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Authors	Silvio Contessi
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Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

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How Does Multinational Production Change International Comovement?

Silvio Contessi*

Federal Reserve Bank of St. Louis

October 2010

Abstract

I study the aggregate implications of the entry of Multinational Firms (MNFs) in a two country Dynamic Stochastic General Equilibrium model in which firms have heterogeneous productivity in the sense of Ghironi and Melitz (2005). Unlike the extant open economy macroeconomics literature, this model endogenizes both multinational production and exports as possible strategies of internationalization of production, a feature that substantially improves the match between model-simulated moments and business cycle data along two dimensions. First, once I allow for concurrent entry (and exit) of MNFs and exporters over the business cycle, the consumption-output anomaly disappears and I can successfully replicate the ranking of cross-country correlations of output and consumption found in the data. Second, I show that the model with heterogeneous MNFs is capable of bringing the simulated volatility of the Real Exchange Rate much closer to the data than previous models with either representative or heterogeneous exporters.

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1 Introduction

The international organization of production is remarkably different today from what it looked like just fifteen or twenty years ago, with the formidable growth of Foreign Direct Investment (FDI)¹ standing out both in data and anecdotal evidence. In terms of growth rates, FDI flows and foreign sales have surpassed international trade and output since the late 1980s, accelerating sharply in the 1990s and 2000s to the extent that such a mode of delivering goods and services abroad has become quantitatively *more important* than trade (Table 1, Figure 1 and 2). Yet, somewhat surprisingly, these developments have received little attention in open economy macroeconomic theory and existing research has not discussed whether the explicit consideration of FDI has the potential of changing the properties of its workhorse models. In this paper, I focus on the role of FDI in altering the transmission of shocks across countries by studying the aggregate implications of firm level heterogeneity in a two country world where firms can engage in FDI in addition to exporting. To do so, I develop a micro-founded Dynamic Stochastic General Equilibrium (DSGE) model of international macroeconomic dynamics where firms have heterogeneous productivity as formulated in Melitz (2003). Unlike the existing open economy macroeconomic literature, this model endogenizes *both* FDI *and* exports as possible modes of accessing foreign markets, a feature that improves substantially the match between model-simulated moments and business cycle data along two dimensions. First, once I allow for concurrent entry (and exit) of MNFs and exporters over the business cycle, the consumption-output anomaly² originally brought to light in Backus, Kehoe, and Kydland (1992) disappears and I can successfully replicate the ranking of cross-country correlations of output and consumption that more traditional models of international co-movement cannot account for. Second, I show that the model with heterogeneous MNFs is capable of bringing the simulated volatility of the Real Exchange Rate much closer to the data than previous models with either representative or heterogeneous exporters (Ghironi and Melitz, 2005). Intuitively, this happens because when MNFs locate production in different destination markets, they diversify

¹According to the I.M.F.-O.E.C.D. Benchmark definition "FDI reflects the objective of obtaining a lasting interest by a resident entity in one economy (direct investor) in an entity resident in an economy other than that of the investor direct investment enterprise. The lasting interest implies the existence of a long-term relationship between the direct investor and the enterprise and a significant degree of influence on the management of the enterprise. Direct investment involves both the initial transaction between the two entities and all subsequent capital transactions between them and among affiliated enterprises, both incorporated and unincorporated." FDI is recorded in the Financial Account of the Balance of Payments of a country.

²In theoretical economies, output is less correlated across countries than is consumption while the data show the opposite.

production risks stemming from local market conditions, in a way that weakens the transmission of shocks across countries but increases price dispersion, and therefore the volatility of the aggregate price indexes.

The quantitative importance of FDI is evident from aggregate data on international commerce. In 2005, global sales by MNFs reached USD 22 trillion, compared with world exports of USD 12 trillion (Table 1 for a historical comparison), while the world stock of inward FDI jumped from 6 to 23 percent of world G.D.P. between 1982 and 2005. The increase of foreign sales of MNFs is the result of sizeable changes at the *extensive margin* (number of firms selling through foreign affiliates) and *intensive margin* (sales per firm), so sizeable indeed, that the value of foreign sales of MNFs in the economy where production takes place is now much larger than the value of the goods exported from one country to another (Figure 2). For example, Barba Navaretti et al. (2002) report that sales of manufacturing products of US subsidiaries in Europe are approximately 3.8 times larger than EU imports from the US and sales of EU subsidiaries in the US are 3.6 times larger than EU exports to the US (Figure 1 and 2).

Economic theory has offered limited predictions as to how the dynamics of the economy at the aggregate level affect the behavior of MNFs, or are influenced by the existence of FDI (Hanson and Slaughter, 2004). However, an infant empirical research has started looking into the issue, using primarily firm-level data. Buch and Lipponer (2005) measure the role of long-term fundamentals as opposed to short-term business cycle developments in driving outward German investment and find that the host countries' business cycle dynamics have a statistically significant impact on MNFs entry and sales. Desai and Foley (2006) show that the co-movement of value added, returns and capital investment between US MNFs and their affiliates is higher than the comovement between aggregate variables of the home and host countries. The Comment by Campa (2006), however, highlights that it is difficult to interpret the sources and implications of these correlations without having a model that is able to provide a theoretical framework of analysis and a quantitative benchmark.

Hence, I adopt Campa's (2006) suggestion and build my argument using a novel approach to model firm dynamics proposed by Ghironi and Melitz (2005), and Bergin, Glick and Taylor (2005), that introduce the type of heterogeneity originally analyzed in the trade literature in a micro-founded DSGE approach to the open economy. International trade research has shown that not all firms within an economy or an industry exports or engage in FDI, nor are internationally

engaged businesses a random subset of the population (Helpman, 2006 and references therein)³. Rather, the relatively small subset of firms that become exporters or multinationals includes the most productive firms of each sector⁴. Therefore, the argument presented in this paper rests on the following two premises: (1) firms are heterogeneous within the same sector and (2) all goods are tradable (but not necessarily traded) and can be produced by foreign affiliates. These assumptions allow me to pin down endogenously the size and the composition of the population of exporters and MNFs so that, in and out of equilibrium, firms that engage in FDI, exporters and local firms can coexist within the same sector, but, over time, respond differently to changes in the domestic and foreign business conditions.

Consistent with microeconomic evidence for flows across industrial countries (Swenson, 2005), I treat FDI sales as a potential alternative to exports (thus, horizontal⁵), in a fashion similar to Brainard (1997) and Helpman et. al. (2004), with the important distinctions that I model FDI in general equilibrium. By adopting a general equilibrium approach, I can show that firms decisions regarding entry, exit and pricing produce substantial dynamic effects on the labor markets of the two countries that, in turn, feed back into the decisions of firms and consumers at subsequent points in time.

I show that foreign demand, relative labor costs, productivity and policy variables are the main factors that trigger FDI entry and expand foreign sales over the business cycle. The introduction of FDI under firm heterogeneity reduces the positive correlation between a country's business cycle and its imports (exports sales for foreign countries), creating a wedge that provides an additional theoretical explanation to the evidence of a lack of increase in measures of international co-movement observed after the late 1980s uncovered in Heathcote and Perri (2004) and Doyle and Faust (2005). In this paper, this is consistent with the case in which a Home increase in demand increases imports less than it increases domestic sales of foreign MNFs and induces entry of new foreign MNFs at the extensive margin of FDI.

³U.S. Census Bureau data, for example, show that most (89 percent) exporting companies are small while a fistful of large companies amounting to 3 percent of the business population, accounts for almost 73 percent of the exports value.

⁴For UK firms, see Criscuolo and Martin (2005) and Head and Ries (2004) for Japanese firms. Helpman, Melitz and Yeaple (2004) find that US MNFs outperform sole exporters by 15% in terms of labor productivity, while exporters have a productivity advantage of 39% relative to domestically oriented producers.

⁵As opposed to vertical FDI. The distinction is based on the motive of affiliate operations. Horizontal MNCs conduct FDI in order to improve access to host country markets, while vertical FDI is undertaken in order to benefit from international factor price differences (see Hanson, Mataloni, and Slaughter, 2005). The bulk of FDI between developed economies is horizontal.

As for relative prices, I show that the existence of FDI in a two country model adds a substantial non-traded component to the output of the host economy, and because MNFs segment production and price-discriminate across countries, the substitution of exports with non-traded overseas sales magnifies the dispersion of prices, the volatility of price indexes and ultimately of the RER. My findings echo the role that Alessandria (2005) attributes to consumer search frictions in breaking the law of one price. Moreover, the analytical results of the model allow me to discuss how FDI sales and entry can modify the predictions of the Harrod-Balassa-Samuelson effect⁶ (HBS) and help explain the weak empirical confirmation it has received (Sarno and Taylor, 2002). The partial substitution of trade with FDI sales modifies the implications of the HBS effect to the extent that MNFs turn some traded goods into *de facto* non-traded (local) goods. By doing so, they can contribute to an increase of the average productivity of the non-traded sector and counter-balance the HBS effect. From this perspective, my research relates to recent work by Bergin et al. (2006) that shows how the HBS effect has emerged only in post-war data and re-inforced over the decades of the second half of the 20th century, suggesting that far from being a universal constant it tends to show time-varying features, linked to the endogenous tradability of goods and services. My results also suggest a potential explanation for the difference in national price levels between countries with a similar level of income (Broda, 2006) due to the different penetration of foreign firms and the exposure to foreign competition.

Finally, as a corollary to these results, I demonstrate that in a world with heterogeneous MNFs and endogenous entry, entrants sort over time according to own productivity, progressively reducing the average productivity of the multinational sector. I argue that this implication of my model might explain the conflicting evidence on the relationship between FDI and growth in aggregate data.

Few papers have proposed models of FDI with firm heterogeneity, namely Russ (2006), Lubik and Russ (2006), in general equilibrium, and Razin et al. (2005), in partial equilibrium, but none of these authors needs to conjecture the co-existence of exporters and MNFs to answer their research questions. The focus of the first two papers is the relationship between *nominal* exchange rates and MNFs entry in general equilibrium. Russ (2006) uncovers the mechanism through which the exchange rate and FDI sales are jointly determined in a monetary model of FDI with price rigidity,

⁶The theory originally formulated in the work of Harrod, Balassa, and Samuelson stating the "tendency for countries with higher productivity in tradeables compared to non tradeables to have higher price levels" (Obstfeld and Rogoff, 1996).

and shows that a MNF's response to increases in exchange rate volatility is ambiguous and differs depending on the source of the volatility. In a similar monetary model of FDI, Lubik and Russ (2006) suggest that MNFs pricing can offer a partial explanation to the exchange rate disconnect puzzle as it tends to make the nominal exchange rate more volatile than relative consumption. In their paper, the nominal ER works as a clearing mechanism to balance the net repatriation of profits across countries, although data show that these flows tend to be less important than trade flows for most industrial countries (Hanson and Slaughter, 2004 and Table 2 in the appendix). Finally, Razin et al. (2005) focus on FDI as international capital *flows* arising from cross-country productivity differences in a partial equilibrium setting, and develop a model with lumpy setup costs of new investment where an exogenous productivity shock in the host country affects both FDI entry and sales. They use the model to estimate the determinants of FDI flows and demonstrate that threshold barriers play an important role in determining the extent of horizontal FDI across O.E.C.D. countries. However, none of these papers can discuss export dynamics and their effects as they all assume that MNFs sales are the only way to serve foreign markets.

The paper is structured as follows. I illustrate the model in section 2 and a few selected analytical results in section 3. I describe the impulse response graphs of the artificial economy in section 4 and compare the second moments of the simulated model with US data and other relevant papers in the literature in section 5. Section 6 concludes.

2 The Model

I build a Dynamic Stochastic General Equilibrium flexible-price model representing a world composed of two identical countries (Home and Foreign), each populated by homogeneous consumers and a mass of potential entrepreneurs who write contracts in nominal terms. Labor mobility within countries ensures domestic wage equalization across producers and international immobility of workers allows wage differentials across countries. As for international financial integration, I assume market incompleteness and I analyze the case of financial autarky as a special case of the more general specification of the model in which I allow international trade of Home and Foreign bonds (B_t^H and B_t^F). This section describes the maximization problems of consumers and producers and explains how their outcomes are embedded into the general equilibrium structure of the economies.

2.1 Consumption

Preferences are identical across countries and consumers solve both an intertemporal and an intratemporal optimization problem.

INTERTEMPORAL PROBLEM. On date t , the Home representative consumer maximizes the lifetime expected utility subject to the intertemporal budget constraint

$$\begin{aligned} \max_{\{C_t, x_t, B_t\}_{t=0}^{\infty}} & \left[E_t \sum_{t=0}^{\infty} \beta^t U_t \right] \\ \text{s.t. } & C_t + \tilde{v}_t (N_{D,t} + N_{E,t}) x_{t+1} + B_{t+1}^H + Q_t B_{t+1}^F + \frac{\eta}{2} (B_{t+1}^H)^2 + \frac{\eta}{2} Q_t (B_{t+1}^F)^2 = \\ & = w_t L_t + (\tilde{v}_t + \tilde{\pi}_t) N_{D,t} x_t + (1 + r_t) B_t^H + (1 + r_t^*) Q_t B_t^F + T_t \end{aligned}$$

Utility takes a C.R.R.A. functional form $U_t = (1 - \gamma)^{-1} C_t^{1-\gamma}$ over aggregate consumption C_t ; $\gamma > 0$ is the parameter of relative risk aversion, the inverse of the inter-temporal elasticity of substitution and $\beta \in (0, 1)$ is the subjective discount factor. Agents can trade three types of assets: (1) the shares (x_{t+1}) of a mutual fund that owns a stock of $N_{D,t} + N_{E,t}$ firms whose average price is \tilde{v}_t and yield profits $\tilde{\pi}_t$ from the sale of $N_{D,t}$ existing goods⁷; (2) domestic and (3) foreign risk-free bonds $(B_{t+1}^H$ and $B_{t+1}^F)$ that yield interest rates r_t and r_t^* , and bear a transaction cost $(B_{t+1}^H)^2 \eta/2$ and $(B_{t+1}^F)^2 \eta/2$, eventually rebated to consumers in each country, for an amount equal to T_t . The nominal and Real Exchange Rate are denoted by e_t and $Q_t \equiv e_t P_t^*/P_t$ ⁸, P_t^* and P_t being the Foreign and the Home consumption price indexes.

Hence, the sources of income for consumers include labor income $w_t L_t$, the risk-free bonds bought in the previous period and the interest they carry $[(1 + r_t) B_t^H$ and $(1 + r_t^*) B_t^F]$, the proceeds from the sale of the share in the mutual fund $\tilde{v}_t x_t$ and the associated profit $\tilde{\pi}_t x_t$ they earn from the producing firms $N_{D,t}$ ⁹. The assumptions of zero population growth and rigid labor supply imply that employment is always equal to the fixed endowment of workers ($L_t = \bar{L}$), so that the aggregate labor income is entirely driven by the dynamic of wages.

In the case of financial autarky, $B_t^F = B_{t+1}^F = 0$, $x_t = x_{t+1} = 1$, so that the Home budget constraint reduces to $C_t + B_{t+1}^H = \tilde{\pi}_t N_{D,t} + w_t L_t - \tilde{v}_t N_{E,t} + (1 + r_t) B_t^H$, according to which the

⁷In monopolistic competition models and in this paper, firms produce an individual variety of a differentiated good, so I shall use the words good, variety, product, firm, and plant as substitutes.

⁸ e_t is equal to units of Home currency necessary to buy one unit of Foreign Currency, while Q_t indicates the units of the Home consumption basket per unit of Foreign consumption basket.

⁹Notice that the budget constraint is expressed in real terms by dividing nominal variables through the aggregate price index P_t and defining real variables as follows $\tilde{\pi}_t \equiv \tilde{\Pi}_t/P_t$ and $\tilde{v}_t \equiv \tilde{V}_t/P_t$, and $w_t \equiv W_t/P_t$.

per-period consumption must be equal to labor income summed to the profits yielded by domestic firms, minus the investment cost of financing new firms. Net of domestic bond trading, the equation can be interpreted as describing the national accounts of an autarkic economy where spending (consumption, C_t plus investment to finance new ventures, $\tilde{v}_t N_{E,t}$) must be equal to labor income $w_t L_t$ plus dividends $\tilde{\pi}_t N_{D,t}$. In this case, consumers can trade consumption intertemporally by buying and selling domestic bonds and shares of the mutual fund, so that the solution of the maximization problem yields two Euler equations for bond and for share holdings

$$E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 + r_{t+1}) \right] = 1 \quad (1)$$

$$E_t \left[\beta(1 - \delta) \left(\frac{C_{t+s}}{C_t} \right)^{-\gamma} (\tilde{v}_{t+1} + \tilde{d}_{t+1}) \right] = \tilde{v}_t \quad (2)$$

Notice that the inter-temporal discount factor β is reduced by a component δ that captures the probability of firms' death discussed in section 2.3. In the case of bond trading, there are two equations for bond holdings, one for each bond, as discussed in the appendix.

INTRATEMPORAL PROBLEM. In each period consumption takes place over a continuum of goods¹⁰ $C_t = \left(\int_{\omega \in \Omega} c_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$ indexed by $\omega \in \Omega$ to which is associated a standard C.E.S. price aggregator $P_t = \left(\int_{\omega \in \Omega} p_t(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$ where $\sigma > 1$ is the symmetric constant elasticity of substitution across goods. As usual in the C.E.S. demand system, the demand for a single variety is $c_t(\omega) = [p_t(\omega)/P_t]^{-\sigma} C_t$ and the expenditure on an individual good is proportional to total expenditure. Analogous optimization problem, resource constraints, Euler equations, price aggregators and demand system can be defined for the Foreign economy.

2.2 Production

On the production side, each variety is produced by a different entrepreneur under increasing returns to scale with a fixed cost and constant but heterogeneous marginal cost of the single factor of production, labor. Not only is the output of each manufacturer influenced by aggregate (or common across firms) productivity Z_t , but also by an idiosyncratic (relative) productivity parameter z that

¹⁰Goods can be produced domestically or abroad, thus should be thought of as a continuum of goods available to production in the whole world. In every period only a subset will be actually produced.

enters a generic cost function as follows¹¹:

$$\Gamma_t(\omega) = W_t l_t(z) = \underbrace{\frac{W_t f_t}{Z_t}}_{\text{fixed cost}} + \underbrace{\frac{W_t}{Z_t z} y_t(\omega)}_{\text{variable cost}} \quad (3)$$

Here, $l_t(z) = [f_t/Z_t + y_t(\omega)/zZ_t]$ is the total labor requirement needed to produce a quantity $y_t(\omega)$ of output. Notice that Z_t is common for all firms and changes over time, while z is firm specific and time invariant. This cost function implies that productivity differences across products translate into differences in the marginal cost of production. The latter is measured in units of consumption good and equals w_t/zZ_t where w_t is the real wage W_t/P_t . Firm level heterogeneity is a key ingredient of this model as it plays a role in determining firm entry at the domestic level and the extent of their global engagement as described in the next two sections.

2.3 Domestic Entry and Exit

Consider the Home country. In each period, a mass of potential entrants decides whether to enter production. The timing of entry is depicted in Figure 3.

Before entering production of a specific variety, each entrepreneur faces a sunk entry cost of $f_{E,t}$ effective labor units equal to $W_t f_{E,t}/Z_t$ units of the home consumption good. Upon entry, he draws the idiosyncratic relative productivity level z from a time-invariant distribution $G(z)$, common across firms and identical across countries. Each of the $N_{E,t}$ new entrants in t , becomes one of the $N_{D,t+1}$ producers only in $t+1$ (), with a one period time-to-build. The manufacturer of each variety maintains its relative productivity until it exits the market, namely until it is hit by a "death" shock, an event that occurs with probability δ in every period, including the very first t when the entrepreneur draws the productivity parameter. Thus, the exit of firms/varieties is independent of the productivity level. Thus, in each period, the number of producers is equal to the number of survivors from the pool of firms existing in the previous period, i.e. established producers and new entrants:

$$N_{D,t} = (1 - \delta)(N_{D,t-1} + N_{E,t-1}) \quad (4)$$

I assume that entrepreneurs are forward looking and have perfect information on the structure of the economy; under this assumption expected profits are exactly the realized average profit,

¹¹See Appendix.

$E_t[\pi_s(z)] = \tilde{\pi}_s(z)$, $s > t$. Potential entrants evaluate their expected post-entry value with the present discounted value of the expected stream of future profits $\{\tilde{\pi}_s\}_{s=t+1}^{\infty}$

$$\tilde{v}_t = E_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left(\frac{C_{t+s}}{C_t} \right)^{-\gamma} \tilde{\pi}_s \quad (5)$$

equal to the average value after entry and production¹². The Free Entry Condition requires that the firm value is equalized with the entry cost

$$\tilde{v}_t = \frac{w_t f_{E,t}}{Z_t} \quad (6)$$

[Figure 3]

In every period, a group of domestic producers (indexed by D) produces and sells domestically, each determining the optimal price $p_{D,t}(\omega)$ for the variety ω by maximizing the profit function

$$\max_{p_{D,t}(\omega)} \Pi_{D,t} \quad \text{where} \quad \Pi_{D,t} = p_{D,t}(\omega) y_t(\omega) - \frac{W_t}{Z_t z} y_t(\omega)$$

that has solution $p_{D,t}(\omega) = \mu W_t / z Z_t$ where $\mu = \sigma / \sigma - 1$ is the mark-up over marginal cost. Notice that firms with higher marginal cost $1/z Z_t$ charge lower prices and one can easily show that higher productivity also implies larger sales, revenues and profit, a result due to Melitz (2003).

2.4 Global engagement

In any t, each Home producer sells in the domestic market and possibly expands to the foreign market, choosing between two alternative business strategies: exports (a strategy indexed by X) or sales through foreign affiliates (a strategy indexed by I)¹³. Hence, exporters produce only domestically but sell both domestically and in the foreign country while MNFs establish a foreign production unit to produce and sell in the foreign market in addition to produce and sell domestically¹⁴. Each of these strategies implies a different structure of costs.

In order to be able to export, firms have to pay a fixed cost of exporting in every period, equal to $W_t f_{X,t} / Z_t$ units of the Home consumption good, and face variable costs in the form of an iceberg

¹²Notice that agents use the stochastic discount factor modified to take into account the probability of firms' exit, that affect expected future profits, in a fashion similar to a model in which consumers have finite lifetime and an exogenously given probability of death.

¹³ I use the same structure as Helpman, Melitz and Yeaple (2004). A more complex set-up is Grossman, Helpman, and Szeidl (2004), who examine integration strategies of MNFs allowed to choose among a larger set of choices to internationally organize their vertically fragmented production.

¹⁴I assume that goods produced abroad by I companies cannot be re-exported to the Home country.

trade cost $\tau_t \geq 1$. In order to establish a multinational network, manufacturers have to hire foreign labor (paid at a foreign wage rate). FDI entails a higher fixed cost of $f_{I,t}$ labor units in every period, equal to $W_t^* f_{I,t}^* / Z_t^*$ units of the Foreign consumption good, but allows firms to save in trade costs. Notice that the decision to become exporters or open overseas affiliates is taken in any period, as $f_{X,t}$ and $f_{I,t}$ are per-period fixed costs, while the decision to enter is taken only once, as $f_{E,t}$ is a once-and-for-all sunk cost.

As for foreign sales, Home firms set the optimal price in the currency of destination of the good by solving the following maximization problems that differ depending on the business strategy:

$$\begin{aligned} \max_{p_{X,t}(\omega)} \Pi_{X,t} \quad \text{where} \quad \Pi_{X,t} &= [e_t p_{X,t}(\omega)] y_t(\omega) - \tau_t \frac{W_t}{Z_t z} y_t(\omega) - \frac{W_t f_{X,t}}{Z_t} \\ \max_{p_{I,t}(\omega)} \Pi_{I,t} \quad \text{where} \quad \Pi_{I,t} &= e_t \left[p_{I,t}(\omega) y_t(\omega) - \frac{W_t^*}{Z_t^* z} y_t(\omega) - \frac{W_t^* f_{I,t}^*}{Z_t^*} \right] \end{aligned} \quad (7)$$

For exporters, prices denominated in foreign currency are converted using the nominal exchange rate e_t . The labor hired in the *Home* country for export production is paid a nominal effective wage rate W_t / Z_t . FDI companies determine the optimal price in the foreign currency, hire workers and pay salaries at the *Foreign* nominal effective wage rate W_t^* / Z_t^* , and maximize operating profits converting both revenues and costs in domestic currency at the nominal exchange rate e_t .

Optimal prices for each strategy are¹⁵

$$p_{X,t}(\omega) = \mu \frac{W_t}{z Z_t} \frac{\tau_t}{e_t}, \quad p_{I,t}(\omega) = \mu \frac{W_t^*}{z Z_t^*} \quad (8)$$

Hence, while Home customers are always charged a price $p_{D,t}(\omega)$, foreign customers are charged either $p_{X,t}(\omega)$ or $p_{I,t}(\omega)$ depending on the choice between becoming an exporter or setting up a foreign production plant for the variety. There are several differences in the optimal price charged by exporters and FDI firms. Exporters incur the iceberg cost that increases their marginal cost, incorporate the exchange rate, and price at a mark-up over *domestic* effective wages. FDI firms set prices independently of the exchange rate, and price at a mark-up over *foreign* effective wages.

The coexistence of heterogeneous marginal costs and sunk costs of exporting or engaging in FDI is key to predicting self-selection of the most productive firms into different business strategies (Exports vs. FDI trade-off). Since higher productivity implies higher operating profits, at any point in time only the most productive firms can amortize the fixed costs and make non-negative total profits, a point that can be understood easily graphically. Consider Figure 3 where I have

¹⁵See Appendix.

drawn profits for each segment of business as a function of idiosyncratic productivity levels (top graph) and the partition of the productivity distribution according to firms' business strategies based on own productivity (bottom graph of figure 3).

The bottom graph in Figure 4 is derived by identifying the cut-off points in the top graph. Take the $\pi_D(z)$ curve on the latter (subscript omitted): for low levels of productivity, firms cannot break even and make the non-negative profits necessary to compensate the fixed cost of entry measured in units of effective labor cost. $(\Theta w_t f_{E,t}/Z$ with $\Theta = [1 - \beta(1 - \delta)] / \beta(1 - \delta)$, is the *annualized* value of the fixed cost of entry. However, any firm with $z > z_{min}$ at time t is a profitable business entity as far as production for the domestic market is concerned. Firms with a very high productivity $z > z_{I,t}$ earn profits high enough to allow them to compensate the fixed cost of investing abroad through FDI ($w_t^* f_{I,t}^*/Z_t^*$). Firms for which idiosyncratic productivity is $z_{X,t} < z < z_{I,t}$ earn profits that are high enough only to cover the fixed cost of exporting ($w_t f_{X,t}/Z_t$) but not large enough to cover $w_t^* f_{I,t}^*/Z_t^*$ and produce overseas. Given the level of trade costs τ_t (that reduces the slope of the profit curve of exporters) and the fixed cost $f_{X,t}$, this intermediate range of firms finds exporting more profitable than investing abroad. The bottom graph shows how cut-off points determine the partition of producers into firms that sell only domestically (only D), firms that export (X) and firms that engage in FDI (I) at any point in time. When cut-off point move as a response to business conditions or policy changes (such as reduction of fixed or trade costs), the share of firms engaged in each mode of business changes accordingly at the extensive margin and each individual firm experiences changes in its per unit sales at the intensive margin. Notice that underlying the structure of the international production I assume an implicit ranking of productivity, by which firms that engage in FDI are more productive than exporters and exporters are in turn more productive than companies that sell only domestically in a fashion similar to Helpman, Melitz and Yeaple (2004). This assumption is easily justified considering that the superior productivity of MNFs at the basis of most theories of FDI has found strong support in studies based on firm-level data, such as Criscuolo and Martin (2005).

To summarize, at any point in time there are three groups of Home firms: $N_{D,t}$ Home firms produce and sell in the Home market, $N_{X,t}$ of these firms produce at Home and sell in Foreign and $N_{I,t}$ of the Home firms produce and sell in the Foreign country. As the composition of expenditure in each country, it is useful to define $S_{D,t} \equiv \tilde{\rho}_{D,t}^{1-\sigma} N_{D,t}$, $S_{X,t} \equiv \tilde{\rho}_{X,t}^{*1-\sigma} N_{X,t}^*$ and $S_{I,t} \equiv \tilde{\rho}_{I,t}^{*1-\sigma} N_{I,t}^*$ as the share of Home expenditure in domestic goods, on imported goods, and on goods produced

in Home by Foreign MNFs ($S_{D,t}^*$, $S_{X,t}^*$, and $S_{I,t}^*$ are defined analogously). To be consistent with the data, my model will be calibrated with the empirical observation that the largest share of the expenditure of Foreign ($S_{D,t}^*$) is allocated to $N_{D,t}^*$ producers, and smaller shares go to imports provided by $N_{X,t}$ Home exporters ($S_{X,t}^*$) and to the sales of $N_{I,t}$ Foreign affiliates of Home MNFs ($S_{I,t}^*$). Symmetric definitions apply to the partition of firms and the composition of expenditure in the Foreign economy.

2.5 Total profits

To see how profits feed back into the budget constraint defined earlier, notice that the price for each variety ω and optimal profits, relative to the price index of the market of *destination* can be expressed as follows:

$$\rho_{D,t}(\omega) \equiv \frac{p_{D,t}(\omega)}{P_t} = \frac{\mu}{zZ_t} w_t, \quad \rho_{X,t}(\omega) = \frac{p_{X,t}(\omega)}{P_t^*} = \frac{\mu}{zZ_t} \frac{\tau_t}{Q_t} w_t \quad (9)$$

$$\rho_{I,t}(\omega) \equiv \frac{p_{I,t}(\omega)}{P_t^*} = \frac{\mu}{zZ_t^*} w_t^* \quad (10)$$

Associated with these optimal prices I can define optimal profits for each segment, relative to the price index of the market of *location* of the mother company, as

$$\pi_{D,t}(\omega) = \frac{1}{\sigma} \rho_{D,t}^{1-\sigma}(\omega) C_t, \quad \pi_{X,t}(\omega) = \frac{Q_t}{\sigma} \rho_{X,t}^{1-\sigma}(\omega) C_t^* - \frac{w_t f_{X,t}}{Z_t} \quad (11)$$

$$\pi_{I,t}(\omega) = \frac{Q_t}{\sigma} \rho_{I,t}^{1-\sigma}(\omega) C_t^* - \frac{Q_t w_t^* f_{I,t}^*}{Z_t^*} \quad (12)$$

where $Q_t = e_t P_t^* / P_t$ is the RER. Combined profits from the two markets, imply that average profit of Home firms are

$$\tilde{\pi}_t \equiv \int_{z_{\min}}^{\infty} \pi_{D,t}(z) dG(z) + \int_{z_X}^{z_I} \pi_{X,t}(z) dG(z) + \int_{z_I}^{\infty} \pi_{I,t}(z) dG(z)$$

This break-down mirrors the fact that out of a total mass of firms $N_{D,t}$ producing domestically in every period in each country, there are a total of $N_{X,t} = [G(z_{I,t}) - G(z_{X,t})] N_{D,t}$ exporters and of $N_{I,t} = [1 - G(z_{X,t})] N_{D,t}$ firms engaging in FDI, where $G(z)$ is a distribution with support $[z_{\min}, \infty)$.

Productivity averages are defined as in Melitz (2003) but I consider separately the average idiosyncratic productivity of firms who produce and sell in Home ($\tilde{z}_{D,t}$), of those that sell domestically and export ($\tilde{z}_{X,t}$), of those that sell domestically and engage in FDI ($\tilde{z}_{I,t}$), and of local firms ($\tilde{z}_{DO,t}$)

$$\tilde{z}_{D,t} \equiv \left[\int_{z_{\min,t}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}, \quad \tilde{z}_{X,t} \equiv \left[\frac{1}{G(z_{I,t}) - G(z_{X,t})} \int_{z_{X,t}}^{z_{I,t}} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}} \quad (13)$$

$$\tilde{z}_{I,t} \equiv \left[\frac{1}{1 - G(z_{I,t})} \int_{z_{I,t}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}, \quad \tilde{z}_{DO,t} \equiv \left[\frac{1}{G(z_{X,t}) - G(z_{\min})} \int_{z_{\min}}^{z_{X,t}} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}} \quad (14)$$

The weights of the averages are proportional to relative firm output shares, and summarize all the relevant information on the productivity distribution, as shown in Melitz (2003)¹⁶. By defining $\tilde{\pi}_{D,t} \equiv \pi_{D,t}(\tilde{z}_{D,t})$, $\tilde{\pi}_{X,t} \equiv \pi_{X,t}(\tilde{z}_{X,t})$, and $\tilde{\pi}_{I,t} \equiv \pi_{I,t}(\tilde{z}_{I,t})$, the total profits earned by the average firm in home can be written as $\tilde{\pi}_t \equiv \tilde{\pi}_{D,t} + \tilde{\pi}_{X,t} [G(z_{I,t}) - G(z_{X,t})] + \tilde{\pi}_{I,t} [1 - G(z_{I,t})]$.

Following the trade literature, I assume that relative productivity parameter z is drawn from a Pareto distribution¹⁷ with lower bound z_{\min} and shape parameter k greater than $\sigma - 1$. Therefore, the p.d.f and c.d.f. of z are $g(z) = k z_{\min}^k / z^{k+1}$ and $G(z) = 1 - (z_{\min}/z)^k$, $k > \sigma - 1$. Under Pareto productivity, the geometric averages defined above become (see Appendix):

$$\tilde{z}_{D,t} = \nabla^{\frac{1}{\sigma-1}} z_{\min} \quad \tilde{z}_{X,t} = \nabla^{\frac{1}{\sigma-1}} \left[\frac{z_{X,t}^{\sigma-1} z_{I,t}^k - z_{X,t}^k z_{I,t}^{\sigma-1}}{z_{I,t}^k - z_{X,t}^k} \right]^{\frac{1}{\sigma-1}} \quad (15)$$

$$\tilde{z}_{I,t} = \nabla^{\frac{1}{\sigma-1}} z_{I,t} \quad \text{with } \nabla = \frac{k}{k - (\sigma - 1)} \quad (16)$$

To determine the extensive margin of export, I identify the marginal firm whose productivity level is such that her operating profits are equal to the fixed cost of exporting. I define such a productivity level $z_{X,t}$ as the cut-off for the exports segment:

$$\pi_{X,t}(z_{X,t}) = 0 \iff z_{X,t} = \left(\frac{f_{X,t}\sigma}{C_t^*} \right)^{\frac{1}{\sigma-1}} \left(\frac{w_t}{Q_t Z_t} \right)^{\mu} \mu \tau_t \quad \text{with } \mu = \frac{\sigma}{\sigma - 1} \quad (17)$$

Analogously, for the FDI segment, I can identify the firm that has productivity $z_{I,t}$, i.e. the firm for which operating profits are equal to the fixed cost of FDI entry and I define such productivity level as the cut-off for the FDI segment:

$$\pi_{I,t}(z_{I,t}) = 0 \iff z_{I,t} = \left(\frac{\sigma f_{I,t}^*}{C_t^*} \right)^{\frac{1}{\sigma-1}} \left(\frac{w_t^*}{Z_t^*} \right)^{\mu} \mu \quad (18)$$

¹⁶ The \tilde{z} s are weighted averages of the productivity levels and are independent of the number of firms while the weights reflect the relative output shares of firms with different productivity levels.

¹⁷ The Pareto distribution of the productivity parameter implies that the distribution of sales is also a Pareto.

Notice that in this case the sunk cost is paid at the Foreign effective wage rate W^*/P^* . Using the cut-off point ($z_{X,t}$ and $z_{I,t}$), I can obtain closed form solutions for the average productivity of exported production and FDI sales ($\tilde{z}_{X,t}$ and $\tilde{z}_{I,t}$) and the average profits from exports¹⁸

$$\tilde{\pi}_{X,t} = \left[\nabla \frac{f_{X,t}^{1+\frac{k}{1-\sigma}} - \left(TOL_t^\sigma \tau_t^{1-\sigma} f_{I,t}^* \right)^{1+\frac{k}{1-\sigma}}}{f_{X,t}^{\frac{k}{1-\sigma}} - \left(TOL_t^\sigma \tau_t^{1-\sigma} f_{I,t}^* \right)^{\frac{k}{1-\sigma}}} - f_{X,t} \right] \frac{w_t}{Z_t}; \quad \tilde{\pi}_{I,t} = (\nabla - 1) \frac{w_t^* f_{I,t}^*}{Z_t^*}$$

with the terms of labor, $TOL_t \equiv \left(c_t \frac{W_t^*}{Z_t^*} \right) / \left(\frac{W_t}{Z_t} \right)$, equal to the relative effective labor cost¹⁹.

3 Analytical Results

3.1 Determinants of foreign sales

In order to discuss the determinants of revenues, costs and profits from foreign activities I define the extensive margin, the composition margin and the scale margin, in a fashion similar to Yeaple (2006)'s analysis of the static partial equilibrium model of Helpman et al. (2004). The extensive margin of exports and FDI sales is defined by the number of exporters and MNFs as a share of the Home producers. The composition margin is defined by the average productivity component of sales, while the scale margin captures country size and factor prices. The interaction between composition and scale margin determines what is commonly known in the literature as intensive margin, the average size of a foreign affiliate's sales. In general equilibrium, only purely exogenous variables (sunk and fixed costs, trade costs, aggregate productivity) have a clearly identifiable impact on endogenous variables, as reported in the following tables of synthesis. Hence, the equations describing foreign affiliates' sales and exports should be read as an abstraction from the general equilibrium structure of the model. Consider the value of the Foreign sales of Home MNFs' affiliates expressed in units of the Home consumption good, $FDIsales_t = N_{I,t} \tilde{\rho}_{I,t}^{1-\sigma} C_t^* Q_t$. Using average relative productivity $\tilde{z}_{I,t} = \nabla^{\frac{1}{\sigma-1}} \left(\sigma f_{I,t}^* / C_t^* \right)^{\frac{1}{\sigma-1}} (w_t^* / Z_t^*)^\mu \mu$ and the relative number of firms $N_{I,t} = (z_{\min} / \tilde{z}_{I,t})^k \nabla^{\frac{k}{\sigma-1}} N_{D,t}$ as derived in the Appendix I have that

¹⁸See Appendix. Correspondingly, the Foreign country host firms are characterized by $N_{D,t}^*$, $N_{X,t}^*$, $N_{I,t}^*$, π_t^* , $\pi_{X,t}^*$, $\pi_{I,t}^*$, $z_{D,t}^*$, $z_{X,t}^*$, $z_{I,t}^*$, $\tilde{z}_{D,t}^*$, $\tilde{z}_{X,t}^*$, $\tilde{z}_{I,t}^*$, and \tilde{v}_t^* .

¹⁹ $TOL_t < 1$ means that Home effective labor is relative cheaper than Foreign.

$$\begin{array}{ccccc}
FDI\ sales_t & N_{I,t} & \times & \tilde{z}_{I,t}^{\sigma-1} & \times & \left(\frac{w_t^*}{Z_t^*}\mu\right)^{1-\sigma} C_t^* Q_t \\
& \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} \\
& extensive & & composition & & scale \\
& margin & & margin & & margin
\end{array}$$

The following relationships between the key exogenous variables in columns and the three margins defined above can be derived from the model:

MNF sales Margins ↓	τ_t	$f_{I,t}^*$	Z	Z^*
Extensive	+	-	-	+
Composition	-	+	+	-
Scale			-	+

The model predicts that when the number of affiliates increases, the average productivity across N_I decreases because new entrants have productivity below the pre-entry cut-off point $z_{I,t}$. Hence, the extensive margin (number of MNFs) is negatively correlated with the composition margin (average productivity of MNFs), a result for which Yeaple (2006) finds evidence based on US micro data, and that will be a feature of my dynamic analysis.

As for exports, using average relative productivity $\tilde{z}_{X,t}$ and the relative number of exporters $N_{X,t}/N_{I,t} = (z_{I,t}/z_{X,t})^k - 1$, the value of Home Exports $X_t = Q_t N_{X,t} \tilde{\rho}_{X,t}^{1-\sigma} C_t^*$ can be decomposed as

$$\begin{array}{ccccc}
X_t & N_{X,t} \tau_t^{1-\sigma} & \times & \tilde{z}_{X,t}^{\sigma-1} & \times & \left(\frac{w_t}{Z_t}\mu\right)^{1-\sigma} C_t^* Q_t^\sigma \\
& \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} \\
& extensive & & composition & & scale \\
& margin & & margin & & margin
\end{array}$$

The model predicts that when the number of exporters increases, the average productivity across $N_{X,t}$ can increase or decrease depending on whether the increase in the number of exporters is due to exit from multinational activity or entry of new exporters. In this case, the former MNFs that return to exports have higher productivity than the average exporter, but new exporters have lower. Hence, the extensive margin (number of exporters) can be positively or negatively correlated

with the composition margin (average productivity of exporters). The relationship between the various exogenous variables and the three margins of exports are as follows.

Export Margins ↓	τ_t	$f_{X,t}$	$f_{I,t}^*$	Z	Z^*
Extensive	-	-	+	+	-
Composition	+	+	+	-	+
Scale				+	-

3.2 Price Indexes and Real Exchange Rate

Following a line of reasoning similar to the one that I used for average productivity, I can define average prices for different market segments: $\tilde{p}_{D,t} \equiv p_{D,t}(\tilde{z}_D)$, $\tilde{p}_{X,t} \equiv p_{X,t}(\tilde{z}_X)$ and $\tilde{p}_{I,t} \equiv p_{I,t}(\tilde{z}_I)$ and re-write the price indexes as

$$P_t = \left(N_{D,t} \tilde{p}_{D,t}^{1-\sigma} + N_{X,t}^* \tilde{p}_{X,t}^{1-\sigma} + N_{I,t}^* \tilde{p}_{I,t}^{1-\sigma} \right)^{\frac{1}{\sigma-1}}; \quad P_t^* = \left(N_{D,t}^* \tilde{p}_{D,t}^{1-\sigma} + N_{X,t} \tilde{p}_{X,t}^{1-\sigma} + N_{I,t} \tilde{p}_{I,t}^{1-\sigma} \right)^{\frac{1}{\sigma-1}}$$

Thus, in each country the domestic price index reflects the numbers of varieties and the average prices for domestic goods sold domestically by domestic producers, for goods produced abroad by foreign producers and goods produced domestically by foreign firms through FDI, both of them sold in the domestic market. By dividing through the price indexes (P_t and P_t^* , respectively), and recalling that $S_D \equiv \tilde{\rho}_D^{1-\sigma} N_D$, $S_X \equiv \tilde{\rho}_X^{1-\sigma} N_X^*$ and $S_I \equiv \tilde{\rho}_I^{1-\sigma} N_I^*$ as the steady state spending on domestic goods, on goods imported from abroad, and on goods produced domestically by MNFs, I re-write each price index in terms of expenditure shares $1 = S_D + S_X + S_I$ and $1 = S_D^* + S_X^* + S_I^*$. In other words, I can think of the amount of goods produced domestically as a combination of goods produced by relatively inefficient domestic producers and relatively efficient foreign investors, where the productivity level of each category is reflected explicitly in lower and higher price indexes for the subset of goods, and (implicitly) in their number and size.

The welfare based price indexes (P_t and P_t^*) can be used to define the welfare-based RER Q_t and its CPI-based transformation q_t that discounts the variety effect and is conceptually closer to the way official statistics report the RER (see Appendix).

The welfare-based RER defined as $Q_t \equiv e_t P_t^* / P_t$, is equal to:

$$Q_t^{1-\sigma} = \frac{N_{D,t}^* \left(TOL_t \frac{\tilde{z}_{D,t}}{\tilde{z}_{D,t}^*} \right)^{1-\sigma} + N_{X,t} \left(\frac{\tau_t \tilde{z}_{D,t}}{\tilde{z}_{X,t}} \right)^{1-\sigma} + N_{I,t} \left(TOL_t \frac{\tilde{z}_{D,t}}{\tilde{z}_{I,t}} \right)^{1-\sigma}}{N_{D,t} + N_{X,t}^* \left(TOL_t \frac{\tau_t^* \tilde{z}_{D,t}}{\tilde{z}_{X,t}^*} \right)^{1-\sigma} + N_{I,t}^* \left(\frac{\tilde{z}_{D,t}}{\tilde{z}_{I,t}^*} \right)^{1-\sigma}} \quad (19)$$

If home effective labor appreciates ($\frac{W_t}{Z_t} \uparrow$) then the terms of labor decrease ($TOL_t \downarrow$), and the Home economy as a whole becomes a less attractive location. The way one should think about the terms of labor is that if $TOL_t > 1$ a firm with a given level of idiosyncratic productivity z can produce at lower cost in the Home country than in the Foreign. The RER q_t constructed using the CPI index (see Appendix) is log-linearized as

$$\begin{aligned} \hat{q}_t = & (2S_{DI} - 1)\widehat{TOL}_t + S_I \left(\hat{z}_{I,t}^* - \hat{z}_{I,t} \right) + (1 - S_{DI}) \left[(\hat{\tau}_t - \hat{\tau}_t^*) + \left(\hat{z}_{X,t}^* - \hat{z}_{X,t} \right) \right] \\ & - \frac{1}{\sigma - 1} \left\{ \left(S_D - \frac{N_D}{N_D + N_X + N_I} \right) \left[\left(\hat{N}_{D,t}^* - \hat{N}_{X,t} \right) - \left(\hat{N}_{D,t} - \hat{N}_{X,t}^* \right) \right] + \right. \\ & \left. + \left(S_I - \frac{N_I}{N_D + N_X + N_I} \right) + \left[\left(\hat{N}_{I,t} - \hat{N}_{X,t} \right) - \left(\hat{N}_{I,t}^* - \hat{N}_{X,t}^* \right) \right] \right\} \end{aligned}$$

where variables with a hat represent percentage deviations from the steady state and $S_{DI} = S_D + S_I$ is the steady state spending in goods produced domestically by Home firms and in goods produced in Home by MNFs

Over time the RER evolves as a function of all the terms above that I discuss in blocks in the following paragraphs, focusing on the case of a decrease in Home effective wage ($\downarrow W_t/Z_t$).

CHANGES IN THE RELATIVE COST OF LABOR (CLASSICAL HBS EFFECT). $(2S_{DI} - 1)\widehat{TOL}_t$. If Home effective labor is less expensive (lower W_t/Z_t , $\widehat{TOL}_t > 0$), then the Home currency depreciates because the price of non-traded goods in the home market becomes relatively lower. This is the channel of RER depreciation in the classical HBS effect due to the existence of a non-traded sector as highlighted, for example, in Obstfeld and Rogoff (1996). Notice that such an effect would be produced in a model without endogenous tradeability or FDI, but here it can be magnified by the presence of FDI if $S_{DI} > S_D$.

ENDOGENOUS MNFS ENTRY $S_I \left(\hat{z}_{I,t}^* - \hat{z}_{I,t} \right)$. Absent changes in TOL_t , entry of new foreign firms into the Home market (endogenous entry) due to lower FDI cut-off points, can reduce the average productivity of foreign FDI firms located in Home, and increase the domestic price index as the entrants charge higher prices. Thus, the home price index increases and the RER appreciates ($q_t \downarrow$).

CHANGES IN THE RELATIVE PRICES OF TRADED GOODS:

- EXOGENOUS TRADE POLICY CHANGES $(1 - S_{DI}) [(\hat{\tau}_t - \hat{\tau}_t^*)]$. The effect of a change in international trade policy or costs can be observed in the second term $(\hat{\tau}_t - \hat{\tau}_t^*)$, capturing relative variations of domestic to foreign trade costs for exporters. For example, if Home exporters

face a reduction of trade costs ($\widehat{\tau}_t < 0$, due to a reduction of tariffs applied by the Foreign customs), then more Home exporters can profitably access the Foreign market with the effect of increasing the average price of Foreign imports (because new exporters are less productive than existing) and this pushes up the Foreign price index and causes a depreciation of the Home currency. Notice the change in trade costs is a truly exogenous phenomenon as it does not hinge on movements of relative effective labor costs, or cut-off points.

- **ENDOGENOUS TRADABILITY** $(1 - S_{DI}) \left[\left(\widehat{z}_{X,t}^* - \widehat{z}_{X,t} \right) \right]$. Changes in the composition of the traded sector in both countries affect the RER when the composition of the traded sector can change as a response to changes in the business conditions (productivity, cut-off points, trade costs the RER) depending on the productivity cut-off levels of Home exporters and Foreign MNFs entry. For example, if the Home economy is hit by a positive productivity shock (lower W_t/Z_t , $\widehat{TOL}_t > 0$), new Home firms decide to start exporting, because the export cut-off lowers at the same time of a reduction of variable costs. Then import prices in the Foreign economy rise as lower productivity firms enter the exports market at Home ($\widehat{z}_{X,t} < 0$), and this makes the RER depreciate, as in the case of $\widehat{\tau}_t < 0$. These effects reinforce the RER depreciation, but less than they would in the case with no FDI ($S_I = 0$). As can be observed in the bottom graph in Figure 4, an asymmetric technological shock that induces Home firms to start exporting also attracts Foreign MNFs into the Home market at the extensive margin of Foreign FDI. Because some foreign producers substitute Foreign exports with sales of Foreign MNFs' affiliates in Home, this decreases the average productivity of exporters abroad ($\widehat{z}_{X,t} < 0$), counterbalancing the depreciation of the RER, through a price effect (average prices of Foreign exporters are now higher). Therefore, the net effect of endogenous tradability on the RER depends on the relative importance of these changes in the composition of the export sectors in the two countries.

EXPENDITURE SWITCHING $\frac{1}{\sigma-1} \left(S_D - \frac{N_D}{N_D+N_X+N_I} \right) \left[\left(\widehat{N}_{D,t}^* - \widehat{N}_{X,t} \right) - \left(\widehat{N}_{D,t} - \widehat{N}_{X,t}^* \right) \right]$. The factor $S_D - \frac{N_D}{N_D+N_X+N_I}$ is negative, as it reflects the difference between the market expenditure share of Home firms and the relative share of these firms/products in the total expenditure as the relative number of D firms is high compared the share of expenditure they capture (numerous small firms with relatively small sales per-capita). It is an implicit indicator of the average productivity difference between domestic and foreign producers, and it is negative under the assumptions on

the productivity distribution and the fixed entry costs that I have made in this paper. When the number of domestic firms $N_{D,t}$ increases slower than the number of Foreign exporters to the Home market $N_{X,t}^*$, then the aggregate price of consumption at Home decreases and the RER depreciates.

MODE OF ENTRY SWITCHING + $\left(S_I - \frac{N_I}{N_D + N_X + N_I}\right) \left[\left(\hat{N}_{I,t} - \hat{N}_{X,t}\right) - \left(\hat{N}_{I,t}^* - \hat{N}_{X,t}^*\right)\right]$. The factor $S_I - \frac{N_I}{N_D + N_X + N_I}$ is positive here, because the number of MNFs out of the total number of firms selling in the domestic economy is smaller than their market share, due to their relatively higher productivity that maps onto lower prices and larger per-capita sales. For lower W_t/Z_t MNFs entry into home increases, because of lower effective labor cost, the number of Home exporters increase and importers decrease. At the same time, Home affiliates of Foreign MNFs increase and overseas branches of Home MNFs decrease, so that the net effect depends on whether $\hat{N}_{I,t} + \hat{N}_{X,t}^* - \hat{N}_{X,t} - \hat{N}_{I,t}^*$ is positive or negative. If the sum of these log deviations is positive, then q_t increases and the Home RER depreciates.

Therefore, the terms that I have labeled as endogenous tradeability, expenditure switching and mode of entry switching can have positive or negative impacts on the RER depending on the relative shares of D, X and I firms in the population of firms selling in each country (extensive margin) and the of average productivity and prices they charge (composition margin) on the aggregate price indexes. This is a theoretical result that would suggest the possibility of an anti-HBS effect, if the sum of the ambiguous contributions to the appreciation or depreciation of the RER exchange rate managed to counterbalance the effect of the non-ambiguous effect of relative labor costs and endogenous MNFs entry. In the simulations I have carried out, the existence of MNFs reinforces the HBS effect, because of its substantial impact on the terms of labor, that dominates the determination of the RER in a traditional fashion. However, the analytical result suggests that empirical work trying to measure the HBS without controlling for the presence and entry of MNFs, might lead to biased inference.

3.3 Current Account and Terms of Trade

CURRENT ACCOUNT. From a national accounts perspective the Current Account can be decomposed in three broad aggregates: Net Exports (NX_t), Net Income Receipts, and Unilateral Transfers²⁰. Under the assumption of Financial Autarky and null unilateral transfers, the Current

²⁰According to the Balance of Payment manual of the IMF Net Income Receipts include interest income, distributed dividends and FDI earnings. Accordingly, there is an entry in the Current Account and an

Account corresponds to the sum of the Trade Balance and the net repatriation of profits from FDI operations, usually not featured in standard open macro models. Given the share of Foreign expenditure on the Home exports goods $N_{X,t}\tilde{\rho}_{X,t}^{1-\sigma}$ and the share of Home expenditure on Foreign goods $N_{X,t}^*\tilde{\rho}_{X,t}^{1-\sigma}$, the balanced Current Account measured in home currency can be written as

$$CA_t = Q_t N_{X,t}(\tilde{\rho}_{X,t})^{1-\sigma} C_t^* - N_{X,t}^*(\tilde{\rho}_{X,t}^*)^{1-\sigma} C_t + N_I \tilde{\pi}_{I,t} - Q_t N_I^* \tilde{\pi}_{I,t}^* + Q_t R_t^* B_t^F - R_t B_t^H$$

where the first two terms are the Trade Balance (Exports minus Imports), and the remaining terms are the Net Income Receipts (Net repatriation of MNFs profits and Net Interest Payment). Hence, in general equilibrium, the RER can be seen as adjusting the Current Account as follows:

$$Q_t = \frac{N_{X,t}^* \tilde{\rho}_{X,t}^{1-\sigma} C_t^* - N_I \tilde{\pi}_{I,t} + R_t B_t^H}{N_{X,t} \tilde{\rho}_{X,t}^{1-\sigma} C_t - N_I^* \tilde{\pi}_{I,t}^* + R_t^* B_t^F}$$

To have an idea of the relative magnitude of these quantities in the US Current Account for 2005 consider that the difference between Exports and Imports (USD 1,275 billion and USD 1.991 billion) generated a negative Trade Balance for 716 billion dollars, while the Net Repatriation of Profits was positive and equal to USD 134 billion, as receipts were USD 251 and payments were USD 117. Finally, receipts and payments on public and private bonds, where approximately USD 340 billion and USD 219 billion. See Table A3 in the Appendix for details.

TERMS OF TRADE (TOT). The terms of trade for a given country is the ratio of the price index for exported goods ($\tilde{p}_{X,t}$) to the price index for imported goods ($\tilde{p}_{X,t}^*$). As export prices arise endogenously, the terms of trade of the Home economy are also endogenously determined and can be expressed as

$$\widetilde{TOT}_t = \frac{e_t \tilde{p}_{X,t}}{\tilde{p}_{X,t}^*} \quad \text{or} \quad \widetilde{TOT}_t = \frac{\tilde{z}_{X,t}^* \tau_t}{\tilde{z}_{X,t} \tau_t^*} \frac{1}{TOL_t} \quad (20)$$

where average Home exports prices are compared to average Home imports prices in units of the Home currency²¹. One can think of these import and export price indices as trade-weighted indices of the prices of goods actually traded with that country's trading partners. Higher \widetilde{TOT}_t implies an improvement of home's average terms of trade, i.e. the price of exports increases relative to the price of imports. This can happen in the cases of a decrease of Home labor cost relative to Foreign ($TOL_t > 0$), an increase of trade costs faced by Home exporters, or a decrease of

offsetting entry in the Financial Account. Unilateral Transfers include net compensation of employees.

²¹ Also, $\widetilde{TOT}_t = Q_t \tilde{\rho}_{X,t} / \tilde{\rho}_{X,t}^*$. Notice that the price of imports needs to be defined in term of the Home currency, so that at the denominator we have $\tilde{p}_{X,t}^* / e_t$, and not $\tilde{p}_{X,t}^*$.

domestic productivity the last two events implying changes at the composition margin, with the new exporters being relatively less productive and thus charging relative higher prices.

NON-TRADED/TRADED PRICE RATIO (NTT). The Non-Traded to Traded Price ratio is the ratio of the average price of goods that are not traded in the Home economy - all varieties produced and sold domestically $N_{D,t}$ and the Home production of Foreign Varieties controlled by Foreign Multinationals - over the average exports price cleared of trade costs and expressed in units of the Home consumption good:

$$\widetilde{NTT}_t = \left(\frac{N_{D,t}}{N_{D,t} + N_{I,t}^*} \tilde{\rho}_{D,t} + \frac{N_{I,t}^*}{N_{D,t} + N_{I,t}^*} \tilde{\rho}_{I,t}^* \right) / \frac{Q_t}{\tau_t} \tilde{\rho}_{X,t}$$

AVERAGE PRODUCTIVITY OF HOME PRODUCERS. I define the average productivity in the Home economy as the weighted average of the productivity of domestic firms and Foreign MNFs in the Home economy.

$$\tilde{Z}_t = \frac{N_{D,t}}{N_{D,t} + N_{X,t} + N_{I,t}^*} \tilde{z}_{D,t} Z_t + \frac{N_{X,t}}{N_{D,t} + N_{X,t} + N_{I,t}^*} \tilde{z}_{X,t} Z + \frac{N_{I,t}^*}{N_{D,t} + N_{X,t} + N_{I,t}^*} \tilde{z}_{I,t}^* Z_t$$

The average productivity of Home firms $\tilde{z}_{D,t}$ is conditioned by the way the parameters of the Pareto distribution interact with the aggregate variables. Entry of MNFs increases the weight $N_{I,t}^* / (N_{D,t} + N_{I,t}^*)$ but also lowers the average productivity of Foreign affiliates located in Home, hence the joint influence of these two effects determines the impact of MNFs entry on average Home productivity. Hence, on top of the volume and value effects on exports and FDI sales, the composition of exporters and FDI firms plays a key role in affecting the dynamics of productivity growth in the destination country. Therefore, in a DSGE framework technology transfer within multinational networks becomes a potential channel of cross-country correlation of productivity changes. Because only firms that own superior technology engage in FDI, entry into the host economy is accompanied by a transfer of technology which alters the average productivity of the host economy, through changes at the extensive and the intensive margin of the FDI sector. At the intensive margin, when overseas branches increase production and sales, the average productivity of the host country increases because MNFs' affiliates have higher productivity than local competitors. At the extensive margin, however, the marginal impact of the new FDI entrants reduces the contribution of FDI to the growth rate of productivity of the host country because the late entrants have lower productivity compared to the existing foreign owned plants²². If entry barriers are lowered, then

²²Yeaple (2006) for microeconomic evidence.

even quite inefficient foreign companies can enter production, an event that can even lower Home productivity, compared to the situation in which very few extremely efficient MNFs operate in the country. This constitutes a potential explanation for the conflicting cross-country evidence on the contribution of MNFs to the growth of their host economies (see Carkovic and Levine, 2005 on this point).

4 System dynamics

4.1 Calibration

I adopt standard parameter choices in line with the literature. The parameter of relative risk aversion (γ) is 2, while the discount factor β is set at 0.99 in order to interpret periods as quarters. I assume the location parameter of the Pareto distribution (z_{min}) to be 1, as well as the endowment of workers, without loss of generalization.

Because the scale factor $\nabla = k/[k - (\sigma - 1)]$ has to be positive, I assume $k + 1 > \sigma$. Notice that in a Pareto distribution, the shape parameter (k) determines the slope of the p.d.f. that in this context mirrors the dispersion of the idiosyncratic productivity. Hence, by assuming that $k + 1 > \sigma$, I implicitly posit a positive relationship between k and the elasticity of substitution (σ) across varieties. With $\sigma = 3.8$, I obtain a constant (average) mark-up over marginal cost of 35.7 percent which is high for a standard monopolistic competition structure but not as high once one considers the existence of fixed costs to production. These costs, along with the trade cost τ , are set in order to replicate stylized facts for the US economy namely the ratio of exporters and FDI firms to producers.

The steady state levels of the variables are listed in Table 3. The ratio of new to existing varieties $\frac{N_E}{N_D}$ is calibrated using a death shock parameter $\delta = 0.025$, and is smaller than the ratio of job destruction for U.S. (10 percent), which is computed based on possibly multiproduct firms. The share of domestic and FDI producers over the total number of firms depends on the fixed cost of FDI (f_I) that I assume to be about 28 percent of the annualized fixed cost of entering production of a new variety $\Theta f_E = 0.036$, the per year fixed cost to export $f_X = 0.0045$ is approximately 10% of Θf_E . Even experimenting with different levels of fixed costs, the model is quite robust to changes in these parameters, except for the fact that the linear rational expectations solution is indeterminate for very low and very large fixed cost.

In steady state, the relative number of firms is as reported in Table 3. Therefore, Home sales of goods produced by domestic firms and foreign MNFs reach 66 percent of total varieties, while imported varieties account for 34 percent of the total number. Imports take up about 13 percent of total expenditure with the remaining 55 percent going to domestic producers and 32 percent going to Foreign MNFs located in the Home country. In order to compare the model with US data, consider that about 20 percent of US plants export, a lower bound considering that some of these plants are multiproduct.

The ranking of productivity averages depends on the cut-off points. The average firm in each economy has productivity 86 percent higher than the most inefficient non-exiter, but the average local firm (selling only domestically), is only 18 percent more efficient than the firm with z_{min} . The average exporter is about 40 percent more productive than the average local firm. As for FDI firms, the cut-off z_I makes the least productive MNF approximately 10 percent more productive than the average exporter in the economy, a figure that is in line with Helpman et al. (2004). Because there is a negative relationship between idiosyncratic prices and productivity, the average price rankings are exactly reversed, with relatively inefficient local producers charging relatively higher prices.

As for aggregate variables, consumption takes up approximately 90 percent of G.D.P. while investment to finance entry of new firms absorbs approximately 10 percent of the aggregate output (compared to 15-20 percent in O.E.C.D. data).

Finally, the share of expenditure of FDI firms (S_I) is higher than the relative number of FDI firms N_I/N_D , because MNFs have larger per-capita sales than local producers or importers.

4.2 Impulse Responses

Figures 7-11 focus on the case of financial autarky and represent the dynamic responses of the main variables of the economy to a one percent temporary cross-sectional productivity (Z) shock in the Home economy, and a negative one percent reduction of the fixed costs of entry f_E , of exporting f_X , and of producing overseas f_I . All shocks are common across firms and all variables referring to profits, prices and idiosyncratic productivity should be read as averages (with tilde).

A TEMPORARY AGGREGATE PRODUCTIVITY INCREASE IN HOME (FIGURE 5). On impact, Z_t increases by one percent, the shock disappearing after approximately 10 years with half-life of 1.5 years. As an immediate consequence, firms observe higher Home labor productivity and start entering different segments of business as follows. First, the number of new domestic producers

(N_E) jumps both because the increase of productivity reduces the fixed cost of entry (measured in units of domestic effective labor), and because the increase in present and future expenditure inflates the present discounted value of expected profits. Hence, the number of producers (N_D) begins to increase over time. Second, Foreign MNFs face a lower entry cost measured in units of *Home* effective labor ($w_t f_{I,t}/Z_t$) and a larger demand/expenditure due to the increase of income (y_t) and consumption (C_t) following the productivity shock. Therefore, the number of foreign controlled firms (N_I^*) in Home jumps on impact. Third, because Home exporters (N_X) pay fixed costs in units of *Home* effective labor ($w_t f_{X,t}/Z_t$) in order to be able to export to Foreign, their number jumps on impact. Fourth, Foreign exporters (N_X^*) now face higher Home expenditure, that expands their exports both at the extensive and the intensive margin.

Because each of these segments of business puts pressure on labor demand (remember that labor is used both for entry and production), excess labor demand triggers a wage spike in home and a more moderate wage increase in Foreign, whose overall effect is to worsen the terms of labor ($TOL_t \equiv e_t W_t^* Z_t / Z_t^* W_t$). The increase of the foreign wages reduces profit opportunities for home MNFs (N_I) that in turn cause exit at the extensive margin of Home FDI.

The average idiosyncratic productivity of different segments of business move in opposite directions, as improved aggregate productivity increases the profit opportunities for firms that were staying out, under the previous productivity level. However, despite the fact that the cut-off levels z_{\min} , z_X , z_X^* , and z_{ff}^* fall, average idiosyncratic productivity of Home and Foreign exporters (\tilde{z}_X and \tilde{z}_X^*) can fall or rise depending on the parametrization of the model.

Looking at aggregate variables, the productivity shock causes a growth of Home consumption and income that increases imports (M). In Foreign, consumption first drops because part of the output is used to finance entry of Foreign Exporters but immediately turns positive as export growth pushes income up and profits from foreign operations increase. Foreign Exports to Home increase (M) more than imports (X) so the trade balance (TB) turns positive (negative for Home). The RER Q_t - based on the welfare base price indexes (P_t and P_t^*) and q_t - based on the CPI (\tilde{P}_t and \tilde{P}_t^*) move together, mostly driven by the entry of new producers. New domestic and FDI firms are relatively less efficient than existing firms, and accordingly charge relatively higher prices. Hence, the price index in Home jumps triggering a reduction of both q_t and Q_t , i.e. an appreciation for Home. The average terms of trade (TOT_t) respond to the entry of relatively inefficient Home producers and increase on impact, as well as the Non-Traded to Traded price ratio (NTT_t) that is

mostly affected by the entry of Foreign MNFs in Home.

Finally, the sixth graph of Figure 5 depicts the dynamics of the average productivity in the Home economy. The exogenous productivity shock has positive repercussions for the average productivity of domestic producers as long as the average productivity of FDI entrants in Home is higher than the average productivity of the firms established in the Home economy prior to the productivity shock.

In the long run, the shock has no long-lasting effects. However, the transitional path is deeply affected by the dynamics of the labor market, in particular the adjustment of wages to post-impact entry. The increase in Home wages is larger than the productivity shock and this worsens the Terms of Labor. Hence, despite the immediate entry of a large number of firms, some of these firms soon start exiting because of the labor costs. This is particularly evident looking at the plot for the number of exporters and domestic firms, and Foreign MNFs in Home. The average productivity levels by types of business, average prices and relative ratios (q_t , Q_t and TOT_t) all move, accordingly bringing the system back to the steady state equilibrium.

A PERMANENT AGGREGATE PRODUCTIVITY INCREASE IN HOME (FIGURE 6). In this case, there is a permanent productivity shock which is common to all firms in the economy (Z_t increases by one percent permanently) and the Home market becomes a relatively more attractive business environment.

The impact on the system of such a shock is similar to the transitory shock case, although here the adjustment and long term effects are substantially different as the economy now moves to a new steady state where productivity is permanently higher. In the long run, Home enjoys higher consumption and income, while Foreign suffers from a short-term drop of C^* , necessary to finance the new exporters, followed by a recovery and an adjustment to a higher level. The transmission of the positive shock from Home to Foreign occurs through higher Foreign export sales, revenues and profits on top of the higher repatriation of profits of Foreign MNFs, all of which accrue to y^* . The dynamics of entry and exit of firms is substantially conditioned by the labor market dynamics.

A SYMMETRIC JOINT REDUCTION OF f_E AND f_E^* , f_X AND f_X^* , f_I AND f_I (FIGURE 7, 8 AND 9). A symmetric reduction of the fixed cost of exporting induces initial entry into the export market for about 2 or 3 years. Because of higher expected profits from exporting, the number of domestic firms increases, which affects the dynamic of wages inducing some wage growth, that puts cost pressure on exporters inducing some exit. Output and consumption also grow in the long run and

this attracts new foreign MNFs later discouraged by higher wages so the initial entry reverts to some exit, alleviating the wage pressure and increasing the average productivity of remaining firms over time.

A symmetric reduction of the fixed cost of FDI induces initial entry into the Home market, so that output and consumption increase on impact, along with wages. The dynamics of the labor market induce domestic firms to exit, a phenomenon that reduces wage growth as well as consumption and output growth. In the long run, the number of Foreign MNFs in Home is higher, but the dynamics of domestic production imply lower consumption and income growth. In this context and under this parametrization, entry of MNFs had detrimental effects on both economies.

5 Properties of the theoretical economy

In this section I examine the second moment properties of the model, in order to discuss the relevance of FDI for business cycle dynamics. I look at the dynamics of the model once it is affected solely by exogenous productivity shocks (\hat{Z}_t, \hat{Z}_t^*) that are common across firms within a country, hence aggregate. The model is simulated for the case of incomplete asset markets as I want to compare this theoretical economy with the relevant literature, in particular with Backus, Kehoe and Kydland (1992, BKK92 henceforth) and GM05. I assume the data generating process originally proposed by BKK92 and used in GM05, with the following stochastic structure:

$$\begin{bmatrix} \hat{Z}_t \\ \hat{Z}_t^* \end{bmatrix} = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix} \begin{bmatrix} \hat{Z}_{t-1} \\ \hat{Z}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_t \\ \xi_t^* \end{bmatrix}$$

where innovations have variance $\sigma_\xi^2 = \sigma_{\xi^*}^2 = 0.0073$ and covariance $cov_{\xi^*\xi} = cov_{\xi\xi^*} = 0.0019$. I shock the Linear Rational Expectations solution of the system with these technological innovations, compute the log-deviations from the steady state, HP-filter (smoothing parameter 1600), iterate and finally calculate the second moments of the endogenous variables. The results reported in the next three tables can be interpreted as percentage terms of the steady state level of each of the variables they refer to, and measured as a ratio of the second moment of output.

Table 4 reports second moments of the main aggregate variables for the US economy from BKK92, and generated moments from GM05, Contessi (2006), and the model with both export and FDI developed in this paper. The moments are reported both as percentages of the steady

state level of the relevant variable and as ratios of the standard deviation of output.

In most of the tables I report moments of the variables as derived in the previous papers and of the variables deflated using the transformation discussed in Feenstra (1994), and Broda and Weinstein (2006) to map welfare-based price indexes of in models of monopolistic competition into price indexes that are closer to the CPI. For example deflated consumption (C_R) and output (y_R) are equal to $C_t P_t / \tilde{P}_t$ and $y_t P_t / \tilde{P}_t$ where P_t and \tilde{P}_t are defined as in the appendix.

Compared to the existing literature, FDI in a model with heterogeneous firms substantially alters the magnitude of the second moments of these variables and several elements deserve attention. The introduction of FDI increases the volatility of both output and consumption compared to standard models and even GM05. One can think of FDI as a device to add a non-tradable component to G.D.P. that increases its volatility, in a fashion similar to purely non-tradable goods in McIntyre (2004). The volatility of consumption in percentage terms or as a ratio of the standard deviation of G.D.P. is also between the two comparison models. Investment in new firms ($\tilde{v}_{R,E} N_E$) is extremely volatile, somewhat disproportionately compared to data, and indeed higher than in the models with solely trade or FDI. I treat the stock of existing producers N_D as the best available proxy for the capital stock, its standard deviation being substantially in line with data (0.63), and definitely higher than the comparison models. The model is somewhat disappointing as regards, the volatility of the trade balance both in absolute terms and as a ratio of G.D.P., as both measures turn out to be much higher than in the data.

However, the model with export and FDI is able to generate an extremely high volatility of the Real Exchange Rate, both measures using welfare based price indexes and CPI, and this constitutes a definite success compared to standard models of business cycle and even compared to GM05. Once computed using CPI-like price indexes the volatility of the RER is very close to the figures reported in CKM03 for US data.

Table 5, reports autocorrelation of the models with trade only and with both trade and FDI. The extended model generates autocorrelations that are somewhat higher than the one in the data and in the GM05

Cross- and within-country correlations in Table 6 are noteworthy. Besides maintaining positive cross-country consumption and output correlations, the model with both export and FDI is able to replicate the *ranking* of their contemporaneous correlations, that emerges from the data but that traditional international business cycle models cannot account for, a problem from which the

GM05 model is not exempt. Introducing horizontal FDI under Financial Autarky allows me to generate a cross-country correlation of consumption that is slightly higher than 50% of the cross country correlation of output between the two countries. Moreover, the model generates a negative correlation between the relative consumption ratio and the RER, both in their theoretical (C_t/C_t^* , Q_t) and their deflated ($C_{R,t}/C_{R,t}^*$, q_t) definition. The latter emerges in the GM05 simulation as well, but not in the model with only FDI and has a magnitude that is fairly close to BKK02. The model however generates too high correlations between investment and saving rates ($1 - C_{R,t}/y_{R,t}$, $\tilde{v}_{ER,t}/y_{R,t}$) as well as a somewhat too persistent RER.

6 Conclusions

Drawing from microeconomic studies that establish a relationship between firm level heterogeneity of productivity and global engagement status, I have proposed the first open economy macroeconomic model that endogenizes *both* FDI *and* export as possible modes of the internationalization of production. In this model, all goods can be potentially traded or produced by MNFs affiliates, but only a subset of firms enter foreign markets, based on the interaction between foreign demand, relative aggregate productivity and labor costs on the one hand, and trade and investment variables - chiefly the fixed cost of exporting, the variable trade cost, and the fixed cost of producing abroad - on the other hand. This paper suggests that introducing MNFs entry and sales in an open macroeconomic setting significantly improves our understanding of the international transmission of shocks and aggregate volatility along two main dimensions.

First of all, I show that the consumption-output anomaly in open economy macroeconomics disappears when I introduce MNFs along with exporters in the two-country model. By locating production in different destination markets, MNFs diversify production risks stemming from aggregate volatility and local market conditions, in a way that weakens the transmission of shocks across countries but increases idiosyncratic output volatility. Admittedly, this is likely to be only one of the elements able to deliver a ranking of consumption-output correlations consistent with the data, but my results explicitly indicate that geographic segmentation of markets must play a role.

Second, I show how the pricing policies of MNFs can affect price indexes, increase the volatility of the Real Exchange Rate and possibly alter the direction of the Harrod-Balassa-Samuelson effect

when exporters are more productive than local producers, but less productive than MNFs. I find that postulating the existence of MNFs leads to a simulated volatility of RER exchange rate that is far closer to the data than in any of the previous open economy models, a result that depends on the ability of MNFs to price-discriminate across countries and relates to their role in increasing the weight of the non-traded component of the aggregate price indexes.

Moreover, a corollary to the analytical results of the model suggests that the transfer of technology of MNFs into host countries contributes to aggregate productivity growth over time at a rate that is decreasing in the number of foreign firms already located the host country. The interaction of fixed costs, demand and labor conditions induces MNFs to sort according to their idiosyncratic productivity, so that late entrants tend to have lower productivity and to be less beneficial to the host economy than the group of early foreign investors. I argue that this result offers a partial solution to the conflicting evidence on the contribution of MNFs to the growth of their host economies and raises doubts about the economic rationale for offering special incentives to attract FDI.

This paper provides an early foray into understanding the dynamics of exports and horizontal FDI in a DSGE setting and suggest a number of directions that are worth future research effort. The trade component neglects the role of technological change (other than exogenous productivity innovation) overlooking the possibility that the distribution of productivity could change in response to learning by producing and exporting, or as the outcome of innovation through research and development (for a contribution in this direction see Atkinson and Burstein, 2006). The macroeconomic component of model would benefit from introducing money and some form of price rigidity in the two-country World (see Russ, 2006 and Lubik and Russ, 2006, for early steps in this direction) as the way monetary policy innovations are transmitted internationally is important for Central Banks and policy makers and there is currently no understanding of the way the existence of MNFs might affect the optimal conduct of monetary policy.

Finally, an extension of the model presented here to the case of a vertically disintegrated production structure would allow to study the North-South transmission of business cycles through vertical fragmentation (Burstein et al. 2006) that appears to be a worthwhile research effort given the spectacular growth of FDI in developing and emerging countries of the early 2000s (UNCTAD, 2006) and recent evidence on the volatility of output generated by US outsourcing in Mexico (Bergin et al., 2006).

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FIGURE 1. SALES BY FOREIGN AFFILIATES AND EXPORTS OF GOODS AND NON-FACTOR SERVICES WORLDWIDE 1982-2003 (BILLIONS USD)

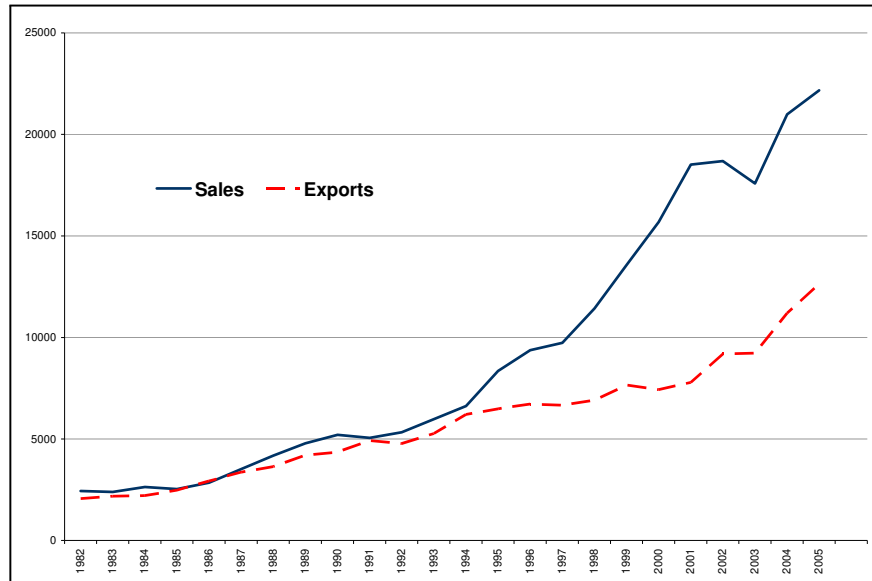


FIGURE 2. WORLD FDI FLOWS, EXPORT AND G.D.P.

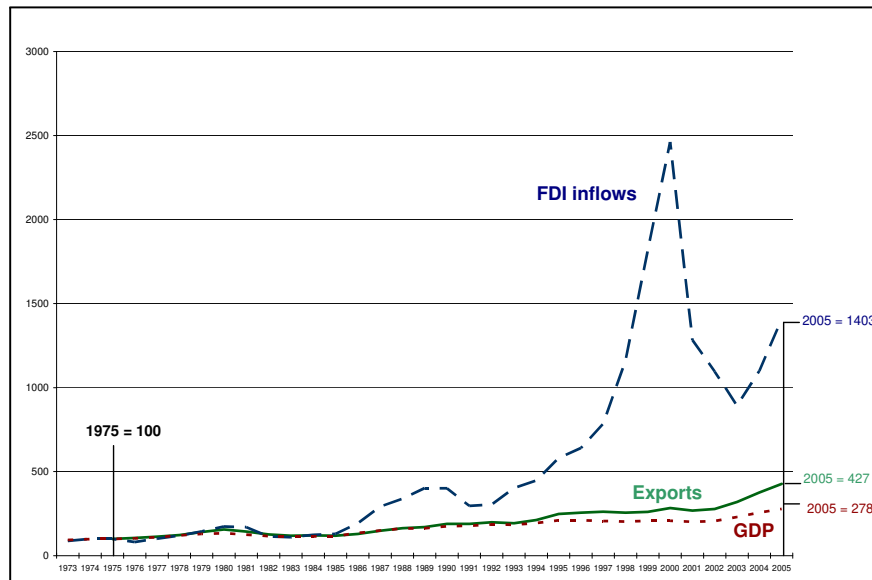


FIGURE 3. TIMING OF ENTRY

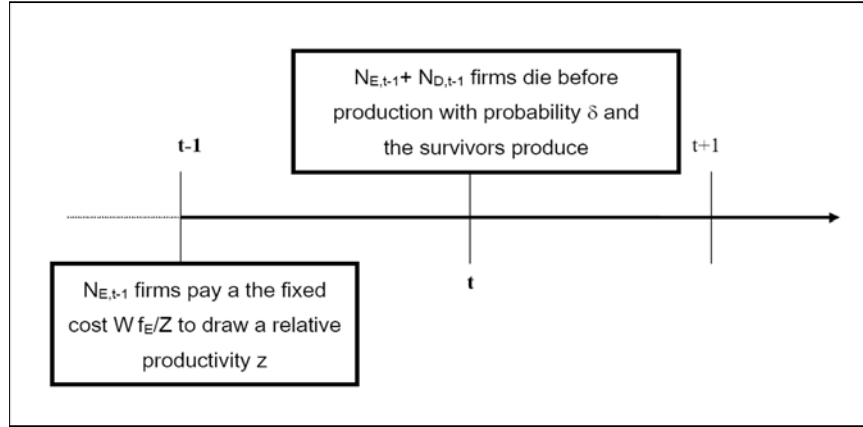


FIGURE 4. PRODUCTIVITY DISTRIBUTION AND ENTRY MODE

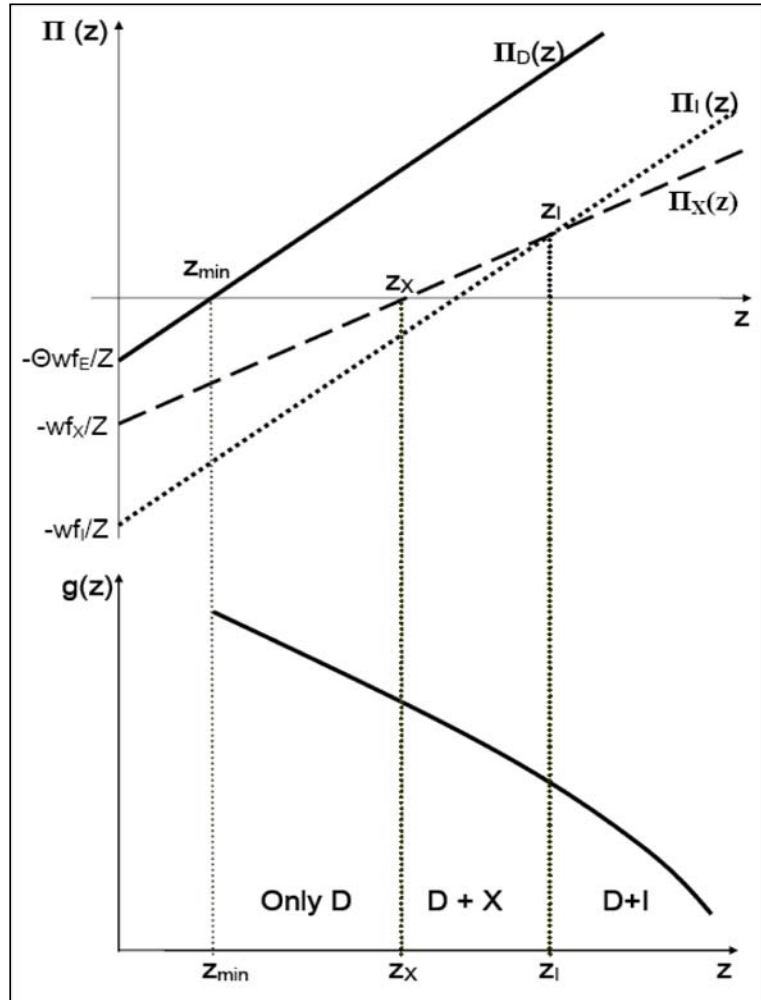


FIGURE 5 - TEMPORARY AGGREGATE PRODUCTIVITY INCREASE IN HOME
(FINANCIAL AUTARKY)

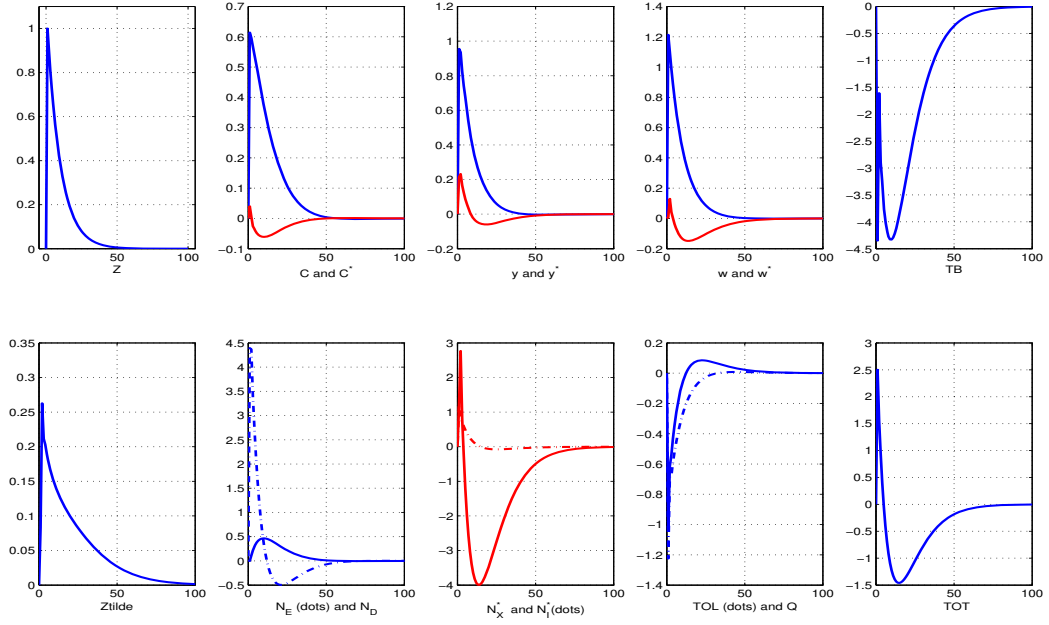


FIGURE 6. A PERMANENT AGGREGATE PRODUCTIVITY INCREASE IN HOME
(FINANCIAL AUTARKY)

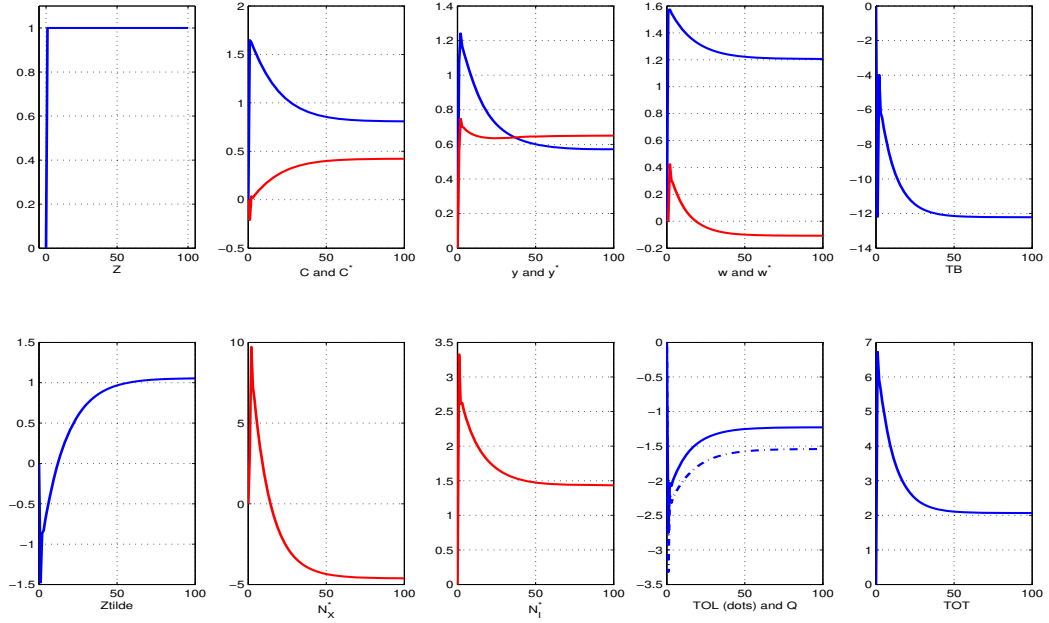


FIGURE 7. A PERMANENT REDUCTION OF f_E IN HOME (FINANCIAL AUTARKY)

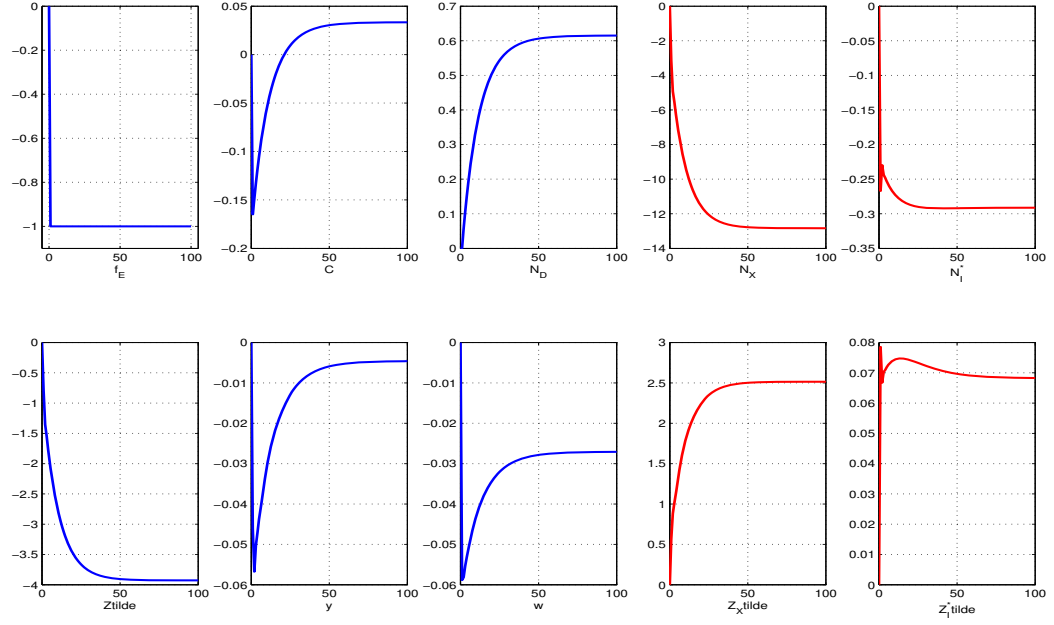


FIGURE 8. A PERMANENT REDUCTION OF f_X IN HOME AND f_X^* IN FOREIGN (BOND TRADING)

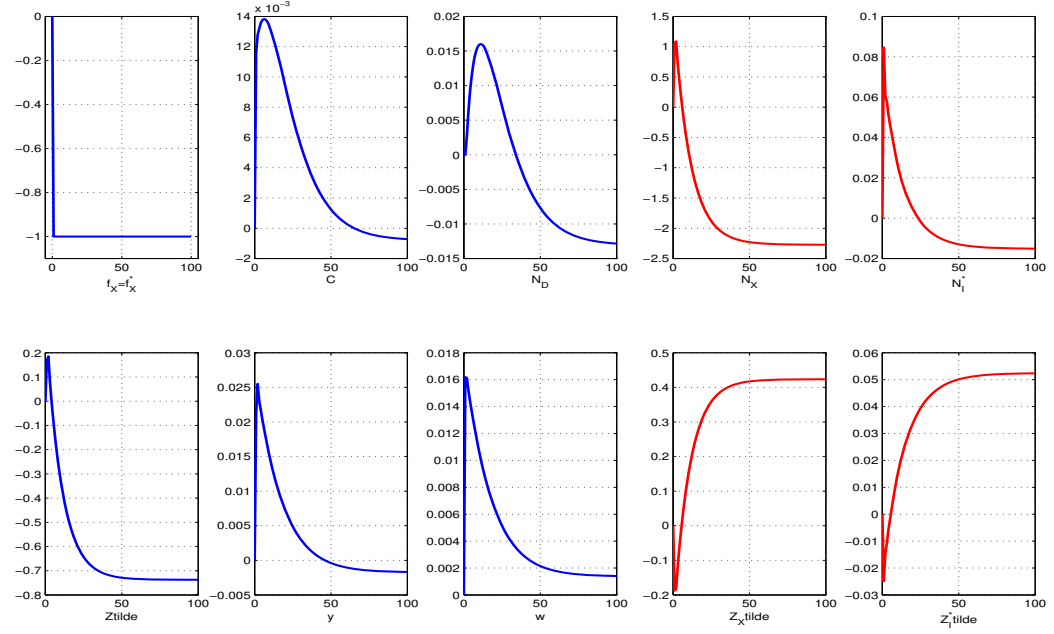


FIGURE 9. A PERMANENT REDUCTION OF f_I IN HOME AND f_I^* IN FOREIGN (BOND TRADING)

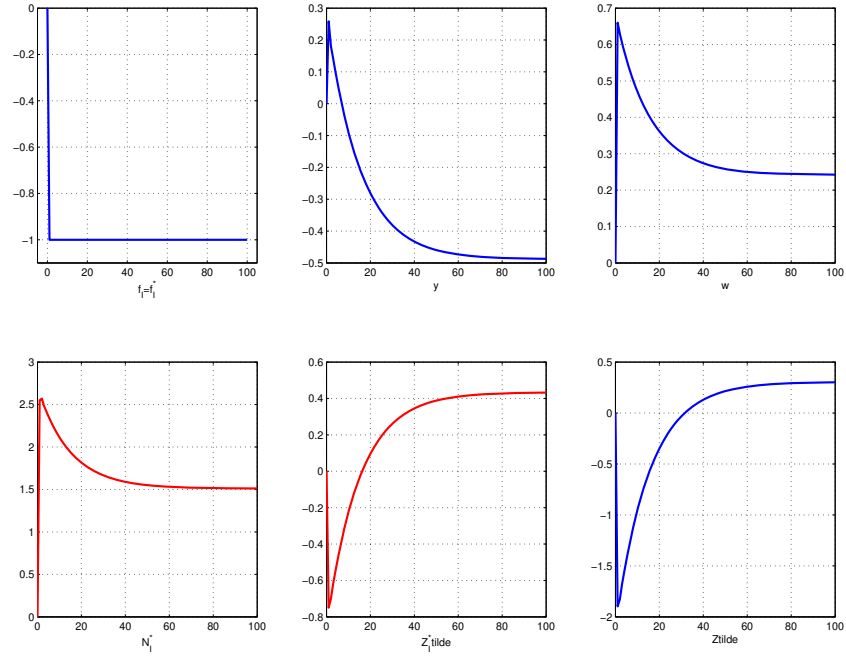


TABLE 1.

Value at current prices (billions of USD) and percentages				
	1982	1990	1996	2005
World G.D.P.	10,899	21,898	29,024	44,674
World sales of foreign affiliates of MNFs	2,620	6,045	9,372	22,171
As a % of world G.D.P.	24.1	27.1	32.3	49.6
World exports of goods	2,247	4,261	6,523	12,641
As a % of world G.D.P.	20.6 %	19.5 %	22.5 %	28.3 %
FDI stock	647	1,789	3,238	10,130
As a % of world G.D.P.	5.9 %	8.2 %	11.2 %	22.7 %
Total Assets of Foreign Affiliates	2,108	5,956	n.a.	45,694
As a % of world G.D.P.	19.3 %	27.2 %	n.a.	102.3 %

Source: UNCTAD, World Investment Report (Various issues)

TABLE 2. US CURRENT ACCOUNT 2005 (MILLIONS USD)

	EXPORTS	IMPORTS	NET
GOODS AND SERVICES	1,275,245	-1,991,975	-716,730
	RECEIPTS	PAYMENTS	NET
DIRECT INVESTMENT	251,370	-116,953	-134,417
OTHER PRIVATE	217,637	-223,612	-5,975
U.S. GOVERNMENT	2,715	-113,559	-110,844
OTHER	5,640	-122,788	-117,148
TOTAL	1,749,892	-2,455,328	-705,436

TABLE 3. STEADY STATE LEVELS OF THE MAIN VARIABLES

STEADY STATE VALUES	INTERPRETATION
$N_E/N_D = 0.026$	New entrants to producers ratio
$N_D/N_{DI} = 0.76$	Home firms as a share of Home producers
$N_D^*/N_H = 0.53$	Home firms as a share of firms selling in Home
$N_X^*/N_H = 0.34$	Importers as a share of firms selling in Home
$N_I^*/N_H = 0.13$	Foreign affiliates a share of firms selling in Home
$N_{DO}/N_D = 0.19$	Local firms as a share of Home firms
$N_I/N_D = 0.31$	MNFs as a share of Home firms
$\tilde{z}_D = 1.86$	Average Productivity of Home producers
$\tilde{z}_{DO} = 1.20$	Average Productivity of local producers
$\tilde{z}_X = 1.59$	Average Productivity of Exporters
$\tilde{z}_I = 2.19$	Average Productivity of Home MNFs
$CONS. / G.D.P = 91\%$	Aggregate Consumption / G.D.P.
$\tilde{v}_{N_E} / G.D.P = 9\%$	Aggregate "Entry" Investment / G.D.P.
$\tilde{\pi} N_D / G.D.P = 24\%$	Dividends / G.D.P.
$S_D = 52\%$	share of domestic sales in Home expenditure
$S_X = 14\%$	share of imports in Home expenditure
$S_I = 34\%$	share of FDI sales in Home expenditure

Appendix

A Firms

A.1 Profit maximization and first order conditions for the firms

1. DOMESTIC PRODUCTION (D). Domestic producers - that who do not incur in fixed cost of production - choose prices by maximizing their following profit function

$$\Pi_{D,t}(\omega) = p_t(\omega)y_t(\omega) - W_t l_{D,t}(\omega) = p_t(\omega)y_t(\omega) - \frac{W_t}{z Z_t} y_t(\omega)$$

where W_t is the wage rate, $l_t(z)$ represents labor use and $y_t(\omega)$ is the level of output. The first order conditions imply that optimal prices and real profits are $\rho_{D,t}(\omega) \equiv \frac{p_{D,t}(\omega)}{P_t} = \frac{w_t}{Z_t z} \mu$ and $\pi_{D,t}(\omega) \equiv \frac{\Pi_{D,t}(\omega)}{P_t} = \frac{1}{\sigma} \rho_{D,t}^{1-\sigma}(\omega) C_t$ where the mark-up $\mu = \frac{\sigma}{\sigma-1}$ and the real wages are $w_t = W_t/P_t$.

2. EXPORT PRODUCTION (X). Exporters maximize the following objective function

$$\Pi_{X,t}(\omega) = e_t p_t(\omega)y_t(\omega) - \tau_t W_t l_t(\omega) - \frac{W_t f_{X,t}}{Z_t}$$

that has first order condition such that $p_{X,t}(\omega) = \mu \frac{\tau_t W_t}{e_t z Z_t}$. Using the definition of $RER_t = e_t P_t^*/P_t$ and the real wage, and dividing by the price index of the market of *destination*, I can derive $\rho_{X,t}(\omega) \equiv \frac{p_{X,t}(\omega)}{P_t^*} = \frac{w_t}{Z_t z} \frac{\tau_t}{Q_t} \mu$, and the optimal profits, in terms of the consumption price index at the *location* of the firm, equal to $\pi_{X,t}(\omega) = \frac{Q_t}{\sigma} \rho_{X,t}^{1-\sigma}(\omega) C_t^* - \frac{w_t f_{X,t}}{Z_t}$.

3. FDI PRODUCTION (I). Producers that engage in FDI maximize the profits from foreign sales expressed as

$$\Pi_{I,t}(z; \omega) = e_t \left[p_t(\omega)y_t^*(\omega) - W_t^* l(\omega) - \frac{W_t^* f_{I,t}^*}{Z_t^*} \right]$$

which implies optimal prices $\rho_{I,t}(\omega) \equiv \frac{p_{I,t}(\omega)}{P_t^*} = \frac{w_t^*}{Z_t^* z} \mu$, and optimal profits from FDI - relative to the price index of the market of *location* of the mother firm - equal to $\pi_{I,t}(\omega) = \frac{Q_t}{\sigma} \rho_{I,t}^{1-\sigma}(\omega) C_t^* - Q_t \frac{w_t^* f_{I,t}^*}{Z_t^*}$.

A.2 Parametrization

Following Helpman, Melitz and Yeaple (2004), I assume that productivity z has a Pareto distribution with lower bound z_{\min} and shape parameter $k > \sigma - 1$, i.e. p.d.f. and c.d.f. of z are $g(z) = k z_{\min}^k / z^{k+1}$ and $G(z) = 1 - (z_{\min}/z)^k$. Using this distribution and replacing the non-parametrized distribution with the Pareto, and defining $\nabla \equiv k/[k - (\sigma - 1)]$, I can obtain a simple explicit solution for the weighted average of the idiosyncratic productivity of firms Domestic Firms (D), Domestic Exporters (X), Home MNFs (I) and Home local firms (DO), by integrating equations (13):

$$\begin{aligned} \tilde{z}_{D,t} &= \nabla^{\frac{1}{\sigma-1}} z_{\min} & \tilde{z}_{X,t} &= \nabla^{\frac{1}{\sigma-1}} \left[\frac{z_{X,t}^{\sigma-1} z_{I,t}^k - z_{X,t}^k z_{I,t}^{\sigma-1}}{z_{I,t}^k - z_{X,t}^k} \right]^{\frac{1}{\sigma-1}} \\ \tilde{z}_{I,t} &= \nabla^{\frac{1}{\sigma-1}} z_{I,t} & \tilde{z}_{DO,t} &= \nabla^{\frac{1}{\sigma-1}} \left[\frac{z_{I,t}^k z_{\min}^{\sigma-1} - z_{I,t}^{\sigma-1} z_{\min}^k}{(z_{I,t} z_{\min})^k} \right]^{\frac{1}{\sigma-1}}. \end{aligned}$$

A.3 Relative number of firms and cut-off points

From $\tilde{z}_{I,t}$, we know that $z_{I,t} = \nabla^{-\frac{1}{\sigma-1}} \tilde{z}_{I,t}$ which allows to simplify the relative number of firms

$$\frac{N_{I,t}}{N_{D,t}} = \frac{[1 - G(z_{I,t})] N_{D,t}}{N_{D,t}} = \left(\frac{z_{\min}}{z_{I,t}} \right)^k = \left(\frac{z_{\min}}{\tilde{z}_{I,t}} \right)^k \nabla^{\frac{k}{\sigma-1}} \quad (21)$$

$$\begin{aligned} \frac{N_{I,t}^*}{N_{D,t}} &= \frac{[1 - G(z_{I,t}^*)] N_{D,t}^*}{N_{D,t}} = \left(\frac{z_{\min}}{z_{I,t}^*} \right)^k \frac{N_{D,t}^*}{N_{D,t}} = \left(\frac{z_{\min}}{\tilde{z}_{I,t}^* \nabla^{\frac{1}{1-\sigma}}} \right)^k \frac{N_{D,t}^*}{N_{D,t}} \\ \frac{N_{X,t}}{N_{I,t}} &= \frac{z_{I,t}^k}{z_{X,t}^k} - 1 \end{aligned} \quad (22)$$

Cut-off points $z_{X,t}$ and $z_{I,t}$ are determined as a zero-net-profit condition for the firm that makes exactly enough operating profit to cover the fixed cost of entry into the export segment and the FDI mode.

MARGINAL FDI FIRM. For the FDI business, $z_{I,t} : \pi_{I,t}(\omega) = 0 \iff \frac{Q_t}{\sigma} [\rho_{I,t}(\omega)]^{1-\sigma} C_t^* = Q_t \frac{w_t^* f_{I,t}^*}{Z_t^*}$. Thus,

$$z_{I,t} = \left(\frac{f_{I,t}^*}{C_t^*} \right)^{\frac{1}{\sigma-1}} \left(\frac{w_t^* \sigma}{Z_t^*} \right)^{\frac{\sigma}{\sigma-1}} \left(\frac{1}{\sigma-1} \right) \quad (23)$$

Using the cut-off point for FDI firms, I can determine average profits of the FDI segment $\tilde{\pi}_{I,t}$ using the definition of $\tilde{z}_{I,t}$,

$$\tilde{\pi}_{I,t} = Q_t (\nabla - 1) \frac{w_t^* f_{I,t}^*}{Z_t^*} \quad (24)$$

MARGINAL EXPORTER. The marginal exporter has a productivity level such that her profits are zero. Hence, $z_{X,t}$ such that $\pi_{X,t}() = 0 \iff \frac{Q_t}{\sigma} [\rho_{X,t}(\omega)]^{1-\sigma} C_t^* = \frac{w_t f_{X,t}}{Z_t}$, is

$$z_{X,t} = \left(\frac{f_{X,t}}{C_t^*} \right)^{\frac{1}{\sigma-1}} \left(\frac{w_t}{Q_t Z_t} \right)^{\mu} \mu \tau_t \quad (25)$$

Using the cut-off point for FDI firms, I can plug $z_{X,t}$ and $z_{I,t}$ into $\tilde{z}_{X,t}$ and derive $\tilde{\pi}_{X,t}$. The average productivity of exporters is then derived from

$$\tilde{z}_{X,t}^{\sigma-1} = \nabla \frac{(\sigma)^\sigma}{(\sigma-1)^{\sigma-1} C_t^{*1}} \frac{\left(\frac{w_t}{Q_t Z_t} \right)^\sigma \tau_t^{\sigma-1} f_{X,t}^{1+\frac{k}{1-\sigma}} - TOL_t^{\frac{\sigma k}{1-\sigma}} \left(\frac{w_t^*}{Z_t^*} \right)^\sigma \tau_t^k f_{I,t}^{*1+\frac{k}{1-\sigma}}}{f_{X,t}^{\frac{k}{1-\sigma}} - TOL_t^{\frac{\sigma k}{1-\sigma}} \tau_t^k f_{I,t}^{*\frac{k}{1-\sigma}}}$$

that implies average profits

$$\tilde{\pi}_{X,t} = \left[\nabla \frac{\Lambda^k - \Lambda^{\sigma-1} (TOL_t^\sigma)^{\frac{1-\sigma+k}{1-\sigma}}}{\Lambda^k - (TOL_t^\sigma)^{\frac{1-\sigma+k}{1-\sigma}}} - 1 \right] w_t f_{X,t}; \quad \Lambda \equiv \left[\left(\frac{f_X}{f_I} \right)^{\frac{1}{\sigma-1}} \tau \right]^{-1} \quad (26)$$

Moreover, the following relationships can be derived using the cut-off points and the definition of the terms of labor $TOL_t \equiv \frac{Q_t Z_t w_t^*}{Z_t^* w_t}$,

$$\frac{z_{X,t}}{z_{I,t}} = \frac{1}{\Lambda TOL_t^\mu}; \quad \text{or} \quad \left(\frac{\tilde{z}_{X,t}}{\tilde{z}_{I,t}} \right)^{\sigma-1} = \frac{(\Lambda TOL_t^\mu)^{k-(\sigma-1)} - 1}{(\Lambda TOL_t^\mu)^k - 1} \quad (27)$$

and this allows me to rewrite the relative number of exporters and FDI firms as a function of the productivity cut-off, hence relative effective labor cost, fixed and trade costs: $N_{X,t}/N_{I,t} = (\Delta TOL_t^\mu)^k - 1$.

B Real Exchange Rate

The welfare based price indexes (P_t and P_t^*) can be used to define the welfare-based RER Q_t and its CPI-based transformation q_t that discounts the variety effect and is conceptually closer to the way official statistics report the RER. Here, one has to pay attention to the fact that the way I aggregate prices into the price index is derived from theory and does not correspond directly to the price indexes available from official statistics. Following GM (2005), I also construct a CPI-based RER q_t , using the transformation suggested by Feenstra (1994) and Broda and Weinstein (2006). The welfare-based Real Exchange Rate is defined as $Q_t \equiv e_t P_t^*/P_t$ and $TOL_t \equiv \left(e_t \frac{W_t^*}{Z_t^*}\right) / \left(\frac{W_t}{Z_t}\right)$ are the Terms of Labor. Hence,

$$Q_t^{1-\sigma} \equiv \left(\frac{e_t P_t^*}{P_t}\right)^{1-\sigma} = \frac{\varepsilon^{1-\sigma} \left[N_{D,t} P_D(\tilde{z}_D)^{1-\sigma} + N_{X,t}^* P_X^*(\tilde{z}_X^*)^{1-\sigma} + N_{I,t}^* P_I^*(\tilde{z}_I^*)^{1-\sigma} \right]}{N_{D,t}^* P_D(\tilde{z}_D^*)^{1-\sigma} + N_{X,t} P_X(\tilde{z}_X)^{1-\sigma} + N_{I,t} P_I(\tilde{z}_I)^{1-\sigma}}$$

By dividing both the numerator and the denominator by $\tilde{p}_{D,t}$, the RER can be expanded to

$$Q_t^{1-\sigma} \equiv \left(\frac{e_t P_t^*}{P_t}\right)^{1-\sigma} = \left(\frac{\frac{e_t P_t^*}{\tilde{p}_{D,t}}}{\frac{P_t}{\tilde{p}_{D,t}}}\right)^{1-\sigma}$$

and

$$\begin{aligned} \left(\frac{e_t P_t^*}{\tilde{p}_{D,t}}\right)^{1-\sigma} &= N_{D,t}^* \left(\frac{\tilde{z}_{D,t}}{\tilde{z}_{D,t}^*} TOL_t\right)^{1-\sigma} + N_{X,t} \left(\tau_t \frac{\tilde{z}_{D,t}}{\tilde{z}_{X,t}}\right)^{1-\sigma} + N_{I,t} \left(TOL_t \frac{\tilde{z}_{D,t}}{\tilde{z}_{I,t}}\right)^{1-\sigma} \\ \left(\frac{P_t}{\tilde{p}_{D,t}}\right)^{1-\sigma} &= N_{D,t} + N_{X,t}^* \left(\tau_t^* \frac{\tilde{z}_{D,t}}{\tilde{z}_{X,t}^*} TOL_t\right)^{1-\sigma} + N_{I,t}^* \left(\frac{\tilde{z}_{D,t}}{\tilde{z}_{I,t}^*}\right)^{1-\sigma} \end{aligned}$$

imply that

$$Q_t^{1-\sigma} = \frac{N_{D,t}^* \left(TOL_t \frac{\tilde{z}_{D,t}}{\tilde{z}_{D,t}^*}\right)^{1-\sigma} + N_{X,t} \left(\tau_t \frac{\tilde{z}_{D,t}}{\tilde{z}_{X,t}}\right)^{1-\sigma} + N_{I,t} \left(TOL_t \frac{\tilde{z}_{D,t}}{\tilde{z}_{I,t}}\right)^{1-\sigma}}{N_{D,t} + N_{X,t}^* \left(TOL_t \frac{\tau_t^* \tilde{z}_{D,t}}{\tilde{z}_{X,t}^*}\right)^{1-\sigma} + N_{I,t}^* \left(\frac{\tilde{z}_{D,t}}{\tilde{z}_{I,t}^*}\right)^{1-\sigma}} \quad (28)$$

If home effective labor appreciates ($\frac{W_t}{Z_t} \uparrow$) then the terms of labor decreases ($TOL_t \downarrow$), and the Home economy as a whole becomes a less attractive location. The way one should think about the terms of labor is that if $TOL_t > 1$ a firm with a given level of idiosyncratic productivity z can produce at lower cost in the Home country than in the Foreign.

Q_t is determined by welfare base price indexes P_t and P_t^* . If I denote CPI price indexes as \tilde{P}_t and \tilde{P}_t^* I use transformation the welfare based price indexes used so far according to $P_t = N_t^{\frac{1}{1-\sigma}} \tilde{P}_t$ and $P_t^* = N_t^{*\frac{1}{1-\sigma}} \tilde{P}_t^*$ I can define a measure of RER that resembles closer the CPI (Broda and

Weinstein, 2005, and references therein). Hence, \tilde{P}_t and \tilde{P}_t^* are thought as average nominal prices for the varieties sold. Accordingly, the CPI-based RER can be derived from

$$q_t^{1-\sigma} = \frac{N_{D,t}^* \left(TOL_t \frac{\tilde{z}_D}{\tilde{z}_D^*} \right)^{1-\sigma} + N_{X,t} \left(\frac{\tau_t \tilde{z}_D}{\tilde{z}_X} \right)^{1-\sigma} + N_{I,t} \left(TOL_t \frac{\tilde{z}_D}{\tilde{z}_I} \right)^{1-\sigma}}{N_{D,t} + N_{X,t}^* \left(TOL_t \frac{\tau_t^* \tilde{z}_D}{\tilde{z}_X^*} \right)^{1-\sigma} + N_{I,t}^* \left(\frac{\tilde{z}_D}{\tilde{z}_{It}^*} \right)^{1-\sigma}} \left(\frac{N_{D,t} + N_{X,t}^* + N_{I,t}^*}{N_{D,t}^* + N_{X,t} + N_{I,t}} \right)^{-1}.$$

C Labor Market Clearing

Labor supply is rigid $L_t^S = \bar{L}$ and labor demand comes from firms that need to cover fixed and variable production costs and sunk costs of entry as follows:

	Entry	Domestic Production	Export to F	FDI from F
Variable labor cost		$w_t l_{D,t}(\omega)$	$w_t l_{X,t}(\omega)$	$w_t l_{I,t}(\omega)$
Per-period Fixed Cost			$\frac{w_t f_{X,t}}{Z_t}$	$\frac{w_t f_{I,t}}{Z_t}$
Once and for all Cost	$\frac{w_t f_{E,t}}{Z_t}$			

PRODUCTION LABOR. A generic firm produces zZ_t units of variety ω per worker. Considering separately work that is used for domestic, export and FDI production, $l_{D,t}(\omega)$ and $l_{X,t}(\omega)$ are the number of workers hired in the domestic economy to produce good for the domestic market and export market, while $l_{I,t}(\omega)$ is the number of workers hired in the domestic economy by foreign firms to produce for the domestic market.

LABOR USED AS INVESTMENT. New entrants hire $f_{E,t}$ workers as an entry cost. Each exporter hires $f_{X,t}$ workers per period to cover export costs. Each foreign firm producing domestically hires $f_{I,t}$ domestic workers per period to carry out FDI production.

Profits from domestic sales for a domestic firm with idiosyncratic productivity z are

$$\pi_{D,t}(\omega) = \underbrace{\rho_{D,t}(\omega)}_{\frac{\sigma}{\sigma-1} \frac{w_t}{zZ_t}} \underbrace{y_{X,t}}_{zZ_t l_{D,t}(\omega)} - w_t l_{D,t}(\omega) = \frac{1}{\sigma-1} w_t l_{D,t}(\omega) \quad (29)$$

Profits from export sales for a domestic firm with productivity z are

$$\pi_{X,t}(\omega) = \underbrace{\rho_{X,t}(\omega)}_{\frac{\tau_t}{Q_t} \frac{\sigma}{\sigma-1} \frac{w_t}{zZ_t} \frac{zZ_t}{\tau_t}} \underbrace{y_{X,t}}_{l_{X,t}(\omega)} - w_t \left[l_{X,t}(\omega) + \frac{f_X}{Z_t} \right] = \frac{1}{\sigma-1} w_t l_{X,t}(\omega) - \frac{w_t f_X}{Z_t} \quad (30)$$

Profits from domestic FDI sales for a foreign firm with productivity z are

$$\pi_{I,t}^*(\omega) \equiv \frac{\Pi_{I,t}^*(\omega)}{P^*} = \frac{1}{e_t P_t^*} \frac{P_t}{P_t} \left[\frac{\sigma}{\sigma-1} W_t l_{I,t}(\omega^*) - W_t \left(l_{I,t}(\omega^*) + \frac{f_I}{Z_t} \right) \right] \quad (31)$$

$$= \frac{1}{Q_t} \left(\frac{1}{\sigma-1} w_t l_{I,t}(\omega^*) - w_t \frac{f_I}{Z_t} \right) \quad (32)$$

From the optimal profits above, I can derive the average amount of labor hired to cover domestic sales, export sales of the domestic average firm and FDI sales of the foreign average firm:

$$\tilde{l}_{D,t}(\omega) = (\sigma-1) \frac{\tilde{\pi}_{D,t}}{w_t}$$

$$\begin{aligned}\tilde{l}_{X,t}(\omega) &= (\sigma - 1) \frac{\tilde{\pi}_{X,t}}{w_t} + (\sigma - 1) \frac{f_{X,t}}{Z_t} \\ \tilde{l}_{I,t}(\omega) &= (\sigma - 1) \frac{Q_t \pi_{I,t}^*}{w_t} + (\sigma - 1) \frac{f_{I,t}}{Z_t}\end{aligned}$$

Therefore, the total amount of production labor hired in Home in every period is

$$(\sigma - 1) N_{D,t} \left[\frac{\tilde{\pi}_{D,t}}{w_t} \right] + (\sigma - 1) N_{X,t} \left[\frac{\tilde{\pi}_{X,t}}{w_t} + \frac{f_{X,t}}{Z_t} \right] + (\sigma - 1) N_{I,t}^* \left[\frac{Q_t \tilde{\pi}_{I,t}^*}{w_t} + \frac{f_{I,t}}{Z_t} \right]$$

The demand for labor used as investment in the economy originates from new entrants ($N_{E,t} f_{E,t}/Z_t$), exporters ($N_{X,t} f_{X,t}/Z_t$), and foreign affiliates $N_{I,t}^* f_{I,t}/Z_t$ and adds to the total amount of production labor as follows:

$$\begin{aligned}L_t^D &= (\sigma - 1) N_{D,t} \left(\frac{\tilde{\pi}_{D,t}}{w_t} \right) + (\sigma - 1) N_{X,t} \left(\frac{\tilde{\pi}_{X,t}}{w_t} + \frac{f_{X,t}}{Z_t} \right) + (\sigma - 1) N_{I,t}^* \left(\frac{Q_t \tilde{\pi}_{I,t}^*}{w_t} + \frac{f_{I,t}}{Z_t} \right) \\ &\quad + \frac{N_{E,t} f_{E,t}}{Z_t} + N_{X,t} \frac{f_{X,t}}{Z_t} + N_{I,t}^* \frac{f_{I,t}}{Z_t}\end{aligned}$$

Since labor supply is fixed at $L_t^S = \bar{L}$, equilibrium is such that

$$\bar{L} = \frac{\sigma - 1}{w_t} (N_{D,t} \tilde{\pi}_{D,t} + N_{X,t} \tilde{\pi}_{X,t} + Q_t N_{I,t}^* \tilde{\pi}_{I,t}^*) + \frac{\sigma}{Z_t} \left(\frac{N_{E,t} f_{E,t}}{\sigma} + N_{X,t} f_{X,t} + N_{I,t}^* f_{I,t} \right) \quad (33)$$

and w_t can be determined solving this equation. The foreign country has an identical labor market equilibrium.

C.1 Balanced Current Account.

Even in the case of Financial Autarky, the Current Account does not correspond to Trade Balance as in standard models, due to the existence of repatriated profits. In order to show that balanced current account allows to close the model, I rewrite the average prices as indirect functions of profits and plug them in the price index ($N_{D,t} \tilde{\rho}_{D,t}^{1-\sigma} + N_{X,t}^* \tilde{\rho}_{X,t}^{1-\sigma} + N_{I,t}^* \tilde{\rho}_{I,t}^{1-\sigma} = 1$)

$$\begin{aligned}\tilde{\pi}_{D,t} &= \frac{1}{\sigma} \tilde{\rho}_{D,t}^{1-\sigma} C_t \implies \tilde{\rho}_{D,t}^{1-\sigma} = \frac{1}{C_t} \sigma \pi_{D,t} \\ \tilde{\pi}_{X,t} &= \frac{Q_t}{\sigma} \tilde{\rho}_{X,t}^{1-\sigma} C_t^* - \frac{w_t f_{X,t}}{Z_t} \implies \tilde{\rho}_{X,t}^{1-\sigma} = \frac{1}{Q_t C_t^*} \sigma \pi_{X,t} + \frac{\sigma}{Q_t C_t^*} \frac{w_t f_{X,t}}{Z_t} \\ \tilde{\pi}_{I,t}^* &= \frac{1}{Q_t \sigma} \tilde{\rho}_{I,t}^{*1-\sigma} C_t - \frac{w_t f_{I,t}}{Q_t Z_t} \implies \tilde{\rho}_{I,t}^{*1-\sigma} = \frac{Q_t}{C_t} \sigma \tilde{\pi}_{I,t}^* + \frac{\sigma}{C_t} \frac{w_t f_{I,t}}{Z_t}\end{aligned}$$

to derive

$$C_t = \left[N_{D,t} \sigma \pi_{D,t} + \sigma N_{X,t} \left(\pi_{X,t} + \frac{w_t f_{X,t}}{Z_t} \right) \right] \quad (34)$$

$$+ \left[N_I \tilde{\pi}_{I,t} + N_{I,t}^* Q_t \sigma \tilde{\pi}_{I,t}^* + N_{I,t}^* \frac{\sigma w_t f_{I,t}}{Z_t} - Q_t N_{I,t}^* \tilde{\pi}_{I,t}^* \right] \quad (35)$$

Now, combining the definition of total profits ($\tilde{\pi} \equiv \tilde{\pi}_D + \frac{N_X}{N_D} \tilde{\pi}_X + \frac{N_I}{N_D} \tilde{\pi}$), aggregate accounting ($C_t = w_t L + N_{D,t} \tilde{\pi}_D + N_X \tilde{\pi}_X + N_I \tilde{\pi}_I - N_{E,t} \tilde{v}_t$) and the free entry condition ($\tilde{v}_t = w_t f_{E,t}/Z_t$), some algebra leads to the labor market equilibrium:

$$L = \frac{\sigma - 1}{w_t} (N_{D,t} \tilde{\pi}_{D,t} + N_{X,t} \tilde{\pi}_{X,t} + Q_t N_{I,t}^* \tilde{\pi}_{I,t}^*) + \frac{\sigma}{Z_t} \left(\frac{N_{E,t} f_{E,t}}{\sigma} + N_{X,t} f_{X,t} + N_{I,t}^* f_{I,t} \right).$$

D International Trade in Bonds

The version of the model allowing for international trade in bonds is a customary term of comparison for two-country economies, although the framework is still one of markets incompleteness. I extend the model to the case in which the representative household can trade Home and Foreign risk-free bonds (B^H and B^F) subject to portfolio adjustment costs that are rebated to consumers. Thus, the budget constraint of Home becomes:

$$\begin{aligned} B_{t+1}^H + Q_t B_{t+1}^F + \frac{\eta}{2} (B_{t+1}^H)^2 + \frac{\eta}{2} Q_t (B_{t+1}^F)^2 + \tilde{v}_t N_{H,t} x_{t+1} + C_t = \\ = (1 + r_t) B_t^H + Q_t (1 + r_t^*) B_t^F + (\tilde{v}_t + \tilde{\pi}_t) N_{D,t} x_t + w_t L_t + T_t \end{aligned}$$

The risk free interest rate is measured in units of the consumption basket of the issuing country's and $T_t = \frac{\eta}{2} (B_{t+1}^H)^2 + \frac{\eta}{2} Q_t (B_{t+1}^F)^2$ is the revenue from adjustment costs. Hence, there are now four Euler equations for bonds holdings, besides the two Euler equations for the shares of the mutual fund:

$$C_t^{-\gamma} = \frac{\beta(1 + r_{t+1})}{(1 + \eta B_{t+1}^H)} E_t [(C_{t+1})^{-\gamma}] ; \quad C_t^{-\gamma} = \frac{\beta(1 + r_{t+1})}{(1 + \eta B_{t+1}^F)} E_t \left[\frac{Q_{t+1}}{Q_t} (C_{t+1})^{-\gamma} \right] \quad (36)$$

$$C_t^{*- \gamma} = \frac{\beta(1 + r_{t+1}^*)}{(1 + \eta B_{t+1}^{H*})} E_t \left[\frac{Q_t}{Q_{t+1}} (C_{t+1})^{-\gamma} \right] ; \quad C_t^{*- \gamma} = \frac{\beta(1 + r_{t+1}^*)}{(1 + \eta B_{t+1}^{F*})} E_t [(C_{t+1}^*)^{-\gamma}] \quad (37)$$

The multiplicative term before the expectation terms includes the adjustment cost of bonds used to ensure stationarity of the model. Moreover, because of zero net supply of bonds worldwide we have two additional conditions, i.e. $B_{t+1}^H + B_{t+1}^{H*} = 0$ and $B_{t+1}^F + B_{t+1}^{F*} = 0$. I will focus on a symmetric equilibrium in which the identical household in the two countries make identical choices so that $B_t^H = B_t^F$ and $B_t^{*F} = B_t^{*H}$ under perfect foresight. Hence, the budget constraint reduces to

$$\frac{B_{t+1}^{H*}}{Q_t} + B_{t+1}^{F*} = (1 + r_t) \frac{1}{Q_t} B_t^{H*} + (1 + r_t^*) B_t^{F*} + N_{D,t}^* \tilde{\pi}_t^* - N_{E,t}^* \tilde{v}_t^* + w_t^* L_t^* - C_t^*. \quad (38)$$

Moreover, using the equations for the two countries I can obtain an expression for the dynamics of home Net Foreign Asset accumulation:

$$\begin{aligned} B_{t+1}^H + Q_t B_{t+1}^F = (1 + r_t) B_t^H + Q_t (1 + r_t^*) B_t^F + \frac{1}{2} (N_{D,t} \tilde{\pi}_t - Q_t N_{D,t}^* \tilde{\pi}_t^*) \\ - \frac{1}{2} (N_{E,t} \tilde{v}_t - Q_t N_{E,t}^* \tilde{v}_t^*) - \frac{1}{2} (N_{E,t} \tilde{v}_t - Q_t N_{E,t}^* \tilde{v}_t^*) - \frac{1}{2} (C_t - Q_t C_t^*) \end{aligned}$$

The Financial Account defined as the changes of aggregate bond holdings in the two countries, is by definition equal to the Current Account

$$\begin{aligned} CA_t &\equiv B_{t+1}^H - B_t^H + Q_t (B_{t+1}^F + B_{t+1}^F) \\ CA_t^* &\equiv \frac{B_{t+1}^{H*} - B_t^{H*}}{Q_t} + (B_{t+1}^{F*} + B_{t+1}^{F*}) \end{aligned}$$

Finally, the sum of the countries' Current Accounts is zero ($CA_t + Q_t CA_t^* = 0$) and world consumption is the sum of World labor income and net investment income.

E The System

TABLE A1. MODEL SUMMARY - FINANCIAL AUTARKY

Price Indexes (1, 2)	$N_{D,t}\tilde{\rho}_{D,t}^{1-\sigma} + N_{X,t}^*\tilde{\rho}_{X,t}^{1-\sigma} + N_{I,t}^*\tilde{\rho}_{I,t}^{1-\sigma} = 1$ $N_{D,t}\tilde{\rho}_{D,t}^{1-\sigma} + N_{X,t}\tilde{\rho}_{X,t}^{1-\sigma} + N_{I,t}\tilde{\rho}_{I,t}^{1-\sigma} = 1$
Total Profits (3,4)	$\tilde{\pi}_t = \tilde{\pi}_{D,t} + \frac{N_{X,t}}{N_{D,t}}\tilde{\pi}_{X,t} + \frac{N_{I,t}}{N_{D,t}}\tilde{\pi}_{I,t}$ $\tilde{\pi}_t^* = \tilde{\pi}_{D,t}^* + \frac{N_{X,t}^*}{N_{D,t}^*}\tilde{\pi}_{X,t}^* + \frac{N_{I,t}^*}{N_{D,t}^*}\tilde{\pi}_{I,t}^*$
Free Entry Conditions (5,6)	$\tilde{v}_t = \frac{w_t f_{E,t}}{Z_t}; \quad \tilde{v}_t^* = \frac{w_t^* f_{E,t}^*}{Z_t^*}$
Profits for the average exporter (7, 8)	$\tilde{\pi}_{X,t} = \left[\nabla \left(f_{X,t}^{1+\frac{k}{1-\sigma}} - \left(TOL_t^\sigma \tau_t^{1-\sigma} f_{I,t}^* \right)^{1+\frac{k}{1-\sigma}} \right) \right. \\ \left. \left(f_{X,t}^{\frac{k}{1-\sigma}} - \left(TOL_t^\sigma \tau_t^{1-\sigma} f_{I,t}^* \right)^{\frac{k}{1-\sigma}} \right)^{-1} - f_{X,t} \right] \frac{w_t}{Z_t}$ $\tilde{\pi}_{X,t}^* = \left[\nabla \left(f_{X,t}^{*1+\frac{k}{1-\sigma}} - \left(TOL_t^{-\sigma} \tau_t^{*1-\sigma} f_{I,t}^* \right)^{1+\frac{k}{1-\sigma}} \right) \right. \\ \left. \left(f_{X,t}^{*\frac{k}{1-\sigma}} - \left(TOL_t^{-\sigma} \tau_t^{*1-\sigma} f_{I,t}^* \right)^{\frac{k}{1-\sigma}} \right)^{-1} - f_{X,t}^* \right] \frac{w_t^*}{Z_t^*}$
Profits for the average fromFDI (9, 10)	$\tilde{\pi}_{I,t} = (\nabla - 1) Q_t \frac{w_t^* f_{I,t}}{Z_t^*}$ $\tilde{\pi}_{I,t}^* = (\nabla - 1) \frac{1}{Q_t} \frac{w_t^* f_{I,t}^*}{Z_t^*}$
Share of Exporting Firms (11, 12)	$\frac{N_{X,t}}{N_{D,t}} = z_{\min}^k \left(z_{X,t}^{-k} - z_{I,t}^{-k} \right)$ $\frac{N_{X,t}^*}{N_{D,t}^*} = z_{\min}^{*k} \left(z_{X,t}^{*-k} - z_{I,t}^{*-k} \right)$
Share of FDI Firms (13, 14)	$\frac{N_{I,t}}{N_{D,t}} = \left(\frac{z_{\min,t}}{z_{I,t}} \right)^k \nabla^{\frac{k}{\sigma-1}}$ $\frac{N_{I,t}^*}{N_{D,t}^*} = \left(\frac{z_{\min,t}^*}{z_{I,t}^*} \right)^k \nabla^{\frac{k}{\sigma-1}}$
Euler Equations for Bonds (15, 16)	$(C_t)^{-\gamma} = \beta(1 + r_{t+1})E_t \left[(C_{t+1})^{-\gamma} \right]$ $(C_t^*)^{-\gamma} = \beta(1 + r_{t+1}^*)E_t \left[(C_{t+1}^*)^{-\gamma} \right]$
Euler Equations for Shares (17, 18)	$\tilde{v}_t = \beta(1 - \delta)E_t \left[\left(\frac{C_{t+s}}{C_t} \right)^{-\gamma} (\tilde{v}_{t+1} + \tilde{\pi}_{t+1}) \right]$ $\tilde{v}_t^* = \beta(1 - \delta)E_t \left[\left(\frac{C_{t+s}^*}{C_t^*} \right)^{-\gamma} (\tilde{v}_{t+1}^* + \tilde{\pi}_{t+1}^*) \right]$
Aggregate Accounting (19, 20)	$C_t = w_t L + N_{D,t} \tilde{\pi}_t - N_{E,t} \tilde{v}_t$ $C_t^* = w_t^* L^* + N_{D,t}^* \tilde{\pi}_t^* + N_{E,t}^* \tilde{v}_t^*$
Number of Firms (21, 22)	$N_{D,t} = (1 - \delta)(N_{D,t-1} + N_{E,t-1})$ $N_{D,t}^* = (1 - \delta)(N_{D,t-1}^* + N_{E,t-1}^*)$
Balanced Trade (23)	$Q_t N_{X,t} \tilde{\rho}_{X,t}^{1-\sigma} C_t^* + N_I \tilde{\pi}_{I,t} = N_{X,t}^* \tilde{\rho}_{X,t}^{*1-\sigma} C_t + N_I^* \tilde{\pi}_{I,t}^*$

Under Financial Autarky, there are 25 endogenous variables. Of these, fifteen are not predetermined at t : $w_t, w_t^*, \tilde{\pi}_t, \tilde{\pi}_t^*, N_{E,t}, N_{E,t}^*, N_{I,t}, N_{I,t}^*, \tilde{z}_{I,t}, \tilde{z}_{I,t}^*, \tilde{v}_t, \tilde{v}_t^*, C_t, C_t^*, Q_t$, four are predetermined at t : $N_{D,t}, N_{D,t}^*, r_t, r_t^*$, and there are six endogenous expectational errors: $\eta_t^{\tilde{v}}, \eta_t^{\tilde{v}^*}, \eta_t^{\tilde{\pi}}, \eta_t^{\tilde{\pi}^*}, \eta_t^C, \eta_t^{C^*}$. Under Bond Trading, there are 29 endogenous variables. Of these, the predetermined group does not change but the 8 variables predetermined at t now include, $B_t^H, B_t^{H*}, B_t^F, B_t^{F*}$. In addition, there is an additional endogenous expectational error, η_t^Q . In both cases, the model also features 10 exogenous variables ($Z_t, Z_t^*, f_{E,t}, f_{E,t}^*, f_{X,t}, f_{X,t}^*, f_{I,t}, f_{I,t}^*, \tau_t, \tau_t^*$). Equation 1-10, 13-14 and 17-18 are the same as in the case of Financial Autarky, the other equations are replaced or introduced ex-novo as in Table A2. The systems are loglinearized around the steady state, represented in canonical form and solved using the Gensys algorithm.

TABLE A2. MODEL SUMMARY - BONDS TRADING

Euler Equations for bonds issues by H (11, 12)	$(1 + \eta B_{t+1}^H) C_t^{-\gamma} = \beta(1 + r_{t+1}) E_t [(C_{t+1})^{-\gamma}]$ $(1 + \eta B_{t+1}^{H*}) C_t^{-\gamma} = \beta(1 + r_{t+1}^*) E_t \left[\frac{Q_{t+1}}{Q_t} (C_{t+1})^{-\gamma} \right]$
Euler Equations for bonds issues by F (15, 16)	$(1 + \eta B_{t+1}^F) C_t^{*- \gamma} = \beta(1 + r_{t+1}) E_t \left[\frac{Q_t}{Q_{t+1}} (C_{t+1}^*)^{-\gamma} \right]$ $(1 + \eta B_{t+1}^{F*}) C_t^{*- \gamma} = \beta(1 + r_{t+1}^*) E_t [(C_{t+1}^*)^{-\gamma}]$
Net Foreign Assets (formerly Balanced Current Account) (19)	$B_{t+1}^H + Q_t B_{*,t+1}^F = (1 + r_t) B_t^H + Q_t (1 + r_t^*) B_{*,t}^F$ $+ \frac{1}{2} (w_t L_t - Q_t w_t^* L_t^*) + \frac{1}{2} \left(N_{D,t} \tilde{\pi}_{D,t} - Q_t N_{D,t}^* \tilde{\pi}_{D,t}^* \right)$ $- \frac{1}{2} \left(N_{E,t} \tilde{v}_t - Q_t N_{E,t}^* \tilde{v}_t^* \right) - \frac{1}{2} (C_t - Q_t C_t^*)$
Labor market equilibrium (20, 21)	$L_t = \frac{\sigma-1}{w_t} (N_{D,t} \tilde{\pi}_{D,t} + N_{X,t} \tilde{\pi}_{X,t}) + \frac{1}{Z_t} (N_{E,t} f_{E,t} + \sigma N_{X,t} f_{X,t})$ $L_t^* = \frac{\sigma-1}{w_t^*} \left(N_{D,t}^* \tilde{\pi}_{D,t}^* + N_{X,t}^* \tilde{\pi}_{X,t}^* \right) + \frac{1}{Z_t^*} \left(N_{E,t}^* f_{E,t}^* + \sigma N_{X,t}^* f_{X,t}^* \right)$
Bond Market Equilibrium (22, 23)	$B_{t+1}^H + B_{t+1}^{H*} = 0$ $B_{t+1}^F + B_{t+1}^{F*} = 0$