ECONOMIC RESEARCH

FEDERAL RESERVE BANK OF ST. LOUIS

WORKING PAPER SERIES

Multivariate Markov Switching With Weighted Regime Determination: Giving France More Weight than Finland

Multivariate Markov Switching With Weighted Regime Determination: Giving France More Weight than Finland

December 2007

Michael Dueker*
Federal Reserve Bank of St. Louis
P.O. Box 442, St. Louis, MO 63166
mdueker@stls.frb.org; fax (314) 444-8731

Martin Sola
Universidad Torcuato DiTella
Birkbeck College
msola@econ.bbk.ac.uk

I

Multivariate Markov Switching With Weighted Regime Determination: Giving France More Weight than Finland

Abstract

This article deals with using panel data to infer regime changes that are common to all of the cross section. The methods presented here apply to Markov switching vector autoregressions, dynamic factor models with Markov switching and other multivariate Markov switching models. The key feature we seek to add to these models is to permit cross-sectional units to have different weights in the calculation of regime probabilities. We apply our approach to estimating a business cycle chronology for the 50 U.S. States and the Euro area, and we compare results between country-specific weights and the usual case of equal weights. The model with weighted regime determination suggests that Europe experienced a recession in 2002-03, whereas the usual model with equal weights does not.

JEL classifications: **F42, C25, C22**

Key words: **Vector autoregression, Regime switching, Business Cycle Turning Point**

II

Markov regime switching models are a popular way to estimate business cycle chronologies (recessions /expansions) because they are reproducible and they often match judgmental chronologies. In particular, multivariate Markov switching models have been developed in recent years as a way to ensure that the inferred business cycle downturns display cross-sectional breadth, in addition to the depth and duration observed in the aggregate. These models include Markov switching vector autoregressions [Clements and Krolzig (2003); Sims and Zha (2006)], dynamic factor models with Markov switching [Chauvet (1998); Kim and Nelson (1998); Chauvet and Hamilton (2005)] and other multivariate models, such as the uncorrelated panel Markov switching model of Asea and Blomberg (1998).

One feature that multivariate Markov switching models have not had to date is a way to permit cross-sectional units to receive different weights when inferring a regime that is common to the entire cross section. Because the data in business-cycle-oriented Markov switching models are usually expressed in terms of growth rates (which are free of any unit of measurement) of real output or employment, a positive spike of two standard deviations of the growth rate in Finland has as much influence on the probability of a European recession as an identical spike in France. Moreover, unlike weighted regressions, one cannot simply scale the data in a regime-switching context to apply a set of cross-sectional weights to affect the regime probabilities. Artis et al. (2004) note that the regime probabilities in Markov switching models are invariant to re-scaling the data because the mean parameters and the covariance matrix adjust commensurately. Nevertheless, given that the economy of France is about ten times as large as that of Finland, we would like to be able to give France's data more weight in determining the state of the European business cycle. This article presents a straightforward method to implement a desired set of cross-sectional weights in multivariate Markov switching models. Because the weighted calculation of regime probabilities alters the inferred business cycle chronology for Europe, we can shed new

1

light on the chronology adopted by the CEPR business cycle dating committee. In particular, we can re-examine the question as to whether the Euro area experienced a recession during 2002-03.

This article also considers the possibility that the desired set of cross-sectional weights might not be based solely on exogenous factors, such as size. Endogenous factors, such as how well a country's data is described by the Markov switching process, could also play a role when choosing country-specific weights. As an extreme case, consider a country whose data do not help discriminate at all between the regimes. Just as it might be sensible to give more weight to the data from larger countries, it likewise might make sense to give more weight to countries whose data are better described by the Markov model. We also consider the possibility of combining such exogenous and endogenous factors when choosing country-specific weights for the calculation of regime probabilities.

We mention at the outset, however, that there is no "correct" set of weights and, therefore, no value-free approach to reject statistically one set of weights in favor of another (or in favor of the usual case of equal weights). In principle, within the context of the Bayesian estimation approach we use, one could compare models in terms of marginal likelihoods.[1] The likelihood function, however, gives equal weight to all counties, so the marginal-likelihood selection procedure would be somewhat contrary to the idea of wanting to weight countries unequally when inferring the regime changes. Instead, we suggest that it is incumbent on the modeler to argue that an adopted set of weights best helps the model capture the desired phenomenon. A useful analogy comes from the choice of a value-weighted versus an equal-weighted portfolio of individual stocks to measure the stock market. Market watchers do not attempt to use statistical tests to decide whether to follow a value-weighted or equal-weighted portfolio to answer a given question about

---

[1]Chib and Albert (1993) pioneered the use of Bayesian methods to estimate Markov switching models.

the stock market. Instead, they suggest why one might be more useful or appropriate in a given context. For example, one could take a panel of rates of return on individual assets to infer when the stock market as a whole has entered a high-volatility regime. It might make sense to use the same weights that go into the construction of a value-weighted portfolio to infer—through weighted regime determination—when the stock market is in the high-volatility regime. In the same way, weighted regime determination of a business cycle chronology utilizes a chosen value-weighted portfolio of country-level data, whereas the usual equal-weighted model uses an equal-weighted portfolio. For the same reasons that value-weighted portfolios are often preferred measures of the stock market, value-weighted portfolios of country-level data can provide useful inferences of the European business cycle.

With respect to our application to business cycle turning points, a standard definition of a recession is that it has breadth, depth and duration. One approach is to define the business cycle in terms of specified changes in a single aggregate series, such as GDP [Zellner, Hong and Min (1991)]. Committees at the National Bureau of Economic Research (NBER) and the Centre for Economic Policy Research (CEPR) take an eclectic, judgmental approach, following many series to date recessions. In the past, the NBER committee calculated diffusion indices, for which the NBER tabulated turning points for many series and took the percentage of series in a downturn at a given moment. Today the CEPR business cycle committee can consult timely aggregate indices, such as the EURO-CoIN coincident indicator, in addition to the country-level data [Altissimo et al. (2001) ; Forni et al. (2005)].

The next section of the paper presents a Markov switching vector autoregressive model and the weighting scheme. It discusses an additional innovation of this article: in our multivariate Markov switching model, the business cycle regime switching takes place in the mean growth rate, not

the intercept. The third section presents estimation results for European country-level data and the difference that various weighting schemes make regarding turning points in the European business cycle. The final section concludes.

Markov switching VAR with weighted regime determination

The vector autoregressive model we use differs from previous business cycle Markov switching VARs from Clements and Krolzig (2003) and Artis et al. (2004) and dynamic factor models from Chauvet (1998) and Chauvet and Hamilton (2005) in that the mean is subject to regime switching, not just the intercept. The potential advantage of this approach is that switching in the intercept would have a greater tendency to appear spuriously if, for example, an autoregressive coefficient experienced a structural break. Moreover, this alternative source of apparent switching in the intercept likely would not reflect the business cycle. The cost of mean switching versus intercept switching is that exact conditional distributions are much more cumbersome to calculate. For this reason, we do not attempt to work with the exact conditional distributions for Bayesian estimation of the model; instead, we introduce two Metropolis-Hastings steps, as outlined below.

In the Markov switching VAR, the number of countries in the cross section is N, y are the growth rate data from all N countries and p is the number of lags.

$$y_{1t} = \mu_{S1_t}^{(1)} + \sum_{i=1}^{N}\sum_{j=1}^{p}\phi_{ij}^{(1)}(y_{i,t-j} - \mu_{S1_{t-j}}^{(i)}) + e_{1t} \tag{1}$$

$$\vdots = \vdots \tag{2}$$

$$y_{Nt} = \mu_{S1_t}^{(N)} + \sum_{i=1}^{N}\sum_{j=1}^{p}\phi_{ij}^{(N)}(y_{i,t-j} - \mu_{S1_{t-j}}^{(i)}) + e_{Nt}$$

4

where $S1_t = 0$ or 1 and

$$Pr(S1_t = 0 \mid S1_{t-1} = 0) \quad = \quad p_1 \qquad (3)$$
$$Pr(S1_t = 1 \mid S1_{t-1} = 1) \quad = \quad q_1$$

To allow for the possibility that the volatility of output growth has changed over time, we include a second Markov state variable so that the covariance matrix of the shocks, e, is also subject to Markov switching governed by a state variable that is independent of $S1$: Ω_{S2_t}, $N \times N$, where

$$Pr(S2_t = 0 \mid S2_{t-1} = 0) \quad = \quad p_2 \qquad (4)$$
$$Pr(S2_t = 1 \mid S2_{t-1} = 1) \quad = \quad q_2.$$

Estimation methods

MCMC Sampling Scheme

A key reason for Bayesian estimation of this Markov switching model is that the number of covariance matrix parameters is $N(N-1)/2$ or 36 when $N = 9$ for both Ω_0 and Ω_1. A numerical search algorithm for maximum-likelihood estimation of so many parameters would be very cumbersome.

The parameters were divided into five blocks for MCMC sampling via Gibbs: (i) Regime-dependent means, denoted μ_{S1}; (ii) autoregressive coefficients, ϕ (iii) Regime-dependent covariance matrices Ω_{S2_t} that are functions of the cross-sectional weights; (iv) the time series of the unobserved binary

Markov state variables $S1$ and $S2$, for mean and variance switching, respectively; (v) transition probability parameters for the first-order Markov processes.

Conditional and/or proposal densities for the parameters are as follows, where the distributions for the means, μ, are understood to be a proposal density.

$$
\begin{aligned}
\mu_j \quad &\sim \quad \mathrm{N}\left(B_j^{-1}\left(B_j^{pr}\mu_j^{pr} + \sum_{t=1}^{T} I(S1_t = j)i_N'\Omega_{S2_t}^{-1}(\mu_{S1_t} + e_t)\right), B_j^{-1}\right) \\
&\text{where } B_j = B_j^{pr} + \sum_{t=1}^{T} I(S1_t = j)i_N'\Omega_{S2_t}^{-1}i_N, j = 0, 1 \quad (5) \\
\phi^{(n)} \quad &\sim \quad \mathrm{N}\left(D_n^{-1}\left(D_n^{pr}\phi^{pr(n)} + \sum_{t=1}^{T} \tilde{Y}_t'\Omega_{S2_t}^{-1}(y_{n,t} - \mu_{n,S1_t})\right), D_n^{-1}\right) \\
&\text{where } D_n = D_n^{pr} + \sum_{t=1}^{T} \tilde{Y}_t'\Omega_{S2_t}^{-1}\tilde{Y}_t, n = 0, .., N \quad (6) \\
\Omega_k^{-1} \quad &\sim \quad \mathrm{Wishart}(\nu_k^{pr} + T_k, R_k) \\
R_k \quad &= \quad \left[(R_k^{pr})^{-1} + \sum_{t=1}^{T} I(S2_t = k)(\mu_{S1_t} + e_t)(\mu_{S1_t} + e_t)'\right]^{-1}, k = 0, 1 \quad (7)
\end{aligned}
$$

$I(S1_t = j)$ is an indicator function that equals one when $S1_t = j$, $\tilde{Y}_t$ is a vector of lags $(y_{t-k} - \mu_{S1_{t-k}})$, $k = 1, ..., p$ and i_N is an N-dimensional vector of ones. The parameters B_j^{pr} and ν_k^{pr} are hyperparameters that determine the strength of the prior. R_k^{pr}, μ_j^{pr} determine the location of the prior distribution.

In the proposal density for the means, μ, the calculated residual, e_t, takes values of the lagged values of $(y_{t-k} - \mu_{S1_{t-k}})$ as given at the previous iteration's draw. For this reason, it is not an exact conditional distribution because it ignores the fact that the value of μ will affect density of the data for the next p periods as well as the current period. Nevertheless, it is valid to treat eq. (6) as a proposal density and subject the draw from it to a Metropolis-Hastings step, using the target density of the data based on the

full model.

A similar Metropolis-Hastings step is needed when drawing the states $S1$ because the exact conditional distribution would require us to follow $p + 1$ values of the state variable in the filtering: $P(S1_t, S1_{t-1}, ..., S1_{t-p} \mid y_t, y_{t-1}, ...)$. By taking the lagged values of $S1$ as given at their values from the previous iteration, we can draw a proposal value for $S1$ as if only the contemporaneous value of $S1$ mattered in the conditional density of the data in the filtered value $P(S1_t \mid y_t, y_{t-1}, ...)$. We draw proposal values from the states using this latter probability and the Bayesian algorithm from Chib (1996). Again we subject this proposal draw to a Metropolis-Hastings step, using the target density based on the true model specification of equation (1).

Weighting the cross section when determining regime probabilities

One novel feature that we want to include in this panel Markov switching model is the ability to weight cross-sectional units: A shift in the employment growth rate of two standard deviations in California ought to have more influence on the regime probabilities than an equal-sized shift in Connecticut.

The weights are implemented by scaling the sample covariance matrix $\widetilde{\Omega}$ with generated weight parameters, ω,

$$\Omega_{ij} = \widetilde{\Omega}_{ij}/(\omega_i \omega_j)^{.5}$$

where the weights are normalized to equal 1.0 on average. In this way, cross-sectional units with below-average weight have variances that are scaled up to reduce their influence in the likelihood function that is used to filter the regime probabilities. Because we have a relatively clear prior for the cross-

7

sectional weights, it is easier to specify the generated covariance matrix in terms of the sample covariance matrix and generated weights than it is to derive a prior for the covariance matrix directly. Furthermore, we would like to apply the same weights to the covariance matrix in both the low- and high-volatility states, Ω_0 and Ω_1. Thus, we would not wish to generate randomized values of Ω_0 and Ω_1 separately.

Another way to illuminate our weighting scheme is to express it equivalently in the regime probability filter as:

$$P(S_t = 0 \mid y_t) = \tag{8}$$
$$\frac{P(S_t = 0 \mid y_{t-1}) \prod_{n=1}^{N} f(y_{nt} \mid y_{1t}, ..., y_{n-1,t}, S_t = 0)^{\omega_n}}{\sum_{j=0}^{1} P(S_t = j \mid y_{t-1}) \prod_{n=1}^{N} f(y_{nt} \mid y_{1t}, ..., y_{n-1,t}), S_t = j)^{\omega_n}}$$

This equivalency holds because for the normal density

$$\phi(\mu, \sigma)^{\omega} \propto \phi(\mu, \sigma/\omega)$$

The idea is to use weights to bring to bear prior information about how we want the regimes to be determined. We also subject the draws of the regime states to a Metropolis-Hastings step.

For the weights, we us Beta updates such that the hyperparameters in the prior reflect country or state size: France vs. Finland or California vs. Connecticut. The posterior update is based on how well the region is captured by the Markov process; that is, how closely the regime probabilities based on that region adhere to either 0 or 1. Regions that discriminate well between the two regimes will receive greater weight in the update.

The particular form of the conditional distribution for the ω weight is

$$\text{weight}_n \sim \textbf{Beta}(80(\text{size share} + \text{ regime discrim. share}), \tag{9}$$
$$80\beta((1 - \text{size}) + (1 - \text{discrim}))),$$

where the hyperparameter β is designed to give considerable weight to the size prior, s.t.

$$\text{weight}_n \approx 0.8\text{size} + 0.2\text{discrim}.$$

In this way, the weighting scheme has the ability to weight the cross section based on characteristics that are unrelated to the Markov switching model (Exogenous weights, denoted exg) and/or characteristics related to the Markov switching model (Endogenous weights, denoted end). A natural candidate for an exogenous weight would be the size of the cross-sectional region, such as population or employment. A natural candidate for an endogenous weight would be how well a region is captured by the Markov process; that is, how closely the regime probabilities for that region adhere to either 0 or 1. In particular, an endogenous weighting scheme is based on how well a region discriminates between the two regimes.

Note again that if all regions were identical with regard to their exogenous and endogenous weighting properties, then ω_n would equal 1.0 for all regions n and the weighted update would be identical to the unweighted update equation

$$
\begin{aligned}
f(y_t \mid S_t = 0) &= f(y_{1t} \mid S_t)f(y_{2t} \mid y_{1t}, S_t = 0) \\
&\times \cdots \times f(y_{Nt} \mid y_{1t}, ..., y_{N-1,t}, S_t = 0),
\end{aligned}
\tag{10}
$$

where S is either of the two state variables, $S1$ or $S2$. Note that our code allowed for the possibility of linear dependencies across the N countries, which could hinder straightforward calculation of the factorization of the likelihood in eq. (9), but this problem did not present itself in this set of data on either European GDP or U.S. State employment growth rates.

Estimation Results for Weighted Business Cycle Chronologies

We apply the Markov switching seemingly unrelated Markov Switching model to employment growth rates for the U.S. States since 1956. We also apply the Markov switching VAR with weighted regime determination to quarterly GDP growth data since 1977 for nine Euro-area countries. Quarterly GDP data from the OECD are availble for Austria, Italy, Denmark, Finland, France, Germany, Portugal, Spain and the Netherlands. Data for Belgium and Greece were only available at a later starting date. The data are in real terms, expressed in base year 2000 Euros, and are seasonally adjusted.

As discussed above, we chose to emphasize country size, largely because this type of heterogeneity appears to be of greater magnitude: the largest economy is easily more than 10 times the size of the smallest, but no country is ten times better at discriminating between the two regimes than the worst country. Table 1 shows the posterior means of the endogenous weights, ω_n^{end} for the nine countries.

Figure 1 plots the posterior mean of the probability of the recession state from the model with weighted regime determination. Recession periods determined by the NBER business cycle committee are shaded. Figure 2 has the corresponding results for the usual case of equal weights ($\omega_n = 1 \forall n$). A comparison of the two sets of recession probabilities does show some differences at interesting junctures. In the last two comparatively mild recessions, which were characterized by jobless recoveries, the weighted model shows a longer recession, whereas the equally-weighted model has trouble finding recessions in 1990 and in 2001.

Figure 3 presents the posterior mean probabilities for the second Markov state variable that governs regime switching in the covariance matrix. The model estimates show that an essentially permanent volatility reduction took

place in 1981. Aggregate data series, in contrast, tend to date the Great Moderation to 1984. Perhaps this owes to the deep recession in the early 1980s, which could have masked the volatility reduction for a time. By including both mean and variance switching, this model helps disentangle these two phenomena. Figure 4 shows the corresponding results for the equally weighted model and they show much less difference than the mean switching finds.

With respect to the European results, in the recession in the early 1980s, for example, the equal-weighted probabilities might lead one to question why the CEPR did not date the trough date earlier. Figures 5 and 6 present corresponding charts for the European GDP growth rates. To convert the regime probabilities into a business cycle chronology, consider the following as a possible rule: to qualify as a recession, the regime probability $S1 = 0$ must reach at least 80 percent at some point and the starting and ending dates are determined by the dates when the probability of $S1 = 0$ initially rises above 50 percent and then below 50 percent on a sustained basis. This method of creating a chronology from the regime probabilities leads to a high degree of overlap on average between the Markov switching chronologies and the CEPR chronology.

In the early 1980s, the trough date based on the equal-weighted model would be 1981Q1. The weighted-regime model, in contrast, would imply a trough date of 1982Q3, which exactly matches the CEPR chronology. Both models differ from the CEPR peak date of 1980Q1 by placing the peak one quarter later.

In the 1992-93 CEPR-dated recession, the both regime-switching models again are one quarter later than the CEPR with respect to the peak date of 1992Q1. For the trough date, the weighted model is one quarterly early, relative to the CEPR date of 1993Q3, whereas the equal-weighted model matches the CEPR.

11

With respect to the post-2001 period, in which the CEPR has declined to declare a business cycle peak, the equal-weighted model would concur with the CEPR business cycle committee, whereas the weighted model would date a recession that lasted a year and a half from 2002Q1 through 2003Q3. This result is a key difference on a controversial question regarding whether Europe also experienced a recession during 2002-03. If the CEPR business cycle committee consulted a multivariate switching model with weighted regime determination, then perhaps they would have declared peak and trough dates in this period.

Table 1: Comparison of Euro Area Business Cycle Chronologies

	Peak	Trough
CEPR	1980 Q1	1982 Q 3
Weighted Regime Model	1980 Q2	1982 Q 3
Equal-Weighted Model	1980 Q2	1981 Q 1
CEPR	1992 Q1	1993 Q 3
Weighted Regime Model	1992 Q2	1993 Q 2
Equal-Weighted Model	1992 Q2	1993 Q 3
CEPR	None	None
Weighted Regime Model	2002 Q1	2003 Q 3
Equal-Weighted Model	None	None

Figure 7 presents the posterior mean probabilities for the second Markov state variable that governs regime switching in the covariance matrix. The weighted regime model suggest that Europe, like the United States, underwent a Great Moderation by the mid-1980s, but the timing is somewhat unclear due to the macroeconomic volatility associated with recessions between 1980 and 1983. Notwithstanding uncertainty regarding the date of the transition, we can say that Europe has been firmly embedded in the low-volatility GDP growth regime since at least the late 1980s.

Conclusions

This article introduces for the first time a cross-sectional weighting scheme for the determination of regime probabilities across countries in multivariate Markov switching models. Essentially what has been missing from these multivariate models has been a way to make the inferred regimes reflect the behavior of the 'value-weighted' cross-sectional portfolio. Until now large countries and small countries have been treated symmetrically in terms of determining the inferred regimes in multivariate Markov switching models of the business cycle.

We apply weighted regime determination to a Markov switching vector autoregression of GDP growth rates from nine European countries. The weighted and symmetric regime probabilities are compared and one point of disagreement is whether the economic slowdown in 2002-03 was severe enough to be classified as a recession. The weighted model suggests that it was, whereas the symmetric model indicates otherwise.

References

Altissimo, Filippo; Antonio Bassanetti; Riccardo Cristadoro; Mario Forni; Marc Hallin; Marco Lippi; Lucrezia Reichlin and Giovanni Veronese, "EuroCOIN: A Real-Time Coincident Indicator of the Euro Area Business Cycle," CEPR Discussion Paper 3108.

Artis, Mike, Hans-Martin Krolzig and Juan Torio, "The European Business Cycle," *Oxford Economic Papers* (2004) 56, 1-44.

Asea, Patrick K. and Brock Blomberg, "Lending Cycles," *Journal of Econometrics* (1998) 83, 89-128.

Chauvet, Marcelle, "An Econometric Characterization of Business Cycle Dynamics with Factor Structure and Regime Switching," *International Economic Review* (1998) 39, 969-996.

Chauvet, Marcelle and James D. Hamilton, "Dating Business Cycle Turning Points," NBER working paper no. 11422 (2005).

Chib, Siddhartha "Calculating Posterior Distributions and Modal Estimates in Markov Mixture Models," *Journal of Econometrics* (1996) 75, 79-97.

Chib, Siddhartha and James Albert "Bayesian Analysis via Gibbs Sampling of Autoregressive Time Series Subject to Markov Mean and Variance Shifts" *Journal of Business and Economic Statistics*, (1993) 11, 1-15.

Clements, Michael P. and Hans-Martin Krolzig, "Business Cycle Asymmetries: Characterization and Testing Based on Markov-Switching Autoregressions" *Journal of Business and Economic Statistics* (2003) 21, 196-211.

Forni, Mario; Marc Hallin; Marco Lippi and Lucrezia Reichlin, "The Generalized Dynamic Factor Model: One-Sided Estimation and Forecasting," *Journal of the American Statistical Association* (2005) 100, 830-40.

Kim, Chang-Jin and Charles R. Nelson, "Business Cycle Turning Points, a New Coincident Index and Tests of Duration Dependence Based on a Dynamic Factor Model with Regime Switching," *Review of Economics and Statistics* (1998) 80, 188-201.

Monch, Emmanuel and Harald Uhlig, "Towards a Monthly Business Cycle

Chronology for the Euro Area," *Journal of Business Cycle Measurement and Analysis* (2005) 2, 43-69.

Sims, Christopher and Tao Zha, "Were There Regime Switches in U.S. Monetary Policy?" *American Economic Review* (2006) 96, 54-81.

Zellner, Arnold, Chansik Hong and Chung-ki Min, "Forecasting Turning Points in International Output Growth Rates Using Bayesian Exponentially Weighted Autoregression, Time-Varying Parameter and Pooling Techniques," *Journal of Econometrics* (1991) 49, 275-304.
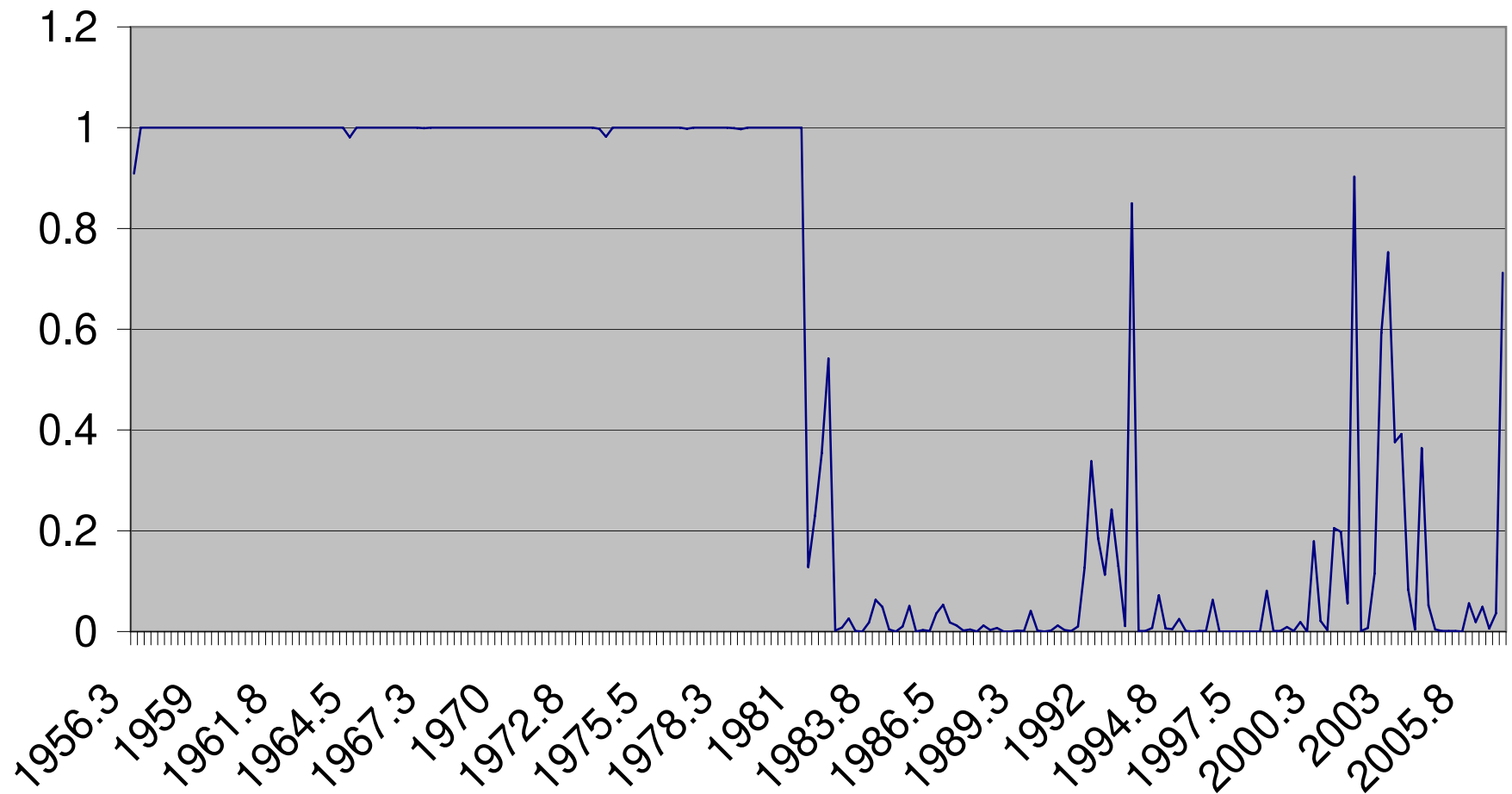
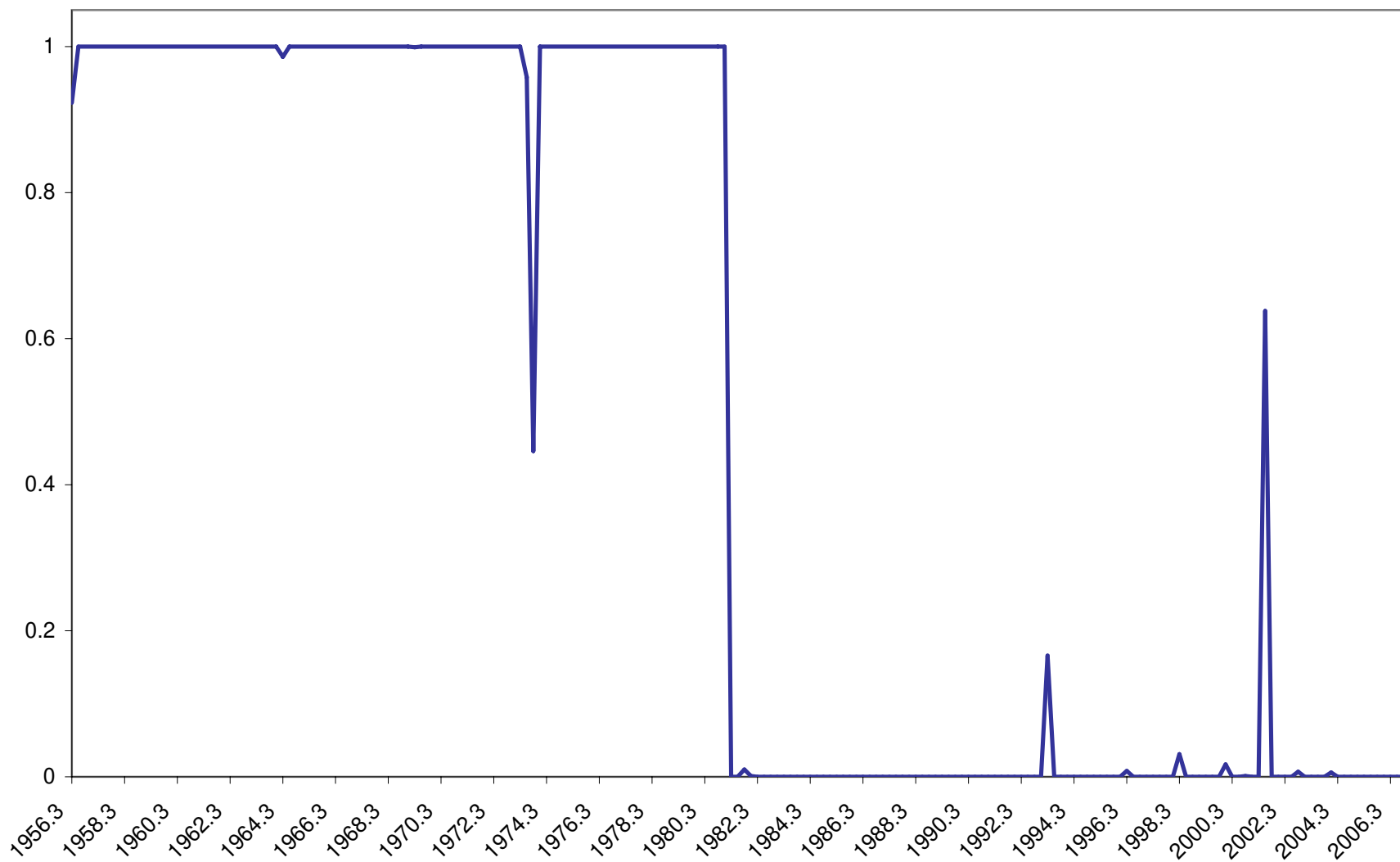**Posterior Mean of Recession State---Weighted Model**
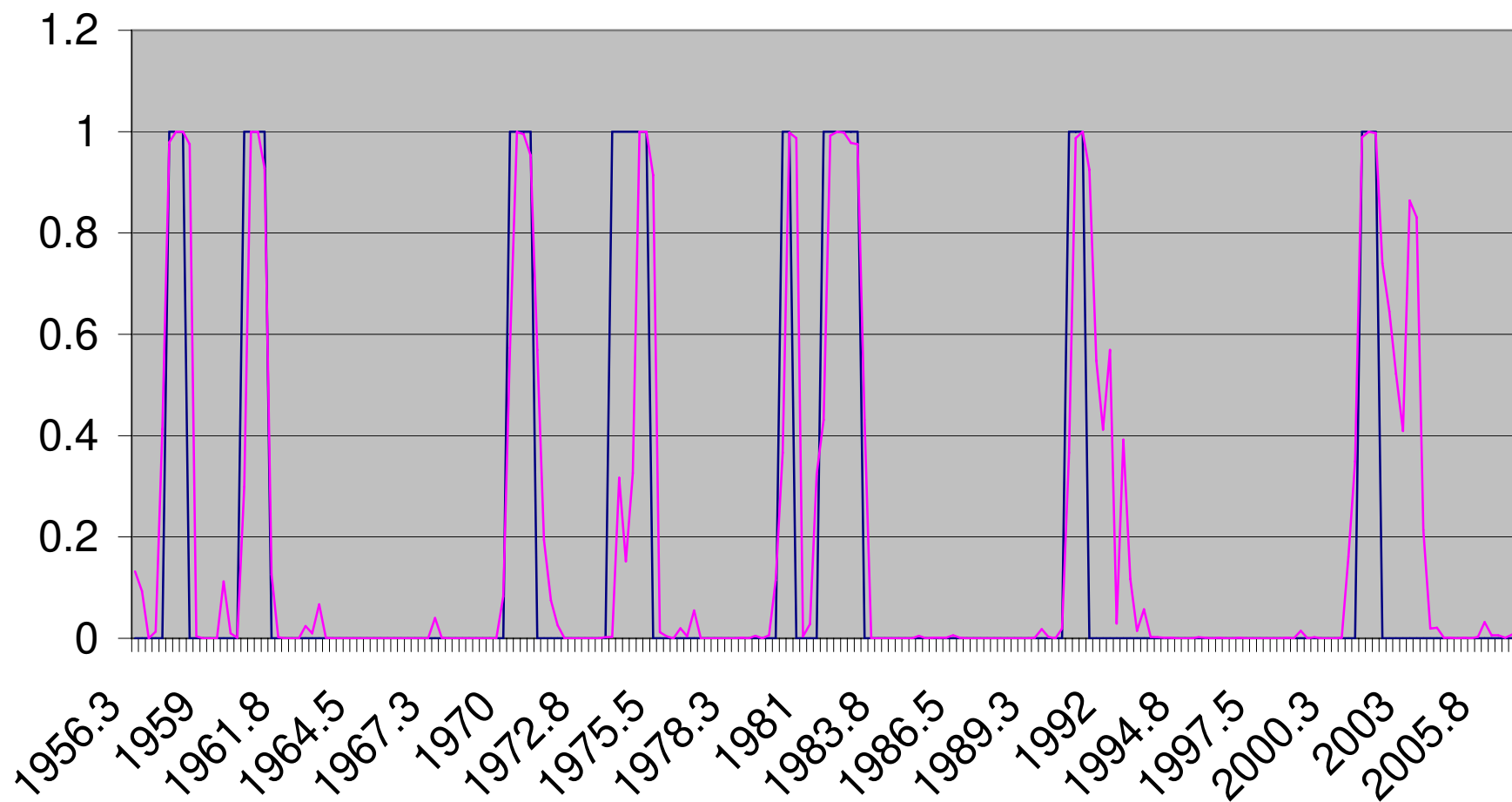
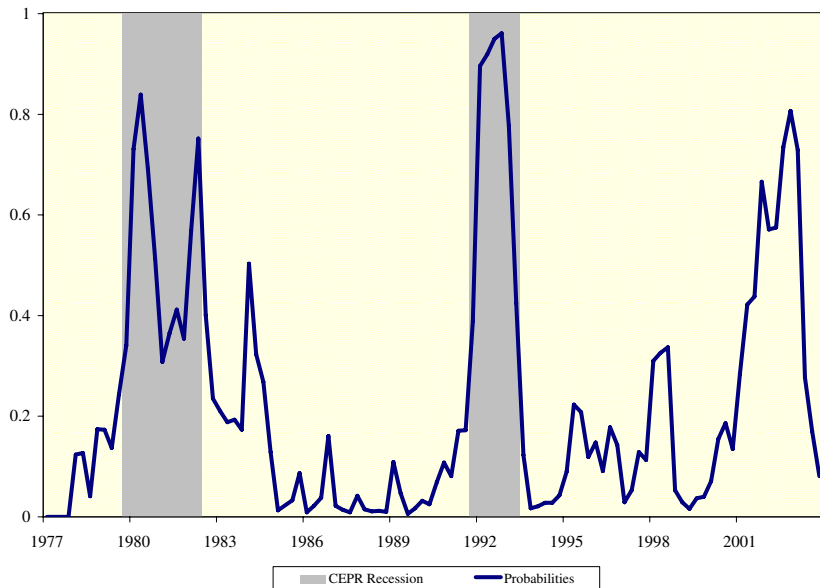**Posterior mean of recession state---Equal-weighted model**
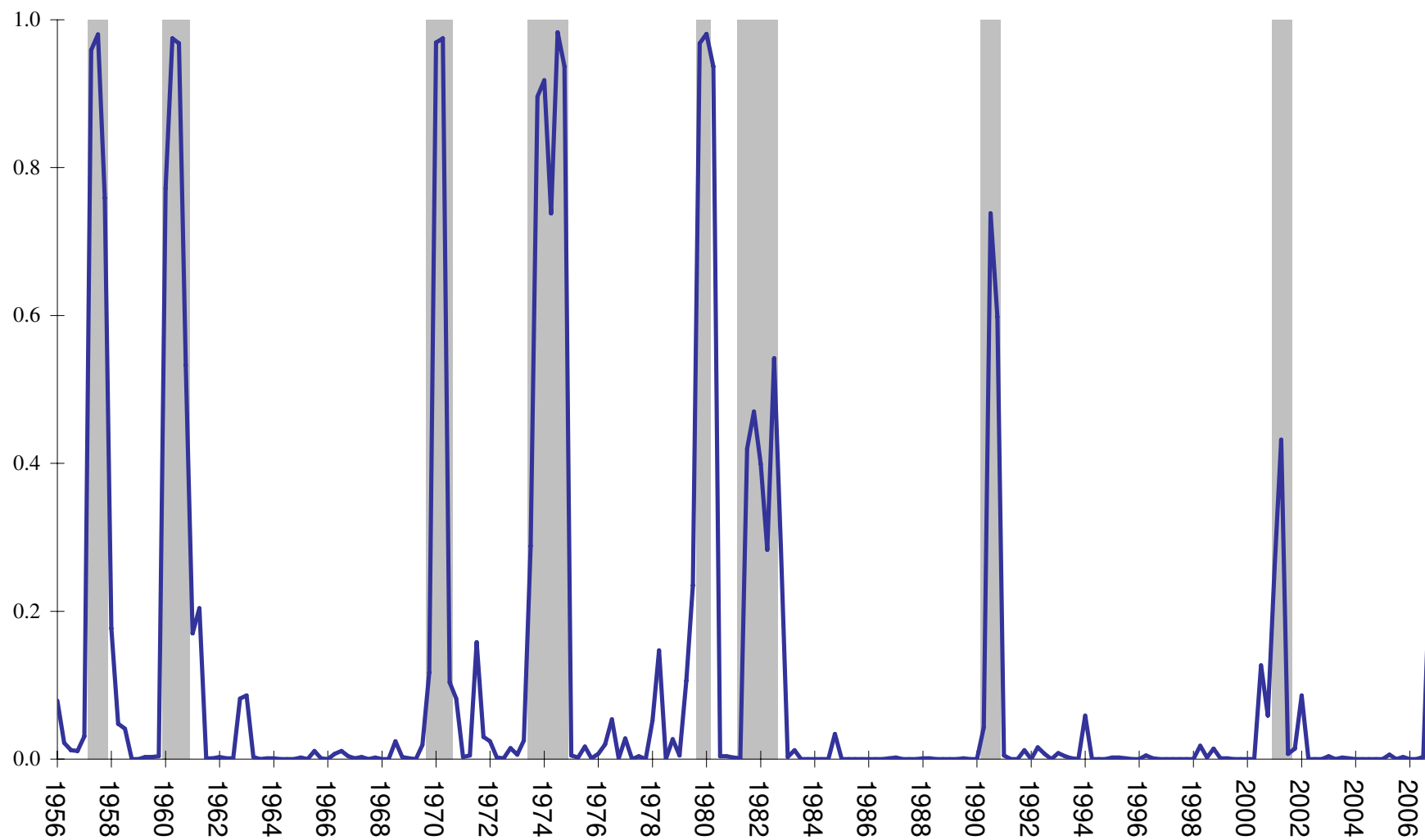
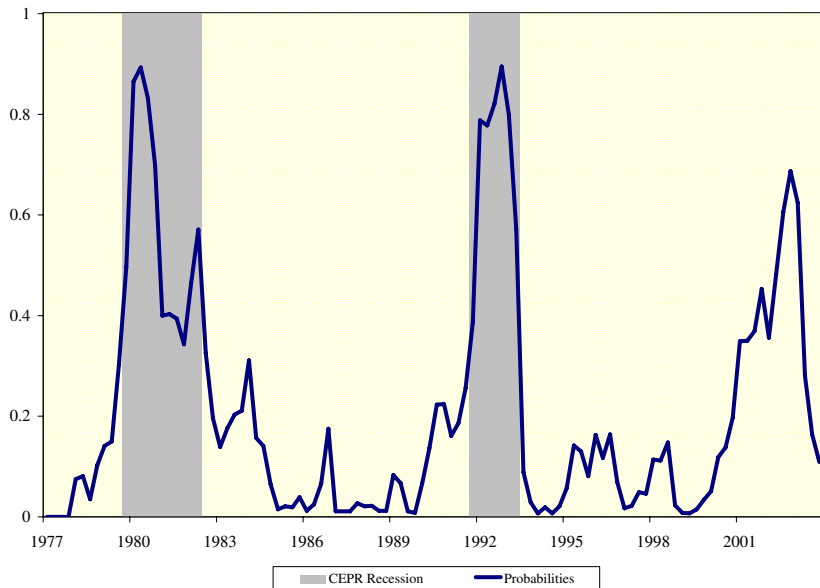**Posterior Mean of High Volatility Regime---Weighted Model**

Great Moderation volatility break---Equal-weighted model

CEPR vs. Recession Probability Under Weighted Regime Determination

CEPR vs. Recession Probability With Equal Weights

Probability of the High Variance State