RESEARCH DIVISION

Working Paper Series

Input and Output Inventory Dynamics

Yi Wen

Working Paper 2011-008A
https://doi.org/10.20955/wp.2011.008

March 2011

FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
P.O. Box 442
St. Louis, MO 63166

Input and Output Inventory Dynamics

Yi Wen*

Abstract

This paper develops an analytically tractable general-equilibrium model of inventory dynamics based on a precautionary stockout-avoidance motive. The model's predictions are broadly consistent with the U.S. business cycle and key features of inventory behavior. It is also shown that technological improvement of inventory management can increase, rather than decrease, the volatility of aggregate output. Key to this seemingly counterintuitive result is that a stockout-avoidance motive leads to a procyclical shadow value of inventories, which acts as an automatic stabilizer that discourages sales in booms and encourages demand in recessions, thereby reducing the variability of GDP.

Keywords: Input-and-Output Inventories, Stockout Avoidance, Countercyclical Stock-to-Sales Ratio, Great Moderation, Business Cycle.

JEL codes: D57, E13, E22, E32.

---

*Yi Wen, Research Department, Federal Reserve Bank of St. Louis, P.O. Box 442, St. Louis, MO, 63166 (e-mail: yi.wen@stls.frb.org). The author is also a professor at School of Economics and Management, Tsinghua University, Beijing, China. This article is a significantly revised version of an earlier working paper (Yi Wen, 2008). I thank three anonymous referees, Mark Bils, Jim Kahn, Lou Maccini, Scott Schuh, Pengfei Wang, and the audience at the 2009 ISIR-AEA Meetings for comments. I also thank Mingyu Chen and Luke Shimek for research assistance and George Fortier for editorial assistance. The usual disclaimer applies.

Industrial economies typically hold large stocks of inventories relative to sales. More importantly, inventory investment is procyclical over the business cycle. For example, for the postwar period, the stock of finished goods inventories is about 60% of quarterly GDP and 83% of aggregate consumption, even though the change of inventory stocks (inventory investment) accounts for less than 1% of GDP. In the meantime, aggregate inventory investment is about 20 times more volatile than GDP and accounts for the bulk of fluctuations in output (see, e.g., Alan S. Blinder, 1981; Blinder and Louis J. Maccini, 1991). It has also been noted that finished goods inventories are procyclical only at the business cycle frequency but countercyclical at higher frequencies (Andreas Hornstein, 1998; and Wen, 2005a). Moreover, in spite of the procyclical inventory investment over the business cycle, the ratio of inventory stock to sales is countercyclical (Mark Bils and James A. Kahn, 2000).

These stylized facts of inventory behaviors cannot be explained by the traditional production-smoothing hypothesis (see, e.g., Blinder, 1986) and have been taken in the literature as indicating alternative but often mutually conflicting theories. For example, procyclical inventory investment may be viewed as supporting either increasing returns to scale or technology-shock models (e.g., Finn E. Kydland and Edward C. Prescott, 1982). Yet the countercyclical stock-to-sales ratio may suggest procyclical marginal cost or imperfect competition with countercyclical markups (Bils and Kahn, 2000).

Despite the importance of inventories in economic activity and their potential role in understanding the business cycle, comprehensive general-equilibrium analysis of inventories with explicit microfoundations has been surprisingly rare. The bulk of the inventory literature uses partial-equilibrium models to analyze inventory behavior, and, in the analyses, interactions between input and output inventories are often neglected (as noted and emphasized by Valerie A. Ramey, 1989; Brad R. Humphreys, Maccini, and Scott Schuh, 2001). This neglect of input inventories may be a serious drawback in business-cycle studies because the average inventory-to-sales ratio for intermediate goods (including raw materials and work in process) is two times larger than that of finished goods, and input inventory investment can be three times more volatile (Humphreys et al., 2001).[1]

Partial-equilibrium analysis is not satisfactory for addressing some important macroeconomic questions because it treats prices, marginal costs, and sales as exogenous. Such a

[1]Important empirical works based on partial-equilibrium analysis include Olivier J. Blanchard (1983), Blinder (1986), Daniele Coen-Pirani (2004), Martin Eichenbaum (1989), John Haltiwanger and Maccini (1988), Kahn (1987, 1992), Ramey (1989, 1991), Ramey and Kenneth D. West (1999), Wen (2005a), and West (1986), among many others.

practice fails to take into account the dynamic interactions between supply and demand and the impact of inventories on sales and prices. There have been attempts in the literature to include inventories in general-equilibrium models; however, this line of general-equilibrium research relies on reduced-form analysis rather than on the microfoundations of inventory behavior. For example, inventories are treated as a factor of production (equivalent to fixed capital) by Kydland and Prescott (1982) and Lawrence Christiano (1988), whereas they are treated as a source of household utility (equivalent to durable consumption goods) by Kahn, Margaret M. McConnell, and Gabriel Perez-Quiros (2002) and Matteo M. Iacoviello, Schiantarelli Fabio, and Schuh (forthcoming). In such reduced-form inventory models, the crucial question of why firms hold inventories is sidestepped and inventories are by definition essential to the economy (Aubik Khan and Julia K. Thomas, 2007a).[2]

This paper develops an analytically tractable general-equilibrium model of inventories with explicit microfoundations. Inventories exist in the model because of a precautionary stockout-avoidance motive in situations where production/delivery takes time and demand is uncertain (Kahn, 1987). It is shown that the general-equilibrium stockout-avoidance theory is broadly consistent with the stylized facts of inventory behavior, including (i) a large stock-to-sales ratio and a small inventory investment-to-GDP ratio in the steady state, (ii) excess volatility of production relative to sales, (iii) input inventories that are more volatile than output inventories, (iv) procyclical inventory investment but a countercyclical inventory-to-sales ratio at the business cycle frequency, and (v) countercyclical inventory investment at the high frequencies (i.e., frequencies shorter than the business cycle) for final consumption goods.

The main intuition behind the success of the model is as follows: (i) To prevent stockout, firms produce to meet an optimal target-inventory level based on the expectation of idiosyncratic demand shocks. Production then moves more than one-for-one with sales so as to replenish inventories on the one hand and prevent anticipated future stockout on the other hand. This results in procyclical inventory investment and confirms the partial-equilibrium analysis of Kahn (1987). (ii) The optimal inventory target-to-sales ratio itself is decreasing in the marginal cost of production because the shadow rate of return to inventory investment (a liquidity premium) is determined by the probability of stockout. Under aggregate

[2] As another example, the reduced-form inventory model of Bils and Kahn (2000) is similar to the cash-in-advance model of money, which assumes that the "inventory-in-advance" constraint always binds. But, as is clear from the analysis in this paper, such constraints do not bind most of the time under the stockout-avoidance motive.

demand shocks, a higher marginal cost requires a higher rate of return to inventory assets (i.e., a higher liquidity premium) for firms to carry inventories, thus calling for a higher probability of stockout. Hence, even with perfect competition and zero markups, inventory stock does not keep pace with sales one-for-one, leading to a countercyclical stock-to-sales ratio. (iii) The steady-state aggregate inventory-to-sales ratio can be large without a large variance of aggregate shocks because the fraction of firms with positive inventories can be large, depending on the strength of the stockout-avoidance motive.[3] This prediction is in contrast to the claims made by Khan and Thomas (2007b). Also, a large aggregate inventory stock-to-sales ratio is consistent with a small aggregate inventory investment-to-sales ratio if the rate of depreciation of inventories is small, so that the need of replenishment is small in the steady state. (iv) Input inventories are more volatile than output inventories because procyclical inventory investment amplifies the volatility of production relative to sales along input-output chain of production. A one percent increase in final sales can trigger a more than one-for-one increase in the production of finished goods because of procyclical inventory investment under the optimal target-inventory policy. This leads to a much larger increase in the demand for intermediate goods because of diminishing marginal product. Hence, orders of intermediate goods and input inventory investment have to increase even more under the stockout-avoidance motive. This finding provides general-equilibrium support to the empirical analysis of Humphreys, Maccini, and Schuh (2001). (v) Because finished goods inventories are a better buffer than fixed capital in meeting unexpected consumption demand, they tend to be countercyclical at the very high frequencies (i.e., during the impact period of a demand shock). This is consistent with the findings of Hornstein (1998) and Wen (2005a).

This paper is closely related to Khan and Thomas (2007b) and Iacoviello et al. (forthcoming). Khan and Thomas (2007b) evaluate two explanations for inventories—the (S,s) and stockout-avoidance motives—within dynamic stochastic general-equilibrium environments.[4] They find that the (S,s) model is consistent with the cyclical behavior of aggregate inventories in the data, whereas the converse is true for a stockout-avoidance model. In particular, they argue that the (S,s) model succeeds in explaining the average magnitude of inventories in the U.S. economy and the stockout-avoidance model does not and appears incapable of sustaining inventories alongside capital. This paper shows that the stockout-avoidance motive is capable of explaining inventory behavior in a general-equilibrium framework. Under

[3] According to Bils (2004), the probability of stockout at the firm level is very small, about 8%. This suggests that firms have strong incentives to hold large amounts of inventories.

[4] Also see Jonas D. Fisher and Hornstein (2000) and Oleksiy Kryvtsov and Midrigan Virgiliu (2009) for general-equilibrium analyses of the (S,s) model. .

the stockout-avoidance motive, idiosyncratic demand shocks are crucial for inducing firms to hold a large amount of inventory despite the rate of return dominance by capital, because inventories yield a liquidity premium (like money) in facilitating sales but capital does not. Such a liquidity value of inventories is not well captured in the stockout models of Khan and Thomas.

Iacoviello et al. (forthcoming) study input and output inventories in general equilibrium. An interesting innovation of their work is to differentiate the service sector (which does not hold inventories) from the goods sector (where inventories are important). However, their model is a reduced-form model in which finished-good inventories enter the utility function as a durable consumption good and input inventories enter the production function as a capital good. To address certain questions, such short cuts may be useful abstractions and simplifications. However, to address some business-cycle issues, such as how the stockout-avoidance motive influences inventory investment decisions and output fluctuations, specific microfoundations of why firms hold inventories may be needed. For example, if inventories yield utility or enter the production function, it rules out the possibility of stockout by assumption. Yet, how firms adjust inventories to avoid stockout in response to business-cycle shocks requires that the stockout-avoidance motive be modeled explicitly and rigorously. It is precisely through such a rigorous approach to the stockout-avoidance motive that this paper is able to uncover an important role of inventories in the business cycle—that inventories can help stabilize, rather than destabilize, the macroeconomy.

In particular, when the general-equilibrium stockout-avoidance model is used as a laboratory to assess the contributions of inventory fluctuations to output volatility, it is found that technological improvement in inventory management that eliminates the information frictions (or production/delivery lags) can *increase* (not *decrease*) the variance of aggregate output. Key to this seemingly counterintuitive result is that a stockout-avoidance motive leads to a procyclical shadow value of inventories (and hence a procyclical relative price of final goods), which acts as an automatic stabilizer that discourages sales in booms and encourages demand in recessions, thereby reducing the variability of GDP.[5] For example, when the model is calibrated to match the inventory-to-sales ratio of the U.S. economy, further reducing inventories by eliminating the production/delivery lags from the model could increase the variance of output by as much as 6% to 30% under aggregate demand shocks. This provides a counterexample to the widely held belief that inventories are destabilizing

[5] That is, production being more volatile than sales could imply that inventories stabilize sales rather than destabilize production.

and played a key role in the "Great Moderation" of the U.S. economy after the mid 1980s (see, e.g., Kahn, McConnell, and Perez-Quiros, 2002).

This result is akin to the finding of Khan and Thomas (2007a). Using a general-equilibrium model based on the (S,s) inventory theory, they show that inventory fluctuations have little impact on the business cycle. However, the analysis of Khan and Thomas (2007a) is based on only one of the possible microfoundations of inventories. It is not immediately clear whether their results and explanations are generalizable and applicable to models based on other microfoundations, such as the stockout-avoidance mechanism considered in this paper. My analysis suggests that it is important to develop and investigate alternative general-equilibrium inventory models with different microfoundations. While Khan and Thomas' (2007a) analysis indicates that inventories destabilize the economy insignificantly and are thus inessential for understanding the business cycle, my analysis suggests that inventories are important for the business cycle, albeit for the opposite reason: They stabilize rather than destabilize the macroeconomy. Nonetheless, both our results share one thing in common: The general-equilibrium effect of procyclical inventory investment reduces the variability of final sales (although for fundamentally different reasons). This suggests that, if inventories are indeed destabilizing in the real world as many people have believed, some unknown forms of market structures or distortions must be important but not captured by Khan and Thomas (2007a) and this paper.

I. The Model

A. Household

A representative household has preferences over a spectrum of finished goods indexed by $j \in [0, 1]$. From the producer's point of view, these goods are homogenous because they are produced by the same production technology with the same costs. However, the goods have different colors and yield different utilities to the household because they are not perfect substitutes in the household's utility function. The household purchases finished goods in a competitive market and is able to store them in refrigerators if needed (refrigerator j stores good j).[6] The costs for storing goods include depreciation and the discounting of the

[6]Refrigerators in the model are a metaphor for retail stores in the real world. According to Blinder (1981), most of finished goods inventories are held by the retail sector rather than by the manufacturing sector. However, this setup can be easily decentralized further by separating the household (who purchases consumption bundles $\{c_t(j)\}_{j=0}^{1}$) from a continuum of retail firms (each firm orders, sells, and stores inventory good j). The results would be very similar.

future. The marginal utility of good j is subject to idiosyncratic taste shocks $\theta_{1t}(j)$ with distribution $F(\theta) \equiv \Pr[\theta_1 \leq \theta]$. Taste shocks are not known to the household when orders (purchases) are made. This is a simple way of capturing the production/delivery lags in the stockout-avoidance theory. Hence, to cope with the idiosyncratic uncertainty and delivery lags, the household has incentives to store inventories of different colors to avoid stockout. The problem of the representative household is to solve

$$\max E \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\Phi_t}{1-\gamma} \left[\int_0^1 \theta_{1t}(j) c_t(j)^\rho dj \right]^{\frac{1-\gamma}{\rho}} - a \frac{N_t^{1+\gamma_n}}{1+\gamma_n} \right\}$$

subject to

$$c_t(j) + s_{1t}(j) = (1-\delta) s_{1t-1}(j) + y_t(j) \tag{1}$$

$$s_{1t}(j) \geq 0 \tag{2}$$

$$\int_0^1 y_t(j) dj + W_{t+1} \leq (1 + r_t) W_t + w_t N_t + \Pi_t, \tag{3}$$

where Φ_t denotes aggregate preference (demand) shocks, ρ the elasticity of substitution across consumption goods with different colors, $c_t(j)$ consumption of good j, $s_{1t}(j)$ inventory of good j, $y_t(j)$ purchase (orders) of good j, N_t aggregate labor supply, $\delta \in [0, 1]$ the common depreciation rate of inventories, r_t the market interest rate on household savings (W_t), w_t the real wage, and Π_t total profit income distributed from firms. The parameters in the utility function satisfy standard restrictions: $\rho \in (0, 1), \gamma \geq 0$, and $\gamma_n \geq 0$.

B. Firms

Final Goods. All colors of final goods are produced by a representative firm using the same production technology. Hence, from the producer's perspective, these goods are homogeneous, which can be either consumed or saved as capital by the household.[7] The production technology is

$$\tilde{Y}_t = A_t K_t^\alpha \tilde{M}_t^{1-\alpha}, \tag{4}$$

[7] We can think of a homogeneous final good being produced and distributed to different retailers. Each retailer j paints the good with color j so it becomes a differentiated good from the viewpoint of the consumer. Incidentally, the classical production-smoothing inventory model used by Charles C. Holt, Franco Modigliani, and Herbert A. Simon (1955) was formulated and estimated based on their study of a paint factory. So the "colors" abstraction in the general-equilibrium model was very concrete in their studies. I thank an anonymous referee for pointing out this interesting incidental connection to me.

where $\tilde{M}_t$ is a composite of intermediate goods, K_t the capital stock, and A_t an aggregate total factor productivity (TFP) shock. The price of the composite intermediate good is P_t^m. The representative firm's problem is to solve

$$\max \left\{ A_t K_t^{\alpha} \tilde{M}_t^{1-\alpha} - (r_t + \delta_k) K_t - \frac{\xi}{2\bar{K}} (K_t - \bar{K})^2 - P_t^m \tilde{M}_t \right\},$$

where $(r_t + \delta_k)$ is the user's cost of capital, with δ_k as the depreciation rate of capital, and $\xi \geq 0$ is the coefficient for a quadratic adjustment cost of capital relative to its steady state $(\bar{K})$. This particular form of adjustment costs is simpler than the conventional one used in the literature, but the effects are very similar. Notice that $\int y_t(j) dj \neq \tilde{Y}_t$ since the left-hand side (LHS) represents only a subset of aggregate demand (i.e., aggregate consumption) while the right-hand side (RHS) is the aggregate supply of the final good. The other components of final demand include capital investment, for example.

Intermediate Goods. In the intermediate-goods sector there are two types of firms, upstream and downstream firms. Upstream firms use labor to produce intermediate goods (or raw materials) $m(i)$. These intermediate goods are produced by identical technologies but are painted afterward with different colors indexed by $i \in [0, 1]$. Intermediate goods are sold to a downstream firm as inputs for synthesizing the composite good $\tilde{M}$ according to the aggregation technology

$$\tilde{M}_t = \left[\int \theta_{2t}(i) m_t(i)^{\rho} di \right]^{\frac{1}{\rho}}, \tag{5}$$

where the elasticity of substitution across intermediate goods is governed by the same parameter $\rho \in (0, 1)$. This production technology indicates that the marginal revenue product of each intermediate good $m_t(i)$ is subject to an idiosyncratic shock, $\theta_{2t}(i)$, which generates idiosyncratic uncertainty for the demand of intermediate good i by the downstream firm. Assume θ_{2t} has the same distribution $F(\theta)$.

Upstream firms' production technology for intermediate goods are given by the linear function, $z_t(i) = n_t(i)$, where $z_t(i)$ denotes the supply of intermediate good i. However, $z_t(i)$ must be determined before observing demand $m_t(i)$ (or the idiosyncratic demand shock $\theta_{2t}(i)$) in each period. Again, this is a simple way of capturing the production/delivery lags. Therefore, the downstream firm (which produces $\tilde{M}$) has incentives to keep inventories (s_{2t}) of intermediate goods in all colors as work-in-process to maximize expected profits.

Both the labor market and the intermediate-goods market are perfectly competitive and the labor used in producing intermediate good i is a perfect substitute for that used in producing other colors of intermediate goods. Hence, we can use a stand-in social planner to represent both the upstream and downstream firms in the intermediate-goods sector (i.e., a big corporate company that uses labor to produce intermediate materials and use the produced materials to synthesize good $\tilde{M}$ for the final-good sector). The problem of the planner is then to solve

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\mu_t}{\mu_0} \left\{ P_t^m \left[\int \theta_{2t}(i) m_t(i)^{\rho} di \right]^{\frac{1}{\rho}} - w_t \int n_t(i) di \right\}$$

subject to

$$m_t(i) + s_{2t}(i) = (1-\delta) s_{2t-1}(i) + n_t(i), \tag{6}$$

$$s_{2t}(i) \geq 0, \tag{7}$$

where μ_t in the objective function denotes the marginal utility of the final good (i.e., $\frac{\mu_t}{\mu_{t-1}} = 1 + r_t$ is the real interest rate) and w_t the real wage. Equation (6) states that in each period t, the usage of a particular intermediate good i ($m_t(i)$) plus the accumulation of work-in-process ($s_{2t}(i) - (1-\delta)\, s_{2t-1}$) equal the orders (or production) of good i ($z_t(i) = n_t(i)$). The total cost of producing all intermediate goods is $w_t \int n_t(i) di$ and the total revenue by selling the synthesized good to the final-good sector is $P_t^m \tilde{M}_t$.

C. First-Order Conditions

Assume that idiosyncratic shocks are orthogonal to aggregate shocks and that all decisions are made after observing aggregate shocks in each period. Denoting $\tilde{C}_t \equiv \left[\int \theta_{1t}(j) c_t(j)^{\rho} dj \right]^{\frac{1}{\rho}}$ as the composite consumption good and $\{\lambda_{1t}, \pi_{1t}, \mu_t\}$ as the Lagrangian multipliers of equations (1) through (3) for the household, respectively, the first-order conditions of the household for $\{N_t, W_t, c_t(j), y_t(j), s_{1t}(j)\}$ are given, respectively, by

$$a N_t^{\gamma_n} = \mu_t w_t \tag{8}$$

$$\mu_t = \beta E_t \mu_{t+1} (1 + r_{t+1}) \tag{9}$$

$$\Phi_t \tilde{C}_t^{1-\rho-\gamma} \theta_{1t}(j) c_t(j)^{\rho-1} = \lambda_{1t}(j) \tag{10}$$

$$\mu_t = E_t^j \lambda_{1t}(j) = \int \lambda_{1t}(j) dF(\theta_{1t}) \tag{11}$$

$$\lambda_{1t}(j) = \beta(1-\delta) E_t \lambda_{1t+1}(j) + \pi_{1t}(j), \tag{12}$$

plus the transversality condition $\lim_{T\to\infty} \beta^T E\mu_T W_{T+1} = 0, \lim_{T\to\infty} \beta^T E\lambda_{1T} s_{1T+1} = 0$, and the complementary slackness conditions, $s_{1t}(j)\pi_{1t}(j) = 0$ for all j.

The operator E_t^j in equation (11) denotes expectations based on the information set of period t excluding $\theta_{1t}(j)$. It reflects the information lag in ordering consumption goods without observing the shock $\theta_{1t}(j)$. The multiplier μ_t is not subject to this information friction because the constraint (3) depends only on aggregate shocks. Without the information lag, equation (11) becomes $\mu_t = \lambda_{1t}(j)$. Equation (12) then implies $\pi_{1t}(j) = \mu_t - \beta(1-\delta)E_t\mu_{t+1} > 0$ and $s_{1t}(j) = 0$ for all j.[8] Hence, it is not optimal to carry inventories when the value of $\theta_{1t}(j)$ is known. The key of the stockout-avoidance theory is the assumption that production/orders must take place in advance before observing the actual demand. In the setup, aggregate shocks do not play a role in the existence of inventories.[9]

This feature (having idiosyncratic demand shocks) avoids the classical problem of the rate of return dominance by capital so inventories are better able to smooth demand than capital. This feature also makes the model analytically tractable because the decision rules for inventories can be solved by taking the aggregate variables as given. Then in equilibrium and by the law of large numbers, there is always a positive measure of finished-goods inventories in any time period. Hence, the aggregate inventory stock is strictly positive and the log-linearization technique can be applied to analyzing the model's aggregate dynamics.

The first-order conditions for the final-good firm with respect to $\left\{K_t, \tilde{M}_t\right\}$ are given, respectively, by

$$r_t + \delta_k + \frac{\xi}{\bar{K}}(K_t - \bar{K}) = \alpha A_t K_t^{\alpha-1} \tilde{M}_t^{1-\alpha}, \quad P_t^m = (1-\alpha) A_t K_t^{\alpha} \tilde{M}_t^{-\alpha}. \tag{13}$$

These equations state that the marginal cost equals the marginal product for each production factor.

[8] Suppose this is not true and $\pi(j) = 0$ instead; then $\mu_t = \beta(1-\delta)E_t\mu_{t+1}$, which implies $\mu_t \to 0$ as time goes to infinity. Since the utility function is strictly increasing, the resource constraint (3) binds with equality in equilibrium, implying $\mu_t > 0$. This is a contradiction.

[9] This is a consequence of the lack of information friction with respect to aggregate shocks. Introducing information frictions at the aggregate level is possible but it may not have significant value added to the results.

For the intermediate-goods sector, denoting $\{\lambda_{2t}, \pi_{2t}\}$ as the Lagrangian multipliers for equations (6) and (7), respectively, the first-order conditions for $\{m_t(i), n_t(i), s_{2t}(i)\}$ are given, respectively, by

$$P_t^m \tilde{M}_t^{1-\rho} \theta_{2t}(i) m_t(i)^{\rho-1} = \lambda_{2t}(i) \tag{14}$$

$$w_t = E_t^i \lambda_{2t}(i) = \int \lambda_{2t}(i) dF(\theta_{2t}) \tag{15}$$

$$\lambda_{2t} = \beta(1-\delta) E_t \frac{\mu_{t+1}}{\mu_t} \lambda_{2t+1}(i) + \pi_{2t}(i), \tag{16}$$

plus a transversality condition, $\lim_{T\to\infty} \beta^T E \lambda_{2T} s_{2T} = 0$, and the complementary slackness conditions, $s_{2t}(i)\pi_t(i) = 0$ for all i.

D. Decision Rules

The decision rules are solved by a guess-and-verify strategy. Because it is possible to obtain closed-form solutions for firms' inventory decision rules, it is worthwhile to detail the steps of the solution process. The key to solving the decision rules of inventory investment is to determine the optimal stock (inventories carried from last period plus new orders or production) based on the distribution of θ. For example, the first-order condition for $y_t(j)$ (equation 11) suggests that the optimal size of orders depends on the expected shadow value of inventory, $E_t^j \lambda_{1t}(j)$. Under the law of iterated expectations, we have $E_t \lambda_{1t+1}(j) = E_t E_{t+1}^j \lambda_{1t+1}(j) = E_t \mu_{t+1}$; hence, equations (11) and (12) imply

$$\lambda_{1t}(j) = \beta(1-\delta) E_t \mu_{t+1} + \pi_{1t}(j). \tag{17}$$

Since the Lagrange multiplier $\pi_{1t}(j)$ may be either positive or zero, the decision rules for inventory investment, sales, and production are then characterized by a cutoff strategy featuring an optimal cutoff of the idiosyncratic shock, θ_{1t}^*, such that the non-negativity constraint on inventory is slack if $\theta_{1t}(j) \leq \theta_{1t}^*$, and it binds if $\theta_{1t}(j) > \theta_{1t}^*$. Thus, in the anticipation that the optimal cutoff is independent of j, consider two possible cases for the finished-goods sector:

Case A: $\theta_{1t}(j) \leq \theta_{1t}^*$. In this case, the realized demand is low, so it is optimal to carry any excess supply of goods (relative to a target) as inventories, in the anticipation that future demand may be high. Thus, we have $\pi_{1t}(j) = 0$, $s_{1t}(j) \geq 0$, and $\lambda_{1t}(i) = \beta(1-\delta)E_t\mu_{t+1}$. The resource constraint (1) implies $c_t(j) \leq (1-\delta)s_{1t-1}(j) + y_t(j)$. Since equation (10) implies $c_t(j) = \left[\frac{\Phi_t \tilde{C}_t^{1-\rho-\gamma}\theta_{1t}(j)}{\beta(1-\delta)E_t\mu_{t+1}}\right]^{\frac{1}{1-\rho}}$, we have

$$\theta_{1t}(j) \leq \left[(1-\delta)\, s_{1t-1}(i) + y_t(j)\right]^{1-\rho} \left[\frac{\beta(1-\delta)E_t\mu_{t+1}}{\Phi_t \tilde{C}_t^{1-\rho-\gamma}}\right] \equiv \theta_{1t}^*, \tag{18}$$

which defines the cutoff θ_{1t}^*. So the optimal stock can be expressed as a function of the cutoff: $y_t(j) + (1-\delta)\, s_{1t-1}(j) \equiv \left[\frac{\Phi_t \tilde{C}_t^{1-\rho-\gamma}\theta_{1t}^*}{\beta(1-\delta)E_t\mu_{t+1}}\right]^{\frac{1}{1-\rho}}$.

Case B: $\theta_{1t}(j) > \theta_{1t}^*$. In this case, the realized demand is high, so it is optimal to deplete (exhaust) inventories to satisfy demand. Thus, $\pi_{1t}(j) > 0, s_{1t}(j) = 0$, and $c_t(j) = y_t(j) + (1-\delta)\, s_{1t-1}(j) \equiv \left[\frac{\Phi_t \tilde{C}_t^{1-\rho-\gamma}\theta_{1t}^*}{\beta(1-\delta)E_t\mu_{t+1}}\right]^{\frac{1}{1-\rho}}$. Equation (10) then implies $\lambda_{1t}(j) = \frac{\theta_{1t}(j)}{\theta_{1t}^*}\beta(1-\delta)E_t\mu_{t+1} > \beta(1-\delta)E_t\mu_{t+1}$. Equation (12) then confirms that $\pi_{1t}(j) > 0$.

Combining these two possible cases, the shadow utility-value of inventories is given by $\lambda_{1t}(j) = \max\left\{\frac{\theta_{1t}(j)}{\theta_{1t}^*}, 1\right\}\beta(1-\delta)E_t\mu_{t+1}$, and equation (11) can then be written as

$$\mu_t = \int_{\theta_{1t}(j)\leq\theta_{1t}^*} \beta(1-\delta)E_t\mu_{t+1} dF(\theta) + \int_{\theta_{1t}(j)>\theta_{1t}^*} \beta(1-\delta)E_t\mu_{t+1}\frac{\theta_{1t}(j)}{\theta_{1t}^*} dF(\theta), \tag{19}$$

where the LHS (μ_t) is the marginal cost of ordering inventory good j and the RHS are the expected benefits of having one more unit of inventory in hand. The first term on the RHS is the continuation value of inventory if the realized demand is low, in which case it reduces the future cost of new orders; and the second term is the shadow value of inventory if the realized demand is high (in the case of a stockout). Thus, the optimal cutoff θ_{1t}^* is determined by equating the marginal cost and the expected marginal benefits. Since aggregate variables are independent of idiosyncratic shocks, equation (19) can be written as

$$\mu_t = \beta(1-\delta)E_t\mu_{t+1}R(\theta_{1t}^*), \tag{20}$$

where the function $R(\theta^*) \equiv F(\theta^*) + \int_{\theta>\theta^*} \frac{\theta}{\theta^*} dF(\theta) > 1$ measures the shadow rate of return (liquidity premium) of inventory investment. This rate exceeds 1 because inventories provide liquidity when the urge to consume (demand shock) is high. On the other hand, if the urge to consume is low, the rate of return to inventory (before depreciation) is just 1. In other words, inventory has an option value and that value is greater than 1. This liquidity premium ($R(\theta^*) > 1$) explains why firms invest in inventories even though the rate of return is seemingly "dominated" by that of capital.

By equation (9), it is clear that a no-arbitrage condition holds between two forms of asset investment in a steady state: $(1-\delta) R(\theta_1^*) = (1+r)$. That is, in equilibrium the liquidity premium (endogenously determined in the model) must exceed the real interest rate by the factor $\frac{1}{(1-\delta)}$ in order for firms to hold inventories.

Notice that $\frac{dR(\theta^*)}{d\theta^*} < 0$. The liquidity premium depends negatively on the cutoff because a higher cutoff implies a larger probability of excess supply and a smaller probability of stockout, which lowers the shadow value (return) of inventories. Given aggregate economic conditions (μ_t, μ_{t+1}), equation (20) solves the optimal cutoff as $\theta_{1t}^* = R^{-1}\left(\mu_t / \beta(1-\delta) E_t \mu_{t+1}\right)$, which is independent of j, confirming the earlier assumption. Also, the cutoff is countercyclical with respect to the current-period marginal cost (μ_t) and procyclical with respect to the expected future marginal cost ($E_t \mu_{t+1}$).

Thus, equation (20) provides the key to understanding why the inventory-to-sales ratio is countercyclical in the model (e.g., under aggregate demand shocks). A rise in aggregate consumption demand leads to a rise in the shadow price (the current-period marginal cost) of orders relative to future marginal cost.[10] A higher marginal cost calls for a higher liquidity premium (or rate of return) for holding inventories. Since the sale price of goods ($\lambda_{1t}(j)$) is higher in the case of stockout, this induces retail stores to increase the probability of stockout by not replenishing the inventory stock as fast as the increase in sales. In doing so, the retail stores break even between marginal cost and benefits of holding inventories. Hence, the average inventory stock of a firm (or the aggregate inventory stock across all firms) does not keep up with sales, leading to a countercyclical stock-to-sales ratio.

Alternatively, since the shadow utility-value of inventory j is determined by

$$\lambda_{1t}(j) = \beta(1-\delta) E_t \mu_{t+1} \times \max\left\{1, \frac{\theta_{1t}(j)}{\theta_{1t}^*}\right\}, \tag{21}$$

it is downward sticky with respect to the idiosyncratic demand shock $\theta_{1t}(j)$.[11] That is, the competitive price of inventory does not decrease to "clear" the market when demand is low

[10] To see this, log-linearizing equation (20) around a steady state gives $\hat{R}_t = \frac{1}{1-\beta(1-\delta)}\hat{\mu}_t - \frac{\beta(1-\delta)}{1-\beta(-\delta)} E_t \hat{\mu}_{t+1}$, where a hat denotes the percentage deviation from steady state. The movement in $\hat{\mu}_t$ must dominate that in $\hat{\mu}_{t+1}$ in a stationary environment. Even in the case where the marginal cost follows a random walk, $E_t \hat{\mu}_{t+1} = \hat{\mu}_t$, we still have $\hat{R}_t = \hat{\mu}_t$. So the dynamics in the liquidity premium $\hat{R}_t$ is always dictated by movements in the current marginal cost $\hat{\mu}_t$.

[11] As noted by Angus Deaton and Guy Laroque (1992), among others.

$(\theta_{1t}(j) \leq \theta_{1t}^*)$. Rather than choosing to sell the good at a price below marginal cost $(\lambda_{1t}(j) = \beta(1-\delta) E_t\mu_{t+1} < \mu_t)$, retailers opt to hold any excess supply as inventories $(s_{1t}(j) > 0)$, speculating that demand may be stronger in the future. On the other hand, when demand is high $(\theta_{1t}(j) > \theta_{1t}^*)$, retailers deplete inventories until stockout and the price rises with $\theta_{1t}(j)$ to clear the market $(\lambda_{1t}(j) = \frac{\theta_{1t}(j)}{\theta_{1t}^*}\beta(1-\delta) E_t\mu_{t+1})$. The optimal cutoff θ_{1t}^* determines the probability of stockout, so that on average the profit is zero $(E_t^j \lambda_{1t}(j) - \mu_{1t} = 0)$. The asymmetric price behavior will not be averaged out across a large number of firms but will instead be captured by movements in the cutoff θ_{1t}^*. That is, when aggregate demand is high, the number of retail stores (refrigerators) that run out of inventories increases, so the value of inventories (liquidity premium) rises to clear the market. This is equivalent to a decrease in the cutoff, suggesting that inventory stock does not track sales one-for-one in booms—a countercyclical stock-to-sales ratio over the business cycle. So again, the countercyclical movement in the cutoff and hence the procyclical movement in liquidity premium holds the key to understanding why the inventory-to-sales ratio in the model is countercyclical despite perfect competition with zero markups and strongly procyclical inventory investment.

This result does not necessarily contradict the arguments of Bils and Kahn (2000). They argue that the countercyclical stock-to-sales ratio in a stockout-avoidance model can imply either (i) procyclical marginal cost or (ii) countercyclical markups. Since they are not able to find procyclical movements in marginal cost in the data, they conclude that countercyclical markups (not present in this paper) must be moving inventory-sales ratios around.

The decision rules for the household sector are summarized by

$$y_t(j) + s_{1t-1}(j) = \tilde{C}_t \left[\frac{\Delta_t \theta_{1t}^*}{\beta(1-\delta)E_t\mu_{t+1}}\right]^{\frac{1}{1-\rho}}, \tag{22}$$

$$c_t(i) = \tilde{C}_t \left[\frac{\Delta_t}{\beta(1-\delta)E_t\mu_{t+1}}\right]^{\frac{1}{1-\rho}} \times \min\left\{\theta_{1t}(i)^{\frac{1}{1-\rho}}, \theta_{1t}^{*\frac{1}{1-\rho}}\right\}, \tag{23}$$

$$s_{1t}(i) = \tilde{C}_t \left[\frac{\Delta_t}{\beta(1-\delta)E_t\mu_{t+1}}\right]^{\frac{1}{1-\rho}} \times \max\left\{\theta_{1t}^{*\frac{1}{1-\rho}} - \theta_{1t}(i)^{\frac{1}{1-\rho}}, 0\right\}, \tag{24}$$

where $\Delta \equiv \Phi_t \tilde{C}_t^{-\gamma}$ denotes the marginal utility of the composite consumption good $\tilde{C}_t \equiv \left[\int \theta c_t(i)^\rho di\right]^{\frac{1}{\rho}}$.

The decision rules for the intermediate-goods sector can be derived analogously and summarized by the following equations[12]:

$$w_t = \beta(1-\delta)E_t\tilde{\mu}_{t+1}R(\theta_{2t}^*) \tag{25}$$

$$n_t(i) + s_{2t-1}(i) = \tilde{M}_t\left[\frac{P_t^m\theta_{2t}^*}{\beta(1-\delta)E_t\tilde{\mu}_{t+1}}\right]^{\frac{1}{1-\rho}}, \tag{26}$$

$$m_t(i) \quad = \quad \tilde{M}_t\left[\frac{P_t^m}{\beta(1-\delta)E_t\tilde{\mu}_{t+1}}\right]^{\frac{1}{1-\rho}} \times \min\left\{\theta_{2t}^{*\frac{1}{1-\rho}}, \theta_{2t}(i)^{\frac{1}{1-\rho}}\right\}, \tag{27}$$

$$s_{2t}(i) \quad = \quad \tilde{M}_t\left[\frac{P_t^m}{\beta(1-\delta)E_t\tilde{\mu}_{t+1}}\right]^{\frac{1}{1-\rho}} \times \max\left\{\theta_{2t}^{*\frac{1}{1-\rho}} - \theta_{2t}(i)^{\frac{1}{1-\rho}}, 0\right\}, \tag{28}$$

where $\tilde{\mu}_{t+1} \equiv \frac{\mu_{t+1}}{\mu_t}w_{t+1}$ denotes the next-period marginal cost of labor discounted by the interest rate (the ratio of the marginal utilities of the final good) and $R(\theta_{2t}^*) > 1$ denotes the liquidity premium of holding intermediate-good inventories. The optimal cutoff θ_{2t}^* in the input inventory industry is determined by equation (25), which is analogous to equation (20) and is independent of i.

II. General-Equilibrium Dynamics

A. Aggregation

Define aggregate variables $C \equiv \int c(i)di, Y \equiv \int y(i)di$, $S_1 \equiv \int s_1(i)di$, $S_2 \equiv \int s_2(i)di$, and $M \equiv \int m(i)di$. By the law of large numbers, the aggregate decision rules for the final good sector are given by

$$\mu_t = \Delta_t R(\theta_{1t}^*)G(\theta_{1t}^*)^{\frac{1-\rho}{\rho}} \tag{29}$$

$$C_t = \tilde{C}_t D(\theta_{1t}^*)G(\theta_{1t}^*)^{-\frac{1}{\rho}} \tag{30}$$

$$Y_t + (1-\delta)S_{1t-1} = C_t\frac{D(\theta_{1t}^*) + H(\theta_{1t}^*)}{D(\theta_{1t}^*)} \tag{31}$$

$$S_{1t} = C_t\frac{H(\theta_{1t}^*)}{D(\theta_{1t}^*)}, \tag{32}$$

[12] The readers are referred to Wen (2008) for more details.

where equation (30) is derived by using the definition of $\tilde{C}$ and the decision rule for $c(j)$, and the functions $\{G(\theta), D(\theta), H(\theta)\}$ are defined as

$$
\begin{aligned}
D(\theta^*) &\equiv \int_{\theta\leq\theta^*} \theta^{\frac{1}{1-\rho}} dF(\theta) + \int_{\theta>\theta^*} \theta^{*\frac{1}{1-\rho}} dF(\theta) > 0, \\
H(\theta^*) &\equiv \int_{\theta\leq\theta^*} \left[\theta^{*\frac{1}{1-\rho}} - \theta^{\frac{1}{1-\rho}}\right] dF(\theta) > 0, \\
G(\theta^*) &\equiv \int_{\theta\leq\theta^*} \theta^{\frac{1}{1-\rho}} dF(\theta) + \int_{\theta>\theta^*} \theta\theta^{*\frac{\rho}{1-\rho}} dF(\theta) > D(\theta^*).
\end{aligned}
\tag{33}
$$

Notice that $\theta^{*\frac{1}{1-\rho}} = D(\theta^*) + H(\theta^*)$.

The aggregate decision rules for the intermediate-goods sector are similarly given by

$$w_t = P_t^m R(\theta_{2t}^*) G(\theta_{2t}^*)^{\frac{1-\rho}{\rho}} \tag{34}$$

$$M_t = \tilde{M}_t D(\theta_{2t}^*) G(\theta_{2t}^*)^{-\frac{1}{\rho}} \tag{35}$$

$$N_t + (1-\delta) S_{2t-1} = M_t \frac{D(\theta_{2t}^*) + H(\theta_{2t}^*)}{D(\theta_{2t}^*)} \tag{36}$$

$$S_{2t} = M_t \frac{H(\theta_{2t}^*)}{D(\theta_{2t}^*)}, \tag{37}$$

where the functions $\{D(\cdot), H(\cdot), G(\cdot)\}$ take the same forms as in the finished-goods sector. Using the market-clearing conditions in the capital and labor markets, $W_t = K_t$ and $N_t = \int n(i)di$, and substituting out the factor income and aggregate profits, the aggregate resource constraints for finished goods and intermediate goods can be written, respectively, as

$$C_t + S_{1t} - (1-\delta) S_{1t-1} + K_{t+1} - (1-\delta_k) K_t = A_t K_t^\alpha \tilde{M}_t^{1-\alpha} - \frac{\xi}{2\bar{K}} (K_t - \bar{K})^2 \tag{38}$$

$$M_t + S_{2t} - (1-\delta) S_{2t-1} = N_t, \tag{39}$$

where equation (38) is derived from equations (1) and (3) and equation (39) is derived from equation (6).

For both input and output inventories, the stock-to-sales ratio is determined by the function $\frac{H(\theta^*)}{D(\theta^*)}$, which in turn is a function of the cutoff $\{\theta_t^*\}$. Thus, the cyclicality of the stock-to-sales ratio in each sector is determined by the movements of marginal cost of inventories in that sector. The aggregate resource constraint in equation (38) suggests that finished goods inventories are a perfect buffer for aggregate consumption and are substitutable for capital investment, whereas input inventories in equation (39) are not directly substitutable for either consumption or capital goods. This difference gives rise to different inventory behavior across finished and unfinished goods, especially at the high frequencies.

A general equilibrium is defined as the sequences of 17 endogenous aggregate variables, $\{\tilde{Y}_t, N_t, \mu_t, r_t, P_t^m,\ \theta_{1t}^*, \theta_{2t}^*, \tilde{C}_t, C_t, Y_t,\ S_{1t}, w_t, \tilde{M}_t, N_t, S_{2t}, K_{t+1}, M_t\}_{t=0}^{\infty}$, that maximize household utilities and firms' profits and satisfy the market clearing conditions, the transversality conditions, the initial conditions $\{K_0, S_{10}, S_{20}\} > 0$, the distribution of idiosyncratic shocks, and the law of large numbers for any given aggregate shocks. The system of equations that solves for the 17 variables includes equations (4), (8), (9), (13), (20), (25), and equations (29) through (39) except (33). It can be confirmed by the eigenvalue method that this system of 17 dynamic equations has a unique saddle-path steady state, suggesting the existence of a unique equilibrium around the steady state. The model can thus be solved by the standard log-linearization method.

B. Structural Parameters

Inventory behavior in the model depends on structural parameters. For analytical tractability, assume that idiosyncratic shocks follow the Pareto distribution,

$$F(\theta) = 1 - \theta^{-\sigma}, \tag{40}$$

with support $\theta \in (1, \infty)$ and the shape parameter $\sigma > 0$. Although the influence of parameters on the model are complex and intertwined, their major roles are easy to distinguish. For example, the parameters $\{\rho, \sigma\}$ affect primarily the steady-state stock-to-sales ratio because they influence the variance of sales at the micro level. When ρ is large, there is more substitutability across goods with different colors, making sales of each colored good more volatile. The shape parameter σ in the Pareto distribution is negatively associated with the variance of the distribution of θ. Hence, a smaller σ also implies more volatile sales. Since a larger variance of sales increases the possibility of stockout, firms have stronger incentives to keep a larger inventory-to-sales ratio for either a larger ρ or a smaller σ.

All aggregate shocks in the model are assumed to be permanent shocks following random walk process:

$$\log \Phi_t = \log \Phi_{t-1} + \varepsilon_{\Phi t}, \qquad \log A_t = \log A_{t-1} + \varepsilon_{At}. \tag{41}$$

Although the random-walk assumption of aggregate demand shocks is not necessary for the model to explain inventory dynamics of the data, it allows the model to better match business-cycle comovements of capital investment.[13]

The parameters in the utility function $\{\gamma, \gamma_n\}$ affect inventory behavior by primarily affecting the relative strength of the income effect and the substitution effect on hours worked when the economy is subject to TFP shocks. For example, the smaller the γ, the more responsive aggregate consumption is to aggregate shocks. In this case, finished goods inventories are more likely to play the role of a buffer stock in the face of consumption changes. Consequently, output inventory investment is more likely to be countercyclical at the high frequencies. On the other hand, larger values of γ or γ_n are more likely to generate negative responses of labor supply to technology shocks because of the increased income effect. Consequently, input inventories are more likely to be countercyclical under TFP shocks.

The adjustment cost parameter, ξ, affects primarily the substitutability between capital investment and inventory investment in finished goods. Hence, as consumption increases under either preference shocks or supply shocks, the buffer-stock roles of capital investment and inventory investment are different. For example, a larger value of ξ tends to attenuate the initial response of capital investment and make finished-goods inventory investment more responsive to aggregate shocks on impact.

Before showing the impulse responses, the general dynamic properties of the model can be summarized as follows:

- Under aggregate demand shocks (including transitory $AR(1)$ demand shocks) and with a wide range of parameter values, the model exhibits the following general properties: (i) inventory investment (for both finished and intermediate goods) is procyclical at the business cycle frequencies; (ii) their respective stock-to-sales ratios are countercyclical; (iii) input inventories are more volatile than output inventories; and (iv) finished-goods inventories have a tendency to be countercyclical at high frequencies. By the accounting identity for input and output inventories (i.e., production = inventory investment + sales), production/usage is more volatile than sales/orders because inventory investment is procyclical. These predictions are all consistent with the data.

- TFP shocks can also generate similar results as those under demand shocks, provided

[13] Near random-walk preference shocks are supported by the empirical analysis of Marianne Baxter and Robert G. King (1991). As noted by Wen (2006), transitory preference shocks generate countercyclical capital investment due to the crowding out effect from consumption, but permanent shocks can avoid this problem.

that the intertemporal substitution effect for households is strong enough (e.g., $\gamma < 1$). Otherwise, input inventory investment is countercyclical because TFP shocks generate a lower demand for intermediate goods when the income effect dominates (since hours worked decline). However, regardless of the parameter values, input inventories are less volatile than output inventories under TFP shocks, which is inconsistent with the data.

The main intuition behind the effects of aggregate demand shocks can be analyzed using the aggregate resource equations (38) and (39), which reveal the demand-supply chain of the production process. First, a persistent or permanent aggregate preference shock increases the marginal utilities of consumption not only in the present period but also for future periods. This encourages the household to accumulate both finished-goods inventories and capital to meet the anticipated higher consumption demand in the future. Such a strong increase in aggregate demand raises the shadow price of finished goods and stimulates finished-goods production; hence, the demand for intermediate goods also increases persistently. This in turn stimulates production of intermediate goods and the accumulation of intermediate-goods inventories. Therefore, a persistent shock to aggregate consumption demand from downstream can generate synchronized business cycles across sectors toward the upstream. Furthermore, since an increase in the demand of finished goods requires more than a one-for-one increase in intermediate goods because of diminishing marginal products of intermediate goods in producing the final good, production upstream must increase more than that downstream. This multiplier effect causes input inventory investment to be more volatile than output inventory investment under the stockout-avoidance motive. Finally, increases in demand at all stages of the production process raise the marginal costs of production at each stage, making the stock-to-sales ratio countercyclical for both input and output inventories.

The mechanism and dynamic effects of TFP shocks are different from those of demand shocks. A shock to TFP serves as a supply-push shock for the final-goods sector but a demand-pull shock for the intermediate-goods sector. However, the magnitude of the supply-side effect is larger than that of the demand-side effect. A one-unit increase in intermediate good $\tilde{M}$ under a positive TFP shock translates into at most a one-unit increase in demand for intermediate goods, but it represents more than a one-for-one increase in the supply of finished goods because of the compounded effect from a higher TFP. This explains why input inventories are in general less volatile than output inventories under TFP shocks. Also, if the income effect dominates the substitution effect on hours worked, then a positive shock to TFP leads to a decline in labor supply and a decrease in the production of intermediate goods, causing input inventory investment to be countercyclical. Hence, the effects of TFP shocks on inventory behavior are more sensitive to structural parameters than those of demand shocks.

The reason why the stock-to-sales ratio for finished goods can also be countercyclical under TFP shocks can be understood by equation (9). A higher TFP reduces the marginal cost of production and thus lowers the value of μ_t. However, it also raises the real interest rate. Hence, the expected future marginal cost (μ_{t+1}) must decline more than the current marginal cost to make equation (9) hold. That is, a permanent TFP shock causes the marginal cost to decline gradually toward a permanently lower (new) steady state. Hence, even though the current marginal cost drops on impact, the expected future marginal cost declines even more, giving rise to a procyclical liquidity premium ($R(\theta^*_{1t})$) and countercyclical stock-to-sales ratio.

Finally, since finished goods inventories held by retailers (i.e., stored in the refrigerators) are a better buffer than capital goods for unexpected idiosyncratic increases in consumption demand, finished-goods inventories tend to be countercyclical on impact at the high frequencies under demand shocks. On the other hand, since finished-goods inventories are substitutable for capital investment in terms of aggregate resource allocation, an unexpected rise in the marginal product of capital (because of a higher TFP) also tends to crowd out orders of finished goods from the household and reduce inventory investment. Thus, countercyclical final-goods inventory investment at the high frequencies can be generated by either aggregate demand shocks or aggregate TFP shocks. This is consistent with the stylized fact documented and analyzed by Wen (2005a).

C. Calibration and Impulse Responses

Table 1. Parameter Values

α	β	δ_k	δ	γ_n	ξ	ρ	σ
0.3	0.99	0.025	0.015	0.25	0.1	0.1	3.0

The structural parameters of the model are set according to Table 1. In particular, the time period is a quarter, the capital's share of income $\alpha = 0.3$, the time-discounting rate $\beta = 0.99$, the inverse labor supply elasticity parameter $\gamma_n = 0.25$ (which corresponds to a log utility function on leisure),[14] the rate of capital depreciation $\delta_k = 0.025$ (which implies

[14] With a log function of leisure, $\log(1 - N_t)$, the corresponding elasticity of hours supply in the log-

the capital stock depreciates about 10% a year), the rate of inventory depreciation $\delta = 0.015$ (which implies a 6% annual rate of depreciation for inventories),[15] the shape parameter $\sigma = 3$, and the substitution parameter $\rho = 0.1$. These values of $\{\beta, \delta, \rho, \sigma\}$ imply an inventory-to-sales ratio of 1.0, an inventory investment-to-GDP ratio of 0.01, and a 7% probability of stockout in the steady state.[16] The adjustment cost parameter is set to $\xi = 0.1$.[17] The risk aversion parameter γ plays an important role in determining the aggregate inventory dynamics (especially under TFP shocks), so different values of γ are experimented in the impulse response analysis below.

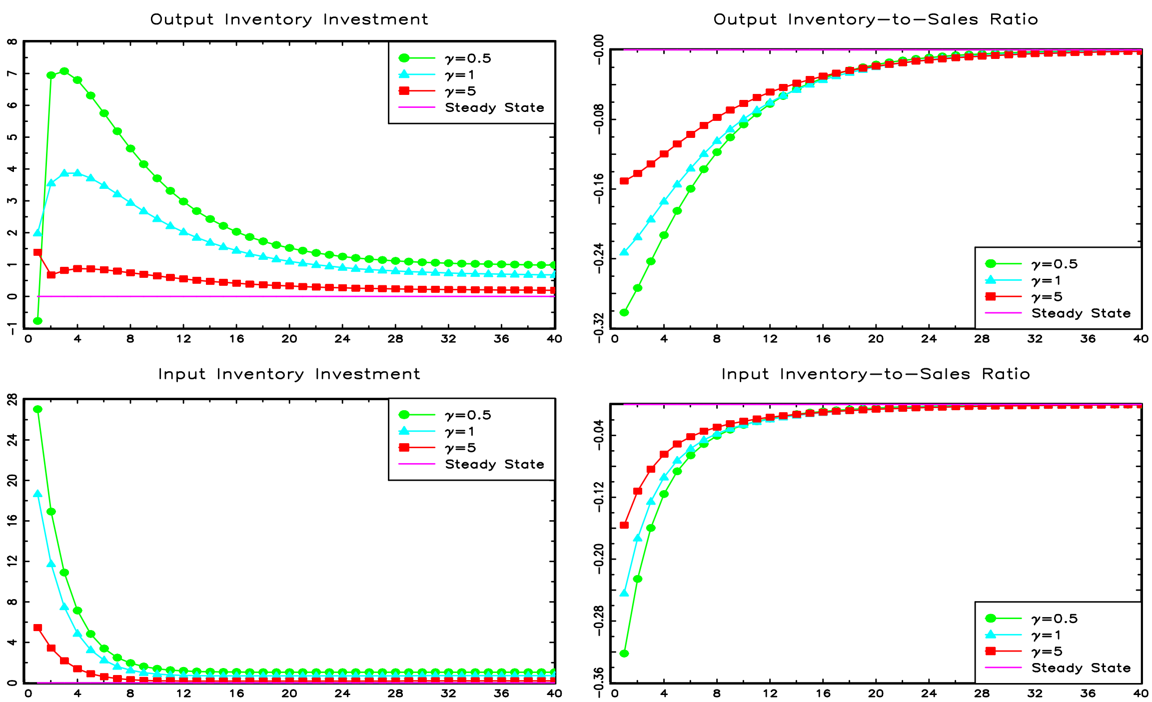

Figure 1. Impulse Responses to Demand Shock.

The impulse responses of the model to a one-standard-deviation shock to aggregate demand are graphed in Figure 1, where the three curves in each panel correspond to three

linearized first-order condition with respect to labor (equation 29) is given by $\frac{N}{1-N}$. Suppose the weekly hours worked are 35, then the fraction of hours worked is given by $N = \frac{35}{7\times 24} = 0.2$, which implies $\frac{N}{1-N} = 0.25$.

[15] Because of wear and tear in use, capital stock depreciates faster than inventory stock.

[16] The results are similar if we target an inventory-to-sales ratio of 0.5 instead of 1. Bils (2004) emphasizes the importance of matching the probability of stockout data at the firm level since it reveals information about inventory supply from a different angle than the stock-to-sales ratio. Since the parameters $\{\delta, \rho, \sigma\}$ are assumed to be the same for both input and output inventory sectors, the implied steady-state stock-to-sales ratios and probability of stockout are the same for both sectors.

[17] Without the adjustment cost, the model can still generate similar inventory dynamics, except that the finished goods inventory investment would have a higher tendency to be negative on impact. However, this negative initial response can be partially countered by a higher value of γ. See Wen (2008) for discussions about the empirical estimates of adjustment costs.

different values of γ. The two panels in the left column show responses of inventory investment and those in the right column show responses of inventory-to-sales ratios. The top-row panels represent the final-goods sector (output inventory) and the lower-row panels the intermediate-goods sector (input inventory). Under aggregate demand shocks, inventory investment in both sectors is procyclical and far more volatile than sales (left panels). However, the inventory-to-sales ratio is countercyclical (right panels). In the meantime, the liquidity premium (rate of return) to inventory investment (not shown in the figure) is procyclical because the probability of stockout $1 - F(\theta_t^*)$ rises in booms; and this explains why the stock-to-sales ratio is countercyclical. Also note in the figure that input inventory investment is more volatile than output inventory investment and that output inventory investment has a tendency to be countercyclical on impact (at high frequencies). In particular, input inventory investment is at least 4 times more volatile than output inventory investment, and both types of inventories are significantly more volatile than their respective sales. Different values of γ are used in generating Figure 1 and the results are robust. A lower value of γ makes consumption more responsive on impact because of lower risk aversion, which crowds out inventories in the short run. In the longer run, however, finished-goods inventories always comove with final sales because of the desire to replenishing inventories under the stockout-avoidance motive. In the meantime, other aggregate variables—including total output, consumption, capital investment, and labor—all increase and comove. These predictions are consistent with the data.

Under TFP shocks (Figure 2), the predicted inventory dynamics in the intermediate-goods sector are consistent with the data if γ is sufficiently small (e.g., $\gamma = 0.5$, see the lines with circles in Figure 2). In this case, both input and output inventory investment are procyclical and the corresponding inventory-to-sales ratios are countercyclical. However, if γ is large (i.e., $\gamma = 1$), input inventory investment becomes countercyclical because a large income effect on labor supply reduces the demand for intermediate goods and input inventories (see, e.g., the lines with triangle symbols in Figure 2). When the income effect is so strong (e.g., $\gamma = 5$), not only is input inventory investment negative (lower-left panel), but the inventory-to-sales ratio in the intermediate-goods sector also becomes procyclical (lower-right panel). This happens because sales in the intermediate-goods sector decline so sharply (due to a sharp reduction in hours worked and supply of intermediate goods) that the stock of intermediate-goods inventories fail to decline as fast as sales, rendering the stock-to-sales ratio procyclical. Therefore, in terms of inventory behaviors, aggregate supply shocks are not as successful and robust as aggregate demand shocks in explaining the data.

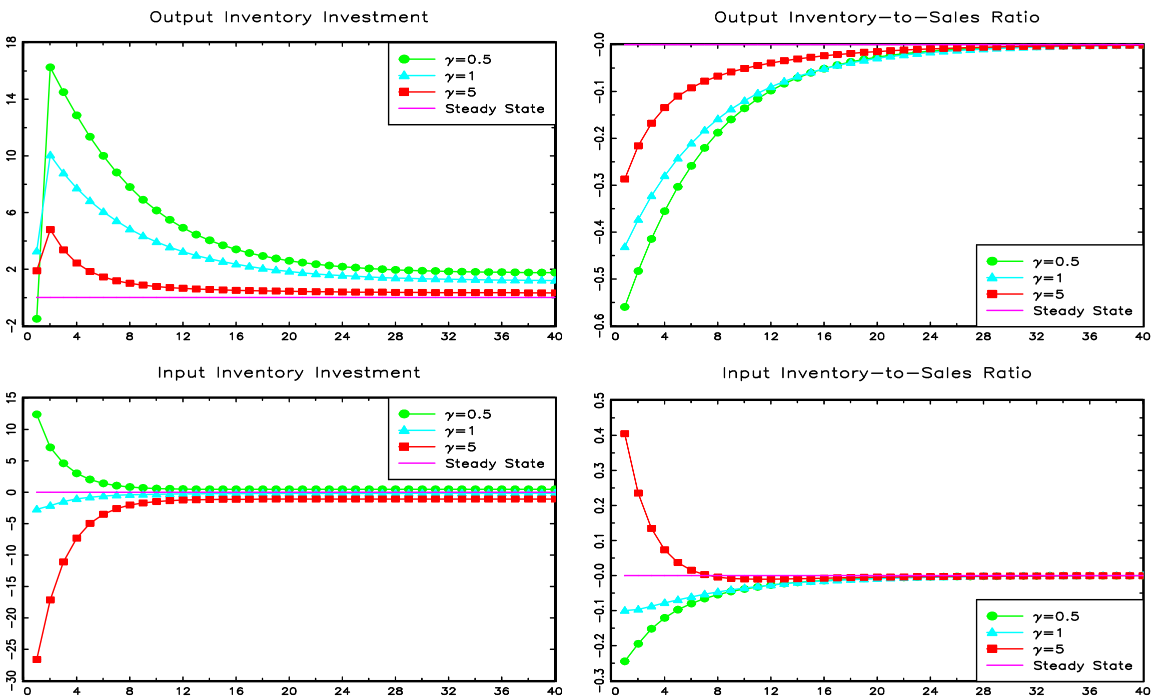

Figure 2. Impulse Responses to TFP Shock.

D. Matching Data

The model has no problem matching the steady-state ratios of inventory stock to sales and inventory investment to sales by properly choosing the parameter values of $\{\sigma, \rho\}$, as well as matching the other major ratios of the U.S. economy. This section, therefore, focuses instead on the ability of the model to match the second moments of the data.

To ensure consistency between the data and the model in the definition of variables, all variables in the data are transformed into percentage deviations from their respective long-run trends according to the definition $\hat{X}_t \equiv \log X_t - \log X_t^*$, where the long-run trend (X^*) is defined as the HP trend. This is consistent with the log-linearization solution method of the model. The relationship between a stock variable S and its corresponding flow I is defined as

$$S_t - (1-\delta)S_{t-1} = I_t. \tag{42}$$

Hence, the log-linearized relationship between stock and flow is given by

$$\hat{S}_t - (1-\delta)\hat{S}_{t-1} = \delta \hat{I}_t. \tag{43}$$

Based on this definition, if a flow variable I takes both positive and negative values and thus cannot be "log-linearized" directly, and if data for the stock S are not available, then the flow variable's percentage deviation from trend can be constructed according to equation (43). For example, to compute percentage changes of aggregate inventory investment in finished goods (I_t), which has negative values sometimes, we can construct first the inventory stock variable S_t according to equation (42) by assuming $\delta = 0.015$. The initial value of S_0 is set such that the imputed stock variable shares a common growth trend with GDP or the stock-to-GDP ratio is stationary over time (at least for the pre-"great moderation" period).[18] The constructed stock variable is then logged and HP filtered, yielding the series $\hat{S}_t$. Using equation (43), we can then obtain the flow series $\hat{I}_t$.[19]

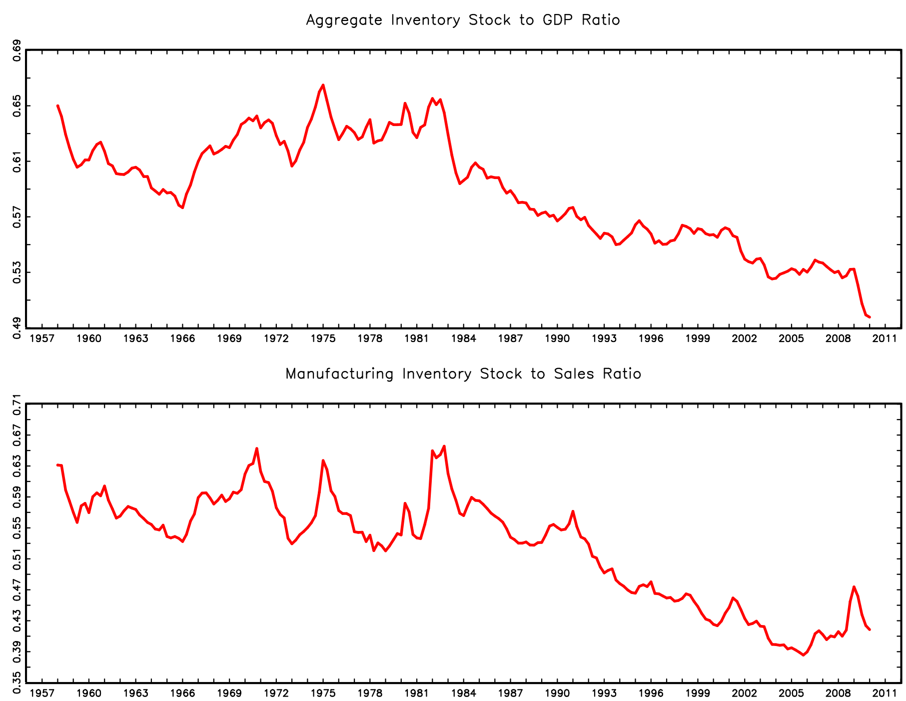

Figure 3. Output and Input Inventory Behavior.

Figure 3 shows the aggregate inventory-to-GDP ratio based on the constructed aggregate inventory stock, along with the total inventory stock-to-sales ratio in the manufacturing

[18]Data for inventory stock in the manufacturing sector are available, so the initial value of S_0 can be further narrowed down by ensuring that the constructed inventory-to-sales ratio of the aggregate finished goods looks similar to that of the manufacturing sector. Using this method, the initial value is set at $S_0 = 0.65GDP_0$, where GDP_0 is the initial value of GDP for the U.S. data sample.

[19]The variance of $\hat{I}_t$ based on this construction is sensitive to the value of δ. To make sure that $\delta = 0.015$ does not exaggerate the variance of inventory investment, we have used this procedure to construct the series of log-linearized fixed capital investment under the value $\delta = 0.015$ and found that the variance of fixed investment is not significantly higher than the original series under direct log-linearization.

sector. Clearly, the constructed aggregate inventory stock series mimics that of the manufacturing sector very closely over the business cycle. The inventory-to-sales ratio for both types of inventories has exhibited a downward trend since the early 80s, coinciding with the great moderation of the U.S. economy.[20] The average inventory-to-GDP ratio is 59% for the entire sample period (1958:1-2010:1), 62% for the pre-1984 period (1958:1-1983:4), and 56% for the post-1984 period. The average inventory-to-consumption ratio is much higher: 90%, 97%, and 82%, respectively, for the different sample periods. For the manufacturing sector, the average inventory-to-sales ratio is 53% for the entire sample period, 58% for the pre-1984 period, and 48% for the post-1984 period.[21]

Table 2 reports some selected business cycle statistics of the U.S. economy (1958:1-2010:1). All data are measured in billions of 2005 dollars. Aggregate consumption (C), fixed capital investment (dK), and inventory investment (dS_1) are from NIPA tables and they correspond to the final-goods sector in the model. Since there is no government and international trade in the model, aggregate production is defined as $Y = C + dK + dS_1$ and aggregate sales is defined as $Y - dS_1$.[22] Based on this definition, the aggregate inventory-to-output ratio is about 75% on average for the entire sample period (82% for the pre-1984 period and 68% for the post-1984 period).

We use data from the manufacturing sector of the U.S. economy as a proxy for the intermediate-goods sector of the model, where total manufacturing production is denoted by Z, total sales (shipments) by M, and the input inventory stock by S_2 (which includes only inventories of raw materials and work in process). Comovements are measured by correlations with sales, as in Khan and Thomas (2007a). Given the extremely high correlation between sales and output, the reported statistics change very little if they are measured instead by

[20] According to Ramey and Daniel J. Vine (2004), however, the downward trend in the finished-goods inventory-to-sales ratio is due to the secular increase in the relative price of services that appear only in the denominator of the inventory-to-GDP ratio. Ramey and Vine (2006) also document a stationary stock-to-sales ratio in the automobile industry. Maccini and Adrian Pagan (2010) on the other hand show that the stock-to-sales ratio for input inventories has declined significantly since the early 1980s.

[21] Data on inventory stocks for the manufacturing sector are available from the Bureau of the Census. The data on production are from the Federal Reserve Board. The BEASIC (Standard Industrial Classification) and NAICS (North American Industrial Classification System) are two numeric systems used for coding industry data. The SIC system started in the 1930s and became less useful in modern times. For this reason, some of the SIC categorized data are unavailable after 2001. After that, data based on the NAICS are used. Because of differences in industry classification, switching from one source to the other causes slight changes to the data. This is why the original version of this paper (Wen, 2008) used the 1958:1-2000:4 sample. For more details, see http://www.mb-journal.com/2001_Q4/sic.htm

[22] There are no separate data on consumption-good or investment-good inventories. Hence, the data and the model's final-goods sector are not a perfect match because in the model there are only consumption-good inventories. In addition, services are not considered in this paper. See Iacoviello et al. (forthcoming) for treatment of services in a general-equilibrium input-output inventory model.

correlations with output.

In Table 2, two statistics of each times series are reported, including standard deviation relative to production (*std./prod*) and correlation with sales (*cor./sales*). That is, the standard deviation of total output is normalized to 1 in each sector. The HP-filtered data series correspond to the "All Frequencies" column, movements isolated by the Band-Pass filter at the business cycle frequencies (8-40 quarters per cycle) correspond to the "8-40 Quarters" column, and those by the Band-Pass filter at the high frequencies correspond to the "2-3 Quarters" column.[23] Standard deviations of the final-goods sector relative to production (*std./y*) are reported in the top panel in the first column under each frequency band, and their correlations with sales in the final-goods sector (*cor./sales*) are reported in the next column under the corresponding frequency band. Similarly, statistics for the intermediate-goods sector are reported in the lower panel under each frequency band.

Table 2. Business Cycle Statistics (U.S. 1958:1 - 2010:1)

Variables	All Frequencies		8-40 Quarters		2-3 Quarters	
Final Good	*std./y*	*cor./sales*	*std./y*	*cor./sales*	*std./y*	*cor./sales*
Y	1	0.97	1	0.99	1	0.56
C	0.61	0.96	0.60	0.97	0.84	0.94
dK	2.50	0.94	2.47	0.95	2.08	0.69
dS_1	22.1	0.43	17.8	0.57	75.1	-0.37
S_1	0.77	0.49	0.71	0.38	0.61	-0.33
$\frac{S_1}{C}$	0.83	-0.59	0.89	-0.68	1.23	-0.89
Z	2.10	0.58	1.75	0.60	2.40	0.31
Interm. Good	*std./z*	*cor./sales*	*std./z*	*cor./sales*	*std./z*	*cor./sales*
Z	1	0.96	1	0.998	1	0.94
M	0.94	1	0.96	1	0.83	1
dS_2	25.7	0.58	23.7	0.73	91.0	0.30
S_2	0.90	0.58	0.94	0.42	0.73	0.31
$\frac{S_2}{M}$	0.85	-0.49	1.02	-0.55	0.92	-0.66

Several stylized facts in Table 2 are worth emphasizing: (i) Inventory investment is extremely volatile and procyclical over the business cycle. For example, over the 8-40 quarter

[23] If the frequency band is 2-40 quarters, the results under the band-pass filter are almost identical to those under the HP filter.

frequency band, its volatility is 18 times that of production in the final-goods sector and 24 times that of production in the intermediate-goods sector; and its correlation with sales is 0.57 in the final-goods sector and 0.73 in the intermediate-goods sector. (ii) Despite the strongly volatile and procyclical inventory investment with respect to sales, the inventory-to-sales ratio is countercyclical. Its correlation with sales is -0.68 in the final-goods sector and -0.55 in the other sector. (iii) Intermediate-goods inventories are more than twice as volatile as those for finished goods. To see this, notice that the standard deviation of production in the intermediate-goods sector (Z) is 1.75 times the final-goods sector; hence, the volatility of inventory investment in intermediate goods relative to final-goods production is $23.7 \times 1.75 = 41.5$, which makes it more than twice as large as the volatility of finished-goods inventory investment (which is 17.8). (iv) Finished-goods inventories are countercyclical at high frequencies. For example, their correlation with sales is -0.37 for inventory investment and -0.33 for inventory stock. However, these correlations are positive for intermediate-goods inventories (0.30 and 0.31, respectively).

Table 3. Model Predictions under Demand (Technology) Shocks

Var.	All Frequencies		8-40 Quarters		2-3 Quarters	
Final	*std./$\tilde{y}$*	*corr./sales*	*std./$\tilde{y}$*	*corr./c*	*std./$\tilde{y}$*	*corr./c*
$\tilde{Y}$	1	0.98 (0.97)	1	0.97 (0.97)	1	0.99 (0.99)
C	0.83 (0.81)	1	0.87 (0.85)	1	0.82 (0.72)	1
dK	1.47 (1.60)	0.82 (0.75)	1.28 (1.37)	0.75 (0.71)	2.28 (2.71)	0.98 (0.98)
dS_1	10.3 (10.9)	0.69 (0.71)	9.61 (10.2)	0.84 (0.86)	11.9 (13.8)	-0.85 (-0.80)
S_1	0.51 (0.52)	0.39 (0.46)	0.65 (0.62)	0.58 (0.59)	0.11 (0.12)	-0.76 (-0.86)
$\frac{S_1}{C}$	0.79 (0.73)	-0.79 (-0.77)	0.73 (0.69)	-0.68 (-0.70)	0.91 (0.83)	-0.99 (-0.99)
Z	1.74 (0.53)	0.93 (0.90)	1.61 (0.51)	0.91 (0.88)	2.16 (0.62)	0.99 (0.99)
Interm.	*std./z*	*corr./m*	*std./z*	*corr./m*	*std./z*	*corr./m*
Z	1	0.97 (0.99)	1	0.98 (0.99)	1	0.99 (1.00)
M	0.82 (0.87)	1	0.88 (0.90)	1	0.68 (0.75)	1
dS_2	17.0 (12.5)	0.66 (0.74)	13.9 (10.5)	0.57 (0.67)	22.3 (17.8)	0.99 (0.99)
S_2	0.57 (0.44)	0.89 (0.82)	0.70 (0.52)	0.92 (0.85)	0.18 (0.14)	0.97 (0.98)
$\frac{S_2}{M}$	0.42 (0.56)	-0.75 (-0.90)	0.36 (0.53)	-0.66 (-0.86)	0.51 (0.61)	-0.99 (-0.99)

Table 3 reports the business cycle statistics predicted by the model (with $\gamma = 0.5$) under aggregate demand shocks (numbers in parentheses are predictions under TFP shocks).[24] The production in the final-goods sector is denoted by $\tilde{Y}$, total sales by C, capital investment by dK, inventory investment by dS_1, and inventory stock-to-sales ratio by $\frac{S_1}{C}$. The production

[24]The statistics are based on simulated time series with 2000 observations and are filtered in the same way as for the U.S. data.

in the intermediate-goods sector is denoted by Z, sales by M, inventory by S_2, and stock-to-sales ratio by $\frac{S_2}{M}$.

Under aggregate demand shocks, the model qualitatively replicates the stylized facts in Table 3. Namely, (i) inventory investment is very volatile and procyclical over the business cycle. Over the 8-40 quarter frequency band, its volatility is about 10 times that of production in the final-goods sector and 14 times that of production in the intermediate sector; and it is positively correlated with sales in both sectors (the correlation is 0.84 in the final-goods sector and 0.57 in the intermediate-goods sector). (ii) The inventory stock-to-sales ratio is countercyclical. Its correlation with sales is -0.68 in the final-goods sector and -0.66 in the other sector. (iii) Intermediate-goods inventories are more than twice as volatile as those for finished goods. The standard deviation of production in the intermediate-goods sector is 1.61 times that of the final-goods sector; hence, the volatility of inventory investment in intermediate goods relative to final-goods production is $14 \times 1.6 = 22$, which makes it more than twice as large as the volatility of finished-goods inventory investment (which is 9.61). (iv) Finished-goods inventories are countercyclical at high frequencies. For example, their correlation with sales is -0.85 for inventory investment and -0.76 for inventory stock. In the meantime, the respective correlations are positive for intermediate-goods inventories, as in the data.

The predictions under TFP shocks are also reported in Table 3 (numbers in parentheses). Most of the predictions are consistent with the data, except the volatility of input inventories relative to output inventories. For example, over the 8-40 quarters frequency band, the standard deviation of production in the intermediate-goods sector is only 0.51 times the final-good sector; hence, the volatility of inventory investment in intermediate goods relative to final-goods production is $10.5 \times 0.51 = 5.4$, which makes it only half as large as the volatility of finished-goods inventory investment (which is 10.2). The reason is precisely the lack of a multiplier effect for TFP shocks on the intermediate-goods sector relative to the final-goods sector. An increase in TFP raises final-goods production (supply) more than intermediate-goods production (demand). That is, the supply-side effect on final goods is the combination of changes in TFP and $\tilde{M}$, whereas the demand-side effect on intermediate goods is only changes in $\tilde{M}$. In addition, if the risk aversion parameter γ is large enough, the effect on intermediate-goods demand is even negative. This problem does not arise under aggregate demand shocks (which originate from the bottom of the production chain).[25]

[25] Cost push shocks originating from upstream are also discussed in Wen (2008).

Notice that the inventory model is qualitatively consistent with the U.S. business cycle. For example, the model is able to explain procyclical aggregate consumption, capital investment, and hours worked across all cyclical frequencies. The model is also able to explain the stylized fact that consumption is less volatile but capital investment is more volatile than GDP at different frequency bands.

III. Inventories and Aggregate Volatility

An important question in the business cycle literature has been whether inventories are stabilizing or destabilizing to the aggregate economy. Because overwhelming empirical evidence indicates that inventory investment is procyclical (consequently, production is more volatile than sales), the consensus view has been that inventory behavior is destabilizing (see, e.g., Blinder, 1981, 1986, 1990). But this view may be false.

A counterexample is provided here. Suppose the development of information technology enables firms to better forecast demand so that the need to carry inventories (due to production/delivery lags) is eliminated from the model (as suggested by Kahn, McConnell, and Perez-Quiros, 2002). Namely, agents are able to anticipate idiosyncratic demand shocks when making ordering/production decisions. In this case, equation (11) then becomes $\mu_t = \lambda_{1t}(j)$, where the expectation operator E_t^j drops out because of the improved information technology. Equation (12) then implies $\pi_{1t}(j) = \mu_t - \beta(1-\delta)E_t\mu_{t+1}$, which is strictly positive in a stationary equilibrium around the steady state. Hence, $s_{1t}(j) = 0$ for all j. Therefore, the advancement of information technology reduces the need to carry inventories.

Table 4 reports changes in the variance of the final-goods supply in the model ($AK^\alpha \tilde{M}^{1-\alpha}$) when information frictions regarding the idiosyncratic shocks are eliminated so that the realized values of $\{\theta_1(j), \theta_2(i)\}$ are known to agents when decisions of $\{y(j), x(j)\}$ are made. The model without information frictions is called RBC in the table. The model with information friction is simulated with different parameter values of σ, which corresponds to different steady-state inventory-to-sales ratios (called Model A,B,C in the table).[26] The results under aggregate demand shocks are reported in the middle column. According to the table, if information frictions exist so that the steady-state inventory-to-sales ratio is 3.0 (which matches some durable-goods industry data in the United States), then an improvement in information technology that eliminates the production/delivery lag will increase the variance

[26]The counterfactual experiments are conducted under aggregate preference shocks, and the simulated time series (with sample size 10000) are all HP filtered.

of GDP by 30%. Even if the steady-state inventory-to-sales ratio is 0.5 (Model C), improvement in information technology can still increase the variance of GDP significantly—by 6%. The results are even more dramatic if the shocks are not permanent (see the numbers in parentheses in the middle column). For example, under AR(1) demand shocks with persistence parameter 0.9, inventories can reduce the volatility of aggregate output by 13% even when the stock-to-sales ratio is 0.5. These results run counter to the claims of Kahn, McConnell, and Perez-Quiros (2002), who use a reduced-form inventory model to argue that a reduction in inventories due to improved information technology (that enables agents to better forecast demand) will stabilize the economy.

However, if the source of aggregate uncertainty is TFP shocks, the stabilizing effect of inventories is substantially reduced. The last column in the table shows that under TFP shocks, inventories make no detectable difference for output variations when the stock-to-sales ratio is 0.5. Even when the stock-to-sales ratio is 3, inventories reduce the variance of output by only 2%. The stabilizing effect becomes significant only if the stock-to-sales ratio is very large—e.g., the variance reduction could be 12% if the inventory-to-sales ratio is 10. The results under TFP shocks are similar to the findings of Khan and Thomas (2007a) but in the opposite direction (Khan and Thomas found inventories to be slightly destabilizing, rather than stabilizing, to the economy under TFP shocks).

Table 4. Contribution of Inventories to Stability (Relative Variance of $\tilde{Y} = AK^{\alpha}\tilde{M}^{1-\alpha}$)

	Inventory-sales Ratio	Agg. Demand Shock	Agg. TFP Shock
Model A	3.0	0.70 (0.47)	0.98
Model B	1.0	0.88 (0.76)	1.00
Model C	0.5	0.94 (0.87)	1.00
RBC	0.0	1.00 (1.00)	1.00

The explanation for the counter-intuitive result is that inventories stabilize final demand (or sales) in general equilibrium. This stabilizing effect on demand is rooted in a procyclical liquidity premium (or asset price) of inventories. Inventories provide liquidity services to demand (because they enable firms to meet demand faster than factors of production due to time lags), and the liquidity premium is procyclical under demand shocks because the shadow value of inventories rises with the probability of stockout (recall that the stock-to-sales ratio is countercyclical). Therefore, agents pay disproportionately higher prices in case of a liquidity shortage (stockout). Hence, even though a procyclical inventory investment raises the variability of production *given* sales, it nonetheless reduces the volatility of sales in general equilibrium. In other words, the procyclical asset value of inventories under a countercyclical stock-to-sales ratio acts as an endogenous stabilizer to aggregate demand and (hence) aggregate output. Similar results hold under TFP shocks, albeit to a significantly less degree.[27]

An alternative way to understand the result is to analyze the relative price $q_t \equiv D(\theta_{2t}^*)G(\theta_{2t}^*)^{-\frac{1}{\rho}}$ in equation (35), which reflects the relative price of the final good in terms of intermediate goods and thus the wedge between the value of the final good and that of the intermediate goods. Recall that a countercyclical stock-to-sales ratio requires θ_{2t}^* to be countercyclical (i.e., the liquidity premium and the probability of stockout are procyclical). Since $\frac{\partial q(\theta_2^*)}{\partial \theta_2^*} < 0$, q_t is thus procyclical. This implies that the final goods (consumption and capital investment) are more expensive relative to inputs in a boom and less expensive in a slump. Thus, the procyclical movements in q_t act as an automatic stabilizer, which discourages final sales in booms and encourages final demand in recessions, thus reducing the variability of final demand $(C_t + K_{t+1} - (1 - \delta)K_t)$ over the business cycle.

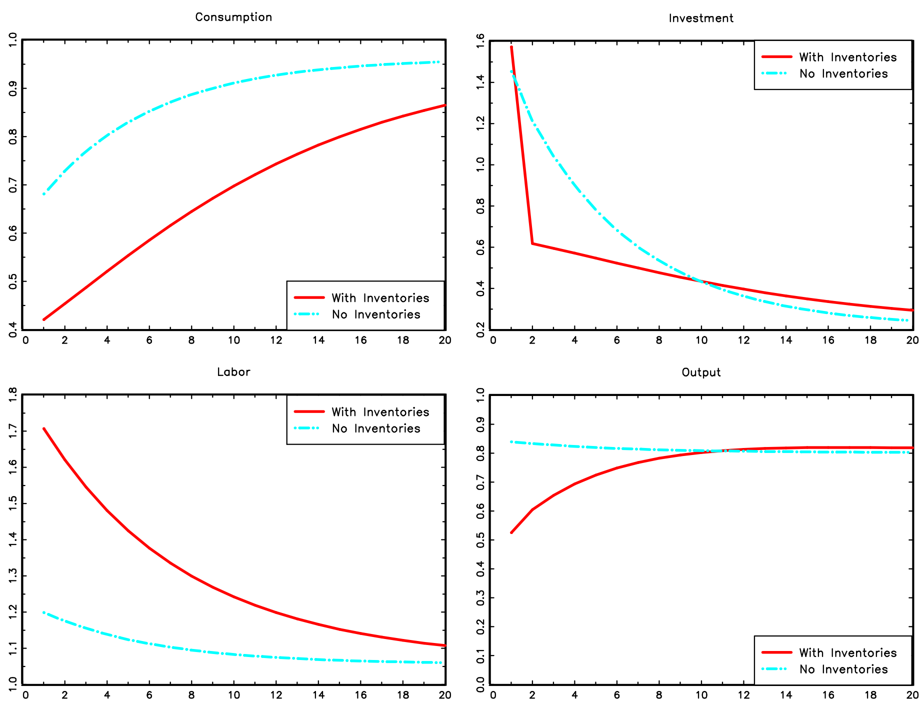

Figure 4. Impulse Responses to Demand Shock.

Figure 4 compares impulse responses of the model (with a high inventory-to-sales ratio of

[27] In the case of TFP shocks, agents anticipate lower prices in the future, hence reducing current demand relative to future demand.

5) and those of a control model without inventories (RBC). The top row panels indicate that both consumption (C_t) and capital investment ($K_{t+1} - (1-\delta)K_t$) have a lower volatility with inventories (solid lines) than without (dashed lines), revealing the stabilizing role of inventories.[28] On the other hand, the bottom-left panel indicates that labor (N_t) is more volatile when inventories exists (solid line), revealing the destabilizing role of inventories (procyclical inventory investment implies more volatile production). However, because of the tradeoff between the lowered variability of the final demand and the increased variability of intermediate-goods production, in net the stabilizing role dominates the destabilizing role. Consequently, the variance of final output ($\tilde{Y}_t$) is reduced (solid line in the bottom-right panel).

Table 5. Standard Business-Cycle Moments

	Relative Volatility to $\tilde{Y}_t$			Correlation with $\tilde{Y}_t$			Auto-correlation			
	C_t	dK_t	N_t	C_t	dK_t	N_t	$\tilde{Y}_t$	C_t	dK_t	N_t
US Data	0.61	2.50	1.08	0.93	0.94	0.89	0.89	0.88	0.91	0.88
Aggregate Demand Shocks										
Model A	0.75	1.56	2.21	0.98	0.77	0.94	0.80	0.78	0.36	0.69
Model B	0.83	1.45	1.72	0.98	0.90	0.97	0.77	0.78	0.61	0.68
Model C	0.86	1.52	1.58	0.98	0.91	0.99	0.76	0.78	0.63	0.69
RBC	0.88	1.63	1.41	0.98	0.92	0.99	0.72	0.78	0.65	0.72
Aggregate TFP Shocks										
Model A	0.69	2.16	0.64	0.97	0.68	0.97	0.73	0.76	0.10	0.65
Model B	0.80	1.64	0.54	0.97	0.87	0.98	0.71	0.76	0.44	0.64
Model C	0.84	1.63	0.50	0.98	0.90	0.98	0.70	0.76	0.52	0.64
RBC	0.88	1.63	0.46	0.98	0.92	0.99	0.69	0.75	0.62	0.66

Adding inventories into a multiple-sector RBC model changes the model's predictions for the business cycle, especially under aggregate demand shocks. For example, Table 5 shows that, in comparison with a similarly structured RBC model without inventories (called "RBC" in the middle panel), inventories under aggregate demand shocks reduce the relative volatility of consumption and capital investment significantly (first and second columns under the subtitle "relatively volatility") but increase the relative volatility of labor significantly (third column).[29] However, inventory models behave very similarly to the RBC model along other dimensions, such as the correlations with output. While the increased labor volatility deteriorates an inventory model's performance in matching labor volatility in the U.S. data,

[28] The kink in investment in the inventory model is due to substitution between capital and inventories.

[29] The models (A,B,C,RBC) in Table 5 correspond to those in Table 4. Employment in the U.S. data is defined as total number of employees times the average weekly hours.

the reduced relative volatility in consumption nonetheless improves RBC model's empirical fit, notwithstanding the fact that RBC models are silent about inventory cycles. Under TFP shocks (the lower panel in Table 5), inventories improve the RBC model's fit for consumption, capital investment, and labor in terms of relative volatilities, but deteriorate the model's fit for the autocorrelation of investment. Notice that the multi-sector RBC model does not quite match the relative volatility of investment in the U.S. data because capital adjustment costs reduce investment volatility under aggregate shocks.

Robustness Analysis. The ability of inventories to stabilize the business cycle, as discovered in this paper, is not due to some peculiar features of the multisector model, but the sole consequence of a procyclical shadow value (cost) of inventories under the stockout-avoidance motive. For example, a legitimate concern is that the result may depend on how convex short-run costs are. If firms benefit greatly from smoothing production relative to avoiding stockouts, wouldn't better information allow them to do so? Second, in the world of this model, especially with preference shocks, fluctuations are beneficial. Consumers are better off with more volatile output. That suggests that perhaps the model in its current setup is not quite the right vehicle for answering the question. It would seem particularly telling that labor input is actually more volatile with inventories than without. These factors suggest that there might be something peculiar about the structure of the model, e.g. that labor is confined to the intermediate-goods sector with a linear technology.

To show that the result is not driven by these specific features of the model, the Appendix (available upon requests by readers) considers a simpler general-equilibrium model with inventories—which is a modified version of the multisector inventory model. The essential differences between this simplified model and the more complicated input-output inventory model is (i) the introduction of government spending shocks (which by design are *not* beneficial to consumers) and (ii) labor is not confined to the intermediate-goods sector with a linear technology but instead serves as a second production factor along with capital in a Cobb-Douglas production technology. The simplified model abstracts from output inventories and features only intermediate-goods inventories. The analysis shows that final demand (consumption and capital investment) is always less volatile while labor input is more volatile with inventories than without inventories, and that inventories significantly reduce the variability of GDP, as in the more complicated model.

IV. Conclusion

This paper develops an analytically tractable general-equilibrium model of input and output inventories with an explicit microfoundation—the stockout-avoidance motive. The model is shown to be broadly consistent with stylized inventory behaviors in the U.S. economy, such as, among other things, (i) excessively volatile production relative to sales, (ii) procyclical inventory investment and a countercyclical inventory-to-sales ratio, (iii) more volatile input inventories than output inventories, and (iv) countercyclical final-goods inventory investment at high frequencies.

This paper also uncovers an important general-equilibrium effect of inventories on the business cycle: the procyclical asset value (or liquidity premium) of inventories under the stockout-avoidance motive. On the one hand, inventories are destabilizing because they magnify the variance of production through procyclical inventory investment; on the other hand, inventories are stabilizing because they reduce the variance of demand through a procyclical liquidity-premium effect—which raises the relative price of final goods in booms, thus dampening final sales over the business cycle. When the stock-to-sales ratio is countercyclical because of a procyclical probability of stockout, the stabilizing effect on final demand dominates the destabilizing effect on production, leading to a less volatile aggregate GDP. Without a general-equilibrium analysis based on an explicit microfoundation of inventory behaviors, such a stabilizing role of inventories through a time-varying liquidity premium of inventories is extremely difficult to imagine and detect.

Although the model may have shortcomings because of its extreme simplicity, its analytical tractability makes it a convenient framework for studying inventories in more complicated DSGE models than the one studied in this paper, such as models with imperfect competition, firm entry and exit, money and sticky prices, international trade, and so on.[30] Also, the approach can be used to study durable goods inventory behavior, which is another important long-standing puzzle of the business cycle (see, e.g., Martin Feldstein and Alan Auerbach, 1976). Given the sheer magnitude of inventory stocks in the economy and their widely believed role in understanding the business cycle, a business-cycle model without inventories is clearly incomplete and unsatisfactory. Microfounded general-equilibrium analysis of the business cycle with inventories is still in its infant stage. Hopefully this paper will contribute to further research and development in this area.

[30] See, e.g., Pengfei Wang and Wen (2009) for analysis of inventories in a model with imperfect competition and idiosyncratic cost shocks. They show that inventory investment can destabilize the economy and generate hump-shaped output dynamics if imperfect competition, financial frictions, and different motives of holding inventories are introduced. This study offers a potential explanation of the Great Moderation based on a declining inventory stock-to-sales ratio, because the empirical studies of Stephen J. Davis, Haltiwanger, Ron Jarmin, and Javier Miranda (2006) and Davis, Lason R. Faberman, Haltiwanger, Jarmin, and Miranda (2010) suggest that the variance of idiosyncratic shocks facing firms have been decreasing over time along side the Great Moderation.

References

[1] Baxter, Marianne, and Robert G. King. 1991. "Productive Externalities and Business Cycles." Institute for Empirical Macroeconomics at Federal Reserve Bank of Minneapolis Discussion Paper 53.

[2] Bils, Mark. 2004. "Studying Price Markups from Stockout Behavior." Draft Paper.

[3] Bils, Mark, and James A. Kahn. 2000. "What Inventory Behavior Tells Us about Business Cycles?" *American Economic Review*, 90(3): 458-481.

[4] Blanchard, Olivier J. 1983. "The Production and Inventory Behavior of the American Automobile Industry." *Journal of Political Economy*, 91(3): 365-400.

[5] Blinder, Alan S. 1981. "Retail Inventory Behavior and Business Fluctuations." *Brookings Papers on Economic Activity*, 12(2): 443-520.

[6] Blinder, Alan S. 1986. "Can the Production Smoothing Model of Inventory Behavior be Saved?" *The Quarterly Journal of Economics*, 101(3): 431-454.

[7] Blinder, Alan S. 1990. *Inventory Theory and Consumer Behavior*. Ann Arbor: University of Michigan Press.

[8] Blinder, Alan S., and Louis J. Maccini. 1991. "Taking Stock: A Critical Assessment of Recent Research on Inventories." *Journal of Economic Perspectives*, 5(1): 73-96.

[9] Christiano, Lawrence. 1988. "Why Does Inventory Investment Fluctuate So Much?" *Journal of Monetary Economics*, 21(2-3): 247-280.

[10] Coen-Pirani, Daniele. 2004. "Markups, Aggregation, and Inventory Adjustment." *American Economic Review*, 94(5): 1328-53.

[11] Davis, Stephen J., John Haltiwanger, Ron Jarmin, and Javier Miranda. 2006. "Volatility and Dispersion in Business Growth Rates: Publicly Traded Versus Privately Held Firms." In *NBER Macroeconomics Annual 2006*, ed. Daron Acemoglu, Kenneth Rogoff, and Michael Woodford, 107–180. Cambridge, MA: MIT Press.

[12] Davis, Stephen J., Jason R. Faberman, John Haltiwanger, Ron Jarmin, and Javier Miranda. 2010. "Business Volatility, Job Destruction, and Unemployment." *American Economic Journal: Macroeconomics*, 2(2): 259-287.

[13] Deaton, Angus, and Guy Laroque. 1992. "On the Behaviour of Commodity Prices." *Review of Economic Studies*, 59(1): 1-23.

[14] Eichenbaum, Martin. 1989. "Some Empirical Evidence on the Production Level and Production Cost Smoothing Models of Inventory Investment." *American Economic Review*, 79 (4): 853-64.

[15] Feldstein, Martin, and Alan Auerbach. 1976. "Inventory Behavior in Durable-Goods Manufacturing: The Target-Adjustment Model." *Brookings Papers on Economic Activity*, 7(2): 351-408.

[16] Fisher, Jonas D. M., and Andreas Hornstein. 2000. "(S,s) Inventory Policies in General Equilibrium." *Review of Economic Studies*, 67(1): 117-45.

[17] Haltiwanger, John, and Louis J. Maccini. 1988. "A Model of Inventory and Layoff Behaviour Under Uncertainty." *The Economic Journal*, 98(392): 731-45.

[18] Holt, Charles C., Franco Modigliani, and Herbert A. Simon. 1955. "A Linear Decision Rule for Production and Employment Scheduling." *Management Science*, 2(1): 1-30.

[19] Hornstein, Andreas. 1998. "Inventory Investment and the Business Cycle." *Economic Quarterly*, 84(2): 49-71.

[20] Humphreys, Brad R., Louis J. Maccini, and Scott Schuh. 2001. "Input and Output Inventories." *Journal of Monetary Economics*, 47(2): 347-75.

[21] Iacoviello, Matteo M., Schiantarelli Fabio, and Scott Schuh. Forthcoming. "Input and Output Inventories in General Equilibrium." *International Economic Review*.

[22] Kahn, James A. 1987. "Inventories and the Volatility of Production." *American Economic Review*, 77(4): 667-79.

[23] Kahn, James A. 1992. "Why is Production More Volatile than Sales? Theory and Evidence on the Stockout-Avoidance Motive for Inventory-Holding." *The Quarterly Journal of Economics*, 107(2): 481-510.

[24] Kahn, James A., Margaret M. McConnell, and Gabriel Perez-Quiros. 2002. "On the Causes of the Increased Stability of the U.S. Economy." *Economic Policy Review*, 8(1): 183-202.

[25] Kahn, Aubhik, and Julia K. Thomas. 2007a. "Inventories and the Business Cycle: An Equilibrium Analysis of (S,s) policies." *American Economic Review*, 97(4): 1165-88.

[26] Kahn, Aubhik, and Julia K. Thomas. 2007b. "Explaining Inventories: A Business Cycle Assessment of the Stockout Avoidance and (S,s) Motives." *Macroeconomic Dynamics*, 11(5), 638-64.

[27] Kryvtsov, Oleksiy, and Virgiliu Midrigan. 2009. "Inventories, Markups, and Real Rigidities in Menu Cost Models." NBER Working Paper No. w14651.

[28] Kydland, Finn E., and Edward C. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica*, 50(6): 1345-70.

[29] Maccini, Louis J., and Adrian Pagan. 2010. "Inventories, Fluctuations and Goods Sector Cycles." Johns Hopkins University Working Paper.

[30] Ramey, Valerie A. 1989. "Inventories as Factors of Production and Economic Fluctuations." *American Economic Review*, 79(3): 338-54.

[31] Ramey, Valerie A. 1991. "Nonconvex Costs and the Behavior of Inventories." *Journal of Political Economy*, 99(2): 306-34.

[32] Ramey, Valerie A., and Kenneth D. West. 1999. "Inventories." In *Handbook of Macroeconomics*, ed. John B. Taylor, and Michael Woodford, 863-923. Elsevier.

[33] Ramey, Valerie A., and Daniel J. Vine. 2004. "Why Do Real and Nominal Inventory-Sales Ratios Have Different Trends?" *Journal of Money, Credit, and Banking*, 36(5): 959-63.

[34] Ramey, Valerie A., and Daniel J. Vine. 2006. "Declining Volatility in the U.S. Automobile Industry." *American Economic Review*, 96(5): 1876-89.

[35] Wang, Peng-fei, and Yi Wen. 2009. "Inventory Accelerator in General Equilibrium." Federal Reserve Bank of St. Louis Working Paper 2009-010A.

[36] Wen, Yi. 2005a. "Understanding the Inventory Cycle." *Journal of Monetary Economics*, 52(8): 1533-55.

[37] Wen, Yi. 2005b. "The Multiplier: A General Equilibrium Analysis of a Multi-Stage-Fabrication Economy with Inventories." Federal Reserve Bank of St. Louis Working Paper 2005-046A.

[38] Wen, Yi. 2006. "Demand Shocks and Economic Fluctuations." *Economics Letters*, 90(3): 373-83.

[39] Wen, Yi. 2008. "Input and Output Inventory Dynamics." Federal Reserve Bank of St. Louis Working Paper 2008-008A.

[40] West, Kenneth D. 1986. "A Variance Bounds Test of the Linear Quadratic Inventory Model." *Journal of Political Economy*, 94(2): 374-401.