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Inflation in the G7: Mind the Gap(s)?*

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ABSTRACT: We investigate the importance of trend inflation and the real-activity gap for explaining observed inflation variation in G7 countries since 1960. Our results are based on a bivariate unobserved-components model of inflation and unemployment in which inflation is decomposed into a stochastic trend and transitory component. As in recent implementations of the New Keynesian Phillips Curve, it is the transitory component of inflation, or “inflation gap”, that is driven by the real-activity gap, which we measure as the deviation of unemployment from its natural rate. Even when allowing for changes in the contributions of trend inflation and the inflation gap, we find that both are important determinants of inflation variation at business cycle horizons for all G7 countries throughout much of the past 50 years. Also, the real-activity gap explains a large fraction of the variation in the inflation gap for each country, both historically and in recent years. Taken together, the results suggest the New Keynesian Phillips Curve, once augmented to include trend inflation, is an empirically relevant model for the G7 countries. We also provide new estimates of trend inflation for the G7 that incorporate information in the real-activity gap for identification and, through formal model comparisons, new statistical evidence regarding structural breaks in the variability of trend inflation and the inflation gap.

Keywords: inflation gap, inflation persistence, natural rate, Phillips Curve, trend inflation

JEL Classification: C32, E31, E32

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Introduction

The Phillips Curve is one of the most recognizable theoretical constructs in macroeconomics. In its modern form, the Phillips Curve relates observed inflation to expected inflation and a measure of excess demand, where the latter is most commonly expressed as the gap between the actual and potential level of real activity. This relationship between inflation and the “real-activity gap” is a primary channel through which monetary policy is assumed to affect the inflation rate in most contemporary macroeconomic models.

Despite its theoretical appeal, the empirical evidence linking the real-activity gap to inflation is mixed. A large literature, typified by the work of Robert Gordon (1982, 1997, 1998), estimates Phillips Curve equations for which the expected inflation term is replaced by lags of inflation. In the “accelerationist” version of this model, the coefficients on lagged inflation are constrained to sum to unity. Such “backward looking” implementations of the Phillips Curve typically find that the real-activity gap, whether measured using output or the unemployment rate, is strongly statistically significant as a driver for U.S. inflation. However, if one instead assumes rational expectations, as in the “New Keynesian” version of the Phillips Curve (NKPC), the evidence in favor of the real-activity gap as an inflation driver is lessened. A number of studies, including Fuhrer and Moore (1995), Fuhrer (1997), Roberts (2001), and Estrella and Fuhrer (2002), find that the estimated effect of the real-activity gap in NKPC equations for U.S. inflation is insignificant, and in some cases has a counterintuitive sign.¹

Another empirical shortcoming of the NKPC relates to its inability to generate substantial inflation persistence. The NKPC implies that inflation is a discounted present value of expected

¹ In the NKPC, the theoretical driving variable for inflation is real marginal cost, which is then proxied for with the real-activity gap. Gali and Gertler (1999) and Gali, Gertler, and Lopez Salido (2001) consider an alternative proxy, the average labor share of national income, and report a better fit for modeling U.S and Euro area inflation rates. However, use of the average labor share as a proxy for real marginal cost is not without criticism (see, e.g., Rudd and Whelan, 2005).

real-activity gap terms, which, assuming the real-activity gap is covariance stationary, implies that inflation itself is covariance stationary. Further, estimates of the discounted present value of expected gap terms display low levels of persistence. This is counterfactual, as it is difficult to reject the null hypothesis of a unit root for the inflation rates of many countries. Indeed, it is now standard for univariate statistical characterizations of inflation to include a stochastic trend.² In response to this, recent contributions, including Cogley and Sbordone (2008) and Goodfriend and King (2009), augment the NKPC to allow for trend inflation. In these models, it is the “inflation gap”, or the difference between inflation and its trend, that is influenced by the real-activity gap. Empirical implementations of the NKPC with trend inflation provide more evidence in favor of the real-activity gap as an inflation driver. For example, Lee and Nelson (2007), Harvey (2008), Piger and Rasche (2008), and Manopimoke (2010) find that the real-activity gap is a statistically significant driver of the U.S. inflation gap, while Cogley and Sbordone (2008) find that the fit of the NKPC for the inflation gap is improved over that for inflation itself.

In this paper, we provide evidence regarding the importance of trend inflation and the real-activity gap for explaining inflation variation in the G7 economies. We work with a bivariate unobserved-components (UC) model of inflation and unemployment that is a reduced form of the NKPC with trend inflation. The real-activity gap is measured using the “unemployment gap”, defined as the deviation of unemployment from its natural rate. As in Staiger, Stock and Watson (1997) and Laubach (2001), the natural rate is defined as the stochastic trend in unemployment. The unemployment gap is assumed to drive the inflation gap, measured as the deviation of actual and trend inflation. Following a large recent literature, for example Stock and Watson (2007), we measure trend inflation using the stochastic trend in

² Stock and Watson (2007) and Kang, Kim and Morley (2010) estimate univariate models in which U.S. inflation is decomposed into stochastic trend and cyclical components. Cecchetti, et al. (2007) applies the Stock and Watson (2007) model to G7 inflation data.

inflation and allow for instability, in the form of multiple discrete structural breaks of unknown timing, in the variance of shocks to this stochastic trend. The model is estimated using a Bayesian framework, and posterior probabilities used to formally incorporate uncertainty regarding alternative numbers of structural breaks.

The bivariate UC model is fit to inflation and unemployment rates of G7 countries over sample periods that range from the past 40-50 years depending on country. The estimation results suggest that both the inflation trend and the inflation gap have been important drivers of actual inflation at business cycle horizons for all countries over these sample periods. In particular, both the inflation trend and inflation gap have contributed significantly to the variation of inflation changes at horizons ranging from one quarter to five years. There have also been significant changes in the contributions of these components over time.

Turning to the determinants of the inflation gap, the results suggest that the unemployment gap has been an important contributor to variation in the inflation gap in G7 countries. The average percentage of inflation gap variance attributed to the unemployment gap over the entire sample exceeds 30% for all G7 countries, and 50% for five countries. For most countries, this percentage is at or near historic highs in recent years. Further, the level of the variance contributed by the unemployment gap to the variance of the inflation gap is very stable for all countries over this sample period. Overall, the results suggest that the NKPC with trend inflation is an empirically relevant model for the G7, in that the unemployment gap is a consistent and substantial contributor to the variation of the inflation gap in these countries.

The results also provide new estimates of trend inflation for the G7 economies, as well as shed new light on the possible presence of structural changes in the variance of shocks to inflation and unemployment. For most countries considered, the level of trend inflation has

varied substantially over time in a hump-shaped pattern, with low trend inflation in the 1960s, high trend inflation in the 1970s and early 1980s, and low trend inflation thereafter. Also, for all countries, trend inflation is near historical lows by the end of the sample period. For many countries, the level of trend inflation has been on average above actual inflation for significant periods in the 1980s and 1990s, which is driven by unemployment rates that are above the estimated natural rate. This result highlights the information that the real-activity gap adds for identification of trend inflation. Finally, model comparisons find strong evidence of multiple structural breaks in the variance of shocks to inflation and unemployment for France, Italy, the United Kingdom, and the United States, but less evidence of multiple breaks for Canada, Japan and Germany. For all countries considered, the volatility of shocks to trend inflation is near historic lows at the end of the sample period.

As trend inflation in our model represents the permanent variation in inflation, it is closely tied to long-horizon expectations of inflation. Indeed, as in Beveridge and Nelson (1981), the long-run expectation of inflation at time t is equivalent to the expectation of trend inflation formed using time t information. Thus, our results could alternatively be interpreted as suggesting that long-horizon inflation expectations have played an important, although not dominant, role in explaining inflation variation at business cycle horizons in most G7 countries, and that long-horizon expectations are currently “anchored” at low levels across the G7. This recent similarity exists despite differences in monetary institutions across G7 countries, primarily the choice of whether or not the monetary authority will formally adopt inflation targeting.

Our paper is most closely related to several recent studies of inflation dynamics that incorporate trend inflation. Lee and Nelson (2007), Harvey (2008), Piger and Rasche (2008), and Manopimoke (2010) estimate similar bivariate UC models of inflation and unemployment and

investigate the statistical significance of the real-activity gap.³ However, they do not consider data outside the United States. Cecchetti, Hooper, Kasman, Schoenholtz, and Watson (2007), estimate a UC model that separates each G7 inflation series into a stochastic trend plus a cycle, and allow for time variation in the variance of shocks to the inflation trend. However, these authors focus on univariate analysis in which the cyclical component of inflation is not influenced by the real activity gap, and so do not provide evidence regarding the relative importance of the real-activity gap for explaining inflation variation in the G7. Further, we find that incorporating information from the real-activity gap for identification of trend inflation makes for significant differences in the estimated pattern of trend inflation in several countries. Finally, these authors do not conduct formal testing or model comparisons regarding the statistical importance of parameter changes. For several countries, most notably Canada, our results support different conclusions regarding structural change from those reached in Cecchetti et al. (2007).

The remainder of this paper is organized as follows. Section 2 motivates and details the bivariate UC model used in our analysis. Section 3 discusses the G7 inflation data, and describes the Bayesian techniques we use for estimation and model comparison. Section 4 presents posterior model probabilities comparing models with alternative numbers of structural breaks, while Section 5 provides new estimates of trend inflation, the natural rate, and the associated inflation and real-activity gaps. Section 6 presents results regarding the contribution of trend inflation and the inflation gap to the variability of inflation changes, as well as the contribution of the unemployment gap to the variance of the inflation gap. Section 7 concludes.

³ An earlier literature, for example Kuttner (1994), Gerlach and Smets (1999) and Basistha and Nelson (2007), also work with Phillips Curve models that have a bivariate UC structure. These models differ from that used in this paper in that they assume inflation is covariance stationary.

2. Model Specification

We assume the quarterly inflation rate, π_t , is an I(1) process with trend / cycle representation:

$$\pi_t = \bar{\pi}_t + \pi_t^g, \quad (1)$$

where the trend component, $\bar{\pi}_t$, represents the stochastic trend in inflation, and π_t^g is a zero mean, covariance stationary process that, following Cogley, Primiceri and Sargent (2010), we label the “inflation gap”. As is standard in the recent literature, trend inflation is modeled as a driftless random walk:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + v_t, \quad (2)$$

where v_t represents a stochastic shock to trend inflation. Stock and Watson (2007) and Piger and Rasche (2008) find that the variability of shocks to trend inflation in the United States have varied substantially over time, while Cecchetti et al. (2007) document similar patterns for some G7 economies. To capture the possibility of changes to the volatility of shocks to trend inflation, we assume that v_t is a Gaussian random variable with time-varying variance:

$$v_t \sim N(0, \sigma_{v,t}^2),$$

where $\sigma_{v,t}^2$ follows a discrete structural break process with m structural changes, so that

$\sigma_{v,t}^2 = \sigma_{v,i}^2$, $i = 1, \dots, m + 1$. In the empirical implementation of the model, we treat the dates of the

structural changes, $\tau_1, \tau_2, \dots, \tau_m$, as unknown parameters, and the selection of m as a problem of model selection.⁴

The trend inflation component has strong links to the long-horizon forecast of inflation, which is equivalent to “core inflation” as defined by Bryan and Cecchetti (1994). Since π_t^g is covariance stationary with zero mean, and $\bar{\pi}_t$ follows a random walk, the long horizon inflation expectation can be written as:

$$\lim_{h \rightarrow \infty} E_t(\pi_{t+h}) = E_t(\bar{\pi}_t),$$

where E_t is an expectation formed using information available at time t . Thus, the minimum mean-squared error estimate of trend inflation at time t is equivalent to the long-horizon forecast of inflation arising from the model. Also, as discussed in Bernanke (2007), since trend inflation in the model captures permanent changes to the inflation rate, it is unlikely that trend inflation would display substantial variation that was not mirrored in long-horizon forecasts of inflation. Finally, several studies, including Cecchetti et al. (2007), Clark and Davig (2008) and Piger and Rasche (2008) show that survey measures of long-horizon inflation expectations are closely aligned with estimates of trend inflation in the United States.

The modern Phillips Curve posits a short-run tradeoff between inflation and the real-activity gap. In our framework, this suggests that the real-activity gap should be a driver of the inflation gap, which represents the temporary deviation of inflation from its stochastic trend. To

⁴ Stock and Watson (2007) and Cecchetti et al. (2007) model the variance of the innovation to trend inflation as following a stochastic volatility process, where the change to the variance is a stochastic shock that comes from a high or low volatility regime. In the implementation of their model, they fix the probability of the high volatility regime to be small, suggesting that large changes to the volatility of trend inflation occur only infrequently. Thus, their model is not inconsistent with the structural break model that we employ here.

capture this, we specify the following linear relationship between the inflation gap and the real-activity gap:

$$\pi_t^g = \sum_{j=0}^{p_x} \delta_j x_{t-j} + z_t. \quad (3)$$

In (3), π_t^g is partially determined by a distributed lag of the real-activity gap, denoted x_t . We augment (3) with a residual component, z_t , meant to capture variation in the inflation gap not related to the real-activity gap. We assume that this residual component has an autoregressive representation:

$$\psi(L)z_t = \omega_t, \quad (4)$$

where $\psi(L)$ is an invertible lag polynomial and ω_t is a Gaussian stochastic shock with time varying variance:

$$\omega_t \sim N(0, \sigma_{\omega,t}^2).$$

Consistent with the variance of trend inflation, $\sigma_{\omega,t}^2$ is assumed to follow a discrete structural break process with m structural changes, where the timing of the structural changes is shared with those in the variance of trend inflation.

We measure the real-activity gap as the unemployment gap, or the deviation of the unemployment rate from its natural rate. In particular, and similarly to inflation, we assume that the quarterly unemployment rate is an I(1) process with a trend / cycle representation:

$$u_t = \bar{u}_t + x_t. \quad (5)$$

Following Staiger, Stock and Watson (1997) and Laubach (2001), we assume that the natural rate, \bar{u}_t , is equivalent to the stochastic trend in the unemployment rate, modeled as a random walk with drift:

$$\bar{u}_t = \mu + \bar{u}_{t-1} + \eta_t, \quad (6)$$

$$\eta_t \sim N(0, \sigma_{\eta,t}^2).$$

Finally, the unemployment gap is modeled as an autoregressive process:

$$\phi(L)x_t = \varepsilon_t, \quad (7)$$

$$\varepsilon_t \sim N(0, \sigma_{\varepsilon,t}^2),$$

where $\phi(L)$ is an invertible lag polynomial. Again, to model time varying volatility, $\sigma_{\eta,t}^2$ and $\sigma_{\varepsilon,t}^2$ are assumed to follow a discrete structural break process with m structural changes, where the timing of the structural changes is shared with those for the other shocks in the model. We assume that the four shocks in the model are mutually independent of one another, both contemporaneously and at all leads and lags.⁵

Taken together, equations (1)-(7) form a bivariate UC model for inflation and unemployment. As discussed in Harvey (2008) and Manopimoke (2010), this model can be interpreted as the reduced form of the NKPC with trend inflation described in Goodfriend and

⁵ A substantial literature finds evidence for a statistically significant correlation between shocks to trend real-activity and shocks to the real-activity gap in the United States (see, e.g., Morley, Nelson and Zivot, 2003). For all countries, we have estimated a version of the model that allows for such a correlation, which in this case is between shocks to the natural rate and shocks to the unemployment gap. Overall, the results from this model regarding the importance of the various model components to explaining inflation and inflation gap variance are qualitatively similar to the model that assumes a zero correlation. Thus, for simplicity of presentation, we focus on the results assuming uncorrelated shocks.

King (2009). In particular, the NKPC with trend inflation implies that the inflation gap has the following dynamics:

$$\pi_t^g = \kappa \sum_{j=0}^{\infty} \beta^j E_t(x_{t+j}), \quad (8)$$

where β is the discount rate. Assuming that x_t follows an autoregressive process as in (7), the expectations in (8) have a simple, recursive, structure that yields a linear expression for π_t^g in terms of current and lagged values of the real-activity gap. For example, if $x_t = \phi x_{t-1} + \varepsilon_t$, then we have the following reduced form for (8) upon substituting expectations:

$$\pi_t^g = \delta x_t,$$

$$\delta = \frac{\kappa}{1 - \beta\phi}.$$

Thus, the inflation gap equation in (3) is a reduced form of (8), augmented to include a role for a residual component (z_t).

The model in (1)-(7) also has connections to the accelerationist version of backward-looking Phillips Curve models. In these models, inflation dynamics are described by:

$$\pi_t = \sum_{j=1}^{p_\pi} \alpha_j \pi_{t-j} + \sum_{j=0}^{p_x} \delta_j x_{t-j} + z_t, \quad (9)$$

where $\sum_{j=1}^{p_\pi} \alpha_j = 1$. As discussed in Harvey (2008) and Piger and Rasche (2008), equations (1)-(3)

above result from replacing the lags of inflation in (9) with the trend inflation component in (2).

However, while related, these models have significant differences in their implications for inflation dynamics. In the accelerationist Phillips Curve, inflation persistence is a structural

feature of the model, in that all variation in inflation becomes mechanically imbedded in the permanent component of inflation. By contrast, in the model in (1)-(7), only events that influence the shock to trend inflation have permanent effects. Further, Stock and Watson (2007) and Cecchetti et al. (2007) present evidence that the first difference of G7 inflation series contain important moving average dynamics. The additive structure of the trend / cycle decomposition in (1) generates such dynamics, regardless of the influence of the unemployment gap.

3. Data and Estimation

We estimate the bivariate UC model in (1)-(7) for each of the G7 countries, which requires data on inflation (π_t) and the unemployment rate (u_t). To measure the inflation rate, we use the log first difference, multiplied by 400, of the quarterly consumer price index, while the unemployment rate is a household survey-based measure. All data was obtained from the OECD database. For each country, we use the longest sample for which all three variables were available. Data for Canada, Germany, Italy, the United Kingdom, and the United States begins between 1957 and 1963, and for France and Japan in 1968 and 1970 respectively. All data series end in either the first or second quarter of 2010. Table 1 details the exact sample periods used in estimation for each country.

We have identified a few specific cases in which exogenous events, such as shifts in VAT or other sales tax rates, resulted in large transitory fluctuations in the inflation series. The dates of these events are listed in Table 2. So as not to allow such outliers to dominate our results regarding the contribution of the transitory component of inflation to the inflation process, we replace these outliers with interpolated values (the median of the six adjacent observations that were not themselves outlier observations).

The bivariate UC model is based on the assumption that both the inflation and unemployment rate follow a unit root process. To provide some evidence regarding the validity of this assumption, Table 3 shows the results of unit root tests for each series by country. The results of these tests are largely consistent with the assumptions of the model. In particular, we cannot reject the null hypothesis of a unit root at the 5% level for any of the unemployment rates and for six of the seven inflation rates, the exception being Japanese inflation.

We estimate the bivariate UC model using a Bayesian framework, which requires prior specifications for each of the model parameters. Our prior densities are independent across parameters. Turning to individual parameters, our prior density for each shock variance, $\sigma_{\omega,i}^2$, $\sigma_{\eta,i}^2$, $\sigma_{\varepsilon,i}^2$, and $\sigma_{v,i}^2$, $i = 1, \dots, m + 1$, is inverse gamma with shape parameter 2.5 and scale parameter 0.5. For each slope parameter in the inflation gap equation, δ_i , each autoregressive parameter in the residual component of the inflation gap, ψ_i , and the drift in the natural rate, μ , our prior density is standard normal. For each autoregressive parameter in the unemployment gap equation, ϕ_i , our prior density is normal with mean zero and variance $\left(\frac{0.5}{i}\right)^2$. This prior shrinks the autoregressive terms toward zero, which reflects our prior belief that the real-activity gap should be a clearly stationary process, and avoids potential identification issues associated with a UC model in which the transitory component displays near unit root behavior. Finally, for the dates of the structural breaks to shock variances, $\tau_1, \tau_2, \dots, \tau_m$, we assume a uniform distribution across all admissible combinations of m break dates. As admissible, we consider all break dates that do not occur in the first or final 10% of the sample, and have at least 5% of the sample between breaks.

To simulate samples from the posterior distribution of the model parameters, we use a Metropolis-within-Gibbs sampling algorithm. The model parameters are divided into two blocks, the first holding the m structural break dates, and the second consisting of all other model parameters. To sample from the posterior distribution of the breaks dates, conditional on the other parameters of the model, we use the Gibbs Sampling steps detailed in Wang and Zivot (2000). To sample from the posterior distribution of the other model parameters, conditional on the structural break dates, we use a Metropolis-Hastings step with proposals generated from a random walk chain. The variance-covariance matrix of innovations to the random walk chain is calibrated using a multivariate normal approximation to the posterior distribution of the relevant parameters.⁶ Denoting the variance-covariance matrix of this approximating distribution as \hat{V} , the variance-covariance matrix for the innovations to the random walk chain is then set equal to $c\hat{V}$, where c is a scalar calibrated to yield acceptable proposal acceptance rates.⁷ Conditional on each draw of all model parameters, we also draw a realization of trend inflation, $\bar{\pi}_t$, and the natural rate, \bar{u}_t , from their respective posterior distributions using the multi-move sampler of Carter and Kohn (1994). All results are based on 20,000 draws after discarding an initial 5000 draws. To check that the sampler had converged, we ran the algorithm multiple times from dispersed sets of starting values and obtained very similar summary statistics regarding the sampled posterior distributions.

For each country, we estimate alternative versions of the model that differ by the number of structural breaks in the variance of innovations to the model shocks, where we consider from $m = 0$ to 4 breaks. To compare alternative values of m , we use the posterior probability of the

⁶ It is worth emphasizing that this multivariate normal approximation is only used for calibrating the variance of innovations to the MH random walk chain, and is not a restriction enforced on the posterior distribution.

⁷ Following the recommendation of Koop (2003), we calibrate c to yield acceptance rates between 0.2 and 0.5.

model with m breaks, $\Pr(m | Y)$, where Y represents the data used in estimation. From Bayes Rule, this probability is proportional to the marginal likelihood of the model with m structural breaks multiplied by the prior probability of m structural breaks:

$$\Pr(m|Y) \propto f(Y|m)\Pr(m).$$

To calculate the marginal likelihood, we use an asymptotic approximation provided by the Schwarz Information Criterion (SIC).⁸ Under fairly general conditions, the SIC statistic is a consistent estimate of the log of the marginal likelihood, and is a popular choice to approximate the marginal likelihood in applied work.⁹ Using this approximation, we then have the following equation for the posterior model probability:

$$\Pr(m | Y) = \frac{e^{SIC_m} \Pr(m)}{\sum_{m=0}^4 e^{SIC_m} \Pr(m)}.$$

Finally, to set the prior model probability, $\Pr(m)$, we give equal prior weight to the case of constant parameters and changing parameters, so $\Pr(m = 0) = 0.5$ and $\Pr(m > 0) = 0.5$. We then assign equal prior probability to each value of $m > 0$ considered, or $\Pr(m) = \frac{1}{8}$, $m = 1, \dots, 4$.

⁸ The SIC is defined in terms of the maximized value of the likelihood function. Exact maximization of the likelihood function for our bivariate UC model with multiple structural breaks is very computationally intensive, as it requires numerical optimization of the likelihood function for all possible combinations of potential break dates. For models with breaks, we instead maximize the likelihood function over the twenty break date combinations that received the highest posterior weight in the Bayesian estimation. We then define the SIC in terms of the maximum likelihood value achieved over these twenty maximizations. This is a generalization of the strategy followed by Wang and Zivot (2000), who define the SIC in terms of the maximized value of the likelihood function evaluated at the median of the posterior distribution for the structural break dates.

⁹ See, for example, Brock, Durlauf and West (2003) and Doppelhofer, Miller and Salai-Martin (2004). For additional discussion of the SIC-based approach to model averaging, see Raftery (1995).

4. Evidence for Structural Breaks in Volatility

We begin by comparing models with alternative numbers of structural breaks in the volatility of model shocks.¹⁰ For each country, Table 4 presents the posterior model probability for alternative numbers of structural break from 0 to 4, constructed as discussed in Section 3. The probabilities provide strong evidence for structural breaks in the volatility of model shocks to inflation and unemployment over the sample periods considered here. For six of the G7 countries, the posterior probability of at least one break is close to 100%. An interesting exception is Canada, for which the model with no structural breaks is strongly preferred. Of the remaining countries, the posterior probability is mixed across different numbers of breaks. For Germany there is strong evidence of a single break, while for the United Kingdom and United States the evidence favors three and four breaks respectively. For France, Italy and Japan, the posterior probability is spread across alternative numbers of breaks. In the results presented below, we average posterior distributions for objects of interest from models that assume a specific number of breaks according to the posterior model probabilities in Table 4. This Bayesian model averaging (BMA) produces inferences that are not conditioned on a particular number of structural breaks, and is the standard Bayesian solution to incorporate model uncertainty.

5. Estimates of Trend Inflation and the Natural Rate

In this section, we first present estimates of trend inflation and estimates of the (potentially) time varying variance of shocks to trend inflation for the G7 countries. We then turn

¹⁰ All reported results are based on models for which the number of lags of the real-activity gap in the inflation gap equation (3) is set equal to four, the residual component of the inflation gap in equation (4) is an AR(1), and the real-activity gap in equation (7) is an AR(2).

to estimates of the natural rate and the unemployment gap, as well as provide some initial evidence regarding the relationship between the inflation gap and the unemployment gap.

5.1 Trend Inflation

Figure 1 displays the actual inflation rate along with the median of the BMA posterior distribution of $\bar{\pi}_t$. There are three similarities across countries that we highlight here. First, there is a general reduction in trend inflation that begins in the late 1970s to early 1980s and continues to near the end of the sample, where trend inflation is at or near its lowest sample level. Second, the estimates of $\bar{\pi}_t$ follow a “hump-shaped” pattern in which trend inflation is lower in the 1960s, higher in the 1970s, and lower again since the early 1980s. The magnitude of these changes varies across countries, and is least pronounced for Germany. Third, for most countries, there are examples of substantial and persistent deviations of trend inflation from actual inflation, which reflects the influence of the inflation gap. The primary exception is France, for which the estimates of trend inflation follow actual inflation rather closely.

Figure 2 displays the median of the BMA posterior distribution of $\sigma_{v,t}^2$, the variance of shocks to trend inflation. This posterior distribution integrates out uncertainty regarding the number and location of structural breaks, which explains the smoother pattern to the posterior median of $\sigma_{v,t}^2$ in some cases than would be suggested by a model with known number and locations of break. The results in Figure 2 can usefully be divided into three groups. First, for the United States, the volatility of shocks to trend inflation display a pattern similar to that observed for the level of trend inflation in Figure 1. In particular, when the level of trend inflation is high, the volatility of shocks to trend inflation is also high. Second, for France, Italy, and the United Kingdom, the volatility of shocks to trend inflation follows a pattern of being high early in the

sample and lower thereafter. Finally, for Canada, Germany and Japan, the volatility of shocks to trend inflation is fairly stable throughout the sample. For all countries, the estimated variance of shocks to trend inflation is near a sample period low at the end of the sample. There is remarkable similarity in the estimated variance of trend inflation in the final sample period, with all variances lying in a tight range from 0.2 (United States) to 0.7 (France).

As discussed earlier, trend inflation is likely closely linked to long-horizon inflation expectations, suggesting the results in Figures 1 and 2 could alternatively be interpreted as results regarding the evolution and uncertainty of long-horizon inflation expectations. Thought of this way, Figures 1 and 2 suggest that long-horizon inflation expectations have played an important role in the determination of actual inflation paths over the sample periods considered here, and that uncertainty regarding long-horizon expectations of inflation is currently at historically low levels in all G7 countries. Given the important role that the credibility of the monetary authority likely has in the determination of the uncertainty regarding long-horizon inflation expectations, it is notable that this recent “anchoring” of inflation expectations exists despite the fact that there are substantial differences in the monetary institutions across these countries, most notably the presence or absence of an inflation targeting framework.

An interesting feature of Figures 1 and 2 is that there are several countries with similar estimated patterns for the level of trend inflation that do not have similar estimated patterns for the volatility of trend inflation, with Canada, the United States, and the United Kingdom providing a leading example. Each of these countries have estimates of trend inflation that follow a hump-shaped pattern over the sample. However, while the United States shows strong evidence of a similar hump-shaped pattern for trend inflation volatility, this pattern is not a feature of the preferred model for the United Kingdom and Canada. In other words, Figures 1 and 2 do not

provide consistent evidence for a link between the level of trend inflation and its volatility, or, cast in terms of inflation expectations, between the level of trend inflation and the uncertainty associated with long-horizon inflation expectations. Notably, these results are not consistent with the traditional hypothesis, prominently argued for in Friedman (1977) and Okun (1971), that there is a strong positive relationship between inflation levels and uncertainty about future inflation, at least not within the G7 countries for the sample periods considered here.

Our modeling strategy uses unemployment to inform estimates of trend inflation and trend inflation volatility. To see the value added of this bivariate approach, it is useful to compare the results in Figures 1 and 2 to those in Cecchetti et al. (2007), who estimate trend inflation in the G7 countries using a univariate UC model of inflation. The deviations of inflation from the Cecchetti et al. (2007) estimates of trend inflation, as displayed in Figure 3.1 of their paper, are in many cases smaller and less persistent than is true for the estimates from our model. For example, for Italy and the United States their estimates of trend inflation track inflation very closely, while our estimates of trend inflation are relatively smooth. In Canada, Germany and the United Kingdom, the deviations of inflation from their measure of trend inflation are much less persistent than the deviations from our estimates displayed in Figure 1. In a number of countries, there are large, persistent deviations of inflation below our measure of trend inflation during periods in the 1980s and 1990s when these economies were in recessions, which is not a feature of the Cecchetti et al. (2007) estimates. As we will show below, this result is driven by substantial deviations of unemployment above the estimated natural rate over these periods, and highlights the role the unemployment gap plays in the identification of trend inflation for our model.

Cecchetti et al. (2007) also find that the variance of trend inflation follows a pattern very similar to the level of trend inflation for most countries, while, as discussed above, our results find much less such similarity. In addition to the information provided by the unemployment rate, another reason for this difference is our consideration of model uncertainty regarding the number of structural changes. As a leading example, Cecchetti et al. (2007) document trend inflation volatility in Canada that follows a hump shaped pattern, undergoing multiple structural breaks. However, the model comparisons presented here suggest that the model with no breaks in Canadian trend inflation volatility is strongly preferred to the model that includes breaks. Thus, consideration of uncertainty regarding the number of structural changes can have very significant effects on the results.

5.2 The Natural Rate and Unemployment Gap

We next present estimates of the natural rate of unemployment and the unemployment gap, which are estimated inside of the bivariate UC model and therefore informed by the behavior of inflation and the NKPC. Figure 3 displays the actual unemployment rate along with the median of the BMA posterior distribution of the natural rate, \bar{u}_t , for each country. For all countries, the natural rate evolves relatively smoothly as compared to actual unemployment, taking persistent, substantial, deviations from the unemployment rate. In Canada, Italy, the United Kingdom, and the United States, the natural rate follows a hump shaped pattern, while for France, Germany and Japan the natural rate drifts upward over the entire sample. In several countries, the natural rate rises noticeably during the late 2000's global recession. This is especially true in the United States, where the natural rate at the end of the sample is at a sample period high of 7.4%

Figure 4 displays the median of the BMA posterior distribution of the unemployment gap, which is a primary theoretical driver of the inflation gap in the NKPC with trend inflation. As a first pass to gauge the empirical relationship between the unemployment gap and the inflation gap, Figure 4 also presents the median of the BMA posterior distribution of the inflation gap. From Figure 4, there is strong visual evidence of such a relationship in many G7 countries. In particular, for most countries, there is a striking pattern of persistent positive unemployment gaps being associated with persistent negative inflation gaps and vice-versa. This pattern appears strongest in Canada, France, Germany, the United Kingdom and United States, and weaker in Japan and Italy. Overall, this is suggestive evidence of an important role for the unemployment gap in determining the inflation gap in G7 countries. In the next section we expand on this evidence, providing quantitative measures of the importance of this relationship.

6. Contribution of Inflation Components to Inflation Variation

This section presents results regarding the contribution of the various inflation drivers in the model of (1)-(7) to inflation volatility. We begin by documenting the relative contribution of the inflation trend and inflation gap to the variance of inflation changes. We then turn to the relative contribution of the unemployment gap and the residual component to the variance of the inflation gap.

6.1 Determinants of Inflation Volatility

To measure the relative importance of the inflation trend vs. the inflation gap for inflation volatility, we construct variance decompositions. As the model assumes that inflation contains a unit root, we focus on explaining the volatility of inflation changes at various horizons. Then, to

measure the contribution of the inflation trend to variation in j -quarter inflation changes, we construct the following ratio:

$$V_{j,t} = \frac{v(\bar{\pi}_t - \bar{\pi}_{t-j})}{v(\pi_t - \pi_{t-j})},$$

where $v(\cdot)$ denotes a variance. For models with structural changes in the volatility of model shocks, $V_{j,t}$ may vary with t . To obtain draws from the posterior distribution for $V_{j,t}$ based on a model with a particular value of m , we compute $V_{j,t}$ analytically for each draw from the posterior distribution of the model parameters. We then obtain draws from the BMA posterior distribution for $V_{j,t}$ using the model-specific draws and the posterior model probabilities in Table 4. Given that inflation is determined by trend inflation and the inflation gap only, and shocks to these components are independent, the contribution of the inflation gap to the variance of j -quarter inflation changes is simply $1 - V_{j,t}$.

Figure 5 presents the median of the posterior distribution for $V_{j,t}$ at horizons of $j = 1, 4,$ and 20 quarters. There are several points from Figure 5 that we highlight here. First, there is a general increase in $V_{j,t}$ as the horizon increases. This is not surprising, as we would expect the inflation trend, which represents the unit root process in inflation, to eventually dominate the variation of inflation changes as the horizon grows. Second, there is substantial variation in $V_{j,t}$ over time for many countries. This implies that changes in the variance of trend inflation shocks documented in Figure 2 were not mirrored by proportional changes in the variability of the inflation gap. As an example, in some cases, notably Italy and the United Kingdom, $V_{j,t}$ is far

from sample period lows at the end of the sample, despite the variance of trend inflation shocks being at sample period lows.

Third, both trend inflation and the inflation gap have played an important role in the variance of inflation changes at business cycle horizons. Focusing on four-quarter inflation changes, the average value of $V_{4,t}$ ranges from a minimum of 0.17 (Japan) to a maximum of 0.61 (France), and is between 0.2 and 0.4 for all other countries. Thus, on average over the past 50 years, both the inflation trend and inflation gap has contributed considerable amounts to the variance of four-quarter inflation changes. Looking at only the most recent sample period, $V_{4,t}$ ranges from between 0.04 (United States) to 0.69 (France), and lies between 0.2 and 0.5 for all other countries. Thus, for most countries, the United States being the primary exception, both the inflation trend and inflation gap continue to contribute to the variance of inflation changes in recent years. In the United States, the inflation gap dominates the variance of one and four-quarter inflation changes at the end of the sample period.

6.2 Determinants of Inflation Gap Volatility

The results in Figure 5 suggest the inflation gap plays an important role in the determination of actual inflation in G7 countries. Given this, we now turn to results regarding the determination of the inflation gap. We are interested in the relative contribution of the drivers of the inflation gap in equation (2), namely the unemployment gap and the residual component, to the variance of the inflation gap. To measure these contributions, we again construct variance decompositions. As the inflation gap is covariance stationary in our model, we focus on explaining the variance of the level of the inflation gap. In particular, we construct the following ratio:

$$R_t = \frac{v\left(\sum_{j=0}^{p_x} \delta_j x_{t-j}\right)}{v(\pi_t^g)},$$

which gives the proportion of total inflation gap variance accounted for by the unemployment gap component. Draws from the BMA posterior distribution for R_t are obtained as described for $V_{j,t}$ above. As the inflation gap is determined by the unemployment gap and residual component only, and shocks to these components are independent, the contribution of the residual component to inflation gap variance is given by $1 - R_t$.

Figure 6 presents the median of the BMA posterior distribution of R_t , and provides evidence that the unemployment gap has been an important contributor to inflation gap variance for all G7 countries. The average value of R_t over the entire sample period ranges from a minimum of 0.31 (Japan) to 0.73 (United States), and exceeds 0.5 for all countries except Japan and Italy. For most countries, the importance of the unemployment gap is at or near historic highs in recent years. The exception is the United States, for which R_t declines steadily over the sample period and obtains the lowest end-of-sample value in the G7 countries. However, even this minimum value suggests that the unemployment gap provides more than 30% of the variance in the U.S. inflation gap.

To isolate the sources of time variation in the value of R_t , Figure 7 presents the BMA posterior distribution of the variance of the inflation gap, the variance of the unemployment gap component of the inflation gap, and the variance of the residual component of the inflation gap. For most countries, the variance of the unemployment gap component is relatively stable over the sample period as compared to the variance of the residual component, meaning that significant historical variation in R_t in Figure 6 is driven primarily by changes in the variance of

the residual component. An exception is the United States, where the variances of both components have displayed significant variation over the sample period. Even here however, Figure 7 demonstrates that the secular decline in R_t documented for the United States is due primarily to a steady increase in the variance of the residual component, rather than a decline in the variance of the unemployment gap component.

Although the contribution of the real-activity gap to inflation gap variance is substantial, the value of R_t in Figure 6 also reveals a significant role for the residual component. Further, the results in Figure 7 show that the residual component has been an important contributor to time-variation in the variance of the inflation gap. As the residual component represents inflation variation that is explained by factors outside the theoretical framework provided by the NKPC with trend inflation, it is worth asking what these factors might be. A primary candidate is supply shocks, which are not directly measured in our bivariate UC model. To investigate this possibility, Figure 8 plots the BMA posterior distribution of the residual component of the inflation gap along with the (demeaned) inflation rate of the food and energy price component of the CPI for each country. This “non-core” component of inflation is often singled out as being disproportionately affected by supply shocks. For most countries, there is substantial comovement between the estimated residual component of the inflation gap and the non-core inflation rate. Although there are clear examples where the two series diverge, many of the swings in non-core inflation are mirrored in the residual component of the inflation gap. In recent years this is particularly true, during which time the residual component mirrors the relatively large swings in non-core inflation very closely. The full-sample correlation coefficient for the two series ranges from 0.26 (United Kingdom) to 0.72 (United States), and is between 0.42 and 0.69 for all other countries. The relative volatility between the series is also similar, with a

primary exception being France, for which the estimated residual component is substantially smoother than the non-core inflation rate.

Taken together, these results suggest the inflation gap has contributed significantly to the variability of inflation changes in all of the G7 countries over the past 40-50 years, and that the unemployment gap has been a substantial and consistent driver of the inflation gap for these countries. This suggests that the NKPC, once augmented to include a trend inflation component, is an empirically relevant model for the G7 countries. That said, there is also a substantial portion of the inflation gap, measured by the residual component in our model, that is driven by factors external to the NKPC theory. This residual is highly correlated with food and energy price inflation in most countries, suggesting supply shocks as an important additional driver of transitory fluctuations in inflation.

7. Conclusion

We have estimated a bivariate unobserved components model of inflation and unemployment in the G7 countries using Bayesian techniques, and used it to shed light on the relative importance of trend inflation and the real-activity gap for explaining variability in realized inflation. Our results reveal that both trend inflation and the deviation of inflation from trend inflation, or the so-called inflation gap, have contributed significantly to variation in inflation changes at business cycle horizons in the G7 countries. Further, we find that the real-activity gap, measured as the deviation of the unemployment rate from its natural rate, is an important determinant of the inflation gap for these countries. Overall, the results suggest that the NKPC with trend inflation is an empirically relevant model for the G7 economies.

We have also provided new estimates of trend inflation in the G7 countries that take into account information in the real-activity gap for identification, as well as formal comparisons of models with and without time-variation in the volatility of model shocks. These comparisons reveal important changes in the volatility of trend inflation in some countries but not others. Both the level and volatility of trend inflation is quite low in all countries near the end of the sample period, which is suggestive that long-horizon inflation expectations are anchored at low levels across the G7 economies.

Our analysis is based on in-sample estimation, and does not address the issue of out-of-sample forecasting, where the performance of traditional Phillips Curve type models has been disappointing (Stock and Watson, 2009). Even if the NKPC with trend inflation is a correct characterization of inflation dynamics, producing out-of-sample forecasts from the bivariate UC model is complicated by the need for end-of-sample unemployment gap estimates, which, as is detailed in Orphanides and Van Norden (2002, 2005), can be unreliable. However, there is some encouraging recent evidence in this regard for the United States, as both Stock and Watson (2010) and Manopimoke (2010) find that versions of the NKPC with trend inflation improve out of sample forecasts of the U.S. inflation gap over univariate models.

A primary focus of our analysis has been on explaining the determinants of the inflation gap. However, for many countries, trend inflation has also been an important contributor to the variability of observed inflation. This result suggests it is important to understand the determination of trend inflation in order to adequately explain the historical path of actual inflation in most countries. Given the link between trend inflation and long-horizon inflation expectations, one approach to understand the evolution of trend inflation is to understand the determinants of long-horizon inflation expectations. To this end, a number of recent studies,

including Clark and Davig (2008) and Kiley (2008) for U.S. inflation and Barnett, Groen and Mumtaz (2009) for U.K. inflation have investigated the effects of various types of shocks on long-horizon inflation expectations. Further research on this topic is likely to be an important avenue for improved understanding of the inflation process in the G7.

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Table 1
Sample Periods

	Data Sample
Canada	1961:Q2 – 2010:Q2
France	1968:Q1 – 2010:Q2
Germany	1962:Q2 – 2010:Q1
Italy	1960:Q2 – 2010:Q1
Japan	1970:Q2 – 2010:Q2
United Kingdom	1963:Q2 – 2010:Q2
United States	1957:Q2 – 2010:Q2

Table 2
Dummy Variable Dates

	Date	Event
Canada	1991:1	Cigarette Tax Change
	1994:1 – 1994:2	Cigarette Tax Change
Germany	1991:1-1991:4	Reunification
	1993:1	VAT Introduction
Japan	1997:2	Consumption Tax Increase
United Kingdom	1990:2	Poll Tax Introduction

Table 3
Augmented Dickey Fuller Tests

	Unemployment Rate	CPI Inflation
Canada	0.45	0.27
France	0.85	0.29
Germany	0.12	0.06
Italy	0.46	0.57
Japan	0.22	0.00
United Kingdom	0.14	0.30
United States	0.18	0.06

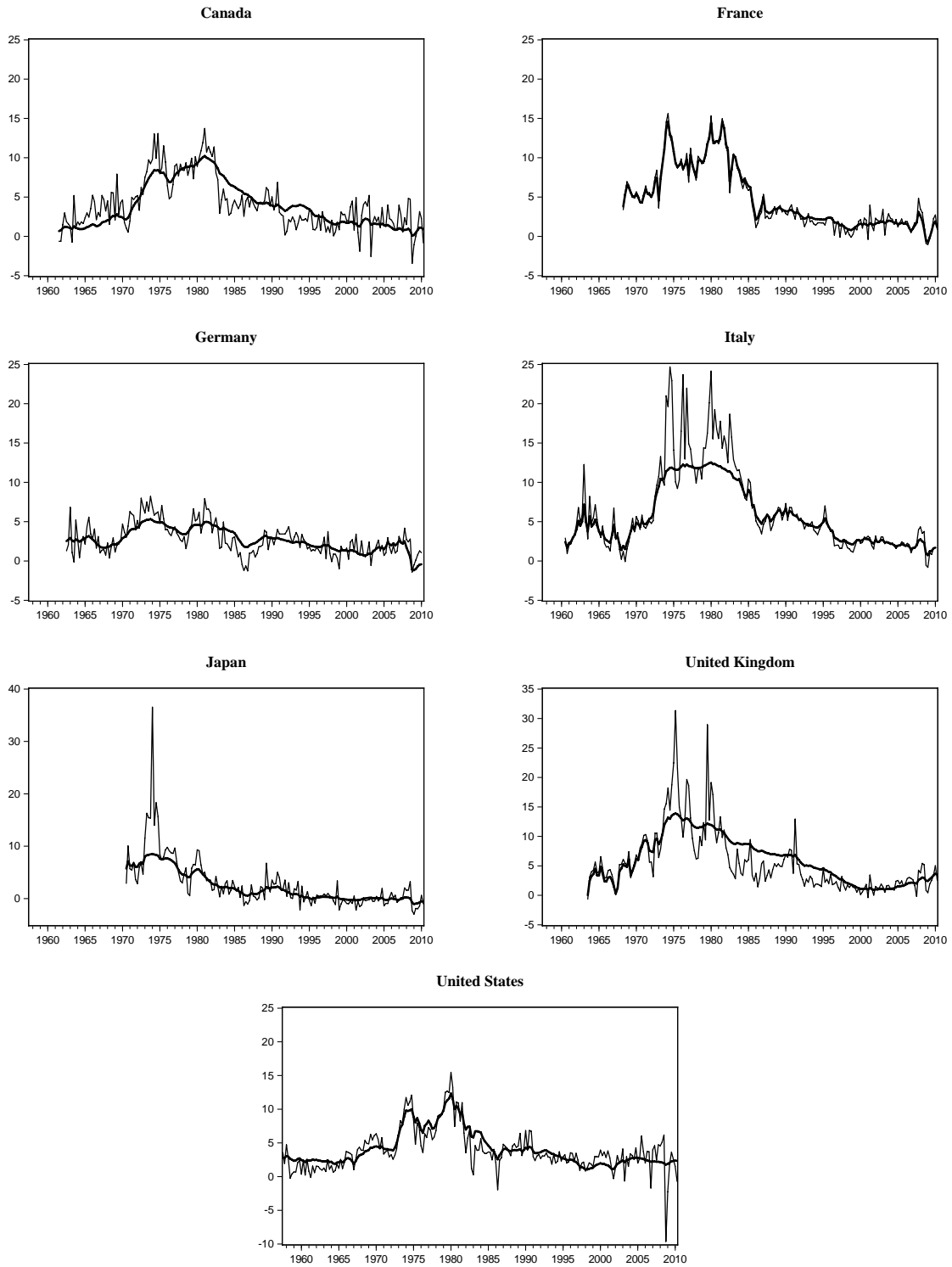
Notes: The table contains the MacKinnon (1996) p-values for the Augmented Dickey Fuller (ADF) test for a unit root in the indicated series. Lag lengths were selected using the Akaike Information Criteria based on a maximum lag length of 12 quarterly lags. The ADF test regression included an intercept for all series, and included a deterministic trend for French and Japanese inflation and French, German, and Japanese unemployment.

Table 4
Posterior Probability for Number of Breaks in Variance of Model Shocks

	Number of Breaks				
	0	1	2	3	4
Canada	99.2%	0.8%	0.0%	0.0%	0.0%
France	0.0%	1.3%	54.3%	44.4%	0.0%
Germany	2.2%	93.9%	0.0%	0.0%	3.9%
Italy	0.0%	0.0%	10.7%	17.8%	71.5%
Japan	0.0%	69.6%	18.5%	11.9%	0.0%
United Kingdom	0.0%	0.1%	0.0%	99.9%	0.0%
United States	0.0%	0.0%	0.0%	1.8%	98.2%

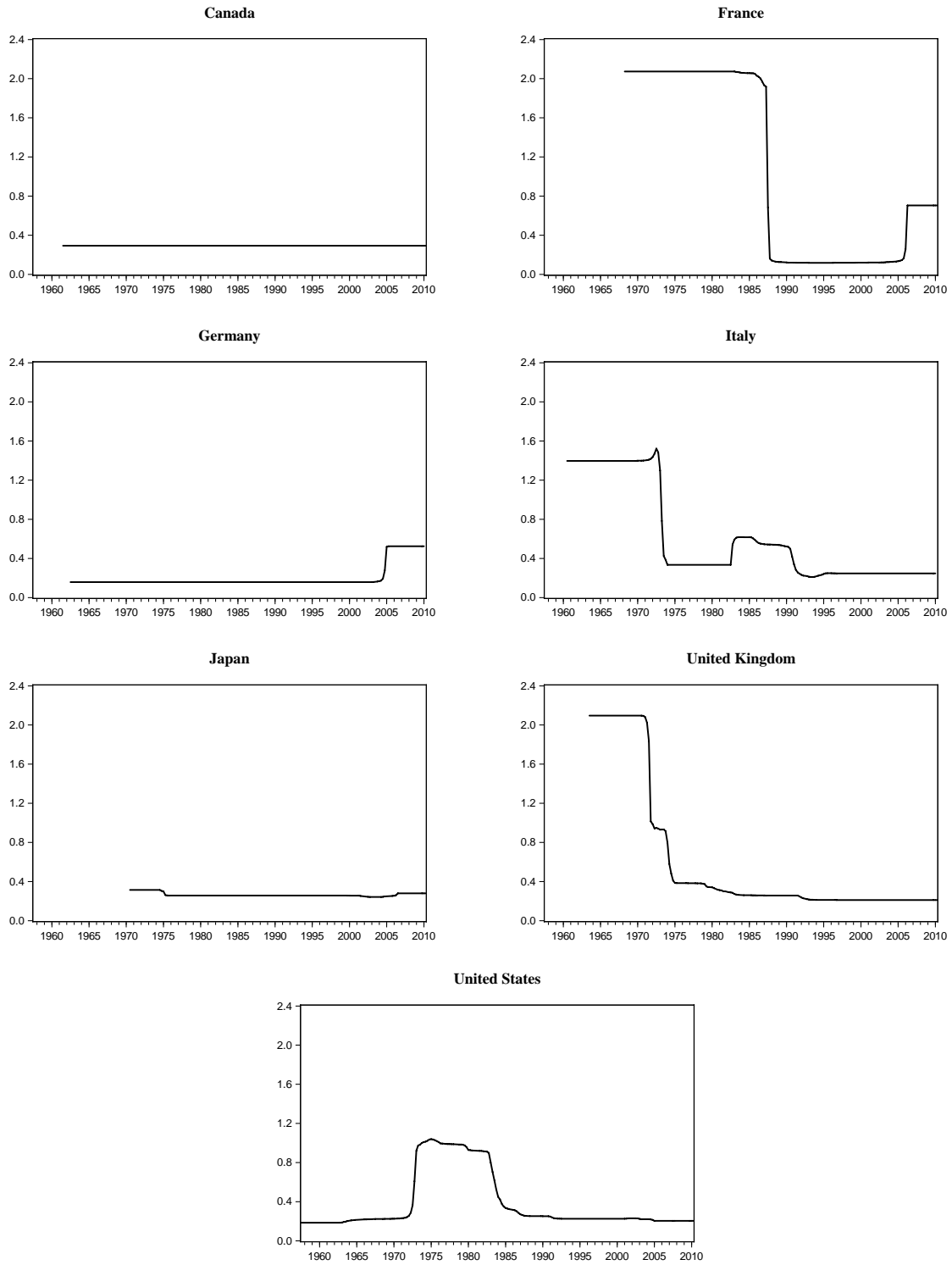
Notes: The table contains the posterior probability of alternative numbers of structural breaks in the variances of model shocks. Posterior probabilities are based on the asymptotic approximation given by the SIC, as discussed in Section 3.

Figure 1
Inflation and Estimated Trend Inflation



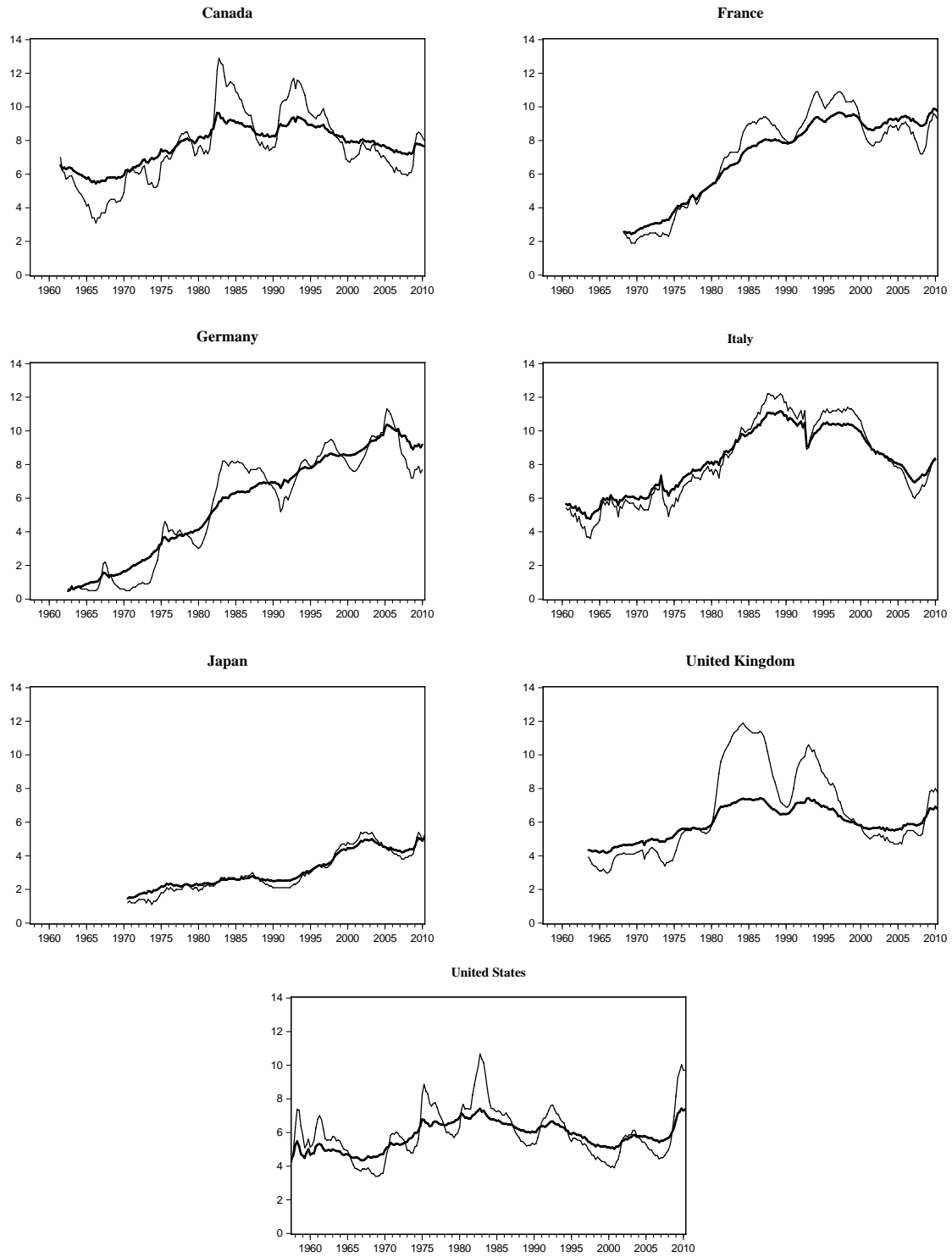
Notes: The figures plot the quarterly inflation rate (thin line), measured using the CPI, along with the median of the Bayesian posterior distribution for trend inflation (thick line). Note that the vertical scale is unique for Japan, the United Kingdom and the United States.

Figure 2
Variance of Shocks to Trend Inflation



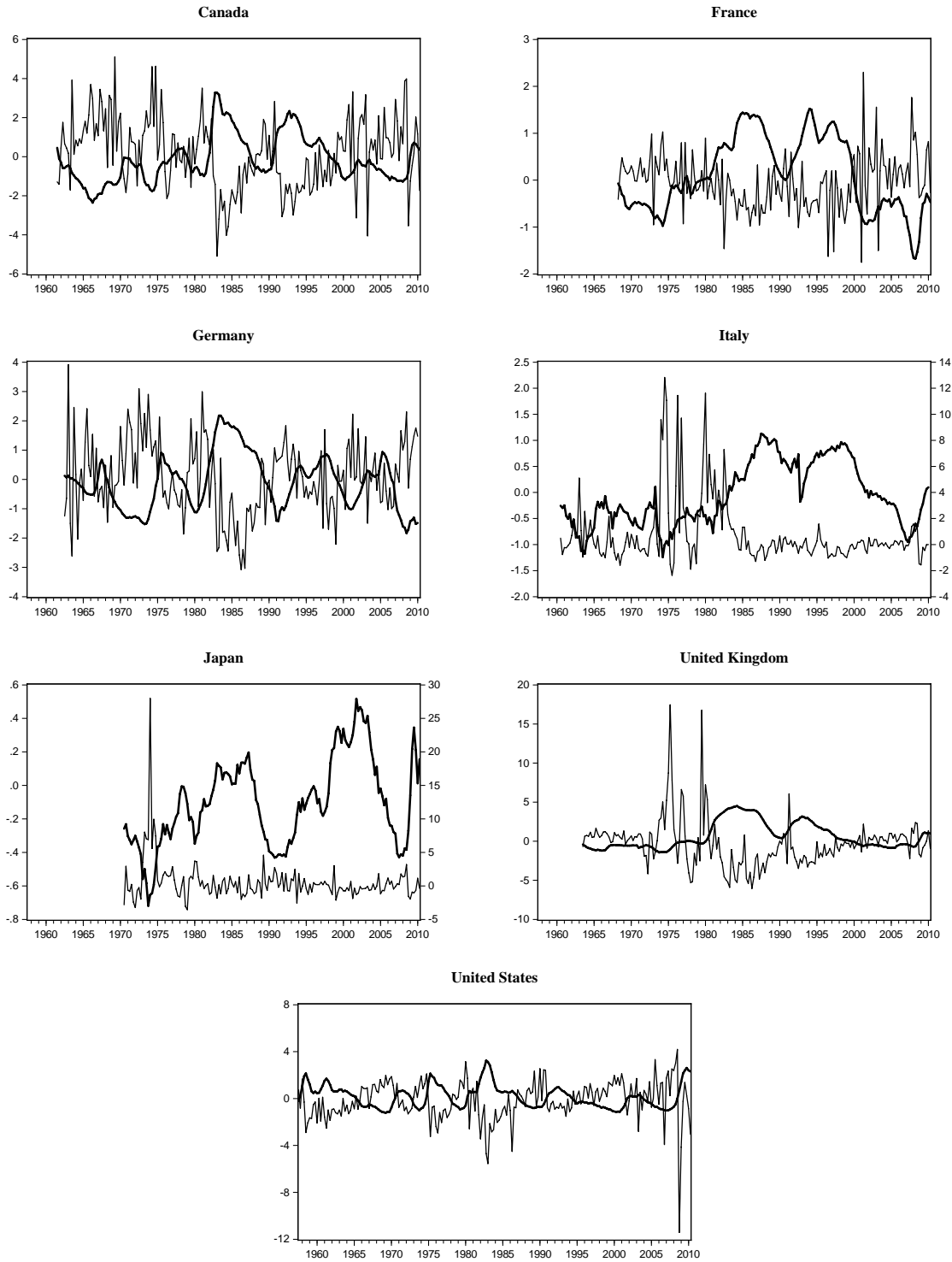
Notes: The figures plot the median of the Bayesian posterior distribution of the variance of shocks to trend inflation.

Figure 3
Unemployment Rate and Estimated Natural Rate



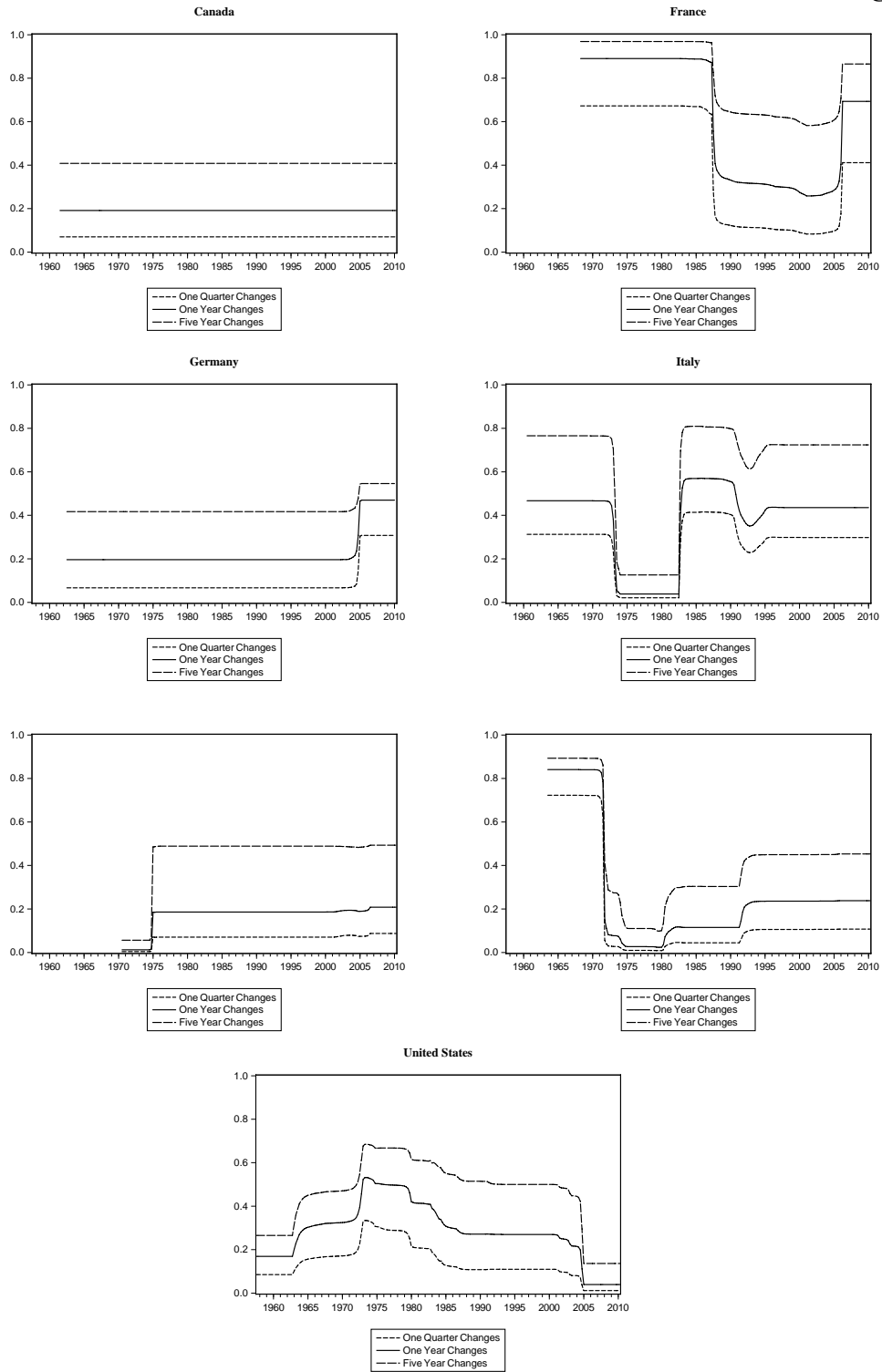
Notes: The figures plot the quarterly unemployment rate (thin line) and the median of the Bayesian posterior distribution for the natural rate of unemployment (thick line).

Figure 4
Estimated Unemployment Gap and Inflation Gap



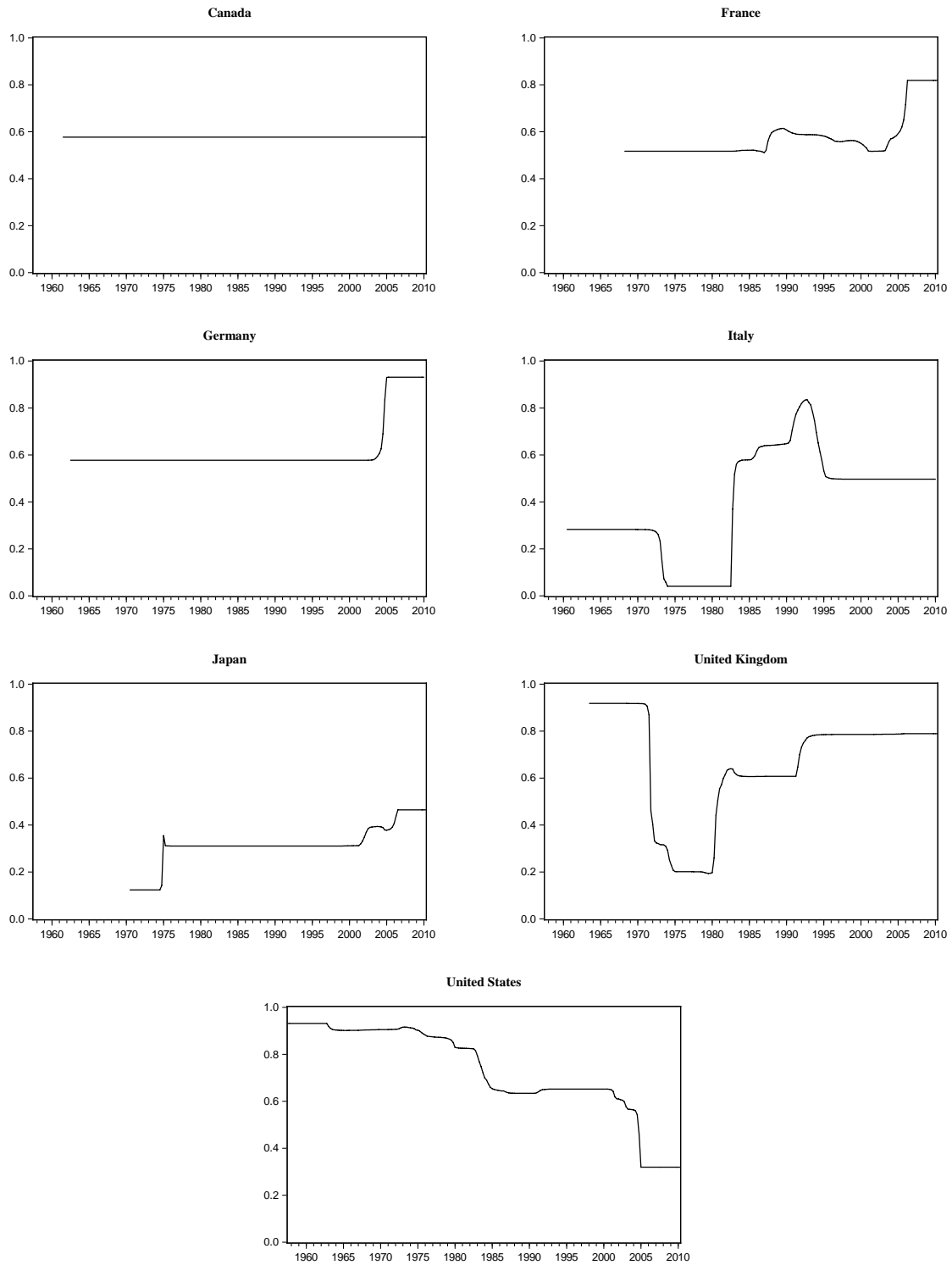
Notes: The figures plot the median of the Bayesian posterior distribution for the inflation gap (thin line) and the unemployment gap (thick line). For Italy and Japan, the left vertical axis refers to the unemployment gap, while the right vertical axis refers to the inflation gap.

Figure 5
Contribution of Inflation Trend to Variance of Inflation Changes



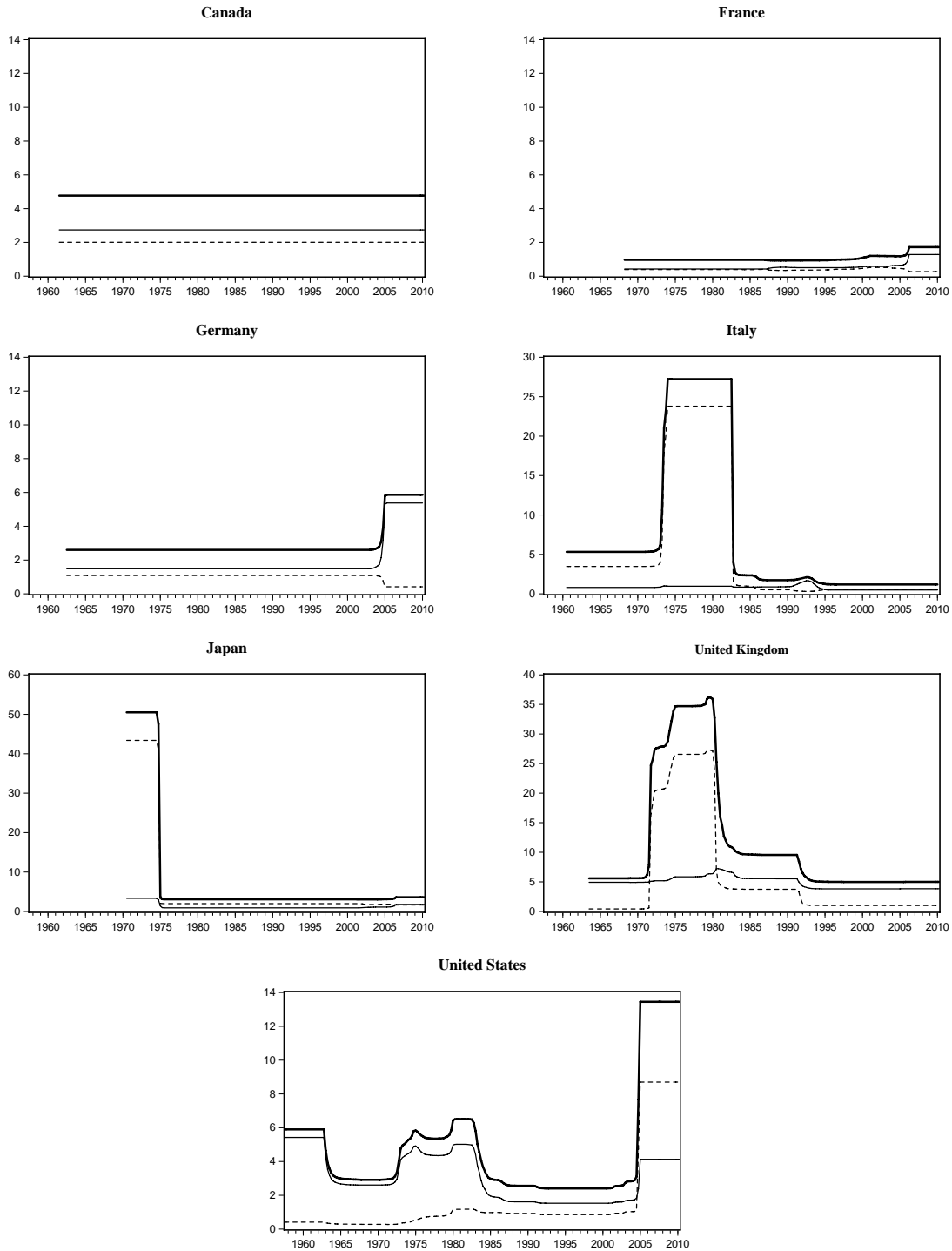
Notes: The figures plot the median of the Bayesian posterior distribution of the proportion of the variance of actual inflation changes at alternative horizons accounted for by the variance of changes to trend inflation.

Figure 6
Contribution of Unemployment Gap to Variance of Inflation Gap



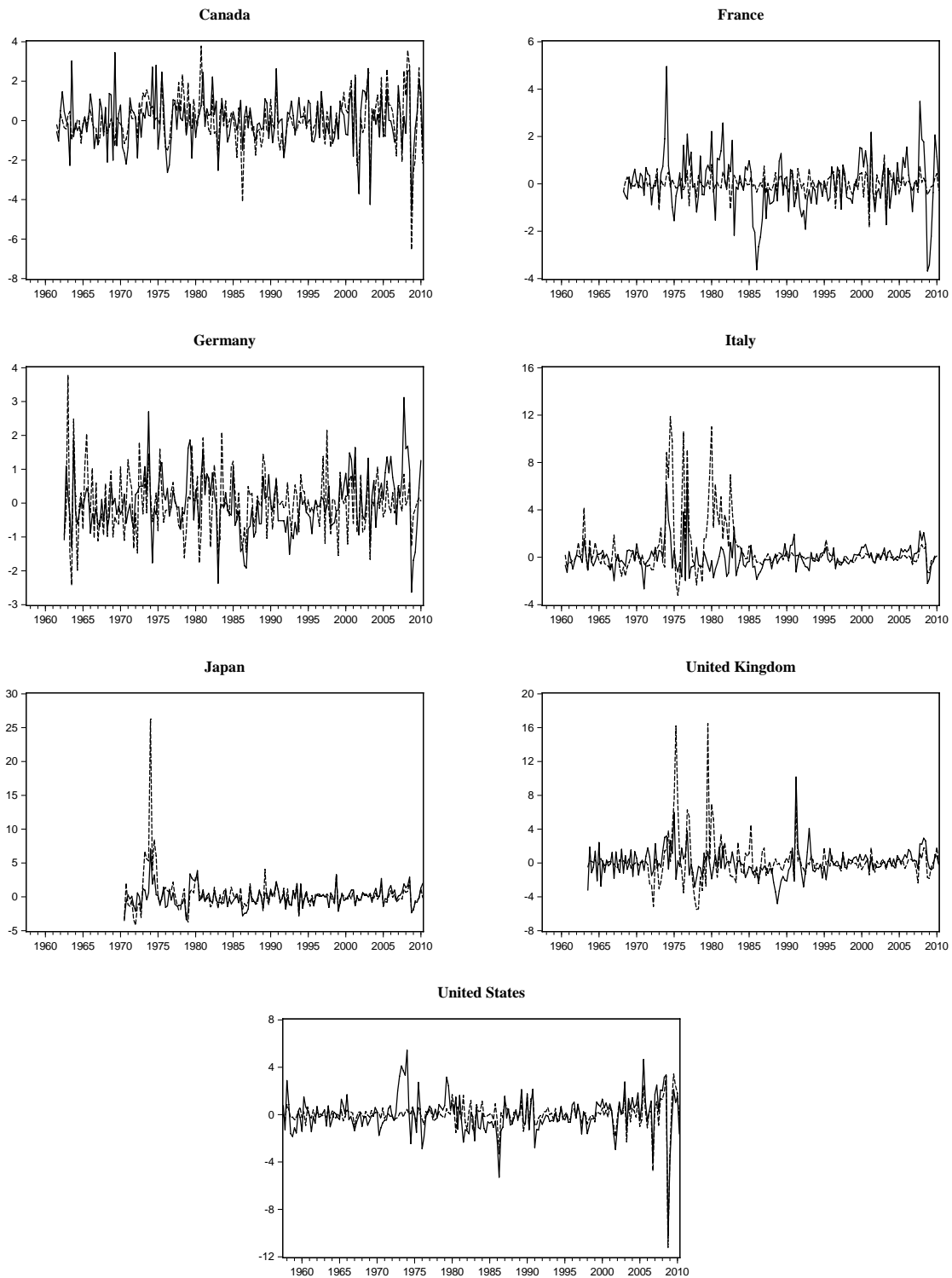
Notes: The figures plot the median of the Bayesian posterior distribution of the proportion of the variance of the inflation gap accounted for by the variance of the unemployment gap.

Figure 7
Variance of Inflation Gap and Inflation Gap Components



Notes: The figures plot the median of the Bayesian posterior distribution of the variance of the inflation gap (thick solid line), the variance of the unemployment gap component of the inflation gap (thin solid line), and the variance of the residual component of the inflation gap (thin dashed line). Note that the vertical scale is unique for Italy, Japan, and the United Kingdom.

Figure 8
Inflation Residual Component and Food and Energy Price Inflation



Notes: The figures plot the demeaned quarterly rate of CPI food and energy price inflation (solid line), along with the median of the Bayesian posterior distribution for the residual component of the inflation gap (dotted line). Note that the vertical scale differs across countries.