks occurred in the previous period, or decreased by one if a high shock occurred. The minimum counter value is one, the maximum is chosen such that this state occurs

\(^{19}\)Retirees have a lot of free time.
infrequently. We choose a maximum of 5, implying with the transition probabilities of the Markov process that the economy will in any of the aggregate states $S_{sc}\%$ of the time, where

$$S = \begin{pmatrix} 50.2 & 16.7 & 5.6 & 1.9 & 0.6 \\ 16.7 & 5.6 & 1.9 & 0.6 & 0.2 \end{pmatrix}.$$

We then solve this model economy with the standard tools for heterogeneous agent economies, that is value function iterations followed by iterations on the invariant distribution (defined over the aggregate states as well). The equilibrium is reached by finding the set of lending rates $r_l$ and loan eligibility rules $m^*$ that balance all markets and satisfy all constraints in all aggregate states.

References


D Figures

Figure 1: Benchmark economy as it cycles through all possible aggregate states
Figure 2: Benchmark and policy with interest rate reduction in worst case only
Figure 3: Benchmark and policy with gradual interest rate reduction in bad return situations
Figure 4: Benchmark and policy with interest rate increase in worst case only

- Lending rate (%)
- Bond rate (%)
- \( m^* \)
- Total loans
- Total deposits
- \( r_e - r_d \) (%)
- Equity/loans (%)
- Equity/loans (%)
- Loans/assets (%)
- Portfolio return (%)
- Entrepreneurs (%)
- Average loan
Figure 5: Benchmark and policy with gradual interest rate increase as aggregate states worsen

- Lending rate (%)
- Bond rate (%)
- \( m^* \)
- Total loans
- Total deposits
- \( r^e - r^d \) (%)
- Equity/loans (%)
- Loans/assets (%)
- Portfolio return (%)
- Entrepreneurs (%)
- Average loan

*Values and graphs are approximate and illustrative.*
Figure 6: Benchmark and policy with relaxing of capital requirements in worst case only
Figure 7: Benchmark and policy with gradual relaxing of capital requirements as aggregate states worsen
Figure 8: Benchmark and policy with tightening of capital requirements in worst case only
Figure 9: Benchmark and policy with gradual tightening of capital requirements as aggregate states worsen