Money and Capital: A Quantitative Analysis

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Abstract

We study the effects of money (anticipated inflation) on capital formation. Previous papers on this adopt reduced-form approaches, putting money in the utility function or imposing cash in advance, but use otherwise frictionless models. We follow a literature that is more explicit about the frictions that make money essential. This introduces several new elements, including a two-sector structure with centralized and decentralized markets, stochastic trading opportunities, and bargaining. We show how these elements matter qualitatively and quantitatively. Our numerical results differ from findings in the reduced-form literature. The analysis also reduces the gap between monetary theory and mainstream macro .

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1   Introduction

We study the relation between fully anticipated inflation and capital formation. This is a classic issue, going back to Tobin (1965), Sidrauski (1967a,1967b), Stockman (1981), Cooley and Hansen (1989,1991), Gomme (1993), Ireland (1994) and many others. All these papers adopt reduced-form approaches: they put money in the utility function, or impose cash in advance, in an attempt to capture implicitly the role of money in the exchange process, but in other respects they ignore frictions. An alternative literature on money, going back to Kiyotaki and Wright (1989,1993), Aiyagari and Wallace (1991), Shi (1995), Trejos and Wright (1995), Kocherlakota (1998), Wallace (2001) and others, strives to be more explicit about the frictions that make a medium of exchange essential. In doing so, these papers introduce new elements into monetary economics, including detailed descriptions of specialization, information, matching, alternative pricing mechanisms, etc., and show these ingredients matter in theory. We show here that they also matter for quantitative analysis.

We use the two-sector model in Lagos and Wright (2005), where some economic activity takes place in centralized and some in decentralized markets. In addition to providing micro-foundations for money, decentralized markets allow us to introduce ingredients like stochastic trade opportunities and bargaining, while centralized markets allow us to incorporate capital as in standard growth theory. This constitutes a step toward integrating theories with decentralized trade and mainstream macro, which has been a challenge for some time. As Azariadis (1993) put it, "Capturing the transactions motive for holding money balances in a compact and logically appealing manner has turned out to be an enormously complicated task. Logically coherent models such as those proposed by Diamond (1984) and Kiyotaki and Wright (1989) tend to be so removed from neoclassical growth theory as to seriously hinder the job of integrating rigorous monetary theory with the rest of macroeconomics." And as Kiyotaki and Moore (2001) put it, "The matching models are without doubt ingenious and beautiful. But it is quite hard to integrate them with the rest of macroeconomic theory – not least because they jettison the basic tool of our trade, competitive markets."

We think our framework makes progress by combining interesting components from both standard models in macro and models in monetary theory that strive for better microfounda-

tions.[1] To explain how this works, relative to reduced-form models, here are the ingredients that matter most.

- Stochastic trading opportunities, like those in search models, are critical for matching observations on velocity. These observations are notoriously hard to capture in cash-in-advance models, especially when calibrating to a shorter period length (see Telyukova and Visschers 2009 for a recent discussion).

- Our two-sector structure highlights a channel not in previous theories. When capital produced in the centralized market is used in decentralized production, since inflation is a tax on decentralized trade, monetary policy affects centralized market investment.

- The above effect depends a lot on what one assumes about price formation in decentralized trade. If we use bargaining, inflation has little impact on investment, although it still has a sizable impact on welfare: going from 10% inflation to the Friedman rule barely changes the capital stock, but is worth around 3% of consumption. Alternatively, if we use price taking, the same experiment increases long-run capital between 3% and 5%, and has a welfare effect of 1.5% across steady states or 1% when we take into account transitions.

- Some other elements of the specification also matter – e.g. given our fiscal policy, we do not get the first best outcome even at the optimal monetary policy, which increases the cost of inflation under either bargaining or price taking – but the list above describes the main innovations relative to past work.[2]

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[1]A previous attempt to put capital into a monetary model by Aruoba and Wright (2003) lead to some undesirable implications, including the following dichotomy: one can solve independently for allocations in the centralized and decentralized markets. This implies monetary policy has no impact on investment, employment or consumption in the centralized market. Other attempts to study money and capital in models with frictions include Shi (1999), Shi and Wang (2006), and Menner (2006), who build on Shi (1997), and Molico and Zhang (2005), who build on Molico (2006). Those models have only decentralized markets. It is much easier to connect with mainstream macro in a model with some centralized trade. In particular, in nonmonetary equilibrium, our model reduces to the textbook growth model (see fn.8 below), while those models reduce to something quite different.

[2]It is worth emphasizing that with either bargaining or price taking our numerical results differ from the reduced-form literature. Here is a short survey: Cooley and Hansen (1989, 1991) find much smaller effects, with welfare numbers substantially below 1%. Gomme (1993) gets even smaller effects in an endogenous growth setup. Ireland (1994) gets welfare numbers around 0.67%. Lucas (2000) (without capital) gets welfare

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The intuition for why it matters whether one assumes price taking or bargaining is the following. When agents invest in capital they not only earn income in the centralized market, they also lower their production cost in the decentralized market. But there is a holdup problem, well known to practitioners of bargaining theory, but previously neglected in macro. Suppose the buyer gets a big share of the surplus in bilateral trade. Then the seller does not reap much of a return on his investment above what he gets in standard models, so the demand for capital does not depend much on what happens in decentralized trade, and inflation does not affect investment much. Now suppose the buyer has low bargaining power. Then the seller does get a big share of the surplus, but the surplus is small, due to a holdup problem on money demand. So whether buyer bargaining power is high or low, inflation has a small impact on investment. This depends on calibration, of course, but we find the impact is quite small for a wide range of parameters. Nonetheless, due to these same holdup problems, decentralized market consumption is very low, so even though inflation does not have a huge effect on decentralized trade it does have a sizable welfare impact.

With price taking, these holdup problems vanish. This means investment demand depends much more on what happens in the decentralized market. Since inflation is a tax on decentralized trade, it acts as an important tax on investment. Thus monetary policy can have a big impact on capital formation. However, without the holdup problems, decentralized market consumption is not nearly so low, and thus when it decreases with inflation the net effect is less painful with competitive price taking than bargaining. We use the theory to provide measures of the cost of bargaining inefficiencies, and find that they are sizable. This is true even though we have bargaining only in the decentralized market, which accounts less than 8% of aggregate output for our parameter values. These results about bargaining in models that are otherwise similar to standard macro theory are novel and suggestive. However, to be clear, the goal is not to take a stand on whether bargaining or price taking is more reasonable, or which better matches the data. Our intent is to lay out a model with each mechanism and document that it makes a difference.

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numbers below 1%; earlier efforts at this approach by Lucas (1981) and Fischer (1981) get 0.3% to 0.45%. Imorhoroglu and Prescott (1991) also get less than 1%. A few papers find larger effects, such as Dotsey and Ireland (1996), because even though inflation does not affect capital very much it does affect the amount of resources used in intermediation.

The rest of the paper proceeds as follows. In Section 2 we describe the model. In Section 3 we discuss calibration. In Section 4 we present quantitative results. In Section 5 we conclude. The Appendix discusses details and alternative specifications.

2 The Basic Model

2.1 General Assumptions

A $[0,1]$ continuum of agents live forever in discrete time. To combine elements of standard macro and search theory, we adopt the sectoral structure in Lagos and Wright (2005), hereafter LW. Each period agents engage in two types of economic activity. Some activity takes place in a frictionless centralized market, called the CM, and some takes place in a decentralized market, called the DM, with two main frictions: a *double coincidence problem*, and *anonymity*, which combine to makes a medium of exchange essential.[3] Given that some medium of exchange is essential, one issue in monetary theory is to determine endogenously which objects serve this function (e.g. Kiyotaki and Wright 1989). In order to focus on other questions, however, other papers avoid this issue by assuming there is a unique storable asset that qualifies for the role. Since we obviously cannot assume a unique storable asset in a paper called "Money and Capital," we need to say a few words about the issue.

What we have to offer is a story along the lines of the "worker-shopper pair" used to motivate cash-in-advance constraints by Lucas (1980), although we extend it based on time-honored ideas about currency having advantages in terms of *portability* and *recognizability*. First, in terms of portability, in the DM our agents have their capital physically fixed in place at production sites. Thus, when you want to buy something from someone you must visit their location, and since you cannot bring your capital, it cannot be used in payment, while currency can. This use of spatial separation is in the spirit of the "worker-shopper" idea, but one really should go beyond this, in any model, and ask why *claims* to capital cannot overcome this friction. One approach is to invoke recognizability. A stark assumption that works is that agents can costlessly counterfeit claims, other than currency, say because the monetary authority has a monopoly on the technology for producing hard-to-counterfeit

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[3]For formal discussions of essentiality and anonymity we refer readers to Kocherlakota (1998), Wallace (2001) or Aliprantis et al. (2007).

notes. Given this, sellers no more accept claims to capital from anonymous buyers in the DM than they accept personal IOU's. Thus, money has a role even while capital is a storable factor of production.[4]

As in standard growth theory, in the CM there is a general good that can be used for consumption or investment, produced using labor H and capital K hired by firms in competitive markets. Profit maximization implies $r = F_K(K, H)$ and $w = F_H(K, H)$, where F is the technology, r the rental rate, and w the real wage. Constant returns implies equilibrium profits are 0. In the DM these firms do not operate, but an agent's own effort e and capital k can be used with technology $f(e, k)$ to produce a different good. Note that k appears as an input the DM, because when you go to a seller's location he has access to his capital, even though you do not have access to your capital. This is important – it is the fact that capital produced in the CM is productive in the DM that breaks the dichotomy mentioned in fn. 1, and this means money has interesting effects on investment and other CM variables.

In the DM, each period with probability σ an agent discovers he is a *buyer*, which means he wants to consume but cannot produce, so he visits the location of someone that can produce; with probability σ he is a *seller*, which means he can produce but does not want to consume, so he waits at his location for someone to visit him; and with probability $1 - 2\sigma$ he is a *nontrader*, and he neither produces nor consumes. This taste-and-technology-shock specification is basically equivalent to bilateral matching, as in many money models, where there is a probability σ of meeting someone that produces a good that you like. We think our specification perhaps fits better with the idea of spatial separation, with buyers visiting sellers' locations, but this is not otherwise important. In any case, in some buyer-seller meetings, the former is able to pay with credit due in the next CM. We think of these meetings as *monitored*. Let ℓ (for loan) be the payment made in the CM, measured in dollars, and assume it is costlessly enforced (no strategic default). But credit is only available in meetings with probability $1 - \omega$. With probability ω, the buyer is anonymous, or the meeting

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[4]While we by no means consider this the last word on the coexistence of money and other assets, we think the story is logically coherent. See Lester et al. (2008) and references therein for attempts to analyze these ideas more rigorously.

not monitored, and the seller requires cash.

Instantaneous utility for everyone in the CM is $U(x) - Ah$, where x is consumption and h labor; as in most applications of LW, linearity in h reduces the complexity of the analysis considerably, although Rocheteau et al. (2008) show how to get the same simplification with general preferences by assuming indivisible labor and lotteries à la Rogerson (1988). In the DM, with probability σ you are a buyer and enjoy utility $u(q)$, and with probability σ you are a seller and get disutility e, where q is consumption and e labor (normalizing the disutility of DM labor to be linear is a choice of units with no implications for our results). Assume u and U have the usual monotonicity and curvature properties. Solving $q = f(e, k)$ for $e = c(q, k)$, we get the utility cost of producing q given k. One can show that $c_q > 0$, $c_k < 0$, $c_{qq} > 0$ and $c_{kk} > 0$ under the usual assumptions on f, and additionally $c_{qk} < 0$ if k is a normal input.

Government sets the money supply so that $M_{+1} = (1 + \tau)M$, where $+1$ denotes next period. We use τ as our policy instrument, but we could instead target inflation or nominal interest rates. In steady state, inflation equals τ and the nominal rate is defined by the Fisher equation $1 + i = (1 + \tau)/\beta$. Government also consumes G, levies a lump-sum tax T, labor income tax t_h, capital income tax t_k, and sales tax t_x in the CM (we omit sales taxes in the DM to ease the presentation, but it makes little difference for the results). Letting δ be the depreciation rate of capital, which is tax deductible, and p the CM price level, the government budget constraint is $G = T + t_h wH + (r - \delta) t_k K + t_x X + \tau M/p$ if we interpret M as currency. When we interpret it as currency plus inside money, as in some of the discussion below, we have to adjust the budget (but this does not matter if we can change T since with quasi-linear utility the wealth effect of changing T is nil).

Let $W(m, k, \ell)$ be the value function for an agent in the CM holding m dollars and k units of capital who owes ℓ from the previous DM. Let $V(m, k)$ be the DM value function. Assuming agents discount between the CM and DM at rate β, but not between the DM and CM, we have

$$W(m, k, \ell) = \max_{x, h, m_{+1}, k_{+1}} \{U(x) - Ah + \beta V_{+1}(m_{+1}, k_{+1})\} \tag{1}$$

$$\text{s.t. } (1 + t_x) x = w(1 - t_h) h + [1 + (r - \delta)(1 - t_k)] k - k_{+1} - T + \frac{m - m_{+1} - \ell}{p}.$$

Eliminating h using the budget and taking FOC, assuming interiority, we get[5]

$$x \quad : \quad U'(x) = \frac{A(1 + t_x)}{w(1 - t_h)}$$

$$m_{+1} \quad : \quad \frac{A}{pw(1 - t_h)} = \beta V_{+1,m}(m_{+1}, k_{+1}) \tag{2}$$

$$k_{+1} \quad : \quad \frac{A}{w(1 - t_h)} = \beta V_{+1,k}(m_{+1}, k_{+1}).$$

Since (m, k, ℓ) does not appear in (2), for any distribution of (m, k, ℓ) across agents entering the CM, the distribution of (m_{+1}, k_{+1}) exiting is degenerate. Also, W is linear:

$$\begin{aligned}
W_m(m, k, \ell) &= \frac{A}{pw(1 - t_h)} \\
W_k(m, k, \ell) &= \frac{A[1 + (r - \delta)(1 - t_k)]}{w(1 - t_h)} \\
W_\ell(m, k, \ell) &= \frac{-A}{pw(1 - t_h)}
\end{aligned} \tag{3}$$

Moving to the DM, we have

$$V(m, k) = \sigma V^b(m, k) + \sigma V^s(m, k) + (1 - 2\sigma)W(m, k, 0), \tag{4}$$

where the values to being a buyer and a seller are

$$V^b(m, k) = \omega[u(q_b) + W(m - d_b, k, 0)] + (1 - \omega)[u(\hat{q}_b) + W(m, k, \ell_b)] \tag{5}$$

$$V^s(m, k) = \omega[-c(q_s, k) + W(m + d_s, k)] + (1 - \omega)[-c(\hat{q}_s, k) + W(m, k, -\ell_s)]. \tag{6}$$

In these expressions, q_b and d_b (q_s and d_s) denote the quantity of goods and dollars exchanged when buying (selling) for money, while $\hat{q}_b$ and ℓ_b ($\hat{q}_s$ and $-\ell_s$) denote the quantity and the value of the loan for the buyer (seller) when trading on credit. We write these expressions as though no money changes hands in credit matches; this is without loss in generality. Similarly, the fact that the loan ℓ is nominal is irrelevant.

Using the linearity of W, write V as

$$\begin{aligned}
V(m, k) = {} & W(m, k, 0) + \sigma\omega\left[u(q_b) - \frac{d_b A}{pw(1 - t_h)}\right] + \sigma\omega\left[\frac{d_s A}{pw(1 - t_h)} - c(q_s, k)\right] \\
& + \sigma(1 - \omega)\left[u(\hat{q}_b) - \frac{A\ell_b}{pw(1 - t_h)}\right] + \sigma(1 - \omega)\left[\frac{A\ell_s}{pw(1 - t_h)} - c(\hat{q}_s, k)\right]
\end{aligned} \tag{7}$$

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[5]One can adapt the discussion in LW to guarantee the concavity of the problem and interiority of the solution; in quantitative analysis, we can check it directly.

This yields

$$V_m(m,k) = \frac{A}{pw(1-t_h)} + \sigma\omega\left[u'\frac{\partial q_b}{\partial m} - \frac{A}{pw(1-t_h)}\frac{\partial d_b}{\partial m}\right]$$
$$+\sigma\omega\left[\frac{A}{pw(1-t_h)}\frac{\partial d_s}{\partial m} - c_q\frac{\partial q_s}{\partial m}\right] \tag{8}$$

$$V_k(m,k) = \frac{A[1+(r-\delta)(1-t_k)]}{w(1-t_h)} + \sigma\omega\left[u'\frac{\partial q_b}{\partial k} - \frac{A}{pw(1-t_h)}\frac{\partial d_b}{\partial k}\right]$$
$$+\sigma\omega\left[\frac{A}{pw(1-t_h)}\frac{\partial d_s}{\partial k} - c_q\frac{\partial q_s}{\partial k} - c_k\right] \tag{9}$$
$$+\sigma(1-\omega)\left[u'\frac{\partial\hat{q}_b}{\partial k} - \frac{A}{pw(1-t_h)}\frac{\partial\ell_b}{\partial k}\right]$$
$$+\sigma(1-\omega)\left[\frac{A}{pw(1-t_h)}\frac{\partial\ell_s}{\partial k} - c_q(\hat{q}_s,k)\frac{\partial\hat{q}_s}{\partial k} - c_k(\hat{q}_s,k)\right].$$

Once we specify how the terms of trade (q, d, $\hat{q}$ and ℓ) are determined, we can substitute for the derivatives in (8) and (9) to get the equilibrium conditions.

First, as a benchmark, consider the planner's problem when money is *not* essential, say, because $\omega = 0$ and all meetings are monitored:

$$J(K) = \max_{X,H,K_{+1},q}\{U(X) - AH + \sigma[u(q) - c(q,K)] + \beta J_{+1}(K_{+1})\} \tag{10}$$
$$\text{s.t. } X = F(K,H) + (1-\delta)K - K_{+1} - G$$

Eliminating X, and again assuming interiority, we have the FOC

$$q : u'(q) = c_q(q,K)$$
$$H : A = U'(X)F_H(K,H) \tag{11}$$
$$K_{+1} : U'(X) = \beta J'_{+1}(K_{+1}).$$

The envelope condition $J'(K) = U'(X)[F_K(K,H) + 1 - \delta] - \sigma c_k(q,K)$ implies

$$U'(X) = \beta U'(X_{+1})[F_K(K_{+1},H_{+1}) + 1 - \delta] - \beta\sigma c_k(q_{+1},K_{+1}). \tag{12}$$

From the first condition in (11), $q = q^*(K)$ where $q^*(K)$ solves $u'(q) = c_q(q,K)$. Then the paths for (K_{+1},H,X) satisfy the Euler equation (12), the second FOC in (11), and the constraint in (10).

This characterizes the first best, or FB for short.[6] Note the presence of the term $-\beta\sigma c_k(q_{+1}, K_{+1}) > 0$ in (12), which reflects the fact that investment affects DM as well as CM productivity because K is used in both sectors. If K did not appear in $c(q)$ the system would dichotomize: we could first set $q = q^*$, where q^* solves $u'(q) = c'(q)$, and then solve the other conditions independently for (K_{+1}, H, X). The fact that K is used in the DM and produced in the CM breaks this dichotomy. Here we assume it is the *same* K used in both sectors, but the Appendix contains a version with two *distinct* capital goods in the CM and DM, as well as a version where K is used only in the CM but is produced and traded in the DM. As discussed in Section 4.3, these variations do not affect the main quantitative results much: i.e., while the one- and two-capital models behave differently if we hold parameters constant, if we recalibrate to fit the same targets the results are similar.

2.2 Bargaining

Assume the DM terms of trade are determined by bargaining. Consider a nonmonitored meeting where trade requires cash. If the buyer's and seller's states are (m_b, k_b) and (m_s, k_s), we assume (q, d) solves the generalized Nash bargaining problem with bargaining power for the buyer θ and threat points given by continuation values. Since the buyer's payoff from trade is $u(q) + W(m_b - d, k_b, 0)$ and his threat point is $W(m_b, k_b, 0)$, by the linearity of W, his surplus is $u(q) - Ad/pw(1 - t_h)$. Similarly, the seller's surplus is $Ad/pw(1 - t_h) - c(q, k_s)$. Hence the bargaining solution is

$$\max_{q,d} \left[u(q) - \frac{Ad}{pw(1 - t_h)} \right]^\theta \left[\frac{Ad}{pw(1 - t_h)} - c(q, k_s) \right]^{1-\theta} \quad \text{s.t. } d \leq m_b.$$

As in LW, it is easy to show that in equilibrium $d = m_b$. Inserting this and taking the FOC with respect to q,

$$\frac{m_b}{p} = \frac{g(q, k_s)w(1 - t_h)}{A}, \tag{13}$$

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[6]Standard methods imply the solution is characterized by the FOC and envelope condition, or, we can replace the FOC for K_{+1} and envelope condition by the Euler equation and transversality condition. One can check when there is a unique steady state to which the planner's solution converges under the usual kind of assumptions. Things are more complicated for equilibria because of distortions. In the working paper we show analytically there is a unique steady state under price taking; in the bargaining version we rely on numerical results.

where

$$g(q, k_s) \equiv \frac{\theta c(q, k_s)u'(q) + (1 - \theta)u(q)c_q(q, k_s)}{\theta u'(q) + (1 - \theta)c_q(q, k_s)}. \tag{14}$$

Writing $q = q(m_b, k_s)$, where $q(\cdot)$ is given by (13), the relevant derivatives in (8) and (9) are $\partial d/\partial m_b = 1$, $\partial q/\partial m_b = A/pw\,(1 - t_h)\,g_q > 0$ and $\partial q/\partial k_s = -g_k/g_q > 0$, where

$$g_q = \frac{u'c_q[\theta u' + (1 - \theta)c_q] + \theta(1 - \theta)(u - c)[(u'c_{qq} - c_q u'')}{[\theta u' + (1 - \theta)c_q]^2} > 0$$

$$g_k = \frac{\theta u'c_k[\theta u' + (1 - \theta)c_q] + \theta(1 - \theta)(u - c)u'c_{qk}}{[\theta u' + (1 - \theta)c_q]^2} < 0.$$

Now consider a meeting where credit is available, assuming the buyer has the same bargaining power θ. Then $(\hat{q}, \ell)$ is determined just like (q, d) above, except there is no constraint on ℓ, the way we had $d \leq m_b$ in monetary trades. Hence,

$$u'(\hat{q}) = c_q(\hat{q}, k_s)$$

$$\frac{A\ell}{pw\,(1 - t_h)} = (1 - \theta)\,u(\hat{q}) + \theta c(\hat{q}, k_s).$$

Given $k_s = K$, notice $\hat{q}(K)$ is the same as the solution to the planner's problem $q^*(K)$. Hence, $\partial \hat{q}_b/\partial k_b = \partial \ell_b/\partial k_b = 0$ and

$$\frac{\partial \hat{q}_s}{\partial k_s} = \frac{c_{qk}(\hat{q}, k_s)}{u''(\hat{q}) - c_{qq}(\hat{q}, k_s)} > 0$$

$$\frac{\partial \ell_s}{\partial k_s} = \frac{pw\,(1 - t_h)}{A}\left[\frac{u'(\hat{q})c_{qk}(\hat{q}, k_s)}{u''(\hat{q}) - c_{qq}(\hat{q}, k_s)} + \theta c_k(\hat{q}, k_s)\right]$$

Inserting these results and imposing $(m, k) = (M, K)$, (8) and (9) reduce to

$$V_m(M, K) = \frac{(1 - \sigma\omega)A}{pw\,(1 - t_h)} + \frac{\sigma\omega A u'(q)}{pw\,(1 - t_h)\,g_q(q, K)} \tag{15}$$

$$V_k(M, K) = \frac{A\,[1 + (r - \delta)\,(1 - t_k)]}{w\,(1 - t_h)} - \sigma\omega\gamma(q, K) - \sigma\,(1 - \omega)\,(1 - \theta)\,c_k(\hat{q}, K) \tag{16}$$

where it is understood that $q = q(M, K)$ and $\hat{q} = \hat{q}(K)$, while

$$\gamma(q, K) \equiv c_k(q, K) + c_q\frac{\partial q}{\partial K} = c_k(q, K) - c_q\,(q, K)\frac{g_k(q, K)}{g_q(q, K)} < 0. \tag{17}$$

The last two terms in (16) capture the idea that if a seller has an extra unit of capital it affects marginal cost in the DM, which augments the value of investment in the CM. The expression in (17) captures non-price-taking behavior in the model: the first term reflects

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the cost reduction due to extra capital, and the second reflects the change in cost due to the change in the terms-of-trade when sellers have more capital.

Substituting (15) and (16), as well as prices $p = AM/w\,(1 - t_h)\,g(q, K)$, $r = F_K(K, H)$, and $w = F_H(K, H)$, into the FOC for m_{+1} and k_{+1}, we get the equilibrium conditions

$$\frac{g(q, K)}{M} = \frac{\beta g(q_{+1}, K_{+1})}{M_{+1}} \left[1 - \sigma\omega + \sigma\omega \frac{u'(q_{+1})}{g_q(q_{+1}, K_{+1})}\right] \tag{18}$$

$$U'(X) = \beta U'(X_{+1}) \{1 + [F_K(K_{+1}, H_{+1}) - \delta]\,(1 - t_k)\} \tag{19}$$

$$-\beta\,(1 + t_x)\,\sigma\,[\omega\gamma(q_{+1}, K_{+1}) + (1 - \omega)\,(1 - \theta)\,c_k(\hat{q}_{+1}, K_{+1})].$$

Two other conditions come from the FOC for X and the resource constraint,

$$U'(X) = \frac{A\,(1 + t_x)}{(1 - t_h)\,F_H(K, H)} \tag{20}$$

$$X + G = F(K, H) + (1 - \delta)K - K_{+1}. \tag{21}$$

An *equilibrium with bargaining* is defined as (positive, bounded) paths for (q, K_{+1}, H, X) satisfying (18)-(21), given policy and the initial condition K_0.[7]

If capital is not used in the DM, then $c(q, K) = c(q)$ and $\gamma(q, K) = c_k(q, K) = 0$. This version dichotomizes, and since M appears in (18) but not (19)-(21), monetary policy affects q but not (K_{+1}, H, X) or $\hat{q}$. Equilibrium does not dichotomize when K enters $c(q, K)$. Notice however that if $\theta = 1$ then, although K enters $c(q, K)$, (19)-(21) can be solved for (K_{+1}, H, X), then (18) determines q since $\gamma(q, K) = 0$. So if $\theta = 1$ money still does not influence CM variables, even though anything that affects the CM (e.g. taxes) influences q. Intuitively, when $\theta = 1$ sellers do not get any of the surplus from DM trade, and so investment decisions are based solely on returns to K that accrue in the CM. Looking at (17), when $\theta = 1$, the cost reduction due to having more capital is exactly matched by the increase in cost due to higher production. This is an extreme version of a holdup problem in the demand for capital.

More generally, for any $\theta > 0$, sellers do not get the full return on capital from DM trade, and hence they underinvest. This holdup problem is not present in standard macro, and

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[7]We are mostly interested in *monetary* equilibrium, with $q > 0$ at every date. But consider for a moment *nonmonetary* equilibrium, with $q = 0$ at all dates. In this case, (K_{+1}, H, X) solves (19)-(21) with $\gamma = 0$, which is exactly the equilibrium for a standard neoclassical growth model.

constitutes a distortion over and above those from taxes and monetary inefficiencies. If we run the Friedman Rule (FR) by setting $i = 0$ and levy only lump-sum taxes, we are left with the holdup problem on capital and a related problem on money emphasized in LW. In some models all holdup problems can be resolved if one sets bargaining power θ correctly. This is *not* possible here: $\theta = 1$ resolves the problem in the demand for money, but this is the worst case for investment; and $\theta = 0$ resolves the problem in the demand for capital, but this this is the worst case for money. There is no θ that can eliminate the double holdup problem, which has implications for both the empirical performance of bargaining models and their welfare implications.

2.3 Price Taking

While our holdup problems cannot simultaneously be solved by bargaining, some other solution concepts work much better. For example, it is by now well known that *competitive search equilibrium*, based on directed search and price posting, rather than bargaining, resolves multiple holdup problems (Moen 1997; Acemoglu and Shimer 1999). And *competitive equilibrium* with Walrasian price taking also does the job here, even though this is not true in all models (e.g. Rocheteau and Wright 2005 show that competitive search equilibrium can do better than competitive equilibrium in environments with search externalities, but we have no such externalities). Since it is easier to present, relative to price posting with directed search, in this section we consider price taking in the DM.

We assume for simplicity that there are two distinct markets – one for anonymous traders where cash is needed, and where credit is available. The DM value function has the same form as (4), but now, in the market with anonymous buyers,

$$
\begin{aligned}
V^s(m, k) &= \max_q \left\{ -c(q, k) + W(m + \tilde{p}q, k, 0) \right\} \\
V^b(m, k) &= \max_q \left\{ u(q) + W(m - \tilde{p}q, k, 0) \right\} \text{ s.t. } \tilde{p}q \le m
\end{aligned}
$$

where $\tilde{p}$ is the price (which generally differs from the CM price level p). Similarly,

$$
\begin{aligned}
\hat{V}^s(m, k) &= \max_{\hat{q}} \left\{ -c(\hat{q}, k) + W(m, k, -\hat{p}\hat{q}) \right\} \\
\hat{V}^b(m, k) &= \max_{\hat{q}} \left\{ u(\hat{q}) + W(m, k, \hat{p}\hat{q}) \right\}.
\end{aligned}
$$

The FOC for the sellers in the two DM markets are

$$c_q(q, k) = \tilde{p}W_m = \tilde{p}A/pw(1 - t_h)$$
$$c_q(\hat{q}, k) = -\hat{p}W_\ell = \hat{p}A/pw(1 - t_h).$$

Market clearing implies buyers and sellers choose the same q and $\hat{q}$ As with bargaining, in the anonymous market, buyers spend all their money so $q = M/\tilde{p}$. Inserting $\tilde{p} = M/q$, we get the analog to (13) from the bargaining model

$$\frac{M}{p} = \frac{qc_q(q, k)w(1 - t_h)}{A}. \tag{22}$$

Similarly, when credit is available, $\hat{q} = \hat{q}(K)$, as in the bargaining model, but now $\ell = pw(1 - t_h)u'(\hat{q})\hat{q}/A$. Then the analogs to (15) and (16) are

$$V_m(M, K) = \frac{(1 - \sigma\omega)A}{pw(1 - t_h)} + \frac{\sigma\omega u'(q)}{\tilde{p}}$$
$$V_k(M, K) = \frac{A + A(r - \delta)(1 - t_k)}{w(1 - t_h)} - \sigma\omega c_k(q, K) - \sigma(1 - \omega)c_k(\hat{q}, K).$$

Inserting these into (2) yields the analogs to (18) and (19)

$$\frac{c_q(q, K)q}{M} = \frac{\beta c_q(q_{+1}, K_{+1})q_{+1}}{M_{+1}}\left[1 - \sigma\omega + \sigma\omega\frac{u'(q_{+1})}{c_q(q_{+1}, K_{+1})}\right] \tag{23}$$
$$U'(X) = \beta U'(X_{+1})\{1 + [F_K(K_{+1}, H_{+1}) - \delta](1 - t_k)\} \tag{24}$$
$$-\beta(1 + t_x)\sigma[\omega c_k(q_{+1}, K_{+1}) + (1 - \omega)c_k(\hat{q}_{+1}, K_{+1})].$$

The other equilibrium conditions (20)-(21) are the same as above. Then an *equilibrium with price taking* is given by (positive, bounded) paths for (q, K_{+1}, H, X) satisfying (23)-(24) and (20)-(21), given policy and K_0. The difference between bargaining and price taking is the difference between (18)-(19) and (23)-(24). Notice the equilibrium condition for q here looks like the one from the bargaining model when $\theta = 1$, and the condition for K looks like the one from the bargaining model when $\theta = 0$, indicating that price taking avoids both holdup problems.[8]

---

[8]To show this formally, set $t_k = t_h = t_x = 0$. Then under price taking the equilibrium conditions for (K_{+1}, H, X) are the same as those for the planner problem. Hence the equilibrium coincides with the FB iff $u'(q) = c_q(q, K)$. From (23), this means $c_q(q, K)q/M = \beta c_q(q_{+1}, K_{+1})q_{+1}/M_{+1}$. Using (22) this reduces to $1/pw = \beta/p_{+1}w_{+1}$. Since $w = A/U'(X)$, it further reduces to $p/p_{+1} = U'(x)/\beta U'(X_{+1})$. Since in any equilibrium the slope of the indifference curve $U'(x)/\beta U'(X_{+1})$ equals the slope of the budget line $1 + \rho$, with ρ equal to the real interest rate, the relation in question finally reduces to $p_{+1}/p = 1/(1 + \rho)$. Using the Fisher equation, this holds and hence $q = q^*(K)$ solves (23) iff we set the nominal rate to $i = 0$. We conclude that under price taking with lump-sum taxes, setting $i = 0$ yields efficiency.

2.4   A Digression on Banking

At first blush, it might seem the relevant notion of money here is $M0$, but that is not the only interpretation. Without going into detail, we mention that one can introduce banks following the approach in Berentsen et al. (2007) (see also He et al. 2008 and Chiu and Meh 2009). Thus, assume that after production stops in the the CM, and agents have decided their m_{+1}, it is revealed which ones want to consume and which are able to produce while banks are still open but before agents go to the DM. As the sellers have no use for money, they deposit it in banks, who lend it to buyers at interest. One can imagine them lending out the same physical currency, or as keeping that in the vault and issuing bank-backed securities that can be used in payments, at least as long as these are not easily counterfeitable. This changes some details, but the basic structure remains fairly close to what we have here.

More needs to be done to address many interesting issues related to financial intermediation in these kinds of models, but we mention banks here for the following reasons. First, we do not necessarily want to take M to be currency – we present results below for several measures of money, including $M0$, but we want to argue that not only $M0$ fits with our environment. Second, consistent with this, when we appeal to micro data to calibrate the fraction of DM trades where credit is available, we aggregate cash, check and debit card, but not credit card, purchases into money trades. This is based on two criteria: (a) we think of checks and debit cards as simply convenient ways to access demand deposits, which like cash are very liquid but pay virtually 0 interest; and (b) we think the most relevant feature of credit cards is that they allow you to consume now and work later, while with either money or demand deposits you have to work first, as discussed in Dong (2008).

3   Quantitative Analysis

3.1   Preliminaries

We begin with some accounting. The price levels in the CM and DM are p and $\tilde{p} = M/q$, respectively, where p satisfies

$$p = \frac{AM}{(1 - t_h) \, g \, (q, K) \, F_H(K, H)} \tag{25}$$

in the bargaining version of the model by (13), and

$$p = \frac{AM}{(1 - t_h)\, qc_q\,(q, K)\, F_H(K, H)} \qquad (26)$$

in the price-taking version by (22). Nominal output is $pF(K, H)$ in the CM, and $\sigma \omega M + \sigma(1-\omega)\ell$ in the DM. Using p as the unit of account, real output in the CM is $Y_C = F(K, H)$ and in the DM is $Y_D = \sigma \omega M/p + \sigma(1 - \omega)\sigma\ell/p$. Total real output is $Y = Y_C + Y_D$.

Define the share of output produced in the DM by $s_D = Y_D/Y$, the share of output where money is essential by $s_M = Y_M/Y$ where $Y_M = \sigma \omega M/p$, and the share where credit is used by $s_\ell = Y_\ell/Y$ where $Y_\ell = \sigma(1 - \omega)\ell/p$. We do not calibrate these shares, but compute them indirectly from other variables. to see how, note that velocity is $v = pY/M = \sigma \omega Y/Y_M$. Hence, $s_M = Y_M/Y = \sigma \omega/v$. The maximum σ can be is $1/2$, and the maximum ω can be is 1, so given $M1$ velocity is around 5, s_M is bounded above by 10%. With our benchmark calibrated parameters, s_M is actually closer to 8%. There are two points to emphasize. First, to think about the size of the different sectors, one does not have to take a stand on which goods are traded in each. Second, the results presented below do not depend on having an excessive amount of monetary trade – around 92% of economic activity looks just like what one sees in nonmonetary models.

We will also discuss the *markup* μ, defined by equating $1 + \mu$ to the ratio of price to marginal cost. The markup in the CM market is 0, since it is competitive. The markup in the DM under price taking is also 0. With bargaining, the markup in the DM is derived as follows. First consider monetary trades. Marginal cost in terms of utility is $c_q(q, K)$. Since a dollar is worth $A/p\,(1 - t_h)\,w$ utils, marginal cost in dollars is $c_q(q, K)\,p\,(1 - t_h)\,w/A$. Since the price is $\tilde{p} = M/q$, the markup in monetary trade is given by

$$1 + \mu_M = \frac{M/q}{c_q(q, K)\,p\,(1 - t_h)\,w/A} = \frac{g(q, K)}{qc_q(q, K)},$$

after eliminating M using (25). Similarly, the markup in credit trade in the DM is

$$1 + \mu_\ell = \frac{\ell/\hat{q}}{c_q(\hat{q}, K)\,p\,(1 - t_h)\,w/A} = \frac{(1 - \theta)\,u(\hat{q}) + \theta c(\hat{q}, k_s)}{\hat{q}c_q(\hat{q}, K)}.$$

The average markup in the DM is then $\mu_D = \omega \mu_M + (1 - \omega)\,\mu_\ell$, while the average markup for the whole economy is $\mu = s_D \mu_D$.

3.2 Calibration

Consider the following functional forms for preferences and technology:

$$\text{CM:} \qquad U(x) = B\frac{x^{1-\varepsilon} - 1}{1 - \varepsilon} \text{ and } F(K, H) = K^{\alpha}H^{1-\alpha}$$

$$\text{DM:} \qquad u(q) = C\frac{(q + b)^{1-\eta} - b^{1-\eta}}{1 - \eta} \text{ and } c(q, k) = q^{\psi}k^{1-\psi}$$

The cost function $c(\cdot)$ comes from the technology $q = e^{1/\psi}k^{1-1/\psi}$; if $\psi = 1$ then the model dichotomizes. The parameter b in $u(q)$ is introduced merely so that $u(0) = 0$, which is useful for technical reasons. This means relative risk aversion is not constant, but if $b \approx 0$, it is approximately constant at $\eta q/(q+b) \approx \eta$. We set $b = 0.0001$ and $\varepsilon = \eta = 1$ as a benchmark, but we show that the results are robust to these choices in Section 4.3. We then normalize $C = 1$, with no loss in generality.

In terms of calibrating the remaining parameters, we begin with a heuristic description, and then provide details. We first point out that our approach is a natural extension of standard methods. To pick a typical application, Christiano and Eichenbaum (1992) study the one-sector growth model, parameterized by

$$U = \log(x) + A(1 - h) \text{ and } Y = K^{\alpha}h^{1-\alpha}$$

for their indivisible-labor version; for their divisible-labor version replace $A(1-h)$ by $A\log(1-h)$. One calibrates the parameters as follows: Set the discount factor $\beta = 1/(1 + \rho)$ where ρ is the observed average interest rate. Then set depreciation $\delta = I/K$ to match the investment-capital ratio. Then set α to match *either* labor's share of income LS or the capital-output ratio K/Y, since these yield the same result given there are no taxes (see below). Finally, set A to match observed average hours worked h.

This method can be adapted to many scenarios. For example, Greenwood et al. (1995) calibrate a two-sector model, with home production, as follows. Consider

$$U = \log(x) + A(1 - h_m - h_n), \quad Y_m = K_m^{\alpha_m}h_m^{1-\alpha_m} \text{ and } Y_n = K_n^{\alpha_n}h_n^{1-\alpha_n},$$

where $x = [Dx_m^{\kappa} + (1 - D)x_n^{\kappa}]^{1/\kappa}$, and x_m, h_m and k_m are consumption, hours and capital in the market while x_n, h_n and k_n are consumption, hours and capital in the nonmarket or

home sector. The two-sector version of the standard method is this: again set $\beta = 1/(1+\rho)$; set δ_m and δ_n to match I_m/K_m and I_n/K_n; set α_m and α_n to match K_m/Y_m and K_n/Y_m; and set A and D to match h_m and h_n. We are left with κ, which is hard to pin down based on steady state observations, and is therefore typically set based on direct estimates of the relevant elasticities.

Since we also have a two-sector model, we use a variant of the home-production method. Thus, first set β, δ and A as above. Then set α and ψ to match *both* K/Y and LS. As we said, in the standard one-sector model, without taxes, it does not matter if one calibrates α to LS or K/Y, but with taxes calibrating α to LS yields a value for K/Y that is too low (Greenwood et al. 1995; Gomme and Rupert 2005). The idea here is to set α to match LS, then try to use ψ to match K/Y, since DM production provides an extra kick to the return on K. Given this, we set the utility parameter B and probabilities σ and ω to match some money demand observations, as discussed below, which is the analog of picking κ in home production framework, and is similar to what is done in any calibrated monetary model. This completes the heuristic description.

We now go into more detail. Our benchmark model is annual, but as we discuss below, the results are basically the same for quarterly and monthly calibrations (which is a big advantage over the typical cash-in-advance model, as mentioned in the introduction). We pin down $\beta = 1/(1+\rho)$ with $\rho = 0.025$.[9] We set $t_h = 0.242$ and $t_k = 0.548$, the average effective marginal tax rates in McGrattan et al. (1997) (Gomme and Rupert 2005 report similar numbers). We compute $t_x = 0.069$ directly as the average of excise plus sales tax revenue divided by consumption. We set $G/Y = 0.25$. We set $\delta = I/K = 0.070$, using residential and nonresidential structures plus producer equipment and software for K. We set $\alpha = 0.288$ to get $LS = 0.712$, using the method in Prescott (1986).

In order to pin down the fraction ω of DM trades where credit is not available, we look at two sources. First, Klee (2008) finds that shoppers use credit cards in 12% of total transactions in the supermarket scanner data. The remaining transactions use cash, checks

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[9]This is the annual after-tax real interest rate in the 1951-2004 U.S. data, based on an average pre-tax nominal rate on Aaa-rated corporate bonds of 7.2%, an inflation rate from the GDP deflator of 3.6%, and a tax on bond returns of 30% from the NBER TAXSIM model. As is standard, we do not explicitly incorporate a bond market in the definition of equilibrium, but we can still price bonds in the usual way.

and debit cards which, we recall from our digression on banking, fit with our notion of money. We do not literally think the DM corresponds to supermarket shopping, but since this is the best available data, we take it as representative. Second, using earlier consumer survey data, Cooley and Hansen (1991) come up with a similar measure of around 16%. We take 15% to be a good compromise and set $\omega = 0.85$, but when we discuss robustness it turns out that over a reasonable range ω does not matter much.

So far we have directly pinned down all the parameters in panel (a) of Table 1. The remaining in panel (b) are: A and B from utility, the cost parameter ψ, the probability of being a buyer σ, and, in the bargaining model, θ. These parameters are determined simultaneously to match the following targets. First, the standard measure of work as a fraction of discretionary time $H = 1/3$. Second, average velocity $v = 5.29$. Third, $K/Y = 2.32$ when we measure K as discussed above. Fourth, a money demand elasticity of $\xi = -0.226$, which we estimate in the Appendix. Fifth, in models with bargaining we target the DM markup, which we set to $\mu = 0.30$, as discussed below. We choose these parameters simultaneously to minimize the distance between the targets in the data and model. Sometimes we add as an additional target the long-run elasticity of investment with respect to inflation, which we estimate on quarterly data as $\zeta = -0.023$.[10] Targeting ζ is like tageting the labor supply elasticity in a standard business cycle model, which one may or may not like; we report results for both cases.

These targets are all fairly standard with one possible exception: our DM markup of 30%. To get this, we use the evidence discussed by Faig and Jerez (2005) from the Annual Retail Trade Survey on markups across retail establishments. At the low end, Warehouse Clubs and Superstores come in around 17%, Automotive Dealers 18%, and Gas Stations 21%. At the high end Specialty Foods come in at 42%, Clothing and Footware 43%, and Furniture 44%. These retailers are examples of what one might have in mind for the DM. We pick $\mu_D = 0.3$, in the middle of the data. In the robustness discussion, however, we argue the exact choice does not matter too much.

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[10]We use the same method here as we use for money demand. Although the estimated -0.023 may appear small, it is statistically significant and economically relevant: raising inflation from our benchmark value to 7% reduces investment by 2.3%, which is nothing to scoff at.

3.3 Decision Rules

We first scale all nominal variables by M, so that $\hat{m} = m/M$, $\hat{p} = p/M$ etc. Then the individual state becomes $(\hat{m}, k, K)$, where in equilibrium $\hat{m} = 1$ and $k = K$. Although the above presentation was more general, now we are interested in recursive equilibrium, given by time-invariant decision rules $[q(K), K_{+1}(K), H(K), X(K)]$ and value functions $[W(K), V(K)]$ solving the relevant equations – e.g. (18)-(21), (1) and (7) in the bargaining model. We solve these equations numerically using a nonlinear global approximation, which is important for accurate welfare computations. Figure 1 plots the decision rules and value function for two preferred parameterizations (Models 3 and 5 as described in the next section) for four scenarios: the planner's problem; monetary equilibrium at the FR; monetary equilibrium at 10% inflation; and nonmonetary equilibrium. We discuss the economic content of these pictures below.

4 Results

4.1 Calibration Results

In Table 2, one column lists the relevant moments in the data, while the others list moments from five specifications of the model. Model 1 uses bargaining in the DM with bargaining power $\theta = 1$, giving up on the DM markup μ_D as a target; it is presented mainly as a benchmark since we already proved that $\theta = 1$ implies money cannot affect the CM variables at all. Models 2 and 3 use bargaining with θ calibrated along with the other parameters; the difference between Models 2 and 3 is that the latter adds the investment elasticity ζ as a target while the former does not. Models 4 and 5 use price taking in the DM, so there is no θ, calibrating the rest of parameters to match the targets other than the DM markup; the difference between Models 4 and 5 is again that the latter adds ζ as a target while the former does not.

We do well matching the targets with two exceptions. First, we match the DM markup μ_D only if we assume bargaining and calibrate θ, rather than fixing it at 1 or assuming price taking, for obvious reasons. Second, we do a good job matching K/Y and ζ only in the price-taking model, for reasons that we now explain. Intuitively, our calibration sets the

CM technology parameter α to match LS and then tries to hit K/Y using the technology parameter ψ (although we think this way of looking at things is instructive, it is meant only to be suggestive, since in fact we pick all of the parameters simultaneously). When $\psi = 1$, K is not used in the DM, and K/Y is too low, as in the standard model once taxes are introduced. As we increase ψ above 1 the return on K from its use in the DM increases and hence so does K/Y. But, in practice, with bargaining, this effect is tiny because the holdup problem eats up most of the DM return on K. Of course, this depends on bargaining power, but even if we pick θ to maximize K/Y we cannot get it big enough.

Intuitively, if θ is big then buyers have all the bargaining power, which makes q big, other things being equal, but gives little return from DM trade to sellers; and if θ is small then sellers have all the bargaining power, which gives them a big share of the return, but only on a very small q. There is no way around it with bargaining. With price taking, however, the holdup problems vanish, and we can pick ψ to match K/Y exactly. The same intuition about how holdup problems affect the level of K/Y also explains how they affect the elasticity ζ: with bargaining, any extra return on K from DM production due to lower inflation will not increase aggregate K much, since the DM return is a small fraction of the overall return to investment. Again, this is not a problem with price taking, and we can hit ζ perfectly.

Earlier we alluded to the fact that we back out a DM share s_D of only around 8%, as seen in Table 2. We think this is reasonable, since it means we are not so far from the standard growth model. Because s_D is relatively small, however, our aggregate markup is only around 2%. This is lower than the numbers some macroeconomists use, but note that we abstract from any markups in the CM.[11] In any case, as we will see in the robustness analysis below, the results do not hinge much on μ_D – e.g. we can recalibrate to match an aggregate markup of 10%, which requires a much bigger DM markup, and the results are quite similar, as will be explained in Section 4.3.

Finally, one might ask how we match the empirical money demand curve, which is often taken to be a measure of 'fit' in monetary calibration exercises, usually using $M1$ (e.g. Lucas 2000). A plot of i versus M/pY from the data and our model looks similar to what sees with

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[11] Aruoba and Schorfheide (2008) introduce markups in the CM by incorporating monopolistic competition, calibrating to around 15% in each sector.

other models in the literature. As with all those approaches, it is not easy to match both the observations with very low i and high M/pY from the first decade and those with low M/pY from the last decade in the sample. But we do not put too much weight on plots of i versus M/pY, anyway, since this specification for money demand assumes a unit income elasticity, which is rejected in the regression results reported in Appendix B.3. From those regression results, we think our money demand specification fits quite well.

4.2 Experiments

Here we consider experiments where, starting in a steady state, we make a once-and-for-all change in the growth rate of money τ and track the behavior of the economy over time. Since inflation in steady state equals τ, we abuse language slightly and describe our experiments as a change in inflation, but note that it actually does not jump to the new steady state level in the short run (i.e. inflation may not equal τ during the transition). Table 3 contains results for each of the five models when we perform a common experiment in the literature and change $\tau^1 = 0.1$ to the FR, which is $\tau^2 = -0.0239$ in the baseline calibration. For now, we make up any change in government revenue with the lump-sum tax T, and consider other fiscal options below. Table 3 presents ratios of equilibrium values of several variables at the two inflation rates.[12]

The first thing to note is that q^1/q^2 is considerably less than 1, varying between 0.67 and 0.84, depending on the model. Inflation is a tax on DM activity, and these results show that this tax is quantitatively very important for q. In Model 1 this is the only effect, since $\theta = 1$ implies monetary policy has no impact on the CM. In Models 2 and 3, monetary policy does affect the CM, in principle, but the impact is tiny, as one should expect from the discussion in Section 4.1. Models 1-3 predict that going to the FR increases aggregate output Y by 2%, essentially all due to the change in q. In Models 4 and 5 the effects are very different. First, q actually changes by more; and second, now K changes and by quite a lot – either 3% or 5%, according to Model 4 or 5. This makes CM consumption X change by about 1%, and the net impact on Y is now 3%.

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[12]When a *1* appears in italics, the true number is not exactly unity but shows up this way due to rounding, to distinguish effects that are theoretically 0 from those that not exactly 0 but numerically very small.

Before discussing the intuition for these results, consider welfare. As is standard, we solve for Δ such that agents are indifferent between reducing τ and increasing total consumption by a factor Δ. We report the answer comparing across steady states – jumping instantly from τ^1 and K^1 to τ^2 and K^2 – as well as the cost of the transition from K^1 to K^2 and the net gain to changing τ starting at K^1. This net gain is the true benefit of the policy change, although we think the steady state comparison is also interesting (it tells us how much an agent facing τ^1 and K^1 would pay to trade places with someone facing τ^2 and K^2). In Model 1 there is no transition since τ does not affect K, and in Models 2 and 3 we expect it to be unimportant, since τ does not affect K much, but in Models 4 and 5 the transition is significant. We also report the net gain to reducing τ to 0, instead of all the way to FR, to check how much of the gain comes from eliminating inflation and how much comes from deflation (most comes from the former).

In Model 1, with $\theta = 1$, going from 10% inflation to the FR is worth around 3/4 of 1% of consumption, commensurate with earlier findings. In Models 2 and 3, with $\theta \approx 0.9$, this policy is worth just under 3% of consumption. Intuitively, at $\theta \approx 0.9$ the money holdup problem makes q very low, so any additional reduction is very costly. In Models 4 and 5 the steady state gain is about half that in Models 2 and 3, since the economy is closer to the first best with price taking. In Models 4 and 5 inflation has a sizable impact on K and X, but since much of the gain accrues in the long run, and agents work more and consume less during the transition, the net gain is closer to 1%. Figure 2 shows the transitions for Models 3 and 5. In Model 5, e.g., in the short run H increases around 1.5% and X falls slightly before settling down to the new steady state, while DM output jumps on impact around 35% and quickly settles down. The difference between the two panels of Figure 2 is the size of the adjustment in CM variables: with bargaining, K changes only about 0.4% in in the long run, while with price-taking K changes over 4%, and this makes all CM effects bigger.

Table 4 compares the FR and FB allocations. The differences are big, mainly due to taxation (McGrattan et al. 1997 find similar results in standard nonmonetary models). We also report the gain to moving from the FR to the FB after setting $t_h = t_k = t_x = 0$ and recalibrating parameters. In Models 4 and 5, the gain in this case is 0 because as we showed the FR implements the FB. In Model 1, with capital holdup but no money holdup, the steady

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state gain is around 4%, although much is lost in transition. In Models 2 and 3, with both holdup problems, the steady state gain is around 16% and 22%.[13] These calculations provide measures of the impact of holdup problems: based on the steady state comparisons, e.g., one could say 4% of consumption is the cost of capital holdup and an additional $12\% - 18\%$ is the cost of money holdup. Although there is no single 'correct' way to decompose these effects, this suggests holdup may be quantitatively important, even though bargaining occurs only in the DM and s_D is only around 8%.[14]

Table 5 reports the actual allocations, not just the ratios of the allocations, at different τ, to facilitate comparisons across models. Notice that q is considerably lower in Models 2 and 3 than in other models, showing the impact of the money holdup problem. The table also reports the allocation in the nonmonetary equilibrium, which can be considered the limit as inflation goes to ∞. Although we can of course compute the cost of very high inflation – e.g., going from 100% inflation to the FR is worth around 13% in Model 3 and 9% in Model 5 – one should take these calculations cautiously for two reasons. First, at very high inflation, agents may well devise other ways to trade in the DM (e.g. ω may vary with policy). Second, our numerical results are more sensitive to parameter choices at very high inflation.

We can also discuss results using the decision rules. In Figure 1, for Model 5 we see that as we lower τ the decision rule for K_{+1} shifts up, and steady state K increases, although it is still far from the FB even at the FR (the symbols on each curve show the location of the steady state, but the FB steady state $K = 3.59$ is off the chart). Also, the decision rule for q shifts up, increasing q in the short run and more in the longer run as we move along the decision rule for q. The latter effect is important here, since K grows a lot. In Model 3 the decision rule for K_{+1} and hence steady state K change little. The decision rule for q shifts,

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[13]When Model 2 is recalibrated with no taxes, we set ψ equal to its calibrated value in the benchmark calibration as the calibration routine increases it without bound.

[14]The calibrated parameters differ across the columns in Table 4. Suppose we instead fix the parameters as in Model 3, and consider three cases: (i) $\theta = 1$; (ii) θ calibrated; and (iii) price taking. With taxes, going from the FR to FB in these three scenarios is worth, in terms of steady state (net) comparisons: (i) 39.52 (22.30); (ii) 43.95 (26.53); and (iii) 7.62 (4.19). With taxes set to 0 we get: (i) 11.32 (3.99); (ii) 15.86 (8.42); and (iii) 0 (0). Looking at the results without taxes, one could say the cost of capital holdup in terms of steady state is 11.32, or 3.99 with transition, and the cost of money holdup is 4.54, or 4.43 with transition. With taxes the cost of capital holdup including transitions is 18.11 while the cost of money holdup is 4.23. Again, there is no single way to measure these effects, but all of this indicates that holdup problems may well be important.

giving a short-run effect, but there is little additional long-run effect. Still, inflation is very costly in Model 3 because the decision rule for q at the FR is quite far from the decision rule at the FB, so any change in q matters a lot, while in Model 5 the decision rules for q at the FR and FB are almost coincident.

One can also consider lowering τ and making up the revenue with proportional taxes. Table 6 reports results when we make up revenue with lump-sum taxes, reproducing Table 3, and with labor or consumption taxes.[15] Since we are using $M1$ in this calibration, government seigniorage revenue is only 10% of τ times the change in M. Going to the FR and making up the revenue with labor taxes requires raising t_h from 24.2% to between 24.4 and 24.7%. On net, the overall impact of lower inflation is positive in all models. In the last two columns of Table 6, we report results for the extreme assumption that the government is able to collect seigniorage revenue from all of $M1$ (as in Cooley and Hansen 1991). For Model 4 e.g. the new t_h is 29.3% and the net effect of lower inflation is a welfare *loss* of about 1.2%. However, for Model 5, even though t_h increases to 30.3%, there is a welfare gain of 0.01%. In general, these results are sensitive to details, but we still think they are interesting.[16]

4.3   Robustness

We redid all the calculations for many alternative specifications, but in the interest of space, in Table 7 we report the results in terms of one statistic: the net welfare gain of going from 10% inflation to the FR. The first row is the benchmark model. The first robustness check involves shutting down the distorting taxes, both for the case where other parameters are kept at benchmark values, and when they are recalibrated. Most of the results are similar to the benchmark calibration, although the cost of inflation is somewhat lower, especially under price taking. This is because the FR achieves the FB under price taking without distortionary taxes, and hence the cost of moderate inflation is low, by the envelope theorem. It is no surprise that some results depend on what one assumes about taxation, and since taxes are a fact of life, we trust the benchmark calibration.

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[15]We could not solve the case where we make up revenue with capital taxaes, since increasing t_k lowered K by so much that sufficient revenue was not forthcoming.

[16]Note that we are not saying anything here about optimal monetary policy when fiscal policy is *also* set to maximize utility, since here we are taking the existing tax rates as given from the data; see Aruoba and Chugh (2008).

We then varied the preference parameters b, ε and η. One can look at the numbers for oneself, but we conclude the results are not overly sensitive.[17] We then consider changing our target for the DM markup. Perhaps surprisingly, it does not matter much – lowering μ_D to 10%, only reduces the welfare cost from 2.70% to 2.43% in Model 2 and from 2.95% to 2.83% in Model 3. This can be understood as follows. First, note that when $\theta = 1$ the markup is actually negative in Table 2, because take-it-or-leave-it offers by buyers means $p = AC < MC$. Thus, just to get $\mu_D > 0$ we need θ significantly below 1: e.g. $\mu_D = 0.01$ requires $\theta = 0.927$, which implies the money holdup problem is already important enough to generate a sizable welfare cost of around 2.43%. We also show results where we increase μ_D to 50% and 100% and those where we target an aggregate markup of 10%. The results are similar, although the numbers increase slightly as markup increases.

The table also shows that the results are not very sensitive to using different time periods for the calibration, and not at all sensitive to assuming a different length for a period (quarterly, monthly and annual models deliver very similar predictions). This is easy to understand: to go from an annual to a quarterly or monthly model, we simply adjust inflation, velocity, interest rates, K/Y and I/K by the relevant factor. The calibrated σ declines, because a shorter period reduces the probability of consuming in any given DM, but the welfare conclusions do not change. We find this important because changing frequency typically *does* change the results in some models, including standard cash-in-advance models, where agents generally spend all their money every period.

We also consider different values for the payment parameter ω. Perhaps surprisingly, the results are robust to this choice within a wide range. Even when only 25% of DM trades require cash, the welfare costs are similar, and in fact somewhat higher than the benchmark. To understand this, first note it is certainly true that a reduction in ω reduces the cost of inflation when we fix other parameters. But when we recalibrate parameters, as we change ω, in order to match the calibration targets, σ increases and B falls. On net, this renders DM activity just about as important for welfare as before. Obviously, $\omega = 0$ means money

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[17]Lowering η generally increases the welfare cost of inflation as it makes money demand more responsive to changes in inflation, but otherwise results are in line with the benchmark calibration. One can also vary β, δ etc. over reasonable ranges without affecting things too much (not reported).

is not valued and hence inflation is irrelevant, but if $\omega = 0$ then we could not match our calibration targets. For values of ω in a reasonable range, as long as we match the same targets, the net effects are very similar.

What does matter is the empirical measure of money, $M0$, $M1$, $M2$ or $M3$. These alternatives imply different values for average velocity, and given our calibration method, this changes the cost of inflation. Intuitively, consider the traditional method of computing the cost of inflation by the area under the money demand curve.[18] With a broader definition of M (i.e. lower velocity), the curve shifts up and increases the estimated cost. An apparent puzzle is that using $M3$ yields a smaller welfare cost than using $M2$. This is due to the fact that, in going from $M2$ to $M3$, while velocity is falling, the calibrated money demand elasticity ξ is also falling; the latter effect, which makes the money demand curve flatter, happens to dominate. Overall, the results indicate that the measure of money does matter, as it should, and as it will in any monetary theory. If forced to choose, we think $M1$ is most appropriate for reasons discussed in Section 2.4, but this is open to further discussion.

One can go beyond these parameter or measurement issues and consider robustness with respect to larger modeling choices. As we mentioned earlier, we also studied a version of the model with two capital stocks, K_C and K_D (see Appendix A.1). Tables 8 and 9 report results for this model with bargaining and with price taking, called Models 6 and 7. These two-capital analogs of Models 3 and 5 do about as well as in matching the targets. In Models 6 and 7, q actually increases by more than in the baseline models when we reduce τ, tending to make inflation more costly. However, there are also other effects, and the net cost of inflation is slightly lower in Model 7 than 5. These other effects occur because in Model 5 the same K is used to produce all output, while in Model 7 q is produced with K_D while X is produced with K_C. Despite these details, the overall picture from the two-capital-stock version is similar to the base case.

Tables 8 and 9 also report results from another extension where K is used in the CM only, but produced and traded in the DM (see Appendix A.2). The bargaining and price-taking versions are called Models 8 and 9. These are potentially interesting because now inflation

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[18]This is not meant as an endorsement of that method. Craig and Rocheteau (2008) show that when $\theta < 1$ it underestimates the true welfare cost.

taxes capital accumulation directly (as in earlier models by Stockman 1981 and Shi 1999), and not only indirectly via q. Now τ has a sizable effect on K under bargaining, not only under price taking. Overall, the results are not so different from the base case, however, even if the welfare cost estimates are affected somewhat. It may be worth studying these alternative models in more detail in the future, although to do so one might want to rethink the calibration strategy. We presented them here mainly to show that the basic results carry over to alternative formulations.

4.4   Summary of Results

Here is what we think we learn from all of this:

- One can integrate elements from models with explicit trading frictions into capital theory in a way that in principle generates interesting effects of money on investment.

- One can use standard methods to calibrate the model, even though it contains some parameters like σ or θ that are not in standard models.

- We do a good job matching most targets, although with price taking we cannot match the markup, and with bargaining we cannot match K/Y or the elasticity of K with respect to i very well.

- When we back out the size of the two sectors from observables, our DM accounts for around 8% of total output.

- Inflation is a tax on DM consumption q, and its impact is big.

- Qualitatively, given K is useful for producing q, inflation reduces investment; quantitatively, this effect is tiny under bargaining but big (3 to 5%) under price taking.

- Under price taking, reducing inflation from 10% to the FR is worth 1.5% across steady states, and 1% taking into account the transition; it is worth around 3% under bargaining.

- With either price taking or bargaining, much of the gain is achieved by reducing inflation to 0 rather than going all the way to the FR.

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- The holdup problems for both money and investment are is important.

- The costs of fiscal distortions are big.

- Most of these results are robust, but the empirical measure of M does matter.

- Many of these results differ from findings in the literature (recall fn. 2).

- A key element of the framework is the explicit two-sector structure, although it does not matter much if the same or different capital stocks are used in the two sectors, or whether capital is traded in one sector or the other.

Perhaps the most surprising results are that the impact of inflation is so different under bargaining and price taking, and that in the latter case the effects on capital and output are quite large, while in the former case the effects on capital are small but the effects on output are still sizable. The model predicts that going from the FR to 10% inflation decreases output by up to 3%, and decreases investment by up to 5%, depending on the mechanism. How plausibile are these findings? When one looks at time-series for the US, the relationships do not appear overly strong: our estimate of the elasticity of investment with respect to inflation was reported in Table 2 as $\zeta = -0.023$. But this is statistically and economically significant. In any case, the model has no problem matching this number, at least under price taking, which is the relevant case since that is where we get the most dramatic results. That is, when we target $\zeta = -0.023$ in the calibration, we hit it. This model is by construction consistent with the time series elasticity.

However, one might think our analysis is more related to long- rather than short-run effects, in the sense that although we study transitional dynamics, we focus on responses to one-time changes in inflation. So, rather than time-series, we can consider cross-sectional evidence. Figure 3 plots average real GDP and investment expenditures versus inflation for 22 developed countries over the period 1950-1999 (GDP and investment are from Penn World Tables 6.1; inflation is from IFS). This is not meant to be a rigorous econometric analysis, and we realize there are issues of endogeneity here, but we still think these data are striking: the correlation between real GDP and inflation is -0.81 and between investment

and inflation is -0.74.[19]

While more econometric analysis is important, our point is simply that there is nothing obvious in the time-series or cross-section data on investment and inflation or on output and inflation to suggest we are way off in our predictions concerning potentially big effects from monetary policy. Finally, it is also true that our theory predicts an upward-sloping Phillips curve – a positive correlation between inflation and unemployment, or at least a negative correlation between inflation and employment, since we do not model unemployment per se. This is not a problem: whatever one believes about the Phillips curve in the short run, it is documented in Berentsen et al. (2009) and Haug and King (2009) that in the long run (after filtering out business cycle frequencies) the US data displays a clear positive correlation between inflation and unemployment and negative correlation between inflation and employment. Again, while we welcome more work on this, we see nothing obvious in this data inconsistent with the model.

5 Conclusion

We already summarized the findings. Our overall conclusion is that it is quantitatively relevant to incorporate elements from the microfoundations of money – including bargaining, alternating centralized and decentralized markets, and stochastic trading opportunities – into theories of capital formation. In terms of future work, it may be interesting to consider more general preferences, still quasi-linear but nonseperable between x and q. This allows one to parameterize more flexibly the degree of substitutability between CM and DM goods, and breaks the dichotomy even if K is not used in the DM. Here we wanted to focus the effect of money on investment coming from the fact that K is used in DM production, so we used seperable utility. In terms of other ideas, one could try to take financial intermediation more seriously, or study optimal monetary and fiscal policy, or examine business cycle properties of the model. All of this is left to other research.

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[19]There are potentially many effects at work here, including the notion that poor countries have trouble relying on taxation and thus must resort to inflation, as the editor points out. However, this sample includes only developed countries, where this is less of an issue.

A   Appendix

We sketch the two extensions discussed in the robustness section, and provide details concerning money demand elasticity. To reduce notation, we set $\omega = 1$.

A.1   Two Capital Goods

Suppose now that k_C is used in production in the CM and k_D is used in the DM, but both are produced in the CM. They depreciate at rates δ_C and δ_D. Neither k_C nor k_D can be used as a medium of exchange in the DM. For illustration, there is no tax on k_D, and we present only the bargaining version (price taking is similar). The CM problem is

$$W(m, k_C, k_D) = \max_{x, h, m_{+1}, k_{C+1}, k_{D+1}} \{U(x) - Ah + \beta V(m_{+1}, k_{C+1}, k_{D+1})\}$$

$$\text{s.t. } (1 + t_x) x = w (1 - t_h) h + [1 + (r - \delta_C) (1 - t_k)] k_C - k_{C+1} - T + \frac{m - m_{+1}}{p}$$

$$+ (1 - \delta_D) k_D - k_{D+1}.$$

Eliminating h using the budget equation, we have the FOC

$$x \; : \; U'(x) = \frac{A (1 + t_x)}{w (1 - t_h)}$$

$$m_{+1} \; : \; \frac{A (1 + t_x)}{pw (1 - t_h)} = \beta V_m(m_{+1}, k_{C+1}, k_{D+1})$$

$$k_{+1} \; : \; \frac{A}{w (1 - t_h)} = \beta V_k(m_{+1}, k_{C+1}, k_{D+1})$$

$$z_{+1} \; : \; \frac{A}{w (1 - t_h)} = \beta V_z(m_{+1}, k_{C+1}, k_{D+1}).$$

The envelope conditions for W_m, W_k and W_z are derived in the obvious way, and the usual logic implies the distribution of (m, k_C, k_D) is degenerate leaving the CM. The DM is as before, except we replace $c(q, k)$ with $c(q, k_D)$ and $g(q, k)$ with $g(q, k_D)$. The value function in the DM and the envelope conditions for V_m, V_k and V_z are derived in the obvious way. This leads to

$$\frac{g(q, K_D)}{M} = \frac{\beta g(q_{+1}, K_{D+1})}{M_{+1}} \left[1 - \sigma + \sigma \frac{u'(q_{+1})}{g_q(q_{+1}, K_{D+1})}\right] \tag{27}$$

$$U'(X) = \beta U'(X_{+1}) \{1 + [F_K(K_{C+1}, H_{+1}) - \delta_C] (1 - t_k)\} \tag{28}$$

$$U'(X) = \beta U'(X_{+1}) \left[1 - \delta_D - \frac{(1 + t_x) \sigma \gamma(q_{+1}, K_{D+1})}{U'(x_{+1})}\right] \tag{29}$$

$$U'(X) = \frac{A (1 + t_x)}{F_H(K_C, H) (1 - t_h)} \tag{30}$$

$$X + G = F(K_C, H) + (1 - \delta_C) K_C - K_{C+1} + (1 - \delta_D) K_D - K_{D+1} \tag{31}$$

where $\gamma(\cdot)$ is defined in (17). An *equilibrium* is given by (positive, bounded) paths for $(q, K_{C+1}, K_{D+1}, H, X)$ satisfying (27)-(31).

A.2 Capital Acquired in the DM

Here new k is acquired in the DM. Agents do not consume DM output q, but use it as an input that is transformed one-for-one into k, an input to CM production. Each period a fraction σ of agents in the DM can produce q, and a fraction σ can transform it into k. Although agents cannot acquire new capital in the CM, they are allowed to trade used capital. Let k be the amount of capital held by an agent entering the CM and k'_{+1} the amount of capital taken out, into the next DM. We show how to construct equilibrium where the distribution of (m, k') coming out of the CM is degenerate, even though the distribution going in is not.

The CM problem is

$$W(m, k) = \max_{x, h, m_{+1}, k'_{+1}} U(x) - Ah + \beta V_{+1}(m_{+1}, k'_{+1})$$

$$\text{s.t. } (1 + t_x) x = w(1 - t_h) h + [r - (r - \delta) t_k] k + (1 - \delta) \phi k - \phi k'_{+1} - T + \frac{m - m_{+1}}{p}$$

where ϕ is the goods price of used capital in terms of x. The FOC are:

$$x : \quad U'(x) = \frac{A(1 + t_x)}{w(1 - t_h)}$$

$$m_{+1} : \quad \frac{A}{pw(1 - t_h)} = \beta V_{+1,m}(m_{+1}, k'_{+1}) \tag{32}$$

$$k'_{+1} : \quad \frac{A\phi}{w(1 - t_h)} = \beta V_{+1,k}(m_{+1}, k'_{+1})$$

The envelope conditions are obtained as usual. Buyers in the DM spend all their money, and bring $k = k' + q$ to the CM. The bargaining solution implies q solves $m_b/p = g(q, r, w, \phi)$ where

$$g(q, r, w, \phi) \equiv \frac{(1 - t_h) w [\theta c(q) + (1 - \theta) c'(q) q] [r - (r - \delta) t_k + (1 - \delta) \phi]}{\theta A [r - (r - \delta) t_k + (1 - \delta) \phi] + (1 - \theta)(1 - t_h) w c'(q)}.$$

In the DM, we have

$$V(m, k') = W(m, k') + \sigma \left\{ \frac{A[r - (r - \delta) t_k + (1 - \delta) \phi] q(m)}{w(1 - t_h)} - \frac{Am}{pw(1 - t_h)} \right\} + \sigma E \left\{ \frac{A\tilde{m}}{pw(1 - t_h)} - c[q(\tilde{m})] \right\},$$

where the expectation is with respect to the money holdings $\tilde{m}$ of agents and we assume you visit one at random (we will establish, but have not yet established, that $\tilde{m} = M$ is

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degenerate). Then

$$V_m(m, k') = \frac{(1-\sigma)A}{pw(1-t_h)} + \frac{\sigma\left[r - (r-\delta)t_k + (1-\delta)\phi\right]}{pw(1-t_h)g_q(q, r, w, \phi)}$$

$$V_k(m, k') = \frac{A\left[r - (r-\delta)t_k + (1-\delta)\phi\right]}{(1-t_h)w}.$$

Since V_m is independent of k', the FOC for m_{+1} in (32) implies m_{+1} is independent of k'_{+1} and hence degenerate. Now the analog to (18) is

$$\frac{\hat{g}(q, K, H, \phi)}{F_H(K, H)M} = \frac{\beta\hat{g}\left(q_{+1}, K_{+1}, H_{+1}, \phi_{+1}\right)}{F_H(K_{+1}, H_{+1})M_{+1}}\left[1 - \sigma + \sigma\Xi(q_{+1}, K_{+1}, H_{+1}, \phi_{+1})\right] \quad (33)$$

where

$$\hat{g}(q, K, H, \phi) \equiv g\left[q, F_K(K, H), F_H(K, H), \phi\right]$$

$$\Xi(q, K, H, \phi) \equiv \frac{F_K(K, H)(1-t_k) + \delta t_k + (1-\delta)\phi}{\hat{g}(q, K, H, \phi)}.$$

The FOC for k'_{+1} is

$$\frac{\phi}{F_H(K, H)} = \frac{\beta\left[F_K(K_{+1}, H_{+1})(1-t_k) + \delta t_k + (1-\delta)\phi_{+1}\right]}{F_H(K_{+1}, H_{+1})},$$

which is an arbitrage condition that implies the demand for k'_{+1} is indeterminate. Hence we can set $k'_{+1} = (1-\delta)K$ for all agents, so (m_{+1}, k'_{+1}) is degenerate. The other conditions are

$$K_{+1} = (1-\delta)K + \sigma q_{+1} \quad (34)$$

$$U'(X) = \frac{A(1+t_x)}{(1-t_h)F_K(K, H)} \quad (35)$$

$$X + G = F(K, H) \quad (36)$$

An *equilibrium* is given by paths for (q, ϕ, K_{+1}, H, X) satisfying (33)-(36).

A.3 Money Demand Elasticity

The interest elasticity of money demand is $\xi = \frac{\partial(M/P)}{\partial i}\frac{i}{M/P}$. To compute this in the bargaining model (price taking is similar) we need to determine $\partial q/\partial i$, $\partial K/\partial i$ and $\partial H/\partial i$. Eliminating X, we can write the steady state as 3 equations in (q, K, H):

$$\frac{i}{\sigma\omega} = \frac{u'(q)}{g_q(q, K)} - 1$$

$$\rho = [F_K(K, H) - \delta](1-t_k) - \frac{\sigma(1+t_x)}{U'[F(K, H) - \delta K - G]}\left[\omega\gamma(q, K) + (1-\omega)(1-\theta)c_k(\hat{q}, K)\right]$$

$$U'[F(K, H) - \delta K - G]F_H(K, H) = \frac{A(1+t_x)}{(1-t_h)}$$

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where $\hat{q}$ solves $u'(\hat{q}) - c_q(\hat{q}, K) = 0$ with $d\hat{q}/dK = c_{qk}(\hat{q}, K) / [u''(\hat{q}) - c_{qq}(\hat{q}, K)]$.

We take the total derivative of this system to obtain

$$
B \begin{bmatrix} dq \\ dK \\ dH \end{bmatrix} = \begin{bmatrix} di \\ 0 \\ 0 \end{bmatrix}
$$

where

$$
B = \begin{bmatrix} \frac{\sigma\omega(g_q u'' - u' g_{qq})}{g_q^2} & -\frac{\sigma\omega u' g_{qk}}{g_q^2} & 0 \\ -\frac{\sigma\omega(1+t_x)\gamma_q U'}{U'^2} & \Theta & \frac{(1-t_k)U'^2 F_{KH} + (1+t_x)U'' F_H[\sigma\omega\gamma + \sigma(1-\varepsilon)(1-\theta)c_k(\hat{q},K)]}{U'^2} \\ 0 & (F_K - \delta) F_H U'' + F_{KH} U' & F_H^2 U'' + F_{HH} U' \end{bmatrix}
$$

and

$$
\begin{aligned}
\Theta \;=\; & (1 - t_k) F_{KK} - \frac{(1 + t_x)}{(U')^2} \left\{ \sigma\omega\gamma_k U' + \sigma(1-\omega)(1-\theta) \left[c_{qk}(\hat{q}, K) \frac{d\hat{q}}{dK} + c_{kk}(\hat{q}, K) \right] \right. \\
& \left. - (F_K - \delta)[\sigma\omega\gamma + \sigma(1-\varepsilon)(1-\theta)c_k(\hat{q}, K)] U'' \right\}
\end{aligned}
$$

and all DM objects without an explicit argument refer to those with q. We can now compute the partials as

$$
\frac{\partial q}{\partial i} = B_{11}^{-1} \quad \frac{\partial K}{\partial i} = B_{21}^{-1} \quad \frac{\partial H}{\partial i} = B_{31}^{-1}
$$

where B_{ij}^{-1} refers to the (i, j) element of B^{-1}.

We now clarify how we get the empirical elasticity of money demand with respect to the nominal rate, ξ. Following the literature (e.g. Goldfeld and Sichel 1990), write the log of real money $(\tilde{m}_t)$ as a linear function of log nominal interest $(\tilde{\imath}_t)$ and log real output $(\tilde{y}_t)$, allowing for first-order autocorrelation in the residuals. We estimated this using levels and first differences, but since the relevant results are statistically identical we report only the latter:

$$
\Delta\tilde{m}_t = \beta_y \Delta\tilde{y}_t + \beta_i \Delta\tilde{\imath}_t - \rho\beta_y \Delta\tilde{y}_{t-1} - \rho\beta_i \Delta\tilde{\imath}_{t-1} + \rho\Delta\tilde{m}_{t-1} + \nu_t
$$

$$
\beta_y = 0.369 \ (0.124), \ \beta_i = -0.226 \ (0.045), \ \rho = 0.347 \ (0.131), \ R^2 = 0.423
$$

Here ρ is the AR(1) coefficient for the residuals in the original equation in levels and the numbers in parentheses are standard errors. The long-run interest elasticity is $\xi = -0.226$, with a relatively small standard error of 0.05.

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Table 1 - Benchmark Calibration

(a) 'Simple' Parameters

Parameters	b	$\varepsilon = \eta$	β	t_h	t_k	t_x	G/Y	δ	α	ω
Targets	0.0001	1	0.976	0.242	0.548	0.069	0.25	0.070	0.288	0.85

(b) Remaining Parameters

Parameters	A	B	ψ	σ	θ
Targets	H	v	K/Y	$-\xi$	μ_D
Target Values	0.33	5.29	2.32	0.23	0.30

Table 2 - Calibration Results

	Data	Model 1 $\theta = 1$	Model 2 calibrate θ	Model 3 calibrate θ calibrate ζ	Model 4 price taking	Model 5 price taking calibrate ζ
Calibrated Parameters						
σ		0.25	0.26	0.27	0.25	0.26
B		1.28	0.99	0.80	2.35	2.32
ψ		1.87	2.35	2.60	1.16	1.29
A		3.41	2.67	2.18	6.38	6.37
θ		–	0.90	0.90	–	–
Calibration Targets						
μ_D	30.00	-46.48 (*)	29.99	29.72	0.00 (*)	0.00 (*)
K/Y	2.32	2.16	2.19	2.18	2.32	2.40
H	0.33	0.33	0.33	0.33	0.33	0.33
v	5.29	5.30	5.31	4.85	5.29	5.28
ξ	-0.23	-0.23	-0.23	-0.22	-0.23	-0.23
ζ	-0.023	0 (*)	-0.002(*)	-0.002	-0.014 (*)	-0.023
Miscellaneous						
s_D		6.88	6.88	8.08	5.21	5.39
s_M		3.95	4.10	4.64	4.06	4.12
μ		-3.20	2.06	2.40	0.00	0.00
$q/\hat{q}$		0.87	0.56	0.57	0.80	0.83
Sq. Error		0.0024	0.0030	0.8541	0.0000	0.0011

Note: The calibration targets marked with (*) are not targeted in the corresponding model and is not included in the computation of the squared error.

Table 3 - $\tau = 0.1$ vs. FR

	Model 1	Model 2	Model 3	Model 4	Model 5
Allocation					
q^1/q^2	0.78	0.82	0.84	0.67	0.69
$\hat{q}^1/\hat{q}^2$	1.00	*1.00*	*1.00*	*1.00*	0.99
K^1/K^2	1.00	*1.00*	*1.00*	0.97	0.95
H^1/H^2	1.00	*1.00*	*1.00*	*1.00*	*1.00*
X^1/X^2	1.00	*1.00*	*1.00*	0.99	0.99
Y_C^1/Y_C^2	1.00	*1.00*	*1.00*	0.99	0.98
Y^1/Y^2	0.98	0.98	0.98	0.97	0.97
Welfare					
ss gain	0.71	2.74	2.99	1.32	1.67
transition	0.00	-0.04	-0.04	-0.30	-0.49
net gain	0.71	2.70	2.95	1.02	1.17
net gain to 0	0.67	2.10	2.29	0.89	0.98

Table 4 - FR vs. FB

	Model 1	Model 2	Model 3	Model 4	Model 5
Allocation					
q^1/q^2	0.65	0.34	0.30	0.92	0.88
$\hat{q}^1/\hat{q}^2$	0.65	0.54	0.48	0.92	0.88
K^1/K^2	0.39	0.35	0.31	0.54	0.56
H^1/H^2	0.73	0.71	0.70	0.76	0.76
X^1/X^2	0.59	0.58	0.56	0.64	0.65
Y_C^1/Y_C^2	0.61	0.58	0.55	0.69	0.69
Y^1/Y^2	0.58	0.55	0.51	0.69	0.70
Welfare					
ss gain	25.84	36.41	43.95	15.31	14.80
transition	-11.57	-14.69	-17.42	-6.95	-6.78
net gain	14.27	21.72	26.53	8.36	8.02
Welfare with no Taxes					
ss gain	3.65	16.19	21.74	0.00	0.00
transition	-2.96	-7.75	-10.13	0.00	0.00
net gain	0.69	8.45	11.61	0.00	0.00

Table 5 - Allocations

	Model 1	Model 2	Model 3	Model 4	Model 5
First Best					
$q = \hat{q}$	1.15	1.35	1.52	0.98	0.98
Y_C	0.76	0.81	0.85	0.70	0.70
Y	0.85	0.93	1.00	0.74	0.75
K	2.75	3.18	3.59	2.18	2.24
H	0.45	0.46	0.47	0.44	0.44
X	0.45	0.46	0.47	0.42	0.42
K/Y	3.24	3.44	3.58	2.95	3.01
Equilibrium at FR					
q	0.74	0.46	0.46	0.90	0.86
$\hat{q}$	0.74	0.73	0.74	0.90	0.86
Y_C	0.46	0.47	0.47	0.48	0.49
Y	0.49	0.51	0.52	0.51	0.52
K	1.08	1.10	1.11	1.18	1.25
H	0.33	0.33	0.33	0.33	0.33
X	0.27	0.26	0.26	0.27	0.27
K/Y	2.18	2.17	2.15	2.32	2.42
Equilibrium at $\tau = 0$					
q	0.70	0.43	0.44	0.82	0.79
$\hat{q}$	0.74	0.73	0.74	0.90	0.86
Y_C	0.46	0.47	0.47	0.48	0.48
Y	0.49	0.50	0.51	0.50	0.51
K	1.08	1.10	1.11	1.17	1.24
H	0.33	0.33	0.33	0.33	0.33
X	0.27	0.26	0.26	0.27	0.27
K/Y	2.19	2.18	2.16	2.32	2.41
Equilibrium at $\tau = 0.1$					
q	0.57	0.37	0.39	0.60	0.60
$\hat{q}$	0.74	0.73	0.74	0.90	0.85
Y_C	0.46	0.47	0.47	0.47	0.48
Y	0.48	0.50	0.50	0.49	0.50
K	1.08	1.10	1.10	1.15	1.19
H	0.33	0.33	0.33	0.33	0.33
X	0.27	0.26	0.26	0.27	0.27
K/Y	2.22	2.21	2.19	2.32	2.38
Nonmonetary Equilibrium					
$\hat{q}$	0.74	0.72	0.72	0.89	0.83
Y_C	0.46	0.46	0.46	0.46	0.46
Y	0.47	0.48	0.48	0.47	0.47
K	1.08	1.07	1.07	1.07	1.07
H	0.33	0.33	0.33	0.33	0.33
X	0.27	0.26	0.26	0.26	0.26
K/Y	2.30	2.25	2.24	2.29	2.29

Table 6 - $\tau = 0.1$ vs FR and...

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 3	Model 4
% of Seignorage	10%	10%	10%	10%	10%	100%	100%
Making up Revenue by T							
q^1/q^2	0.78	0.82	0.84	0.67	0.69	0.84	0.67
K^1/K^2	1.00	*1.00*	*1.00*	0.97	0.95	*1.00*	0.97
H^1/H^2	1.00	1.00	1.00	*1.00*	*1.00*	*1.00*	*1.00*
X^1/X^2	1.00	*1.00*	*1.00*	0.99	0.99	*1.00*	0.99
Y^1/Y^2	0.98	0.98	0.98	0.97	0.97	0.98	0.97
T^1/Y^1	-1.78	-1.02	-0.81	-1.13	-0.71	-2.33%	-2.51
T^2/Y^2	-1.54	-0.80	-0.57	-0.95	-0.57	-0.01%	-0.44
ss gain	0.71	2.74	3.00	1.32	1.67	3.00	1.32
transition	0.00	-0.04	-0.05	-0.30	-0.50	-0.05	-0.30
net gain	0.71	2.70	2.95	1.02	1.17	2.95	1.02
Making up Revenue by t_h							
q^1/q^2	0.78	0.82	0.84	0.67	0.69	0.87	0.67
K^1/K^2	1.01	1.00	*1.00*	0.97	0.95	1.05	1.02
H^1/H^2	1.01	1.00	1.01	*1.00*	*1.00*	1.06	1.05
X^1/X^2	1.01	1.01	1.01	*1.00*	0.99	1.09	1.06
Y^1/Y^2	0.99	0.98	0.98	0.97	0.97	1.04	1.02
New t_h	0.247	0.247	0.247	0.245	0.244	0.303	0.293
ss gain	0.45	2.48	2.70	1.18	1.58	-0.54	-1.36
transition	0.03	0.01	0.01	-0.27	-0.48	0.55	0.18
net gain	0.48	2.49	2.71	0.91	1.11	0.01	-1.18
Making up Revenue by t_x							
q^1/q^2	0.78	0.82	0.84	0.67	0.69	0.86	0.67
K^1/K^2	*1.00*	*1.00*	*1.00*	0.97	0.95	1.04	*1.00*
H^1/H^2	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*	1.04	1.04
X^1/X^2	1.01	*1.00*	1.01	*1.00*	0.99	1.06	1.05
Y^1/Y^2	0.98	0.98	0.98	0.97	0.97	1.02	*1.00*
New t_x	0.075	0.075	0.075	0.072	0.071	0.140	0.129
ss gain	0.50	2.53	2.75	1.21	1.60	0.34	-0.65
transition	0.04	0.00	0.00	-0.28	-0.48	0.40	0.03
net gain	0.53	2.53	2.75	0.93	1.12	0.74	-0.62

Table 7 - Robustness

	Model 1	Model 2	Model 3	Model 4	Model 5
Benchmark	0.71	2.70	2.95	1.02	1.17
Only Lump-sum Tax					
Recalibrated	0.82	3.08	3.20	0.88	0.59
Not	0.71	2.67	2.91	0.67	0.61
Utility Parameters ε and η (Benchmark $\varepsilon = \eta = 1$)					
$\varepsilon = 2, \eta = 1$	0.77	2.07	3.46	0.92	1.05
$\varepsilon = 5, \eta = 1$	0.78	1.55	3.03	0.88	0.95
$\varepsilon = 1, \eta = 1/2$	0.54	3.65	6.27	1.41	1.27
$\varepsilon = 2, \eta = 1/2$	0.74	3.67	6.69	1.22	1.14
$\varepsilon = 5, \eta = 1/2$	0.75	3.81	7.38	1.11	1.07
$\varepsilon = 1, \eta = 2$	0.75	2.38	3.91	0.81	1.04
$\varepsilon = 2, \eta = 2$	0.75	1.55	4.22	0.76	0.90
$\varepsilon = 5, \eta = 2$	0.76	1.82	4.55	0.74	0.80
Utility Parameter b (Benchmark $b = 0.0001$)					
$b = 0.00001$	0.71	2.42	2.87	1.02	1.17
$b = 0.01$	0.71	2.35	3.61	1.03	1.18
$b = 0.1$	0.71	2.46	5.17	1.10	1.20
Markup Target (Benchmark $\mu_D = 30\%$)					
$\mu_D = 10\%$	–	2.43	2.83	–	–
$\mu_D = 50\%$	–	2.49	3.06	–	–
$\mu_D = 100\%$	–	2.86	3.26	–	–
$\mu = 10\%$	–	3.13	3.33	–	–
Measures of Money (Benchmark M1)					
$M0$	0.05	0.36	0.36	0.05	0.05
$M2$	2.05	4.36	8.32	2.62	2.36
$M3$	1.46	4.15	7.58	2.07	1.78
Frequency (Benchmark Annual)					
Quarterly	0.73	1.84	2.74	0.94	1.14
Monthly	0.72	1.51	2.66	0.95	1.18
Period (Benchmark 1951-2004)					
1961-2004	0.62	2.21	1.37	1.18	0.64
1951-1998	0.71	2.49	3.13	0.93	1.19
1986-2004	0.73	2.08	2.79	1.54	0.96
Payment Parameter ω (Benchmark $\omega = 0.85$)					
$\omega = 1$	0.72	2.52	2.91	1.02	1.19
$\omega = 0.5$	0.68	1.51	3.11	1.02	1.11
$\omega = 0.25$	0.94	1.78	3.40	1.18	1.33

Table 8 - More Robustness : Calibration Results

|	Data	Model 6	Model 7	Model 8	Model 9
Calibrated Parameters					
σ		0.23	0.21	0.25	0.19
B		0.83	2.35	0.42	0.13
ψ		2.77	1.96	3.45	7.71
A		2.20	6.41	0.87	0.28
G		0.12	0.12	0.13	0.15
θ		0.84	–	0.42	–
Calibration Targets					
μ_D	30.00	30.00	0.00 (*)	30.06	0.00 (*)
K/Y	2.32	2.22	2.23	2.38	2.69
G/Y	0.25	0.25	0.25	0.25	0.25
H	0.33	0.33	0.33	0.33	0.33
v	5.29	5.07	5.28	5.51	1.53
ξ	-0.23	-0.22	-0.23	-0.08	-0.20
ζ	-0.023	-0.001	-0.023	-0.025	-0.025
Miscellaneous					
s_D		4.48	3.98	4.56	12.45
μ		1.34	0.00	1.37	0.00
Sq. Error		0.902	0.002	0.454	0.552

Note: The calibration targets marked with (*) are not targeted in the corresponding model and not included in the computation of the squared error.

Table 9 - More Robustness : $\tau = 0.1$ vs. FR

	Model 6	Model 7	Model 8	Model 9
Allocation				
q^1/q^2	0.66	0.62	0.81	0.93
K^1/K^2	1.00	0.99	0.81	0.93
Z^1/Z^2	0.67	0.62	–	–
ϕ^1/ϕ^2	–	–	1.10	1.03
H^1/H^2	1.00	0.99	1.02	1.01
X^1/X^2	1.00	1.00	0.94	0.98
Y_C^1/Y_C^2	1.00	0.99	0.95	0.99
Y^1/Y^2	0.98	0.97	0.93	0.92
Welfare				
ss gain	8.35	1.59	6.67	1.41
transition	-0.16	-0.49	-1.11	-0.38
net gain	8.19	1.10	5.57	1.03
net gain to 0	6.24	0.96	4.15	0.83

Figure 1 - Decision Rules and Value Functions

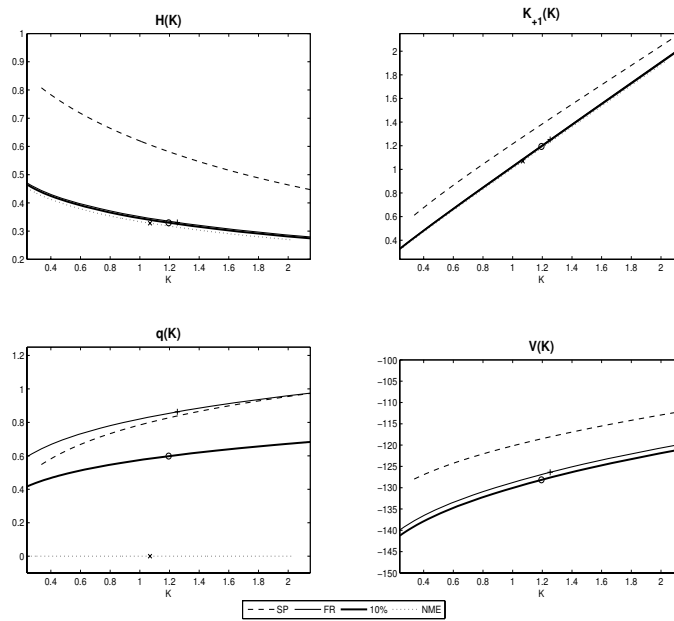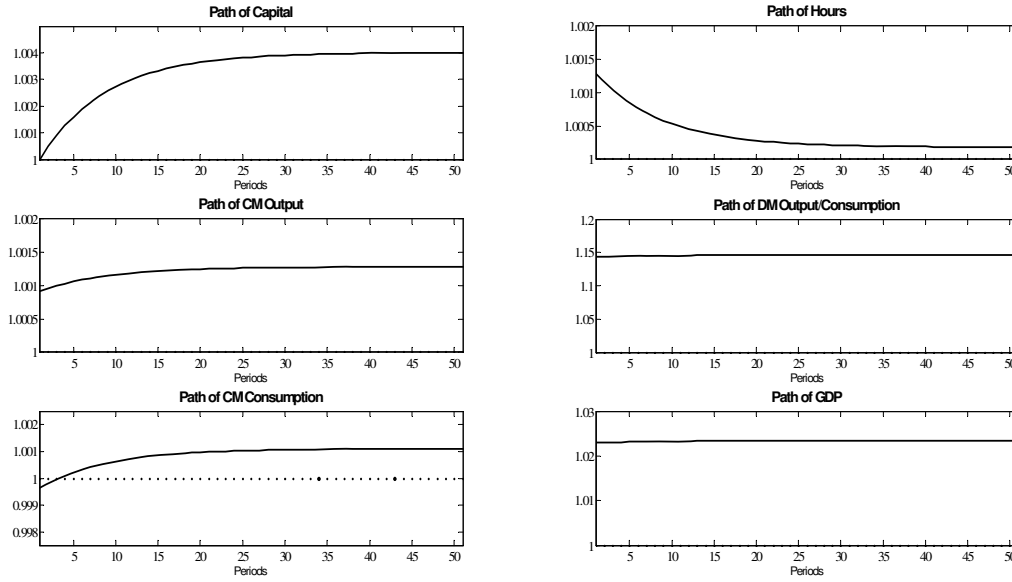## (a) Model 3
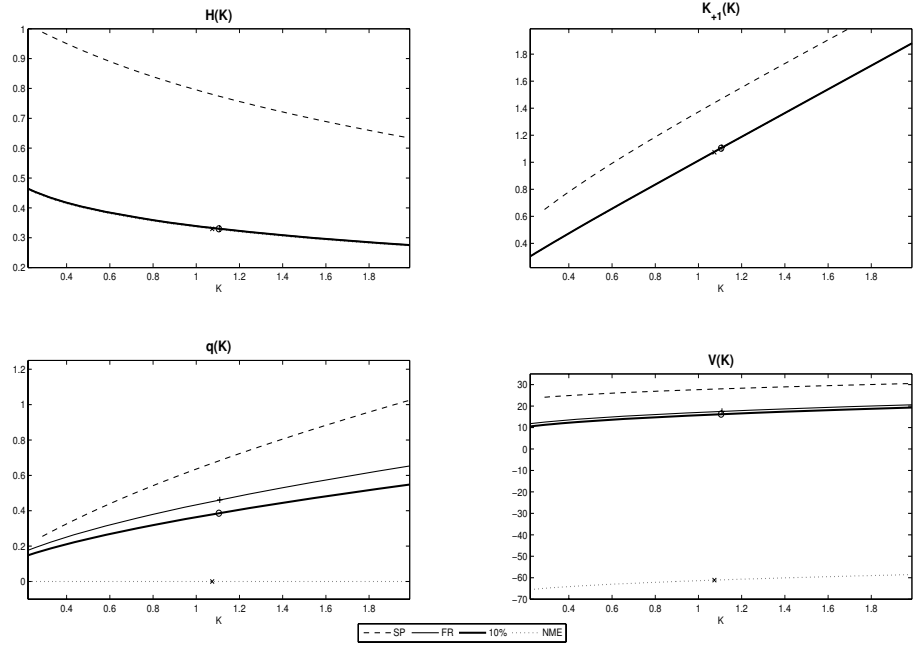
(b) Model 5

Figure 2 -10% to FR: Transitions
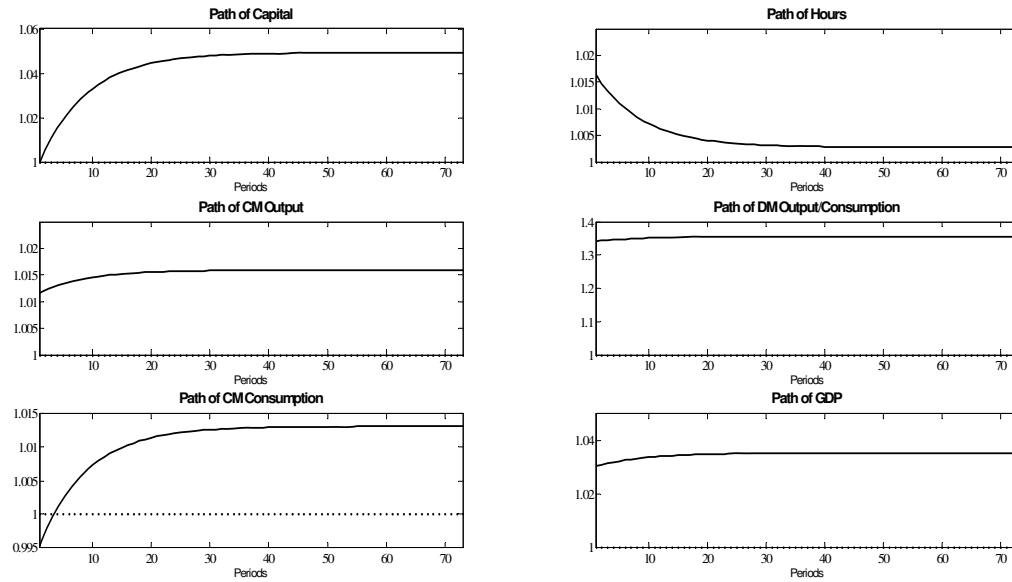
(a) Model 3

(b) Model 5

Figure 3 - Average Real GDP and Investment vs. Inflation for 22 Developed Countries (1950-1999)

GDP vs. Inflation

Investment vs. Inflation