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Externalities, Endogenous Productivity, and Poverty Traps*

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Abstract

We present a version of the neoclassical model with an endogenous industry structure. We construct a distribution of firms' productivity that implies multiple steady-state equilibria even with an arbitrarily small degree of increasing returns to scale. While the most productive firms operate across all the steady states, in a poverty trap less productive firms operate as well. This results in lower average firm productivity and total factor productivity. The distributions of employment by firm size across steady states are consistent with the empirical observation that poor countries have a higher fraction of employment in small firms than rich countries. Differences in output and total factor productivity across steady states are increasing in the degree of returns to scale, the capital share, and the Frisch elasticity of labor supply.

JEL: L16, O11, O33, O40

Keywords: endogenous productivity, multiple equilibria, poverty traps

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1 Introduction

The role of an endogenous industry structure as a powerful amplifying mechanism is well established in the literature, building on the seminal work of Hopenhayn (1992). Endogeneity of the industry structure allows to obtain countercyclical markups and indeterminacy (Jaimovich, 2007). It can rationalize the business cycle characteristics of entry, profits, and markups, while accounting for the standard business cycle moments (Bilbiie, Ghironi, and Melitz, 2012). It provides an amplification mechanism for differences in prices of investment goods to translate into large differences in output across countries (Armenter and Lahiri, 2012). Finally, the endogenous response of the industry structure to various institutional and policy failures, such as barriers to entry, induces misallocation of inputs across firms (for a survey, see Hopenhayn, 2014).

We show that endogenizing the industry structure can lead to an extreme form of amplification. We introduce heterogeneous firms à la Hopenhayn (1992) in a standard neoclassical model. Many ex-ante identical potential firms obtain a random productivity draw upon payment of an entry cost. Only firms productive enough to pay an overhead labor cost choose to operate. We construct a distribution of firms' productivity, characterized by a high fraction of similarly unproductive firms, that implies multiple steady-state equilibria even with an arbitrarily small degree of increasing returns to scale.[1] Consider a steady state with a high productivity cutoff and a large capital stock: These imply that the wage is high, as is the operating cost. A high operating cost makes low productivity firms unprofitable, effectively cleansing the pool of firms. This justifies the cutoff being high in the first place. Since only high productivity firms are operating, TFP is high. Conversely, in a steady state where capital is low and lower productivity firms are operating, the wage is low and lower profits are sufficient to cover the operating cost. Low productivity firms sully the pool of producers, leading to lower TFP and capital. In a good equilibrium high productivity firms produce more than in a bad equilibrium, despite facing a higher wage and the same interest rate, because firms face a higher demand.

As in most models with heterogeneous firms, firms' sizes are proportional to their productivity. Assuming that the distributions by size in the data are

[1] Galí (1995) obtains multiple equilibria and poverty traps in a model where large increasing returns stem from endogenous markups. However, subsequent empirical evidence suggests that the degree of increasing returns is small.

the result of the same underlying distribution of productivity across firms, truncated at different levels, the data are supportive of the kind of distribution that generates multiple equilibria in the model. In fact, the key feature of the distribution of firms by size in poor countries is that it has a much higher share or employment in small firms than the distribution of firms in rich countries (see Tybout, 2000). We use a numerical example that captures this feature of the data to show that differences in TFP and output across steady states are increasing in the degree of returns to scale and in the share of capital in income.

In the benchmark model labor is supplied inelastically. Multiplicity of steady states with an arbitrarily small degree of increasing returns is maintained when the supply of labor is endogenized. Furthermore, differences in capital, labor, TFP, and output across steady states are increasing in the Frisch elasticity of labor supply.

Several models of poverty traps relying on increasing returns have been proposed in the literature, surveyed by Azariadis and Stachurski (2005, Section 5).[2] Previous studies of poverty trap models with endogenous TFP pointed to the failure of adopting the most productive technologies as the cause of low TFP and income in poor countries (Murphy, Shleifer, and Vishny, 1989; Ciccone and Matsuyama, 1996). In contrast, we focus on differences in the usage of the least productive technologies. There is evidence pointing to the fact that differences in TFP across economies are related to the lowest level of firms' productivity. Comin and Hobijn (2010) take a comprehensive look at the uses of various technologies as determinants of TFP and find that the key is not when new, better technologies are adopted, but when old, obsolete ones are relinquished. Also, the empirical evidence on the importance of international knowledge spillovers summarized in Klenow and Rodriguez-Clare (2005) suggests that all countries can easily access frontier technologies. Banerjee and Duflo (2005) cite the McKinsey Global Institute (2001) report on India, which finds that while larger production units (firms) use relatively new technologies, smaller (in home) production units have low productivity.

Models similar to the one we analyze, with constant returns to scale and a unique steady state, have been used in the misallocation literature to

[2]An earlier survey of the literature can be found in Azariadis (1996). Kraay and McKenzie (2014) provides a more recent survey and cast doubts on the empirical relevance of poverty traps models.

study the effects of cross-country differences in entry barriers. Barseghyan and DiCecio (2011) show that the observed differences in entry costs across countries generate sizeable differences in output and TFP. The key difference is that the distribution of productivity is calibrated to match the distribution of firms and employment by size in the U.S. and it does not feature a "mass point" of similar, low-productivity firms. Boedo Moscoso and Mukoyama (2012) show that entry regulations and firing costs have important effects on cross-country differences in TFP and output. Poschke (2010) argues that small differences in entry costs, by affecting firms' technology choices, can explain a substantial part of the TFP differences across similarly developed economies.

The rest of the paper is organized as follows. Section 2 presents the benchmark model with inelastic labor supply. Section 3 studies its steady state and dynamics properties. In Section 4 we present comparative statics of the benchmark model, using a calibrated numerical example. Section 5 discusses the implications of endogenizing the supply of labor. We conclude in Section 6.

2 The Model

Our model is a variant of the neoclassical growth model. The model departs from the standard framework by having a richer structure of the production side of the economy. We model firms following Lucas (1978), Jovanovic (1982), and Hopenhayn (1992). Firms are heterogeneous: each firm has monopoly power over the good it produces, and firms have different productivity levels. Two features of the production side of the economy are crucial for the results of the paper:

1. a sunk entry cost;
2. an operating cost: in addition to capital and labor used directly in production, firms pay for a fixed amount of overhead labor.

A part of the entry costs stems from satisfying different official regulatory requirements (see Djankov, La Porta, Lopez-de-Silanes, and Shleifer, 2002). In addition, in some countries, entry requires significant side payments to local officials.[3] Entry cost may also include expenses related to acquisition

[3] In the case of Peru, this is documented by De Soto (1989).

of firm-specific capital,[4] acquisition of appropriate technology,[5] and market research.

The operating cost typically refers to overhead labor and expenses that are lumpy in nature (e.g., renting a physical location). According to the findings of Domowitz, Hubbard, and Petersen (1988), in U.S. manufacturing plants, the overhead labor accounts for 31 percent of total labor. Ramey (1991) suggests that overhead labor is about 20 percent. The preferred estimate of overhead inputs in Basu (1996) is 28 percent.

We also assume that firms learn their productivity only after a sunk entry cost is paid. This assumption reflects very high uncertainty faced by entering firms and is a stylized fact documented, for example, by Klette and Kortum (2004).

2.1 Households

There is a continuum of households that supply a fixed amount of labor, consume, invest, and own all firms in the economy. The problem of the representative household is given by

$$\begin{gathered}\max \sum_{t=0}^{\infty} \beta^t U(C_t),\ \ \beta \in (0,1) \qquad (1)\\ \text{s.t. } C_t + I_t = r_t K_t + w_t + \Pi_t + T_t,\\ I_t = K_{t+1} - (1-\delta) K_t,\end{gathered}$$

where C_t denotes consumption, I_t is investment, K_t denotes the total household capital, r_t is the rental rate on capital, and w_t is the wage.[6] Π_t is the firms' profits, and T_t is a lump-sum transfer from the government; β and $\delta \in (0,1)$ are the discount rate and depreciation rate, respectively. We assume a constant elasticity of substitution utility function with elasticity $\sigma > 0$.

[4] Ramey and Shapiro (2001) show that in some instances the specificity of firm capital is so extreme that the sale price of such capital after a firm has been dissolved is only a small fraction of the original cost.

[5] See, for example, Atkeson and Kehoe (2005).

[6] We assume that the household inelastically supplies one unit of labor.

2.2 Firms

2.2.1 Final Good Producers

The final consumption good in this economy is produced by perfectly competitive firms, according to the following production function:

$$Y_t = \left[\int_0^{\mu_t} [y_t(i)]^{\frac{1}{\lambda}} \, di \right]^{\lambda},$$

where μ_t is the measure of intermediate goods produced in the economy, $y_t(i)$ is the quantity of the intermediate good i and λ is a constant that satisfies $\lambda \in (1, 1 + \gamma \min(\alpha, 1 - \alpha))$. The parameter λ determines the elasticity of substitution between any two intermediate good, $\varepsilon \equiv \lambda / (\lambda - 1)$. This parameter also determines the level of returns to scale at the aggregate level. Increasing returns, implied by $\lambda > 1$, arise from demand (pecuniary) externalities.[7]

Let $p_t(i)$ be the price of the i^{th} intermediate good relative to the final good. Then, the maximization problem of the final good producer can be written as

$$\Pi_t^{FF} = \max \left[\int_0^{\mu_t} [y_t(i)]^{\frac{1}{\lambda}} \, di \right]^{\lambda} - \int_0^{\mu_t} p_t(i) y_t(i) di, \tag{2}$$

and the first-order optimality condition implies that the demand function for the i^{th} intermediate good is given by

$$p_t(i) = \left[\frac{y_t(i)}{Y_t} \right]^{-\frac{\lambda - 1}{\lambda}}.$$

2.2.2 Intermediate Goods Producers

A firm in the intermediate goods sector lives for one period and is profit maximizing. All firms are ex-ante identical. There is a sunk entry cost, κ.[8]

[7] This is similar to increasing returns to variety (see, e.g., Kim, 2004). In models with identical producers of intermediate varieties increasing reutrns operate at the extensive margin. In our model increasing returns are at the intensive margin, i.e., they apply to the productivity of operating firms and not to the measure of operating firm.

[8] We assume that κ is denominated in consumption units and that all entry-cost payments are rebated to the households in a lump-sum fashion. Alternatively, one can model κ as a sunk investment, i.e., in units of capital. Such a formulation would not change any of our results, but it would make the exposition more cumbersome.

Once the entry cost is paid, a firm gains the ability to produce an intermediate good. The firm has monopoly power over the good it produces. Next, the firm draws a productivity parameter $A(j)$, where j is drawn from an i.i.d. uniform distribution over $[0,1]$. The production function for the good j is given by

$$y(j) = [A(j)]^{1-\gamma} \left[k(j)^{\alpha} n(j)^{1-\alpha}\right]^{\gamma},$$

where $k(j)$ and $n(j)$ denote capital and labor, respectively. The productivity parameter differs among the firms. A firm with a higher index has a higher productivity parameter, i.e., $A(j) > A(i)$ for $j > i$. In addition, function $A(j)$ is assumed to be continuous, differentiable almost everywhere, and $A(0) = 0$.[9] The parameter $\gamma \in (0, \lambda)$ determines the degree of returns to scale in variable inputs,[10] and it satisfies $\alpha\gamma < 1$. The parameter α is between zero and 1.

If a firm decides to produce, it must incur an operating cost in terms of wages paid to ϕ units of overhead labor. Consider the decision of a firm born in time t with a draw j. If it decides to produce, its profits are

$$\begin{aligned} &\pi_t^P(j) = \max_{k_t(j), n_t(j)} p_t(j)\, y_t(j) - r_t k_t(j) - w_t\left[n_t(j) + \phi\right] \\ &\text{s.t. } y_t(j) = [A(j)]^{1-\gamma}\left[k_t^{\alpha}(j) n_t^{1-\alpha}(j)\right]^{\gamma}, \; p_t(j) = \left[\frac{y_t(j)}{Y_t}\right]^{\frac{1-\lambda}{\lambda}}. \end{aligned} \tag{3}$$

The decision to produce or not depends on whether $\pi_t^P(j)$ is positive. Therefore, the j^{th} firm's profits, $\pi_t^F(j)$, are given by

$$\pi_t^F(j) = \max\{\pi_t^P(j), 0\}. \tag{4}$$

Free entry implies that, in equilibrium, firms' expected profits must be equal to the entry cost, κ:

$$\int_0^1 \pi_t^F(j)dj = \kappa. \tag{5}$$

2.2.3 Firms' average productivity

We derive the equilibrium relationship between the firms' average productivity and the operating cost. First, we determine the lowest productivity level necessary for a firm to decide to produce. The existence of economy-wide

[9] Alternatively, one could model firms as drawing productivity instead of an index $j \in [0,1]$. The function $A(j)$ is the inverse of the CDF of the distribution of firms' productivity.

[10] This is what Lucas (1978) calls managers' span of control.

competitive factor markets implies that in equilibrium, the gross profits, capital, and labor ratios of any two firms are equal to their (scaled) productivity ratio:

$$\frac{p_t(j)y_t(j)}{p_t(i)y_t(i)} = \frac{k_t(j)}{k_t(i)} = \frac{n_t(j)}{n_t(i)} = \frac{a(j)}{a(i)},\ \forall i, j, \tag{6}$$

where $a(j) \equiv A(j)^{\frac{1-\gamma}{\lambda-\gamma}}$. The first-order conditions of problem (3) imply that profits from producing are equal to the firm's share of the gross profits $\left(1 - \frac{\gamma}{\lambda}\right)$ minus the operating cost[11]:

$$\pi_t^P(j) = \left(1 - \frac{\gamma}{\lambda}\right) p_t(j)y_t(j) - \phi w_t.$$

Clearly $\pi_t^P(j)$ is increasing in j and, since $a(0) = 0$, there exists a cutoff firm, J_t, which is indifferent between producing or not:

$$\left(1 - \frac{\gamma}{\lambda}\right) p_t(J_t)y_t(J_t) = \phi w_t. \tag{7}$$

Firms with indices higher than J_t will produce, and those with lower indices will not. Thus, firms' zero profit condition in (5) can be written as

$$\kappa = \phi w_t \int_{J_t}^{1} \left[\frac{a(j)}{a(J_t)} - 1\right] dj. \tag{8}$$

The previous equation defines the cutoff J_t as a function of the operating cost ϕw_t. An increase in the cutoff J_t has two effects: Profits of every firm decline, and the measure of producing firms as a fraction of entering firms declines. Therefore, the right-hand side of (8) is decreasing in J_t and increasing in the fixed cost ϕw_t. Hence, the cutoff is increasing in the operating cost and average firm productivity, $\bar{a}\left(J_t\right) = \frac{\int_{J_t}^1 a(j)dj}{1-J_t}$, is an increasing function of the operating cost.

2.2.4 Measures of entering and of operating firms

Entry in this model refers to the number of firms that pay the entry cost, κ. The measure of entering firms differs from the measure of operating firms because only a fraction of entrants will have productivity high enough to

[11] Later on, with some abuse of terminology, we will refer to $(1 - \frac{\gamma}{\lambda})p_t(j)y_t(j)$ as firms' gross profits.

operate: the pool of producers consists only of firms which have an index higher than J_t. In particular, let ν_t denote the measure of entering firms and μ_t the measure of operating firms. Then

$$\mu_t = \nu_t \int_{J_t}^{1} dj. \tag{9}$$

2.3 Aggregate Output and TFP

Aggregate capital, labor, and output can be expressed as

$$K_t = \nu_t \int_{J_t}^{1} k_t(j)dj, \tag{10}$$

$$N_t = \nu_t \int_{J_t}^{1} \left[n_t(j) + \phi\right] dj, \tag{11}$$

$$Y_t = \left[\left(\mu_t \bar{a}\left(J_t\right)\right)^{(\lambda-\gamma)} u_t^{(1-\alpha)\gamma}\right] K_t^{\alpha\gamma} \left(N_t\right)^{(1-\alpha)\gamma}, \tag{12}$$

where u_t is the fraction of labor used in production. Finally, the rental rate on capital, the wage rate, and the equation determining the cutoff J_t can be written as

$$r_t = \alpha \frac{\gamma}{\lambda} \frac{Y_t}{K_t}, \tag{13}$$

$$w_t = (1-\alpha) \frac{\gamma}{\lambda} \frac{Y_t}{u_t N_t}, \tag{14}$$

$$\left(1 - \frac{\gamma}{\lambda}\right) \frac{a(J_t)}{\bar{a}\left(J_t\right)} \frac{Y_t}{(1-u_t)N_t} = w_t. \tag{15}$$

2.4 Closing the Model

The resource constraint is given by

$$C_t + K_{t+1} = Y_t + (1-\delta)K_t. \tag{16}$$

The only role the government has in the model is to collect the entry fees $\nu_t \kappa$ from firms and rebate them lump-sum to the households:

$$T_t = \nu_t \kappa. \tag{17}$$

Profits and the labor market clearing condition are

$$\Pi_t = \Pi_t^{FF}, \tag{18}$$

$$N_t = 1. \tag{19}$$

The definition of equilibrium is standard.

3 Steady States and Dynamics

In this section we analyze the existence and stability of the steady states. The main finding is that, given an arbitrarily small degree of returns scale above constant returns, one can construct a productivity distribution such that the model has multiple stable steady states.

3.1 Steady States

First, note that the measure of firms is proportional to the total amount of labor used to cover the fixed cost:

$$\mu_t = \frac{1-u_t}{\phi} N_t.$$

Therefore, aggregate output is given by

$$Y_t = TFP_t K_t^{\alpha\gamma} N_t^{\lambda-\alpha\gamma}, \tag{20}$$

where total factor productivity is

$$TFP_t = \phi^{\gamma-\lambda} \left[\bar{a}\left(J_t\right)\right]^{\lambda-\gamma} \left(1-u_t\right)^{\lambda-\gamma} u_t^{(1-\alpha)\gamma}. \tag{21}$$

There are two components of TFP: firms' average productivity $\left[\bar{a}\left(J_t\right)\right]^{\lambda-\gamma}$ and the term $u_t^{(1-\alpha)\gamma}\left(1-u_t\right)^{\lambda-\gamma}$, which we call the *labor allocation* component.[12] Firms' average productivity is increasing in J_t. The labor allocation component is a function of J_t as well, though not necessarily monotonic. The effect of J_t on average productivity dominates and TFP_t is increasing in J_t.

The following proposition allows us to present the model economy in a more familiar, neoclassical framework.

[12] The labor allocation component is part of TFP because, given aggregate capital and labor, the number of firms—and hence the labor allocation component—affects aggregate output. Also, the model's TFP is different from the Solow residual, as the later is constructed using factor shares that sum to one.

Proposition 1 *The aggregate production function in (12) and total factor productivity (21) are increasing in the cutoff J. The cutoff J, the wage w, and aggregate output Y are all increasing functions of capital K. The rate of return on capital $R \equiv (r + 1 - \delta)$ is a function of K.*

Proof. Equations (14) and (15) imply that the fraction of labor used in production u is a function only of the cutoff, J :

$$u = \frac{\bar{a}\,(J)}{\bar{a}\,(J) + \frac{\lambda-\gamma}{(1-\alpha)\gamma}a(J)}. \tag{22}$$

Substituting this expression of u into equation (21), we obtain:

$$TFP\,(J) = \phi^{\gamma-\lambda}\left[\frac{\frac{\lambda-\gamma}{(1-\alpha)\gamma}a(J)}{\bar{a}\,(J) + \frac{\lambda-\gamma}{(1-\alpha)\gamma}a(J)}\right]^{\lambda-\gamma} \times$$

$$\times\left[\frac{\bar{a}\,(J)}{\bar{a}\,(J) + \frac{\lambda-\gamma}{(1-\alpha)\gamma}a(J)}\right]^{(1-\alpha)\gamma} [\bar{a}\,(J)]^{\lambda-\gamma} \tag{23}$$

Differentiating the previous expression,

$$\text{signum}\,(TFP_J) = \text{signum}\left[\begin{array}{c} \frac{(\lambda-\gamma)(1-\alpha)\gamma}{(1-\alpha)\gamma\bar{a}+(\lambda-\gamma)a}\frac{a_J}{a}\,(\bar{a}-a) + \\ \bar{a}_J\left(\frac{1}{\bar{a}} - \frac{1}{\bar{a}+\frac{\lambda-\gamma}{(1-\alpha)\gamma}a}\right) \end{array}\right], \tag{24}$$

where $TFP_J = \frac{\partial TFP(J)}{\partial J}$, $a_J = \frac{\partial a(J)}{\partial J}$, $\bar{a}_J = \frac{\partial \bar{a}(J)}{\partial J}$, $a = a\,(J)$ and $\bar{a} = \bar{a}\,(J)$. The terms in parenthesis in (24) are positive and they are multiplied by positive terms. Hence, $TFP_J > 0$. Using the firms' first-order condition in (14) and the zero profit condition in (8) we get that the following relation between the cutoff, J, and capital, K :

$$\kappa = (1-\alpha)\frac{\gamma}{\lambda}\left[\frac{\lambda-\gamma}{(1-\alpha)\gamma}\right]^{\lambda-\gamma}\left[\frac{\bar{a}}{\bar{a}+\frac{\lambda-\gamma}{(1-\alpha)\gamma}a}\right]^{(1-\alpha)\gamma+(\lambda-\gamma)-1} a^{\lambda-\gamma}\frac{\bar{a}-a}{a}\,(1-J)\,K^{\alpha\gamma}.$$

For a given K, the right-hand side of this equation varies with J from $+\infty$ to zero (as J ranges from 0 to 1). Moreover, one can easily show that the right-hand side is decreasing in J. Thus, there exists a unique J which solves the equation. In addition, it is increasing in K. Because J is increasing in

K, so are output, Y, and the wage rate, w. In addition, since, for a given K, output Y is uniquely determined, so is the rate of return on capital, i.e., R is a function of K. ■

The proposition above implies that the dynamics of the economy can be characterized by the following system of difference equations,

$$\begin{aligned} \left(\frac{c_{t+1}}{c_t}\right)^{1/\sigma} &= \beta R(K_{t+1}), \\ C_t + K_{t+1} &= Y(K_t) + (1-\delta)K_t, \end{aligned} \tag{25}$$

plus a transversality condition. We now turn to the existence and multiplicity of steady states.

Proposition 2 *For any $\lambda > 1$, there exists a distribution of productivities, $a(j)$, such that the system (25) has multiple steady-state equilibria.*

Proof. This proof is by construction, i.e., we construct a function $a(j)$. We use equations (13) and (20) to express K as a function of r and J. By substituting this expression of K into equation (23) and by using equation (14), we get

$$\kappa = \zeta\Phi(J) \tag{26}$$

where

$$\Phi(J) \equiv \left[\frac{\bar{a}(J)}{\bar{a}(J) + \frac{\lambda-\gamma}{(1-\alpha)\gamma}a(J)}\right]^{\frac{\lambda-1}{1-\alpha\gamma}} a(J)^{\frac{\lambda-\gamma}{1-\alpha\gamma}} \int_J^1 \left[\frac{a(j)}{a(J)} - 1\right] dj, \tag{27}$$

and ζ is a constant:

$$\zeta = \phi^{\frac{\gamma-\lambda}{1-\alpha\gamma}-1}\left(1-\frac{\gamma}{\lambda}\right)\left(\frac{\alpha\gamma}{\lambda}\right)^{\frac{\alpha\gamma}{1-\alpha\gamma}}\left[\frac{\lambda-\gamma}{(1-\alpha)\gamma}\right]^{\frac{\lambda-\gamma}{1-\alpha\gamma}-1} r^{\frac{\alpha\gamma}{\alpha\gamma-1}}. \tag{28}$$

Since $\Phi(J)$ is continuous, $\lim_{J\to 0^+}\Phi(J) = +\infty$, and $\lim_{J\to 1^-}\Phi(J) = 0$,[13] by the intermediate value theorem there always exists a J^* that satisfies equation (26).

[13] Notice that $\int_J^1 \left[\frac{a(j)}{a(J)} - 1\right] dj = \frac{1-J}{a(J)}\left[\bar{a}(J) - a(J)\right]$. Hence,

$$\Phi(J) = a(J)^{\frac{\lambda-\gamma}{1-\alpha\gamma}-1}\left[\frac{\bar{a}(J)}{\bar{a}(J) + \frac{\lambda-\gamma}{(1-\alpha)\gamma}a(J)}\right]^{\frac{\lambda-1}{1-\alpha\gamma}}\left[\bar{a}(J) - a(J)\right](1-J),$$

and the limits immediately follow.

We have to show that for any J^* satisfying equation (26) there exists a pair (c^*, K^*), both positive, such that $R(K^*) = 1/\beta$ and $c^* = Y(K^*) - \delta K^*$. This is an immediate consequence of Proposition 1.

If there is more than one J^* satisfying equation (26), then the model has multiple steady states. For given parameters λ, γ, and α, the shape of the function $\Phi(J)$ is entirely determined by the shape of the function $a(j)$: If $a(j)$ is such that $\Phi_J > 0$ then (26) can have multiple solutions. To conclude the proof, we construct a function $a(j)$ such that $\Phi_J > 0$. Compute Φ_J:

$$\begin{aligned}\Phi_J &= \frac{\lambda-1}{1-\alpha\gamma}\left[\frac{\bar{a}_J}{\bar{a}} - \frac{\bar{a}_J + \frac{\lambda-\gamma}{(1-\alpha)\gamma}a_J}{\bar{a} + \frac{\lambda-\gamma}{(1-\alpha)\gamma}a}\right]\Phi + \\ &\quad + \left[\frac{\lambda-\gamma}{1-\alpha\gamma} - \frac{\bar{a}}{\bar{a}-a}\right]\frac{a_J}{a}\Phi.\end{aligned}$$

Consider a function $b(\cdot)$ constant on $\left[\underline{J}, \bar{J}\right]$ for some $0 < \underline{J} < \bar{J} < 1$, strictly increasing elsewhere on $[0,1]$, and such that $b(0) = 0$, $b(1) < \infty$. Let $I(\cdot)$ be any increasing function with a continuous, bounded derivative on $[0,1]$, e.g., the identity function. Finally, let

$$a(j) = (1-\varepsilon)b(j) + \varepsilon I(j).$$

It follows that $a(J)$ is differentiable with derivative $a_J \leq \varepsilon M$ for $j \in \left[\underline{J}, \bar{J}\right]$, where $M = \max_{j\in\left[\underline{J},\bar{J}\right]} I_J(j)$ is finite (if $I(\cdot)$ is the identity function, then $a_J = \varepsilon$).

Next, note that $\bar{a}_J = (\bar{a} - a) / (1 - J)$ and hence

$$\lim_{\varepsilon\to 0}\bar{a}_J = \frac{\bar{b}-b}{1-J} > 0,$$
$$\lim_{\varepsilon\to 0}\bar{a} = \bar{b} > b = \lim_{\varepsilon\to 0} a.$$

It follows that

$$\lim_{\varepsilon\to 0}\Phi_J = \frac{\lambda-1}{1-\alpha\gamma}\left[\frac{\bar{b}_J}{\bar{b}} - \frac{\bar{b}_J}{\bar{b} + \frac{\lambda-\gamma}{(1-\alpha)\gamma}b}\right]\Phi > 0.$$

Hence there exists an $\varepsilon > 0$ for which Φ_J is positive at some $\tilde{J}$. For $\tilde{J}$ to be an equilibrium, i.e., $\kappa = \zeta\Phi(\widetilde{J})$, it is sufficient to scale the function $a\,(j)$ appropriately, by multiplying it by $(\kappa/\zeta)^{\frac{1-\alpha\gamma}{\lambda-\gamma}}$. There are at least two more

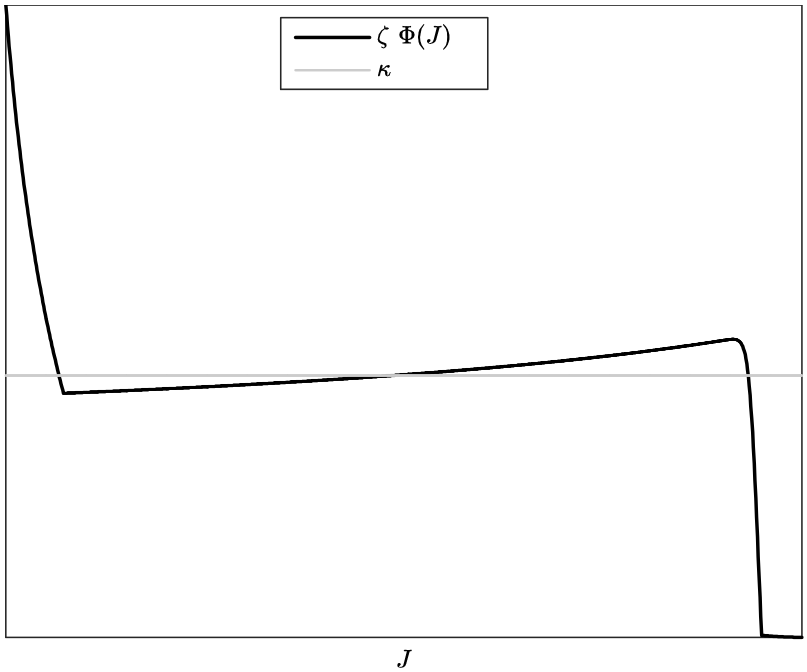

Figure 1: A graphical depiction of equation (26).

values of $J \in (0,1)$ for which the equation $\kappa = \zeta\Phi(J)$ is satisfied. Indeed, recall that $\lim_{J\to 0^+}\Phi(J) = +\infty$, and $\lim_{J\to 1^-}\Phi(J) = 0$. Since $\Phi_J(\widetilde{J}) > 0$, by the intermediate value theorem that there exists $\tilde{J}_L < \tilde{J}$ such that $\zeta\Phi(\tilde{J}_L) = \kappa$ and $\tilde{J}^H > \tilde{J}$ such that $\zeta\Phi(\tilde{J}^H) = \kappa$. ■

To have multiple solutions, it is necessary for the function Φ to be non-monotone (see Figure 1). Note that equation (26) implies that if $\Phi(J)$ is increasing, so is $r(K)$. The necessary condition for the existence of multiple steady states is that for some values of K the return on capital must be increasing. The properties of the function Φ mimic those of firms' expected profits, i.e., the right-hand side of the zero-profit condition (8). A necessary condition for existence of multiple steady state is that firms' expected profits are increasing in J. An increase in the cutoff J has two opposite effects. On one hand the wage rate increases, increasing expected profits. On the other hand, a higher J implies a lower value of the integral on the right-hand side of (8). For expected profits to rise with J, the increase in the wage has to dominate the fall in the value of the integral. A sufficient condition for this is that $\partial \bar{a}(J)/\partial J \gg \partial a(J)/\partial J$: A relatively high derivative of average productivity guarantees a strong positive effect on TFP and the wage rate, while a relatively low derivative of the marginal firm's productivity, $a(J)$, implies a mild negative response of the integral term. A function $a(j)$ which is sufficiently flat on some interval and increases rapidly for higher values of j has this property.

Given Propositions 1 and 2, the "high-J" economy has higher capital stock, higher output, higher TFP, and higher firms average productivity.

3.2 Dynamics

The following proposition characterizes the behavior of the economy around the steady state(s).[14]

Proposition 3 *Steady states with an odd index are saddles. Steady states with an even index can be classified as follows:*

1. *source, if* $Y' - \delta > \frac{\sigma C R'}{R}$ *and* $\left[(Y' - \delta) - \frac{\sigma C R'}{R}\right]^2 > 4\frac{\sigma C R'}{R}$;
2. *unstable spiral, if* $Y' - \delta > \frac{\sigma C R'}{R}$ *and* $\left[(Y' - \delta) - \frac{\sigma C R'}{R}\right]^2 < 4\frac{\sigma C R'}{R}$;
3. *sink, if* $Y' - \delta < \frac{\sigma C R'}{R}$ *and* $\left[(Y' - \delta) - \frac{\sigma C R'}{R}\right]^2 > 4\frac{\sigma C R'}{R}$;
4. *stable spiral, if* $Y' - \delta < \frac{\sigma C R'}{R}$ *and* $\left[(Y' - \delta) - \frac{\sigma C R'}{R}\right]^2 < 4\frac{\sigma C R'}{R}$.

Proof. Linearizing (25) about a steady state:

$$\begin{bmatrix} \hat{K}_{t+1} \\ \hat{C}_{t+1} \end{bmatrix} = \begin{bmatrix} Y' + 1 - \delta & -1 \\ \frac{\sigma R'}{R} C (Y' + 1 - \delta) & 1 - \frac{\sigma R'}{R} C \end{bmatrix} \begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix}$$

The eigenvalues of the transition matrix are given by:

$$\xi_{1,2} = 1 + \frac{(Y' - \delta) - \frac{\sigma C R'}{R} \pm \sqrt{\left[(Y' - \delta) - \frac{\sigma C R'}{R}\right]^2 - 4\frac{\sigma C R'}{R}}}{2}.$$

If $R' < 0$ (odd steady states) both eigenvalues are real and $\xi_1 < 1 < \xi_2$ (saddles). If $R' > 0$ (even steady states) there are four possible cases:

[14] An analysis of the global dynamics of our model is beyond the scope of this paper, and we refer the reader to Galí (1995) and Slobodyan (2005).

1. $Y' - \delta > \frac{\sigma CR'}{R} > 0 \wedge \left[(Y' - \delta) - \frac{\sigma CR'}{R}\right]^2 > 4\frac{\sigma CR'}{R} \Rightarrow \xi_{1,2} \in \mathbb{R}, \left\|\xi_{1,2}\right\| > 1$ (sources);

2. $Y' - \delta > \frac{\sigma CR'}{R} > 0 \wedge \left[(Y' - \delta) - \frac{\sigma CR'}{R}\right]^2 < 4\frac{\sigma CR'}{R} \Rightarrow \xi_{1,2} \in \mathbb{C}, \left\|\xi_{1,2}\right\| > 1$ (unstable spirals);

3. $\frac{\sigma CR'}{R} > Y' - \delta > 0 \wedge \left[(Y' - \delta) - \frac{\sigma CR'}{R}\right]^2 > 4\frac{\sigma CR'}{R} \Rightarrow \xi_{1,2} \in \mathbb{R}, \left\|\xi_{1,2}\right\| < 1$ (sinks);

4. $\frac{\sigma CR'}{R} > Y' - \delta > 0 \wedge \left[(Y' - \delta) - \frac{\sigma CR'}{R}\right]^2 < 4\frac{\sigma CR'}{R} \Rightarrow \xi_{1,2} \in \mathbb{C}, \left\|\xi_{1,2}\right\| < 1$ (stable spirals).

■

For the parameter values we consider in the rest of the paper, we obtain three steady states, with the odd steady state unstable (cases 1 and 2 in Proposition 3). In comparing output and TFP across steady states we will focus on the two stable steady states.

4 Properties of the Model

In this section we discuss some quantitative properties of our model by computing differences in output and TFP in a calibrated version of the model. Also, we perform sensitivity analysis on the degree of increasing returns to scale and capital share parameters. We conclude that the differences between high and low steady states are sizable.

4.1 Calibration

Our model contains seven parameters (β, δ, λ, γ, α, ϕ, κ), plus any additional parameters determining the function $a(j)$. The model's implications are robust to the choice of β and δ for the commonly used values of $\beta \in (0.94, 0.99)$ and $\delta \in (0.08, 0.12)$. Therefore, we set $\beta = 0.95$ and $\delta = 0.10$. The parameters λ, γ, and α deserve more consideration.

The first parameter, λ, governs the degree of increasing returns to scale in the economy. There has been a large debate in the recent literature on the magnitude of increasing returns in the economy. While earlier researchers (most notably, Hall, 1988) suggested that there are large increasing returns to scale in the economy, subsequent work has shown that the returns to scale can be best described as constant or at most moderately increasing. The latest estimates of λ are probably those constructed by Laitner and Stolyarov (2004). Their preferred point estimate is $\lambda = 1.1$, with confidence interval $(1.03, 1.2)$. These figures are close to the estimates of Bartelsman, Caballero, and Lyons (1994), Burnside (1996), Burnside, Eichenbaum, and Rebelo (1995), Basu (1996), Basu and Fernald (1997), and Harrison (2003). Hence, we calibrate our model with $\lambda = 1.1$.

The next parameter, γ, represents the share of output that goes to capital and labor used directly in production, for a given value of λ. Note that in the model there is a difference between aggregate returns to scale and firm-level returns to scale. While at the aggregate level there are increasing returns to scale, at the firm level, as long as $\gamma < 1$, the returns to scale in *variable* inputs are decreasing. In our model, heterogeneous productivity leads to a heterogeneous degree of returns to scale in *all* inputs. For high productivity firms, the decreasing returns to scale in variable inputs dominate the increasing returns to scale effect of the fixed cost; for low productivity firms, it is the opposite. These observations are broadly consistent with empirical findings of Basu (1996), and Basu and Fernald (1997). As a benchmark, we consider $\gamma = 0.85\lambda$, which is the preferred value of Atkeson and Kehoe (2005) and is very close to the estimated value of 0.84 in Basu (1996).

The choice of the next parameter, α, depends on the interpretation of entrepreneurs' income share, $(1 - \gamma/\lambda)$. If part of it is considered capital income, then the share of output that remunerates variable capital, rK/Y, is less than the overall share. A commonly used rule is to apportion to capital a fraction α of the entrepreneurs' income share. Summing up, the capital share of output is $[rK/Y + \alpha(1 - \gamma/\lambda)] = \alpha$. We set $rK/Y = 0.4$, which implies $\alpha = 0.47$.

We have shown that for some functions $a(j)$ there will be multiple stable steady states. The key property of the function $a(j)$ that generates multiplicity of equilibria is that $\bar{a}_J$ strongly dominates a_J for some J.[15] A function that has this property is one that is sufficiently flat on some interval (J_1, J_2). The larger this interval is, the farther apart the stable steady states are from each other. In terms of firms' productivity distribution, this translates into the lower steady state having a large fraction of firms with nearly the same

[15] See proof of Propositon 2.

J_1	J_2	b	M_1	M_2	κ
$1e-6$	0.97	1.65	282.1	105.8	0.50

Table 1: Parameter Values: function $a(j)$ and fixed cost

low productivity. Hence, we parameterize the function $a(j)$ as follows:

$$a(j) = \begin{cases} j/J_1, & \text{if } j \leq J_1; \\ 1 + b\left(\frac{j-J_1}{J_2-J_1}\right)^{M_1}, & \text{if } J_1 < j \leq J_2; \\ (1+b)\left(\frac{j}{J_2}\right)^{M_2}, & \text{if } j > J_2. \end{cases} \tag{29}$$

We normalize ϕ to 1,[16] and we choose κ and the five parameters pinning down the productivity distribution (J_1, J_2, M_1, M_2, b) so that the distribution of firms by size implied by our model in the two stable steady state is as close as possible to the distributions of firms by size in the average Least Developed Country (LDC) and in the U.S. (see Tybout, 2000, Table 1). Notice how in the average LDC the distribution of firms by size is characterized by a much higher share of small firms than in the U.S.

Figure 2 portrays the distributions of firms by size for the U.S. and the average LDC (right column), together with the distributions for the high and low steady states of our calibrated model (left column).

Figure 3 reports the function $a(j)$, which minimizes the distance between the model distributions and their empirical counterparts.

For this calibration, TFP and output differ across steady states by a factor of 1.1 and 1.21, respectively.

4.2 Comparative Statics

In this section we conduct comparative statics by analyzing how varying the degree of increasing returns to scale and the capital share maps into differences across the high and the low steady states of our model.

The limiting case of λ equal to 1 has the least favorable implications for the existence of multiple steady states, because the model essentially collapses to the standard neoclassical model. It is important to see how large the steady state differences can be for λ arbitrarily close to 1. The condition

[16]Notice that for our results only $\kappa/\phi^{\frac{\lambda-\gamma}{1-\alpha\gamma}-1}$ matters: see equations (26) and (28).

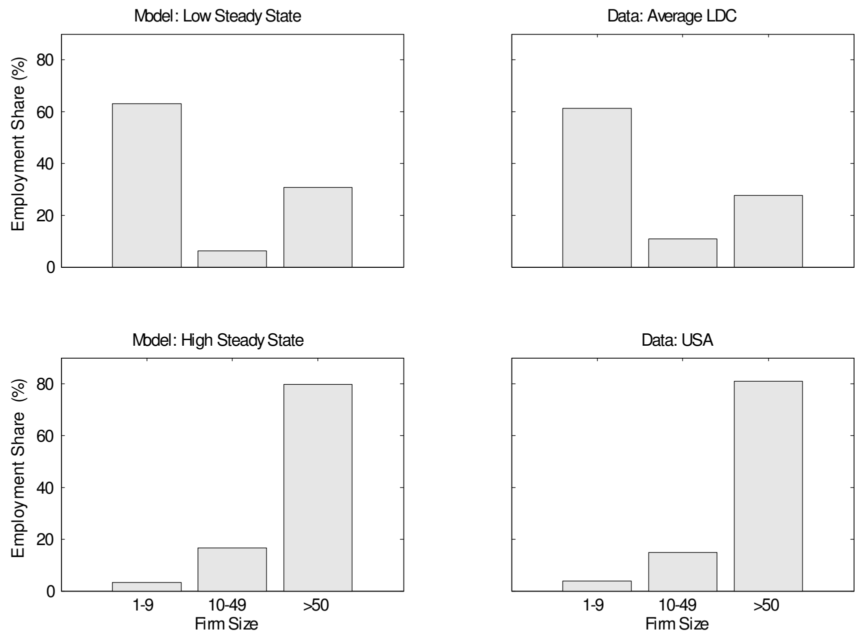

Figure 2: Employment Shares: model versus data.

for the existence of multiple steady states translates to $a(J)$ being flat over some interval. In this case, the extremes of this interval correspond to the two steady-state values of J. This implies that the ratios of total factor productivity, capital, and output levels in the two stable steady states are bounded:

$$\frac{TFP^H}{TFP^L} \leq \left(\frac{1 - rK/Y}{\gamma - rK/Y}\right)^{1-rK/Y}, \tag{30}$$

$$\frac{K^H}{K^L} \leq \frac{1 - rK/Y}{\gamma - rK/Y}, \tag{31}$$

$$\frac{Y^H}{Y^L} \leq \frac{1 - rK/Y}{\gamma - rK/Y}. \tag{32}$$

When our economy approaches constant returns to scale, the endogenous TFP mechanism alone is quite powerful and it can generate differences in TFP and output across steady states of up to 28 and 50 percent, respectively, for low values of γ (see Table 2).

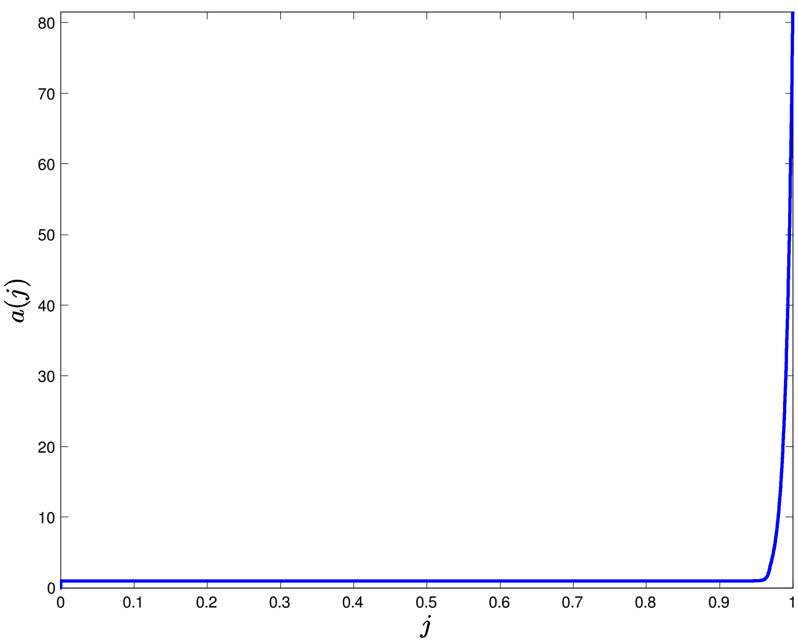

Figure 3: Calibrated function $a\,(j)$.

γ	TFP	Output	rK/Y	TFP	Output
0.8	1.28	1.50	0.36	1.19	1.31
0.85	1.19	1.33	0.4	1.19	1.38
0.9	1.12	1.20	0.5	1.20	1.50
0.95	1.05	1.09	0.6	1.11	1.75

Table 2: Bounds on relative TFP and output for $\lambda \to 1$.

In the studies of the long-run behavior of an economy, using the proper measure of capital share of output is of crucial importance. For example, for the unified theory of Parente and Prescott (2005) to be successful, the capital share of output should be between 0.55 and 0.65. The magnitude of this share depends on the definition of investment (capital). In the context of this paper it is proper to define investment as "any allocation of resources that is designed to increase future productivity" (see Parente and Prescott, 2000). That is, investment should include maintenance and repair, research and development, software, investment in organizational capital, and investment in human capital. Parente and Prescott (2000) find that including these items in investment implies that the capital share of output is larger than 1/2 and can reach as high as 2/3.[17] The capital share is important for two reasons. First, there is a standard neoclassical effect: The higher the capital share is, the higher the effect of TFP is on the economy. For two economies differing only in their TFP, the steady state capital ratio relates to the TFP ratio as follows: $\left(K^H/K^L\right) = \left(TFP^H/TFP^L\right)^{\frac{1}{1-\alpha\gamma}}$. The higher the share of capital is, the higher the difference in steady state capital is between the two economies. Second, the capital share directly impacts TFP, because it enters into the definition of TFP in (21) and into the definition of the function $\Phi(J)$ in (27). Because of the highly non-linear nature of TFP and Φ as functions of the cutoff J, it is not possible to derive analytically the effect of an increase in the capital share on the resulting TFP differences across the steady states. However, when λ tends to 1 the theoretical upper bound on these differences gets larger as the capital share grows (see equation (31) above). The increase in the capital share of output increases the TFP differences. Combined with

[17] For details and references see the original paper. A large portion of the unmeasured capital is organization capital. Findings of Atkeson and Kehoe (2005) imply that the value of organizational capital in the US manufacturing sector is *larger* than the value of physical capital.

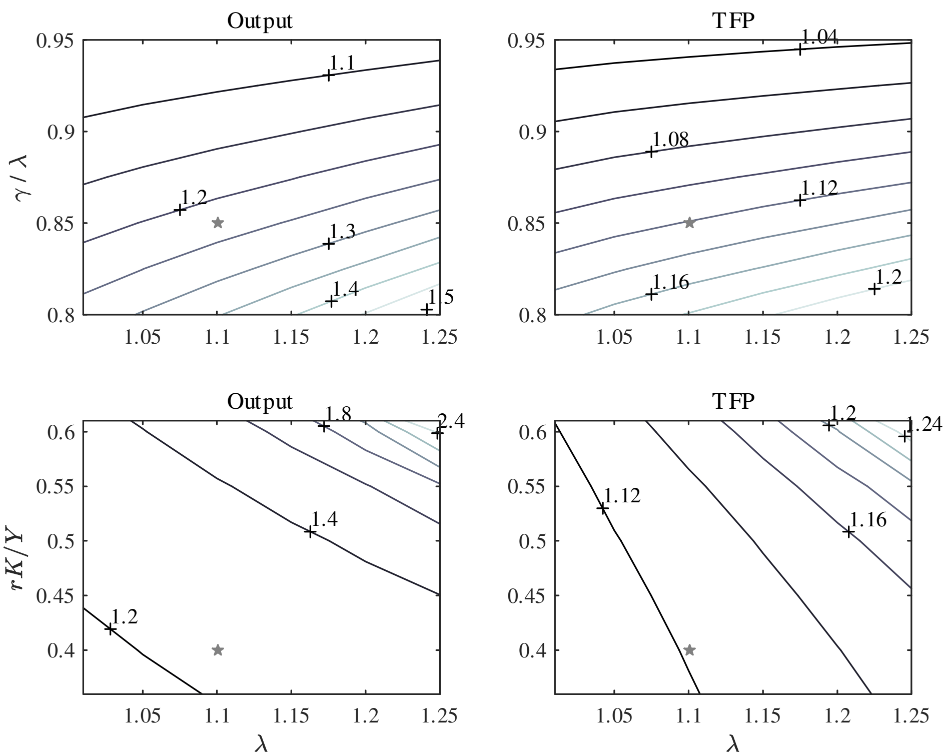

Figure 4: Relative (high/low steady state) output and TFP for different values of γ/λ (top row), rK/Y (bottom row), and λ. Stars correspond to the benchmark paramter values (see Section 4.1).

the "neoclassical effect" described above, this leads to even larger differences in output and in capital across the steady states.

Figure 4 shows the effect of varying λ, γ, and rK/Y on the differences in output and TFP between the high and the low steady state. The higher the degrees of returns to scale at the aggregate level (λ) and the capital share (rK/Y), the larger the differences. The lower the degree of returns to scale at the firm level (γ), the larger are output and TFP in the high steady state relative to the low steady state.

5 Endogenous Labor Supply

Consider a representative agent who values consumption and leisure according to the following GHH preferences (see Greenwood, Herkowitz, and Huffman, 1988):

$$\sum_{t=0}^{\infty} \beta^t \frac{\sigma}{\sigma-1} \left[C_t - \frac{\chi}{1+\eta} \left(N_t^S \right)^{1+\eta} \right]^{\frac{\sigma-1}{\sigma}}, \tag{33}$$

where N_t^S is the household supply of labor, η is the inverse of the Frisch elasticity of labor supply and χ is a parameter determining the steady-state value of labor supply. The inelastic labor model discussed above is a special case, obtained with $\eta \to \infty$. The household's first order conditions are as follows:

$$\left[\frac{C_{t+1} - \frac{\chi}{1+\eta} \left(N_{t+1}^S \right)^{1+\eta}}{C_t - \frac{\chi}{1+\eta} \left(N_t^S \right)^{1+\eta}} \right]^{1/\sigma} = \beta \left(r_{t+1} + 1 - \delta \right), \tag{34}$$

$$\chi \left(N_t^S \right)^{1+\eta} = w_t. \tag{35}$$

The labor market clearing condition equates labor demand and supply, i.e., $N_t = N_t^S$.

Proposition 4 *For any $\lambda > 1$ there exists a distribution of productivities, $a(j)$, and a value of the entry cost, κ, such that the model with endogenous labor supply has multiple steady-state equilibria.*

Proof. From (13) and the equation for aggregate output, $Y = TFP \left(K^{\alpha\gamma} N^{\lambda-\alpha\gamma} \right)$:

$$Y = TFP \left(TFP \frac{\alpha\gamma}{r\lambda} N^{\lambda-\alpha\gamma} \right)^{\frac{\alpha\gamma}{1-\alpha\gamma}} N^{\lambda-\alpha\gamma}.$$

Combining labor demand (14) and labor supply (35) and substituting from Y:

$$N = \left[\frac{(1-\alpha)}{\chi} \frac{\gamma}{\lambda} \left(\frac{\alpha\gamma}{r\lambda} \right)^{\frac{\alpha\gamma}{1-\alpha\gamma}} TFP^{\frac{1}{1-\alpha\gamma}} \frac{1}{u} \right]^{\frac{1-\alpha\gamma}{\eta(1-\alpha\gamma)-(\lambda-1)}}.$$

Substituting in (8) for w from (35) and for N from the previous equation:

$$\kappa = \phi\chi\left\{\frac{(1-\alpha)}{\chi}\frac{\gamma}{\lambda}\left(\frac{\alpha\gamma}{r\lambda}\right)^{\frac{\alpha\gamma}{1-\alpha\gamma}}\left[\frac{\lambda-\gamma}{(1-\alpha)\gamma}\frac{a(J)}{\phi}\right]^{\frac{\lambda-\gamma}{1-\alpha\gamma}}u^{\frac{\lambda-1}{1-\alpha\gamma}}\right\}^{\frac{\eta(1-\alpha\gamma)}{\eta(1-\alpha\gamma)-(\lambda-1)}}\times$$
$$\times\int_J^1\left[\frac{a(j)}{a(J)}-1\right]dj.$$

where u is given by (22). Hence the cutoff equation can be written as $\kappa = \hat{\zeta}\hat{\Phi}(J)$, where

$$\hat{\Phi}(J) = \left\{a(J)^{\frac{\lambda-\gamma}{1-\alpha\gamma}}\left[\frac{\bar{a}(J)}{\bar{a}(J)+\frac{\lambda-\gamma}{(1-\alpha)\gamma}a(J)}\right]^{\frac{\lambda-1}{1-\alpha\gamma}}\right\}^{\frac{\eta(1-\alpha\gamma)}{\eta(1-\alpha\gamma)-(\lambda-1)}}\int_J^1\left[\frac{a(j)}{a(J)}-1\right]dj,$$
$$\hat{\zeta} = \phi\chi\left\{\frac{(1-\alpha)}{\chi}\frac{\gamma}{\lambda}\left(\frac{\alpha\gamma}{r\lambda}\right)^{\frac{\alpha\gamma}{1-\alpha\gamma}}\left[\frac{\lambda-\gamma}{(1-\alpha)\gamma\phi}\right]^{\frac{\lambda-\gamma}{1-\alpha\gamma}}\right\}^{\frac{\eta(1-\alpha\gamma)}{\eta(1-\alpha\gamma)-(\lambda-1)}}.$$

Notice that $\lim_{\eta\to\infty}\frac{\eta(1-\alpha\gamma)}{\eta(1-\alpha\gamma)-(\lambda-1)} = 1$, implies $\lim_{\eta\to\infty}\hat{\Phi}(J) = \Phi(J)$ and $\lim_{\eta\to\infty}\hat{\zeta} = \zeta$. Also, for $\lambda \to 1$ the cutoff equations with endogenous labor supply and inelastic labor are equivalent, i.e., $\lim_{\lambda\to 1}\hat{\zeta}\hat{\Phi}(J) = \lim_{\lambda\to 1}\zeta\Phi(J)$. Differentiating $\hat{\Phi}(J)$, it is evident that by constructing the value of κ and the function $a(\cdot)$ exactly as in Proposition 2, one will guarantee the existence of multiple steady states:

$$\hat{\Phi}_J = \frac{\eta(1-\alpha\gamma)}{\eta(1-\alpha\gamma)-(\lambda-1)}\frac{\lambda-1}{1-\alpha\gamma}\left[\frac{\bar{a}_J}{\bar{a}}-\frac{\bar{a}_J+\frac{\lambda-\gamma}{(1-\alpha)\gamma}a_J}{\bar{a}+\frac{\lambda-\gamma}{(1-\alpha)\gamma}a}\right]\hat{\Phi}+...$$
$$+\left[\frac{\eta(1-\alpha\gamma)}{\eta(1-\alpha\gamma)-(\lambda-1)}\frac{\lambda-\gamma}{1-\alpha\gamma}-\frac{\bar{a}}{\bar{a}-a}\right]\frac{a_J}{a}\hat{\Phi}.$$

■

We set the parameters of the production side of the model, as well as the discount factor and the intertemporal elasticity of substitution to the same values as in Section 4. The remaining two parameters determining the disutility from working are set as follows: χ is such that the steady state value of labor in the low steady state is the same as in the inelastic labor model, i.e., $N^S = N = 1$; We take the benchmark value for η from Jaimovich and Rebelo (2009), $\eta = 0.4$, which corresponds to a Frisch elasticity of 2.5.

	$\eta = 0.4$			$\eta \to \infty$		
	Low	**High**	**High/Low**	**Low**	**High**	**High/Low**
K	3.38	5.51	1.63	3.38	4.10	1.21
N	1	1.27	1.27	1	1	1
TFP	0.74	0.83	1.12	0.74	0.83	1.11
Y	1.27	2.07	1.63	1.27	1.54	1.21
w	0.68	0.75	1.10	0.68	0.71	1.04

Table 3: GHH model: stable steady states with $\eta = 0.4$ and $\eta \to \infty$.

Table 3 contrasts the values of output, labor, TFP, output and wages for the two stable steady states of the model with endogenous labor with the same objects in the benchmark inelastic labor model. Labor is 27% higher in the high steady state when households supply labor elastically. This corresponds to a higher wage when labor is endogenous. The differences in capital and output across steady states are much larger with endogenous labor (63% vs. 21%). The differences in TFP are similar.

Figure 5 illustrates that the differences across steady states are even larger if one is willing to assume a higher Frisch elasticity of labor supply, i.e., a lower η. For example, for $\eta = 0.3$ the high steady state has a level of output 183% higher than the low steady state.

6 Conclusions

We show that allowing for an endogenous industry structure along the lines of Hopenhayn (1992) can yield to multiple locally stable steady states: We construct a distribution of productivity that implies multiple steady state for an arbitrarily small degree of increasing returns. If only the most productive firms operate, TFP and output are high. When the pool of firms is sullied by a large measure of low-productivity firms, the economy is in a low-TFP and low-output steady state. This is consistent, qualitatively, with the empirical evidence: LDC countries feature a much higher share of employment in small (low productivity) firms than the US.

Differences in TFP and output across steady states are increasing in the degree of returns to scale, in the share of capital in income, and in the Frisch elasticity of labor supply.

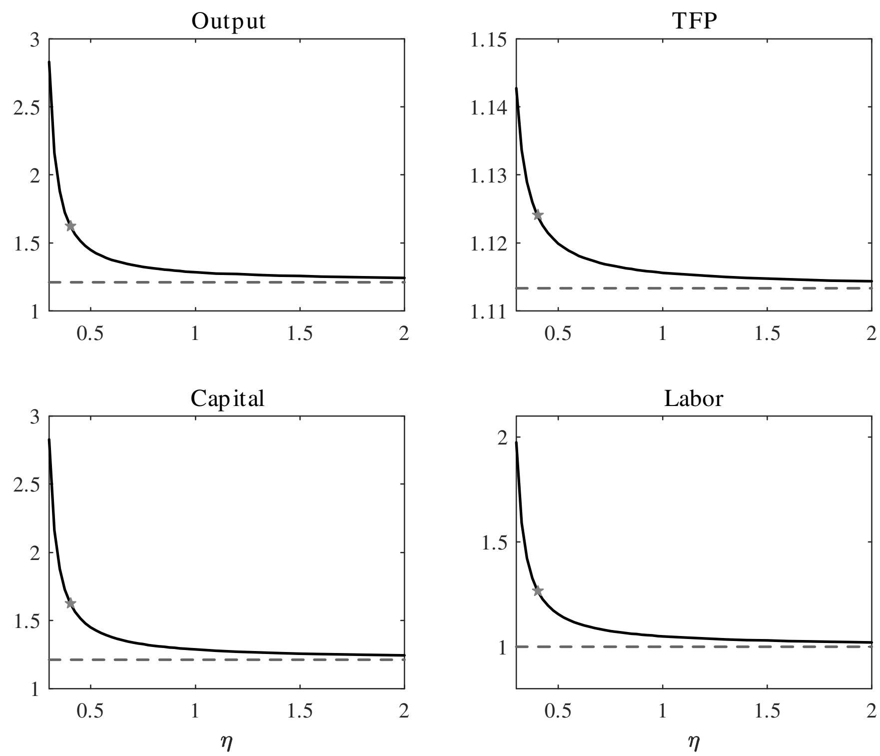

Figure 5: GHH preferences: relative (high/low steady state) output, TFP, capital and labor as a function of η. The dashed lines correspond to $\eta \to \infty$ (i.e., the inelastic labor model). Stars correspond to the benchmark value, $\eta = 0.4$.

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