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Abstract

I examine optimal monetary policy in a Lagos and Wright [A unified framework for monetary theory and policy analysis, J. Polit. Econ. 113 (2005) 463–484] model where trade is centralized and all exchange is voluntary. I identify a class of incentive-feasible policies that improve welfare beyond what is achievable with zero intervention. Any policy in this class necessarily entails a non-negative inflation rate and a strictly positive nominal interest rate. Despite the absence of a lump-sum tax instrument, there exists an incentive-feasible policy that implements the first-best allocation.

Key words: Money; Interest; Friedman rule; Voluntary trade; Incentive-feasible policies; Efficient implementation. JEL codes: E4, E5.

1 Introduction

I examine optimal monetary policy in a version of the Lagos-Wright [7] model where trade among individuals is competitive; see also Rocheteau and Wright [8]. Absent policy intervention, the competitive monetary equilibrium is inefficient. That is, the real rate of return on money is too low; so that individuals are motivated to economize excessively (from a social perspective) on their real money balances. Efficiency is restored when the real rate of return on money is equated to the rate of time-preference (the Friedman rule).

It is standard in this literature to assume that the government has the ability to levy lump-sum taxes. Because money is generally superneutral when it is introduced by way of interest, the Friedman rule may be implemented with virtually any inflation rate when a lump-sum tax instrument is available. This may be accomplished, for example, with zero inflation and a positive nominal interest rate; where interest payments are financed by a lump-sum tax. Alternatively, it may be accomplished with deflation and a zero nominal interest rate; where the deflation financed by a lump-sum tax that is used to contract the money supply. That is, interest-bearing money is not essential when a lump-sum tax instrument is available. [1]

In environments where *all* trade is restricted to be voluntary–including those involving people and their government–lump-sum taxation is unavailable as a policy instrument. The question of interest here is what this restriction implies in terms of efficient implementation. A reasonable conjecture is that the constrained-efficient policy entails zero intervention (at least, this was my own prior). But I demonstrate below that this in fact not the case; i.e., there exist policies that can strictly improve on the allocation associated with zero intervention. Indeed, I demonstrate that the Friedman rule remains implementable under a suitably designed policy.

Naturally, voluntary trade imposes restrictions on the design of an optimal policy. In particular, policies are constrained to be incentive-feasible in that lump-sum payments, if they are to be made, must respect individual rationality. I demonstrate that the standard prescription of deflating at the rate of time-preference is not an incentive-feasible policy. In particular, in the class of incentive-feasible policies I study, a welfare-improving policy necessarily entails a non-negative inflation rate and a strictly positive nominal interest rate. It is in this sense then that interest-bearing money (and inflation) is essential; at least, in the version of the Lagos-Wright model that I study below.

2 Environment

There is a nonatomic measure of infinitely-lived individuals $i \in [0,1]$. Time is discrete; $t = 0, 1, 2, ..., \infty$ and, following Lagos and Wright [7], there are two subperiods at each date labeled *day* and *night*. A distinct nonstorable output is produced and consumed in each subperiod and people meet in a central location at every point in time. [2]

Let $x_t(i) \in \mathbb{R}$ denote the consumption of output during the day in period t by individual i (a negative quantity is interpreted as production). As this output is nonstorable, an aggregate resource constraint implies

$$\int x_t(i)di \leq 0 \tag{1}$$

[1] Nor, one might add, is *fiat* money essential. The ability to lump-sum tax implies that the government has ownership and control over a real asset (e.g., human capital). The government could, in this case, issue paper that is fully backed by the revenue stream generated by this asset.

[2] This is in contrast to the original Lagos and Wright [4] formulation where trade is "centralized" in one subperiod and "decentralized" in the other.

for all $t \geq 0$. Let $\{c_t(i), y_t(i)\} \in \mathbb{R}^2_+$ denote consumption and production, respectively, of output at night in period t by individual i. As this output too is nonstorable, an aggregate resource constraint implies

$$\int c_t(i)di \leq \int y_t(i)di; \tag{2}$$

for all $t \geq 0$.

Individuals have preferences that are linear in $x_t(i)$. At the beginning of every night, each individual experiences an idiosyncratic shock that determines their *type* for the night. There are three possible types. A *consumer* has a desire to consume and an inability to produce. The utility flow for a consumer is given by $u(c_t(i))$, where $u'' < 0 < u'$, $u'(0) = \infty$ and $u(0) = 0$. A *producer* has an ability to produce and no desire to consume. The utility flow for a producer is given by $-h(y_t(i))$, where $h', h'' > 0$, $h'(0) < \infty$ and $h(0) = 0$. An *inactive* individual has neither a desire to consume nor an ability to produce. The utility flow associated with inactivity is normalized to zero.

An individual is inactive with probability $1 - 2\pi$ $(0 < \pi \leq 1/2)$ and is otherwise a consumer or producer with equal probability. For each individual, the stochastic process determining types is *i.i.d.* across time and at each date there is a measure π of consumers and a measure π of producers. The utility function for individual i is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ x_t(i) + \pi \left[u(c_t(i)) - h(y_t(i)) \right] \right\} \tag{3}$$

where $0 < \beta < 1$.

Weighting all individuals equally, a planner maximizes (3) subject to the resource constraints (1) and (2). As utility is linear in $x_t(i)$, individuals are indifferent across any lottery over $\{x_t(i) : t \geq 0\}$ that delivers a given expected value. Without loss of generality, a planner may set $x_t(i) = 0$ for all i and all $t \geq 0$. Since h is strictly convex, all producers will be required to produce the same level of output $y \geq 0$. Given the strict concavity of u, all consumers will be allocated the same level of consumption $c \geq 0$. As the population of active individuals divided equally among producers and consumers at night, the resource constraint (2) implies $c = y$. It follows that, conditional on a given level of y (and invoking the fact that $E_t[x_t(i)] = 0$), *ex ante* welfare is represented by

$$W(y) = \pi(1-\beta)^{-1} \left[u(y) - h(y) \right]. \tag{4}$$

Clearly, there is a unique maximizer $0 < y^* < \infty$ satisfying

$$u'(y^*) = h'(y^*). \tag{5}$$

In what follows, I refer to y^* as the *first-best* allocation. Associated with this allocation is any lottery over $x_t(i)$ that generates $E_t[x_t(i)] = 0$.

I want to impose restrictions on this environment that will render fiat money essential. First, there is no commitment or coercion; so that *all* exchanges–including exchanges involving people and their government–are restricted to be sequentially rational (individually rational at every point in time). Among other things, this rules out the use of coercive lump-sum tax instruments.[3] Second, individuals are anonymous in the sense that their personal trading histories are not monitored. This second restriction, together with the first, rules out private debt. Third, I assume that only society (the government) can create durable, divisible, and non-counterfeitable tokens. This third restriction, together with the second, implies that the economy's record-keeping technology is limited to what can be achieved with the use of these tokens. Finally, I restrict trade among individuals to occur in competitive spot markets (though exchange between individuals and the government is not restricted in this manner).[4]

3 Individual decision-making

Individuals begin time endowed with the economy's supply of tokens; which, anticipating their use as a medium of exchange, will henceforth be labeled *money*. Let (v_1, v_2) denote the (non-zero and finite) price of money in the day and night markets, respectively.

Government policy will be explained in detail below, but it will be useful to describe here the aspects of policy that are relevant for individual decision-making. The government's policy rule operates at the beginning of the day, *prior* to day-market trading. An individual who enters the day with money balances m has the *option* of approaching a "government counter" and transforming these balances into $Rm - T$ units of money. If $R > 1$ and $T > 0$, then money here is like an interest-bearing bond subject to a flat redemption fee. If an individual declines the redemption option, he simply enters the day-market with m units of money.[5]

Following day-market activity, individuals carry money into the night and realize their types (whether producer, consumer, or inactive). Subsequent to

[3] It does not, however, rule out the use of distortionary taxes. These taxes are voluntary in the sense that the tax obligation can be avoided by choosing not to engage in the activity being taxed. In what follows, I abstract from distortionary taxation as the use of such instruments alone cannot improve welfare.

[4] The restriction to competitive trade is not innocuous; see, for example, Hu, Kennan and Wallace [4]).

[5] Andolfatto [1] and Hu, Kennan and Wallace [4] interpret T as a voluntary lump-sum tax. In these latter papers, an agent may avoid a lump-sum tax by not participating in trade. In this sense, the lump-sum tax in these latter papers resembles a "market participation fee," rather than the redemption fee modeled here.

night-market activity, individuals carry any remaining money balances forward to the next day, where they are once again presented with the option of redeeming their money for interest.

3.1 The day-market

Let $m_1 \geq 0$ denote an individual's money balances at the beginning of a day; and let $m_2 \geq 0$ denote the money balances carried forward into the night. Let $\omega \in [0,1]$ denote the probability of exercising the redemption option. Subsequent to the redemption choice, the individual is free to purchase or sell output (utility) x at the going market price v_1; so that the day budget constraint is given by $x = v_1 [\omega(Rm_1 - T) + (1-\omega)m_1 - m_2]$. Define the real quantities $q_1 \equiv v_1 m_1$ and $q_2 \equiv v_2 m_2$. As well, let $\tau \equiv v_1 T$ and define $\phi \equiv v_1/v_2$. The day budget constraint can now expressed as

$$x = \omega(Rq_1 - \tau) + (1-\omega)q_1 - \phi q_2 \tag{6}$$

I seek a recursive representation of the choice problem. Let $D(q_1)$ represent the individual's maximum value function at the beginning of the day with real money balances $q_1 \geq 0$; and let $N(q_2)$ represent the individual's maximum value function at the beginning of the night (prior to realizing his type) with real money balances $0 \leq q_2 \leq \overline{q}$; where $\overline{q}$ is an arbitrarily large, but finite, upper bound. These two value functions must satisfy the following recursion

$$D(q_1) \equiv \max_{\omega, q_2} \{\omega(Rq_1 - \tau) + (1-\omega)q_1 - \phi q_2 + N(q_2)\} \tag{7}$$

I make the following assumption:

[**A1**] *The function* $N : \mathbb{R}_+ \rightarrow \mathbb{R}$ *satisfies* $N'' < 0 < N'$ *and* $\phi < N'(0)$.

Naturally, as N and ϕ are equilibrium objects, I will have to verify later on that the properties assumed in [**A1**] are in fact valid.

The quasi-linear environment simplifies matters considerably. In particular, the redemption choice ω and money demand q_2 can be characterized independently of each other. The optimal redemption choice satisfies

$$\begin{aligned} \omega = 1 \quad & \text{if} \quad (R-1)q_1 > \tau \\ \omega \in [0,1] \quad & \text{if} \quad (R-1)q_1 = \tau \\ \omega = 0 \quad & \text{if} \quad (R-1)q_1 < \tau \end{aligned} \tag{8}$$

It follows that if $R > 1$ and $\tau > 0$, then only individuals with sufficiently large money balances q_1 will find it optimal to pay the redemption fee τ.

[**A1**] implies that the demand for real money balances $0 < q_2 < \overline{q}$ is determined uniquely by,

$$\phi = N'(q_2) \tag{9}$$

As in Lagos and Wright [7], the demand for money at this stage is independent of initial money holdings q_1, so that all individuals enter the night with identical money balances. However, unlike Lagos and Wright [7], the value function D is not (in general) linear in q_1. In particular,

$$D'(q_1) = \begin{cases} R & \text{if} \quad (R-1)q_1 > \tau \\ 1 & \text{if} \quad (R-1)q_1 < \tau \end{cases} \tag{10}$$

By the Theorem of the Maximum, D is continuous in q_1. But if $R > 1$ and $\tau > 0$, D is piece-wise linear (and convex) in q_1 and non-differentiable at the point $q_1 = (R-1)^{-1}\tau$.

3.2 The night-market

3.2.1 Consumers

Let $C(q_2)$ denote the value associated with being a consumer in possession of real money balances q_2. This money is used to make purchases of output $0 \leq c \leq \overline{y} < \infty$ at the prevailing price-level v_2^{-1}. Hence, future money balances are given by $m_1^+ = m_2 - v_2^{-1}c$, with the restriction that $m_1^+ \geq 0$. Expressed in real terms, this constraint is given by $q_1^+ = (v_1^+/v_1)\phi(q_2 - c) \geq 0$. The choice problem can be stated as,

$$C(q_2) \equiv \max_{c, q_1^+} \left\{ u(c) + \beta D(q_1^+) : q_1^+ = (v_1^+/v_1)\phi(q_2 - c) \ \geq 0 \right\} \tag{11}$$

The solution here is at the very least an upper hemi-continuous correspondence; but in what follows, I anticipate that the relevant range for q_1^+ will fall below the critical value $(R-1)^{-1}\tau$. In other words, I anticipate that a consumer will not find it desirable–in equilibrium–to exercise his future redemption option. As D is differentiable below this range, the solution will in this case constitute a pair of functions. In fact, I will go even further here in assuming that the solution to (11) is characterized by a binding debt-constraint; i.e.,

[A2] $c = q_2$ and $q_1^+ = 0$.

If the conjecture [**A2**] is valid, then it implies that consumers returning to the day-market will find it optimal not to exercise the redemption option. By the envelope theorem, we have

$$C'(q_2) = u'(c) \tag{12}$$

3.2.2 Inactive Agents

Let $I(q_2)$ denote the value associated with being inactive and in possession of real money balances q_2. As an inactive individuals neither consume nor produce, they enter the next day with real money balances $q_1^+ = (v_1^+/v_1)\phi q_2$. Hence $I(q_2) \equiv \beta D((v_1^+/v_1)\phi q_2)$. I anticipate that the following condition will hold in equilibrium:

[A3] $q_1^+ > (R-1)^{-1}\tau$ whenever $R > 1$.

If the conjecture **[A3]** is valid, then it implies that inactive individuals returning to the day-market will find it optimal to exercise the redemption option. If this is the case, then

$$I'(q_2) = \phi(v_1^+/v_1)R\beta \tag{13}$$

3.2.3 Producers

Let $P(q_2)$ denote the value associated with being a producer in possession of real money balances q_2. If a producer makes sales of output $0 \leq y \leq \overline{y} < \infty$ at the prevailing price-level v_2^{-1}, his future money balances are given by $m_1^+ = m_2 + v_2^{-1}y$; or, in real terms, $q_1^+ = (v_1^+/v_1)\phi(q_2 + y)$. Clearly, the constraint $q_1^+ \geq 0$ will not bind in this case; and the choice problem may be formulated as,

$$P(q_2) \equiv \max_{y, q_1^+} \left\{ -h(y) + \beta D(q_1^+) : q_1^+ = (v_1^+/v_1)\phi(q_2 + y) \right\}. \tag{14}$$

Note that if condition **[A3]** holds, then producers too must strictly prefer to exercise their redemption option the next day. The supply of output at night $0 < y < \overline{y}$ is therefore characterized by

$$h'(y) = \phi(v_1^+/v_1)R\beta \tag{15}$$

Moreover, by the envelope theorem,

$$P'(q_2) = \phi(v_1^+/v_1)R\beta \tag{16}$$

3.2.4 Gathering restrictions

The *ex ante* value function associated with entering the night-market with money balances q_2 is given by $N(q_2) \equiv \pi C(q_2) + (1-2\pi)I(q_2) + \pi P(q_2)$. Employing the envelope results (12), (13) and (16), we have,

$$N'(q_2) = \pi u'(c) + (1-\pi)\phi(v_1^+/v_1)R\beta \tag{17}$$

Note that if condition **[A2]** holds, then the function N essentially inherits the properties of u; in other words,

Lemma 1 *Condition* **[A2]** *implies* **[A1]**.

Now, combining (9) with (17) yields $\phi = \pi u'(c) + (1-\pi)\phi(v_1^+/v_1)R\beta$. Making use of (15), this latter expression (after some manipulation) can be expressed as

$$(v_1^+/v_1)R\beta\pi u'(c) = [1 - (v_1^+/v_1)R\beta(1-\pi)]h'(y) \tag{18}$$

4 Government policy

Recall that the government's operating procedure is to intervene at the beginning of each day, prior to day-market trading. The policy rule is to pay a nominal interest rate R on money balances presented for redemption to individuals willing to pay the redemption fee T.

Let M denote the supply of money during any given period; with M^- denoting "previous" period's money supply. Condition [**A2**] implies that the entire money supply M^- is held by producers and inactive individuals at the beginning of the day. Condition [**A3**] implies that both producers and inactive individuals will find it optimal to pay the redemption fee T. Hence, the government has an aggregate interest obligation $(R-1)M^-$ along with revenues from redemptions equal to $(1-\pi)T$.

The government may also print new money $M - M^-$. A feasible government policy will have to satisfy the government budget constraint, $(R-1)M^- = M - M^- + (1-\pi)T$; or

$$RM^- = M + (1-\pi)T$$

Let μ denote the (gross) rate of money supply expansion, so that $M = \mu M^-$. The government budget constraint can now be expressed as $T = (R/\mu - 1)(1-\pi)^{-1}M$. Multiply both sides of this latter expression by v_1 and note that $v_1 M \equiv \phi q_2$. Hence, the government budget constraint may alternatively be expressed in real terms by

$$\tau = (R/\mu - 1)(1-\pi)^{-1}\phi q_2 \tag{19}$$

In what follows, I refer to (R, μ, τ) as a government policy. An *incentive-feasible policy* is a government policy that satisfies (19), together with conditions [**A2**] and [**A3**].

5 Equilibrium

In this section, I characterize the steady-state monetary equilibrium given an incentive-feasible government policy.

To begin, market-clearing at night implies $c = y$ and $v_2 = y/M$ or $q_2 = y$ (by condition **[A2]**, consumers exhaust their money balances). As $y = y^+$, it follows that $v_2^+/v_2 = 1/\mu$. Moreover, as $\phi = \phi^+$ and $v_1 \equiv \phi v_2$, it follows that $v_1^+/v_1 = 1/\mu$ as well. These results, together with condition (18), imply

$$\delta\beta\pi u'(y) = [1 - \delta\beta(1-\pi)]h'(y) \tag{20}$$

Condition (20) characterizes the equilibrium level of night-market output y conditional on an incentive-feasible policy $\delta \equiv R/\mu$.

To characterize the equilibrium price ϕ, observe that (9) and (15) together imply

$$\phi = \pi u'(y) + (1-\pi)h'(y) \tag{21}$$

With (y, ϕ) so determined, condition (19) delivers (utilizing the condition $q_2 = y$) an expression for the equilibrium redemption fee

$$\tau = (\delta - 1)(1-\pi)^{-1}\phi y \tag{22}$$

Next, I derive the equilibrium distribution of real money balances at the beginning of the day. By condition **[A2]**, $q_1^+ = 0$ for consumers. As inactive individuals neither consume nor produce, $q_1^+ = (\phi/\mu)y$. And as producers augment their money balances by sales to consumers, $q_1^+ = (\phi/\mu)2y$. Hence, the steady-state distribution for q_1 is given by

$$F(q_1) = \begin{cases} \pi & \text{for} \quad 0 \leq q_1 < (\phi/\mu)y \\ 1-\pi & \text{for} \quad (\phi/\mu)y \leq q_1 < (\phi/\mu)2y \\ 1 & \text{for} \quad (\phi/\mu)2y \leq q_1 < \infty \end{cases} \tag{23}$$

Finally, combine the equilibrium objects above with the budget constraint (6) to derive the consumption allocation across types in each day. For those who were consumers in the previous night, $\omega = 0$ and $q_1 = 0$; so that

$$x = -\phi y \tag{24}$$

For those who were inactive the previous night, $\omega = 1$ and $q_1 = (\phi/\mu)y$; so that

$$x = \left[\frac{1-\delta\pi}{1-\pi}\right]\phi y - \phi y \tag{25}$$

And for those who produced in the previous night, $\omega = 1$ and $q_1 = (\phi/\mu)2y$; so that

$$x = \left[\frac{1+(1-2\pi)\delta}{1-\pi}\right]\phi y - \phi y \tag{26}$$

One can easily verify that the population-weighted sum of (24), (25) and (26) is equal to zero.

5.1 *Zero intervention*

Zero intervention ($R = \mu = 1$) is trivially an incentive-feasible policy. Such a policy implies $\delta = 1$; which, by condition (20) determines an equilibrium level of output $0 < y_0 < y^*$. In a monetary equilibrium, the debt-constraint for consumers will bind tightly so that [**A2**] holds. Condition [**A3**] is irrelevant here, as $R = 1$.

In fact, it should be clear that there exists a class of incentive-feasible policies (R, μ) satisfying $\delta = R/\mu = 1$ and $R > 1$ that implements the zero intervention allocation y_0 as an equilibrium. That is, as the real rate of return on money remains unchanged, the debt-constraint for consumers binds as before; so that [**A2**] holds. As the government's interest obligation is in this case financed entirely by new money creation, the equilibrium fee in this case is zero. As inactive individuals and producers carry strictly positive money balances into the day, condition [**A3**] must necessarily hold when $\tau = 0$. It follows that money is superneutral when it is introduced in the form of interest.

6 Welfare-improving incentive-feasible policies

The interesting question, of course, is whether it might be possible to implement an allocation that improves upon y_0. As *ex ante* welfare $W(y)$ is strictly increasing in y over the range $[y_0, y^*)$, and as (20) implicitly defines a function $\hat{y}(\delta)$ that is strictly increasing in δ, I restrict attention to policies that satisfy $\delta > 1$. In addition, I restrict attention to policies that satisfy $\delta\beta < 1$; since otherwise, a monetary equilibrium will fail to exist.[6] Together then, the relevant range of policies is given by

$$1 < \delta < \beta^{-1} \tag{27}$$

Note that for any such policy, condition (22) implies that $\tau > 0$. That is, it is critical that the government's interest obligation not be financed entirely by money creation.

The characterization of equilibrium derived earlier is predicated on the validity of [**A2**] and [**A3**]. I now turn to checking the validity of these assumptions for any given policy (R, μ) satisfying (27) and (22).

Consider first [**A3**], which asserts that individuals who are inactive at night will find it optimal to exercise the redemption option the next day. In the proposed equilibrium, inactive individuals enter the day-market with real money balances $q_1 = (\phi/\mu)y$. By (8), exercising the redemption option is (strictly) preferred if and only if $(R - 1)(\phi/\mu)y > \tau$; or, by appealing to (22), if and

[6] That is, except in the limit as $\delta \nearrow \beta^{-1}$.

only if

$$\left(\delta - \frac{1}{\mu}\right)\phi y > \left(\frac{\delta - 1}{1-\pi}\right)\phi y$$

Simplifying, we have the following restriction

$$\mu > \left[\frac{1-\pi}{1-\delta\pi}\right] > 1 \text{ for } \delta > 1 \tag{28}$$

Condition (28) implies that deflationary policies are not incentive-feasible. In particular, the "standard" Friedman rule prescription of setting $(R, \mu) = (1, \beta)$ is not incentive-feasible. Under this latter policy, the implied lump-sum tax τ (redemption fee) would not be willingly paid by any individual, so that a contraction of the money supply impossible. Of course, since $R \equiv \delta\mu$, it follows that $R > 1$ will be a necessary property of any welfare-improving incentive-feasible policy.

Consider next **[A2]**, which asserts that consumers at night will be debt-constrained (so that exercising their future redemption option is necessarily suboptimal). Imagine, by way of contradiction, that consumers are not debt-constrained; i.e., so that $q_2 > y$. In this case, we must consider the possibility that consumers carry a sufficiently large quantity of real money balances forward to make exercising the redemption option optimal. Ignoring a razor's edge case, one of two things must be true; i.e.,

$$\begin{aligned} &\text{Case 1:} \quad q_1^+ < (R-1)^{-1}\tau \\ &\text{Case 2:} \quad q_1^+ > (R-1)^{-1}\tau \end{aligned}$$

where $q_1^+ = (\phi/\mu)(q_2 - y)$.

As consumers are not debt-constrained, their demand for output at night as determined by (11) must satisfy $u'(c) = (v_1^+/v_1)\phi\beta D'(q_1^+)$; or, by employing equilibrium prices and quantities, $u'(y) = (\phi/\mu)\beta D'(q_1^+)$. Referring to (10), note that $D'(q_1^+) = 1$ in Case 1 and $D'(q_1^+) = R$ in Case 2.

Let me consider Case 1 first. Appealing to (17), we have $N'(q_2) = \pi(\phi/\mu)\beta + (1-\pi)(\phi/\mu)R\beta$. Existence of a monetary equilibrium requires that $0 < q_2 < \infty$ satisfy condition (9), so that

$$\phi = \pi(\phi/\mu)\beta + (1-\pi)\phi\delta\beta$$

But this can only be true if $\delta = 1/\mu$; a condition that is violated by the fact that $\delta > 1$ and $\mu > 1$; i.e., see condition (28).

Let me now consider Case 2. Appealing to (17), we have $N'(q_2) = \phi\delta\beta$. Once again, existence of a monetary equilibrium requires $0 < q_2 < \infty$ satisfying condition (9), so that

$$\phi = \phi\delta\beta$$

But this condition is violated by the fact that $\delta\beta < 1$; i.e., see (27).

Thus, for the range of policy parameters considered here, [**A2**] must hold. It appears then that for the policy parameters constrained to satisfy (27) and (28), the conjectured properties [**A2**] and [**A3**] are in fact properties of the (stationary) monetary equilibrium characterized above.

Before concluding this section, there is one other matter that deserves some attention. The welfare-improving policy rule studied here is nonlinear in the sense that the real return on money net of the flat fee is increasing in the quantity of money presented for redemption. What is to prevent a coalition of individuals with cash on hand at the beginning of the day to pool their money balances and elect among themselves a representative to exercise the redemption option on behalf of the group? The preceding analysis implicitly assumes that such coalitions are infeasible.[7]

This assumption, however, is better viewed as a direct implication of the properties of the environment. In particular, the redemption phase is assumed to occur prior to day-market trading. This timing assumption prevents any given individual from collecting a group's money balances in exchange for output. Hence, the only way an elected representative can collect money from the group is in exchange for a promise to return it later (post redemption) with interest. But as all individuals are anonymous and lack commitment, a representative can renege on any such promise with impunity.

The main conclusion then is summarized by the following proposition.

Proposition 1 *Under the range of incentive-feasible policies described by (27) and (28), there exists a stationary monetary equilibrium with an allocation* $y_0 < \hat{y}(\delta) < y^*$ *characterized by (20).*

It follows that any welfare-improving policy necessarily requires strictly positive inflation and nominal interest rates. In addition, since $\hat{y}(\delta) \nearrow y^*$ as $\delta \nearrow \beta^{-1}$, it follows as a corollary that under an appropriately designed policy, the equilibrium allocation $\hat{y}(\delta)$ can be made arbitrarily close to the first-best.

7 Relation to the literature

My analysis bears some relation to the work of Berentsen, Camera and Waller [3]. These authors, who examine an environment similar to the one considered here, also make a case for interest-bearing money. In their model, this is accomplished by introducing a "bank" in the day-market that pays interest on deposits of cash from producers, redirecting these funds to consumers in the form of interest-bearing loans. For this solution to work, the bank must be endowed with at least a limited record-keeping technology. While modifying

[7] If such coalitions were feasible, then the allocation associated with zero intervention is constrained-efficient.

the environment in this manner seems entirely reasonable, my Proposition 1 suggests that it is not essential to do so if policy is designed correctly.

Recently, Hu, Kennan and Wallace [4] have examined a Lagos-Wright model that is similar to my own. Among other things, they discover that first-best implementation is possible without intervention; at least, if individuals are sufficiently patient. This result appears to contradict my own claim that an intervention is necessary to improve welfare. In fact, there is no contradiction as our respective environments differ in an important way.

The important difference is that Hu, Kennan and Wallace [4] remain true to the original Lagos and Wright [7] model by assuming pairwise meetings in one of the subperiods. The assumption of pairwise meetings essentially grants maximum freedom in the design of "trading protocols" conducive to efficient implementation. The search friction places limits on coalition formation; that is, it effectively imposes a communication barrier between the members of a match and the rest of the community. As the size of a meeting is increased (say, by replicating the buyer-seller pair in each meeting), the core converges to a competitive equilibrium. As in Tsu and Wallace [10], the Hu, Kennan and Wallace [4] result will fail to hold when trade among individuals is competitive.

There is one other difference between Hu, Kennan and Wallace [4] and my own that is worth mentioning. Like myself, these authors also restrict attention to individually-rational lump-sum taxes; see also Sanches and Williamson [9]. A relevant issue in these settings concerns the penalty for noncompliance. This penalty is typically modeled as a threat of exclusion from trade. For example, in Hu, Kennan and Wallace [4], individuals are free to skip the day-market (the subperiod in which the tax is levied) and in doing so, they escape the tax. The penalty for tax avoidance is the personal welfare loss associated with not having the opportunity to rebalance money holdings during the day. If this welfare loss is not very large then, in a competitive setting, there is an incentive-induced lower bound on the rate of deflation that renders the Friedman rule infeasible.[8] As is frequently the case in these settings, including Hu, Kennan and Wallace [4], better allocations are available for more patient economies.

In contrast, the government in my environment is unable to prevent participation in any market. The penalty for noncompliance in my model is simply the foregone interest that would have been earned in the act of redemption. As the cost-benefit calculation concerning the redemption choice (8) involves no intertemporal trade-off, efficient implementation in my model turns out to be independent of the discount factor.

Finally, my results also bear some relation to Kocherlakota's [6] illiquid (interest-bearing) bond policy. In Andolfatto [2], I recast Kocherlakota's [6] argument

[8] See Andolfatto [1] and Sanches and Williamson [9].

in the context of a Lagos-Wright model. [9] A bond issued in the day represents a risk-free claim to cash the next day and individuals carry money and bonds into the night-market. An asset-market opens at night, subsequent to the realization of types, but prior to the goods-market at night. Kocherlakota [6] assumes that the bond is illiquid in the sense that it cannot be used to purchase goods at night. Moreover, he demonstrates that this trading restriction is necessary to improve welfare. In fact, in [2] I demonstrate that the first-best is implementable with an appropriate choice of the money-bond ratio and inflation rate. Efficient implementation necessarily requires a positive inflation rate and a positive nominal interest rate (bonds must trade at a discount).

Hence, in both my paper here and in Kocherlakota [6], an interest-bearing government asset is essential. Kocherlakota's illiquid bond policy is attractive relative to my own in that it constitutes a linear mechanism. On the other hand, his mechanism can only work if individuals are somehow prohibited from using bonds as a means of payment. My mechanism has an advantage in that it does not rely on the imposition of any trading restriction between individuals.

8 Conclusion

The Lagos and Wright [7] model constitutes the leading framework of analysis in the contemporaeneous monetary theory literature. The framework models individuals exposed to idiosyncratic risk generated by the random arrival of opportunities to produce and consume output over time. This risk may be modeled as the outcome of random search in a decentralized market, or it may be modeled as the outcome of random shocks to preferences and technologies in a centralized market. Either way, the key simplifying property of the framework is quasi-linear preferences, which removes the distribution of wealth as an endogenous state variable.

The modeling choice of centralized versus decentralized trade appears to have important policy implications. Hu, Kennan and Wallace [4] investigate these implications when trade is decentralized. My paper investigates these implications for the case of centralized trade. I identify a class of incentive feasible policies that improve welfare beyond what achievable with zero intervention. When all trade is restricted to be voluntary, any such policy in this class evidently requires a strictly positive nominal interest rate and a non-negative inflation rate. The efficient allocation (the Friedman rule) is implementable for a suitably designed incentive-feasible policy. The traditional prescription

[9] As it turns out, an illiquid bond (or type-contingent transfers) can play no welfare-improving role in the model I study here; see also, Berentsen, Camera and Waller [3]. In Andolfatto [2], I modify the spatial structure of the environment so that it corresponds to the one used in Kocherlakota [6].

of implementing the Friedman rule via deflation is not incentive-feasible.

My proposition 1 also has implications for theorists interested in exploring the foundations of banking in the Lagos-Wright model. For example, the banking sector modeled in Berentsen, Camera and Waller [3] appears not to be essential; at least, not under an optimal monetary policy. This suggests that current formulations of the theory are possibly missing important aspects of reality that render banking an essential institution in monetary economies. Discovering precisely what these missing elements might be deserves further attention.

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References

[**1**] D. Andolfatto, The simple analytics of money and credit in a quasi-linear environment, Manuscript: Simon Fraser University, 2008.

[**2**] D. Andolfatto, On the societal benefits of illiquid bonds, Manuscript: Simon Fraser University, 2009.

[**3**] A. Berentsen, G. Camera, C. Waller, Money, credit, and banking, J. Econ. Theory 135 (2007) 171–195.

[**4**] T. Hu, J. Kennan, N. Wallace, J. Polit. Econ. 117 (2009) 116–137.

[**5**] N.R. Kocherlakota, Money is memory, J. Econ. Theory 81 (1998) 232–251.

[**6**] N. R. Kocherlakota, Societal benefits of illiquid bonds, J. Econ. Theory, 108 (2003) 179–193.

[**7**] R. Lagos, R. Wright, A unified framework for monetary theory and policy analysis, J. Polit. Econ. 113 (2005) 463–484.

[**8**] G. Rocheteau, R. Wright, Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium, Econometrica 73 (2005) 175–202.

[**9**] D. Sanches, S. Williamson, Money and credit with limited commitment and theft, Manuscript: Washington University in St. Louis, 2008.

[**10**] T. Tsu, N. Wallace, Pairwise trade and coexistence of money and higher return assets, J. Econ. Theory 133 (2007) 524–535.