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Incentive-Feasible Deflation

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Incentive-Feasible Deflation*

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Abstract

For economies in which the real rate of return on money is too low, the standard prescription is to deflate prices according to the Friedman rule. Implicit in this recommendation is the availability of a lump-sum tax instrument. In this paper, I view lump-sum tax obligations as a form of debt subject to default. While individuals may agree to honor such obligations *ex ante*, a lack of commitment (the *sine qua non* of modern monetary theory) may prevent them from following through on their promises *ex post*. In such cases, there may exist an incentive-induced limit to deflationary policy. **Key Words:** *Friedman rule, deflation, lump-sum taxation, debt constraint.* **JEL Codes: E4, E5**

1 Introduction

Friedman (1969) famously argued that money should earn a real rate of return equal to the rate of time-preference. When money takes the form of zero-interest currency, the "Friedman rule" implies that the optimal monetary

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policy is contractionary. And indeed, a moderate deflation appears to be optimal in a standard class of monetary models; see, for example, Ljungqvist and Sargent (2000).

Both the desirability and feasibility of the Friedman rule have been questioned when money is used as a form of insurance against consumption risk. Bewley (1983) demonstrates that equilibria with rates of return even close to the Friedman rule may not be possible, as the taxes needed to finance the requisite deflation may exceed available resources.[1] Indeed, Levine (1991) argues that the optimal monetary policy is in any case likely to be inflationary. Intuitively, if the lump-sum taxes that are needed to finance a deflation cannot differentiate between lucky and unlucky individuals, then contractionary monetary policy exacerbates the penalty associated with bad luck. An inflation, on the other hand (generated via lump-sum transfers of cash) will tend to insure unlucky individuals.[2]

In this paper, I propose another reason for policy to depart from the Friedman rule. As in the papers previously cited, in my model, money is used as a form of insurance. In contrast to Bewley (1983), however, it is always resource feasible to finance the Friedman rule. Moreover, in contrast to Levine (1991), the Friedman rule is always a desirable policy. The limit placed on an optimal deflation here is instead driven by a participation constraint that is commonly used for credit economies. Specifically, I assume that individuals cannot commit to repay debt, and that the penalty associated with default is limited. Because a lump-sum tax can be viewed as a debt owed to society, the prospect of individuals reneging on this debt places an upper bound on the resources that can be extracted to finance the requisite deflation.

For my formal analysis, I adopt the Lagos and Wright (2005) framework (absent their search friction). The key property of this setup is that individuals always have an opportunity to recover from a spell of bad luck. In particular, individuals with depleted money balances are granted the opportunity to replenish them (equivalently, debtors are always given an opportunity to discharge their debt). The setup also assumes quasilinear preferences and is therefore highly tractable. Together, these two properties of the environment circumvent the forces responsible for the conclusions reached in Bewley

[1] Taub (1988) elaborates on the results presented in Bewley (1983).

[2] See also Molico (2006), Bhattcharya, *et. al.* (2008), and the references cited therein.

(1983) and Levine (1991). As a consequence, a competitive monetary equilibrium exists at the Friedman rule and it is optimal policy–at least, assuming that the government possesses a lump-sum tax instrument. Of course, it is precisely this last proviso that I wish to relax.

I adopt the following strategy. First, I describe the environment and characterize the efficient allocation. Next, I assume a lack of commitment and characterize the set of incentive-feasible credit-based allocations. As usual, whether the efficient allocation is incentive-feasible depends on parameters. Finally, I dispense with record-keeping so that monetary trade is necessary. Here, I restrict attention to (stationary) competitive monetary equilibria. While the efficient allocation is implementable under the Friedman rule, I show that it may not be incentive-feasible. In impatient economies, individuals may want to avoid participating altogether rather than bearing the finance burden of deflation. Nevertheless, a deflationary policy remains desirable and possible, even if it restricted to operate away from the Friedman rule.

2 The environment

The economy is populated by a continuum of *ex ante* identical individuals distributed uniformly on the unit interval. Each period $t = 0, 1, 2, ..., \infty$ is divided into two subperiods labeled *day* and *night*. individuals meet at a central location in both subperiods; in particular, I abstract from search frictions–since they play no critical role in the point I wish to make.

All individuals have common preferences and abilities during the day. Let $x_t(i) \in \mathbb{R}$ denote the consumption (production, if negative) of output in the day by individual i at date t. The key simplifying assumption is that preferences are linear in this term. The possibility of exchange then implies transferable utility. Output produced in the day is nonstorable, so an aggregate resource constraint implies

$$\int x_t(i)di \leq 0 \tag{1}$$

for all $t \geq 0$.

At night, individuals realize a shock that determines their type for the night. In particular, individuals have either a desire to consume, an ability to produce, or neither. In what follows, I refer to these types as *consumers*, *producers*, and *nonparticipants*, respectively. Types are determined randomly by an exogenous stochastic process. This process is *i.i.d.* across individuals and time; there is no aggregate uncertainty. Let $\pi \in (0, 1/2]$ denote the measure of individuals who become either consumers or producers; thus $(1 - 2\pi)$ denotes the measure of nonparticipants.[3]

A consumer has utility $u(c)$ and a producer has utility $-g(y)$; where $c \in \mathbb{R}_+$ and $y \in \mathbb{R}_+$ denote consumption and production of the night good, respectively. Assume that $u'' < 0 < u'$, $u(0) = 0$ and $g', g'' \geq 0$ with $g(0) = g'(0) = 0$. Nonparticipants do not wish to consume and have no ability to produce output; their utility is normalized to zero. Since the night good is also nonstorable, there is another aggregate resource constraint

$$\int c_t(i)di \leq \int y_t(i)di \tag{2}$$

for all $t \geq 0$.

Individuals are *ex ante* identical and their preferences are represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ x_t(i) + \pi \left[u(c_t(i)) - g(y_t(i)) \right] \right\} \tag{3}$$

where $0 < \beta < 1$. Note that there is no discounting across subperiods.

Weighting all individuals equally, a planner maximizes (3) subject to the resource constraints (1) and (2). Because utility is linear in x, feasibility implies that individuals are indifferent across any lottery over $\{x_t(i) : t \geq 0\}$ that delivers $E_0 x_t(i) = 0$. Anticipating what is to follow later, one such lottery takes the form

$$x_t(i) = \begin{cases} +x & \text{w.p.} \quad \pi \\ 0 & \text{w.p.} \quad 1 - 2\pi \\ -x & \text{w.p.} \quad \pi \end{cases} \tag{4}$$

for any $x \geq 0$.

Consider next how output is allocated at night. If g is strictly convex, all producers will be required to produce a common level of output $y \geq 0$.[4]

[3] At the individual level, these measures represent probabilities.

[4] If g is linear, then y can be interpreted as an expected level of output.

Given the strict concavity of u, all consumers will be allocated a common level of consumption $c \geq 0$. Given that the active population is divided equally among producers and consumers at night, the resource constraint (2) implies $c = y$. Hence, conditional on a given level of y (and invoking the fact that $Ex_t(i) = 0$), *ex ante* welfare is represented by

$$W(y) = \left(\frac{\pi}{1-\beta}\right)[u(y) - g(y)] \tag{5}$$

Clearly, $W(0) = W(\overline{y}) = 0$ for some unique $0 < \overline{y} < \infty$. Moreover, there exists a unique maximizer $y^* \in (0, \overline{y})$ characterized by:

$$u'(y^*) = g'(y^*) \tag{6}$$

I refer to (x^*, y^*) as the *efficient allocation*, where x^* should be understood to satisfy a lottery in the form of equation (4).

As far as a social planner is concerned, the day subperiod is irrelevant, so attention can be restricted to the efficient allocation $(0, y^*)$. The pattern of trades that supports this allocation entails some form of social insurance. Specifically, individuals face the risk of wanting consumption with no means of producing it. The solution entails having those individuals with a contemporaneous ability to produce to satisfy those members of society with a contemporaneous want. Alternatively, the planner's solution may also be interpreted as a type of social credit system wherein individuals borrow resources from society when they have a desire to consume and promise to discharge their debt to society when they have an ability to produce.

3 Incentive-feasible allocations

Assume now that all exchange must be voluntary and that individuals lack commitment. Voluntary exchange means that allocations are recommended rather than imposed. A lack of commitment means that recommended allocations must be sequentially rational. Individuals are free not to accept a recommended allocation, but noncompliant behavior is punished with social ostracism.

Consider the recommended feasible allocation (x, y). The producer is asked to deliver y units of output at night, in exchange for the reward x

promised the next day. This allocation is sequentially rational for the producer if and only if,

$$-g(y) + \beta\,[x + W(y)] \geq 0 \tag{7}$$

That is, $-g(y)$ is the utility cost of producing output at night, the reward of which consists of the discounted future payoff x, plus the continuation value $W(y)$ associated with remaining in the game. Because $g(0) = 0$ and $W(0) = 0$, the reward for noncompliance is zero.

Those individuals who consume y at night are obliged to deliver x units of output the next day. It is sequentially rational to honor this obligation if and only if

$$-x + W(y) \geq 0 \tag{8}$$

Notice that from an *ex ante* perspective, individual are face a lottery over daytime production/consumption x that is equivalent in form to equation (4).

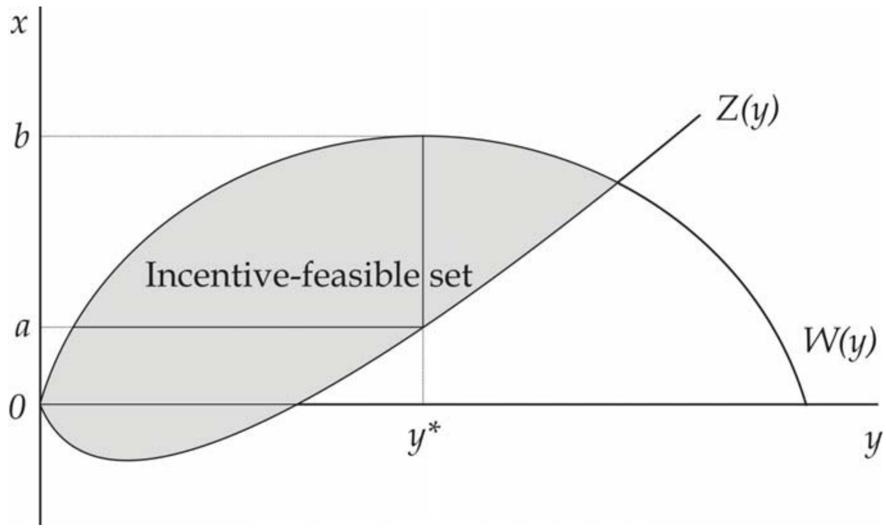
An *incentive-feasible allocation* is defined as a feasible allocation that satisfies the sequential rationality constraints (7) and (8). Let $\mathcal{F}$ denote the set of incentive-feasible allocations; that is,

$$\mathcal{F} \equiv \left\{(x, y) \in \mathbb{R} \times \mathbb{R}_+ : W(y) \geq x \geq \beta^{-1}g(y) - W(y)\right\} \tag{9}$$

Given the properties of u and g, the set $\mathcal{F}$ is clearly non-empty, convex, and compact. It follows as a corollary that the problem of choosing $(x, y) \in \mathcal{F}$ to maximize $W(y)$ is well defined. Moreover, since $W(y)$ is strictly concave, there is a unique solution $y > 0$ to this problem. Associated with this solution is an x (not necessarily unique) that satisfyies (9). The exact nature of the solution depends on parameters–and in particular, the discount factor β.

A diagrammatic characterization of the solution proves helpful in what is to follow. To begin, note that $x = W(y)$ defines a locus of points (x, y) that leave the (historical) consumer just indifferent between participating or not during the day. The properties of $W(y)$ were previously described in the discussion following equation (5). Similarly, note that $x = \beta^{-1}g(y) - W(y) \equiv Z(y)$ defines a locus of points (x, y) that leave the producer indifferent between participating or not during the night. It is easy to verify that $Z(y)$ possesses the following properties: $Z(0) = 0$, $Z'(p) = 0$ for some $0 < p < y^*$, $Z'(y) \lesseqgtr 0$ for $y \lesseqgtr y^*$, and $Z''(y) < 0$. Both $W(y)$ and $Z(y)$ are depicted in Figure 1 for a case in which the efficient allocation is incentive-feasible for any $\underline{x} \leq x^* \leq \overline{x}$.

FIGURE 1
Incentive-Feasible Allocations



I now establish that the efficient allocation does not belong to incentive-feasible set sufficiently impatient economies. First, note that while $W(y)$ is increasing in β at any given $y > 0$, the efficient level of production y^* itself remains invariant to β. By inspection, $Z(y)$ is decreasing in β at any given $y > 0$. Next, consider the intersection of these two curves at some point $y > y^*$ (see Figure 1). As β decreases, the W curve shifts "down" and the Z curve shifts "up" (both curves continue to satisfy $W(0) = Z(0) = 0$) until at $\beta = \beta^*$ we have $W(y^*) = Z(y^*)$, or

$$\left(\frac{\pi}{1-\beta^*}\right)[u(y^*) - g(y^*)] = \left(\frac{1}{2\beta^*}\right) g(y^*). \tag{10}$$

That is, for $\beta = \beta^*$, the sequential rationality constraints for the consumer and producer (respectively) bind at the efficient allocation–with $x^* = W(y^*) = g(y^*)/(2\beta^*)$. Since the left-hand-side (right-hand-side) of equation (10) is increasing (decreasing) in β, it follows that the efficient allocation continues to be incentive-feasible for all $\beta \in [\beta^*, 1)$.

Result 1 *If* $\beta \in [\beta^*, 1)$, *then the allocation* (x^*, y^*) *with* $a \leq x^* \leq b$ *is incentive-feasible, where* $a \equiv Z(y^*)$ and $b \equiv W(y^*)$.

Result 1 is stated as a sufficient condition, but it is clearly necessary as well. It follows as a corollary that when individuals are sufficiently impatient, the efficient allocation is not incentive-feasible. The constrained-efficient allocation is characterized in the following result and is depicted in Figure 2.

Result 2 *If* $\beta \in (0, \beta^*)$*, then the constrained-efficient allocation* (x_0, y_0) *is characterized by* $0 < y_0 < y^*$ satisfying $W(y_0) = Z(y_0) = x_0$.

4 Competitive monetary equilibrium

If allocations cannot be conditioned on individual trading histories, then it will not be possible to support recommended allocations with credit. It may nevertheless be possible to support some trade with exchanges involving fiat money. To this end, assume that the government can produce durable, divisible, and noncounterfeitable tokens. In what follows, I develop the standard treatment of monetary exchange with competitive markets.[5]

4.1 Money supply

Let M^- denote the total stock of fiat money at the beginning of the day-market (prior to any injection or withdrawal). Assume that this stock expands at the gross rate μ so that $M = \mu M^-$, where M denotes the "next" period's money supply. Assume that the initial money stock is distributed evenly across the population. New money $(\mu - 1)M^-$ is injected (or withdrawn) by way of a lump-sum transfer (or tax) τ; the government budget constraint is denoted by

$$\tau = (\mu - 1)M^-$$

Assume that this transfer (or tax) is distributed (collected) in each day-market. Finally, assume that $\mu \geq \beta$ (it can be shown the equilibria do not exist for $\mu < \beta$).

Individuals trade on a sequence of competitive spot markets (with money being exchanged for goods in both the day and night markets). Let (v_d, v_n)

[5] The model in this section is a variant of the competitive market model presented in Rocheteau and Wright (2005).

denote the price of money in the day and night, respectively (measured in units of day and night output, respectively).

4.2 The day market

Let a denote an individual's nominal money balances at the beginning of the day market (exclusive of any transfer) and $m \geq 0$ denote the money this person takes into the night market. In the day market, all individuals are able to buy or sell output x; this gives rise to a day-market budget constraint:

$$x = v_d (a + \tau - m) . \tag{11a}$$

Let $D(a)$ denote the utility value of beginning the day with a units of money and $N(m)$ denote the utility value of beginning the night with m units of money. Note that $N(m)$ denotes the value before knowing whether an individual will have a desire to consume or opportunity to produce in the night-market. The value functions D and N must satisfy the recursive relationship:

$$D(a) \equiv \max_{m \geq 0} \left\{ v_d (a + \tau - m) + N(m) \right\} .$$

Assuming for the moment that $N'' < 0 < N'$ (a property that can be shown to hold for all $\mu > \beta$), the demand for money in the day market is characterized by:

$$v_d = N'(m). \tag{12}$$

As originally highlighted by Lagos and Wright (2005), money demand at this stage is conveniently independent of beginning-of-period money balances a. Furthermore, we see that

$$D'(a) = v_d, \tag{13}$$

so that D is linear in a.

4.3 The night market

Consider an individual who brings m units of money into the night market. The individual subsequently realizes whether he is a producer, a consumer, or a nonparticipant. Let $P(m)$ and $C(m)$ denote the utility value associated with being a producer and consumer, respectively–and let $I(m)$ denote the value of being a nonpariticipant. Then the *ex ante* value of entering the night market satisfies

$$N(m) \equiv \pi C(m) + \pi P(m) + (1 - 2\pi) I(m).$$

4.3.1 Consumers

In the night market, a consumer holding m units of money faces the following budget constraint:

$$a^{+} = m - v_n^{-1} y,$$

where a^{+} denotes money balances carried forward into the next period's day market and y denotes purchases of output at night. Note that the environment prevents the existence of private debt, so $a^{+} \geq 0$. As demonstrated by Rocheteau and Wright (2005), this constraint binds tightly for any inflation rate away from the Friedman rule and just binds (becomes slack) at the Friedman rule. Invoking the latter result, the solution to the consumer's choice problem is simply $y = v_n m$, which yields the value function $C(m) \equiv u(v_n m) + \beta D(0)$, with the result

$$C'(m) \equiv v_n u'(y). \tag{14}$$

4.3.2 Producers

In the night-market, a producer holding m units of money faces the following budget constraint:

$$a^{+} = m + v_n^{-1} y. \tag{15}$$

Note that the constraint $a^{+} \geq 0$ here does not bind as a producer will want to accumulate money balances. Hence, the producer's choice problem is given by

$$P(m) \equiv \max_{y} \left\{ -g(y) + \beta D\left(m + v_n^{-1} y\right) \right\}.$$

The supply of output at night is characterized by

$$v_n g'(y) = \beta v_d^{+}, \tag{16}$$

where the latter expression uses equation (13). Note that producers are willing to produce even in the absence of any explicit future reward promised to them (i.e., the x embedded in the sequential rationality constraint (7). Instead, the future reward for their current sacrifice is embedded in the belief that the money they accumulate today will have purchasing power in the future day market–that is, $v_d^+ > 0$. By the envelope theorem,

$$P'(m) = \beta v_d^+ \; [\; = v_n g'(y) \;]. \tag{17}$$

A nonparticipant faces the trivial choice of simply carrying his money balances forward to the next day. Consequently, we have $I(m) \equiv \beta D(m)$, with

$$I'(m) = \beta v_d^+. \tag{18}$$

4.4 Equilibrium

Combining (14), (17), and (18), we have $N'(m) = v_n \left[\pi u'(y) + (1-\pi) g'(y)\right]$, which when combined with (12), results in

$$v_d = v_n \left[\pi u'(y) + (1-\pi) g'(y)\right].$$

Updating the latter expression by one period and combining with equation (16) yields

$$v_n g'(y) = \beta v_n^+ \left[\pi u'(y^+) + (1-\pi) g'(y^+)\right].$$

Recall that for the consumer, $y = v_n m$. Market clearing requires $m = M$, which implies $v_n = y/M$. Substituting this into the expression above yields

$$g'(y) = \left(\frac{\beta}{\mu}\right)\left(\frac{y^+}{y}\right)\left[\pi u'(y^+) + (1-\pi) g'(y^+)\right].$$

In what follows, I restrict attention to the non-degenerate steady-state ($y = y^+ > 0$), so the expression above simplifies to

$$u'(y^e) = \pi^{-1}\left[\left(\frac{\mu}{\beta}\right) - 1 + \pi\right] g'(y^e). \tag{19}$$

Condition (19) characterizes the equilibrium level of output y^e as a function of parameters. In particular, note that $y^e < y^*$ for any $\mu > \beta$, and that $y^e \nearrow y^*$ as $\mu \searrow \beta$. In other words, we have the following result:

Result 3 *The competitive monetary equilibrium corresponds to the efficient allocation at the Friedman rule ($\mu = \beta$).*

At first glance, Result 3 may seem remarkable in the sense that competitive equilibria appear to dominate constrained optima. That is, Results 1 and 2 claim that the efficient allocation in credit economies is incentive-feasible only if individuals are sufficiently patient. Result 3, on the other hand, appears to hold for all rates of time-preference. This discrepancy is explained by the fact that the monetary economy above assumes a lump-sum tax instrument. What happens if people must be induced to fulfill their tax obligations in the same manner they must be induced to fulfill any debt obligation?[6]

4.5 Incentive-feasible deflation

It is not *a priori* obvious why individuals can commit (be forced) to pay some debts (e.g., public) and not others (e.g., private). Of course, it is possible that the *penalty* for reneging on an obligation differs across creditors. But as long as there are limits to how severely people can be punished for their sins, sequential rationality is a relevant constraint.

In what follows, I assume that the penalty for failing to pay taxes in the day market is exclusion from the market at night. For this to be possible, some record of tax payment is necessary. Observed money holdings are insufficient for this purpose because people are free to acquire any money balance they wish in the day market. I assume that the government can produce a distinct indivisible token that is issued in the day and evaporates at the end of the night. This token is designed to serve as a tax receipt. It's economic function is to serve as a license for the night market. Individuals not in good standing in terms of their tax obligations are precluded from conducting business in the night market.[7]

Let (x^e, y^e) denote the allocation associated with a competitive monetary equilibrium. Conditional on a $\mu > \beta$, the quantity y^e is determined by

[6] Just to be clear, I view linear taxes, like a flat income tax, to be a voluntary tax in the sense that one can avoid paying such a tax by producing zero income.

[7] Alternatively, and equivalently, I could assume that individuals are free to skip the day market (in doing so, they avoid the tax but also the opportunity to replenish money balances). This latter interpretation is adopted by Hu, Kennan, and Wallace (2009).

equation (19). I now solve for the equilibrium quantity x^e. Recall the budget constraint (11a), $x = v_d(a + \tau - m)$. Recall too that $\tau = (\mu - 1)M^-$ and that $m = M$. From the producer's budget constraint (15), $a = y/v_n^- + M^-$. From the consumer's problem recall that $y = v_n^- M^-$. Together, these latter two conditions imply $a = 2M^-$ (consumers sell all of their money to producers), from which we can derive $x = v_d M^-$. Next, combine the market-clearing condition $v_n M = y$ with condition (16) to derive $v_d M^- = \beta^{-1} y g'(y)$. As a result, we have

$$x^e = \beta^{-1} y^e g'(y^e). \tag{20}$$

Condition (20) defines a locus of allocations (x^e, y^e) one of which constitutes the (non-degenerate) competitive monetary equilibrium. Precisely which of these allocations constitutes the equilibrium is determined by condition (19) which, in turn, depends on the monetary policy parameter μ.

An incentive-feasible monetary allocation must satisfy (9), or

$$\left(\frac{\pi}{1-\beta}\right)[u(y^e) - g(y^e)] \geq \left(\frac{1}{\beta}\right) y^e g'(y^e) \geq \left(\frac{1}{\beta}\right) g(y^e) - \left(\frac{\pi}{1-\beta}\right)[u(y^e) - g(y^e)].$$

Notice that the second inequality above holds trivially because $W(y^e) > 0$ and because $yg'(y) \geq g(y)$ for any convex function. Consequently, a competitive monetary equilibrium is always sequentially rational for the producer at night. The relevant constraint pertains to those individuals who enter the day market with depleted money balances (those who consumed in the previous night market).

The first thing I want to determine are the conditions for which the efficient allocation is incentive-feasible. Evidently, there is a unique $\beta = \beta^{**}$ satisfying

$$\left(\frac{\pi}{1-\beta^{**}}\right)[u(y^*) - g(y^*)] \equiv \left(\frac{1}{\beta^{**}}\right) y^* g'(y^*). \tag{21}$$

The following result is immediately apparent.

Result 4 *The competitive monetary equilibrium with $\mu = \beta$ is incentive-feasible for all $\beta \in [\beta^{**}, 1)$.*

It is also of some interest to compare β^{**} with β^* (i.e., see equation (10)). In particular, $0 < \beta^* < \beta^{**} < 1$. In words, the efficient allocation is more easily implementable for the credit economies studied earlier relative to the monetary economy examined here. Notably, the distinction in this case has nothing to do with money versus credit. Instead, the result in this case is entirely the consequence of allowing for a nonlinear mechanism in the credit economy while restricting attention to a linear mechanism for the monetary economy.[8]

It follows as a corollary to Result 4 that the Friedman rule is not incentive-feasible for impatient economies. Figure 2 depicts a case in which the Friedman rule allocation (x^*, y^*) is not incentive-feasible. That is, $E(y^*) \equiv \beta^{-1} y^* g'(y^*) > W(y^*)$. For $\beta \in (0, \beta^{**})$, the best incentive-feasible monetary equilibrium is the one that achieves the highest level of night-output y_1 without violating sequential rationality for the consumer during the day; that is,

$$\left(\frac{\pi}{1-\beta}\right)[u(y_1) - g(y_1)] = \left(\frac{1}{\beta}\right) y_1 g'(y_1) = x_1. \tag{22}$$

Recall from Result 2 that $W(y_0) = Z(y_0)$ determines $y_0 < y^*$ for $\beta < \beta^*$ in the credit economy. Since $\beta^{-1} y g'(y) > Z(y)$, condition (22) implies that $y_1 < y_0$ over the same range of discount factors. Again, this reflects the fact that the linear mechanism is more restrictive than the nonlinear mechanism studied earlier; see Figure 2.

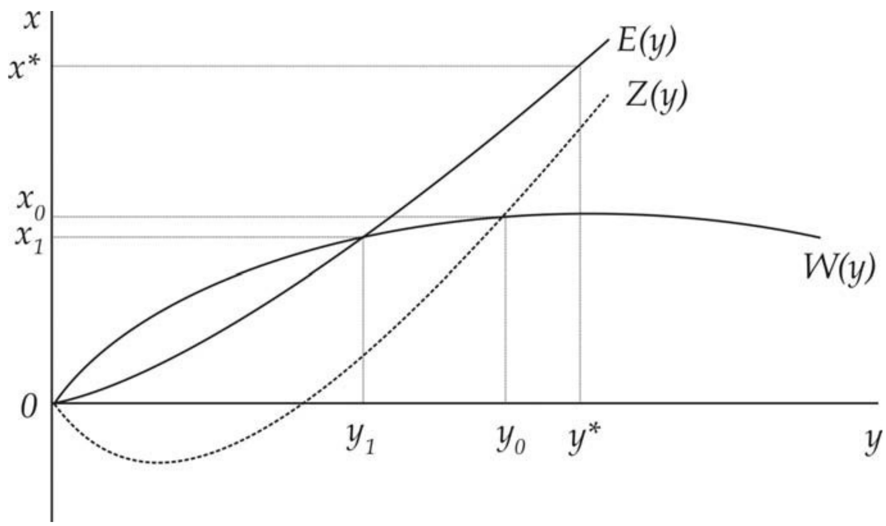
With y_1 so determined, condition (19) can be used to determine the inflation (deflation) rate μ_1 consistent with y_1; that is,

$$\mu_1 = \beta \left[\pi A(y_1) + 1 - \pi\right],$$

where $A(y) \equiv u'(y)/g'(y)$, $A'(y) < 0$ and $A(y^*) = 1$. Since $y_1 < y^*$, it follows that $A(y_1) > 1$ so that $\mu_1 > \beta$ (policy is restricted to operate away from the Friedman rule).

[8] In retrospect, I could have restricted attention to linear (competitive) credit economies with the limited commitment friction. In this case, a distortionary subsidy for night production financed by a daytime lump-sum tax can restore the economy to efficiency. I could have then studied the limits to implementing this policy when individuals can avoid paying lump-sum taxes subject to some penalty.

FIGURE 2
Constrained-Efficient Monetary Equilibrium



The only question remaining is whether I can establish that $\mu_1 \leq 1$. In other words, can I rule out the possibility that the only incentive-feasible monetary policy requires a strictly positive inflation rate? One can in fact show that a strictly positive inflation cannot be part of a constrained optimum in this setting.

Consider, for example, a policy of zero intervention ($\mu = 1$). In this case, there are no taxes or transfers and all trade is necessarily voluntary. Because all trade is voluntary, the consumer's sequential rationality constraint must be satisfied. Increasing the money growth rate from this point implies granting individuals positive transfers of cash in the day market (instead of encumbering them with taxes). As individuals have no incentive to avoid transfers, an inflationary policy remains incentive-feasible. However, the strictly positive inflation rate inefficiently reduces the level of output at night (see equation (19)). Consequently, zero intervention dominates an inflationary policy. And in general, as the analysis above shows, some welfare-improving deflation is incentive-feasible.

5 Conclusion

For economies in which the real rate of return on money is too low, the standard prescription is to deflate prices according to the Friedman rule. Implicit in this policy recommendation is the existence of a lump-sum tax instrument. The availability of this instrument seems inconsistent with the foundations of monetary theory. The prospect of lump-sum taxes should, in my view, be treated as a debt obligation. In the model above, it is an obligation to which all individuals would agree *ex ante*, but one on which individuals may want to renege *ex post*. When this is the case, there is the possibility of an incentive-based limit to deflationary policy.

It is of some interest to ask whether there exist other mechanisms that may circumvent the limits to deflation described above. I can propose at least two.[9]

The first is based on the idea in Kocherlakota (2003), who shows that the introduction of an illiquid bond can improve social welfare. I have previously demonstrated (Andolfatto, 2011) that there is a monetary policy (a strictly positive inflation rate and a money-to-bond ratio) that implements the efficient allocation as a competitive monetary equilibrium in a Lagos-Wright model similar to the one described above. In that equilibrium, all individuals leave the day market with identical money/bond holdings but then trade assets at night before visiting the product market. In the context of the model above, consumers sell their bonds (to producers) for money. Because bonds cannot (by an assumed trading restriction) be used to purchase goods, they trade at a discount. The implied positive interest rate induces a more efficient level of production.

One drawback of the Kocherlakota (2003) mechanism is that it does not explain how the trading restriction (a *de facto* cash-in-advance constraint) is to be enforced. In previous work (Andolfatto, 2010), I bypass this issue by proposing a nonlinear mechanism that effectively pays interest on "large" money balances. Of course, this solution works only to the extent that coali-

[9] Hu, Kennan and Wallace (2009) study a version of the Lagos-Wright model where producers and consumers meet pairwise at night. In contrast, the agents in my model meet in a centralized location. The restriction to bilateral bargaining (agents cannot communicate with others outside the match) allows these authors to consider bargaining protocols that implement the efficient allocation even under zero intervention.

tion formation is somehow prevented.[10] If attention is restricted to linear mechanisms with no trading restrictions, then the best incentive-feasible allocation corresponds to what I have derived herein.

[10] Arguably, the ability of coalitions to exploit a nonlinear price system depends on the ability to commit to promises, which is assumed to be absent.

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