# Dynamic Taxation, Private Information and Money

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### Abstract

The objective of this paper is to study optimal fiscal and monetary policy in a dynamic Mirrlees model where the frictions giving rise to money as a medium of exchange are explicitly modeled. The framework is a three period OLG model where agents are born every other period. The young and old trade in perfectly competitive centralized markets. In middle age, agents receive preference shocks and trade amongst themselves in an anonymous manner. Since preference shocks are private information, in a record-keeping economy, the planner's constrained allocation trades off efficient risk sharing against production efficiency in the search market. In the absence of record-keeping, the government uses fiat money as a substitute for dynamic contracts to induce truthful revelation of preferences. Inflation affects agents' incentive constraints and so distortionary taxation of money may be needed as part of the optimal policy even if lump-sum taxes are available.

# 1 Introduction

Should the inflation tax be used as part of an optimal fiscal policy? The answer to this question has been explored in a substantial body of research using dynamic general equilibrium models. Most of this literature adopts the Ramsey approach to optimal taxation – the government is assumed to be unable to use lump sum taxes and so it chooses distortionary tax rates to maximize the welfare of the representative agent subject to being a competive equilibrium. The best known use of this approach to study the inflation tax is most notably in Chari, Christiano and Kehoe (1996), denoted CCK hereafter. They show that the Friedman rule will be optimal if preferences are homothetic and weakly separable in consumption and leisure regardless of whether money is valued because of money-in-the-utility, cash-in-advance or shopping time motives.

There are two drawbacks to this approach for studying the inflation tax. First, even though non-distortionary (lump-sum) taxes achieve the first-best allocation, the representative agent approach simply prohibits their use for some unmodeled reason. Thus, the first-best is not implementable solely because of an unspecified feature of the environment – the inability to use non-distorting taxes. Second, when it comes to studying the inflation tax, money is always 'forced' into the model via some shortcut such as money-in-the-utility function, shopping time or cash-in-advance. The frictions that create a need for money are never explicitly modeled. So in addition to an unspecified reason for limiting the set of tax instruments, there is also an unmodeled friction in the environment that gives money transaction value.

This suggests that a better approach for studying optimal fiscal and monetary policy is to construct an environment where: 1) the government is free to use lump-sum taxes but *chooses* not to, and 2) the frictions giving rise to money as a medium of exchange are explicitly modeled.

Addressing point one above is the basis for the 'New Dynamic Public Finance' (NDPF) literature.[1] This literature studies optimal taxation in a dynamic version of Mirrlees's (1971) model where agents are heterogeneous and there is private information about agent types. In this framework, the government chooses taxes to maximize welfare subject to providing appropriate incentives for agents to reveal private information regarding their preferences and productivity. There are no restrictions whatsoever on the set of tax instruments available to the government and the standard result in this literature is that distortionary taxation is optimal.

Recently, da Costa and Werning (2008) use a Mirrlees approach to study inflation taxation by assuming labor productivity is heterogenous and there is private information regarding this productivity. However, they assume a money-in-the utility framework to see if inflation taxation is Pareto improving. They find that the Friedman rule is optimal if money and work effort are compliments and labor income is taxed in a non-decreasing manner. While this result is informative for this class of monetary models, the model's lack of micro-foundations for money throws doubt on the robustness of their findings. How could this be the case? If money itself is used as a means to overcome private information frictions, then ignoring this role of money when studying the optimality of positive inflation within a Mirrlees framework may yield misleading results.

Modeling the frictions that provide the microfoundations for money demand is the basis of the "New Monetary Economics" literature. This literature, based on the Kiyotaki and Wright (1989,1993) monetary search framework, explicitly models the trade and information frictions that prevent the use of trade credit between agents thus giving rise to the need for a medium of exchange. As a result, money is 'essential' in that it expands the set of allocations that can be achieved.[2]

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[1] Most notable in this area are the papers by Golosov, Kocherlakota and Tysvinski (2003) and Kocher-lakota (2005b).

[2] See Kocherlakota (1998) and Wallace (2001).

The objective of this paper is to construct a model that combines these two literatures in order to study the dynamic taxation of money. I analyze a model where money is essential as a medium of exchange yet informational frictions induce the government to use distortionary taxation of money balances.[3] The basic model is a three period OLG/search model developed by Zhu (2008) but with the addition of private information about individual preference shocks. In this framework, a new generation is born every other period. In these periods, the young and old agents can trade amongst themselves. Young agents have an endowment of labor but old agents do not. When 'middle aged', agents receive idiosyncratic preference shocks that make some of them producers and others consumers.

Since Kocherlakota (1998) showed that money is fundamentally a form of record-keeping, I construct allocations for three environments with differing assumptions on record-keeping. First, the allocation is derived when no durable asset or record-keeping technology exists (autarky). Second, I consider the case with a record-keeping technology where agents send reports about their preference shocks to a planner at the beginning of middle age and, based on their reports, are given a sequence of consumption/production in middle and old age. Within this environment I consider two cases: 1) when the shocks are public information and 2) when they are private information. For the latter case, the allocation must be incentive compatible with truthful revelation of the shocks. This allocation is referred to as the constrained optimum. In the constrained optimum the planner must create consumption risk for the old to induce agents to produce in middle age. Thus, the planner trades off risk sharing amongst the old against productive efficiency when middle aged. Similar to the NDPF literature, the planner wants to create a wedge between the marginal utility of consumption when young against the expected marginal utility of consumption when old to

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[3]Kocherlakota (2005a) criticizes search theoretic models of money for studying inflation taxation without allowing for an array of other tax instruments. This paper, as well as those by Aruoba and Chugh (2008) and Gomis-Poqueris and Peralta-Alva (2008), is an attempt to rectify that problem.

induce truthful reporting.[4]

Finally, I consider decentralizing the constrained optimal allocation. I assume there is a government with no record-keeping technology and agents do not send in reports. Rather, as a substitute for record-keeping, the government provides fiat currency to agents. Money provides no utility in and of itself and only has value by being generally accepted by agents as a medium of exchange. In this economy, middle-aged, agents search for a suitable trading partner and bargain over the terms of trade when a match occurs. Anonymity in this market makes money essential as a medium of exchange. Most importantly, money acts as a device to induce truthful revelation of one's type and is therefore an imperfect substitute for a dynamic contract. Inflation in turn affects real balances and thus incentive constraints of agents. The government then chooses the growth rate of the money stock, tax rates and lump-sum transfers to achieve a maximize a social welfare function subject to being a competive equilibrium.

Using this framework I address the following questions. First, can the constrained planner allocation be implemented with the use of fiat money and optimally chosen fiscal policy? Second, does the optimal fiscal policy require use of the inflation tax even if lump-sum taxes are available?

Although it is difficult in general to show existence or uniqueness of the constrained optimum or the monetary equilibrium, I show that the constrained optimum can be replicated with a non-linear consumption tax and zero lump-sum taxes. In this sense, the government has the option to use lump-sum taxation but does not do so, instead it relies on distorting consumption taxes. Because the government has a full set of tax instruments, it can use the inflation tax or not but the allocation is unaffected. I then consider the case where the

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[4]An alternative approach would be to make money a part of the environment that gives agents an outside option to trade, as in Aiyagari and Williamson (2000). Since money creates a better outside option for agents and thus makes the planner's problem more difficult, a planner would never introduce money if it was under his control.

government is constrained to using lump-sum taxes and a linear consumption tax. Under this restriction, I construct an example where both the constrained optimum and the monetary equilibrium exists. I am then able to show that as long as the lump-sum taxes are not too large, it is optimal for the government to resort to the inflation tax as part of the optimal policy.[5]

Why do I obain different results than da Costa and Werning? Consider the following interpretation of da Costa and Werning's results. Think of effort as being negative leisure and money as being merely a form of wealth as opposed to an argument in the utility function. Then assuming complimentarity between money and effort is implicitly assuming leisure is an inferior good – the more wealth one has, the less leisure is taken. In my framework, agents have to work today to acquire money for future consumption. So a standard wealth effect occurs – if an agent has more money today, he wants more leisure today (via lower effort/production). Consequently, to induce optimal effort/production, money is taxed via inflation even though lump-sum taxes are available.

What is most striking about this example is that distortionary taxation of money is optimal even though preferences are homothetic and separable in consumption and leisure. This suggests distortionary taxation of money is driven by the informational frictions that make money essential as a medium of exchange as opposed to assumptions on homotheticity or separability of preferences or on the complementarity between money and work effort.

The structure of the paper is as follows. First, the environment is described. Then I derive the autarkic allocation and the constrained optimal stationary allocation for a planner with a record-keeping technology. Then the equilibrium steady state conditions are derived for the monetary equilibrium. I then show how the constrained optimum can be replicated and describe aspects of the optimal tax structure. Finally, I analyze an example where

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[5]Zhu (2008) also shows that inflation is welfare improving in this model. However, he does not allow for any other tax instruments hence, his results are subject to the Kocherlakota (2005a) critique.

the government can use lump-sum taxes and linear consumption taxes. The last section concludes.

# 2  Environment

The basic environment is the three period OG/search model of Zhu (2008). There is a continuum of agents born with unit measure every other period. In these periods, the young and the old come together. Call this location the centralized market or CM. Young agents are endowed with labor at the start of each CM and there is a linear production technology available that converts one unit of labor into one unit of goods. Goods are perishable so there is no ability to store goods across periods. The old have no labor endowment and must receive goods from the young to consume. After meeting in the CM, the old die. In the second period of life, middle-aged agents can trade amongst themselves. Call this location the decentralized market or DM. The CM/DM structure follows from Lagos and Wright (2005). Let $\beta \leq 1$ be the discount factor from the DM to the CM and $\beta_D \leq 1$ be the discount factor from the CM to the DM.[6]

An agent's preferences are assumed to be additively separable across consumption, labor and time. Let $U(C)$ be utility from consuming $C$ units of goods in the CM and $v(h)$ is the disutility of working $h \geq 0$ hours when young. Assume $U', -U'', v', v'' > 0$ and $U'(0) \rightarrow +\infty$. Preferences in middle age are given by $\epsilon_b u(q) - \epsilon_s \psi(q)$ where $\epsilon_b u(q)$ is utility from consuming $q$ units of goods in the DM while $-\epsilon_s \psi(q)$ is the disutility of producing $q$ units of goods in the DM. Assume $u', -u'', \psi', \psi'' > 0, u'(0) \rightarrow +\infty$ and $u(0) = \psi(0) = \psi'(0) = 0$. The variables $\epsilon_b$ and $\epsilon_s$ are preference shocks such that with probability $\sigma_b$, $\epsilon_b = 1$ and $\epsilon_s = 0$, meaning an agent can consume but cannot produce. With probability $\sigma_s$, $\epsilon_b = 0$ and $\epsilon_s = 1$, meaning

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[6]Unlike the dynamic taxation literature, one cannot study a finite economy with fiat money. Thus, the OLG framework provides analytical simplicity in that we can study something akin to a finite economy with valued fiat currency.

an agent can produce but not consume. Finally, with probability $1 - \sigma_b - \sigma_s$, $\epsilon_b = \epsilon_s = 0$ meaning they do neither. Those who are idle in the DM receive zero payoffs in the DM. From here on I will assume $\sigma_b = \sigma_s = \sigma$ with $\sigma \leq 1/2$. I will refer to consumers as buyers, producers as sellers and those doing neither as idle. Note that utility from consumption and disutility from production is allowed to differ across markets.

An agent's lifetime utility born at time $t - 1$ is given by

$$W_{y,t-1} = U\left(C_{y,t-1}\right) - v\left(h_{t-1}\right) + \beta_D V_t$$

where

$$V_t = \sigma\left[u\left(q_{b,t}\right) - \psi\left(q_{s,t}\right)\right] + \sigma\beta U\left(C_{o,t+1}^b\right) + \sigma\beta U\left(C_{o,t+1}^s\right) + \beta\left(1 - 2\sigma\right) U\left(C_{o,t+1}^n\right) \quad (1)$$

and $C_y$ is consumption when young, $q_b$ is consumption of goods in the DM if a buyer, $q_s$ is production if a seller and $C_o^j$ is consumption when old if the agent's trading state, $j$, in the DM was a buyer ($b$), a seller ($s$) or neither ($n$).

# 3    Optimal Allocations and Record-keeping

An important result in monetary theory due to Kocherlakota (1998) is that money is a form of record-keeping. So before looking at the monetary equilibrium, I consider economies with various forms of record-keeping technologies in order to compare them to the allocation that occurs in the monetary equilibrium. Consider the case where there is a complete absence of record-keeping. The age structure and the absence of durable goods or assets means the only allocation is autarky.[7] Consequently, young agents produce and consume for

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[7]The inability of the old to produce will  eliminate the use of trade credit in the DM even if there is record-keeping and enforcement. However, if the old could produce in the CM, then trade credit in the DM is ruled out by anonymity. As usual, the age structure in OLG models prevents the young from extending

themselves and have $q_b = q_s = C_o^j = 0$. Lifetime welfare in the autarkic allocation is given by $W^a = U\left(C_y^a\right) - \upsilon\left(C_y^a\right) + \beta U\left(0\right)$ where the superscript $a$ denotes the autarkic choice and $C_y^{a}$ solves $U'\left(C_y^a\right) = \upsilon'\left(C_y^a\right)$.

## 3.1   Public information and record-keeping

Now consider the case where there is a planner with a record-keeping technology. In this environment, agents send a report to the planner about the realization of their idiosyncratic preference shock in the DM and record-keeping technology allows the planner to keep track of agents' reports. Conditional on the report, the planner gives the agent a sequence of consumption in the DM and CM. If an agent reports himself as a buyer, he is given $q_b$ units of output by the planner in the DM to consume and $C_o^b$ when old. If he reports himself as a producer, he delivers $q_s$ units of output to the planner in the DM and consumes $C_o^s$ when old. If he reports being idle, he neither consumes nor produces in the DM and receives $C_o^n$ when old. As is standard, assume the planner can commit to this sequence of consumption. We would like to know what allocation can be supported by this record-keeping technology. Young agents simply report their age and receive $C_y$ units of consumption and provide $h$ units of labor.

Assume the economy starts in period $t = -1$ with an initial generation in middle-age. The planner chooses sequences of CM consumption and production $\{C_y, h, C_o^j\} \equiv \left\{C_{y,2t}, h_{2t}, C_{o,2t}^b, C_{o,2t}^s, C_{o,2t}^n, \right\}_{t=0}^{\infty}$ and DM consumption and production $\{q_b, q_s\} \equiv \{q_{b,2t+1}, q_{s,2t+1}\}_{t=-1}^{\infty}$

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trade credit to the old.

to maximize a weighted average of current and future generations expected utilities:

$$\mathcal{W} = \sigma\lambda^{-1}\left[u\left(q_{b,-1}\right) - \psi\left(q_{s,-1}\right)\right] + \beta\lambda^{-1}\left[\sigma U\left(C_{o,0}^b\right) + \sigma U\left(C_{o,0}^s\right) + (1-2\sigma)U\left(C_{o,0}^n\right)\right]$$

$$+ \sum_{t=0}^{\infty}\lambda^t\left\{U\left(C_{y,2t}\right) - \upsilon\left(h_{2t}\right) + \sigma\beta_D\left[u\left(q_{b,2t+1}\right) - \psi\left(q_{s,2t+1}\right)\right]\right\}$$

$$+ \sum_{t=0}^{\infty}\lambda^t\beta_D\beta\left[\sigma U\left(C_{o,2t+2}^b\right) + \sigma U\left(C_{o,2t+2}^s\right) + (1-2\sigma)U\left(C_{o,2t+2}^n\right)\right]$$

$$\text{s.t.} \quad h_{2t} \geq \sigma C_{o,2t}^b + \sigma C_{o,2t}^s + (1-2\sigma)C_{o,2t}^n + C_{y,2t} \quad \forall t \geq 0$$

$$\sigma q_{s,2t+1} \geq \sigma q_{b,2t+1} \quad \forall t \geq 0 \tag{2}$$

where $\lambda^t$ is the weight assigned to a generation born at time $2t$ and the weight on the first young generation is normalized to 1.

Suppose that the preference shocks in the DM are public information and are thus observable by the planner. If the planner can force exchange to occur ex post, then the optimal allocation satisfies the resource constraints and

$$
\begin{aligned}
u'\left(q_{b,2t+1}\right) &= \psi'\left(q_{s,2t+1}\right) \quad t \geq 0 \\
U'\left(C_{y,2t}\right) &= \upsilon'\left(h_{2t}\right) \quad t \geq 0 \\
U'\left(C_{o,2t}^b\right) &= U'\left(C_{o,2t}^n\right) = U'\left(C_{o,2t}^s\right) \quad t \geq 0 \quad \Rightarrow C_{o,2t}^j = C_{o,2t} \quad \forall j \\
U'\left(C_{y,0}\right) &= \beta\lambda^{-1}U'\left(C_{o,0}\right) \\
U'\left(C_{y,2t}\right) &= \beta_D\beta\lambda^{-1}U'\left(C_{o,2t}\right) \quad t > 0
\end{aligned}
$$

plus the intertemporal consumption Euler equation

$$
\begin{aligned}
U'\left(C_{y,2t}\right) &= \frac{\upsilon'\left(h_{2t}\right)}{\upsilon'\left(h_{2t+2}\right)}\beta_D\beta\lambda^{-1}\left[\sigma U'\left(C_{o,2t+2}^b\right) + \sigma U'\left(C_{o,2t+2}^s\right) + (1-2\sigma)U'\left(C_{o,2t+2}^n\right)\right] \\
&= \frac{\upsilon'\left(h_{2t}\right)}{\upsilon'\left(h_{2t+2}\right)}\beta_D\beta\lambda^{-1}EU'\left(C_{o,2t+2}\right) \tag{3}
\end{aligned}
$$

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where $C_{o,2t+2}$ is old age consumption. Call this the *unconstrained* optimal allocation.[8] In this allocation, the planner chooses the efficient quantity in the DM and wants to eliminate all CM consumption risk among the old, since $C_{o,2t}^b = C_{o,2t}^s = C_{o,2t}^n$ for all $t$.

Note that, note that if $\beta_D = 1$ then the initial old get the same allocation as future old. In short, the economy can start in a steady-state. Also, if the planner $\lambda = \beta$, then in a steady state (or with linear disutility of labor for the young), the planner equates the marginal utility of CM consumption across young and old at a point in time as well as across time for an individual generation.

## 3.2 Private information and record-keeping

The unconstrained allocation is not feasible if the planner cannot observe agents' preference shocks. Why? Those who are producers in the DM get the same consumption in the next CM as everyone else – hence there is no reward for producing. Consequently, those agents would never reveal their true DM preference shock to the planner. So even being able to force agents to produce is useless since the planner cannot identify who the sellers are in the DM. As a result, with private information, the planner is constrained to implement an allocation that is incentive compatible, i.e., agents truthfully reveal their idiosyncratic shock in the DM and trade voluntarily. This problem is considered next.

With private information the planner has to worry about incentive constraints such that no agent has an incentive to misrepresent their true state in the DM. Truthful reporting

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[8]The planner can also support this allocation without resorting to force if: $\psi(q^*) < U(C_o) - U(0)$ and $W^a < U(C_y) - \upsilon(C_y + C_o) + \beta\sigma[u(q^*) - \psi(q^*)] + \beta^2 U(C_o)$. where $C^*$ solves $U'(C^*) = \upsilon'(C^*)$. The first condition ensures that a middle aged seller prefers to produce than starve when old, while the second ensures the young do not prefer autarky.

requires:

$$u\left(q_{b,2t+1}\right) + \beta U\left(C^b_{o,2t+2}\right) - \beta U\left(C^n_{o,2t+2}\right) \geq 0$$

$$-\psi\left(q_{s,2t+1}\right) + \beta U\left(C^s_{o,2t+2}\right) - \beta U\left(C^n_{o,2t+2}\right) \geq 0$$

$$U\left(C^n_{o,2t+2}\right) - U\left(C^b_{o,2t+2}\right) \geq 0$$

The first two constraints require that buyers and sellers in the DM have no incentive to misreport themselves as idle. The last constraint states that an idle agent must have no incentive to misreport himself as a buyer. Buyers and idle agents cannot misrepresent themselves as sellers since they would be required to deliver $q_s > 0$ units of goods, which they cannot do. Thus, incentive constraints for these cases can be dispensed with. However, an idle agent can declare himself a consumer and freely dispose of the goods (or consume them at zero utility). The third constraint also ensures that a seller would rather report himself as idle than as a buyer.

In general, there also must be a participation constraint on the young to induce them to produce output for both generations rather than go into autarky and produce only for themselves. Suppose a young agent decides not to participate. The worst punishment the planner can impose is to force them into autarky. Participation by the young then requires

$$W^a \leq U\left(\bar{C}_{y,t}\right) - \upsilon\left(\bar{h}_t\right) + \beta\sigma\left[u\left(\bar{q}_{b,2t+1}\right) - c\left(\bar{q}_{s,2t+1}\right)\right] + \beta_D \beta EU\left(\bar{C}_{o,2t+2}\right)$$

The bars denote the planner's allocation. If the constraint does bind then obviously, the planner would prefer autarky as well since his objective is to maximize the lifetime utility of the agents. I will proceed as if this constraint is not binding. Given the resulting allocation, one would then need to impose restrictions on preferences to ensure the constrained optimal allocation makes this constraint non-binding. For example, if $U\left(C\right) = \ln C$ then $U\left(0\right) \to -\infty$

so this will typically be satisfied.

The planner's problem is now given by

$$
\max_{\{C_y,h,C_o^j\}\{q_b,q_s\}} = \sigma\lambda^{-1}\left[u\left(q_{b,-1}\right) - \psi\left(q_{s,-1}\right)\right] + \beta\lambda^{-1}\left[\sigma U\left(C_{o,0}^b\right) + \sigma U\left(C_{o,0}^s\right) + (1-2\sigma)U\left(C_{o,0}^n\right)\right]
$$

$$
+ \sum_{t=0}^{\infty}\lambda^t\left\{U\left(C_{y,2t}\right) - \upsilon\left(h_{2t}\right) + \sigma\beta_D\left[u\left(q_{b,2t+1}\right) - \psi\left(q_{s,2t+1}\right)\right]\right\}
$$

$$
+ \sum_{t=0}^{\infty}\lambda^t\beta_D\beta\left[\sigma U\left(C_{o,2t+2}^b\right) + \sigma U\left(C_{o,2t+2}^s\right) + (1-2\sigma)U\left(C_{o,2t+2}^n\right)\right]
$$

$$
s.t. \quad h_{2t} \geq \sigma C_{o,2t}^b + \sigma C_{o,2t}^s + (1-2\sigma)C_{o,2t}^n + C_{y,2t} \quad \forall t \geq 0
$$

$$
\sigma q_{s,2t+1} \geq \sigma q_{b,2t+1} \quad \forall t \geq 0
$$

$$
u\left(q_{b,2t+1}\right) + \beta U\left(C_{o,2t+2}^b\right) - \beta U\left(C_{o,2t+2}^n\right) \geq 0
$$

$$
-\psi\left(q_{s,2t+1}\right) + \beta U\left(C_{o,2t+2}^s\right) - \beta U\left(C_{o,2t+2}^n\right) \geq 0
$$

$$
U\left(C_{o,2t+2}^n\right) - U\left(C_{o,2t+2}^b\right) \geq 0
$$

The optimal allocation under private information satisfies

$$
\begin{aligned}
\psi\left(q_{,2t+1}\right) &= \beta U\left(C_{o,2t+2}^s\right) - \beta U\left(C_{o,2t+2}^n\right) \quad t \geq -1 \\
\frac{u'\left(q_{,2t+1}\right)}{\psi'\left(q_{,2t+1}\right)} &= \frac{U'\left(C_{o,2t+2}^b\right)}{\sigma U'\left(C_{o,2t+2}^b\right) + (1-\sigma)U'\left(C_{o,2t+2}^s\right)} \geq 1 \quad t \geq -1 \\
U'\left(C_{y,2t}\right) &= \upsilon'\left(h_{2t}\right) \quad t \geq 0 \\
C_{o,2t+2}^b &= C_{o,2t+2}^n < C_{o,2t+2}^s \quad t \geq 0 \\
U'\left(C_{y,0}\right) &= \beta\lambda^{-1}\left[\sigma\frac{1}{U'\left(C_{o,0}^s\right)} + \sigma\frac{1}{U'\left(C_{o,0}^b\right)} + (1-2\sigma)\frac{1}{U'\left(C_{o,0}^n\right)}\right]^{-1} \\
U'\left(C_{y,2t+2}\right) &= \beta_D\beta\lambda^{-1}\left[\sigma\frac{1}{U'\left(C_{o,2t+2}^s\right)} + \sigma\frac{1}{U'\left(C_{o,2t+2}^b\right)} + (1-2\sigma)\frac{1}{U'\left(C_{o,2t+2}^n\right)}\right]^{-1} \quad t > 0
\end{aligned}
$$

From these conditions we see three key results. First, sellers' incentive constraints bind so they get no trade surplus in the DM. It then follows that, for any $q_{,2t+1} > 0$, DM sellers

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have to receive more old age consumption than idle agents to compensate them for producing in the DM. Second, the idle DM agents' incentive constraints bind, meaning they get the same old age consumption as DM buyers. As a result, there is incomplete risk-sharing in old age consumption all for $t \geq 0$. Finally, because the planner must accept incomplete risk-sharing among the old to induce DM sellers to produce, DM output is inefficiently low. Thus, due to incentive constraints, the planner trades off efficiency in DM production against old age risk-sharing. Note that if $\beta_D = 1$ then the allocation between the initial old and the initial young is equivalent to that arising in later generations. i.e., the planner can start the economy off in a steady state.

We also get the following intertemporal Euler equation

$$U'(C_{y,2t}) = \frac{v'(h_{2t})}{v'(h_{2t+2})}\beta_D\beta\lambda^{-1}\left[\sigma\frac{1}{U'\left(C^s_{o,2t+2}\right)} + \sigma\frac{1}{U'\left(C^b_{o,2t+2}\right)} + (1 - 2\sigma)\frac{1}{U'\left(C^n_{o,2t+2}\right)}\right]^{-1}$$

which can be rewritten

$$U'(C_{y,t}) = \frac{v'(h_{2t})}{v'(h_{2t+2})}\beta_D\beta\lambda^{-1}\left\{E\left[\frac{1}{U'(C_{o,2t+2})}\right]\right\}^{-1}. \tag{4}$$

From this expression, we see that the planner wants to equate the marginal utility of young consumption to the *harmonic* mean of old age marginal utility of consumption. This is in constrast to (3) where the planner equates the marginal utility of young consumption to the *arithmetic* mean of marginal utility when old.

Call a solution to these equations the *constrained* optimal allocation. Set $\beta_D = 1$ and consider a steady-state allocation for the constrained optimum. A steady-state constrained

allocation is a list $\left\{ C_y, h, q, C_o^b, C_o^s, C_o^n \right\}$ solving

$$C_o^n = C_o^b \tag{5}$$

$$h = C_y + \sigma C_o^b + \sigma C_o^s + (1 - 2\sigma) C_o^n \tag{6}$$

$$U'(C_y) = v'(h) \tag{7}$$

$$\frac{u'(q)}{\psi'(q)} = \frac{U'(C_o^b)}{(1 - \sigma) U'(C_o^s) + \sigma U'(C_o^b)} > 1 \tag{8}$$

$$\psi(q) = \beta \left[ U(C_o^s) - U(C_o^n) \right] \tag{9}$$

$$U'(C_y) = \beta \lambda^{-1} \left[ \sigma \frac{1}{U'(C_o^s)} + \sigma \frac{1}{U'(C_o^b)} + (1 - 2\sigma) \frac{1}{U'(C_o^n)} \right]^{-1} \tag{10}$$

Note that if $\sigma = 0$ the information problem is effectively eliminated (the only state is idle and publicly known) and the unconstrained allocation $(C_o, C_y)$ can be implemented subject to the young agents' participation constraint being satisfied ($q$ is irrelevant in this case).

# 4   Monetary equilibrium: Absence of recordkeeping

Now consider the case where the government has no record-keeping technology and receives no reports. Instead it provides fiat currency to agents. Money is the only durable object in the economy and it is perfectly divisible and agents can hold unbounded amounts. Money is injected in lump sum fashion to the middle-aged agents. As will be shown, since they all leave the CM with the same amount of money the lump-sum injection has equal value to all young agents.[9] Since monetary injections occur every other period we have $M_{t+2} = \gamma_t M_t$ where $\gamma_t = 1 + \pi_t$ is the gross growth rate of the money supply from $t$ to $t + 2$. From here on, the $t$ subscript is suppressed for notational ease so that $+1$ denotes $t + 2$ and so on.

In the CM, firms hire labor and sell the output in perfectly competitive markets. Given

---

[9]This eliminates welfare gains from inflation due to a non-degenerate distribution of money balances as in Levine (1991), Molico (2006) and Berentsen, Camera and Waller (2005).

the linear production technology the real wage paid to young labor is 1. Firms sell their output at the nominal price $\mathcal{P} = 1/\phi$ where $\phi$ is the goods price of money in the CM. It then follows that the gross real rate of return on money from $t - 1$ to $t + 1$. is $R_m = \phi_{+1}/\phi_{-1}$.

In the DM, the preference shocks create a double coincidence of wants problem, which combined with anonymity means that money is essential for trade – buyers give up money for goods while sellers increase their holdings of money by selling goods. As a result, there is a non-degenerate distribution of money balances among the old. However, death keeps the distribution of money holdings analytically tractable. This DM/CM structure gives money a 'store of value' role from young to old age and a 'medium of exchange' function in middle age. How buyers and sellers are matched in the DM is left unspecified but it could be modeled as the result of random search or a random matching process that pairs each buyer to a seller who then bargain over the terms of trade.

The role of money in the decentralized economy is that it induces truthful revelation of one's type. A seller is willing to reveal his type if the buyer can offer money for goods. By revealing his true type and producing today, a seller acquires money which generates more future consumption. Furthermore, idle agents will not attempt to mimic buyers since doing so means giving up money today (future consumption) for goods they do not desire today. Hence, money is a replacement for the dynamic contract (i.e., record-keeping) used by the planner. The only remaining question is whether the introduction of money and taxations can replicate the constrained optimum.[10]

The government is able to observe an agent's age, hours worked by the young, and consumption. It can impose lump sum taxes/transfers by age, distortionary labor taxes on the young and a non-linear consumption tax on market goods purchased. The consumption

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[10]In this paper, the role of money in overcoming private information frictions is explicitly taken into account. This view of money is quite different than in da Costa and Werning (2008), who use a money-in-the-utility function model of money. They do not ask what information frictions are being overcome via the use of money. Since information frictions are at the heart of the Mirrlees approach to taxation, it seems that being explicit about *all* information frictions is preferred when studying the inflation tax.

tax can be made age dependent.[11] The consumption tax on the old is collected in the form of consumption goods as opposed to payment in cash. Since all market consumption of the old is bought with cash the consumption tax is equivalent to a tax on real monetary wealth.

The government's budget constraint is

$$T_o = \tau^h h + T_y + \tau^c C_y + \int \eta \left( C_o^j - T_o \right) dF \left( C_o^j \right) \tag{11}$$

where $T_y$ is a lump-sum tax of goods on the young, $T_o$ is the lump-sum transfer to the old, $\tau^h$ is the tax rate on real labor income, and $\tau^c$ is the consumption tax rate on young agents' consumption. Furthermore, $\eta \left( C_o^j - T_o \right)$ is the market consumption tax collected from an old agent who acquired $C_o^j - T_o$ units of goods in the CM.[12]

In the monetary equilibrium, at the start of the DM, the lifetime expected utility of a middle-aged agent depends on the amount of money brought into the DM. Let $q_b \left( m, m^s \right)$ and $C_{o,+1}^b \left( m, m^s \right)$ be the quantities an individual $j$ consumes when he is a buyer with $m$ units of money and meets a seller with money balances $m^s$ in the DM and the next CM respectively. Similarly, let $q_s \left( m^b, m \right)$ and $C_{o,+1}^s \left( m^b, m \right)$ denote the quantities individual $j$ produces and consumes when he holds $m$ units of money and meets a buyer with $m^b$ units of money in the DM and next CM respectively. We thus have, $V_t = V_t(m)$ where

$$\begin{aligned} V_t \left( m \right) &= \sigma \int \left\{ u \left[ q_b \left( m, m^s \right) \right] + \beta U \left[ C_{o,+1}^b \left( m, m^s \right) \right] \right\} dF \left( m^s \right) \\ &\quad + \sigma \int \left\{ -\psi \left[ q_s \left( m^b, m \right) \right] + \beta U \left[ C_{o,+1}^s \left( m^b, m \right) \right] \right\} dF \left( m^b \right) \\ &\quad + \beta \left( 1 - 2\sigma \right) U \left[ C_{o,+1}^o \left( m \right) \right]. \end{aligned} \tag{12}$$

---

[11] The use of age-dependent distortionary taxes has been studied in an OG framework by Erosa and Gervais (2002).

[12] Old agents total consumption is $C_o^j = C_o^{CM} + T_o$ where $C_o^{CM}$ is the consumption of goods acquired in the CM. An old agent's real balances must pay for consumption of the CM goods and the sales tax. Thus, $\phi m^j = C_o^{CM} + \eta \left( C_o^{CM} \right) = C_o^j - T_o + \eta \left( C_o^j - T_o \right)$ the consumption tax received on those purchases is $\eta \left( C_o^{CM} \right) = \eta \left( C_o^j - T_o \right)$.

In what follows, I will solve the problem facing a representative agent of a generation born in time $t - 1$.

## 4.1 CM Trading

An agent born at time $t - 1$ chooses $C_{y,-1}, h_{-1}$ and $m$ to

$$\max_{C_{y,-1},h_{-1},m} \quad U\left(C_{y,-1}\right) - \upsilon\left(h_{-1}\right) + V_t\left(m\right)$$

$$\text{s.t.} \quad \left(1 + \tau^c\right) C_{y,-1} + \phi_{-1} m = \left(1 - \tau^h\right) h_{-1} + T_y$$

where $1 - \tau^h$ is the after-tax real wage. The FOC yield

$$U'\left(C_{y,-1}\right) = \frac{1 + \tau^c}{1 - \tau^h} \upsilon'\left(h_{-1}\right) \tag{13}$$

$$\frac{\phi_{-1}}{1 + \tau^c} U'\left(C_{y,-1}\right) = V'\left(m\right) \tag{14}$$

Since the FOC are the same for all young agents they leave with the same amount of money balances. Note that since consumption and money balances are financed by labor, the labor tax rate does not directly appear in (14). Old agents use their real balances to pay for their market consumption plus the consumption tax and then die. Thus we have.

$$\phi_{-1} m^b_{-1} = C^b_{o,-1} - T_o + \eta\left(C^b_{o,-1} - T_o\right)$$

$$\phi_{-1} m^s_{-1} = C^s_{o,-1} - T_o + \eta\left(C^s_{o,-1} - T_o\right)$$

$$\phi_{-1} m^n_{-1} = C^n_{o,-1} - T_o + \eta\left(C^n_{o,-1} - T_o\right)$$

with

$$\frac{dC^j_{o,-1}}{dm^j_{-1}} = \frac{\phi_{-1}}{1 + \eta'\left(C^j_{o,-1} - T_o\right)} \qquad j = b, s, o.$$

## 4.2 DM Trading

In the second period agents receive their idiosyncratic preference shocks and trade.

### 4.2.1 Bargaining

Suppose that after receiving their lump-sum transfer of cash and realizing their idiosyncratic shocks, each buyer is paired with a seller via some process and they bargain over the terms of trade. The terms of trade is a pair $(q, d)$ where $q$ is the quantity of goods exchanged while $d$ is the quantity of money exchanged. It is assumed agent's money balances are observable to their trading partners. Assume buyer-take-all bargaining. The bargaining problem is

$$\max u(q) + \beta U\left(C_{o,+1}^b\right) - \beta U\left(\tilde{C}_{o,+1}^b\right)$$

$$s.t. \quad d \leq m^b$$

$$-\psi(q) + \beta U\left(C_{o,+1}^s\right) - \beta U\left(\tilde{C}_{o,+1}^s\right) \geq 0$$

where $\hat{C}_{o,+1}^s$ is the old age consumption if the seller walks away. If $U'(T_o)$ is sufficiently large, then the cash constraint does not bind and $q$ and $d$ solve

$$\begin{aligned}
\frac{u'(q)}{\psi'(q)} &= \frac{U'\left(C_{o,+1}^b\right)\left[1 + \eta'\left(C_{o,+1}^s - T_o\right)\right]}{U'\left(C_{o,+1}^s\right)\left[1 + \eta'\left(C_{o,+1}^b - T_o\right)\right]} \\
\psi(q) &= \beta U\left(C_{o,+1}^s\right) - \beta U\left(\tilde{C}_{o,+1}^s\right).
\end{aligned} \tag{15}$$

Note that in the absence of distortionary CM consumption taxation, $u'(q) > \psi'(q)$ since $C_{o,+1}^b < C_{o,+1}^s$.

If the cash constraint does bind $q$ and $d$ solve

$$d = m^b$$

$$\psi(q) = \beta U\left(C_{o,+1}^s\right) - \beta U\left(\tilde{C}_{o,+1}^s\right)$$

# 5   Equilibrium

In equilibrium, the threatpoints correspond to the same consumption as idle agents so $\tilde{C}_{o,+1}^b = \tilde{C}_{o,+1}^s = C_{o,+1}^n$. With buyer-take-all bargaining (12) becomes

$$
\begin{aligned}
V_t(m) &= \sigma \int \left\{ u\left[q_b(m, m^s)\right] + \beta U\left[C_{o,+1}^b(m, m^s)\right]\right\} dF(m^s) \\
&\quad + \beta(1 - \sigma) U\left[C_{o,+1}^n(m)\right].
\end{aligned}
$$

with

$$
\begin{aligned}
V_t'(m) &= \sigma \left[ u'\left[q_b(m, m^s)\right] \frac{\partial q_b}{\partial m} + \beta U'\left[C_{o,+1}^b(m, m^s)\right] \left[1 + \eta'\left(C_{o,+1}^b - T_o\right)\right]^{-1} \phi_{+1}\left(1 - \frac{\partial d}{\partial m}\right)\right] \\
&\quad + \beta(1 - \sigma) U'\left[C_{o,+1}^n(m)\right] \left[1 + \eta'\left(C_{o,+1}^n - T_o\right)\right]^{-1} \phi_{+1}.
\end{aligned}
$$

If $U'(T_o)$ is large enough so that the cash constraint does not bind then, in a symmetric equilibrium we have

$$
V_t'(m) = \phi_{+1}\beta \left[ \sigma \frac{U'\left(C_{o,+1}^b\right)}{1 + \eta'\left(C_{o,+1}^b - T_o\right)} + (1 - \sigma) \frac{U'\left(C_{o,+1}^n\right)}{1 + \eta'\left(C_{o,+1}^n - T_o\right)} \right] \tag{16}
$$

If the cash constraint does bind then, in a symmetric equilibrium, we have

$$
V_t'(m) = \phi_{+1}\beta \left[ \sigma \frac{U'\left(C_{o,+1}^s\right)}{1 + \eta'\left(C_{o,+1}^s - T_o\right)} \frac{u'(q)}{\psi'(q)} + (1 - \sigma) \frac{U'\left(C_{o,+1}^n\right)}{1 + \eta'\left(C_{o,+1}^n - T_o\right)} \right] \tag{17}
$$

Combining (16) with (13) and (14) gives

$$\frac{1}{1+\tau^c} U'(C_{y,-1}) = R_m \beta \left[ \sigma \frac{U'\left(C_{o,+1}^b\right)}{1+\eta'\left(C_{o,+1}^b - T_o\right)} + (1-\sigma) \frac{U'\left(C_{o,+1}^n\right)}{1+\eta'\left(C_{o,+1}^n - T_o\right)} \right]. \qquad (18)$$

which using (15) can also be written as

$$\frac{1}{1+\tau^c} U'(C_{y,-1}) = R_m \beta \left[ \sigma \frac{U'\left(C_{o,+1}^s\right)}{1+\eta'\left(C_{o,+1}^s - T_o\right)} \frac{u'(q)}{\psi'(q)} + (1-\sigma) \frac{U'\left(C_{o,+1}^n\right)}{1+\eta'\left(C_{o,+1}^n - T_o\right)} \right].$$

If the constraint binds then using (17) with (13) and (14) gives

$$\frac{1}{1+\tau^c} U'(C_{y,-1}) = R_m \beta \left[ \sigma \frac{U'\left(C_{o,+1}^s\right)}{1+\eta'\left(C_{o,+1}^s - T_o\right)} \frac{u'(q)}{\psi'(q)} + (1-\sigma) \frac{U'\left(C_{o,+1}^n\right)}{1+\eta'\left(C_{o,+1}^o - T_o\right)} \right] \qquad (19)$$

Equation (18) looks like a standard consumption Euler equation. However, note that this is not a standard Euler equation because the term in brackets on the right-hand-side of this expression is not expected marginal utility from consuming when old because $U'\left(C_{o,+1}^s\right)$ is missing due to the buyer-take-all assumption. So, when choosing money balances, young agents ignore any old age consumption value from a marginal unit of money should they be a seller in the DM. Since a buyer does not spend all of his money balances, DM consumption does not appear even though money is essential for trade in this market. In short, the marginal liquidity value of money in the DM is zero. Note that the Euler equations are the same for both the unconstrained and constrained case. Thus, the only difference is in the equilibrium values of the arguments.

Consider a symmetric steady-state with $m^b = m^s = m^o = M_{+1}$ and $\phi_{+1} M_{+1} = \phi_{-1} M_{-1} = z$, e.g., real balances in the CM are constant across time, and $\phi_{+1}/\phi_{-1} = 1/\gamma = R_m$ where $R_m$ is the gross rate of return on money. Furthermore, real spending (measured in the next CM goods price) in the DM is stationary, $\phi_{+1} d = \delta$.

A steady state equilibrium with a non-binding cash constraint is a list $\{z, q, \delta, C_o^b, C_o^s, C_o^n, C_y, h\}$ solving

$$C_o^s = z + \delta - \eta (C_o^s - T_o) + T_o \tag{20}$$

$$C_o^b = z - \delta - \eta (C_o^b - T_o) + T_o \tag{21}$$

$$C_o^n = z - \eta (C_o^n - T_o) + T_o \tag{22}$$

$$h = C_y + \sigma C_o^b + \sigma C_o^s + (1 - 2\sigma) C_o^n \tag{23}$$

$$U'(C_y) = \frac{1 + \tau^c}{1 - \tau^h} v'(h) \tag{24}$$

$$\psi(q) = \beta [U(C_o^s) - U(C_o^n)] \tag{25}$$

$$\frac{u'(q)}{\psi'(q)} = \frac{U'(C_o^b)[1 + \eta'(C_o^s - T_o)]}{U'(C_o^s)[1 + \eta'(C_o^b - T_o)]} \tag{26}$$

$$\frac{1}{1 + \tau^c} U'(C_y) = R_m \beta \left[ \sigma \frac{U'(C_o^b)}{1 + \eta'(C_o^b - T_o)} + (1 - \sigma) \frac{U'(C_o^n)}{1 + \eta'(C_o^n - T_o)} \right] \tag{27}$$

where (23) is the CM aggregate resource constraint and comes from substituting the government's budget constraint into the young's CM budget constraint. Using (20)-(23), the last four equations give us $q, z, \delta$ and $C_y$ as functions of $R_m = 1/\gamma$. With a binding cash constraint, (26) is replaced by $z = \delta$ and (27) is replaced by (19).

Finally, note that if $\sigma = 0$ it is straightforward to show that the unconstrained optimum can be achieved ($q$ is then irrelevant) by setting $\eta'(\cdot) = \tau_h = T_o = 0$ and $R_m = 1/\lambda$ in the monetary economy. However, it is also the case that the unconstrained optimum can be replicated by removing money from the economy and using lump-sum taxes. In short, money is not essential if the government has access to lump-sum taxes and transfers. Thus, the OLG structure is not what makes money essential in this model. Rather it is the search market with private information on preferences that is critical for the results we obtain below regarding the optimal fiscal policy. The OLG structure itself merely keeps the distribution of money balances tractable.

# 6   Replicating the constrained optimum

In general, it is difficult to show existence or uniqueness of the constrained optimal alloca-
tion and the equilibrium allocation. However, examples can be constructed which in which
unique solutions exist. Nevertheless, under the assumption that a constrained optimal allo-
cation exists, we can ask whether or not it can be replicated (decentralized) in the monetary
economy via an appropriately designed system of taxes.

## 6.1   Non-linear consumption taxes

Consider the equilibrium allocation where the cash constraint does not bind. Under Inada
conditions on CM utility, this can always be achieved by setting $T_o$ sufficiently low (or even
zero). For ease of notation, let $\eta \left( C_o^j - T_o \right) = \tau^j C_o^j$ and $\eta \left( C_o^j - T_o \right) = \tau^j$. Although I have
modeled this as a consumption tax it is equivalent to a non-linear tax on the old agents'
monetary wealth.

Comparing (5)-(10) to (21)-(27) we see that replicating the constrained optimal allocation
requires

$$
\begin{align}
\tau_h &= -\tau_c \tag{28} \\
\frac{1+\tau_b}{1+\tau_o} &= \frac{z-\delta}{z} < 1 \tag{29} \\
\frac{1+\tau_s}{1+\tau_b} &= \frac{U'\left(\bar{C}_o^s\right)}{(1-\sigma)\,U'\left(\bar{C}_o^s\right)+\sigma U'\left(\bar{C}_o^b\right)} < 1 \tag{30} \\
R_m \left[ (1-\sigma)\left(\frac{1+\tau_c}{1+\tau_o}\right) + \sigma\left(\frac{1+\tau_c}{1+\tau_b}\right) \right] &= \lambda^{-1} \frac{U'\left(\bar{C}_o^s\right)}{(1-\sigma)\,U'\left(\bar{C}_o^s\right)+\sigma U'\left(\bar{C}_o^b\right)} \tag{31}
\end{align}
$$

The first line ensures that the young agents consumption/work decision is not distorted
and is an application of the uniform commodity taxation principle. The second is needed
for $\bar{C}_o^b = \bar{C}_o^n$, i.e., partial risk-sharing. The third ensures that the quantities in the DM are

constrained optimal. Finally, the last one ensures the optimal amount of risk-sharing occurs. Since there are 6 quantities to replicate and 8 tax instruments available, there are numerous ways of replicating the constrained optimal allocation. But under all possible tax policies we have the following relationship: $\tau_o > \tau_b > \tau_s$ which means there is some progressivity in the tax system and some regressivity. Thus, a linear consumption (or wealth) tax cannot replicate the constrained optimal allocation. Interestingly, the 'rich' old agents (those who sold in the DM) face the lowest tax rate. This is needed to induce them to produce in the DM and to produce the optimal amount.

With a full set of non-linear tax rates, the real return on money $R_m$ can be greater than one, one or less than one. Thus, the government may or may not use the inflation tax. However, it cannot be too small for the following reason. A young agent could work more and take in $z + \delta$ units of money into the DM, then choose not to work if he is a seller and have $z + \delta$ or $z$ when old. This incentive to deviate can be avoided by raising the real value of money and thus $\delta$. This in turn makes it more costly for the young agent to acquire the additional $\delta$ units of real balances. Consequently, $R_m$ cannot be too low.

However, note that if $\tau_c = 0$, then we can interpret $\tau_o, \tau_b$ and $\tau_c$ as tax rates on money balances(or wealth) when old. In this case, the LHS of (31) measures the after-tax real return on money. It then follows that the optimal policy reduces the after-tax rate of return on money, the LHS of (31), below what would be optimal in the standard two-period OG model, which is $R_m = 1/\lambda$. In short, the optimal policy is to reduce the after tax return on money to reduce consumption risk when old.

Note that if $\sigma = 0$, then $\delta = 0$ and $\tau_o = \tau_b = \tau_c = \tau_h = 0$, $R_m = 1/\lambda$ replicates the constrained optimum. However, money is not essential in this case if lump-sum transfers are available since with $\sigma = 0$ the information/trade frictions disappear and $T_o = \bar{C}_o$ is optimal.

## 6.2 Linear tax rates and lump-sum taxes

With linear consumption taxes, the constrained optimum cannot be replicated even if lump-sum taxes are available. If we constrain the government to using linear tax rates and lump sum taxes, will it be optimal to use the inflation tax? In general it is difficult to show however. In what follows I construct an example where the constrained optimum exists and is unique. Furthermore, the equilibrium with unconstrained cash holdings also exists and is unique if $T_o$ is sufficiently low. I then consider a Ramsey problem where the government chooses, consumption tax rates and labor tax rates and lump sum taxes.

Let preferences in the CM be homothetic and given by $U(C) = \ln C$, $v(h) = h$, $u(q) = 1 - \exp^{-q}$ and $\psi(q) = \rho q$ with $\sigma < \rho < 1$. It follows that $C_y^* = 1$ in all examples and $q^* = \ln(1/\rho) > 0$. Since $U(0) = \ln 0 \rightarrow -\infty$, the young's participation constraint will not be binding.

**Example of the constrained optimal allocation**  With these preferences, from (5)-(8) we obtain

$$\bar{C}_o^{\prime s} = \frac{\beta}{\lambda \exp^{\bar{q}}} \qquad \bar{C}_o^b = \bar{C}_o^m = \frac{\sigma\beta}{1-\sigma}\left[\frac{1}{\sigma} - \frac{1}{\lambda \exp^{\bar{q}}}\right] \qquad \bar{C}_y = 1 \quad \bar{h} = 1 + \beta$$

where a unique value $0 < \bar{q} < q^*$ solves

$$\lambda\rho\bar{q} + \beta\ln\left[\rho\exp^{\bar{q}} - \sigma\right] = \beta\ln(1-\sigma).$$

**Example of the non-binding cash constraint**  Assume that $\tau^j = \tau^c$ so that all agents pay the same linear consumption tax and $U'(T_o)$ is sufficiently large that agents do not spend

all of their real balances in middle age. For these functional forms (20)-(27) yield

$$\hat{C}_y = \frac{1 - \tau_h}{1 + \tau^c}$$

$$\hat{C}_o^s = (1 + \hat{\varepsilon}) R_m \beta \left(\frac{1 - \tau_h}{1 + \tau^c}\right) \left(\frac{1 - \hat{\varepsilon} + \sigma\hat{\varepsilon}}{1 - \hat{\varepsilon}}\right)$$

$$\hat{C}_o^m = R_m \beta \left(\frac{1 - \tau_h}{1 + \tau^c}\right) \frac{1 - \hat{\varepsilon} + \sigma\hat{\varepsilon}}{1 - \hat{\varepsilon}}$$

$$\hat{C}_o^b = (1 - \hat{\varepsilon}) R_m \beta \left(\frac{1 - \tau_h}{1 + \tau^c}\right) \left(\frac{1 - \hat{\varepsilon} + \sigma\hat{\varepsilon}}{1 - \hat{\varepsilon}}\right)$$

$$\hat{h} = \left(\frac{1 - \tau_h}{1 + \tau^c}\right) \left[1 + R_m \beta \left(\frac{1 - \hat{\varepsilon} + \sigma\hat{\varepsilon}}{1 - \hat{\varepsilon}}\right)\right]$$

$$\hat{q} = \ln(1/\rho) + \ln\left(\frac{1 - \hat{\varepsilon}}{1 + \hat{\varepsilon}}\right)$$

$$\hat{z} = R_m \beta (1 - \tau_h) \frac{1 - \hat{\varepsilon} + \sigma\hat{\varepsilon}}{1 - \hat{\varepsilon}} - (1 + \tau^c) T_o$$

where $0 < \hat{\varepsilon} < \frac{1 - \rho}{1 + \rho}$ solves

$$\ln(1/\rho) = \left(\frac{\beta + \rho}{\rho}\right) \ln(1 + \hat{\varepsilon}) - \ln(1 - \hat{\varepsilon})$$

and is independent of $R_m$, $\tau_h$, $\tau_c$ and for $T_o$ sufficiently small to ensure the cash constraint is not binding. Surprising since $\hat{\varepsilon}$ is independent of $R_m$ the quantity traded in the DM is independent of any of the tax instruments. This is due to the homothetic nature of CM preferences – although $\hat{C}_o^s$ and $\hat{C}_o^b$ are decreasing functions of $R_m$ the ratios of the two are not. Hence from (26) if $\hat{\varepsilon}$ is independent of $R_m$ then $q$ is as well.

**Optimal fiscal policy**   What is the optimal fiscal policy in this economy?  Assume the government chooses taxes to maximize steady-state welfare subject to these equilibrium outcomes. Under the condition that $T_o$ is small enough such that the cash constraint does

not bind, $\hat{q}$ is independent of policy. So ignore the DM payoff. We then have

$$
\begin{aligned}
\mathcal{W} &= \beta\lambda^{-1}\left[\sigma U\left(C_o^b\right) + \sigma U\left(C_o^s\right) + (1-2\sigma)U\left(C_o^n\right)\right] \\
&\quad + \frac{1}{1-\lambda}\left\{U\left(C_y\right) - \upsilon\left(h\right) + \beta\left[\sigma U\left(C_o^b\right) + \sigma U\left(C_o^s\right) + (1-2\sigma)U\left(C_o^n\right)\right]\right\}
\end{aligned}
$$

Substituting in the expressions above and rearranging yields

$$
\begin{aligned}
\mathcal{W} &= \frac{1}{1-\lambda}\left[\ln\left(\frac{1-\tau_h}{1+\tau_c}\right) - \frac{1-\tau_h}{1+\tau_c}\left(1+R_m\beta\frac{1-\hat{\varepsilon}+\sigma\hat{\varepsilon}}{1-\hat{\varepsilon}}\right)\right] \\
&\quad + \frac{\beta}{(1-\lambda)\lambda}\left[\sigma\ln\left(\frac{1-\hat{\varepsilon}}{1-\hat{\varepsilon}}\right) + \ln\left(\frac{1-\tau_h}{1+\tau_c}\right) + \ln R_m + \ln\beta\frac{1-\hat{\varepsilon}+\sigma\hat{\varepsilon}}{1-\hat{\varepsilon}}\right]
\end{aligned}
$$

The FOC yield

$$
\begin{aligned}
\frac{1-\tau_h}{1+\tau_c} &= 1 \\
R_m &= \frac{1}{\lambda}\left(\frac{1-\hat{\varepsilon}}{1-\hat{\varepsilon}+\sigma\hat{\varepsilon}}\right) < \frac{1}{\lambda}.
\end{aligned}
$$

This implies that there is more inflation in this model than is optimal in the standard two-period OG model. Note that if we set $\sigma = 0$, which effectively shuts down the DM, then $R_m = 1/\lambda$ which is that standard result. If $\sigma = 0$ and $\lambda = 1$ so the government maximizes the SS welfare of the representative generation, then $R_m = 1$ and $\gamma = 1$ is optimal.

So why is inflation preferred when $\sigma > 0$? In their choice of real balances, young agents take into account the marginal value of buying additional goods. However, in equilibrium, this additional unit of money also raises the amount of real wealth they have if they are a seller in a match. This wealth effect means the quantity produced will be lower, ceteris paribus. Hence, in a symmetric equilibrium they overestimate the quantity of goods they receive from this marginal unit of money. As a result, money is overvalued and young agents must work too much to acquire a unit of real balances. Thus, inflation reduces the wealth

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of agents and induces sellers to produce more on the margin for a unit of money.

Finally, if $\tau_c = \tau_h = 0$ then $T_o = -T_y$ but these transfers have no effect on the allocation when they are sufficiently small since $\hat{\varepsilon}$ is constant. Why? If $T_o > 0$, young agents know they will be getting a transfer of goods when old. This lowers the marginal value of carrying a unit of money into old age and therefore decreases the demand for money when young. It follows that the goods price of money in the CM, $\phi$, decreases thereby reducing the equilibrium value of real balances by exactly the value of the transfer. Consequently, consumption when old is unaffected by the transfer. More succinctly, real balances and transfers are perfect subsitutes. While the current young have to work more to provide the transfer to the current old, they work less to earn the needed real balances. On net, labor hours are unchanged. So setting $T_o = T_y = 0$ is consistent with maximizing SS welfare.

# 7    Conclusion

We observe distortionary taxation of wealth by governments to redistribute wealth and to achieve a more equitable distribution of consumption across agents. Since standard Ramsey analyses suggest lump-sum taxation achieves the first best allocation, the use of distorting taxes is puzzling. Research in the NDPF literature has provided important insights as to why governments may use distortionary taxation of wealth, via capital income taxes, even though lump sum taxes are available. This paper has shown that since money is always a form of wealth, even when used for transaction purposes, it is not exempt from the same arguments.

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