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The IT revolution and the unsecured credit market

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The IT Revolution and the Unsecured Credit Market

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Abstract

The information technology (IT) revolution coincided with the transformation of the U.S. unsecured credit market. From 1983 to 2004 households' unsecured borrowing increased rapidly and there was a even faster increase in the number of bankruptcy filings. To study the effect of information costs on debt and bankruptcy a risk of repudiation model with asymmetric information and costly screening is introduced. When information costs are high, the design of contracts under private information prevents some households from borrowing with risk of default. As information costs drop, households borrow more and the number of bankruptcy filings increase. A calibrated version of the model reproduces the main characteristics of the U.S. unsecured credit market in 1983 and 2004. Quantitative exercises suggest that the IT revolution may have played an important role in the transformation of the unsecured credit market.

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1 Introduction

Stylized Facts. The U.S. unsecured credit market has changed dramatically since the beginning of the 1980s. Moss and Johnson (1999) refer to the period starting in the mid-1980s as the "Revolution in Consumer Credit and Consumer Bankruptcy."[1] The period of transformation in the U.S. unsecured credit market coincided with the information technology (IT) revolution.[2] During this period financial firms underwent a technological transformation.[3] Currently the financial sector is one of the largest buyers of IT: According to the 2010 Bureau of Economic Analysis Use Input-Output matrix, about 12 percent of the sales of the information sector as intermediate goods were bought by the financial sector.

Table 1 shows that between 1983 and 2004, households' unsecured credit increased dramatically: From 0.5 to 1.2 percent of mean income. The number of bankruptcy filings increased even faster: From 286,444 to 1,563,145.[4] It also shows the change in the share of households in debt: From 5.6 percent in 1983 to 8.0 percent in 2004. This table also presents the bankruptcy rate, defined as the ratio of the number of filings to the population; this statistic increased significantly: From 0.12 percent in 1983 to 0.53 percent in 2004. Finally, *Table 1* shows the value of loans and leases removed from the books and charged against loss reserves divided by the total amount of debt (charge-off rates); this variable increased from 3 percent in 1983 to 4.5 percent in 2004.[5]

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[1]They also pointed out that from 1929 to 1985 the relationship between real consumer credit and nonbusiness filings remained remarkably stable: Real consumer credit grew at a compound rate of 4.727 percent per year, while nonbusiness filings grew at a compound rate of 4.735 percent per year.

[2]The IT revolution refers to the dramatic change in the cost of electronics, computing, and telecommunication. See Forester (1985).

[3]See Jordan and Katz (1999).

[4]This paper focuses on the period 1983-2004. Starting in 1983 is ideal given the availability of data from the Survey of Consumer Finance (SCF). Ending in 2004 is convenient because the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) of 2005 altered the conditions of bankruptcy.

[5]The appendix describes how each of these variables was created.

This paper. The hypothesis that a drop in the cost of information played a role in the transformation of unsecured credit markets is entertained here. To evaluate this hypothesis I proceed in two steps. First, I present an extension of the risk of repudiation model of Eaton and Gersovitz (1981) to incorporate asymmetric information. This extension is necessary to study to role of information technologies because understanding what would happen if lenders do not directly observe the default risk of households is key for the analysis. In particular, when the cost of information is extremely high, I study the separating equilibrium of the asymmetric information version of Eaton and Gersovitz (1981)'s economy.[6] Second, I calibrate an infinite-horizon version of the model following the work of Krueger and Perri (2005), Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), and Livshits, MacGee, and Tertilt (2007). The quantitative model is useful to further characterize the model with costly information and to study the role of the IT revolution in the transformation of the U.S. unsecured credit market.

Findings. The analysis of a simple two-period model in *Section 2* illustrates the effect of a drop in the cost of information. When information costs are high, the design of self-revelation contracts implies that low interest rates are available only for small amounts of debt. As information costs drop and households' risk of default can be screened, some households can borrow more at relatively low interest rates. These households will default after a low realization of income. Therefore, as information costs drops, both debt and bankruptcy rise. A quantitative model is presented in *Section 3*. The calibrated model reproduces the main characteristics of the unsecured credit market in 1983 and 2004, as shown in *Section 4*. The numerical solution of the model is used to characterize the

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[6]The option of studying the separating equilibrium is convenient because the pooling equilibrium does not exist in this framework. The separating equilibrium may not exist, though. There are several reasons to consider the prices and allocation of the separating equilibrium despite this problem, however. First, if the share of riskier households is small enough, then this is the unique equilibrium. Second, if this equilibrium does not exist, there is no Nash equilibria. And third, slightly different equilibrium definition (as in Riley, 1979) return these prices and allocations as the unique equilibrium.

model introduced in *Section 5*. This analysis shows that the relationship between the cost of information and default is a strong one: As the cost of information drops from the level that would prevent any screening of households' risk of default to zero, the share of the population filing for bankruptcy more than triples. The effect on borrowing is less pronounced: While the debt-to-income ratio more than doubles, the share of people in debt increases little. This occurs because as information costs drop, more borrowers use screening contracts and they are able to borrow with risk of default. Finally, the counterfactual analysis in *Section 6* suggests that the IT revolution alone accounts for a significant part of the unsecured credit market transformation.

Related literature. This paper builds on the versions of Eaton and Gersovitz (1981)'s model introduced to study unsecured credit by Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2007). The use of costly screening in credit markets with informational frictions resembles the arrangement in Wang and Williamson (1998). The study of credit markets under asymmetric information is related to an early literature masterfully surveyed in Hirshleifer and Riley (1992).

The main premise of the paper—determining the effect of the IT revolution in unsecured credit markets—has been recently analyzed by several authors. Narajabad (2011) proposes the IT revolution as the explanation for the rise in bankruptcies. He does not explore the use of revelation contracts or the quantitative explanations of the model.

Drozd and Nosal (2008) present a search model of the market for unsecured credit. They study the effect of a drop in the cost of screening and soliciting credit customers on debt and bankruptcy. The cost of screening is potentially close to the current paper. However, Drozd and Nosal (2008) do not model asymmetric information—lenders have no alternative to paying the cost of screening. Therefore, their cost is related mainly to a transaction cost, as the one analyzed by Livshits, MacGee, and Tertilt (2010).

Livshits, MacGee, and Tertilt (2011) present a stylized model with informational frictions in which lenders must pay a cost to design a contract. In that model, a drop in the cost of designing a new contract affects the share of people borrowing. This differs with the model in this paper, where the cost of information affects not only who borrows but also how much they borrow. Considering borrowing intensity—the amount of debt each household borrows—as a choice variable is necessary to reproduce the stylized facts described previously.

Finally, Athreya, Tam, and Young (2012) present a quantitative model of unsecured debt with informational frictions. They consider signaling equilibria, so lenders' beliefs are crucial. Their analysis differs substantially from the current paper in which, as explained previously, lenders (partially) offset the lack of information by designing debt contracts accordingly.

2 The Story in a Simple Two-Period Model

This section previews the main driving forces at work in the full model presented later using a simple two-period model. Imagine an economy populated by households and lenders. Households live for two periods, $t = 1, 2$, and they derive utility from consumption according to the utility function u with standard properties. They are born with assets a_1, and in each period they are endowed with a quantity of labor measured in efficiency units, l_n, that can take two values, $l_n \in \{l_L, l_H\}$, denoting low or high productivity. Thus, income is $y_n = wl_n$, where the wage w is exogenous. The transition probability between state L and H is $\pi_{L,H}$. Persistence is assumed: $\pi_{H,H} > \pi_{H,L}$ and $\pi_{L,L} > \pi_{L,H}$. Importantly, this implies $\pi_{H,H} > \pi_{L,H}$; that is, households with high productivity this period are more likely to have high productivity next period than those with low productivity this period.

Lenders are risk neutral and compete in offering debt contracts. A contract is a mapping q from promised amount for the next period, a_2, and observable current household characteristics (e.g., productivity, L or H), to the share of the amount promised that is advanced in the current period. Notice that because a_2

represents assets, a household borrowing ($a_2 < 0$) promises to pay back $-a_2 > 0$ in the next period and receives $-a_2 q(a_2, \cdot) > 0$ in the current period.

To incorporate information technologies into the credit process, lending firms are allowed to offer two types of contracts. On the one hand, lenders offer *screening contracts*. These contracts require the use of information technologies to determine a household's productivity. The price function of screening contracts is denoted as $\widetilde{q}$. The cost of screening a household's productivity (also referred to as information costs), $\mathcal{C}$, is proportional to the amount borrowed to simplify the analysis. On the other hand, lenders may offer *revelation contracts*. These contracts are designed to induce households to reveal their productivity. Although these contracts are cheaper because there is no use of information technologies, their design implies that low interest rates will be linked to tight borrowing limits. Thus, when information costs are expensive and only revelation contracts are attractive, some households will be borrowing constrained. The price function of revelation contracts is denoted as $\widehat{q}$.

In the first period of life, a household decides which type of debt contract is preferred and how much to borrow.[7] In the second period, after the realization of the productivity shock, the household decides whether to file for bankruptcy or to pay back the debt normally. The punishment for bankruptcy is a proportion of the household income, τ. Thus, the lifetime utility of a household with first-period assets a_1, income y_n, and facing a (generic) price function q is

$$U(a_1, y_{1,n}; q) = \max_{a_2 \in \mathbf{A}} u(y_{1,n} + a_1 - q(a_2, n, a_1)a_2) + \beta \pi_{n,H} \max\{u(y_{2,H} + a_2), u(y_{2,H}(1 - \tau))\}$$

$$+ \beta \pi_{n,L} \max\{u(y_{2,L} + a_2), u(y_{2,L}(1 - \tau))\}.$$

Notice that above the price function q is used to represent $\widetilde{q}$ or $\widehat{q}$. A priori the bond price may depend on (a_2, n, a_1). Later it will be clear that the price function of screening contracts depends on (a_2, n), whereas the price function of revelation contracts depends on (a_2, a_1). Once the choice of contract is taken into account,

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[7]The fact that households and not lenders choose the contract (screening vs. revelation) is for exposition and without loss of generality.

the lifetime utility of a household can be written as

$$\mathbf{U}(a_1, y_{1,n}) = \max\{U(a_1, y_{1,n}; \widetilde{q}), U(a_1, y_{1,n}; \widehat{q})\}.$$

The analysis in this section is based on Figure 1, which shows a simple numerical example illustrating the main mechanism.[8] Households preferences are represented in Figure 1 by indifference curves over price values, $\mathbf{q}$, and amounts of assets for period 2, a_2. Notice that the lifetime utility associated with a particular combination $(\mathbf{q}, a_2)$ is

$$\mathcal{U}(a_1, y_{1,n}; a_2, \mathbf{q}) \equiv u(y_{1,n} + a_1 - \mathbf{q}a_2) + \beta\pi_{n,H} \max\{u(y_{2,H} + a_2), u(y_{2,H}(1 - \tau))\}$$
$$+ \beta\pi_{n,L} \max\{u(y_{2,L} + a_2), u(y_{2,L}(1 - \tau))\},$$

Indifference curves of household are combinations of $(\mathbf{q}, a_2)$ that provide a household with the same $\mathcal{U}$. First, consider indifference curves in the range in which the risk of default does not change. The level of assets $a_2^*(\mathbf{q})$ corresponds to the level solving the first-order condition of the household's problem given a price function constant at $\mathbf{q}$. By construction, the slope of the indifference curve is zero at that level of debt. Starting from there, it is simple to understand the shape of the indifference curve. To the right of the point $a_2^*(\mathbf{q})$ borrowing more is desirable and to the left borrowing less is desirable. Thus, any deviation from $a_2^*(\mathbf{q})$ reduces the household's utility, implying that any deviation must be compensated with a higher $\mathbf{q}$ to sustain the same utility. Second, consider the points at which the bankruptcy decision changes (the steps in the function q). The abrupt change in the slope of the indifference curves is due to the jump in the default probability that occurs there. Since households know there is a higher probability they will file bankruptcy in the next period, they desire to acquire more debt so they are willing to take a drop in q to do so.

Importantly, indifference curves of households with different current productivity have different slopes at a given amount of debt. Take any value $\{\mathbf{q}, a_2\}$. The

[8]The following values were given to the parameters in this example: $\beta = 0.93$, $r = 0$, u is a constant relative risk aversion utility function with risk aversion parameter 2, $y_{1L} = 0.25$, $y_{1H} = 0.3$, $y_{2L} = 0.2$, $y_H = 0.5$, $\pi_{LH} = 0.75$, $\pi_{HH} = 0.9$, and $\tau = 0.28$.

slope is bigger (steeper) for households with low productivity than for those with high productivity. This result is crucial for the existence of revelation contracts. It means that the trade-off between more borrowing and higher prices (lower interest rates) is different for households with different productivity. Intuitively, this follows because households with low productivity are relatively more affected by larger amounts borrowed—they expect higher income growth and relatively less affected by lower prices —they are more likely to default in the second period.

In equilibrium, competition between lenders implies zero expected discounted profits. The prices satisfying this condition are presented in *Figure 1* with solid black (high current productivity) and red (low current productivity) lines. Notice that if a_2 is close to zero ($a_2 > -0.055$ in *Figure 1*), the price is equal to one (meaning zero interest rate) for both values of current-period productivity. This occurs because if the amount of debt is too small, then the household will prefer to pay debt normally instead of filing for bankruptcy for any value of productivity in the next period.[9] For larger amounts of debt, households will prefer to default in the next period. However, if debt is not too large ($a_2 \in [-0.055, -0.14]$ in the example in *Figure 1*), bankruptcy will occur only if the low productivity is realized. Therefore, the zero-profit price in this range varies for households with different current-period productivity according to their probability of low productivity in the next period. For even larger amounts of debt ($a_2 < -0.14$ in *Figure 1*), default will be beneficial for both levels of productivity and, therefore, prices are zero.

The zero expected profit condition for prices of *revelation contracts* must take into account the cost of information. Thus, these prices will be $\tilde{q}(a_2, n) = \pi_{n,H}(1 + i)^{-1} - \mathcal{C}$ for households borrowing with probability of bankruptcy larger than zero and smaller than one.[10]

Now, focus on *revelation contracts*. In this case, lenders design contracts

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[9]In this simple example the risk-free rate, r, is set at zero. Otherwise, the price would be $1/(1 + r)$.

[10]If the default probability is zero or one, information about current productivity is irrelevant and lenders will offer the zero-profit prices described previously.

under the constraint that they must induce households to reveal their productivity. Using prices and amounts of debt as instruments, it is possible to separate households according to productivity because in order to obtain more debt, low-productivity households are willing to accept a larger increase in interest rates than high-productivity households; (i.e., indifference curves cross, as shown in *Figure 1*). Consider the price of a revelation contract as depending on the amount borrowed, a_2, the household's *report* on productivity, m, and the current stock of assets, a_1. Then, a function q satisfies self-revelation if and only if $\forall a_1$ and $\forall n$,

$$\mathbf{U}(a_1, y_{1,n}) \geq \max_{a_2,m} \mathcal{U}(a_1, y_{1,n}; a_2, q(a_2, m; a_1)).$$

In words, q satisfies the self-revelation constraint if and only if households are better off borrowing at the price designed for their productivity than misrepresenting their productivity.

To determine the limits imposed by self-revelation consider *Figure 1*. Indifference curves of low-productivity households are represented by dashed red lines, while dashed black lines represent indifference curves of high-productivity households. The point $\mathbf{C}$ in the figure corresponds to $a_2 = -0.14$, the value of debt that maximizes the utility of the low-productivity household given zero-profit prices. Notice that this contract can be offered as a revelation contract because high-productivity households will be better off at other offered prices, as point $\mathbf{A}$, at which high-productivity households are borrowing very little and with no risk of default. Now consider point $\mathbf{B}$. At this point, the low-productivity household is borrowing less than in C but at higher prices (lower interest rates). Since both points are on the same indifference curve, this household is actually indifferent between $\mathbf{C}$ and $\mathbf{B}$. Amounts of debt to the left of $\mathbf{B}$ cannot be offered at the zero-profit prices for high-productivity households as revelation contracts. The low-productivity household would prefer to misreport productivity and take those contracts. Thus, a point such as $\mathbf{D}$ cannot be offered under private information. Actually, the point $\mathbf{B}$ determines $\underline{a}_2$: the maximum amount of debt that can be offered as a revelation contract at the zero-profit price for high-productivity

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households ($\underline{a}_2 = -0.08$ in *Figure 1*). Thus, equilibrium prices of revelation contracts are

$$\hat{q}(a_2, m; a_1) = \begin{cases} (1+r)^{-1} & \text{if } a_2 \geq \bar{a}_2, \\ \pi_{m,H}(1+r)^{-1} & \text{if } \bar{a}_2 \geq a_2 \geq \underline{a}_2(a_1) \text{ and } m = H, \\ \pi_{m,H}(1+r)^{-1} & \text{if } \underline{a}_2(a_1) \geq a_2 \geq \hat{a}_2, \text{ and } m = L, \text{ and} \\ 0 & \text{otherwise,} \end{cases}$$

where $\bar{a}_2$ is the minimum a_2 that can be borrowed and households would prefer not to default for either y_L or y_H in the second period, and $\hat{a}_2$ is the maximum a_2 that would imply default in the second period for either y_L or y_H.

Notice that households with low productivity never prefer screening contracts. They would be paying for information to show that they are actually riskier. For households with high productivity, there exists a cost of information $\underline{c}$ such that they are indifferent between screening and revelation contracts. They will prefer screening contracts if and only if $\mathcal{C} < \underline{c}$. That cost can be easily found in *Figure 1*: At point **E**, where the high-productivity household is borrowing more than in **A** but at lower prices (higher interest rates).

It is simple to study the effect of a drop in the cost of information in *Figure 1*. Initially, assume the cost of information $\mathcal{C}$ is high enough ($\mathcal{C} > \underline{c}$). The high-productivity household prefers to borrow small amounts at the risk-free interest rate—point **A** in *Figure 1*. Then, for simplicity, imagine the cost of information drops to zero.[11] High-productivity households would prefer to borrow more at slightly higher interest rates—point **D**. The higher interest rate reflects the higher probability of bankruptcy. Therefore, this change implies a rise in debt and bankruptcy. Does this mechanism deliver the other stylized facts presented above? Was the IT revolution quantitatively important for the transformation in the unsecured credit market? A quantitative model is developed next in an attempt to answer these questions.

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[11] Any change from $\mathcal{C}_0 > \underline{c}$ to $\mathcal{C}_1 < \underline{c}$ would have the same effect on bankruptcy and very similar effects on borrowing.

3 Quantitative Model

Time is discrete and denoted by $t = 0, 1, 2, \ldots$ At any time there is a unit mass of households. They discount the future at the rate β and survive to the next period with probability ϱ. Preferences of households are given by the expected value of the discounted sum of momentary utility

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} (\beta\varrho)^t u(c_t) \right],$$

where c_t is consumption at period t. The utility function u is strictly increasing, strictly concave, and twice differentiable.

Each household is endowed with one unit of time. Productivity is exogenously determined by labor endowments. Labor endowments are

$$e_t = \alpha y_t,$$

where α is the fixed individual component and y_t follows

$$\log(y_t) = n_t + \varepsilon_t,$$

$$n_t = \rho n_{t-1} + \eta_t.$$

Here ρ is time invariant. ε_t and η_t are independent and serially uncorrelated on $N(0, \sigma_{\varepsilon_t}^2)$ and $N(0, \sigma_{\eta_t}^2)$, respectively. Persistency is very important: Households with higher n_t will have lower default risk. Thus, the infinite-horizon model resembles the two-period model presented above.

There is asymmetric information between lenders and borrowers about the latter's persistent component of productivity, n. On one side, households know their n. On the other side, if borrowers are not screened, then the persistent component of productivity is private information. Nevertheless, each lender has access to a technology that can be used to learn a household's persistent component of productivity. The cost of information or the cost of screening borrowers is represented by $\mathcal{C}$, and it is proportional to the amount borrowed. The stock of assets, a_t, is publicly observable.

As in the example, there are two types of debt contracts: *Screening* and *revelation* contracts. For simplicity, lenders can borrow at the risk-free interest rate r from the rest of the world.[12] When using screening contracts, borrowers must pay the screening cost. The price charged is $\tilde{q}(a_{t+1}, n_t)$. This price depends on a_{t+1} because it determines the debt the household will have to repay in the next period, which in turn affects its willingness to repay the debt. The price depends on n_t because it affects the next-period productivity and thereby the probability of bankruptcy.

Revelation contracts must satisfy a "self-revelation" condition, formally stated later. This condition basically states that, given the contract design, borrowers are better off revealing their probability of default (persistent component of productivity). Since lenders offering revelation contracts do not observe n, prices depend on a households' reports on n, m. Additionally, since the current stock of assets affects a household's willingness to borrow, prices satisfying the revelation constraint also depend on this variable.

3.1   The household's problem

Hereafter, period-t variables are expressed without subscripts or superscripts, and period-$t + 1$ variables are represented with superscripts "'". Households decide on consumption, c, and asset accumulation, a'. In addition, they decide which kind of debt contract they prefer and whether to file for bankruptcy or to repay the debt.

Several assumptions determine the advantages and disadvantages of bankruptcy. The key advantage is the discharge of debts: In the period after bankruptcy, debt is set at zero. Thus, a household with too much debt may find it beneficial to file for bankruptcy. There are two disadvantages of doing so, however. In the period of bankruptcy, a proportion of income, τ, is lost.[13] Additionally, in that period,

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[12]See Sanchez (2009) for a general equilibrium analysis. Similar results are obtained.

[13]Chatterjee, Corbae, and Rios-Rull (2008) build a model where no punishment is required after filing bankruptcy. In that model, asymmetric information is crucial to create incentives for debt repayment,

consumption equals income—neither saving nor borrowing is allowed.

In this environment, lifetime utility can be written as

$$G(n, \varepsilon, a; \alpha) = \max\{V(n, \varepsilon, a; \alpha), D(n, \varepsilon; \alpha)\} \tag{1}$$

where V and D (defined below) are lifetime utilities for households repaying the debt and filing bankruptcy, respectively. This means that a household has the choice of filing bankruptcy. The policy function R indicates whether the household repays the debt or not,

$$R(n, \varepsilon, a; \alpha) = \begin{cases} 1 \text{ if } V(n, \varepsilon, a; \alpha) \geq D(n, \varepsilon; \alpha), \\ 0 \text{ otherwise.} \end{cases}$$

Next turn to a household choosing to file for bankruptcy. In this case, lifetime utility is

$$D(n, \varepsilon; \alpha) = u(\alpha \exp(n + \varepsilon)(1 - \tau)) + \varrho\beta\mathbb{E}\left[G(n', \varepsilon', 0; \alpha)|n\right]. \tag{2}$$

This household's consumption equals net income (labor income minus the proportion lost due to bankruptcy). In the period after bankruptcy, the household will have no debt. This can be seen in the zero that appears in the function G in the right hand side of the function equation above.

Now turn to a household that decides to pay debt normally. Then, this household must decides which kind of debt contract to use: screening or revelation. Thus, the value function of repaying debt is

$$V(n, \varepsilon, a; \alpha) = \max\{\widetilde{V}(n, \varepsilon, a; \alpha), \widehat{V}(n, \varepsilon, a; \alpha)\}, \tag{3}$$

where $\widetilde{V}(n, \varepsilon, a; \alpha)$ and $\widehat{V}(n, \varepsilon, a; \alpha)$ (defined below) are lifetime utilities associated with borrowing using screening and revelation contracts, respectively. The policy function S indicates whether the household borrows using screening contracts or not:

$$S(n, \varepsilon, a; \alpha) = \begin{cases} 1 \text{ if } \widetilde{V}(n, \varepsilon, a; \alpha) \geq \widehat{V}(n, \varepsilon, a; \alpha), \\ 0 \text{ otherwise.} \end{cases}$$

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because households signal their type by repaying their debt.

Household using the screening debt contract face the debt price $\widetilde{q}(n, a'; \alpha)$ and its lifetime utility is

$$\widetilde{V}(n, \varepsilon, a; \alpha) = \max_{\{a', c\}} u(c) + \varrho\beta\mathbb{E}\left[G(n', \varepsilon', a'; \alpha)|n\right],$$

subject to

(4)

$$c + a'\widetilde{q}(n, a'; \alpha) = a + e,$$

$$c \geq 0,$$

$$e = \alpha\exp(n + \varepsilon).$$

The key here is that the prices $\widetilde{q}$ incorporate the cost of information. In contrast, suppose the household prefers to use a revelation contract. Then, the relevant debt price is $\widehat{q}(m, a'; \alpha, a)$. Lifetime utility in this case is represented by

$$\widehat{V}(n, \varepsilon, a; \alpha) = \max_{\{a', c\}} u(c) + \varrho\beta\mathbb{E}\left[G(n', \varepsilon', a'; \alpha)|n\right],$$

subject to

(5)

$$c + a'\widehat{q}(n, a'; \alpha, a) = a + e,$$

$$c \geq 0,$$

$$e = \alpha\exp(n + \varepsilon).$$

Here I write the problem assuming $\widehat{q}$ satisfies self-revelation. This will impose limits on how much can be borrowed at low interest rates. [14]

3.2 Equilibrium

How are prices determined? First, they must imply zero expected profits. In general, a price function $q(a', n; \alpha)$ implies zero profits if the following equation is satisfied:

$$q(a', n; \alpha) = \frac{1}{1+r}\varrho\mathbb{E}\left[R(n', \varepsilon', a'; \alpha)|n\right].$$

(6)

Looking at this equation it is very clear why prices (or interest rates) depend on (a', n). They depend on a' because it affects the bankruptcy decision, R, at each

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[14]This condition is formally defined below.

possible state. They depend on n because it determines the transition probability to each n' and therefore the next-period labor endowment, e'.

In the case of screening contracts, zero-profit prices must take into account the cost of information. This implies the following condition:

$$\widetilde{q}(a', n; \alpha) = \frac{1 - \mathcal{C}}{1 + r} \varrho \mathbb{E}\left[R(n', \varepsilon', a'; \alpha)|n\right].\tag{7}$$

Additional notation must be introduced to study revelation contracts. The lifetime utility of a household with productivity n and reporting m (potentially $m \neq n$), choosing next period assets a' given the price function q can be written as

$$V^M(n, \varepsilon, a; \alpha, m, a', q) = u(c) + \varrho\beta\mathbb{E}\left[G(n', \varepsilon', a'; \alpha)|n\right],$$

subject to

$$c + a'q(m, a'; \alpha, a) = a + e,$$
$$c \geq 0,$$
$$e = \alpha \exp(n + \varepsilon).$$

Then, for any n, the function $\widehat{q}(m, a'; \alpha, a)$ satisfies self-revelation if for each n and ε,

$$G(n, \varepsilon, a; \alpha) \geq \max_{a', m} V^M(n, \varepsilon, a; \alpha, m, a', \widehat{q}).\tag{8}$$

This means that households would prefer not to misreport the persistent component of productivity. This condition resembles that situation in the two-period model, but with two differences. First, there are more than two types, so the limit for the zero expected profit prices corresponding to n will be the tightest of those set by households with persistent component of productivity $j < n$. The second difference is that households with any i.i.d component, ε, could misrepresent their persistent component of productivity. Again, this implies that the limit will be the tightest among those set by households with different ε.

An *equilibrium* in this economy is a set of value functions, optimal decision rules for the consumer, default probabilities, and bond prices, such that equations (1) to (5) are satisfied; prices of screening contracts, $\widetilde{q}$, satisfy the zero-profit

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condition (7); and prices of revelation contracts, $\widehat{q}$, satisfy the zero-profit condition (6) and the direct revelation condition (8).

4   Calibration

Most parameters can be obtained directly from data or from previous estimation. Only the discount factor, β, the cost of information in 1983, $\mathcal{C}_{1983}$, and the share of income lost in bankruptcy, τ, are calibrated to replicate specific statistics.

Income process.   Following the methodology in Krueger and Perri (2005), the cross-sectional log-income variance can be decomposed into the fixed (σ_α^2), persistent (σ_η^2, ρ), and i.i.d. (σ_ε^2) components. Fixed characteristics are those that can be easily observed—at no cost—by the lenders. This component is referred to as α. It consists of the head of household's gender, education, and age. The variance of the fixed component, $\sigma_{\alpha t}^2$, is the variance of predicted values from a regression of log-income per household on these characteristics. Then, using the autocovariance of residuals and setting a value for ρ, the variance of the residuals can be decomposed into σ_η^2 and σ_ε^2. The value chosen for ρ is in the range of estimates in Storesletten, Telmer, and Yaron (1999), 0.95, and Storesletten (2004), 0.9989. The results, presented in *Table 2*, are in line with Krueger and Perri (2005)'s estimations.

Parameters determined using a priori information.   The survival probability, ϱ, is determined to match a period of a financially active life of 40 years. The utility function is
$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$
where σ was chosen to match a coefficient of risk aversion of 2.

Evans and Schmalensee (2005) describe the lending process for a credit card company after the IT revolution.[15] Of the total cost of lending, about 70 percent

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[15]See page 224.

corresponds to the cost of funds and charge-offs (or the cost of defaults) and only 4 percent to data processing. In terms of the level, Evans and Schmalensee (2005) mention that the cost of processing an application for a credit card is 72 dollars per approved application. This number may overestimate the cost of information in this paper because it involves marketing and other costs not directly related to information.[16] However, it is already small enough that reducing this cost does not change the predictions of the model. Therefore, I set $\mathcal{C}_{2004} = 0$.

Parameters jointly determined. The discount factor, β, and the fraction of earnings lost when households default, τ, are calibrated to minimize the distance to specific targets for 2004. The same parameters are then used in 1983 and 2004. The calibration of the cost of information in 1983, $\mathcal{C}_{1983}$, is explained in the next section, where the model with costly information is analyzed.

Figure 2 shows how β and τ affect the statistics describing the economy calibrated for the year 2004. The default rate and the charge off rate are monotonically decreasing in the share of income lost in bankruptcy, τ. This is very intuitive: As the cost of default increases, its frequency decreases. The monotonicity implies that it would be convenient to use one of these statistics as a target of calibration to pin down τ. The different lines in these graphs show the effect of the discount factor, β. The share of households in debt and the debt-to-income ratio is monotonically decreasing in β. Therefore, it would be ideal to use one of these statistics to calibrate β. As a consequence, the discount factor, β, and the fraction of earnings lost when households default, τ, are calibrated to minimize the distance to the *bankruptcy rate* and the *debt-to-income ratio*. The bankruptcy rate is the ratio of filings to the population; this ratio is 0.53 percent in 2004. The debt-to-income ratio is 1.23 percent in 2004. To determine the number used as the target of calibration both statistics were prorated because income shocks cause only 53 percent of the bankruptcy cases.[17] The values of the

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[16]Also, it may overestimate the cost of information because it corresponds to 2000 instead of 2004.

[17]This adjustment is necessary because the model has only income shocks; in reality other shocks are also important. Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) applied the same procedure when they

parameters that minimize the distance to these targets are presented in *Table 2*. The fit of the targeted moments is presented in *Table 3*.

5   Understanding the Costly Information Model

This section characterizes the costly information model introduced in this paper beyond the analysis in the two-period example. First, consider how the main statistics depend on the cost of information, $\mathcal{C}$. *Figure 3* displays this relationship.[18] The model simplifies to the full information model when the cost of information is zero, and it is independent of the cost of information when this cost if large enough. The default rate is monotonically decreasing in the cost of information. Why does this occur? The explanation is twofold. First, default diminishes because there is less borrowing. In *Figure 3*, both the share of households in debt and the debt-to-income ratio are decreasing in the cost of information. Second, default drops because there is less debt at risk of default. Initially, of all the households, 27 percent have debt and 25 percent have debt with risk of default. Then, as information costs raise, the drop in the share of households with debt at risk of default drops by 10 percentage points and the share of households in debt drops only 4 percentage points. This change is explained by the large drop in the share of households in debt using screening contracts. This share is 80 percent when information costs are zero and drops to zero for information costs larger than 0.25. The charge off rate is not monotone because it is the ratio of the defaulted debt, decreasing in the cost of information, and the total debt, also decreasing in the cost of information.

To characterize the model with costly information further, I now set the value of $\mathcal{C}_{1983}$. It is determined such that the model represents the U.S. economy in 1983. In particular, it is calibrated to match the default rate in that year, after adjusting the income process to 1983 and keeping discount factor, β, and the

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calibrated their model with only income shocks.

[18]These statistics were computed with the income process calibrated for 1983.

fraction of earnings lost when households default, τ, at the value calibrated for the year 2004. The value that minimizes the distance to the target is 0.12. It implies that the minimum interest rate of screening contracts is about 15%. This is the rate paid by households with high persistent component of productivity that started the current period with a large amount of debt. They have very low default risk but they cannot borrow that much at low interest rates because combination of large amounts of debt and low interest rates do not satisfy the self-revelation constraint: households with lower persistent component of productivity (i.e., with higher default risk) would have incentive to misrepresent their report and use this contract. It turned out that in the economy calibrated to 1983 about sixty percent of the borrowers use screening costs, despite its high cost.

[19]

Now the analysis focuses in the model with costly information calibrated to 1983. The value functions are displayed in *Figure 4*. These functions behave as in the standard quantitative model of bankruptcy (Chatterjee, Corbae, Nakajima, and Rios-Rull, 2007). The lifetime utility of a household that decides to repay debt normally, V, is increasing and concave in asset holdings. At the same time, lifetime utility of a household that decides to default on its debt, D, is independent of the stock of assets. This implies that households with enough negative assets— or large enough debt—find defaulting on their debt beneficial.

Figure 4 also shows that both D and V are increasing in the three components of income: the fixed component, α, the persistent component, z, and the i.i.d. component, ϵ. The level of debt at which V and D cross determines the default threshold: households will file bankruptcy if and only if their debt is larger than the threshold. Importantly, in the three panels in *Figure 4* that threshold is decreasing in α, z, and ϵ. This means that there are some intermediate levels of debt at which households with more income would decide to pay its debt normally while household with less income would file for bankruptcy. This is exactly what occurs in the two-period example.

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[19]the fit of the targeted statistic is presented in *Table 3*.

Figure 5 displays the zero-profit prices of debt as a function of the total amount borrowed for households with different levels of income. These prices would represent the equilibrium prices if the cost of information was zero. They are decreasing in the amount borrowed, indicating that everything else constant, households that want to borrow more must pay a higher interest rate. In contrast, households with higher income, for any of the components, pay lower interest rates. Although in the standard quantitative model of bankruptcy (Chatterjee, Corbae, Nakajima, and Rios-Rull, 2007) these prices represent the equilibrium prices offered to households, here they would be available only after paying the cost of information or if they satisfy the self-revelation constraint. Thus, the households that do not pay the cost of information face borrowing limits on the prices described above imposed by private information.

Figure 6 contains six panels with the revelation prices that agents with different levels of current assets, a, face in any given period. Focus first in the top panel. This is the price that an agent with $a = -0.31$ and low observable component of productivity, $\alpha = \alpha_H$, is offered as revelation contract, independently of the level of the unobservable component of productivity, z and ϵ. Notice that the maximum amount of debt offered is $a' = -0.015$. This is much less than households with high z would be able to borrow in an economy with perfect information. For a simple comparison, look again at the top right panel of *Figure 5*. Notice that the limits are even tighter for households with less debt, as shown in the bottom panels of *Figure 6*. This happens because households with less debt are willing to borrow less and as a consequence are "harder to separate" by offering them more debt at higher interest rates.

6   The Role of the IT Revolution

To isolate the effect of changes in the cost of information from changes in other parameters (income, cost of default, so on), this section presents the results of computing a counterfactual economy. This economy is referred to as counterfac-

tual 1983 and answers the following question: What would the bankruptcy rate (and other statistics) be in 1983 with the information technologies of 2004? The results are presented in *Table 4*.

The first row in *Table 4* shows the only exogenous difference between the two economies: the cost of information. The next four rows display variables of interest generated by the models. The third column shows the change in these variables between the model calibrated for the year 1983 and the economy referred to as counterfactual 1983. The last column shows the actual change in these variables between 1983 and 2004.

Importantly, the default rate would have been 119 percent larger in 1983 if the cost of information were as small as in 2004. As the consequence of the same change, the share of households in debt increased 8 percent, the debt-to-income ratio increased 36 percent, and the charge-off rate increased 22 percent. All changes are in the same direction than in the data, displayed in the last column. Additionally, the model also replicates the fact that the largest increase is in the default, then the debt-to-income ratio and last the share of households in debt. In this sense, the mechanism in the model is able to generate some of the distinguishing changes during this period.

The last two rows present statistics that help in understanding the model: the share of households using risky debt and the share of debtors using screening contracts. As a consequence of the adoption of information technologies in 1983, the share of households borrowing with risk of default would increase from 22 to 25 percent and the share of debtors using screening contracts would increase from 62 to 84 percent. This is the key to understand the mechanism: the increase in bankruptcy occurs because households are allowed to take more risk of bankruptcy by the drop in the cost of screening contracts.

Welfare gains of information. To gain more insight on the effect of information, this section consider welfare gains of moving from the economy calibrated to 1983 to a counterfactual economy with all the same parameters but zero cost of

information. Notice that this comparison is between steady states. It is valid as the measure a household would consider to decide whether to be born (with certain characteristics) in one economy or the other. A consumption equivalent (CE) unit indicates how much consumption should be increased in every period in the economy with costly information so that a household in the state $(n, \varepsilon, a, \alpha)$ is indifferent between "being born" in that economy and in one with zero information costs.

Figure 7 presents the welfare comparison for households in different states. Welfare gains depend on the level of assets. All panels in *Figure 7* show minimal gains for agents with large holdings of assets. Notice also that for negative-enough assets holdings (too much debt), welfare gains are independent of the amount owed. This occurs because such households would default in that state, and the gains come only from the next period and later, when they will start with zero assets.

Perhaps the most interesting result is that the maximal gains, around 0.15%, are achieved by households with high persistent component of income that have some debt and are planning to refinance those obligations. These are the households in need of financing that would pay for the (expensive) screening cost or reduce their consumption drastically in the economy with $\mathcal{C} = 0.12$. They experience an abrupt decline in interest rates in an scenario with perfect information. This explains the spike in welfare gains in the second panel on the left and the third on the right in *Figure 7*.

7 Conclusions

What is the role of the IT revolution in the transformation of the unsecured credit market? Asymmetric information and costly screening are incorporated into a model of consumer debt and bankruptcy to study this question. In a simple example, a drop in information costs allow previously borrowing-constrained households to borrow more. Since the borrowing limits imposed by private infor-

mation prevent households from borrowing with risk of default, as the limits are relaxed not only debt but also the number of bankruptcy filings increase.

Can this model account for the changes in consumer credit markets over the past 20 years? The calibrated model replicates the main characteristics of the unsecured credit market before (1983) and after (2004) the IT revolution. In the economy calibrated for 1983, the lowest interest rate on screening contract is about 15% and revelation contracts imply very tight borrowing limits. As a consequence, some households are borrowing constrained. A drop in the cost of information allow these households to use screening contracts and borrow more, with higher risk of default. A quantitative exercise suggests that the cost of information alone accounts for a significant part of the changes in borrowing and bankruptcy. In particular, the drop in information costs accounts for the disproportional increase in the number of bankruptcy filings, compared with the debt-to-income ratio and mainly the share of households in debt.

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8   Appendix: Data

The measure of debt used here was first used by Chatterjee, Corbae, Nakajima, and Rios-Rull (2007). It is defined as minus net worth for households with a negative net worth , and zero for households with positive net worth. This measure may be preferable because it is actual unsecured debt. For the case of net worth debt, the sample analyzed is restricted to household heads with age 22-65 that have incomes greater that zero. In addition, households whose negative net worth to median income ratio is in the top 1% of the distribution of this ratio are excluded since these households are likely to have a substantial net worth debt due to entrepreneurial activities. For 1983, new worth is obtained from the 1983 Survey of Consumer Finances (SCF) Full Public Data Set as "Net worth" (b3324). For 2004, it is extracted from the 2004 Tabling Wizard as "Total net worth of household" (NETWORTH). Income is defined as "Total 1982 Household income" (b3201) for 1983, and "Total amount of income of household" (INCOME) for 2004. For 1983, sample weights used were the "Extended income FRB weight"

(b3016) and for 2004, they were "sample weight" (WGT) found in the tabling wizard. A few alternative measures of unsecured debt can be computed from the SCF, though.

The first alternative considered is referred to as unsecured debt. In 1983 it is computed as "Total Consumer Debt" (b3319) minus "Total Loans for Automobile Purchase" (b4205). In the 2004 Tabling Wizard, it is the variable "Total value of debt held by household" (DEBT) minus "Total value of debt secured by the primary residence held by household" (MRTHEL) minus "Total value of vehicle loans held by household" (VEH_INST) minus "Total value of debt for other residential property held by households" (RESDBT). The mean-debt-to-mean-income ratio using this measure of debt is 10.1 percent in the 1983 SCF and 11.2 percent in the 2004 SCF.

Other alternative is revolving debt. In 1983, this variable is "Total Revolving Charge Debt" (b4101). In the 2004 Tabling Wizard, it is computed as the variable "Total value of credit card balances held by household" (CCBAL) plus "Total value of other lines of credit held by household" (OTHLOC). This variable includes credit card debt and other lines of credit. The mean debt-to-mean income ratio using this measure of debt is 3.4 percent in 1983 SCF and 4.1 percent in 2004 SCF, a 23 percent increase.

Finally, one could consider credit card debt. The rise is the largest in this case. The debt-to-income ratio is 1.2 percent in the 1983 SCF and 3.3 percent in the 2004 SCF. Notice that the rise in this case will be biased because of the substitution of other types of credit with credit card debt.

Bankruptcy data are found in the American Bankruptcy Institute's non-business U.S Bankruptcy Filings 1980-2010 (from the "Business, Non-Business, Total" Table), while population data are from the U.S Census Bureau. Charged off rates are published in the Federal Reserve Statistical Release of Charge-Off and Delinquency Rates on Loans and Leases at Commercial Banks and correspond to credit card consumer loan charge-off rates in Q4-1985 and Q4-2004 for 1983 and 2004, respectively.

Table 1: Borrowing and bankruptcy, 1983 and 2004

Statistics	1983	2004	% change
Mean debt-to-mean income ratio (net worth), %	0.50	1.23	146.0
Share of households with net worth debt, %	5.64	8.01	42.0
Bankruptcies-to-population ratio, %	0.12	0.53	341.7
Charge-off rate,%	2.98	4.52	51.7

Source: See the Appendix.

Table 2: Parameters values

Parameter	Calibration year	
	1983	2004
σ_α, Standard deviation of fixed effects	0.205	0.294
σ_η, Standard deviation of persistent shocks	0.089	0.114
σ_ε, Standard deviation of i.i.d shocks	0.242	0.324
ρ, Autocorrelation of the persistent component	0.980	0.980
ϱ, Survival rate	0.975	0.975
σ, Relative risk aversion coefficient	2.000	2.000
r, Risk free interest rate	1.0%	1.0%
$\mathcal{C}$, Cost of information	0.0	0.12
τ, Share of income lost in bankruptcy	0.047	0.047
β, Discount factor	0.901	0.901

Table 3: Target and Model statistics.

Statistics	Target	Model
Default rate, 2004 (%)	0.283	0.246
Mean debt-to-mean income ratio, 2004 (%)	0.650	0.663
Default rate, 1983 (%)	0.065	0.064

Table 4: The effect of information costs

Variable	(1) Model, counterfactual 1983	(2) Model, 1983	(1)/(2)-1 Model (% change)	(2004)/(1983)-1 Data (% change)
Cost of information	0	0.120	–	
Default rate	0.140	0.064	118.5	341.7
Share households in debt	26.297	24.383	7.8	42.0
Debt-to-income ratio	0.827	0.609	35.9	146.0
Charged-off rate	0.472	0.388	21.6	51.7
Share of households with risky debt	25.212	22.338	–	
Share of debtors using screening contracts	83.677	61.563	–	

Figure 1: The mechanism in a simple example

Legend:
- - - - Indifference curve, low-income household
- ——— Debt price q, low-income household
- — · — Indifference curve, high-income household
- ——— Debt price q, high-income household

q, debt price

High-income household debt under perfect information

High-income household debt under assymmetric information

Borrowing limit at this price under asymmetric information

Low-income household debt under perfect information and asymmetric information

a_2, next-period assets

Figure 2: Main statistics as a function of β and τ, Perfect information model
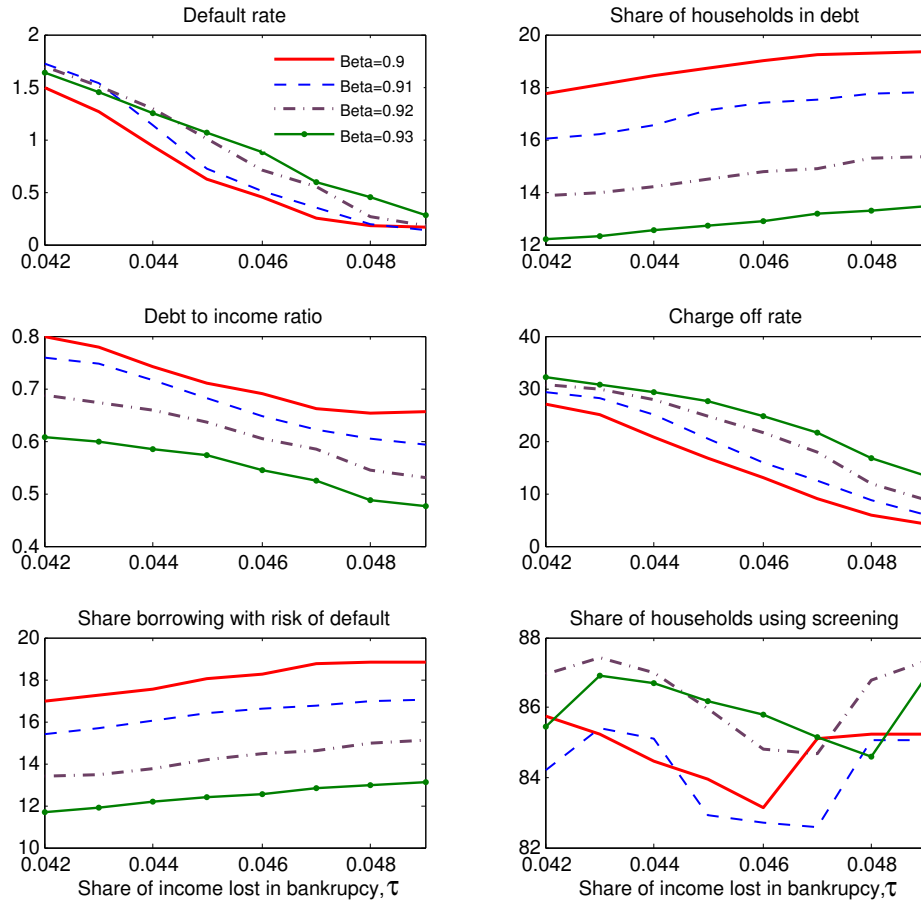
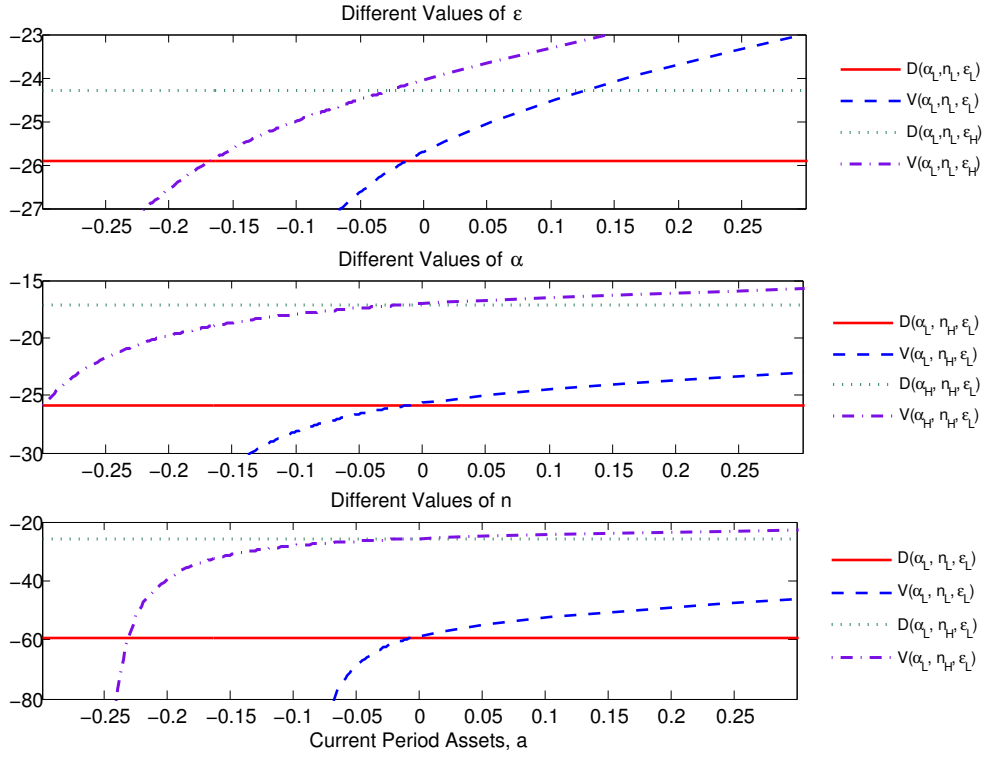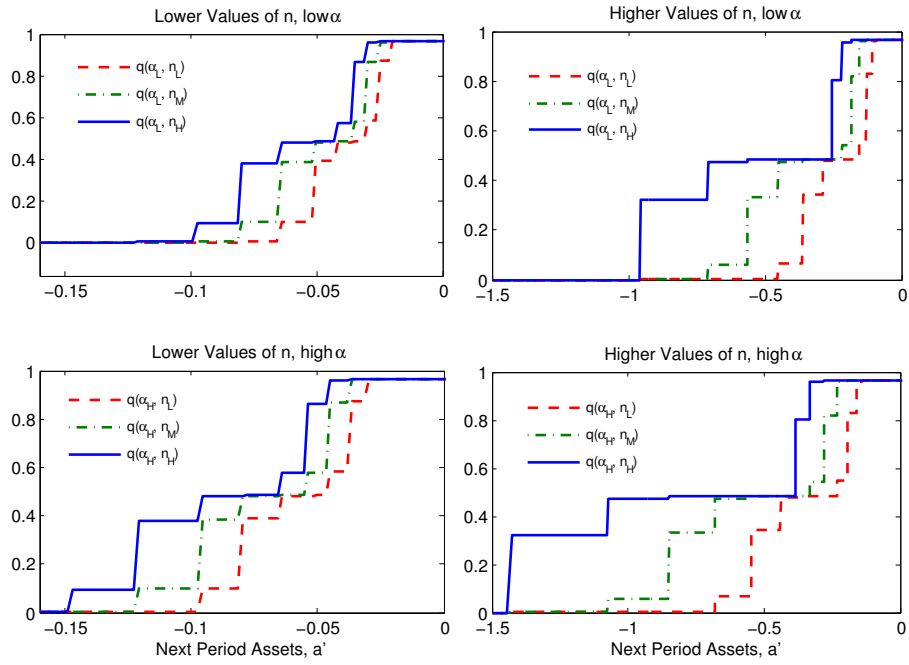**Figure 3: Main statistics as a function of the cost of information, $\mathcal{C}$**
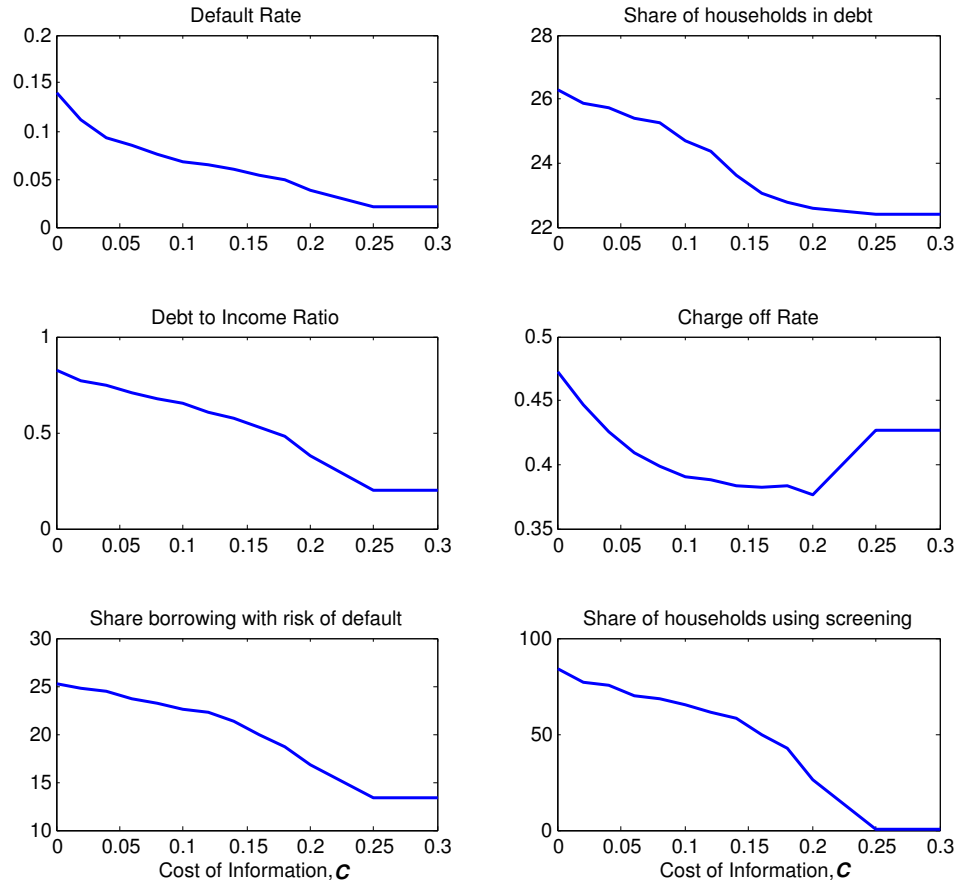
Figure 4: Value functions

Note: Graphs show the effect of changes to α, z, and ϵ while maintaining the rest of the variables constant. z_L has a value of 0.226 while z_H has a value of 0.475.

Figure 5: Zero-expected-profits prices

Note: Left column shows q with respect to small, medium, and large values of z, which are 0.7424, 0.862, and 1, respectively. Right column shows q with respect to small, medium, and large values of z, which are 2.836, 3.2919, and 3.8206, respectively. Top column shows q maintaining α at a low value, while bottom row shows q maintaining α at a high value.

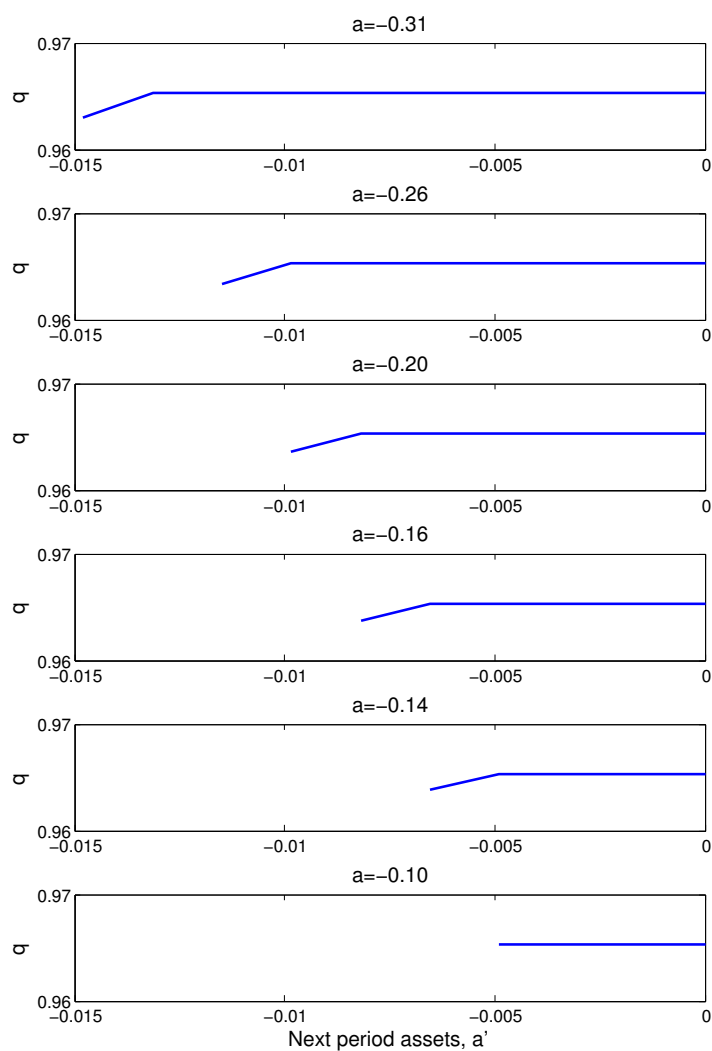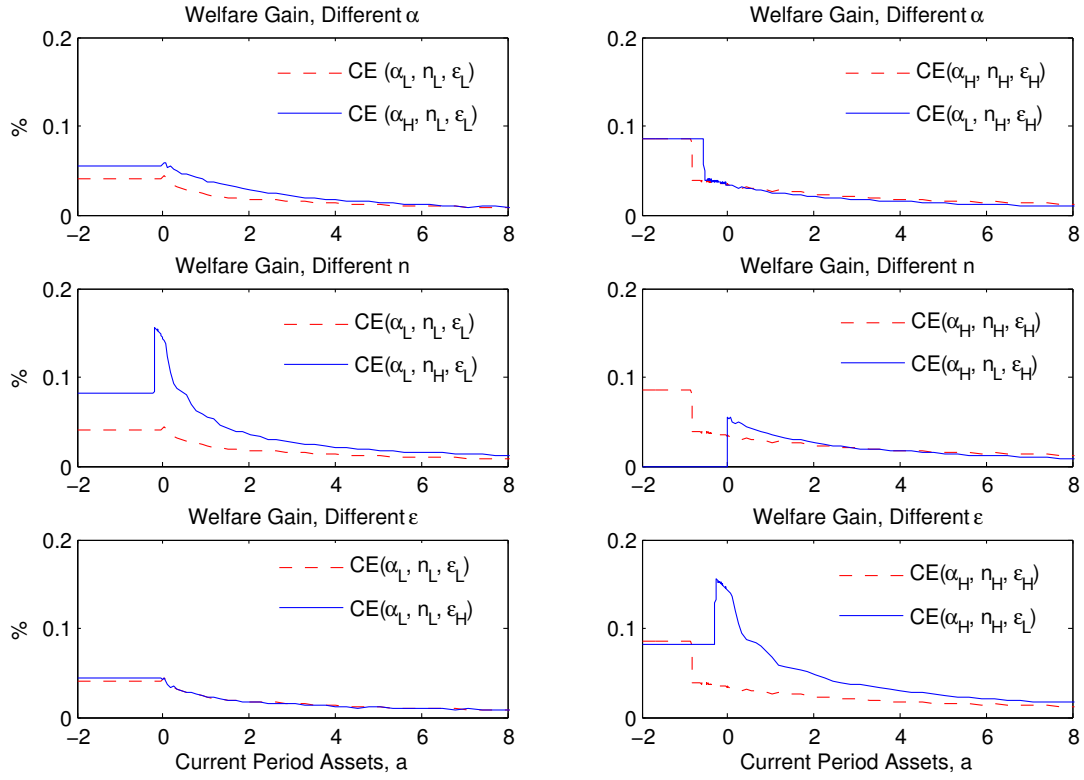Figure 6: Limits on borrowing imposed by private information, examples

Figure 7: Welfare gains of IT revolution

Note: Left column shows changes to α, n, and ϵ while maintaining the rest of the variables constant to a low value. Right column shows changes to α, n, and ϵ maintaining the rest of the variables at a high value. n_L has a value of 0.2256 and n_H has a value of 3.821.

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