An American Inequality Trap

Understanding Permanent Black-White Earnings Inequality

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Abstract

For more than 30 years, the ratio of average black earnings to average white earnings has remained close to 0.6. Additionally, US cities have remained dramatically segregated by race. This paper provides a joint theory of pre-market skills and residential segregation for quantitatively studying these phenomena. It is established that the magnitude of racial and earnings sorting observed in US cities implies 70 percent of observed black-white inequality. While the mechanism posed is intricate, all of its three non-standard components are essential for permanent black-white inequality to arise in a steady state: neighborhood human capital externalities, house price differences across neighborhoods, and preferences over neighborhood racial composition.

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1 Introduction

The ratio of average black earnings to average white earnings has remained close to 0.6 for around 30 years. The gap in educational attainment and test scores of black and white Americans has remained roughly constant for around 30 years as well.[1] Figure 1 shows the dramatic stagnation of two simple indicators: the gap in average years of schooling and the ratio of black to white household earnings. The two supplementary dotted lines

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[1]For a detailed discussion see Neal (2006) or Goldin and Katz (2009).

Figure 1: Persistence of Black-White Inequality

(a) Average Years of Schooling

(b) Average Household Earnings
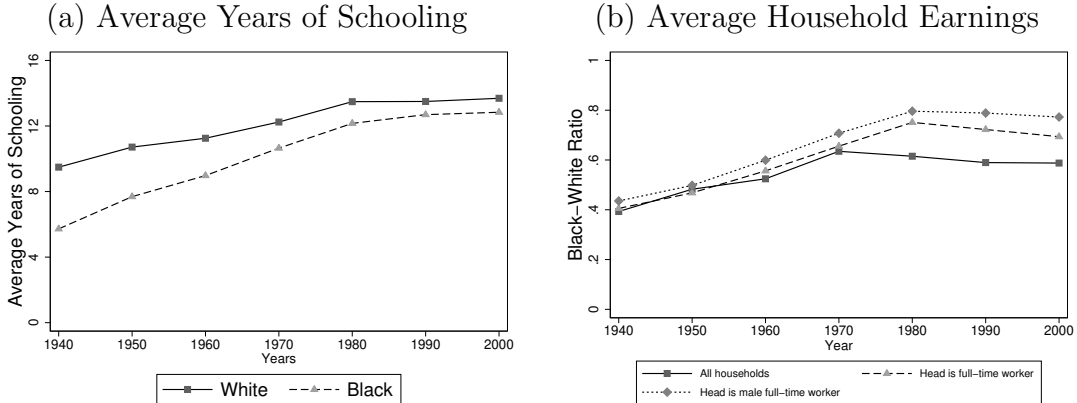
Based on 1940 to 2000 Census data. The base sample consists of single-family households headed by a US-born black or white person between 25 and 64 years of age. Panel (a) plots average years of education for sample heads of household 30 to 35 years of age. Panel (b) plots the black/white ratio of average household earnings in three ways: for all sample households, for households headed by a full-time worker and for households headed by a full-time working male. Household earnings are measured as earnings of head plus earnings of spouse (if present) divided by the following adult equivalence scale: $\sqrt{\#adults + 0.5 \times \#children}$. See the appendix for additional details.

in Figure 1(b) starkly illustrate the fact that black-white earnings inequality is not an artifact of racial differences in labor force attachment or household composition. A large and persistent gap remains among households with strong labor force attachment even if female-headed households are excluded.

There is evidence that pre-market skills are an important source of black-white earnings inequality. A prominent finding by Neal and Johnson (1996, 1998) is that racial differences in Air Force Qualification Test (AFQT) scores (a proxy of pre-market skills) can account for nearly three quarters of the black-white gap in mean *hourly* earnings and half of the gap in mean *annual* earnings.[2]

This paper focuses then on explaining racial differences in pre-market skills.[3] To do so, the paper adopts the dual view that (i) pre-market skills are the single determinant of earnings differences across households and (ii) pre-market skills depend strongly on parental inputs. This dual view is well rooted in the literature.[4]

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[2]The AFQT is administered between ages 16-18 to determine eligibility for the Air Force. AFQT score is considered a racially unbiased measure of pre-market skills. The results cited correspond to a sample of males in their late twenties.

[3]The urgency for such explanation is widely recognized and discussed in the literature. See, for example, Loury (1998) or Neal (2006)).

[4]Keane and Wolpin (1997) and Huggett, Ventura and Yaron (2010) find that the role of labor market risk and post-secondary schooling and training in shaping lifetime earnings is small compared to the role

The basic framework contains a continuum of Loury-Becker dynasties. Each dynasty is an infinitely lived agent that makes human capital accumulation decisions. The innate ability of children is an uninsurable idiosyncratic shock to human capital production technology.[5] An intergenerational consumption smoothing motive drives intergenerational persistence of human capital and earnings beyond what is directly implied by the persistence of shocks.

In the basic framework, individual mean reversion implies there is no black-white mean earnings inequality because every dynasty (regardless of race) displays similar mean earnings over long periods.[6] For this reason, the basic framework is enriched here by adding three elements related to neighborhoods:

1. Neighborhood human capital externalities by which investments in human capital are more productive at neighborhoods with higher human capital.

2. Neighborhood housing markets by which housing services can have different prices across neighborhoods.

3. Preferences over neighborhood racial composition by which, everything else constant, black households prefer predominantly black neighborhoods and white households predominantly white ones.

It is also assumed that agents choose where to live at the beginning of each period and relocation is costless.

The mechanism sustaining racial inequality in steady state depends crucially on a sorting pattern which stems from (1-3). Specifically, the following pair of neighborhoods, denoted I and II, can arise in a steady state equilibrium under rational expectations:

- I is predominantly black, with low average human capital and low housing prices.

- II is predominantly white, with higher average human capital and higher housing prices.

The key assumption for sorting by human capital is that the benefit from externalities in (1) is higher for higher human capital households. Households from the right tail of the human capital distribution "price out" households from the left tail. This is allowed by house price variation allowed by (2).

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of pre-market factors. Cunha and Heckman (2007) and Todd and Wolpin (2004), among others, find a strong impact of parental inputs on pre-market skills.

[5]A common rationale for market incompleteness and zero borrowing constraints in this framework is that a child's future earnings are inadequate as collateral for loans to parents.

[6]A key standard assumption for mean reversion is decreasing returns to scale in human capital production. See Becker and Tomes (1986, p. S7).

If the human capital distribution displays racial inequality, sorting by human capital will already imply some sorting by race. This follows mechanically from the correlation between race and human capital. This type of sorting is not sufficient for black white inequality to persist indefinitely.

The key ingredient for black white inequality to persist over time is the racial preference in (3). Thanks to the racial preference in (3) there is range of human capital values at which black households sort into I but white households sort into II. Black children from parents in this interval will posess lower mean human capital than white children from parents in this interval. Reversion to the mean is broken in a systematic way for households in this interval. Black households drift away from the mean downwards and white households drift away upwards.

It turns out that the sorting pattern described above is also an adequate qualitative description of the US urban landscape in year 2000. This coincidence is the foundation for the paper's first contribution. The models parameters are calibrated to exactly match target neighborhood-level and aggregate-level facts without imposing any degree of racial inequality. Given these parameters, the models ability to reproduce the observed extent of racial inequality, among other additional facts, is evaluated. When the model matches all targets exactly it produces a black-white earnings ratio of 0.72. Therefore, given the empirical ratio of .61, the model produces 72differential in earnings.

In the benchmark equilibrium, racial inequality is not due to racial differences in parental investments within each neighborhood, but due to differences in the residential location of each race. The location decisions of households account for 86% of the difference in average human capital investments and 97% of the difference in average earnings of households. According to the model, the arguable existence of "cultural differences" that lead to racial differences in investment is only an artifact of residential location.

The benchmark calibration reveals that racial preferences are sizable. Within the benchmark equilibrium, in any given period, the aggregate period utilities of black households would be left unchanged by a 3.9% cut of their consumption of housing and non-housing goods if they could enjoy their ideal neighborhood racial configuration instead of the observed one. The aggregate utilities of white households would be unchanged by a 1% decrease in consumption if they could enjoy their ideal neighborhood racial configuration instead of the observed one. An unexpected result is that all racial inequality and racial segregation disappear from the benchmark equilibrium when the weight of racial preferences in the utility function is halved.

The second contribution of the paper is to show that the mechanism is parsimonious. The elimination of any of the three components enumerated above would suffice to eliminate black white inequality from equilibrium. Without (1) there are no incentives for sorting by human capital so there is perfect segregation by race. Under perfect segregation the two neighborhoods behave like two countries in a standard growth model and racial convergence follows. Without (2) there are no incentives for white households to live in I. So full

segregation and equalization follow. Without (3) black and white households make identical location and investment decisions so that mean reversion implies black-white convergence.

Parsimony is important for two reasons. First, because parsimony allows viewing this model as a reasonable starting point for quantitative black-white inequality studies. Second, because persistent black-white inequality can be interpreted as evidence that the (1-3) are quantitatively relevant more broadly for studying heterogeneity in pre-market factors.

A related mechanism based on exogenous segregation was first proposed by Loury (1976) and extended by Lundberg and Startz (1998) and Loury Bowles and Sethi (2008).[7] Introducing endogenous segregation by race solves two issues in exogenous segregation models. The first problem is that some households adhere to a detrimental social group while they, at the same time, are willing to pay to be part of a more advantageous social group. The second issue is that counterfactual exercises and policy experiments are hard to interpret when the researcher is forced to arbitrarily choose the evolution of the key force, which is segregation by race.

To the author's best knowledge, this is the first paper to address the issue of black white earnings inequality within a quantitative equilibrium model. It is also the first paper to integrate race into a multi-community model with endogenous earnings and sorting. Benabou (1993) and Durlauf (1996) provide multi-community models with endogenous earnings distributions. Fernandez and Rogerson (1998) provides a quantitative application. Compared to existing papers in the quantitative multi-community literature this paper is unique in motivating investments in children through perfect altruism. Treating children's future human capital as consumption of a good could be restrictive if one is interested in investigating investment patterns across black and white households. And, finally, this is also the first paper to endogenize the distribution of earnings within a model of racial segregation. Schelling (O) provides models of segregation by race based on (3), while Sethi and Somanathan (2004) allow for (2) and (3) in a model of residential segregation by race and earnings with exogenous earnings distributions.

A third contribution of this paper is to show that the racial inequality and residential segregation patterns observed in the last 30 years are quantitatively consistent with the insight of Loury (1976). In Loury (1976) this mechanism was primarily viewed as a theoretical construct by "equal opportunity" laws would not be enough to guarantee the equalization of outcomes by race over time.

The remainder of the paper proceeds as follows. Section 2 characterizes a set of US urban areas as a single city with two "representative neighborhoods" where households display one of two skin colors (black or white). Census data and a clustering technique are employed

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[7]Loury (1976) allows two types of externalities: "home environment" and "community environment". "Home environment" is akin to the investment decision considered here and "community environment" is specified here as average human capital within a residential area. Lundberg and Startz (1998) considers exogenous residential and workplace segregation. Bowles, Loury and Sethi (1998) allows binary parental investments.

for this purpose. Section 3 describes the model and presents some illustrative theoretical assertions. Section 4 finds model parameter values that generate a steady-state equilibrium that exactly replicates residential racial and earnings segregation, housing prices, intergenerational earnings persistence, cross sectional lifetime earnings dispersion, and aggregate investment in human capital facts from US data. Given the estimated parameters, the section then evaluates the model's ability to predict BW earnings inequality, among other features of the data. Section 5 analyzes the estimation results and uses the estimated model to compute counterfactual equilibria where features of the model are arbitrarily modified. Section 6 presents conclusions and the agenda for future work.

2 Representative Neighborhoods

In this section, the US is characterized as a single city with two neighborhoods. A sample of $17,815$ census tracts from Metropolitan Statistical Areas (MSA) containing 10% or more black households is selected from Census 2000 data. The sample is partitioned into two groups, which are labeled Neighborhood I and Neighborhood II. Whether a tract is labeled I or II depends on its average human capital H, racial configuration, R, and price of housing services, P. Human capital H is measured as average household earnings in the tract. Racial configuration us measured as the percentage of non-Hispanic white households in the tract. The price of housing services P is measured as a constant-quality gross-of-tax unit price of housing in the tract.[8] The K-Means clustering algorithm is used to perform the partition. This algorithm provides a partition of tracts into a fixed number of mutually exclusive groups. The resulting partition minimizes a sum of squared deviations of each tract's $(\log H, R, \log P)$ from its group's mean. Each observation (census tract) receives a weight proportional to the number of households it contains. Each component of $(\log H, R, \log P)$ is normalized by transforming it to a Z-score. The resulting partition therefore minimizes within-group squared deviations and maximizes between-group deviations of an index that equally weights the three variables above.

The resulting ratio of between-group variance to total variance in (H, R, P) is 0.37. This is interpreted as indicating that using an off-the-shelf clustering procedure and two groups, one can capture more than one third of the variation in human capital, racial composition and housing service prices across US neighborhoods.

Table 1 summarizes the characteristics of each group of tracts. Racially, Neighborhood I is mixed, with 37% white, while neighborhood II is 93% white. This reflects strong segregation by race. Focusing only on black and white households, average earnings in Neighborhood I are .61 of those in Neighborhood II. This ratio is explained both by the sorting of each race by income, and by the fact that black earnings are lower.[9] Housing

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[8]See Appendix A1 for details on the construction of H R and P measures and sample selection.

[9]Among black households, the ratio of average earnings in I to average earnings in II is .70. This ratio

Table 1: Characteristics of Representative Neighborhoods in 2000

Number of HH (thousands)[1]	Neighborhood		
	I	II	I/(I+II)
Black	4,451	1,359	.77
White	2,662	18,577	.13
Other	1,152	2,150	.35
Total	8,265	22,085	.27
W/(B+W)	.37	.93	

Household Income ($)	Neighborhood		
	I	II	I/II
Black	40,076	57,124	.70
White	44,727	76,711	.58
Other	41,320	67,166	.62
Total (w/o Other)	41,816	75,376	.55
Total	41,747	74,577	.56

Earnings and Prices ($)	Neighborhood		
	I	II	I/II
Average Earnings ($)	33.591	61.889	.54
Average Earn. B and W only ($)	41.816	75.376	.61
Price of Housing Services	10.405	14.268	.73

prices in Neighborhood I are .7, relative to those in Neighborhood II.

Figure 2 display the geographical location of tracts classified as Neighborhood I and II for Chicago and Detroit which are two of the largest MSA in the sample.[10] Tracts classified into the same group are spatially concentrated forming large parcels within each MSA. This concentration is consistent with the model abstraction whereby each neighborhood defines a separate housing market and a separate residential environment.

The spatial configuration of the MSAs in terms of neighborhood types has not changed much over time, except for an expansion of the area covered by each type of Neighborhood. The paper now turns to the description of the model. The presentation of the model is set in N neighborhoods for convenience and generality. The subsequent sections return to the two neighborhood characterization provided here.

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is .58 among white households.

[10]Maps of other MSA mostly have similar properties and are available from the author. See the appendix for a list of MSA included in the sample.

Figure 2: Geographical Properties of Representative Neighborhoods

Chicago   Detroit
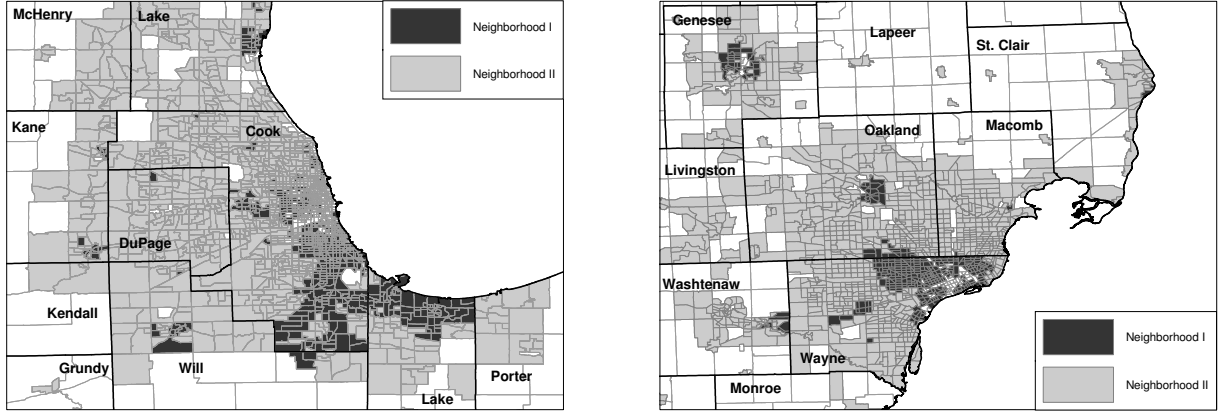
3   Model

The model economy is populated by a unit continuum of dynastic households. Each period, a household is composed by an adult and a child. Households are described by their epidermic color $r = \{B, W\}$, the innate ability of the child z and the parent's stock of human capital h. An exogenous fraction χ_B of households is black (B) and a fraction χ_W is white (W), with $\chi_B = 1 - \chi_W$.

Each household chooses its residential location from within a finite set of neighborhoods. Adults work full time and obtain earnings wh at a rental rate w per unit of human capital h. Black and white households receive an equal market return w per unit of human capital. Households choose consumption of non-housing goods c, housing services l, and the flow i of private investment in the child's future human capital h'.

Exiting every period the adult dies and the child becomes adult. Entering next period, a new child is born from each adult. The new child's innate ability z' is drawn from a probability distribution $\pi(z'|z)$ which depends on parental innate ability z but is independent of the household's skin color.[11] The new child's color r is equal to its parent's color. Each household is altruistic towards its child's future household, discounting its utility at rate $0 < \beta < 1$. Households cannot borrow against the child's future labor income and, for simplicity, they can only transfer resources to the next generation by investing in the child's future human capital.

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[11]It is assumed that the child's ability is observed before the household makes decisions. Ex-ante observation of ability is consistent with model periods representing several years of investment decisions and observation of the child's characteristics. The literature contains examples of ex-ante and ex-post observation of ability. For the former, see Becker and Tomes (1986), for the latter see Loury (1981).

3.1 Neighborhoods

Each neighborhood n is characterized by an exogenous aggregate local supply of housing services L_n and a vector of endogenous neighborhood characteristics. Endogenous characteristics include the local price of housing services P_n, the fraction of the households living in the neighborhood which are of color r, denoted by $R_n(r)$, and the average human capital H_n of adult residents of the neighborhood. For simplicity, I assume that all housing is owned by absentee landlords outside the model.

3.2 Preferences

Households of color r derive utility from the consumption of non-housing goods c, housing services l, and from the fraction R of households living in their neighborhood that have color r.

$$u\left(c, l, R\left(r\right)\right)$$

Function u is jointly concave, twice continuously differentiable, strictly increasing in c and l, and satisfies $u_R\left(c, l, 0\right) > 0$ and $u\left(c, l, 0\right) < u\left(c, l, 1\right)$.

Note that monotonicity in R is not imposed. The last two conditions, which follow those in Sethi and Somanathan (2004), allow an interior satiation point in R. For this reason, households may prefer a neighborhood with some degree of integration over one with an overwhelming majority of their own color. The last condition says, however, that households prefer a neighborhood populated exclusively by their own color to a neighborhood that is exclusively populated by the other color.

3.3 Technology

Consumption Good The production technology of non-housing goods is linear in aggregate human capital, and human capital productivity is equal to w.

Human Capital The child's future stock of human capital h' depends on innate, parental and neighborhood factors. A child's future human capital is determined by its innate ability z, parental human capital h, household investment in human capital i and neighborhood average human capital H_n.

$$h' = \left(1 - \delta\right) h + z F\left(i, H_n\right)$$

Function F is jointly concave, twice continuously differentiable, strictly increasing in each argument and exhibits decreasing returns to scale. Clearly, technology is independent of the household's color.

The specification is broadly consistent with standard theories of intergenerational transmission.[12]

Private investments i are measured in units of human capital. This allows a dual interpretation of wi as the foregone earnings from parental time devoted to the child or as "monetary" investments.

The direct transmission term $(1 - \delta) h$ can be thought of as the part of parental human capital that is passed costlessly across generations. Attitude, personality and social connections are examples of this component.[13]

Innate ability z represents the genetic component of learning ability together with "luck". Finally, the impact of neighborhood average human capital H_n on human capital accumulation captures two main aspects of neighborhoods. First, what is known as neighborhood effects: social connections, positive role models, reduced exposure to violence, and more community resources. Second, differences in local provision of human-capital-enhancing public goods across neighborhoods.[14]

3.4 Household's Problem

The problem is most efficiently described in recursive language. The household's state vector is given by $x = (h, r, z)$. The vector of neighborhood characteristics $\{H_n, R_n, P_n\}_{n=1}^{N}$

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[12]In Becker and Tomes (1986, equation 4), for example, production of human capital depends on public and private investments together with an "endowment". The endowment is given by social environment, genetic, family-culture and luck factors. The H_n argument here can be interpreted as a social environment factor, z as the counterpart of genetic and luck factors, and $(1 - \delta) h$ can be interpreted as the family-culture endowment. Like in that paper, marginal productivity of parental investments rises with environment, genetic and luck endowments here. However, this paper's counterpart to the family culture component does not affect the returns to private investment. Also, social environment is allowed to vary by neighborhood and public investments are abstracted from.

[13]A growing literature (see, for example, Heckman and Rubinstein, 2001) finds an important role for noncognitive skills in the determination of labor market outcomes. Another literature finds that various personality traits are highly heritable (see Loehlin 1985, and references therein). Finally, Groves (2005) finds that intergenerational persistence in the personality trait denominated "fatalism" can explain 4 percentage points of the intergenerational earnings correlation.

[14]Admittedly, this last component is captured here in a rough way. This is not a critical caveat, and even a desirable feature of this analysis given that focus here is on racial inequality. There is evidence suggesting that, on average, schooling expenditures are similar for black and white youth in the US (see Neal 2006, section 4.2). Fernandez and Rogerson (1996) focuses specifically on differences in schooling expenditures across neighborhoods.

is taken as given. The decision problem of a household in state x is given by

$$
\begin{aligned}
V(x) &= \max\{V_1(x),...,V_N(x)\} \\
V_n(x) &= \max_{c,l,i} u(c,l,R_n(r)) + \beta E[V(x')|z] \\
&\quad subject\ to \\
c + P_n l + wi &\leq wh \\
h' &= (1-\delta)h + zF(i,H_n) \\
x' &= (h',r,z').
\end{aligned} \tag{1}
$$

The problem formalizes the verbal description in the preceding parts of Section 3. Function $V(x)$ denotes the optimal value for a household in state x. This value reflects the maximum utility among those provided by neighborhoods 1 to N. These neighborhood specific utilities $V_n(x)$ for $n = 1, ..., N$ are generated by the household's choice over consumption, housing services, and investment in child's human capital, given each neighborhood's racial configuration R_n, price of housing services P_n and average human capital H_n. The next definition summarizes the objects that constitute a solution to the household problem.

Definition 1 *A solution of the household decision problem consists of neighborhood conditional decision rules $(c(x,n), l(x,n), i(x,n))$, neighborhood value functions $\{V_n(x)\}_{n=1}^N$ and a value function $V(x)$ satisfying (1).*

Due to the discrete neighborhood choice involved in this household problem, only some of the standard properties of value functions and decision rules hold. Appendix A-2 reviews these properties. In summary, it possible to prove the Bellman equation above defines a contraction mapping in the space of continuous bounded functions. However, the fixed point $V(x)$ of such mapping will, in general, not be differentiable or concave.

3.5   Location Decision Rules

It was not necessary to specify where households actually choose to live in order to define the solution to the household's problem above. Knowing the utility provided by the best neighborhood was enough. However, specifying a location decision rule is vital for the definition and computation of equilibrium. Location is completely pinned down by the solution of the decision problem when the maximum in the right hand side of the first equation in (1) is unique. However, in states x where the household is indifferent between two or more best neighborhoods, there is no way to determine where the household would actually choose to live. It is assumed that households randomize when indifferent. Therefore, the neighborhood decision rule is defined as a state-conditional probability distribution over neighborhoods.

Definition 2 *A location decision rule $\eta(n|x)$ is consistent with a solution to the household's problem if it satisfies the following conditions:*

1. $\eta(n|x) = 0$ if $V_n(x) < V(x)$

2. $0 \leq \eta(n|x) \leq 1 \; \forall \; n$

3. $\sum_n \eta(n|x) = 1$.

When there are no ties, $\eta(n|x) = 1$ if $V_n(x) = V(x)$ and $\eta(n|x) = 0$ if $V_n(x) < V(x)$. Therefore, $\eta(n|x)$ is completely specified by the decision problem. When there are ties, some probabilities are inevitably left unspecified.

With the solution to the household problem and the location decision rule in hand equilibrium can be defined.

3.6   Equilibrium

Before proceeding it is important to note that the mentioned indeterminacy of some probabilities in the location decision rule $\eta(n|x)$ does not create any problems in the definition of equilibrium, besides a minor modification to the usual definition of the transition function.[15]

Definition 3 *A stationary spatial equilibrium is a probability measure of agents over individual states μ, a vector of neighborhood characteristics $\{(H_n, R_n, P_n)\}_{n=1}^N$, a solution of the household decision problem, and a location decision rule $\eta(n|x)$ that satisfy the following conditions:*

1. The solution of the decision problem takes $\{(H_n, R_n, P_n)\}_{n=1}^N$ as given.

2. The location decision rule $\eta(n|x)$ is consistent with the solution to the household problem.

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[15]Define the transition function as

$$P(x, (A_h, r', z')) = \sum_n \widehat{P}_n(x, (A_h, r', z'))$$

$$\widehat{P}_n(x, (A_h, r', z')) = \begin{cases} \pi(z'|z)\eta(n|x) & \text{if } h(x,n) \in A_h \text{ and } r = r' \\ 0 & \text{otherwise.} \end{cases}$$

$$h(x,n) = (1-\delta)h + zF(i(x,n), H_n)$$

3. Neighborhood characteristics and aggregate demographics implied by μ together with decision rules are consistent with aggregate demographics χ_r and neighborhood characteristics $\{(H_n, R_n, P_n)\}_{n=1}^{N}$.

   $\chi_r = \mu(\{x : r(x) = r\})$ for $r = B, W$

   $H_n = \int h\eta(n|x)\, d\mu$ for all n

   $R_n(r) = \frac{\int_{x:r(x)=r} \eta(n|x)d\mu}{\int \eta(n|x)d\mu}$ for $r = B, W$ and all n.

4. Housing markets clear

   $L_n = \int l(x, n)\, \eta(n|x)\, d\mu$ for all n.

5. The probability measure is stationary and consistent with the solution of the household maximization problem

   $\mu(A) = \int P(x, A)\, d\mu$ for any Borel set A.

I now discuss three propositions that illustrate the mechanics of the model and also highlight important points. First, including more than one neighborhood in the model is essential in generating racial inequality. Second, including racial preferences in the model is essential in generating racial segregation and racial inequality. Third, any equilibrium of the model obtained in the absence of racial preferences can be replicated in the presence of racial preferences.

One Neighborhood When $N = 1$ the decision problem of households collapses to a version of the standard consumption-savings problem with uninsurable idiosyncratic risk. In this case the value function is routinely shown to be concave and differentiable, and the following illustrative first order conditions follow

$$
\begin{aligned}
u_c P_1 &= u_l \\
u_c &\geq \beta F_i E\left[u_{c'}|z\right] \ \ (= \text{if } i > 0).
\end{aligned}
$$

The first condition simply equates the marginal utilities of consumption and housing each period.[16] The second condition is the intertemporal Euler equation, which closely resembles the condition in the incomplete markets economies of Huggett (1993) and Aiyagari (1994). In these economies, households trade a non contingent asset in a centralized market to insure against idiosyncratic employment shocks. Here, households self-insure against low innate ability shocks by investing in human capital, which has deterministic marginal

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[16]In the one-neighborhood model the price of housing serves only a utility scaling role when preferences are homothetic in (c, l). In the multiple neighborhood case with idiosyncratic risk, housing prices have a much richer role, see Section 5.6

returns. However, claims to future consumption cannot be traded here due to the assumption of no intergenerational financial markets. Marginal productivity F_i is a function of each household's choices. Households share the neighborhood productivity component but the marginal product is decreasing in each household's private investment i.

Equilibria in the one-neighborhood economy cannot display racial inequality. When there is only one neighborhood and preferences are additively separable in the racial component, decision rules must be identical for black and white households. Therefore, in well behaved cases, the stationary distribution of human capital and innate ability, conditional on color, should be identical for black and white households. This result is now stated more formally.

Assumption $A1$, below, says that the utility function is additively separable in the racial component. Assumption $A2$ guarantees that for any given solution to the household decision problem, a unique stationary distribution is generated by the decision rules. In particular, $A2$ rules out cases where the Markov process implied by the transition function $P(x, (A, r', z'))$ has multiple ergodic sets as in standard models of a "poverty trap".[17]

$A1.$ $u(c, l, R(r)) = u(c, l) + v(R(r))$

$A2.$ *The sequence of probability measures,* $\{\mu_j\}_{j=1}^{\infty}$ *generated from any initial probability measure* μ_0 *such that* $\mu_0(\{x : r = B\}) = \chi^B$ *by the transition function* $P(x, (A, r', z'))$, *converges weakly to* μ.

Proposition 1 (One Neighborhood) *Suppose A1 holds. Then in any equilibrium* E_0, *the solution of the household decision problem is identical for* $r = W, B$. *If in addition, A2 holds in* E_0, *then* $\mu((A_h, B, z))/\chi_B = \mu((A_h, W, z))/\chi_w$ *for all intervals* A_h *and all* z.[18]

Proof: See Appendix A-2.

The proposition formalizes a crucial point. In this model, neighborhoods are essential in generating racial inequality.

No Color Preference If $u(c, l, R(r)) = u(c, l)$ households do not care about the color composition of their neighborhood when making residential location decisions. In this case it will also be hard to obtain equilibria that display racial disparities or residential racial segregation when there are *two* or more neighborhoods ($N > 1$).

Assumption $B1$, below, implies that households do not care about the color composition of their neighborhood. Assumption $B2$ guarantees that the location decisions of households

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[17]This is a nontrivial concern here, given that decision rules typically exhibit discontinuities. However, equilibria computed in the paper satisfy the uniqueness of the stationary distribution. See Appendix A-3 for details of the computation.

[18]The equality of probability measures here and in the other propositions is defined in the weak sense. See Stokey and Lucas (1986), pag. 337.

are completely specified by the solution to the household's problem. Assumption $B3$ plays the same role of assumption $A2$ above.

B1. $u\left(c, l, R(r)\right) = u\left(c, l\right)$

B2. *The measure assigned by μ to the set of households that are indifferent between two or more neighborhoods equals zero.*

B3. *The sequence of probability measures, $\{\mu_j\}_{j=1}^{\infty}$ generated from any initial probability measure μ_0 such that $\mu_0(\{x : r = B\}) = \chi^B$ by the transition function $P\left(x, (A, r', z')\right)$ converges weakly to μ.*

Proposition 2 (No Color Preference) *Suppose B1-B2 hold. Then in any equilibrium E_N the solution of the household decision problem is identical for $r = W, B$. If in addition, B3 holds in E_N, then $\mu\left((A_h, B, z)\right)/\chi_B = \mu\left((A_h, W, z)\right)/\chi_w$ for all intervals A_h and all z.*

Proof: See Appendix A-2.

The proposition states that preferences over neighborhoods' color composition are a necessary ingredient in generating racial inequality and racial segregation within this model. On one hand, the result provides motivation for this feature of the model. On the other hand, it raises the question of whether one can obtain substantial inequality and segregation with moderate levels of racial preference. This type of consideration has occupied the literature on segregation since Schelling (1971, 1972), and it is explored numerically here (see section 5.4).

Symmetric R Equilibrium The next proposition shows how equilibria without segregation or racial inequality can arise in the presence of any degree of racial preference. In these equilibria, H and P might or might not vary across neighborhoods. However, R is identical across neighborhoods. As in the no color preference case, symmetric R equilibria do not display any racial inequality.

Proposition 3 (Symmetric R Equilibrium) *Suppose B1-B3 hold. Then for any equilibrium with no color preference E_N^* and any additively separable racial component of the utility function $v(R(r))$, there exists an equilibrium E_N under color preference $v(R(r))$ which only differs from E_N in that value functions V and $\{V_n\}$ are additively rescaled.*

The existence of symmetric R equilibria is viewed here as a particular case of the problem of equilibrium multiplicity. However it is interesting to highlight that there exist equilibria for which racial preferences of any magnitude can be unobservable.

4 Fitting the Model to Data

This section first describes the steps taken to find model parameters such that a numerical approximation to an equilibrium of the model produces facts that mimic a set of facts from data. The section then tests the model's performance in its ability to match additional facts.

4.1 Choosing Parametric Forms

The initial step is to choose parametric forms for the process of innate abilities $\pi\left(z'|z\right)$, the utility function and the human capital production function in order to allow computation of equilibria of the model economy.

Innate Ability   Following a standard practice, the process for innate ability is parameterized as a Markov chain taking a small number s of values $(z_1, z_2, ..., z_s)$. These values are equally spaced on the log scale. The range of log values and the transition matrix are chosen to approximate a continuous state Markov process with normally distributed innovations

$$\ln(z') = \rho \ln(z) + \varepsilon \qquad \varepsilon \sim N(0, \sigma_\varepsilon^2).$$

The method in Tauchen (1986) is employed for this purpose.[19]

Utility Function   The parametric specification includes two additively separable terms. The first term captures the period utility from consumption of non-housing and housing goods. This term is parameterized as the composition of a Cobb Douglas and a CRRA utility function.[20] The second term captures utility from neighborhood racial configuration. This term is parameterized as a quadratic loss function where deviations of $R(r)$ from parameter R^* generate disutility.

$$
\begin{aligned}
u\left(c, l, R\right) &= \frac{\left(c^\alpha l^{1-\alpha}\right)^{1-\sigma}}{1-\sigma} - \kappa\left(R - R^*\right)^2 \\
\sigma &\geq 0 \\
0 &\leq \alpha \leq 1 \\
\kappa &\geq 0 \\
0 &\leq R^*
\end{aligned}
$$

---

[19]The values of $ln(z)$ used for computation are chosen to lie in $[-2\sigma_z, 2\sigma_z]$ where $\sigma_z^2 = \frac{\sigma_\varepsilon^2}{1-\rho^2}$. The number of values s is set to 9.

[20]The Cobb Douglas specification follows Davis and Ortalo-Magne (2006) who provide evidence in favor of unit elasticity of substitution between housing and non-housing consumption.

Parameter R^* can be interpreted as a bliss point in neighborhood's racial configuration when $R^* \leq 1$. Parameter κ scales the racial component. Additive separability of the racial component was an assumption in Propositions 1-3. There, separability implied that the neighborhood's racial configuration R does not impact household investment decisions when their residential location is taken as given. This is a desirable discipline in the model.

Production of Human Capital The production of human capital is given by a direct transmission term $(1 - \delta) h$ and the production of new human capital. The latter is modeled as a CES production function with technological parameter zA, elasticity of substitution $1/(1 - \gamma)$, returns to scale χ and share parameter λ.

$$h' = (1 - \delta) h + zA \left[\lambda i^\gamma + (1 - \lambda) H^\gamma \right]^{\frac{\chi}{\gamma}}$$

This function provides a reasonable degree of flexibility. In particular, it allows for flexible degree of substitutability between neighborhood human capital externalities H and private investments i. The marginal rate of substitution between i and H is given by $\frac{1}{1-\gamma}$.

4.2 Parameter Values and Model Fit

Three steps are followed in order to set the model parameters and assess whether the model is correctly specified. First, a subset of 7 pre-specified parameters is set to standard values in the literature or values directly suggested by data. Then, a numerical search algorithm is used to estimate the remaining 9 parameters to exactly match a set of 9 target facts from data. Third the model's performance is analyzed using a set of 4 additional facts.

Estimation Approach The target moments for estimation are chosen in the following spirit: Choose parameters to exactly match segregation by earnings and racial segregation, along with other facts, without imposing any degree of racial inequality. Then ask what degree of racial inequality is generated by such parameter values.

Pre-Specified Parameters The total fraction of black households is set to $\chi_B = 0.21$, which is the value from Census data, derived from 1. This choice implies a fraction of white households $\chi_W = .79$. The rental rate of human capital w and the price of housing services in neighborhood I, P_I, play only a scaling role in the model. Their values are set to 1 without loss of generality. The subjective discount factor β is set to match an annual factor of 0.96 while the duration of model periods is implicitly set to 25 years. This yields $\beta = 0.36$. Finally, the direct transmission term of human capital $(1 - \delta)h$ is set to zero by setting $\delta = 1$, this decision follows preliminary experimentation with the model, which showed that this term can be dropped without affecting the model's ability to match the target facts.

Table 2: Pre-Specified Parameters

Definition	Parameterization	Calibrated Value
Fraction of Black Households	χ_B	$\chi_B = .21$
Human Capital Rental Rate	w	$w = 1$
Housing Service Price Level in I	P_I	$P_I = 1$
Discount Factor	β	$\beta = .36$
Utility Function ((c, l) component)	$u(c,l) = \frac{(c^\alpha l^{1-\alpha})^{1-\sigma}}{1-\sigma}$	$\alpha = .75$ $\sigma = 2.5$
Direct transmission of Human Capital	$(1-\delta)h$	$\delta = 1$

Parameter σ, which determines relative risk aversion and the elasticity of intertemporal substitution, is set to $\sigma = 2.5$. This is a standard value according to micro-econometric estimates (see Browning, Hansen and Heckman, 1999).[21]

Finally, parameter α which controls the share of housing services in current period household expenditures is set to $\alpha = .75$. This value generates the housing share of expenditures found by Charles et al. (2008) in CEX data (.25). The value is also consistent with Census data evidence presented by Davis and Ortalo-Magne (2008). Table 2 summarizes pre-specified parameters and their values.

Estimation Targets The remaining 9 parameters of the model are chosen to exactly match a set of 9 target facts. These parameters and their values are displayed in Table 3. The search is conducted using the simplex routine of Press et. al (1992) in conjunction with the equilibrium search procedure (see Appendix A-3 for further description). The targets and their source are now reviewed.

1. Average Earnings: The average level of earnings, \$54,200, derived from 1, is matched in order to keep the numerical scaling of the problem approximately constant across different parameterizations. However, it has no further effect on the behavior of the model economy.

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[21]Some preliminary experimentation showed this value allows a better fit of the model compared to other alternatives considered (namely $\sigma = 1.5$ and $\sigma = 2$).

Table 3: Estimated Parameters

Definition	Parameterization	Value
Innate Ability Process	$ln(z') = \rho ln(z) + \epsilon$ $\epsilon \sim N(0, \sigma_\varepsilon^2)$	$\rho = .082$ $\sigma_\varepsilon = .678$
Human Capital Production Function	$F(z, i, H_n) =$ $zA(\lambda i^\gamma + (1-\lambda)H_n^\gamma)^{\frac{\chi}{\gamma}}$	$A = 2.761$ $\gamma = -.786$ $\lambda = .027$ $\chi = .769$
Utility Function (racial component)	$v(R(r)) = \kappa(R(r) - R^*)^2$	$\kappa = 3.23 \times 10^{-03}$ $R^* = .812$
Relative Supply of Housing Services	L_I/L_{II}	.28

2. Neighborhood Average Earnings Ratio ($I : II$): The ratio of average earnings in I and II from Table 2 is matched in order to reflect the degree of segregation by earnings in the data. The value of this ratio is 0.54.

3. Fraction of total white households living in Neighborhood I: This target and the next, together with the value of χ_B, completely specify the total number of households in each neighborhood and the racial configuration R of each neighborhood.

4. Fraction of total black households living in Neighborhood I: See the previous numeral.

5. Relative Price of Housing ($I : II$): The relative price of housing services across neighborhoods is taken from Table 2. This price ratio could be imposed directly as a pre-specified parameter. It is included as an estimated parameter because preliminary experimentation showed that the search procedure works better when it can at first deviate from the actual price ratio and then gradually move closer to it.

6. Variance of log lifetime Earnings: This measure is taken from Restuccia and Urrutia (2004), which in turn obtains it from the PSID dataset analyzed by Mulligan (1996). The value is set at 0.36.

7. Intergenerational log lifetime Earnings Correlation: The target value, 0.4, is taken from Solon (1992). This estimate is in line with the Mulligan (1997), whose benchmark value is 0.48. The value 0.4 is used here to maintain comparability with Restuccia and Urrutia (2004).

8. Intergenerational log Consumption Correlation: The target value, .48, comes from Mulligan (1997) table 7.2.[22]

9. Ratio of Human Capital Investment to Average Earnings: This target is set at the value of combined public and private expenditures in primary, secondary and college education divided by GDP. Data comes originally from the Statistical Abstract of the US (1999), table 208. Following Restuccia and Urrutia (2004), the value is set at 0.072.

Panel (a) of Table 4 compares data and model generated facts. Despite its parsimonious structure, the model does remarkably well in matching the targets. Most targets are reached at two or more decimals of precision.

Table 4: Model Fit

(a) Estimation Targets

Definition	Data	Model
Avg. Earnings in I	33,591	33,543
Avg. Earnings Ratio ($I : II$)	.54	.54
Log Earnings Variance	.36	.35
Intergenerational log Earnings Correl.	.40	.40
Intergenerational log Consumption Correl.	.49	.45
Fraction W in I	.13	.13
Fraction B in I	.77	.76
Investment-Income Ratio	.072	.072
Housing Price Ratio $I : II$	.73	.71

(b) Additional Facts

Definition	Data	Model
BW Avg. HH Earnings Ratio		
Overall	.61	.72
In Neighborhood I	.90	1.74
In Neighborhood II	.74	.74
BW Expenditure Ratio	.66	.72

---

[22]The consumption measure employed by Mulligan (1997) is imputed to PSID data using the coefficients from a regression computed with data from the Consumer Expenditure Survey. Household expenditures in nondurable goods are regressed against expenditures in food at home, expenditures in food away from home, rent expenditures and house value.

A few comments on some of the estimated parameter values are in order. First the intergenerational correlation in innate abilities $\rho = 0.08$ is low. A direct comparison with other estimates in the literature would be incorrect since there is variation in the human capital accumulation production functions across papers. However, the standard value in Restuccia and Urrutia (2004), 0.24, is substantially higher, indicating that the "nature vs. nurture" balance will be more inclined towards nurture in this paper.

The parameter $\gamma = -.786$ in the human capital accumulation function implies an elasticity of substitution of $\frac{1}{(1-\gamma)} = .56$. Thus, under the benchmark calibration's technology, private investment i is a poor substitute of neighborhood externality H_n relative to what a standard Cobb Douglas technology would imply (recall that under Cobb Douglas technology, the elasticity of substitution is 1).

The "share" parameter $\lambda = .027$, which seems to attribute almost no importance to private investment, has to be interpreted with caution. The reason why a quick interpretation can be misleading is that i varies widely across households (i approximately ranges from 180 to 13.000), while H_n only displays two values which are in the same order of magnitude (33.591 and 61.889), one for each neighborhood. Examination of model generated data reveals that a unit of investment i produces approximately 30% less output (h') in the low human capital neighborhood. Therefore, the impact of human capital externalities H_n is much more limited than what the value of λ initially suggests.

The racial preference parameter $R^* = .81$ implies a strong preference for diversity where the ideal racial neighborhood configuration (satiation point in R) contains .19 households of the other race. The "econometric" identification of κ and R^* comes from four points in the $(-v, R)$ space (recall that $-v$ is the value of the racial component of period utility). The four points correspond to the utility loss experienced by each color in each neighborhood under the target racial neighborhood racial configurations.

Additional Facts   The ability of the model to match additional facts under the benchmark parameterization is now assessed.

1. Black-White Household Earnings Ratio: The model produces a BW household earnings ratio of .72. This value is consistent with the ratios reviewed in the introduction. The ratio from Census data used to construct Neighborhood facts is .61. In this sense, the model produces 28 out of the 39 percentage points, or 72% of the racial difference in earnings considered in this study. This is one of the main results of the paper.

2. Black-White Household Earnings Ratio within I: The model produces a BW household earnings ratio of 1.72. This reflects, as will be discussed below, that white households deciding to live in neighborhood I are very poor. In the data, this ratio is .9. This is the only dimension in which the benchmark parameterization fails to

qualitatively replicate the data. Given the success of the model in many other dimensions, it is left for future research to investigate whether this is a generic feature of the model and to what extent it is an important feature of the data. However, it is borne to the reader's attention as a non negligible feature of the benchmark equilibrium.

3. Black-White Household Earnings Ratio within II: The model produces a BW household earnings ratio of .74. This value exactly matches the data from Table 2.

4. Black-White Expenditure Ratio: The model produces a BW ratio of current period expenditures of .72. This value is in the vicinity of its data counterpart, which is .66. The latter is calculated from the CEX statistics reported in Charles et al. (2008), Table A.2.

Table 4(b) summarizes additional facts and their model counterparts. In conclusion, the model adequately reproduces the target facts. The model generates segregation by income and earnings similar to those found in the data while abiding to cross sectional earnings variation, earnings dynamics and the size of the education sector of the US economy. The model also does a good job replicating additional facts, remarkably generating a substantial degree of racial inequality.

5 Results

5.1 Economics of the Benchmark Equilibrium

The economics behind the benchmark equilibrium can be summarized by four main forces. First, black and white households with very low human capital need to go to I in search of low housing prices. This is caused by the high marginal utility of current expenditures experienced by these households.

Second, black and white low ability households with medium or high human capital locate in II in search of neighborhood externalities. Households with lower ability are less productive in human capital accumulation and need to increase the inputs into the human capital production function. However, own investment is not a good substitute of neighborhood externalities, requiring these households to move to II (the elasticity of substitution between i and H_n implied by the benchmark parameter values equals 0.57).

Third, high ability black households with medium or high human capital decide to go to neighborhood I due to racial preferences. These households do not care much about housing prices or human capital externalities, due to their stock of human capital and high ability. Since the racial component of preferences is independent of wealth or ability, it becomes more important for these households, leading them to opt for neighborhood I.

Negative selection by ability is not a generic property of the model. Sorting by ability depends on the complementarity between ability and neighborhood quality. An upcoming revision of the paper will features positive selection by abilities into neighborhoods.

Fourth, all white households with medium-low human capital have additional incentives, relative to comparable black households, to go to neighborhood II despite the attractiveness of low house prices in neighborhood I. This is caused by racial preferences.

The first and second forces generate equilibrium differences in human capital and housing prices across neighborhoods. The third and fourth forces generate racial segregation and racial inequality.

Figure 3: Decomposition of Bellman's Equation
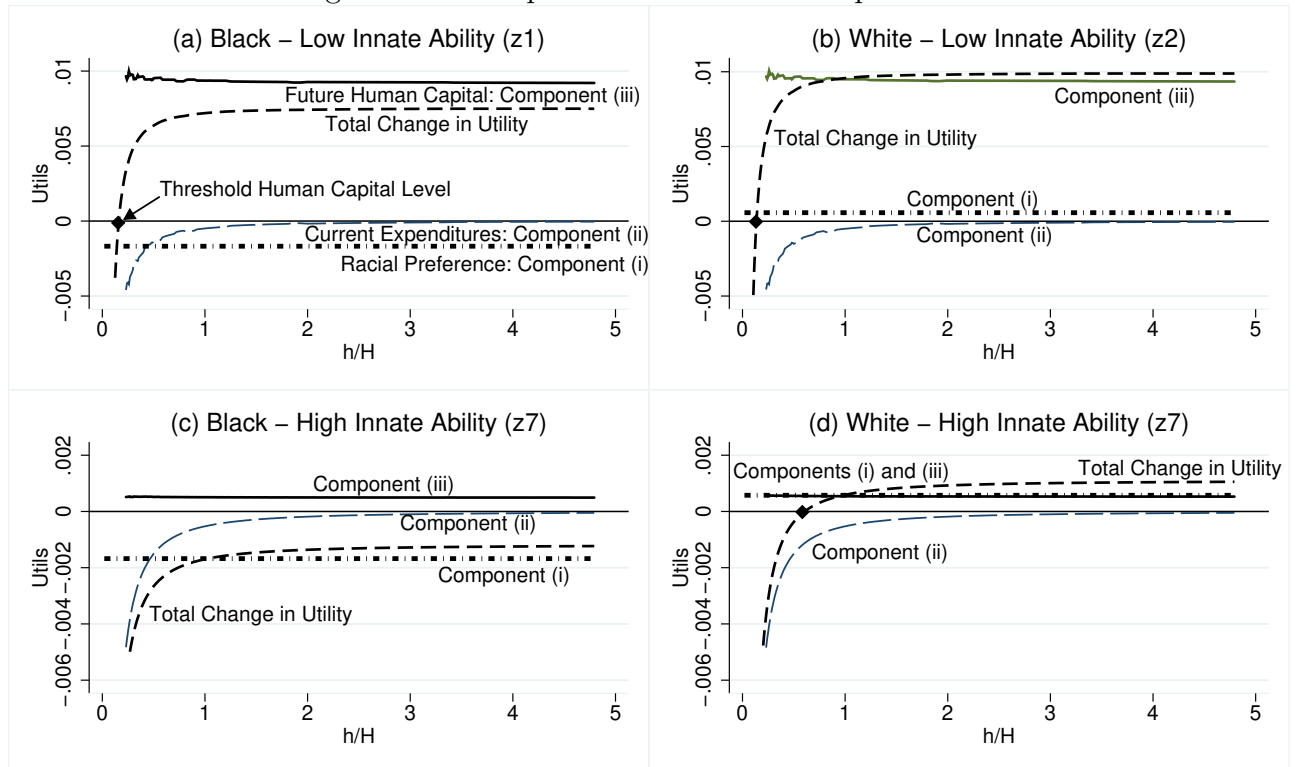
Note: This figure decomposes the total change in utility $V_{II}(x) - V_I(x)$ experienced by a household hypothetically moving from I to II into three components. Component (i) measures the change in period utility from racial preferences $v(R_{II}(r)) - v(R_I(r))$. Component (ii) measures the change in period utility from current expenditures $u(c(x, n = II), l(x, n = II)) - u(c(x, n = I), l(x, n = I))$. Component (iii) measures the change in future utility $E[V(h(x, n = II), r, z')|z] - E[V(h(x, n = I), r, z')|z]$.

These four forces are clearly observed in 3. This figure decomposes the "total change in utility" $V_{II}(x) - V_I(x)$ experimented by a household moving from I to II into three

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components.

Component (i) contains the change in utility coming from racial preferences $v(R_{II}(r)) - v(R_I(r))$. This component is positive for white households and negative for black households. Importantly, the component does not vary with the ability of the household or with its human capital level.

Component (ii) contains the change in utility coming from current period expenditures $u(c(x, n = II), l(x, n = II)) - u(c(x, n = I), l(x, n = I))$. This component reflects the effect of higher housing prices in II, therefore it is always negative. The key aspect of this component is that it becomes infinitely large as parental human capital goes to zero. From the graph it is clear that Component (i) dominates all other components for low human capital households.

Component (iii) contains the change in utility coming from differences in future human capital $E[V(h(x, n = II), r, z')|z] - E[V(h(x, n = I), r, z')|z]$ with $h(x, n) = zAF(i(x, n), H_n)$. This component is large for low ability households and small for high ability households.

For black and white households with low human capital (panels a, b, c and d), current expenditure component (ii) dominates the other two components as described by the first force described in 1, leading these households to locate in I. For black and white low-ability households with medium or high human capital (panels a and b), investment component (iii) dominates the other two components as describe by the second force, leading them to locate in II. For high ability black households with medium to high human capital (panel c) current expenditure and investment components (ii) and (iii) are small compared to racial component (i) as required by the third force, leading them to locate in I. For all white households of medium low ability (panels b and d), the "threshold human capital level" at which they decide to move from I to II is lower than it is for comparable black households (panels a and c) as required by the fourth force, pushing lower human capital white households towards neighborhood II.

The Role of Neighborhoods   Differences in human capital across race are not due to differences in parental investments across races within each neighborhood, but to differences in the residential location of each race. Figure 4 displays the law of motion for human capital that maps parental human capital h, color r and innate ability z into the child's future human capital h'. The horizontal axis contains parental human capital as a fraction of average human capital in the economy. The vertical axis does the same for child's future human capital. Each curve represents the law of motion for one value of the ability shock z. Dotted lines indicate the household decides to live in I while solid lines indicate the household chooses to live in II. In this figure it is clear that the child's future human capital is roughly the same for white and black households of similar ability and parental human capital when they live in the same neighborhood. However, human capital accumulation is very different for similar households located in different neighborhoods. In the benchmark equilibrium, investments in human capital average $2,800$ for black house-

holds and $4,200$ for white households while earnings average $41,700$ for black households and $57,900$ for white households. If black households kept their within-neighborhood investment decision rules but adopted the white households' location decision rules, investments of black households would average $4,000$ and their average earnings would average $57,400$ in partial equilibrium. We can thus say that, within the benchmark equilibrium, the location decisions of households account for 86% of the difference in investment and 97% of the difference in average earnings.

Figure 4: Human Capital Law of Motion by Color (selected z shock values).
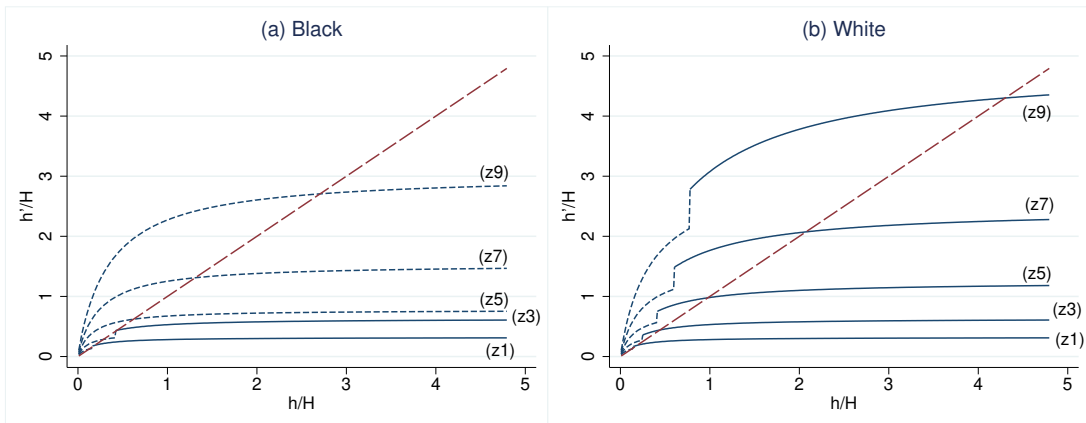
Note: Each curve corresponds to a value of the innate ability shock z. The horizontal axis measures parental human capital while the vertical axis measures the child's future human capital. Parental and child's future human capital are related by the law of motion $h' = zAF(i(x,n), H_n)$ where n is the neighborhood chosen by the household ($\eta(n|x) = 1$). A dotted line is assigned to states x where the household decides to live in neighborhood I and a continuous line represents states where the household decides to live in neighborhood II. Axes are normalized by average human capital H.

5.2 Equilibrium Multiplicity

One can compute a variety of different equilibria of this model under the benchmark parameterization. Appendix A-4 contains a list of equilibria that have been computed and discusses their possible interest for this and other applications of the model.[23] Equilibrium multiplicity is a natural feature of this type of economy. In this sense, it is clear that the story in this paper is ultimately based on initial conditions. In subsequent work it would be

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[23]The list includes equilibria where black and white households swap roles respect to the benchmark equilibrium and equilibria with full segregation where human capital is the same in both neighborhoods.

interesting to assess which type of initial conditions give rise to the benchmark equilibrium and which don't.

5.3   The Magnitude of Racial Preferences

In order to grasp the relative magnitude of this component of preferences a simple, conservative, indicator is computed. Consider the stationary distribution of households μ. In any given period, aggregate period utility is given by $\int u(c(x,n), l(x,n)) + v(R_n(r(x)))d\mu$. Then one can ask what reduction a of current housing and non-housing consumption leaves households indifferent when the racial composition of neighborhoods is set at the ideal fraction of own race, given by R^*. This reduction satisfies

$$\int u(c(x,n), l(x,n)) + v(R_n(r(x)))d\mu = \int u((1-a)c(x,n), (1-a)l(x,n))d\mu.$$

The racial component drops from the right hand side when $R_n(r(x)) = R^*$ under the functional form assumption of Section 4.1. This indicator is conservative since future utility is not accounted for, and households are not allowed to re-optimize on the right hand side of the previous equation.

Using the benchmark equilibrium's stationary distribution the value $a = .015$ is obtained. Aggregate period utility would be unchanged by a reduction of 1.5% in period consumption accompanied by a change in neighborhood's racial configuration that allowed all households to enjoy their ideal neighborhood racial configuration.

When the proposed indicator is computed only for black households $a = .03$ is obtained, while $a = .01$ is obtained for white households . The larger value of a for black households corresponds to the fact that neighborhoods I and II are both farther from the ideal configuration for black households than for white households.

When the indicator is computed for households with earnings above the median $a = .03$ is obtained. The indicator yields $a = .10$ for black households above the median and $a = .01$ for white households above the median. All high earnings households are willing to pay more than low earnings households to enjoy their ideal neighborhood configuration, however, most high earnings black households live in I which has only 63% black households, while high earnings white households live in II, where the 93% of households is white. This explains the difference across race.

5.4   The Role of Color Preferences

Can a lot of segregation and BW inequality be obtained from a small degree of neighborhood color preference? Proposition 2 shows that without any color preference, the model predicts zero BW inequality and no residential segregation. However, a discontinuity could exist whereby adding a small degree of color preferences, substantial segregation and inequality

could arise. In terms of the study of racial segregation, this has been a common line from Schelling (1971) to Sethi and Somanathan (2004).

This section first analyzes the effect of decreasing the extent of racial preference on the benchmark equilibrium. For this purpose a sequence of stationary equilibria with gradually lower values of κ is computed. The main finding is that a 46% reduction in κ is enough to eliminate all racial segregation and all racial inequality from the benchmark equilibrium.[24]

The second part of the section asks if these non-segregated equilibria are robust to arbitrary variations in households' perceptions of color segregation. The main finding is that even if all households perceive neighborhoods to be highly segregated, housing costs and human capital considerations vastly prevail when κ is small, so household decisions still imply little or no color segregation and racial inequality.

A Sequence of Equilibria with Decreasing κ. A sequence of stationary equilibria is computed. At each step of the sequence, parameter κ is decreased by a fixed amount. Then, an equilibrium is computed. Equilibrium $\{H, R, P\}$ values from the preceding step are used as starting values for the numerical equilibrium search procedure (see Appendix A-3). The first equilibrium in the sequence corresponds to the benchmark equilibrium.

Figure 5 displays the results. Panel (a) shows how the steady state equilibrium BW earnings ratio increases monotonically as κ is reduced. The ratio reaches 1 at $\kappa = 1.75 \times 10^{-3}$, which is .54 of the benchmark value. Panel (b) shows how this result follows from increased neighborhood integration. Neighborhoods become monotonically more integrated as κ is reduced, reaching virtually full integration around $\kappa = 1.75 \times 10^{-3}$. The other panels show relatively little variation in other dimensions along the sequence.[25]

Figure 3 provides the key intuition behind this result. For a given ability level z the threshold value of human capital at which households decide to migrate from I to II is lower for white than for black households (the threshold is indicated by the intersection of "Total change in utility" and the horizontal axis). Thus, white residents of I have such low expenditures that the utility cost of paying higher house prices in II is close to infinity (see magnitude of component (ii) in the graph). Therefore, changes in racial preferences barely impact this threshold for whites (in the graph, κ displaces "Total Change in Utility" vertically without any effect on its intersection with the horizontal axis, because its slope is too high at the intersection).

The threshold for black households is higher. Thus, the utility cost of higher house prices in II is lower for them, and a change in racial preference does impact the threshold. However, after κ is low enough, the situation for black households becomes identical to that of white households. The residents of I are so poor that changes in racial preferences

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[24]A similar result holds for other equilibria of the model. In particular, no equilibrium with racial inequality has been found when κ is reduced 46% with respect to its benchmark value. See Appendix A-4.

[25]Panel (c) exhibits an interesting non-monotone behavior in relative housing prices, which is not central to the discussion.

Figure 5: Sequence of Counterfactual Stationary Equilibria
(varying κ)
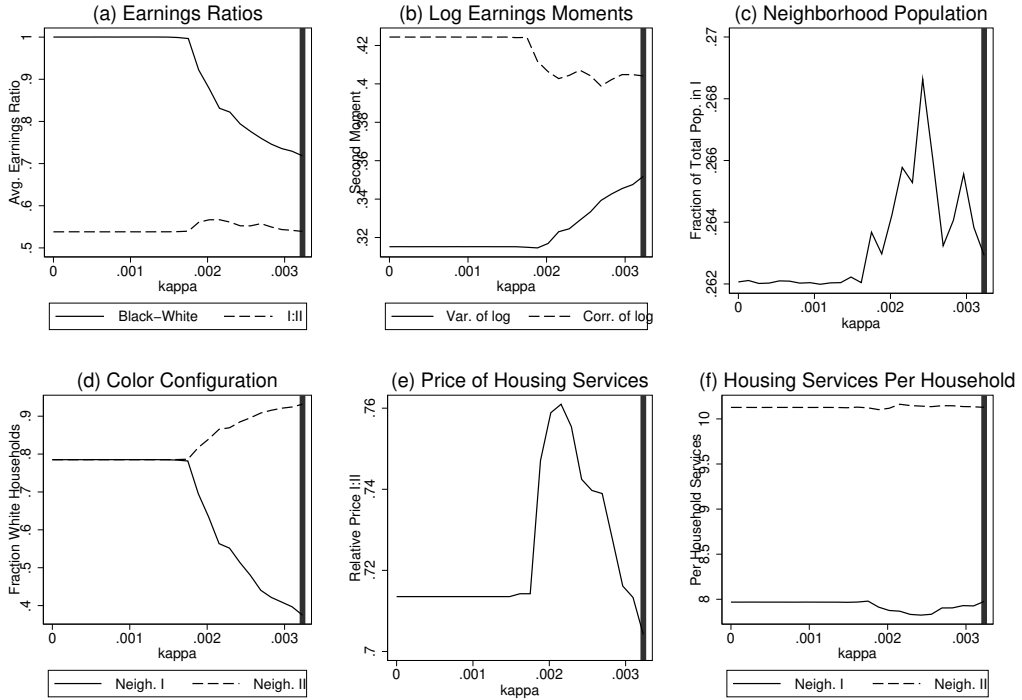
Note: The benchmark value of the parameter is depicted by a vertical bar. A stationary equilibrium is computed for each of 25 equally spaced values of the parameter in the range of the horizontal axis. The vertical axis measures equilibrium values of model outcomes.

play no further role (if "Total change in Utility" is close to vertical at the intersection with horizontal axis, it can be shifted up without any change in the threshold).

Along the described sequence of stationary equilibria, households perceive the neighborhoods as being increasingly integrated. Given this perception, their residential decisions become very similar, effectively implying integration. This is precisely the kind of equilibria discussed in Proposition 3.

5.5 The Role of Externalities

In order to investigate the role played by neighborhood externalities, a sequence of stationary equilibria is computed for gradually decreasing values of the share of externalities in the production of human capital $(1-\lambda)$. Panel (a) Figure 6 shows that with a small reduction

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Figure 6: Sequence of Counterfactual Stationary Equilibria
(varying λ)
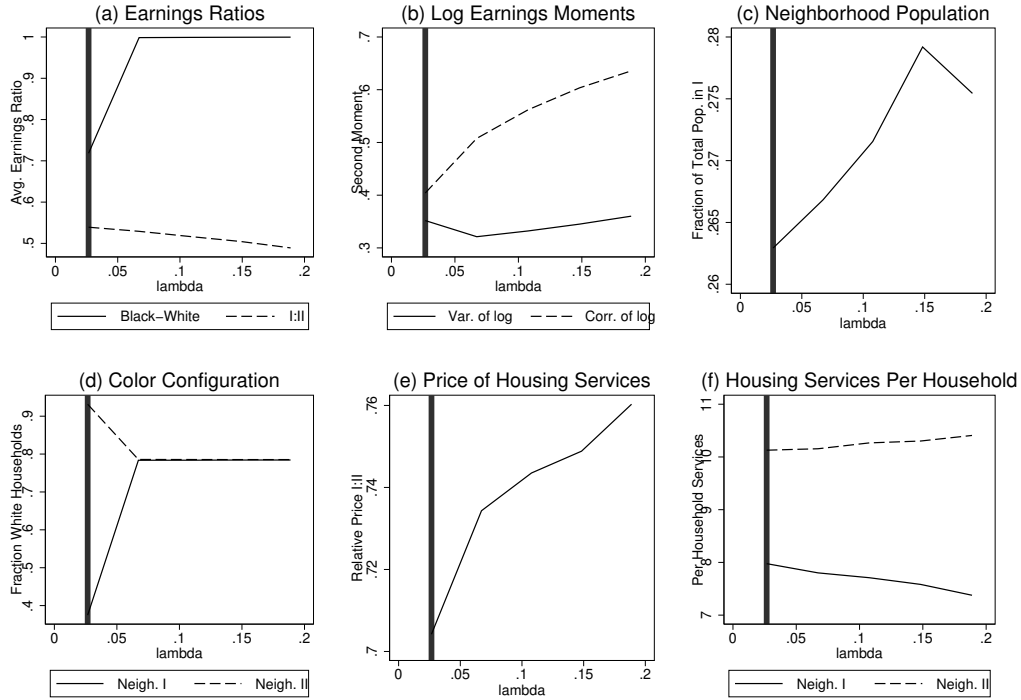
Note: The benchmark value of the parameter is depicted by a vertical bar. A stationary equilibrium is computed for each of 25 equally spaced values of the parameter in the range of the horizontal axis. The vertical axis measures equilibrium values of model outcomes.

of the importance of externalities in the production of human capital, all racial inequality disappears from the equilibrium, while segregation by earnings increases. Panel (b) shows that the reduction in racial inequality responds to the full integration of neighborhoods.

Under a small reduction of the importance of externalities households can rely more on own investments to generate human capital for the next generation. This reduces racial inequality. For this reason, more white households are attracted by the low housing prices in I. As white households migrate from II to I, the fraction of black households in II becomes larger, attracting high-earnings black households. As one decreases $(1 - \lambda)$ this process continues until all racial segregation has been eliminated from the benchmark equilibrium. This result is very interesting for two reasons. First it shows that human capital externalities are necessary in generating BW inequality in the model. Second, it shows that the observed degree of segregation in the model, requires the interplay of housing

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prices, racial preferences *and* human capital externalities.

5.6 The Role of Housing

In order to investigate the role played by housing, a sequence of stationary equilibria is computed for gradually decreasing values of the share of current expenditures in housing services (1-α). This reduction lowers the importance of housing price differences across neighborhoods in the behavior of households.

Figure 7: Sequence of Counterfactual Stationary Equilibria
(Varying α)
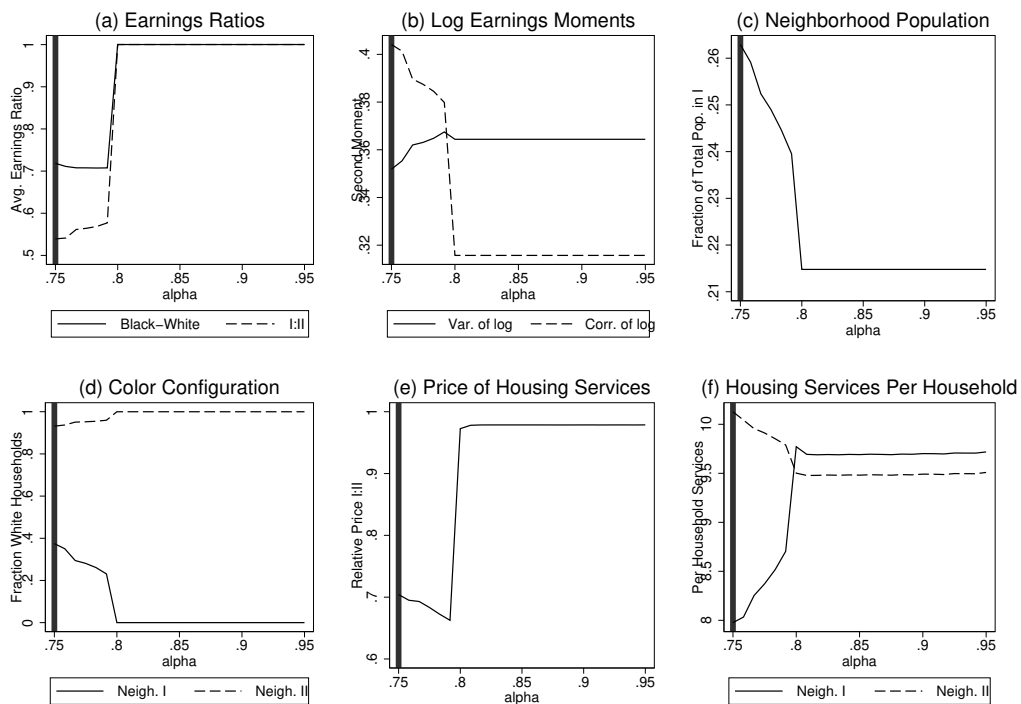
Note: The benchmark value of the parameter is depicted by a vertical bar. A stationary equilibrium is computed for each of 25 equally spaced values of the parameter in the range of the horizontal axis. The vertical axis measures equilibrium values of model outcomes.

Figure 7 contains implications for equilibrium. When housing markets become relatively unimportant, low human capital white households no longer have a reason to live in neighborhood I. The low housing prices of this neighborhood are no longer a large source of current utility. Therefore, the fraction of white households in I decreases (panel b) and

30

total population in I falls (panel b) until full segregation is obtained. With full segregation, average human capital becomes identical in each neighborhood, eliminating racial earnings inequality.

Neighborhood I supplies .27 of total housing services, while the total fraction of black households is .21. Thus, each color group fits nicely into a neighborhood, with only a small difference in per-household consumption of housing services across neighborhoods (panel c). This difference is compensated by a small differential in housing prices across neighborhoods.

This result is analogous to that in Proposition 2. The proposition states that without racial preferences, segregation by race and racial inequality cannot be obtained in this model. This section has established that under the benchmark parameterization, the model is unable to predict any racial inequality, while producing full segregation by race, when housing markets are unimportant. This suggests that housing is an indispensable element of the model.

6   Conclusions

This paper has crystallized common intuition about the relationship between residential segregation and racial inequality in a parsimonious model economy. In this endeavor it has extended the scope of a popular class of heterogeneous agents models by introducing original modifications to the standard framework.

The paper has achieved success in taking the posed mechanics to the data and replicating important facts of the US economy. The calibrated model exactly replicates US residential segregation by race and earnings, intergenerational correlations of earnings, and the size of the US education sector with respect to GDP, within a stationary equilibrium. The mapping of a parsimonious two-neighborhood model to real life data is allowed by the application of clustering techniques to characterize US neighborhood data through two "representative neighborhoods". Clustering is a standard method in itself, but its application to quantitative economic modeling is novel, and evidently useful.

The calibrated model produces 72% of the observed BW family income gap. This important result is obtained while abiding to a set of current stylized facts from the US economy. Prominently, the model reflects the fact that black and white workers seem to receive similar compensation for observable skills. In other words, the model generates large black white differences without appealing to labor market discrimination or informational frictions, which are not strong features of current US data.

It is important to note that no asymmetries by race, beyond a preference over neighborhood color composition, have been assumed. Furthermore, in equilibrium, the decisions of black and white households of similar human capital and ability are similar within each neighborhood. Therefore, differences in human capital across races are generated by dif-

ferences in the residential location of races and not by BW differences in human capital investment behavior within each location. Within the benchmark equilibrium, the location decisions of households account for 86% of the BW difference in average human capital investments and 97% of the BW difference in average earnings across races.

In summary, the paper has made progress towards a compelling answer to the crucial question it addresses: Why are the earnings of black households persistently so low compared to those of white households? When the pattern of residential segregation by earnings and color found in data is viewed as generated by the mechanisms in the benchmark equilibrium of the model proposed here, a significant portion of the observed extent of inequality can be explained. The central message of the paper is that all three ingredients: human capital externalities, racial neighborhood preferences and local housing markets are necessary in producing this result.

Additionally, the paper has interesting implications for the study of racial segregation. A common interpretation of Schelling (1971, 1972) is that the extent of segregation observed in the US could be caused by a "small degree" of racial preference. In contrast to this ingrained line of thought, this paper finds that strong racial preferences are required to match US facts. Furthermore, the paper measures the sensibility of the equilibrium of interest to a change in the magnitude of racial preferences, finding that segregation and inequality become negligible when one halves the importance of race in utility.

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A-1 Empirical Appendix - IPUMS Data

Figure 1 uses 1940 to 2000 Decennial Census data. The data is obtained from the Integrated Public Use Microdata Series (IPUMS) one-percent samples, available at www.ipums.org. The base sample consists of 2'883, 885 single-family households headed by a US-born non-hispanic white or black person between 25 and 64 years of age. Households with missing or inconsistent income responses for either head or spouse (if present) are deleted.[26] Earnings and hours are considered inconsistent if (i) earnings are zero and hours are positive, (ii) earnings are positive and hours are zero or (iii) the implied wage rate is below the federal minimum wage. The clean sample contains 1'967, 153 households.

Education   Panel (a) of Figure 1 displays average years of education by race for household heads 30 to 35 years of age. Years of education are derived from IPUMS categorical variable (EDUC) by assigning numerical values to each category.[27]

Household Earnings   Panel (b) of Figure 1 plots the black-white ratio of average (single-family) household earnings per adult equivalent for the clean sample and for a restricted sample. The restricted sample excludes households where the head was not a full-time worker. A head of household is considered a full time worker if he/she worked 35 or more hours per week and 40 or more weeks in the year. Household earnings are constructed by aggregating the inflation-adjusted wage and salary income (INCWAGE*CPI99) of the head and the spouse (if present).[28] Household earnings are divided by $\sqrt{\#adults + 0.5 \times \#children}$ to obtain a per-adult-equivalent measure.

A-2 Empirical Appendix - Tract Data

Data for this study comes from the US Census Bureau, 2000 Census of Population and Housing, Summary File 3. Each table comprising SF3 is available at the Census Tract aggregation level.[29]

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[26]Spouse information is not available in 1950 so only head information is used in cleaning the sample in that year.

[27]"Zero or non reported" equals 0 years. "Nursery to grade 4"equals 2.5 years. "Grade 5, 6, 7, or 8" equals 6.5 years. "Grade 9","Grade 10","Grade 11","Grade 12" equal 9, 10, 11 and 12 years, respectively. "1 year of college","2 years of college","3 years of college" and "4 years of college" equal 13, 14, 15 and 16 years, respectively. "5 years of college or more" equals 17 years.

[28]Spouse earnings are not available in 1950 so a proxy is used. The proxy is the IPUMS variable (FWAGE2) which aggregates the earnings of all family members excluding the household head.

[29]Census tracts are small geographical subdivisions of the US designed by the Census Bureau. The primary purpose of tracts is to provide a unit for presentation of decennial census data. Census tracts generally contain between 1,500 and 8,000 people. The design of tracts aims at generating areas with homogeneous populations in demographic and economic terms, containing around 4,000 people. In contrast

A-2.1 Variables

The choice of variables seeks to obtain measures of neighborhood's (H, R, P). The operational definition of a neighborhood will be the census tract.

Racial Composition. For clustering purposes, the fraction of white households is measured as the number of non-Hispanic white households divided by the total number of households in a census tract. For modeling purposes only black and non-Hispanic black households are included in total number of households.

Human Capital. Under standard competitive labor market assumptions, measures of household earnings capture differences in human capital. The log of mean household earnings in a census tract is employed as a measure of log average human capital.[30]

Price of Housing Services. In order to measure P, two steps are taken. First, median tract house values are converted into median annual Implicit Rental Values IRV using a procedure based on Poterba (1992).[31] This procedure consists simply on applying an annual user-cost factor to the house value. Here the factor is set to 8.93% of the house value.[32]

Second, the price component (as opposed to the quantity component) of housing expenditures is extracted as the residual of a regression of log median IRV against a set of housing and neighborhood characteristics variables.[33]

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with population size, the spatial size of census tracts varies widely depending on local population density.

[30]Badel (2008a) considers an additional measure of human capital based on mean tract income and two additional measures that control for age and other factors finding similar results.

[31]Badel (2008a) considers two additional measures based on median rental values to renters and Census Bureau's measure of median housing user costs finding similar results.

[32]Calabrese et. al (2006) employs this approach. The user cost of housing for homeowners is calculated by letting implicit rental values IRV be given by $IRV = \kappa_p V$, where V is the market value of the home. The annual user-cost factor is given by

$$\kappa_p = (1 + t_y)(i + t_v) + \psi,$$

where t_y is the income tax rate, i is the nominal interest rate, t_v is the property tax rate, and ψ contains the risk premium for housing investments, maintenance and depreciation costs, and the inflation rate. Calabrese et. al (2006) uses $\kappa_p = 8.93\%, 13.93\%$. This procedure's main criticism comes from evidence suggesting that risk premia vary significantly across locations (see Campbell et al., 2007). This concern is mitigated in Badel (2008a) by employing alternative measures of housing expenditures, which yield similar results.

[33]This set of housing characteristics contains the median number of rooms in the unit, a distribution of the number of units in the housing structure (10 categories), a distribution of the number of bedrooms (6 categories), fraction of units with telephone service, fraction of units with complete plumbing facilities, fraction of units with complete kitchen facilities and distribution of travel time to work in the tract (12

Badel (2008a) provides sufficient conditions for this procedure to identify and estimate the relative price of housing services across tracts.

A-2.2   Sample Selection

The sample selection criteria aim at providing a comprehensive picture of the interaction of (H, R, P) in US data while being selective enough to yield plausibility to the microeconomic mechanisms of the model.

Only areas with significant potential for the existence of strong human capital externalities are considered. Thus, only MSA with population of at least 1 million are considered. Also the sample focuses on tracts with a relatively high population density. Only those census tracts that contain at least 200 people per square kilometer are kept.[34]

Only large metropolitan areas where arguably similar social interaction mechanics may exist are included. The sample is restricted to MSA where 10% of the population or more is of black race. In order to restrict attention to BW inequality and abstract from interactions with other groups, only census tracts with less than 50% of "other race" households are included. In order to avoid atypical observations due to small samples, attention is restricted to tracts with 200 households or more.

Some Census Tracts contain a large number of individuals living in group quarters.[35] The sample is limited to Census Tracts with less than 25% population in group quarters.

Application of these sample selection criteria results in a set of 28 MSA in 25 states, containing 80.7 million people and 17,815 Census Tracts. The largest MSA in the sample is New York-Northern New Jersey-Long Island with 3850 tracts, the smallest is Raleigh-Durham-Chapel Hill, with 157 tracts (see table 8). See Badel (2008a) for additional details.

A-3   Theoretical Appendix

Proposition 4 *The household problem defines a contraction mapping in the space of continuous bounded functions. There exists a unique bounded continuous function $V(x)$ satisfying (1) and the optimal policy correspondence is nonempty and u.h.c.*

Proof *This proof follows directly from the standard proof for the one-sector growth model in Stokey and Lucas (1989, exercise 5.1).*

Proof (Proposition 1) *(i) The household's problem defines a contraction mapping. Therefore, the solution to the household's problem in E^0 is unique. If A1 holds, the period utility*

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categories). Regression residuals are then rescaled to match the mean of the original variable in levels.

[34]This is a standard threshold in the housing literature above which an area is considered urban.

[35]Correctional institutions, nursing homes, juvenile institutions, college dormitories, military quarters, and group homes are considered group quarters.

function differs by a constant for black and white households. Direct comparison shows the value functions for households in states (h, B, z) and (h, W, z) differ by $\frac{v(R(W)) - v(R(B))}{1-\beta}$. Thus, by uniqueness, human capital investment decision rules will be identical for black and white households.

(ii) Since decision rules are identical for white and black households, the definition of the transition function implies that $P\left((h, B, z), (A_h, B, z')\right) = P\left((h, W, z), (A_h, W, z')\right)$ for all h, z, z', and all intervals $A_h \subset [0, \bar{h}]$.[36] Assumption A2 implies μ(the limit of a sequence probability measures $\{\mu_j\}_{j=1}^\infty$ generated from any initial measure μ_0 by the transition function $P\left(x, (A_h, r', z')\right)$) is unique.[37]. Since households never change color, the limit of a sequence of color conditional probability measures $\{\mu_j(.|r)\}$ generated from any initial probability measure $\mu_0(.|r)$ by the transition function $P\left((h, r, z), (A_h, r, z')\right)$ converges weakly to $\mu(.|r)/\chi^r$, the equilibrium measure conditional on color. Finally, suppose by way of contradiction that $\mu(.|r = B)/\chi^B \neq \mu(.|r = W)/\chi^W$. Then one can set the initial probability measure for black households to equal the initial measure used for white households $\mu_0(.|r = B) = \mu_0(.|r = W)$ and apply $P\left((h, B, z), (A_h, B, z')\right)$ recursively, generating a sequence $\{\mu_j(.|r = B)^*\}$. Since $P\left((h, B, z), (A_h, B, z')\right) = P\left((h, W, z), (A_h, W, z')\right)$, then $\mu_j(.|r = B)^* \rightarrow \mu(.|r = W)/\chi^W$. This contradicts the uniqueness of $\mu\left(.|r = B\right)/\chi^B$.

Proof (Proposition 2)*(i) By assumptions B1-B2 and the contraction mapping property of the household problem, the location decision rules $\eta(n|x)$ and neighborhood conditional decision rules $(c(x, n), l(x, n), i(x, n))$ are identical for black and white households. Therefore, by definition, the transition function $P(\cdot)$ is also independent of color. Assumption B3 guarantees uniqueness of the stationary distribution μ which together with (i) implies the color-conditional stationary distributions are identical (see part (ii) of previous proof).*

Proof (Proposition 3)*(i) By Proposition 2, without color preferences $\mu(.|r = B)/\chi^B = \mu(|r = W)/\chi^W$. By assumption B2, $\eta(n|x)$ is pinned down by the solution to the household problem almost everywhere. This implies residential decisions of white and black households are identical almost everywhere. Identical residential decisions imply that in equilibrium E_N, $R_n(r) = \chi^r \ \forall \ n$, so all neighborhoods are fully integrated. Under full integration any racial preference $\nu(R(r))$ just adds a constant to period utility. Thus, the solution to the household problem from E_N also solves the decision problem with racial preference $\nu(R(r))$ given $\{H_n, R_n, P_n\}_{n=1}^N$ from E_N when the value functions are additively rescaled.*

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[36]Decreasing returns to scale in the production of human capital h guarantee that $h \in [0, \bar{h}]$ for some finite $\bar{h}$.

[37]See Stokey and Lucas 1989, pg. 353

A-4 Numerical Algorithm

This appendix describes the main equilibrium search algorithm used in the paper and then discusses its application to model estimation and computation of counterfactual equilibria. The algorithm calculates a Pseudo-Equilibrium of the model.[38] Pseudo-Equilibrium differs from the Stationary Spatial Equilibrium of Definition 3 in allowing demand for housing services in each neighborhood to take any (positive) value, eliminating the market clearing condition for housing services.

This concept is useful to estimate the model. Neighborhood supply of housing services $\{L_n\}_{n=1}^N$ is not observed in the data, so it has to be treated as a free parameter. Fortunately, each neighborhood's population is known. The model can therefore be estimated by including neighborhood *populations* in the set of estimation targets and then setting model parameters to match these targets under Pseudo-Equilibrium (without considering the housing market clearing condition). Once the benchmark parameters of the model have been estimated one can take the Pseudo-Equilibrium housing *demands* at each neighborhood D_n^{bench} and interpret them as an estimate of housing supply, setting $L_n^{bench} \equiv D_n^{bench}$ for each neighborhood n. Supply, estimated in this way, can then be used to check the housing market clearing conditions in computations of counterfactual equilibria.

Algorithm 1 *Pseudo-Equilibrium*

1. *Set $\{(H_n, R_n)\}_{n=1}^N$ at starting values $\{(H_n^0, R_n^0)\}_{n=1}^N$. Fix the parameter vector at some starting value $\Theta^{pseudo} = \Theta^0$.[39]*

2. *Taking $\{(H_n, R_n)\}_{n=1}^N$ from the previous step and $\{P_n\}_{n=1}^N$ from the parameter vector Θ^{pseudo}, solve the household's decision problem. Use the solution to the household's problem to simulate the path of the individual state vector x for one W and one B households over 500,000 model periods. Assuming stationarity, calculate implied average human capital and racial configuration of each neighborhood, $H_n^{'}$ and $R_n^{'}$, from the simulated time series by employing the location decision rules and appropriate time series averages.*

3. *If $\max_n[(H_n - H_n^{'})^2] < \varepsilon_H$ and $\max_n[(R_n - R_n^{'})^2] < \varepsilon_R$ go to step 4. Otherwise, for each neighborhood n update*

$$
\begin{aligned}
H_n &= \nu H_n + (1 - \nu) H_n^{'} \\
R_n &= \nu R_n + (1 - \nu) R_n^{'}
\end{aligned}
$$

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[38]Define a Pseudo-Equilibrium in the same way as in Definition 3, but suppress the housing market clearing condition (Condition 4).

[39]The parameter vector Θ contains the parameters listed in Table 4 (b). These parameters include the housing price vector (P_n).

and go back to step 2.[40]

4 Check for multiple stationary distributions.[41] *Terminate.*

If the algorithm terminates successfully (i.e. reaches step 4 and there is a unique stationary distribution) one has found a Pseudo-Equilibrium of the model under parameter vector Θ^{pseudo}.

Estimating Model Parameters Parameter estimation proceeds by searching over values of the parameter vector Θ^{pseudo} in order to find a Pseudo-Equilibrium that matches target moments from data. The AMOEBA minimization routine of Press et. al (1992) is used to minimize the squared percentage deviation of model implied moments to estimation targets. These targets are described in section 4.2 and summarized in Table 3(b). Since the number of targets equals the number of parameters one should expect the terminal value of the procedure to be close to zero, implying an exact match. The starting value for the vector Θ^{pseudo} was obtained by picking the vector providing the best fit from a large set of randomly generated vectors.[42]

Computing Counterfactual Equilibria The computation of counterfactual equilibria proceeds by imposing changes to the benchmark value of Θ^{pseudo} in order to obtain $\Theta^{counter}$. Supply of housing is taken from the benchmark equilibrium by setting $L_n^{bench} \equiv D_n^{bench}$. With these elements in hand, a full equilibrium of the model is computed by adjusting all components of $\{(H_n, R_n, P_n)\}_{n=1}^N$. Mechanically, this is achieved by applying the AMOEBA minimization routine in order to search over the subset of parameters in $\Theta^{counter}$ that determine housing prices $\{P_n\}_{n=1}^N$ to minimize the percentage squared deviation of housing demand $D_n^{counter}$ from housing supply L_n^{bench} in each neighborhood n under Pseudo-Equilibrium.

A-5 Multiple Equilibria

The search for additional equilibria fixes parameter values at the benchmark values estimated in section 4.2. Given these parameter values, the procedure for computing counterfactual equilibria described in Appendix A-3 is applied. This procedure results in the

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[40]Constants ε_H and ε_R control the error tolerance, these are set to the value 0.0001. Constant ν is a "relaxation parameter" that facilitates convergence. This parameter is set to 0.8.

[41]Markov processes sometimes have multiple stationary distributions. A sufficient condition for the uniqueness of the stationary distribution is that the long run averages from the simulations in (2) do not depend on starting values.

[42]Parallel computation using Georgetown's Zappa computer cluster was crucial in computing equilibrium for a large number of randomly generated parameter vectors in reasonable time.

benchmark equilibrium if starting values for $\{(H_n, R_n)\}_{n=1}^N$ are set to benchmark values in step 1 of Numerical Procedure 1 of Appendix A-3. Additional equilibria are obtained by varying these starting values. Equilibria are computed for a total of 30 starting values at the benchmark parameter values. Four new equilibria, described below, arise from this exercise.

After examining multiple equilibria at the benchmark parameter values, κ is reduced 46% with respect to its benchmark value, and the same 30 starting values are used to compute multiple equilibria. This exercise results in a total of three equilibria, none of which exhibits racial inequality, and two which exhibit no racial segregation. One of the equilibria exhibits complete racial segregation. When κ is reduced to 25% of the benchmark, only two equilibria are obtained, and both exhibit no racial segregation or racial inequality.

Five starting values for (H_I, H_{II}) are considered. In case 1 (H_I, H_{II}) is set to the benchmark value. In case 2 H_I is set to the benchmark value of H_{II} and H_{II} is set to the benchmark value of H_I. In the third case H_I and H_{II} are both set to the overall average earnings of the benchmark case. In the fourth case H_I is set to 1/3 of its benchmark value while H_{II} is set to 3 times its benchmark value. In the fifth case H_I is set to 3 times the benchmark value of H_{II} while H_{II} is set to 1/3 of the benchmark value of H_I.

Six starting values are considered for (R_I, R_{II}). The first three cases vary (R_I, R_{II}) to maintain neighborhood populations constant in each neighborhood with respect to the benchmark equilibrium. The first case starts with all households of I white. The second case starts with all households in II white. The third case sets the fraction of white households in I to the benchmark fraction of white households in II, which is .93.

The second set of three cases explores starting values that alter total populations in each neighborhood with respect to the benchmark equilibrium. In the first of these cases, all black households live in I while all white households live in II. The second case starts with all white households living in I while all black households live in II. In the third case the fraction of white households in I equals the benchmark fraction of white households in II, and the fraction of white households in II equals the benchmark fraction of white households in I.

The five starting values for (H_I, H_{II}) coupled with the six starting values for (R_I, R_{II}) generate the set of 30 starting values. While many of these starting values result in the benchmark equilibrium, four additional equilibria of the model arise from the configurations of starting values described above.

Reversed-Roles Equilibrium Five of the 30 starting value configurations resulted in an equilibrium where the role played by black and white households in the benchmark equilibrium is reversed. In this equilibrium, white households represent 85% and 75% of the populations of neighborhoods I and II, respectively. Therefore, most black households (82%) reside in II. Average earnings in neighborhood I represent .52 of those in neighborhood II while the relative price of housing services in I, relative to II is .68. These two

ratios are roughly in line with data. Of white households .28 is located in neighborhood I, whereas only .17 of black households is. This implies that black households enjoy lower costs of human capital accumulation. The ratio of *white to black* earnings is .93. The white-black gap in this reversed-roles equilibrium is just .07, compared to BW gap of .39 in the benchmark equilibrium. This asymmetry is a consequence of the different black and white population sizes, combined with the housing supplies in each neighborhood. Housing supply in I relative to II is roughly one third, while black population is 21% of total. Therefore, when black households are concentrated in I the ratio of black to white households in I tends to be closer to 1 than the ratio in II when black population is concentrated in II. Thus, the correlation between the fraction of black households with human capital across neighborhoods is lower, leading to milder racial inequality in the reversed-roles equilibrium.

Equilibrium with Full Segregation   Two of the 30 configurations result in an equilibrium where all black households live in I, all white households live in II and average human capital is identical in each neighborhood. In this equilibrium there is no racial inequality and the relative price of housing in neighborhood I with respect to neighborhood II is .97, reflecting a slight difference in housing services per household across neighborhoods.

Privileged-Minority Equilibrium   One of the 30 configurations results in an interesting equilibrium where the smallest neighborhood (I) becomes the wealthiest one and the one with high housing prices. The ratio of average earnings in I with respect to II is 1.89, and relative price of housing in I with respect to II is 2.7. In this equilibrium, .60 of all black households live in I while .2 of all white households live in I. This implies that the racial minority (black households in this case) faces lower costs of investing in human capital, leading to racial inequality with a "privileged" minority. In this equilibrium, the BW earnings ratio is 1.26, this corresponds to a white-black ratio of .79. This equilibrium immediately suggests the relevance of the model for developing countries, where cities are characterized by small, exclusive residential areas, inhabited by an elite and large surrounding "poverty belts".

Equilibrium with Full Racial Integration   Two of the 30 starting values considered result in an equilibrium where neighborhoods are fully racially integrated, but where there is segregation by earnings. In this equilibrium, the ratio of average earnings in I with respect to II is 1.83, while the relative price of housing services in neighborhood I, relative to II is 2.44. This equilibrium presumably corresponds to the *Symmetric R* equilibrium devised in Proposition 3.