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When does Heterogeneity Matter?*

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Abstract

How do movements in the distribution of income affect the macroeconomy? Krusell and Smith (1998) analyzed this question in a neoclassical growth model, and their results show that the representative-agent assumption provides a good approximation for aggregate behaviors of heterogeneous agents. This paper extends their analysis to a cash-in-advance model with heterogeneous money demand. It is shown that movements in the distribution of monetary income can have significant impact on the macroeconomy. For example, the dynamic responses of aggregate output to monetary shocks behave very differently from those of a representative agent; the welfare costs of moderate inflation are much higher than previously thought, up to 20% of consumption when the inequality of cash distribution is sufficiently large. This is in sharp contrast to the findings of Cooley and Hansen (1989) and Lucas (2000) based on representative-agent models.

Keywords: Cash-in-Advance, Heterogeneity, Distribution, Money Demand, Velocity, Welfare Costs of Inflation.

JEL codes: E12, E13, E31, E32, E41, E43, E51.

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1 Introduction

The infinite-horizon, representative-agent neoclassical growth model and its monetized versions remain the paradigm of business-cycle and monetary policy analysis in macroeconomics. The major reason is not only the framework's analytical tractability; the accuracy of the representative-agent assumption for approximating aggregate dynamics of heterogeneous agents has also gained strong support from the literature, most notably the work of Krusell and Smith (1998). They use a numerical-computation method to show that taking heterogeneous households into consideration makes little difference with respect to the model's predictions for aggregate dynamics. Similar results are also obtained by other papers in different contexts. For example, Imrohoroglu (1989) and Krusell and Smith (1999) show that the welfare costs of business cycles with uninsurable individual risks are small and similar to those calculated by Lucas (1987) based on a representative-agent model; Imrohoroglu (1992), Attanasio, Guiso, and Jappelli (2002), and Costa and Werning (2008) show that the welfare costs of inflation with heterogeneous agents are on the same order as those in representative-agent models[1]; Thomas (2002) shows that firm-level lumpy investment does not change the aggregate dynamics of a representative-agent RBC model.[2]

This paper extends the analysis of Krusell and Smith (1998) to a heterogeneous-agent growth model with cash-in-advance (CIA) constraints and compares the model with its representative-agent counterpart (e.g., Cooley and Hansen, 1989), which is the canonical framework for monetary policy analysis.[3] I show that heterogeneity can lead to dramatically different implications for business cycle and monetary policies. These differences include: (i) The velocity of money is highly variable in the heterogeneous-agent model, in contrast to the findings of Hodrick, Kocherlakota, and Lucas (1991) based on a representative-agent model. (ii) Transitory lump-sum monetary injections can increase aggregate output, unlike the implications under the representative-agent assumption. (iii) Inflation can be extremely costly: under an annual inflation rate of 10% and sufficiently large inequality in cash holdings, households are willing to reduce up to 20% of average consumption each year in exchange for the Friedman-rule inflation rate, in sharp contrast to the findings of Cooley and Hansen (1989) and others in the literature.

However, consistent with the findings of Krusell and Smith (1998), the model's aggregate dynamics under technology shocks are similar to those of a representative-agent model. The reason is

[1] This literature tends to find higher welfare costs of inflation in heterogeneous-agent models; but the absolute magnitude is still small, just a few percentage points of consumption under moderate inflation rates.

[2] Miao and Wang (2009) explain why firm-level lumpiness may not matter for aggregate dynamics. Wang and Wen (2009) show that firm-level heterogeneity reduces aggregate investment volatility under technology shocks.

[3] See, e.g., Lucas and Stokey, 1987; Ireland, 1996; Mankiw and Reis, 2002; and Woodford, 2003. For a comprehensive literature review on recent development in monetary theories, see Williamson and Wright (2008).

that technology shocks have no effect on the equilibrium distribution of income over time; hence, the median individual's dynamic behaviors are the same as the aggregate dynamics. But this is not the case under monetary shocks.

Krusell and Smith (1998, p.870) base their explanations for their findings on the intuition "that the utility costs from fluctuations in consumption are quite small and that self-insurance with only one asset is quite effective." By the same token, one would expect that similar results hold true in a heterogeneous-agent CIA model with capital accumulation because the welfare cost of inflation is also found to be quite small in the literature based on representative-agent models (see, e.g., Cooley and Hansen, 1989; Dotsey and Ireland, 1996; and Lucas, 2000). However, this intuition does not apply in general. For example, the welfare costs of moderate inflation in my model are several orders larger than those obtained by the literature. There are three reasons behind this result: (i) Agents with large money holdings suffer disproportionately more from inflation tax than do agents with smaller real balances. (ii) Anticipated inflation reduces everyone's money demand; hence, the fraction of the population with a binding CIA constraint increases significantly with inflation. This generates additional welfare costs along the extensive margin. (iii) Agents opt to switch from "cash" goods (consumption) to "credit" goods (leisure), thereby reducing labor supply and aggregate output. These three channels reinforce each other. In representative-agent models, only the third channel exists; hence, the implied welfare costs of inflation are small. However, the first two channels are extremely important for business-cycle and welfare analysis, yet are completely missing from representative-agent models. This also explains why the existing literature based on heterogeneous-agent models (e.g., Imrohoroglu, 1992) also obtains small welfare costs of inflation: To reduce computational costs this literature typically assumes that the probability of a binding borrowing constraint is exogenously fixed rather than endogenously determined (e.g., by assuming a binary distribution of idiosyncratic shocks). This makes the distribution of income not responsive to environmental changes, dampening the welfare costs of inflation.

My model is a generalization of the heterogeneous-agent CIA model of Lucas (1980) to a dynamic stochastic general-equilibrium setting with capital accumulation. Since the Lucas model is not analytically tractable, representative-agent versions of this model are routinely used in the literature for business-cycle and policy analysis. However, I show that under quasi-linear preferences (or indivisible labor) and a certain form of information/timing structure, the Lucas model becomes analytically tractable and the distribution of real balances can be characterized by closed forms. Consequently, both the short-run and long-run implications of monetary policies and aggregate dynamics can be examined by standard solution methods available in the real-business-cycle (RBC) literature, without the need to use complicated computational methods such as that of Krusell and Smith (1998). Analytical tractability is a great virtue because it makes the model's mechanisms transparent.[4]

The key factor in generating results that differ from those of the literature is the sensitivity of the distribution of income in response to environmental changes. As pointed out by Aiyagari (1994) and Krusell and Smith (1998), under borrowing constraints, agents have strong incentives to self-insure against idiosyncratic shocks through precautionary savings. Hence, in equilibrium the probability of a binding borrowing constraint is very small or the fraction of the liquidity-constrained population is very small. This implies that aggregate technology shocks have little impact on the distribution of income because households are already nearly perfectly self-insured against such shocks. However, the same precautionary-saving mechanism works against the individuals under inflation or monetary shocks, because self insurance implies that agents opt to hold too much cash in hand to avoid a binding CIA constraint. Thus, they are heavily taxed by inflation, leading to large welfare costs. Also, the aggregate dynamics of the model react to monetary shocks significantly because transitory monetary injections stimulate consumption for the liquidity-constrained individuals without much impact on aggregate prices. Consequently, aggregate prices are "sticky" and money is expansionary for aggregate output and employment. This is in sharp contrast to representative-agent CIA models where monetary shocks are contractionary because prices move nearly one-for-one with the money supply so that the inflation tax effect dominates.

The rest of the paper is organized as follows. Section 2 presents the model and shows how to obtain closed-form decision rules for money demand at the individual level. Section 3 characterizes general equilibrium, and Section 4 presents the control model. The short- and long-run implications of heterogeneity are studied in Sections 5 and 6. Section 7 concludes the paper.

2 The Model

There is a continuum of households indexed by $i \in [0, 1]$. As in Lucas (1980) and Krusell and Smith (1998), each household is subject to an idiosyncratic preference shock to the marginal utility of consumption, $\theta(i)$, which has the distribution $F(\theta) \equiv \Pr[\theta(i) \le \theta]$ with support $[\theta_l, \theta_h]$. Leisure enters the utility function linearly as in Wen (2009).[5] A household chooses consumption $c(i)$, labor supply $n(i)$, a non-monetary asset $s(i)$ that pays the real rate of return $r > 0$, and nominal balance $m(i)$ to maximize lifetime utility,

[4] In a different paper (Wen, 2009), I study welfare implications of financial intermediation and monetary policies in a generalized Bewley (1980) model where money serves solely as a store of value and is not required as a medium of exchange. The welfare costs of moderate inflation in the two models are similar but those of hyperinflation are different because of the CIA constraints. In Wen (2009), agents opt to not hold money as a store of value when the inflation rate is too high. In addition, the velocity of money is bounded by unit in this paper, whereas it is not the case in Wen (2009). However, the solution techniques used in this paper are similar to that used in Wen (2009).

[5] The linearity assumption simplifies the model by making the distribution of wealth degenerate. However, the distribution of money holdings is not degenerate. This setup also makes the results regarding the welfare costs of inflation comparable to those of the CIA model of Cooley and Hansen (1989) based on indivisible labor.

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \theta_t(i) \frac{c_t(i)^{1-\tau}}{1-\tau} - a n_t(i) \right\}$$

subject to

$$c_t(i) + \frac{m_t(i)}{P_t} + s_t(i) \leq (1+r_t)s_{t-1}(i) + \frac{m_{t-1}(i) + \tau_t}{P_t} + W_t n_t(i) \tag{1}$$

$$c_t(i) \leq \frac{m_t(i)}{P_t}, \tag{2}$$

where P denotes aggregate price, W the real wage, and τ a lump-sum per-capita nominal transfer. Without loss of generality, assume $a = 1$ in the utility function.

To make the model analytically tractable, assume that cash accumulated in the current period, $m_t(i)$, can be used immediately to facilitate consumption transactions, instead of waiting for one period as in the standard CIA literature. This assumption is also more realistic because business-cycle models are typically calibrated to quarterly or yearly frequencies; hence, requiring households to hold cash for that long in order to purchase consumption goods is unrealistic. To capture the liquidity role of money under this assumption, we also need to assume that the decisions for labor supply and investment on interest-bearing assets (such as capital) must be made before observing the idiosyncratic preference shock $\theta(i)$ in each period. Thus, if there is an urge to consume in period t, money stock is the only asset that can be adjusted to meet the liquidity demand. Borrowing of liquidity (money) from other households is not allowed. These assumptions imply that households may find it optimal not to spend all cash in hand in each period because carrying the excess money balances to the next period may be beneficial when the current marginal utility of consumption is low and future marginal utilities may be high.[6] As in the standard literature, any aggregate uncertainty is resolved at the beginning of each period and is orthogonal to idiosyncratic uncertainty.

Denoting $\{\lambda(i), \pi(i)\}$ as the Lagrangian multipliers for constraints (1) and (2), respectively, the first-order conditions for $\{c(i), n(i), m(i), s(i)\}$ are given, respectively, by

$$\theta_t(i) c_t(i)^{-\tau} = \lambda_t(i) + \pi_t(i) \tag{3}$$

$$1 = W_t E_t^i \lambda_t(i) \tag{4}$$

$$\frac{\lambda_t(i)}{P_t} = \beta E_t \frac{\lambda_{t+1}(i)}{P_{t+1}} + \frac{\pi_t(i)}{P_t} \tag{5}$$

$$E_t^i \lambda_t(i) = \beta E_t (1 + r_{t+1}) \lambda_{t+1}(i), \tag{6}$$

[6] See, e.g., Svensson (1985).

where the expectation operator E^i denotes expectations conditional on the information set of time t excluding $\theta_t(i)$. By the law of iterated expectations and the orthogonality assumption of aggregate and idiosyncratic shocks, equations (5) and (6) can be written as

$$\lambda_t(i) = \beta E_t \frac{P_t}{P_{t+1}W_{t+1}} + \pi_t(i) \tag{7}$$

$$\frac{1}{W_t} = \beta E_t (1 + r_{t+1}) \frac{1}{W_{t+1}}, \tag{8}$$

where $\frac{1}{W}$ is the marginal utility of consumption in terms of labor.

The decision rules at the individual's level are characterized by a cutoff strategy, taking as given the aggregate environment. Consider two possible cases:

Case A: $\theta(i) \leq \theta^*$. In this case the urge to consume is low. Hence, it is optimal not to spend all cash in hand but to carry the excess money as inventories. Thus, $c_t(i) \leq \frac{m_t(i)}{P_t}$, $\pi_t(i) = 0$, and the shadow value of good $\lambda_t(i) = \beta E_t \frac{P_t}{W_{t+1}P_{t+1}}$. Equation (3) implies $c(i) = \left[\theta(i)\left(\beta E \frac{P}{P_{t+1}W_{t+1}}\right)^{-1}\right]^{\frac{1}{\tau}}$. Defining

$$x(i) \equiv (1 + r)s_{t-1}(i) + \frac{m_{t-1}(i) + \tau_t}{P_t} + W_t n_t(i) - s_t(i) \tag{9}$$

as real wealth net of asset investment, the budget identity (1) then implies

$$\frac{m_t(i)}{P_t} = x(i) - \left[\theta(i)\left(\beta E \frac{P_t}{P_{t+1}W_{t+1}}\right)^{-1}\right]^{\frac{1}{\tau}}. \tag{10}$$

The requirement $\frac{m_t(i)}{P_t} \geq c_t(i)$ then implies $x(i) \geq 2\left(\theta(i)\left[\beta E \frac{P_t}{P_{t+1}W_{t+1}}\right]^{-1}\right)^{\frac{1}{\tau}}$, or

$$\theta(i) \leq \left(\frac{1}{2}x(i)\right)^{\tau} \beta E \frac{P_t}{P_{t+1}W_{t+1}} \equiv \theta^*, \tag{11}$$

which defines the cutoff θ^*. Notice that the cutoff is independent of i because the wealth $x(i)$ is determined before the realization of $\theta_t(i)$ and all households face the same distribution of idiosyncratic shocks. This property simplifies the computation of the general equilibrium of the model tremendously. Given the definition for θ^*, we have

$$x(i) = 2\left[\theta^*\left(\beta E \frac{P_t}{P_{t+1}W_{t+1}}\right)^{-1}\right]^{\frac{1}{\tau}} \tag{12}$$

and $\frac{m(i)}{P} = \left(2\theta^{*\frac{1}{\tau}} - \theta(i)^{\frac{1}{\tau}}\right)\left(\beta E \frac{P_t}{P_{t+1}W_{t+1}}\right)^{-\frac{1}{\tau}}$.

Case B: $\theta(i) > \theta^*$. In this case the urge to consume is high. It is then optimal to spend all cash in hand, so $\pi_t(i) > 0$ and $c(i) = \frac{m(i)}{P_t}$. By the resource constraint (1), we have $c_t(i) = \frac{1}{2}x(i)$, which by (12) implies $c_t(i) = \left(\theta^* \left[\beta E \frac{P}{P_{t+1}W_{t+1}}\right]^{-1}\right)^{\frac{1}{\tau}}$. Equations (3) and (5) then imply $\pi(i) = \frac{\theta(i)}{\theta^*}\beta E \frac{P}{P_{t+1}W_{t+1}} - \lambda(i) = \left(\frac{\theta(i)}{\theta^*} - 1\right)\beta E \frac{P}{P_{t+1}W_{t+1}} - \pi_t(i)$, which gives

$$\pi(i) = \frac{1}{2}\left(\frac{\theta(i)}{\theta^*} - 1\right)\beta E \frac{P_t}{P_{t+1}W_{t+1}} > 0. \tag{13}$$

Hence, the shadow value of goods is given by

$$\lambda_t(i) = \frac{1}{2}\left(\frac{\theta(i)}{\theta^*} + 1\right)\beta E \frac{P_t}{P_{t+1}W_{t+1}}. \tag{14}$$

Notice that the shadow value $\lambda(i)$ is higher under case B than under case A.

The above analyses imply that the shadow price $\lambda(i)$ takes two possible functional forms; hence, the expected shadow value of goods, $E^i\lambda(i)$, and also the optimal cutoff value θ^*, are determined by the following asset-pricing equation based on (4):

$$\frac{1}{P_t W_t} = \beta E_t \frac{1}{P_{t+1}W_{t+1}} R(\theta_t^*), \tag{15}$$

where

$$R_t \equiv \left[\int_{\theta(i)\leq\theta^*} dF(\theta) + \int_{\theta(i)>\theta^*} \frac{1}{2}\left(\frac{\theta(i)}{\theta^*} + 1\right) dF(\theta)\right] \tag{16}$$

measures the (shadow) rate of return to liquidity or cash inventory. The left-hand side of equation (15) is the utility cost of holding one unit of real balances as inventory. The right-hand side is the expected return to inventory, which takes two possible values: The first is simply the discounted next-period utility cost of inventory ($\beta E_t \left[P_{t+1}w_{t+1}\right]^{-1}$) in the case of low liquidity demand ($\theta \leq \theta^*$), which has probability $\int_{\theta(i)\leq\theta^*} dF(\theta)$. The second is the marginal utility of consumption ($\frac{\theta_t(i)}{\theta_t^*}\beta E_t \left[P_{t+1}w_{t+1}\right]^{-1}$) in the case of high liquidity demand ($\theta > \theta^*$), which has probability $\int_{\theta(i)>\theta^*} dF(\theta)$. The optimal cutoff θ^* is chosen so that the marginal cost equals the expected marginal gains. Hence, the rate of return to inventory investment in money (liquidity) is determined by $R(\theta^*)$. Notice that $R(\theta^*) > 1$ as long as $\theta^* < \theta_h$. The fact that $R > 1$ implies that the option value of one dollar exceeds one because it provides liquidity in the event there is the urge to consume. This inventory-theoretic formula of the rate of return to liquidity is similar to that derived by Wen (2008, 2009).

The cutoff strategy implies that the optimal level of wealth in period t is determined by a "target" given by (12), which specifies that wealth (real money balances plus labor income net of asset investment) is set to a target level depending on the cutoff and the expected future utility. This implies that labor supply will adjust so that the wealth level meets the target. Since all individuals face the same distribution of idiosyncratic preference shocks, wealth distribution in this model is degenerate. This greatly simplifies the computation of general equilibrium and makes the model analytically tractable. However, heterogeneity still matters because the distribution of money holdings is not degenerate.

Utilizing equation (15), the decision rules of household i's consumption and money holdings are summarized by

$$x(i) = 2\left[\theta^* W_t R(\theta_t^*)\right]^{\frac{1}{\tau}} \tag{17}$$

$$c(i) = \begin{cases} \left[\theta(i) W_t R(\theta_t^*)\right]^{\frac{1}{\tau}} & \text{if } \theta(i) \leq \theta^* \\ \left[\theta^* W_t R(\theta_t^*)\right]^{\frac{1}{\tau}} & \text{if } \theta(i) > \theta^* \end{cases} \tag{18}$$

$$\frac{m(i)}{P_t} = \begin{cases} \left(2\theta^{*\frac{1}{\tau}} - \theta(i)^{\frac{1}{\tau}}\right)\left[W_t R(\theta_t^*)\right]^{\frac{1}{\tau}}. & \text{if } \theta(i) \leq \theta^* \\ \left[\theta^* W_t R(\theta_t^*)\right]^{\frac{1}{\tau}} & \text{if } \theta(i) > \theta^* \end{cases}. \tag{19}$$

Denoting $C \equiv \int c(i)di, M \equiv \int m(i)di, S = \int s(i)di, N = \int (n(i)di,$ and $X = \int x(i)di$ and integrating the household decision rules over i by the law of large numbers, the aggregate variables are given by

$$X_t = 2\theta^{*\frac{1}{\tau}} (WR)^{\frac{1}{\tau}} \tag{20}$$

$$C_t = (WR)^{\frac{1}{\tau}} D(\theta^*) \tag{21}$$

$$\frac{M_t}{P_t} = (WR)^{\frac{1}{\tau}} H(\theta^*), \tag{22}$$

where

$$D(\theta^*) \equiv \int_{\theta \leq \theta^*} \theta(i)^{\frac{1}{\tau}} dF(\theta) + \int_{\theta > \theta^*} \theta^{*\frac{1}{\tau}} dF(\theta) \tag{23}$$

$$H(\theta^*) \equiv \int_{\theta \leq \theta^*} \left(2\theta^{*\frac{1}{\tau}} - \theta(i)^{\frac{1}{\tau}}\right) dF(\theta) + \int_{\theta > \theta^*} \theta^{*\frac{1}{\tau}} dF(\theta). \tag{24}$$

Notice $D(\theta^*) + H(\theta^*) = 2\theta^{*\frac{1}{\tau}}$.

The Quantity Theory. The aggregate relationship between consumption (21) and money demand (22) implies the quantity equation,

$$P_t C_t = M_t V_t, \tag{25}$$

where the consumption velocity of money is given by

$$V_t = \frac{D(\theta^*)}{H(\theta^*)}, \tag{26}$$

which has the support $\left[E\theta^{\frac{1}{\tau}}/(2\theta_h^{\frac{1}{\tau}} - E\theta^{\frac{1}{\tau}}), 1\right]$ and is thus bounded above by unity and below away from zero.[7]

Rule-of-Thumb Consumption Behavior. Notice that aggregate consumption follows the linear rule

$$C_t = \tilde{\alpha}_t X_t, \tag{27}$$

where $\tilde{\alpha} \equiv \frac{D(\theta^*)}{2\theta^{*(1/\tau)}} \in (0,1)$ is the marginal propensity to consume. However, here the marginal propensity to consume is time varying and depends endogenously on aggregate economic conditions. This type of consumption behavior is the consequence of the borrowing (CIA) constraint. When the constraint is not binding, consumption essentially follows permanent income; and when it binds, consumption is determined by the current real balances. Since the CIA constraint binds in only some states of the world, the average consumption depends on current wealth-income with the marginal propensity to consume being time-varying. The tendency for consumption smoothing is captured by the fact that consumption is a function of wealth and the fact that $\frac{\partial \tilde{\alpha}(\theta^*)}{\partial \theta^*} < 0$.[8] For a given increase in the real wage, consumption rises by D units but wealth (X) rises by $\theta^* > D$ units. In addition, if the cutoff θ^* also increases,[9] $D(\theta^*)$ will rise by $1 - F(\theta^*) < 1$ units while θ^* can rise one for one. Hence, consumption rises proportionally less than wealth-income not only because $\tilde{\alpha} < 1$ but also because $\tilde{\alpha}$ decreases. Similarly, the demand for real balances obeys the same linear rule, $\frac{M_t}{P_t} = \tilde{\beta}_t X_t$, where $\tilde{\beta}_t \equiv \frac{H(\theta^*)}{2\theta^{*(1/\tau)}}$ and the marginal propensity to hold money is also less than one and countercyclical with respect to θ^*. Demand on interest-bearing assets (S_t), on the other hand, does not obey such simple rules.

Distributional Effects. Notice that aggregate shocks will in general affect the distribution of money holdings across households by affecting the cutoff θ_t^*. Equation (15) is the key to understanding such effects. For example, consider the situation without aggregate uncertainty. Equation

[7] Alternatively, we can also measure the velocity of money by aggregate income, $PY = M\tilde{V}$, where $\tilde{V} \equiv V\frac{Y}{C}$ is the income-velocity of money.

[8] The proof is straightforward by using the definition of $D(\theta^*)$.

[9] As will be shown shortly, under a transitory monetary injection, θ^* is procyclical.

(15) implies that the cutoff θ^* is determined by the following relation:

$$R(\theta^*) = \frac{1+\pi}{\beta}, \tag{28}$$

where $\pi \equiv \frac{P_t - P_{t-1}}{P_{t-1}}$ is the steady-state rate of inflation.[10] Hence, the distribution of money holdings depend on inflation. In particular, since $\frac{\partial R}{\partial \theta^*} < 0$, an increase in the rate of inflation will decrease the cutoff, hence shifting the distribution of money holdings towards one with more agents being liquidity constrained. This suggests that with heterogeneous agents the welfare costs of inflation can be significantly different from those implied by representative-agent models because inflation affects the distribution of real balances.

With aggregate uncertainty, the strength of the distributional effects depends on the movements of the real wage and prices. For example, the real wage behaves like consumption and is hence smooth (because the movements in labor and output tend to cancel each other). In addition, with money supply fixed, the aggregate price moves against consumption nearly one for one under technology shocks, thus further dampening any movement in the real wage. Consequently, the nominal wage and the cutoff variable θ^* do not respond to technology shocks. This explains why under technology shocks the model behaves similarly to a representative-agent model. This also explains why Krusell and Smith (1998) find that consumption is proportional to wealth with the marginal propensity to consume ($\tilde{\alpha}(\theta^*)$) remaining constant under aggregate technology shocks. However, the cutoff θ^* will respond to monetary shocks. This is the key for understanding why money is not neutral in this model.

3 General Equilibrium

This heterogeneous-agent model outlined above can be easily embedded into a standard real business cycle (RBC) framework with capital accumulation. For example, assume that capital is the only non-monetary asset and is accumulated according to $K_{t+1} - (1-\delta) K_t = I_t$, where I is gross aggregate investment and δ rate of depreciation; the production technology is given by $Y_t = A_t {K_t}^{\alpha} N_t^{1-\alpha}$, where A denotes total factor productivity (TFP). Under perfect competition, factor prices are determined by marginal products, $r_t + \delta = \alpha \frac{Y_t}{K_t}$ and $W_t = (1-\alpha)\frac{Y_t}{N_t}$. Market clearing implies $S_t = K_{t+1}$, $\int n_t(i) = N_t$, and $M_t = \overline{M}_t = \overline{M}_{t-1} + \tau_t$, where $\overline{M}_t$ denotes aggregate money supply in period t. Notice that equations (21), (22), and (20) with money market clearing ($M = M_{-1} + \tau$)

[10] The quantity relation (25) implies $\frac{P_t}{P_{t-1}} = \frac{M_t}{M_{t-1}}$ in the steady state, so the steady-state inflation rate is the same as the growth rate of money.

imply the aggregate goods-market clearing condition,

$$C_t + K_{t+1} - (1-\delta)K_t = Y_t. \tag{29}$$

A general equilibrium is defined as the sequence $\{C_t, Y_t, N_t, K_{t+1}, M_t, P_t, W_t, r_t, \theta_t^*\}$, such that all households maximize utility subject to their resource and CIA constraints, firms maximize profits, all markets clear, the law of large numbers holds, and the set of standard transversality conditions is satisfied. The equations needed to solve for the general equilibrium are (8), (15), (21), (22), (29), the production function, firms' first-order conditions with respect to $\{K, N\}$, and the law of motion for money, $M = M_{-1} + \tau$. The aggregate model has a unique steady state. The aggregate dynamics of the model can be solved by log-linearizing the aggregate model around the steady state and then applying the method of Blanchard and Kahn (1980) to find the stationary saddle path as in King, Plosser, and Rebelo (1988).

Monetary Policy. We consider two types (regimes) of monetary policies. For the short-run dynamic analysis, money supply shocks are purely transitory without affecting the steady-state stock of money,

$$\tau_t = \rho\tau_{t-1} + \overline{M}\varepsilon_t, \tag{30}$$

$$M_t = \overline{M} + \tau_t, \tag{31}$$

where $\rho \in [0,1]$ and $\overline{M}$ is the steady-state money supply. This policy implies the percentage deviation of money stock follows an $AR(1)$ process, $\frac{M_t - \overline{M}}{\overline{M}} = \rho\frac{M_{t-1} - \overline{M}}{\overline{M}} + \varepsilon_t$. Under this policy regime, the steady-state inflation rate is zero, $\pi = 0$.

For the long-run (steady-state) analysis, money supply has a permanent growth component with,

$$\tau_t = (\mu_t - 1)\, M_{t-1} \tag{32}$$

$$\log\mu_t = \rho\log\mu_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iid(\bar{\varepsilon}, \sigma^2), \tag{33}$$

where μ_t is the gross growth rate of money with mean $\bar{\mu}$ and $\bar{\varepsilon} = (1-\rho)\log\bar{\mu}$ is the mean of the innovation ε.

4 The Control Model

The control model is a version of that in Cooley and Hansen (1989), where a representative agent solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\tau}}{1-\tau} - N_t \right\}$$

subject to

$$C_t + \frac{M_t}{P_t} + K_{t+1} - (1-\delta)K_t \leq A_t K_t^{\alpha} N_t^{1-\alpha} + \frac{M_{t-1} + \tau_t}{P_t} \tag{34}$$

$$C_t \leq \frac{M_t}{P_t}. \tag{35}$$

The first-order conditions with respect to $\{C, N, M, K'\}$ are given, respectively, by

$$C_t^{-\tau} = \Lambda_t + \Pi_t \tag{36}$$

$$1 = \Lambda_t (1-\alpha)\frac{Y_t}{N_t} \tag{37}$$

$$\frac{\Lambda_t}{P_t} = \beta E_t \frac{\Lambda_{t+1}}{P_{t+1}} + \frac{\Pi_t}{P_t} \tag{38}$$

$$\Lambda_t = \beta E_t \Lambda_{t+1}\left(\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta\right). \tag{39}$$

As argued by Cooley and Hansen (1989) and numerically illustrated by Hodrick, Kocherlakota, and Lucas (1991), the CIA constraint (35) will almost always bind in all states, as long as the inflation rate is above the Friedman rule, $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} > \beta - 1$. Hence, as in Cooley and Hansen (1989), we assume the constraint holds with equality and the Lagrangian multiplier $\Pi_t > 0$ for all t.

5 Steady-State Comparison

To facilitate quantitative analysis, we assume the idiosyncratic shocks $\theta(i)$ follow the Pareto distribution, $F(\theta) = 1 - \theta^{-\sigma}$, with $\sigma > 1$ and the support $\theta \in (1, \infty)$. Since the support is not bounded above, monetary equilibrium with a strictly positive price level $P > 0$ does not exist under the Friedman rule. Hence, our analysis in this part of the paper treats the Friedman rule as a limiting case.[11] We choose a log utility function ($\tau = 1$) and a sufficiently high degree of heterogeneity by setting the shape parameter $\sigma = 1.5$.[12] We set the time period to a quarter of a year, and the discounting factor $\beta = 0.99$.

[11] With the Pareto distribution, as $1 + \pi$ approaches β, the demand for real balances approaches infinity. Since in equilibrium money demand must equal money supply (which is finite), this implies that the price level must approach zero (or the value of money must approach infinity).

[12] The variance of the Pareto distribution is inversely related to σ. The empirical literature based on distributions of income and wealth typically finds $\sigma \in (1.1, 3.5)$ or centered around $1.5 \sim 2.5$ (see, e.g., Wolff, 1996; Fermi, 1998; Levy and Levy, 2003; Clementi and Gallegati, 2005; and Nirei and Souma, 2007). The Gini index for money holdings is about 0.8, indicating very high inequality (see Jellou, 2007). Hence, $\sigma = 1.5$ is within the empirical estimates.

For the representative-agent control model, equation (39) implies $\frac{K}{Y} = \frac{\alpha\beta}{1-\beta(1-\delta)}$; the budget constraint (34) implies $\frac{C}{Y} = 1 - \frac{\delta\alpha\beta}{1-\beta(1-\delta)}$. Equation (38) implies $\Lambda(1+\pi-\beta) = (1+\pi)\Pi$; (36) implies $C^{-1} = \Lambda\left(1 + \frac{1+\pi-\beta}{1+\pi}\right)$; (37) then implies

$$C = \frac{1+\pi}{2(1+\pi)-\beta}W, \tag{40}$$

where $W = \left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}}$ is the marginal product of labor, which is independent of inflation. Hence, consumption is decreasing in π. Under the Friedman rule, $1+\pi = \beta$, we have the maximum consumption given by $C^* = W$. Given W, we have $N = \frac{(1-\alpha)}{W}Y = \frac{(1-\alpha)}{W}\frac{1-\beta(1-\delta)}{1-\beta(1-\delta)-\beta\delta\alpha}C$.

For the heterogeneous-agent model, the capital-output and consumption-output ratios are given by $\frac{K}{Y} = \frac{\beta\alpha}{1-\beta(1-\delta)}$ and $\frac{C}{Y} = 1 - \frac{\delta\beta\alpha}{1-\beta(1-\delta)}$, respectively. Since $r+\delta = \alpha\frac{Y}{K}$ and $W = (1-\alpha)\frac{Y}{N}$, the factor prices are given by $r = \frac{1}{\beta} - 1$ and $W = (1-\alpha)\left(\frac{\beta\alpha}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}}$, respectively. These results are the same as in the control model. Hence, heterogeneity does not alter the steady-state saving rate and the real factor prices. However, the levels of income, consumption, employment, and capital stock will be affected by heterogeneity. With the Pareto distribution, we have $D(\theta^*) = \frac{\sigma}{\sigma-1} - \frac{1}{\sigma-1}\theta^{*1-\sigma}$ and $R(\theta^*) = 1 + \frac{1}{2}\frac{1}{\sigma-1}\theta^{*-\sigma}$, which implies (see equation 28) $\theta^* = \left[\frac{1+\pi-\beta}{\beta}2(\sigma-1)\right]^{-\frac{1}{\sigma}}$. Hence, the steady-state aggregate consumption is

$$C = W\frac{\sigma}{\sigma-1}\frac{1+\pi}{\beta}\left(1 - \frac{1}{\sigma}\left[\frac{1+\pi-\beta}{\beta}2(\sigma-1)\right]^{\frac{\sigma-1}{\sigma}}\right),$$

which is decreasing in π. The maximum consumption is obtained under the Friedman rule, $C^* = W\frac{\sigma}{\sigma-1}$.

Interior solution requires $\theta^* > 1$, which implies $1+\pi < \frac{2\sigma-1}{2(\sigma-1)}\beta$. If a inflation rate exceeds this upper bound, all agents will have a binding CIA constraint in all states and the model degenerates to the representative-agent control model. This implies that the two models are identical up to a constant at two polar points: one under the Friedman rule and another at $1+\pi_{\max} = \frac{2\sigma-1}{2(\sigma-1)}\beta$. These two points converge to one point as the variance of the distribution of $\theta(i)$ decreases to zero. In this case, the distribution of $\theta(i)$ is degenerate (i.e., $\sigma \to \infty$), and the two models become exactly identical for all rates of inflation.

Following Cooley and Hansen (1989), we measure the welfare costs of inflation by the consumption ratio,

$$\lambda = \frac{C^*}{C} - 1, \tag{41}$$

which is the percentage reduction in consumption due to inflation rising above the Friedman rule. The welfare costs of inflation and the velocity of money in the two different models are graphed in Figure 1, where dashed lines represent the control model.

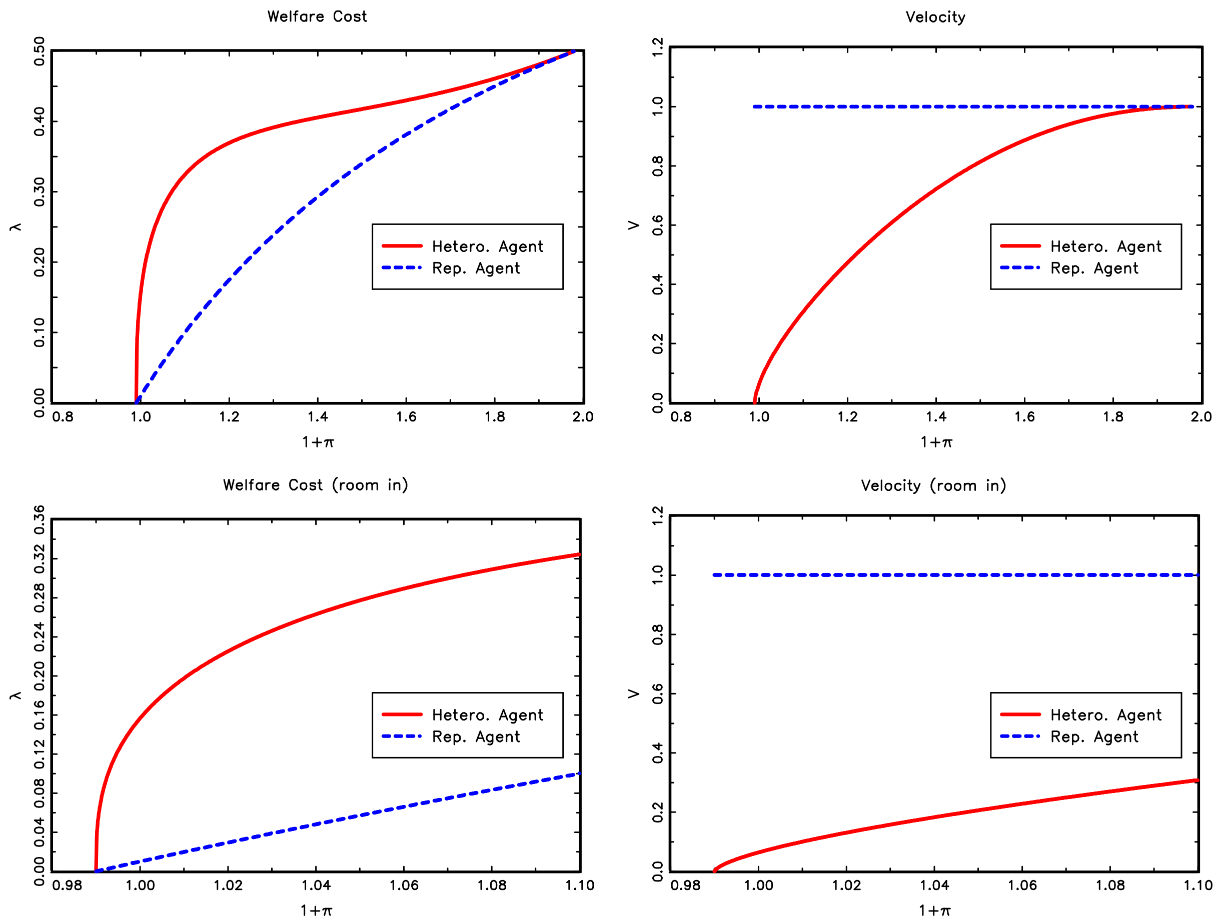

Figure 1. Heterogeneous-Agent versus Representative-Agent Model.

Notice how heterogeneity alters the model's implications for welfare costs. For example, at the moderate inflation rate of 1% a quarter (or 4% a year), the welfare cost is about 2% of consumption in the control model; but it is about 20% in the heterogeneous-agent model. At $\pi = 2.5\%$ a quarter (or 10% a year), the welfare cost is 3.4% of consumption in the representative-agent model, but 23.6% in the heterogeneous-agent model. These are astonishingly large welfare loses, which are comparable to the marginal income taxes in the U.S. With such a high welfare cost of moderate inflation, one starts to understand why so many times in human history violent revolutions were triggered by or associated with inflation.[13]

Why does heterogeneity make such a large difference? One crucial reason is that, with idiosyncratic shocks, the CIA constraint does not bind for many agents (or not very often for the same agent) because of precautionary saving motives under idiosyncratic risk. This is very different from representative-agent models where the CIA constraint almost always binds under aggregate risks (see the analysis of Hodrick, Kocherlakota, and Lucas, 1991). Figure 2 plots the probability of a binding CIA constraint in the heterogeneous-agent model as a function of the inflation rate. It shows that the probability of running out of cash inventories is very low under moderate inflation (i.e., less than 10% of the time), suggesting that most agents opt to hold an excessive amount of liquidity as a precautionary device for self insurance most of the time. This explains why inflation is so costly in the heterogeneous-agent model: (i) agents suffer disproportionately more from inflation tax when they hold more cash reserves as self-insurance; and (ii) as inflation increases, more agents will be liquidity constrained, further reducing aggregate consumption along the extensive margin. These two mechanisms magnify the substitution effect between cash goods (consumption) and credit goods (leisure), further reducing labor supply and output.

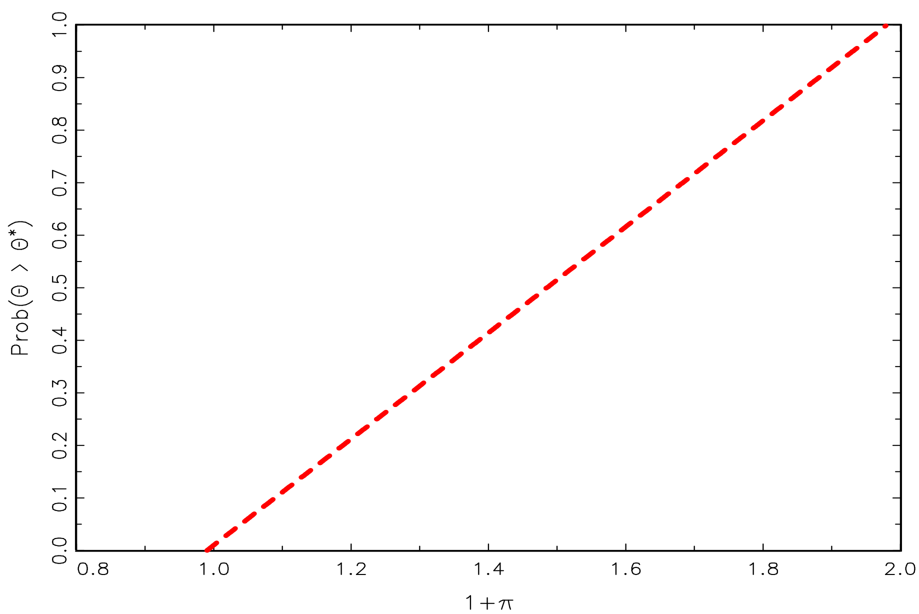

Figure 2. Probability of a Binding CIA Constraint.

The realism of the model can be tested by the U.S. "money demand" curve estimated by Lucas (2000). Using long-term time series data for GDP, money stock (M1), and the nominal interest rate, Lucas (2000) showed that the ratio of M1 to nominal GDP is downward sloping against the

[13] For empirical studies on the relationship between moderate inflation and revolutions in recent world history, see Cartwright, Delorme, and Wood (1985) and Looney (1985).

nominal interest rate. Lucas interpreted this downward relationship as a "money demand" curve and argued that it can be rationalized by the representative-agent Sidrauski (1967) model of money-in-the-utility. Analogous to Lucas, the money demand curve implied by the heterogeneous-agent CIA model of this paper takes the form

$$\frac{M}{PY} = A\frac{H(\theta^*)}{D(\theta^*)}, \tag{42}$$

where A is a scale parameter influenced by the definition of money in the data and the cutoff θ^* is a function of the nominal interest rate implied by equation (28). Figure 3 shows a good fit of the theoretical model to the U.S. data.[14]

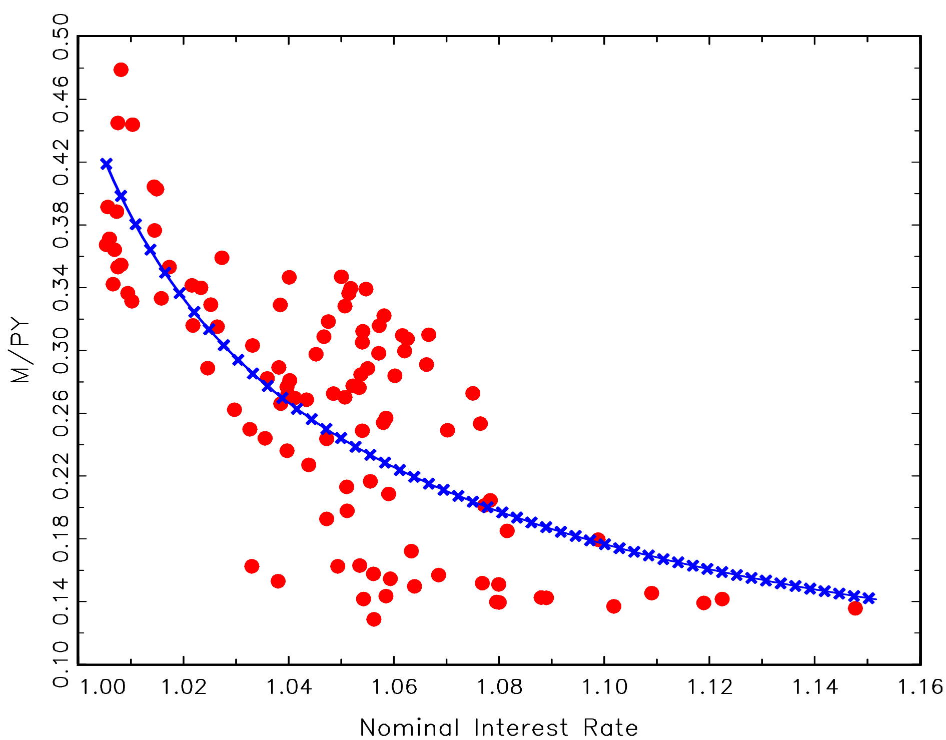

Figure 3. Predicted Money Demand Curve and U.S. Data.

6 Impulse Responses

Following the standard RBC literature, we set the capital depreciation rate $\delta = 0.025$ and capital's income share $\alpha = 0.3$. The impulse responses of the model to a 1% transitory increase in the money

[14]The circles in Figure 3 show plots of annual time series of a short-term nominal interest rate (the commercial paper rate) against the ratio of M1 to nominal GDP for the United States for the period 1892–1997. The data are taken from the online Historical Statistics of the United States–Millennium Edition. The solid line with the cross ($\times$) symbols is the model's prediction calibrated at annual frequency with $\beta = 0.97$, $\delta = 0.1, \alpha = 0.3$, and $\sigma = 1.5$. The nominal interest rate in the model is defined as $\frac{1+\pi}{\beta}$. The scale parameter is set to $A = 0.08$. Notice the predicted money demand curve is close to that in Lagos and Wright (2005) and Wen (2009).

stock under the first policy regime (30), $\frac{M_t - \overline{M}}{\overline{M}} = \rho \frac{M_{t-1} - \overline{M}}{\overline{M}} + \varepsilon_t$, where $\rho = 0.9$, are shown in Figure 4 by the red symbols (circles), where the blue symbols (triangles) represent the control model.

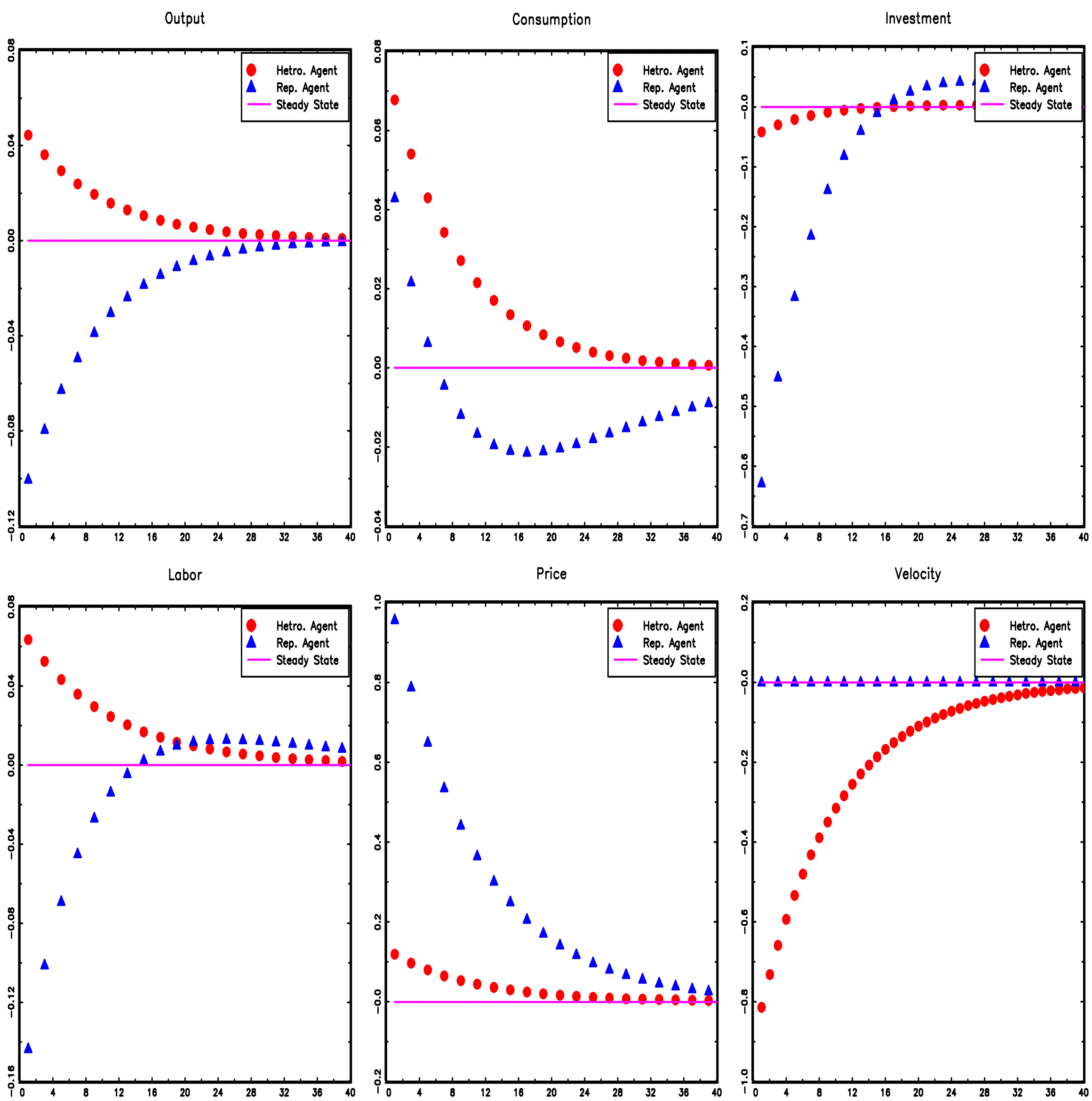

Figure 4. Responses to Money Shock.

The figure shows that under heterogeneity the economy responds to transitory monetary injections very differently from its representative-agent counterpart. In particular, money is expansionary for aggregate output, consumption, and labor in the heterogeneous-agent model, but contractionary in the control model.[15] The price level appears very "sticky" and velocity is highly countercyclical in the heterogeneous-agent model, whereas the price increases almost one-for-one

[15] Permanent increases in money, however, are no longer expansionary in the heterogeneous-agent model because of anticipated inflation.

with money supply and velocity remains constant in the control model. The "sticky" price behavior and countercyclical movements in velocity under transitory monetary shocks are consistent with the data (see, e.g., Alvarez, Atkeson and Edmond, 2008). The reason is as follows: Since only a small fraction of the agents face a binding CIA constraint, a monetary injection stimulates consumption significantly only for the liquidity-constrained agents; hence, aggregate price does not increase proportionately with money. On the other hand, because a large fraction of the population are not cash constrained, their individual monetary velocity decreases, reducing the aggregate velocity of money.[16]

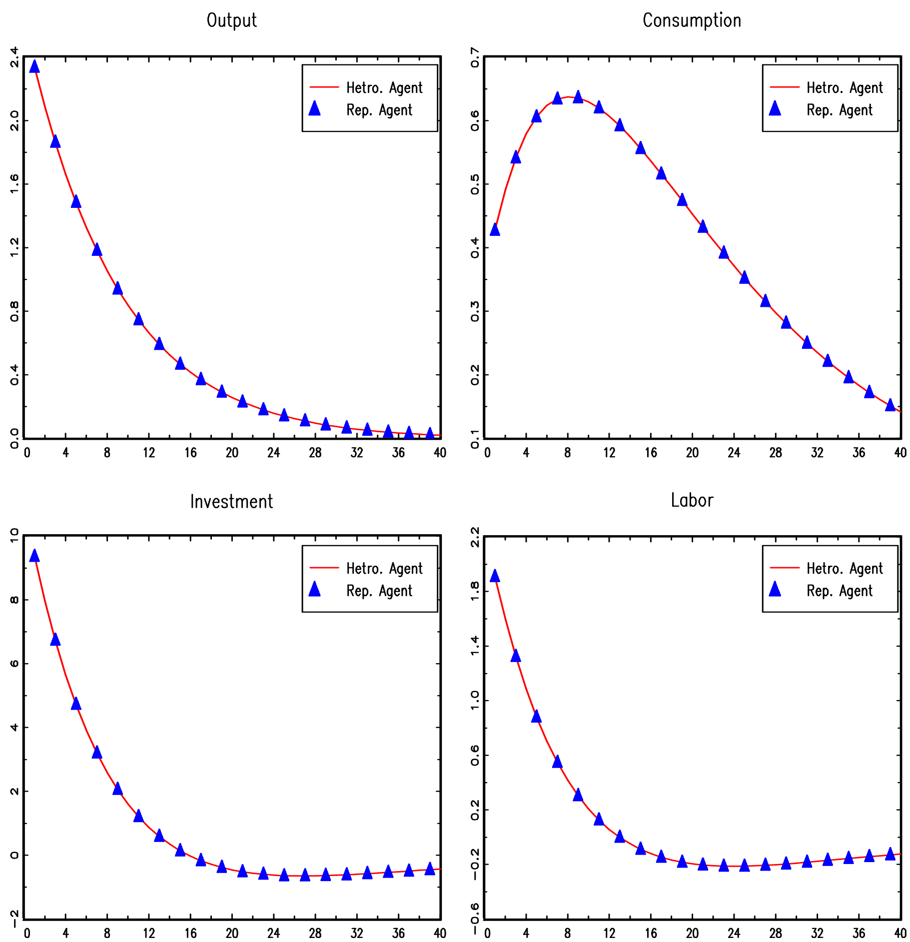

Figure 5. Responses to Technology Shock.

The situation changes dramatically under technology shocks. Figure 5 shows that the two models behave exactly the same under technology shocks. This is consistent with Krusell and Smith's (1998) finding that heterogeneity does not matter: The aggregate dynamics of heterogeneous agents behave just like those of a representative agent under technology shocks. Two crucial factors help

[16]Similar results are also obtained by Alvarez, Atkeson, and Edmond (2008) in a Baumol-Tobin model and Wen (2009) in a generalized Bewley (1980) model.

explain this result. First, the representative-agent model and the heterogeneous-agent model have the same aggregate saving rates in the steady state (i.e., the same capital-output ratio). In this regard, heterogeneity does not matter. Second, technology shocks have no effect on the cutoff value θ_t^*; hence, they do not alter the income distribution of the population along the dynamic path.[17] Consequently, the aggregate dynamics are the same in the two models. However, monetary shocks do affect the income distribution by impacting the cutoff variable. A temporary increase in money supply makes agents richer in cash balances and reduces the probability of a binding CIA constraint, thereby raising the value of θ_t^*.[18]

7 Conclusion

This paper provides an analytically tractable general-equilibrium model of heterogeneous money demand with the cash-in-advance constraints. The model makes dramatically different predictions about monetary business cycles and the welfare costs of inflation from the existing literature. Such findings are in contrast to those obtained by Krusell and Smith (1998), where they show that the representative-agent assumption provides a very good approximation for aggregate behaviors of heterogeneous agents. But my analysis shows that the results depend on the source of shocks and whether it can significantly affect the distribution of income. In both the Krusell-Smith model and my model, aggregate technology shocks do not change an individual's probability of having a binding borrowing constraint; hence, heterogeneity does not matter. However, monetary shocks in my model can significantly alter an individual's marginal propensity to save, thereby affecting the business cycle dynamics and policy implications by shifting the distribution of real balances.

Being able to obtain closed-form solutions for the distribution of individuals' money demand functions is an additional contribution of this paper. The analytical intractability of the original Lucas (1980) model has prevented its applicability in the literature and hence induced researchers to use representative versions of that model for policy analysis. Consequently, the literature has relied almost exclusively on the Baumol (1952) and Tobin (1956) framework to study the issue of heterogeneous money demand and its policy implications.[19] Hopefully, the model provided in this paper may serve as an alternative to the Baumol-Tobin model for policy and business-cycle analysis.

[17] The nominal wage remains constant under technology shocks in both models.

[18] In Krusell and Smith (1998), technology shocks do affect the income distribution (or the probability of a binding borrowing constraint) because aggregate shocks are correlated with idiosyncratic shocks by assumption, but this channel is not significant. In my model, the probability of a binding borrowing (CIA) constraint is significantly affected by monetary shocks, albeit not by technology shocks; hence, money injection can have a significant aggregate effect.

[19] For the monetary literature based on the Baumol-Tobin model, see Jovanovic (1982), Grossman and Weiss (1983), Rotemberg (1984), Romer (1986), and Chatterjee and Corbae (1992). For the more recent literature, see Alvarez, Atkeson and Kehoe (1999), Alvarez, Atkeson and Edmond (2008), Chiu (2007), and Khan and Thomas (2006), among others.

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