Negative Correlation between Stock and Futures Returns: An Unexploited Hedging Opportunity?

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Abstract

The negative correlation between equity and commodity futures returns is widely perceived by investors as an unexploited hedging opportunity. A Lucas (1982) asset-pricing model is adapted to analyze the fundamentals driving equity and commodity futures returns. Using the model we argue that such a negative correlation could arise as an equilibrium relationship which reflects traders' perceptions about the shocks driving the fundamentals such as energy and consumables, and does not necessarily indicate any hedging opportunity.

Keywords: Futures, Equity, Hedging, Beta

JEL Classification: G12, Asset Pricing

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1 Introduction

 The Commodity Futures Modernization Act in 2000 gave large financial firms wide latitude in trading commodity derivatives.[1] The institutional fund managers shifted out of equities into commodity futures partly in the belief that commodity futures represented a previously unrecognized hedge for the business cycle risk. Greer (2000) argues that commodity index funds are an asset class that is underused because commodity index futures returns were negatively correlated with stocks and bonds over the period 1970-99. Gorton and Rowenhorst (2006) also found that the returns on long positions in commodity futures are negatively correlated with the returns from comparable bond and equity portfolios. Erb and Harvey (2006) report a similar historical record but caution against using historical correlations to make prospective portfolio allocations. Boyuksahin, Haigh and Robe (2010) provide detailed evidence of the correlation between equity and commodity returns and find that commodities did not provide enough diversification when it was needed. Likewise Daskalaki and Skiapoulos (2011) provide out-of-sample evidence that commodities as an asset class do not improve returns over portfolios which include only traditional asset classes.

A common question that arises in all these extant studies is: Does the negative correlation between commodity and equity returns provide an unexploited hedging opportunity? This question cannot be effectively answered without an asset pricing model that identifies the common macroeconomic fundamentals driving both commodity and equity returns. To the best of our knowledge, there is no theoretical treatment of the common macroeconomic fundamentals driving returns to both equities and commodity futures using general equilibrium principles.

In this paper, we adapt a Lucas (1982) asset-pricing model to analyze the fundamentals driving equity and commodity futures returns. We show that in a frictionless complete market setting, even though households are fully hedged, a negative correlation could arise as an equilibrium phenomenon. Such a negative correlation by itself cannot be used as a hedging motive.

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[1] See, Basu and Gavin (2011) for a documentation of the rise in commodity trading.

In the model, the representative household is exposed to two types of endowment risks. The first is the business cycle risk of its own consumable output. The second is the commodity supply risk arising from fluctuations in the oil endowment. The model is kept quite simple and stylized where oil, used as a stand-in for commodities generally, is treated as a consumption item, and all returns are real. We demonstrate that the correlation between equity and futures real returns depends crucially on the variance-covariance matrix of these two economic fundamentals, oil and consumption growth.

A central implication of our asset pricing model is that the *ex post* equity return is positively related to the growth rate of non-oil output while the *ex post* oil futures return is determined by the relative growth rates of oil and non-oil outputs as well as the *news* about the growth of future oil output. In the log linear model with separable utility, the sign of the correlation between stock and futures returns thus crucially hinges upon the sizes of the variance of non-oil output and the covariance between oil and non-oil outputs. If the latter covariance term exceeds the variance, the correlation between stock and futures returns is negative although agents are fully hedged. Such a negative correlation between equity and futures returns can be also understood as an inverse association between the systematic risks (*beta*) of futures and oil. The same equilibrium variance-covariance structure of stock and futures return continues to hold with a more general utility function which is nonseparable in oil and non-oil consumption items.

2   A Lucas Tree Model

The economy is endowed with two types of goods, a composite good that includes all consumables except oil and is generically called non-oil (indexed as a) and a commodity generically called *oil* (indexed as b). In a similar spirit as in Boldenstein et al. (2011) and Gavin et al. (2015), oil is introduced as a commodity in the utility function to motivate the pricing of the commodity. At date t, the representative agent is endowed with y_t^a and y_t^b units of non-oil and oil goods respectively. The growth rates of these endowments evolve stochastically as a Markov process with a stationary distribution.

In view of the complete market nature of the financial environment, all conceivable Arrow-Debreu securities can be traded. However, we shall focus only on three financial instruments which traders hold in equilibrium: (i) equity claims (z_t^a) to future flows of non-oil y_{t+1}^a which sell at the price q_t^a today, (ii) equity claims (z_t^b) to future flows of oil y_{t+1}^b which sell at the price q_t^b today, (iii) claims to future delivery of oil at a price of oil contracted today. Let f_t^j be the price of a binding *forward contract* for delivery of one barrel of oil at date $t + j$, n_t^j be the number of barrels of oils contracted at date t for delivery at date $t + j$. Let there be k such forward contracts which means $j = 1, 2...k$, and let s_t be the spot price of oil. Since the barrels of oil upon delivery at date $t + j$ can be sold at the spot price s_{t+j}, by the definition of such forward contract, it follows that f_t^j is nothing but the price of a claim to a payoff at time $t + j$ which equals the spot price s_{t+j}

The flow budget constraint facing the household is:

$$c_t^a + s_t c_t^b + q_t^a(z_t^a - z_{t-1}^a) + q_t^b(z_t^b - z_{t-1}^b) + \sum_{j=1}^{k} f_t^j n_t^j$$

$$= z_{t-1}^a y_t^a + z_{t-1}^b s_t y_t^b + s_t \sum_{j=1}^{k} n_{t-j}^j. \tag{1}$$

The representative household derives direct utility from consumption of these goods which is represented by the instantaneous utility function, $u(c_t^a) + v(c_t^b)$. The household maximizes the discounted stream of utilities:

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t^a) + v(c_t^b)],$$

subject to (1), where $0 < \beta < 1$ and E_0 is the expectation operator at date 0.

In equilibrium, long and short purchases of the commodity oil, n_t^j add up to zero for each j. The first order conditions are:

Equities:

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$$z_t^a : u'(c_t^a)q_t^a = \beta E_t u'(c_{t+1}^a)\{q_{t+1}^a + y_{t+1}^a\}. \tag{2}$$

$$z_t^b : u'(c_t^a)q_t^b = \beta E_t u'(c_{t+1}^a)\{q_{t+1}^b + s_{t+1}y_{t+1}^b\}. \tag{3}$$

Forward:

$$n_t^j : f_t^j u'(c_t^a) = \beta^j E_t s_{t+j} u'(c_{t+j}^a), \quad j = 1, 2, ...k. \tag{4}$$

Spot :

$$s_t = \frac{v'(c_t^b)}{u'(c_t^a)}. \tag{5}$$

The equity price equations (2) and (3) are standard. The pricing equation (4) of the forward contract basically means that if a trader buys such a forward contract j at the price f_t^j, it entitles him to a payoff s_{t+j} at date $t+j$ which is evaluated in discounted utility terms to equate to the utility cost of buying such a claim at date t. Spot price (5) is given by the intratemporal marginal rate of substitution between c_a and c_b.

Using (4) and (5), one gets the following equation of the forward contract:

$$f_t^j u'(c_t^a) = \beta^j E_t v'(c_{t+j}^b) \text{ for } j = 1, 2, ...k \tag{6}$$

2.1   Correlation between returns on equities and futures

For the sake of illustration, consider first the case when the utility function is separable and logarithmic: $u(c_t^a) + v(c_t^b) = \ln c_t^a + \ln c_t^b$. This simplification enables us to get a clean second moments condition for a negative correlation between stock and futures returns.

The equilibrium equity prices are proportional to non-oil production as follows:

$$q_t^a = q_t^b = \frac{\beta}{1 - \beta} y_t^a. \tag{7}$$

Note that the equity price of oil is also proportional to non-oil production because by virtue of eq. (5), the spot price s_t is y_t^a/y_t^b which means $s_t y_t^b$

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$= y_t^a.$

The *ex post* returns on oil and nonoil stocks are equal to the *ex post* intertemporal marginal rate of substitution in non-oil consumption (c_a). Call this equity return R_{t+1}^E. Using (7) we have:

$$R_{t+1}^E = \beta^{-1} \frac{y_{t+1}^a}{y_t^a}. \tag{8}$$

which means

$$\ln R_{t+1}^E = -\ln \beta + \ln(\frac{y_{t+1}^a}{y_t^a}). \tag{9}$$

In other words, the *ex post* equity return is proportional to the growth rate of non-oil output.

The *ex post* return on the jth futures (call it $R_{t+1}^{F,j}$) is: f_{t+1}^{j-1}/f_t^j . Using (6) one can rewrite $R_{t+1}^{F,j}$ as:

$$R_{t+1}^{F,j} = \beta^{-1} \left[\frac{y_{t+1}^a}{y_t^a} \right] \cdot \frac{E_{t+1} \left[\frac{1}{y_{t+j}^b} \right]}{E_t \left[\frac{1}{y_{t+j}^b} \right]} \quad \text{for } j = 1, 2...k. \tag{10}$$

Noting that $\frac{y_{t+1}^b}{y_t^b}$ is already realized at date t+1, (10) can be rewritten in a log return form as:

$$ln R_{t+1}^{F,j} = -\ln \beta + \ln \frac{y_{t+1}^a}{y_t^a} + \left[\ln E_{t+1} \left(y_{t+j}^b/y_t^b \right)^{-1} - \ln E_t \left(y_{t+j}^b/y_t^b \right)^{-1} \right] \tag{11}$$

The *ex post* one period futures return depends positively on the growth rates of non-oil output, $\ln(y_{t+1}^a/y_t^a)$ and the *news* about the future production of oil shown in the square bracket term. Everything else equal, better news about future oil production depresses the expected return to oil futures because the news about higher oil production signals a lower future spot price.

The correlation between equity and oil futures returns depends on the covariance matrix of shocks to oil and non-oil production. In general the

sign of the correlation can be positive or negative. For illustration, consider a special case where the log of the oil and non-oil production levels are random walk processes with a drift as follows:

$$\ln y_{t+1}^a = \mu^a + \ln y_t^a + \xi_{t+1}^a \tag{12}$$

$$\ln y_{t+1}^b = \mu^b + \ln y_t^b + \xi_{t+1}^b \tag{13}$$

where ξ_{t+1}^a and ξ_{t+1}^b are normal white noises with zero means, variances equal to σ_a^2 and σ_b^2 respectively, and covariance equal to σ_{ab}. Given this assumption, one can rewrite (11) as:

$$lnR_{t+1}^{F,j} = -\ln \beta + (\mu_b - 0.5\sigma_b^2) + \ln \frac{y_{t+1}^a}{y_t^a} - \ln \frac{y_{t+1}^b}{y_t^b} \tag{14}$$

Using (9) and (14), one obtains:

$$cov(\ln R_{t+1}^E, \ln R_{t+1}^{F,j}) = var(\ln \frac{y_{t+1}^a}{y_t^a}) - cov(\ln \frac{y_{t+1}^a}{y_t^a}, \ln \frac{y_{t+1}^b}{y_t^b}) \tag{15}$$

where $cov(.)$ and $var(.)$ stand for unconditional covariance and variance respectively. If $cov(\ln \frac{y_{t+1}^a}{y_t^a}, \ln \frac{y_{t+1}^b}{y_t^b}) > 0$ and it exceeds $var(y_{t+1}^a/y_t^a)$, futures and equity returns are negatively correlated. This happens in an equilibrium where traders are fully hedged.[2]

2.2   A beta based intuition

The negative correlation between equity and futures returns can be understood as an inverse association between the systematic risks of oil futures and oil. To see this divide both sides of (15) by $var_t(\ln R_{t+1}^E)$ and use (8) to

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[2] The random walk assumption makes the growth rates of oil and nonoil iid processes. This assumption is made for simplicity to make the key point. Allowing serial correlation in growth rates such as:$\ln \frac{y_t^a}{y_{t-1}^a} = (1 - \rho_a)\mu_a + \rho_a \ln \frac{y_{t-1}^a}{y_{t-2}^a} + \varepsilon_t^a$ and $\ln \frac{y_t^b}{y_{t-1}^b} = (1 - \rho_b)\mu_b + \rho_b \ln \frac{y_{t-1}^b}{y_{t-2}^b} + \varepsilon_t^b$ where $0 < \rho_a < 1$, $0 < \rho_b < 1$ changes the covariance term in (15) to: $cov_t(lnR_{t+1}^E, lnR_{t+1}^F) = \sigma_a^2 - \left[\frac{1-\rho_b^j}{1-\rho_b}\right]\sigma_{ab}$ which now involves an additional *news effect* shown in the square bracket term. The sign of the covariance between equity and futures returns is more likely to be negative with this "*news effect*".

get:[3]

$$\frac{cov(\ln R_{t+1}^E, \ln R_{t+1}^{F,j})}{var(\ln R_{t+1}^E)} = 1 - \frac{cov(\ln R_{t+1}^E, \ln \frac{y_{t+1}^b}{y_t^b})}{var(\ln R_{t+1}^E)}. \tag{16}$$

The left hand side of (16) is nothing but the *beta* of the jth futures (call it $\beta^{F,j}$) given that R_{t+1}^E is the market portfolio. The right hand side second term may be interpreted as the *beta* of the quantity of oil (referred as $beta^{Oil}$) with respect to the market portfolio given that non-oil output captures all aggregate risk. Thus equation (16) basically means the following tight relationship between these two betas:

$$beta^{F,j} = 1 - beta^{Oil}. \tag{17}$$

Note that $beta^{F,j}$ in (17) does not depend on j and it represents the systematic risk in the oil futures market while $beta^{Oil}$ summarizes the systematic risk in the oil output. The model predicts an inverse relation between $beta^{F,j}$ and $beta^{Oil}$. If the systematic risk of oil is quite substantial ($beta^{Oil} > 1$), a predictable relationship (a negative correlation) emerges between oil futures return and equity returns which means a negative $beta^{F,j}$. However, such a negative relationship cannot be exploited by investors because it arises as an equilibrium condition.

2.3   Case of a non-separable utility function

Until now we assumed that the utility function is additively separable in oil and non-oil output. How restrictive is this assumption? We analyze now the case where the instantaneous utility function is nonseparable, $V(c_t^a, c_t^b)$. The Euler equation for stock a (i.e. eq. (2)) thus changes to:

$$V_{1t}q_t^a = \beta E_t V_{1t+1}\{q_{t+1}^a + y_{t+1}^a\} \tag{18}$$

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[3]Note that (8) implies that $var_t(\ln R_{t+1}^E) = var_t(\ln \frac{y_{t+1}^a}{y_t^a})$

where $V_{it}(y_t^a, y_t^b)$ is the first derivative of the utility function with respect to the ith argument $(i = 1, 2)$ evaluated at date t endowments.

The price equation (6) for the forward contract now changes to:

$$f_t^j V_{1t} = \beta^j E_t s_{t+j} V_{1t+j}, \quad j = 1, 2, ...k. \tag{19}$$

The spot price equation (5) changes to:

$$s_t = \frac{V_{2t}}{V_{1t}} \tag{20}$$

Using (19) and (20) the ex post futures return equation is written as:

$$R_{t+1}^{F,j} = \beta^{-1} . \frac{V_{1t}}{V_{1t+1}} . \frac{V_{2t+1}}{V_{2t}} \frac{E_{t+1}(V_{2t+j}/V_{2t+1})}{E_t(V_{2t+j}/V_{2t})} \tag{21}$$

Without imposing any further restrictions on the preference and endowment processes, one cannot characterize the correlation between stock and futures returns. Assume further that the utility function is homothetic of the form:

$$V(c_{at}, c_{bt}) = \frac{(c_{at}^\theta c_{bt}^{1-\theta})^{1-\gamma} - 1}{1 - \gamma} \tag{22}$$

where $0 < \theta < 1$, γ is the relative risk aversion parameter[4]. With the same random walk processes for oil and non-oil endowments as in (12) and (13), the price:dividend ratio q_t^a/y_t^a is a constant which implies that the ex post stock return R_{t+1}^E is proportional to the non-oil output growth, y_{t+1}^a/y_t^a. On the other hand, the ex post futures return expression (21) becomes proportional to the relative output growth, $(y_{t+1}^a/y_t^a)/(y_{t+1}^b/y_t^b)$. The immediate implication is that the unconditional covariance between stock and futures return takes the same form as in (15). Thus the modification of the utility

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[4]The case of a separable power utility function with the same relative risk aversion coefficient (λ) for nonoil and oil consumption is straightforward. Let $v(c_t^a, c_t^b) = c_{at}^{1-\lambda}/(1 - \lambda) + c_{bt}^{1-\lambda}/(1 - \lambda)$. It is easy to verify that with the same i.i.d specification for growth rate of nonoil, the stock return of nonoil is proportional to the growth rate of nonoil endowment. In this case, the covariance between stock return and futures return is simply λ times the right hand side expression of (15). The correlation between stock and futures return is thus unaffected by this specification.

function to a nonseparable form has no effect on the variance-covaraince matrix of the stock and futures return.[5]

2.4   An Example

In this section, we provide an illustrative example that it is plausible that the model correlation between returns to equity and returns to commodity futures could be negative for the observed variance-covariance matrix of oil and non-oil output. Therefore, it need not represent an unexploited opportunity for hedging the stock market or the business cycle as many analysts claimed before the financial crisis in 2008. To demonstrate this, we use crude oil from the Federal Reserve Bank of St Louis website as a stand-in for commodities based on the consideration that crude oil dominates the stochastic properties of the commodity futures index funds (See Figure 6 in Basu and Gavin, 2011). From a practical perspective, the volatility in crude oil drove the volatility in the commodity futures index funds that were being marketed between 2003 and 2007 as a previously unexploited hedging opportunity. For non-oil, we use the quarterly real GDP series from Bureau of Economic Analysis website. Since the entire debate about the correlation between stock and futures returns refers to the pre-crisis period, our sample period is restricted to 1973Q1-2007Q4.[6]

For our sample, the variances of real GDP growth and oil growth are 0.63% and 9.82% respectively. The covariance between oil and GDP growth rates is 1.05%. Using these observed variance-covariance matrix of real GDP and oil, we compute the model correlation between stock and futures returns based on the covariance formula (15). We find that the correlation is -0.18 which is statistically significant at a 5% level.

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[5]We are grateful to a referee to guide us to this result. Details of the derivation are available from the authors upon request.

[6]In fact, commodity futures failed miserably as a hedge against equity risk during the financial crises.

3 Conclusion

The negative correlation between equity and future returns is often interpreted as a potential hedging opportunity for investors. In this short paper, we establish that such a negative correlation can arise in equilibrium when all investors are fully hedged against aggregate risk. We illustrate this point using a variant of the Lucas (1982) consumption CAPM model. The model shows that the correlation between equity and oil futures returns stems from the variance and covariance properties of non-oil and oil production.

We use a general equilibrium perspective to understand the implications for the correlation between equity and futures returns. The lesson that we learn from this exercise is that commodity and equity markets are integrated and should not be studied in isolation. Thus a negative correlation between these two returns should not necessarily be construed as a hedging opportunity to common macroeconomic shocks. Rather, it reflects the equilibrium response of equity and futures markets to fundamental shocks driving the economy. We do not claim that forward contracts can never be used as a hedge. The rationale for doing so would require heterogeneous agents and an explicit specification of the incomplete market environment with a storage technology that might give rise to the hedging opportunity. This could be a possible extension of this paper.

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