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Abstract

What is the optimal policy response to a negative sectoral shock? How do frictions in goods and labor markets affect the nature and speed of the process of reallocating resources across alternative uses? Should government controlled inputs be allocated to compensate for frictions faced by the private sector or, rather, should they be deployed to complement private sector decisions? In this paper we make a first attempt to understand what features of an economy determine the answers to the previous questions. We study a model in which the drop in the private demand for structures frees up resources that can be used to produce government capital. For a reasonable calibration, we find that government spending increases in response to the drop in private demand, but that the size of the increase is inversely related to the level of frictions: the larger the costs that the economy faces to reallocate resources (capital and labor) across sectors, the smaller the optimal level of government spending.

Keywords: Optimal Infraestructure Spending, Sectoral Demand Shocks, Reallocation frictions.

JEL Codes: E2, E3, H54

1 Introduction

What is the optimal policy response to a negative sectoral shock? How do frictions in goods and labor markets affect the nature and speed of the process of reallocating resources across alternative uses? Should government controlled inputs be allocated to compensate for frictions faced by the private sector or, rather, should they be deployed to complement private sector decisions? In this paper we make a first attempt to understand what features of an economy determine the answers to the previous questions. We study a model in which a drop in the demand for housing lowers the demand for "structures" that can also be used to produce a non-residential investment as well as government provided capital (e.g. roads, ports). In addition to the basic no frictions case we consider two types of frictions: irreversible capital —essentially a non-negativity constraint on each type of capital— and labor reallocation costs — basically training requirements before workers can be employed in another sector— and we study the optimal response of government spending in infrastructure to a shock that lowers the demand for housing.

In this setting a simple intuition is that, in the presence of reallocation costs, a drop in the demand for housing lowers the opportunity cost of investing in (productive) government capital and, hence, that the optimal response to a negative housing shock is to increase government investment in infrastructure. Moreover, the magnitude of the response should be larger the more severe the reallocation costs as the opportunity cost is lower. This argument, however, fails to take into account general equilibrium effects. In particular, it fails to acknowledge that the productivity of public capital in the production of non-housing consumption —the "other" sector that we model— depends on what other, privately chosen, inputs are allocated to that sector. In particular if it is costly to reallocate workers to the consumption sector it may be the case that additional public capital also has a relatively low return and, hence, that frictions actually call for a smaller optimal response. The two forces that we have described, which we might describe as compensation and coordination, must be present in any reasonable specification of an economy subject to sectoral shocks and with productive government spending, and our model makes a first attempt at identifying the key frictions and at quantifying their importance.

To understand the mechanism through which frictions affect optimal government policy, we construct a three sector growth model with two final goods, consumption and housing, and a sector producing structures which can be used either for private residential and non-residential construction or government infrastructure projects. As a natural first step we study the optimal allocation in this economy that would be attained if the government had access to lump sum taxes and all necessary Ramsey taxes. In order to highlight the role played by frictions we study a sequence of economies indexed by the difficulty of reallocating resources. In the reference economy —which we label "Reversible"— both labor and capital can be **costlessly** and **instantaneously** moved across sectors. The second economy —denoted by the "Irreversible" label— adds a **non-negativity constraint** on sector specific investment, that is, we view capital as sector specific and, hence, investment decisions as irreversible. Finally, the third economy that we study —which we call "Costly Labor"— displays the most frictions: capital is sector specific and irreversible and it is **costly to reallocate workers** across sectors.

We calibrate the model so that it matches some moments of the U.S. economy and then subject the model to a demand shock that, in equilibrium, results in a (short run) decrease in the price of existing houses of 15%. We find that, across all three specifications, such a shock results in a small drop in output. At the same time, the behavior of other aggregates, e.g. sectoral investment and employment, depends in an essential way on the details of the frictions in the economy. In general, economies in which it is more costly to reallocate resources display smaller fluctuations and longer transitions. Interestingly, we find that the optimal government response is not a simple function of macro aggregates like GDP and private investment: In our experiments the larger the change in output the smaller the optimal government response.

In our calibrated economies we find that the general equilibrium effect is critical for understanding optimal government investment policy in infrastructure. In all cases an efficient policy requires a balanced mix between government provided and privately accumulated inputs. Thus, in economies with few frictions, it is efficient to rapidly reallocate private inputs and, hence, government investment is highly responsive and is characterized by rapid convergence to the new steady state level. In economies with frictions, optimal policies instruct the government not to "lean against the wind." Rather, an efficient allocation requires government inputs to "coordinate" with the private sector so as not to create imbalances in the input mix.

Several studies have explored the implications of sectoral shocks for macroeconomic fluctuations. A paper closely related in spirit to ours is Phelan and Trejos (2000). These authors consider a model with costly labor reallocation frictions given by search and matching, and show that a one-time permanent change in technologies or preferences can have a large impact on key macro aggregates. Their analysis, however, abstracts from capital and investment, and does not consider the role of the government in response to sectoral shocks. It turns out that these features suffice to limit the impact that costly labor reallocation has on dynamics. In our model, capital reallocations —even though limited by non-negativity constraints— in some sense compensate for frictions in reallocating workers and, hence, we find that the marginal contribution of imposing limited labor mobility is minimal. Put it differently:

the key friction is the impossibility of reallocate capital across uses (e.g. structures cannot be turned into machines), while training requirements add very little —both qualitatively and quantitatively— to the dynamics of the equilibrium response.

Our model is also related to the literature on the propagation of sectoral shocks. For example, Horvath (2000) considers a multisector real business cycle model and calibrates it to the US input output matrix using 2 digit-SIC data. His analysis shows that sectoral shocks can be important driving forces behind aggregate fluctuations. Unlike Horvath's work, however, our focus is on the optimal response of government spending and the nature of the dynamic response of temporary and permanent demand shocks

Our paper is also related to the literature on housing. Unlike most work in that area we abstract from frictions emerging from incomplete markets and collateralized borrowing since our emphasis is on understanding the optimal choice of government investment as a response to a shock, and not to market imperfections[1]. Chambers, Garriga and Schlagenhauf (2009), and Favilikus, Ludvigson and Van Nieuwerburgh (2010) build general equilibrium models where a relaxation of borrowing limits and an increase in foreign ownership of U.S. treasuries can partially account for recent expansions in housing prices and ownership rates. Essentially, they study models in which the shock that accounts for the drop in housing prices is driven by changes in market variables while, given our strategy, we have to rely in a preference shock. We believe that more work is needed —in the form of description and analysis of the dynamics— before we can reasonably feel confident that we understand the ultimate source of the shock.

[1] We do not mean to imply that market imperfections are irrelevant for the design of optimal policies, rather we believe that a first step in understanding efficient government investment requires understanding its behavior in a simple economy without market imperfections.

The reminder of this paper is structured as follows. Section 2 describes the model economy. In Section 3 we characterize optimal allocations and we discuss some issues related to the decentralization of the planner's allocation. Section 4 presents our calibration and our benchmark quantitative experiments. Section 5 concludes.

2 The Model

There is a continuum of households who derive utility from consumption, c_t, and housing services, h_t. Preferences of the representative household over sequences of consumption and housing services can be represented by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t). \tag{1}$$

u denotes the instantaneous utility function, which satisfies standard properties; the intertemporal discount factor is $0 < \beta < 1$.

The economy has three sectors. The first produces consumption and equipment and requires private capital (equipment, k_t^0, and non-residential structures, k_t^s), publicly provided (rival) capital, k_t^g, and labor, n_t^0 as inputs. There is also a construction sector which produces investment goods, m_t^g, m_t^s, m_t^h, that can be used to add to the stock of public capital, non-residential structures, or residential structures, respectively. Production of these investment goods requires equipment, k_t^1, and labor, n_t^1. Finally, there is a third sector that produces residential services employing structures, k_t^h, and land, L_t^h, as inputs.

We label the sector producing consumption goods as sector 0, the sector producing structures as sector 1, and we denote by h the sector that produces housing services. Investment in equipment in sector i is denoted by x_t^i, $i = 0, 1$. Hence, the aggregate resource constraints of this economy are:

$$c_t + x_t^0 + x_t^1 \leq z_t^0 F^0(k_t^g, k_t^0, k_t^s, n_t^0), \tag{2}$$

$$m_t^g + m_t^s + m_t^h \leq z_t^1 F^1(k_t^1, n_t^1), \tag{3}$$

$$h_t \leq z_t^h F^h(k_t^h, L_t^h), \tag{4}$$

where we assume that all the F^j functions are concave and homogeneous of degree one. The laws of motion for the four capital stocks follow the usual specification,

$$k_{t+1}^j \leq (1-\delta^j)k_t^j + x_t^j, \qquad j \in \{0,1\} \tag{5}$$

$$k_{t+1}^j \leq (1-\delta^j)k_t^j + m_t^j, \qquad j \in \{s,h,g\}. \tag{6}$$

Since labor is free to move across sectors, equilibrium requires that aggregate labor demand must equal the labor endowment, which is normalized to 100,

$$n_t^0 + n_t^1 \leq 100. \tag{7}$$

Our formulation captures the idea that sectors are interconnected: sector one produces the capital stock that is used to build structures, both for public and private use. Public infrastructure, in turn, increases productivity of private agents (sector 0). Note that, relative to a more standard formulation, we have not, up to now, imposed that investment be non-negative. This is the "Reversible" economy.

The second economy that we study —"Irreversible"— adds a simple (and realistic) friction to our basic framework: sectoral investment cannot be negative. This, of course, is equivalent to assuming that capital is totally sector specific. Formally, we impose

$$\begin{aligned} x_t^i, m_t^j &\geq 0, \text{ for } i = 1,2; \\ j &= s,h,g. \end{aligned} \tag{8}$$

Finally, we consider a third economy —"Costly Labor"— subject, as above, to investment irreversibility, and also to labor reallocation frictions. In particular, we take the view that workers that switch sectors require training in order to be productive in their new employment. We follow Phelan and Trejos (2000) and posit that firms that want to increase employment need to allocate resources in order to train workers. Training uses as inputs time on the part of the worker who has to change sectors, as well as "trainer's time;" that is, labor input from an individual who has already been trained (and whose opportunity cost is producing). This specification implies that if the residential construction sector shrinks, it maybe optimal to slow down the rate at which workers who specialize in residential structures are retrained and to use already trained workers who face low demand for their skills to produce government infrastructure.

Formally, the economy with costly labor reallocation is subject to investment irreversibility, and the additional constraints:

$$n_t^j + v_t^j + b_t^{j,i} \leq \bar{n}_t^j + q(s_t^j, v_t^j),$$

where $\bar{n}_t^j$ is the stock of workers trained in sector j at the beginning of period t, and $q(s_t^j, v_t^j)$ is the number of newly trained individuals available for production. This quantity in turn, depends on the number of untrained agents, s_t^j, as well as the number of trainers, v_t^j. The available stock of all trained workers can be used either directly in production activities, n_t^j, as trainers, v_t^j, or sent to sector $i \neq j$ to be trained, and ultimately used in the production of good i. We denote the latter by $b_t^{j,i}$.

The stock of trained workers in a given sector evolves according to

$$\bar{n}_{t+1}^j \leq \bar{n}_t^j - b_t^{j,i} + q(s_t^j, v_t^j). \tag{9}$$

For this formulation to produce a well specified model it is necessary to assume that the function q is such that $q(s_t^j, v_t^j) \leq s_t^j$, i.e. that the number of trained workers cannot exceed the number of trainees. There are several ways in which we can interpret the function q. One view is that, following the ideas of the matching literature, it captures that number of potential trainees that are trained and employed. From this perspective, $s - q(s, v)$ is the fraction that remain unemployed.

Our requirement that, for all values of (s, v), $q(s, v) \leq s$, implies that, if the function is differentiable, $\partial q/\partial s \leq 1$. In other words, the training technology cannot produce more than one trained worker per trainee. These conditions imply that firms will not be laying of workers and, at the same time, training workers, since if $b_t^{ij} > 0$ and $s_t^i > 0$ then, given $\partial q/\partial s \leq 1$, a decrease in b_t^{ij} matched with an decrease in s_t^i increases total employment in the next period. This substitution will take place until either b_t^{ij} or s_t^i equal zero.

In our quantitative model we assume that the function q is Leontief and given by

$$q(s, v) = \min(s, v^* v), \text{ with } v^* > 1.$$

In this formulation, it takes $1/v^*$ hours of a trained individual to turn a trainee into a trained worker.

3 Planner's Problem

Since the model is convex standard arguments imply that a competitive equilibrium is optimal. Furthermore, optimal allocations can be easily characterized by the solution to a planner who aims at maximizing the utility of the representative household. In the reference economy, the planner solves

$$\max U = \sum_{t=o}^{\infty} \beta^t u(c_t, h_t). \tag{10}$$

subject to,

$$c_t + x_t^0 + x_t^1 \leq z_t^0 F^0(k_t^g, k_t^0, k_t^s, n_t^0), \tag{11}$$

$$m_t^g + m_t^s + m_t^h \leq z_t^1 F^1(k_t^1, n_t^1), \tag{12}$$

$$h_t \leq z_t^h F^h(k_t^h, L_t^h), \tag{13}$$

$$n_t^0 + n_t^1 \leq \bar{n}. \tag{14}$$

and the laws of motion of the capital stocks

$$k_{t+1}^j \leq (1-\delta^j)k_t^j + x_t^j, \qquad j \in \{0,1\} \tag{15}$$

$$k_{t+1}^j \leq (1-\delta^j)k_t^j + m_t^j, \qquad j \in \{s,h,g\}. \tag{16}$$

Our intermediate economy adds non-negativity constraints on investment given by

$$x_t^i, m_t^j \geq 0, \quad \text{for } i = 1,2; j = s,h,g. \tag{17}$$

Finally, our third economy includes the cost of reallocating labor across sectors. The additional constraints are

$$\bar{n}_{t+1}^j \leq \bar{n}_t^j - b_t^{j,i} + q(s_t^j, v_t^j), \text{ for } i,j \in \{0,1\}, \ i \neq j, \tag{18}$$

$$n_t^j + v_t^j + b_t^{j,i} \leq \bar{n}_t^j + q(s_t^j, v_t^j), \text{ for } i,j \in \{0,1\}, \ i \neq j, \tag{19}$$

$$\bar{n}_t^0 + \bar{n}_t^1 \leq \bar{n}. \tag{20}$$

3.1 Decentralization

As it is frequently the case, there are alternative market arrangements that can be used to support the optimal allocation. There are two features of this model that require some discussion. First, it is necessary to specify how changes in the stock of government provided infrastructure appear from the perspective of private firms.

One possibility is to assume that the firms' production functions are homogeneous of degree one in all inputs —including the stock of government capital— which implies that they display decreasing returns to scale in the privately chosen inputs. This, of course, requires taking a stand on the number of firms and on how to allocate the rents associated with public inputs.

A second view —and the one that we adopt following Judd (1999)— is to model changes in government supplied inputs as akin to changes in total factor productivity from the point of view of the firm. To be precise, we assume that publicly provided inputs are subject to congestion and that firms face constant returns to scale technologies in the privately chosen inputs.

We assume that firm j in the goods producing sector takes the following production function as given

$$y^j = z_j (k_j^0)^{\alpha_0} (k_j^s)^{\alpha_s} n_j^{1-\alpha_0-\alpha_s},$$

where z_j is, from the perspective of the firm, an exogenous level of productivity. In our formulation, firm level productivity is a function of "true" TFP and the stock of aggregate public capital, K^g, corrected by a congestion effect. Specifically, we assume that

$$z_j = z^0 \left(\frac{K^g}{(K^0)^{\alpha_0} (K^s)^{\alpha_s} N^{1-\alpha_0-\alpha_s}} \right)^{\gamma},$$

where z^0 is TFP and capital letters denote aggregate variables (e.g. $K = \sum_{i=1}^{N} k_i$).

In a symmetric equilibrium where all firms hire the same amount of capital and hours, (k, n), we obtain that aggregate output satisfies

$$
\begin{aligned}
Y &= z^0 \left(\frac{K^g}{(K^0)^{\alpha_0} (K^s)^{\alpha_s} N^{1-\alpha_0-\alpha_s}} \right)^{\gamma} \sum_{j=1}^{N} (k_j^0)^{\alpha_0} (k_j^s)^{\alpha_s} n_j^{1-\alpha_0-\alpha_s} \\
&= z^0 \left(\frac{K^g}{(K^0)^{\alpha_0} (K^s)^{\alpha_s} N^{1-\alpha_0-\alpha_s}} \right)^{\gamma} N (k^0)^{\alpha_0} (k^s)^{\alpha_s} n^{1-\alpha_0-\alpha_s} \\
&= z^0 \left(\frac{K^g}{(K^0)^{\alpha_0} (K^s)^{\alpha_s} N^{1-\alpha_0-\alpha_s}} \right)^{\gamma} (K^0)^{\alpha_0} (K^s)^{\alpha_s} N^{1-\alpha_0-\alpha_s}.
\end{aligned}
$$

It follows that the aggregate technology is:

$$
Y = z^0 (K^g)^{\gamma} (K^0)^{(1-\gamma)\alpha_0} (K^s)^{(1-\gamma)\alpha_s} N^{(1-\gamma)(1-\alpha_0-\alpha_s)} = z^0 F^0(K^g, K^0, K^s, N).
$$

Here, F^0 is the constant returns to scale production function in the planner's problem.

This view of the firms specific and aggregate technologies has the advantage that both firms and the planner face problems with homogeneous of degree one technologies. The main disadvantage is that congestion needs to be priced and this requires the appropriate Ramsey taxes. If we assume that firms face a tax τ, their after tax marginal product is

$$
\begin{aligned}
(1-\tau)\frac{\partial y_j}{\partial k_j^0} &= (1-\tau)\alpha_0 z_j (k_j^0)^{\alpha_0} (k_j^s)^{\alpha_s} n_j^{1-\alpha_0-\alpha_s} \\
&= (1-\tau)\alpha_0 z^0 \left(\frac{K^g}{(K^0)^{\alpha_0} (K^s)^{\alpha_s} N^{1-\alpha_0-\alpha_s}} \right)^{\gamma} (K^0)^{\alpha_0 - 1} (K^s)^{\alpha_s} N^{1-\alpha_0-\alpha_s} = \\
&= (1-\tau)\alpha z^0 (K^g)^{\gamma} K^{(1-\gamma)\alpha - 1} N^{(1-\alpha)(1-\gamma)}
\end{aligned}
$$

$$
(1-\tau)\frac{\partial y_j}{\partial n_j} = (1-\tau)(1-\alpha_0-\alpha_s) z^0 (K^g)^{\gamma} (K^0)^{(1-\gamma)\alpha_0} (K^s)^{(1-\gamma)\alpha_s} N^{(1-\gamma)(1-\alpha_0-\alpha_s)-1}.
$$

Thus, by setting $\tau = \gamma$ we guarantee that social and private marginal products are equated.

The second important feature is the training technology. There is more than one way to decentralize the planner's solution but a particularly simple one is to view trainees as employees of the firm who are paid a trainee wage. Let w^j be the wage of a trained worker in sector j. Then both active workers and trainers get paid w^j while trainees receive a wage equal to $\tilde{w}^j = w^j(1 - 1/v^*)$. That is, their wages are reduced by the cost of training.

If we consider only trained individuals —the only type that exists in the steady state— labor share in the goods producing sector is (measured as a percentage of output in that sector)

$$\frac{(1-\tau)(1-\alpha)z^0(K^g)^\gamma(K^0)^{(1-\gamma)\alpha_0}(K^s)^{(1-\gamma)\alpha_s}N^{(1-\gamma)(1-\alpha_0-\alpha_s)}}{z^0(K^g)^\gamma(K^0)^{(1-\gamma)\alpha_0}(K^s)^{(1-\gamma)\alpha_s}N^{(1-\gamma)(1-\alpha_0-\alpha_s)}} = (1-\tau)(1-\alpha_0-\alpha_s).$$

Using similar arguments, one can show that labor share in the structure producing sector is given by the appropriate parameter of the Cobb-Douglas specification since, by assumption, that sector does not benefit from publicly provided capital.

Finally, the price of the housing stock is given by the present discounted value of rents minus the costs of investing in structures to compensate for depreciation.

$$P_t^h = \sum_{s=1}^{\infty} \Pi_{j=1}^{s} \left(\frac{1}{1+r_{t+j}}\right) \left[p_{t+j}^h h_{t+j} - p_{t+j}^1 m_{t+j}^h\right],$$

where $p_t^h = \partial u/\partial h_t / \partial u/\partial c_t$ is the rental price of housing and p_t^1 is the price of structures.

4 Quantitative Analysis

In order to study the quantitative impact of a shock that lowers the private demand for housing, we calibrate the model so that its steady state variables match long-run averages of key macroeconomic aggregates of the US economy. Our analysis focuses on the transition path after a shock to the parameter that captures preference for housing. Since our interest is in the dynamic properties of the optimal allocation, we consider both a permanent and a temporary shock, and study their impact across the three specifications of the frictions in the basic economy.

4.1 Functional Forms

Our benchmark model assumes the following functional forms.

1. Preferences

$$u(c, h) = \ln(c) + \theta \ln(h).$$

2. Technology: Consumption

$$F^0 = (k^g)^\gamma (k^0)^{(1-\gamma)\alpha_0} (k^s)^{(1-\gamma)\alpha_s} n^{(1-\gamma)(1-\alpha_0-\alpha_s)}$$

3. Technology: Housing

$$F^h = (k^h)^{\alpha_h} (L^h)^{(1-\alpha_h)}.$$

4. Technology: Construction sector

$$F^1 = (k^1)^{\alpha_1} (n^1)^{1-\alpha_1}.$$

5. Training technology

$$q(s, v) = \min(s, v^* v).$$

4.2 Calibration

We choose parameters of the model so that several theoretical moments match the corresponding values in the U.S. data. As a first step we describe the mapping between model variables and the appropriate concepts in the U.S. National Income and Product Accounts (NIPA) —Table 1— and to the U.S. Fixed Asset Tables — Table 2. Our model economy is closed, while the U.S. trades with the rest of the world. To make the data and the model compatible, we consider net exports as another component of private consumption.

We assume that housing consumption in the model corresponds to housing services as reported in the NIPA. We take the rest of consumption as corresponding to consumption, c, in the model.

We focus on government structures because the negative demand shock we explore is to the output of the construction sector. It seems reasonable to assume that the construction sector can build either private or government structures. It would seem more unrealistic to assume the construction sector can also produce non-defense equipment and software.

Table 1: Model Variables vis-a-vis U.S. NIPA

	Model	NIPA
Expenditure Acct.		
Private consumption	c	Personal consumption exp. - housing services + net exports
+ Housing services	$+p^h h$	+ housing services
+Government consumption	$+G$	+ govt consumption exp. + govt investment in defense structures+ govt investment in non-defense equipment and software
+ Private investment	$+x^0+x^1$ $+p^1(m^h+m^s)$	+ gross private domestic investment
+ Public investment	$+p^1 m^g$	+govt investment in non-defense structures
= GDP	= GDP	= GDP

Table 2: Model's Fixed Assets vs. U.S. Data*	
Model	**Data**
k^0	Current-cost net stock of fixed assets − $[(k^1 + p^1(k^g + k^s + k^h) + p^L L^h]$
$+k^1$	private fixed assets in the construction industry
$+(p^1 k^g)$	+govt fixed asset structures
$+(p^1 k^s)$	+Nonresidential structures
$+(p^1 k^h)$	+FF replacement value of household and non-profit structures
$+(p^L L^h)$	FF value of household and non-profit land
= Total Fixed assets	= Total Fixed assets

*Sources: U.S. Fixed Asset Tables and the Flow of Funds of the U.S. (indicated by FF)

We set the parameters of the model as follows. First, we impose an annual depreciation rate (in physical units) for residential capital, non-residential and government structures equal to 2%, $\delta^h = \delta^s = \delta^g = 0.02$. Further, we assume that the depreciation rate for equipment is the same across sectors, $\delta^0 = \delta^1 = 0.06$. The reminder parameters are set so that the equilibrium variables of the model, in a balanced growth path, match the average value of the corresponding U.S. data during the 1965-2000 period. In particular, we match, as ratios to GDP: private investment (16%), consumption other than housing services (55%), housing services (9.5%), and government consumption (16.4%). The parameters of the production technologies are chosen such that the model matches the ratios of the net stocks of government and non-residential structures (current cost) to the net stock of equipment (1.35 and 1.03, respectively) in sector 0. Our parameterization implies that labor share in sector 0 is 0.66, and the share of land in the value of housing is 0.32.

Employment in the construction sector is set to match the fraction of total hours worked by full and part-time employees in the construction sector (5%). Productivity parameters normalize the pre-shock value of GDP to 100 and housing services to 1 (and thus the relative price of housing to 9.5 so as to match its expenditure share). For the benchmark experiment we use a Leontieff matching function and set parameter v^* so as to make reallocating labor relatively expensive. In particular, we set $1/v^* = 0.35$ which corresponds to assuming that training a worker requires 17 weeks of an already trained individual. Even though this value is on the high side, we chose it to highlight the fact that costs of labor reallocation do not play a major role in our results.[2]

A summary of calibration targets and resulting parameter values are listed in the

[2]Phelan and Trejos use a value of $1/v^* = 0.07$. Thus our choice implies that reallocating labor is almost five times as costly in our economy relative to Phelan and Trejos' model.

two tables below.

Calibration targets	

Model Variable	**US Average** (% of GDP)
Consumption	55.2
Housing Services	9.5
Private Investment	16.3
Govmt consumption	16.4
$(p^1k^g)/k^0$	1.0
$(p^1k^s)/k^0$	1.3
Value of Land/Value Housing	32.0
GDP	100
Employment in construction sector	5.0

Parameter	calibrated value
α_0	0.15
γ	0.09
α_s	0.12
α_1	0.65
α_h	0.75
β	0.96
θ	0.17
z_0	0.75
z_1	0.37
z_h	0.02

4.3 Results

In this section we report the results of subjecting our model economy to a once and for all unanticipated shock. We distinguish between a permanent and a temporary shock. The shocks we consider are large and thus we must employ nonlinear methods to solve for the equilibrium time series. Since the model is deterministic, we approximate equilibrium time series by solving a modified version of the planner's problem where steady state values are imposed after a long enough period, T. We let $T = 130$, which results in a highly accurate solution (our maximum Euler equation residuals along simulations are of order 10^{-12}).

4.3.1 Permanent Shocks

House price indices for the United States display a decline from peak to bottom of 15% (FHFA Index). In our quantitative experiments we lower θ —the parameter determining preferences for housing— to match this drop (which requires lowering θ by 25%) in the short run.[3]

The fundamental force driving the model after the shock is the desire for a higher ratio of non-housing consumption to housing services. Producing more non-housing consumption requires increasing (all types of) capital and labor in sector 0. Producing fewer housing services can only be achieved by lowering the stock of residential structures since the other input, land, is in fixed supply. Thus, the shock to the demand for housing has, in principle, an ambiguous effect on the demand for construction services as the private demand for housing related construction decreases but the private demand for non-residential construction and government infrastructure —both inputs in the production of non-housing consumption— could potentially increase. The strength of these factors turns out to depend on the frictions associated with the economy's ability to reallocate factors across sectors.

Aggregate Output Since our model possesses a rich structure it is not surprising that the results are difficult to summarize. One major finding is that the relatively large preference shock that we study —which, as indicated above, results in a 15% decrease in the market price of existing houses— has a small impact on aggregate output in the short run across all three specifications. The drop in output is about 0.5% within the first three years after the shock. The differences across models appear in the medium run —between 3 and 15 years— and in the speed of adjustment.

[3]This parameterization implies that, in the steady state, housing prices decrease by 6.5%.

Even then, those differences are driven mostly by changes in relative prices. Figure 1 shows GDP at base year prices. In all three cases the dynamics suggest smooth and monotonic adjustment to the new long run level that is about 2% below the previous steady state. The larger drop is associated with the "Irreversible" economy. For the behavior of GDP at constant prices adding labor frictions ("Costly Labor") results in a smaller drop in output.

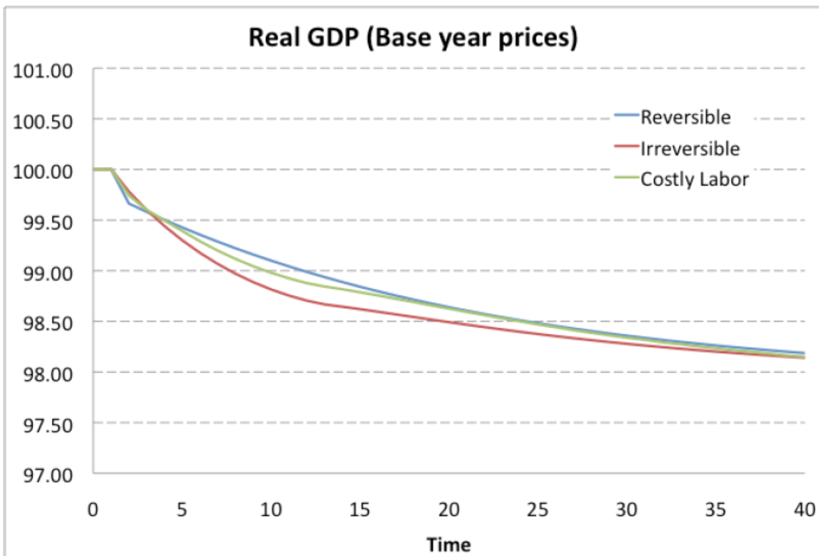

Figure 1

Figure 2 displays GDP using current period prices[4]. In this case the "Reversible" economy overshoots its long run level before it smoothly adjusts to the new long run level. The other two economies, one characterized by irreversible investment only and the other by an additional cost of reallocating labor, display different dynamics. The initial low output period lasts for approximately ten years. Moreover, there is a small increase in output before it slowly converges to its long run level. Even though

[4]Chain linked GDP should be close to an average between the two series.

the timing differs, in all cases there is a short lived recovery followed by a long (but very shallow) decline toward the new steady state.

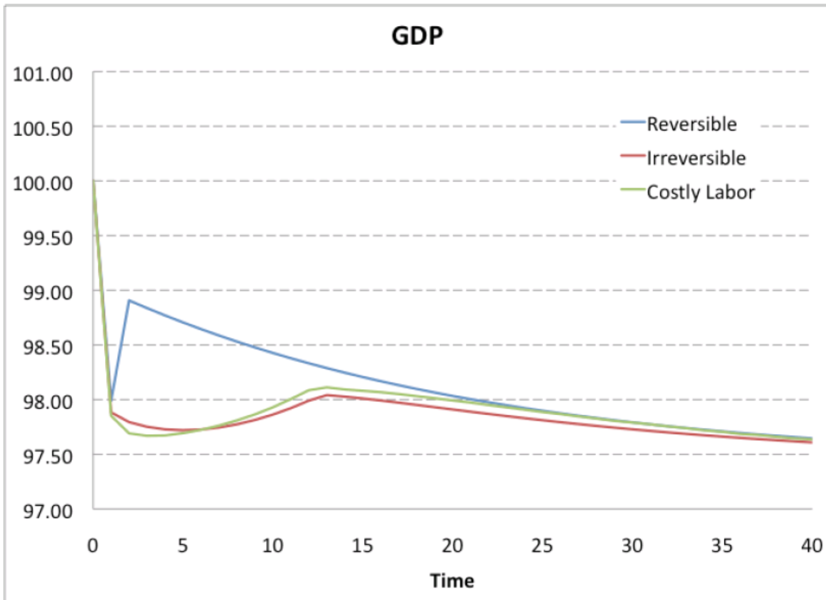

Figure 2

Private Investment The equilibrium behavior of investment illustrates both the heterogeneity across sectors and the general pattern across specifications (see Figures 3a-d below). There is a clear shift in the allocation of output from sector one. In all cases there is a temporary decrease in investment in residential structures and an equally temporary increase in investment in non-residential structures. The main differences across specifications are associated with the degree of reversibility. In the "Reversible" economy the changes are abrupt (the flows change by a factor of four) and short lived with most of the adjustment completed within three years of the shock. The "Irreversible" economy displays much longer transitions as the housing sector remains stagnant (zero investment) for almost ten years. Investment in non-residential structures behaves almost as a mirror image. Adding frictions to labor mobility has almost no impact on the results. Here the key driving force is the small depreciation rate of structures: If capital is irreversible adjustment to a new lower steady state level requires no investment and time to let the stock reach its new level.

The model's predictions for investment in equipment in sectors 0 (consumption) and 1 (structures) are also close to mirror images of each other: equipment investment in consumption increases while investment in the structure producing sector decreases. In the reversible economy the changes are large and short lived. In the economies with frictions the changes are spread out over a decade and this is independent of the existence of labor reallocation costs.

The model implies that the equilibrium level of residential services (not shown) decreases monotonically towards its new steady state while non-housing consumption (not shown) initially overshoots its long run level but then converges monotonically to it.

The picture that emerges from private sector choices is that the shock induces a significant reallocation of investment away from housing related sectors and into the production of inputs used in the production of non-housing consumption. In terms of the implications for the duration of the adjustment period, the key feature is specificity of capital: given small depreciation rates it takes about a decade to generate the desired adjustment. In terms of private sector choices, adding costs of reallocating labor has no additional impact on the predictions of the model.

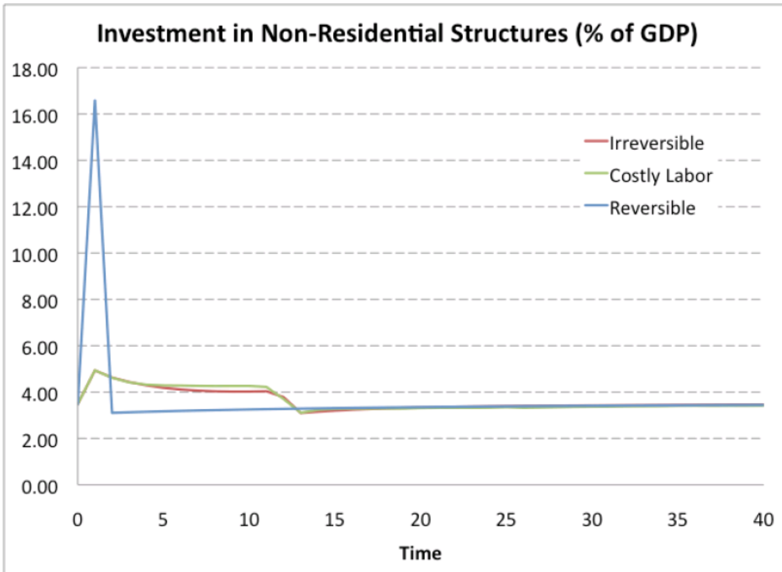

Figure 3a

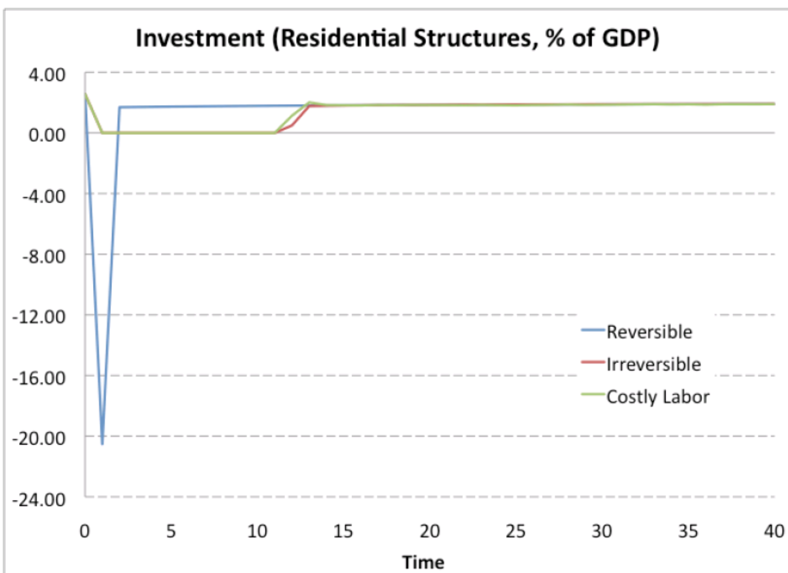

Figure 3b

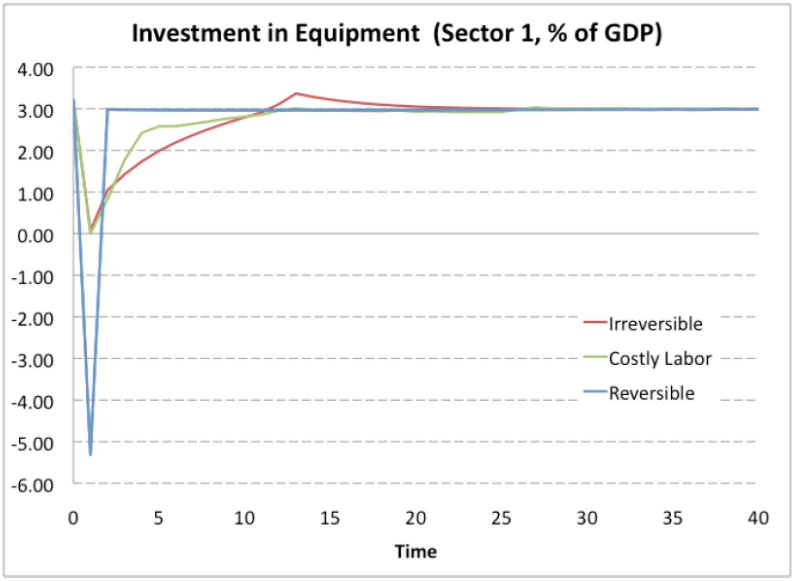

Figure 3c

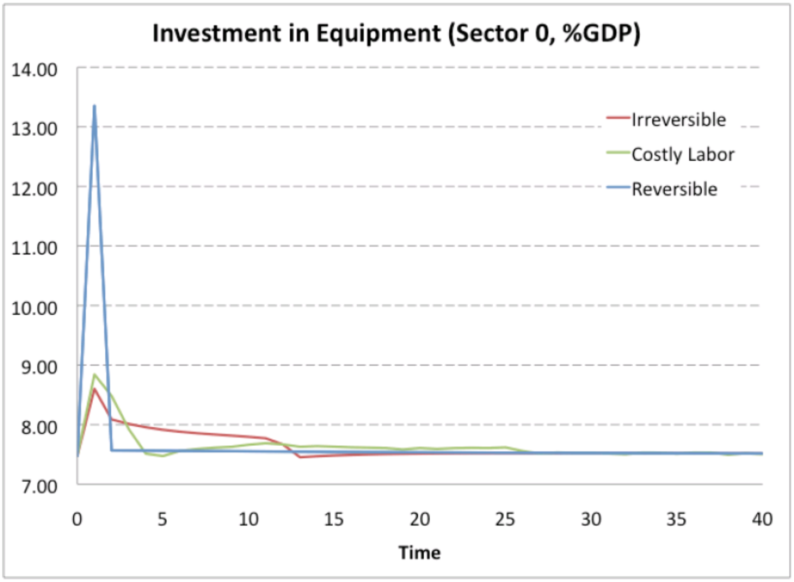

Figure 3d

Government Investment The most salient feature of optimal level of government expenditure (Figure 5) is that it increases by a substantial amount in the short run as a result of the shock. In the economies subject to irreversibility it increases by 44% on impact, while in the frictionless economy the increase is by a factor of over 4.

Contrary to the intuition that specifies that government spending should be more responsive in an economy with frictions so as to partially compensate for the cost of moving reallocating factors, we find that the opposite is the optimal response. In our models the lower opportunity cost of using underemployed construction workers does not compensate for the low productivity of government structures in the consumption sector and the optimal policy calls for complementing the allocation of resources in the private sector. Thus in the economies with frictions we find that optimal government spending is smaller and hence that irreversibilities result in a lower level of stimulus.

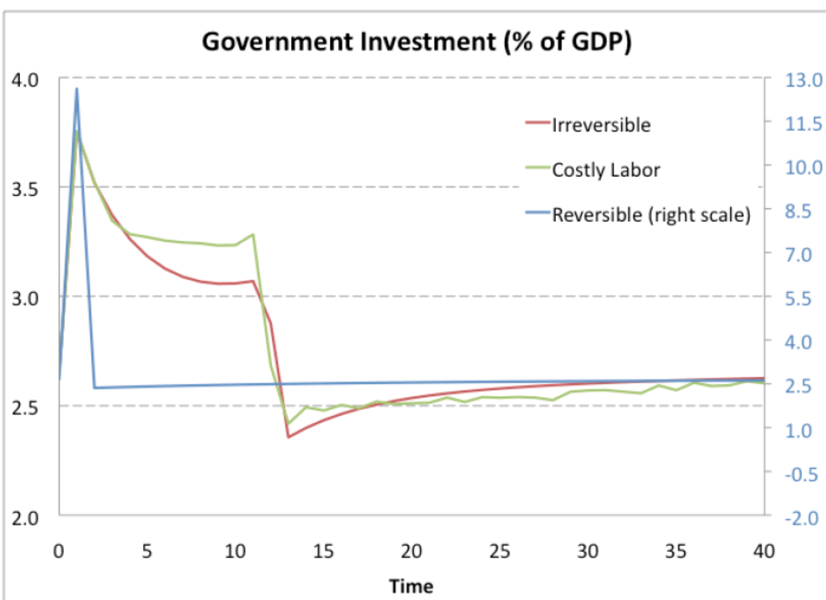

Figure 5

The dynamics of the optimal level of government spending also differ across economies. In the absence of frictions the optimal level displays a large but short lived increase. In the economies with frictions the level of government investment in infrastructure remains high for a much longer period of time (12 years). In this economy, the change in preferences requires a permanent increase in government spending. However, the speed at which this happens under the optimal policy depends on the ability of the private sector to reallocate resources. From a quantitative perspective, labor frictions do not seem to matter much since the patterns of government expenditure in the two economies subject to irreversibility are extremely similar. Again, this reinforces the fact that more frictions do not necessarily translate in significantly higher government investment and it illustrates the role that capital mobility —even restricted by the requirement that there is no disinvestment— has in compensating for partial labor immobility and it accounts for the small role that we find for costly labor reallocations relative to the results in Phelan and Trejos (2000).

Employment As expected, the one variable whose equilibrium behavior depends on the nature of the frictions is sectoral employment. The shock that we study has a significant negative impact on the construction sector. Employment in this sector goes down by up to 20% in the economies with no labor reallocation frictions. The largest drop in employment in the construction sector occurs one period after the shock in the economy with reversible capital (as soon as capital is adjusted), and 10 years later in the economy with irreversibility (See Figure 6). Labor reallocation frictions have an important quantitative impact on employment trends. In particular, labor frictions result in a substantially smaller drop in employment of about 12%. The intuition for this is also straightforward. It is relatively more expensive to move labor resources to increase consumption, and therefore less labor is reallocated on impact. Also note that the drop in employment in the economies without labor frictions overshoots its long run trend by a substantial amount. With labor frictions, this overshooting is too expensive to be optimal.

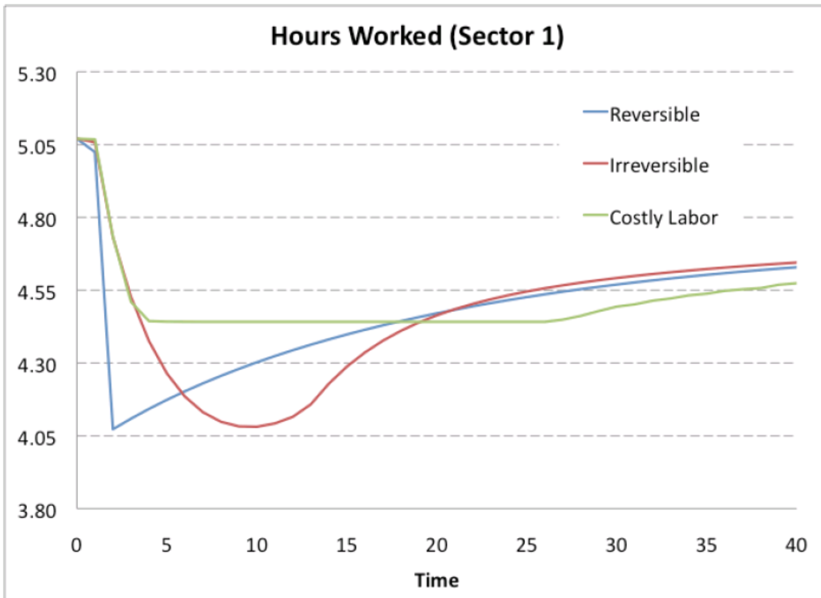

Figure 6

In summary, our quantitative analysis illustrates several interesting equilibrium responses to a housing demand shock. The first is that the percentage change in public investment is quite large and its dynamics highly dependent on the frictions associated with reallocating other inputs in the economy. Second, the optimal size of a government stimulus is not a simple function of observed changes in standard macroeconomic indicators such as GDP or investment. Thirdly, because of the very low depreciation rate of structures, the transition to the new steady state takes a long time. This suggests that the dynamic response to a housing shock may be very different from that of standard productivity shocks. Finally, labor reallocations matter for the labor market, but have a small impact on the behavior of other macroeconomic aggregates. In the model that we study capital mobility —even restricted— is a good substitute for labor mobility to implement the optimal amount of consumption smoothing. Put it differently, dramatic differences in the allocation of labor resources need not signal large changes in aggregate output, or point to the need of implementing different government spending policies.

4.3.2 Temporary shocks

The analysis of the previous section assumed that the preference shock is permanent. In this section we study the optimal response to a temporary shock. We consider the following simple modification to our model economies that allows for transitory change in housing preferences. Let $\theta_0 = (1-\epsilon)\bar{\theta}$ be the initial shock to housing preferences, where $0 \leq \epsilon < 1$, and $\bar{\theta}$ is the calibrated value for this preference parameter previous to the shock. Then, we construct a sequence of preference parameters for housing services as follows:

$$\theta_{t+1} - \bar{\theta} = \rho(\theta_t - \bar{\theta}),$$

with $0 \leq \rho \leq 1$. The case $\rho = 1$ corresponds to the experiments of the previous subsection where the preference shock is permanent.

In order to economize space, we only report here the results for the economy subject to irreversibility (but no labor reallocation costs) and we only discuss the most important results.[5] We set all parameter values as in the benchmark experiments reported above. The values of the new parameters, ρ and ϵ, are determined as follows: We pick $\rho = 0.7$ such that preferences for housing have recovered 80% of

[5] Of course, all time series are available upon request.

their long-run value ($\bar{\theta}$) five years after the shock hits. For ease of comparison with the benchmark experiment, we set ϵ such that the drop in house values that results from this shock, taking as given given the value of ρ, is also 15%. House values equal the present discounted value of residential services and, given the low persistence of the process driving housing preferences, matching such drop in values requires a very large shock, $\epsilon = 0.925$. Figure 7 displays the equilibrium path of the price of residential services in the original irreversible economy and in this new economy subject to a temporary shock. The stock of residential structures and land are predetermined when the lower preference for housing shock hits. In equilibrium, the price of residential services —a measure of the rental price of the housing stock— must go down. In the irreversible economy this price converges to a new lower steady state, while in the temporary shock economy the rent-price ratio declines significantly and it remains 15% lower 5 years after the shock hits. Thus, the more temporary the shock the larger the decrease in the rent-price ratio. In the case of a permanent shock the model implies a post shock decrease —taking a two year average— of the rent-price ratio of approximately 8%, while in the economy subject to a temporary shock the decrease is almost 80%

Figure 7

This dramatic decrease in the price of residential services results in a substantial decline in GDP (-9%) for the economy subject to the temporary shock (see Figure 8). Most of this decline comes from relative price changes as real GDP barely moves in the economy subject to the temporary shock.

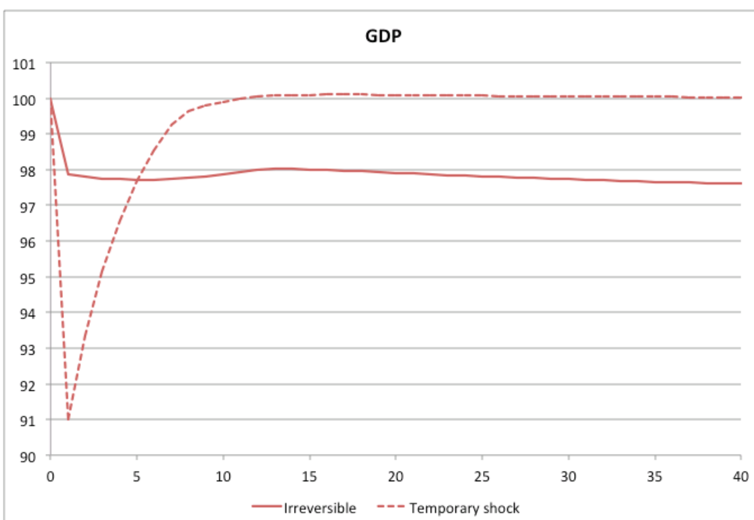

Figure 8

The fundamental force driving the dynamics of this economy with temporary shocks is still a desire to increase non-housing consumption relative to residential services. Of course, given that the shock is temporary, non-housing consumption and residential services respond considerably less than in the irreversible economy. Surprisingly enough, the magnitude of the optimal increase in government investment at the time the shock hits is the same as that of the economy where the shock is permanent (Figure 9). Further, the optimal stimulus with a temporary shock is kept higher than in the benchmark for up to 6 years after the shock first hits, but the stimulus is withdrawn earlier than in the benchmark case (optimal government spending falls substantially 7 years after the shock). Finally, the economy subject to a temporary shock displays a much larger undershooting of long-run government spending (equivalent to up to one percent of GDP) than the benchmark economy subject to irreversibility only. Similar trends can be found in equilibrium investment in non-residential structures.

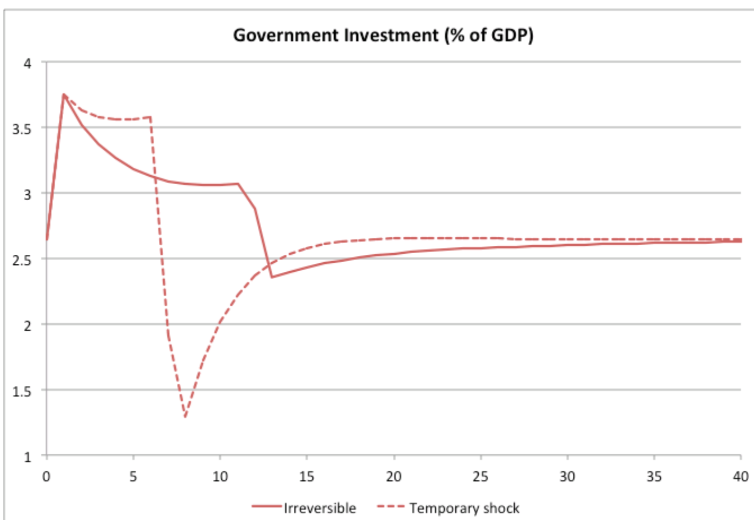

Figure 9

Employment in the construction sector does not decline on impact as much in the economy subject to a temporary shock (Figure 10), and it also recovers much earlier. The reasons for these patterns are as follows: For the first few years after the shock, investment in non-residential and government structures is kept higher than in the irreversible economy, while the drop in residential structures investment is comparable. Furthermore, the shock is only temporary. Hence, residential services and the stock of residential structures must go back to their preshock levels. This brings, roughly 7 years after the initial shock, a strong recovery (and even overshooting) in the overall demand for structures and employment in the construction sector.

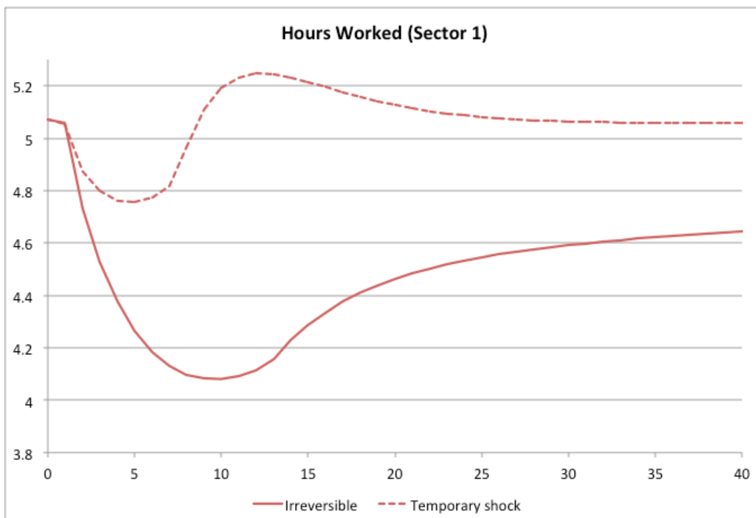

Figure 10

5 Concluding Comments

In this paper we study the role that frictions in reallocating resources play in determining the optimal response of the government to a drop in demand. Overall we find that the stronger the costs of reallocating resources the smaller the optimal level of intervention by the government and the more persistent the level of government purchases. Moreover, the optimal response is not a simple function of macro aggregates: the largest drop in output does not trigger the largest increase in government spending; rather, it results in the smallest level of infrastructure investment. In terms of welfare, it is trivially true that the economy with no frictions welfare is highest although it is the one that displays the most significant fluctuations in response to a shock.

The objective of our study is to run a controlled thought experiment and derive the implications of a demand shock for key macroeconomic aggregates, as well as the properties of the optimal policy response to such a shock. Hence, we have purposely not contrasted the predictions of the theory with the data. Periods when house prices have fallen have also coincided with recessions and financial crises. Contrasting theory against data would therefore require a model that allows for several types of shocks, and a complex identification method to determine which shocks hit the economy, and when. Such work may be certainly interesting but it differs in nature from our objective in this paper. We therefore leave it for future research.

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