



**ECONOMIC RESEARCH**  
FEDERAL RESERVE BANK OF ST. LOUIS  
WORKING PAPER SERIES

**A General Equilibrium Theory of College with Education Subsidies,  
In-School Labor Supply, and Borrowing Constraints**

<b>Authors</b>	Carlos Garriga, and Mark P. Keightley
<b>Working Paper Number</b>	2007-051A
<b>Creation Date</b>	October 2007
<b>Citable Link</b>	<a href="https://doi.org/10.20955/wp.2007.051">https://doi.org/10.20955/wp.2007.051</a>
<b>Suggested Citation</b>	Garriga, C., Keightley, M.P., 2007; A General Equilibrium Theory of College with Education Subsidies, In-School Labor Supply, and Borrowing Constraints, Federal Reserve Bank of St. Louis Working Paper 2007-051. URL <a href="https://doi.org/10.20955/wp.2007.051">https://doi.org/10.20955/wp.2007.051</a>

Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

The views expressed in this paper are those of the author(s) and do not necessarily reflect the views of the Federal Reserve System, the Board of Governors, or the regional Federal Reserve Banks. Federal Reserve Bank of St. Louis Working Papers are preliminary materials circulated to stimulate discussion and critical comment.

# A General Equilibrium Theory of College with Education Subsidies, In-School Labor Supply, and Borrowing Constraints\*

Carlos Garriga

Federal Reserve Bank of St. Louis

Mark P. Keightley<sup>†</sup>

Florida State University

November 2007

## Abstract

This paper analyzes the effectiveness of three different types of education policies: tuition subsidies (broad based, merit based, and flat tuition), grant subsidies (broad based and merit based), and loan limit restrictions. We develop a quantitative theory of college within the context of general equilibrium overlapping generations economy. College is modeled as a multi-period risky investment with endogenous enrollment, time-to-degree, and dropout behavior. Tuition costs can be financed using federal grants, student loans, and working while at college. We show that our model accounts for the main statistics regarding education (enrollment rate, dropout rate, and time to degree) while matching the observed aggregate wage premiums. Our model predicts that broad based tuition subsidies and grants increase college enrollment. However, due to the correlation between ability and financial resources most of these new students are from the lower end of the ability distribution and eventually dropout or take longer than average to complete college. Merit based education policies counteract this adverse selection problem but at the cost of a muted enrollment response. Our last policy experiment highlights an important interaction between the labor-supply margin and borrowing. A significant decrease in enrollment is found to occur only when borrowing constraints are severely tightened *and* the option to work while in school is removed. This result suggests that previous models that have ignored the student's labor supply when analyzing borrowing constraints may be insufficient.

**Keywords:** Student Loans, Education Subsidies, Higher Education

**J.E.L. classification codes:** E0, H52, H75, I22, J24

---

\*Mark Keightley has benefited from the hospitality of the Congressional Budget Office and the Federal Reserve Bank of St. Louis. Carlos Garriga is grateful to the financial support of the National Science Foundation for Grant SES-0649374 and the Spanish Ministerio de Ciencia y Tecnología through grant SEJ2006-02879. The views expressed herein do not necessarily reflect those of the Federal Reserve Bank of St. Louis, the Federal Reserve System, National Science Foundation, or the Congressional Budget Office.

<sup>†</sup>Corresponding author: Mark Keightley, Department of Economics, Florida State University, 271-E Bellamy Building, Tallahassee, FL 32306-2180. E-mail: mpk03d@garnet.fsu.edu. Tel.: 850-644-8145. Fax: 850-644-4535.

# 1 Introduction

Public policy as it relates to the subsidizing of higher education has been a focal point of empirical and theoretical economists for some time. Becker (1964) points out that young individuals often lack adequate amounts of capital to pledge to private investors. Without government intervention only individuals with access to sufficient resources would be able to pursue higher education. This observation has driven macroeconomist to understand the role education subsidies play in reducing economic inequality.<sup>1</sup> At the same time, the empirical microeconomic literature has consistently debated the existence and magnitude of borrowing constraints as well as using new data from various educational programs in order to identify the most effective policy tools for enhancing the aggregate skill level in the economy. The goal of this paper is to make a first step in combining these interrelated research agendas in order to understand the microeconomic mechanism through which higher education subsidies work within the context of a macroeconomic model. The key elements of the college investment process are isolated and examined in an attempt to better understand the interaction between available financing options and the decision to enroll in college *and* complete college in a timely manner. Rather than using a welfare criteria for selecting the optimal policy, we focus on discussing the mechanism through which each policy leads to the predicted results.

Our quantitative theory of college behavior and financial aid features endogenous enrollment, time-to-degree, and dropout decisions made by individuals that differ in their innate ability and initial wealth. College is modeled as a multi-period risky investment that requires a commitment of both physical resources and time in order to complete. Risk is introduced primarily through uncertainty over ones college ability which we correlate with innate ability according to micro-level education data. The same data is used to account for the empirical correlation between innate ability and available financial resources, a feature absent in Gallipoli, Meghir, and Violante (2006). Students learn their college ability after enrolling in college but before dropping out. This implies that there is an option value embedded in college as argued by Comay, Melnik, and Pollatschek (1973), Manski (1989), and Altonji (1991). Our model is unique in that we model all three major college decisions as the result of optimal decision making on the part of rational individuals. We feel that allowing for such intricate college behavior is necessary for studying policy proposals designed to specifically alter these three behavioral margins. In contrast to our research, the existing literature most frequently accounts for dropouts with the introduction of an exogenous "dropout shock" as

---

<sup>1</sup>There has also been an extensive literature in macroeconomics and growth theory that tries to understand the role of human capital acquisition as an engine of growth. See Uzawa (1965) and Lucas (1988) among others.

in Caucutt and Kumar (2003) or Akyol and Athreya (2005). To our knowledge no one has accounted for the time-to-degree dimension of the college investment process. In order to account for important general equilibrium effects we embed the college investment decision within an overlapping generations production economy.

The labor supply of both full-time worker and college students in our model is endogenous. While allowing for college students to work during their college years greatly increases the computational complexity of our model, we feel that it is essential for understanding the influence of borrowing constraints. A report from National Center for Education Statistics reveals that the percentage of full-time students employed increased from 34 percent to 49 percent between 1970 and 2005.<sup>2</sup> In addition, the percent of full-time students working 20 or more hours per week more than doubled over the same period, increasing from 14 percent in 1970 to 30 percent in 2005. If borrowing constraints begin to bind labor income becomes a viable financing alternative, but only at the cost of a reduction in the amount of time remaining for studying. In addition, college students may choose to work only a few hours in order to reduce the burden associated with large student loan payments in the event of dropping out. In the absence of this mechanism low income students would be forced to rely solely on grants and loans to finance the cost of education. This has the potential to overestimate the sensitivity to proposed subsidy policies.

The model accounts for the main statistics regarding education such as enrollment rate, dropout rate, and time to degree while matching the observed aggregate wage premiums consistent with the labor and macro literature. We use our model to study three types of education policies: tuition subsidies (broad based, merit based, and flat tuition), grant subsidies (broad based and merit based), and loan limit restrictions (with and without endogenous in-school labor supply). The effectiveness of some of these programs depends in part on the quantitative importance of the income and substitution effects, as well as the general equilibrium effects that determine the skill premium. Tuition subsidies effectively change the relative price (cost) of education. At the individual level the substitution and income effects work together to encourage students to register for more credits. Enrollment increases only moderately because poorer students cannot afford to put forth the effort required to reap the benefits of cheaper tuition. Grants increase the disposable income of students. However, the pure income effect of grants does not necessarily incentivize all students to significantly tilt their expenditures towards education as they also value the consumption of goods and leisure.

Also driving a number of the model's predictions is the correlation between ability and wealth. We find that broad based tuition and grant policies cannot simultaneously increase

---

<sup>2</sup>NCES: The Condition of Education 2007

enrollment and reduce dropouts because students incentivized to enroll are from the lower end of the ability and wealth distribution. These results are consistent with those of Cameron and Heckman (1998) who find that failure to account for ability heterogeneity leads to the biased conclusion that policy interventions late in the life-cycle are effective at raising skill levels. This type of adverse selection is also present in Akyol and Athreya (2005). Merit based programs and flat tuition policies serve as a screening mechanism. As such, they are successful at significantly reducing dropouts but only marginally improving enrollment.

Allowing for endogenous college labor supply has important implications for understanding the role of borrowing constraints. Interestingly, we concluded that borrowing constraints must be severely tightened *and* working in college prohibited for there to be a sizeable affect on enrollment. These findings are a generalization of those found in Keane (2002) who uses a stylized example to shows that while borrowing constraints may not impact the enrollment decision, they do affect the work behavior of students. The dropout rate is also reduced when working is removed from the student's choice set and the magnitude of borrowing constraints increased. This is a result of two reinforcing effects. On the one hand removing the work decision forces more time to be devoted to studying (and leisure). Additionally, the inability to work prohibits some poorer students from enrolling in school. Since wealth and ability are correlated these students were also those that were most likely to dropout. While borrowing constraints have only a small effect on the enrollment decision, they provide an important insurance mechanism for currently enrolled students. The model shows that the majority of college students that are subject to borrowing constraints have already completed at least two years of college. Once the borrowing constraint binds, the students have to rely on labor income to fund the remaining years of college, hence the increase in the time to degree. When students are not allowed to work while in college (or have severe restrictions) the time to completion decreases substantially.

In the only empirical study addressing the time-to-degree dimension of college, Bound, Lovenheim, and Turner (2006) have found that during the time period covering the high school graduating classes of 1972 and 1992 the percent of students receiving a degree in 4 years fell from 57.6 percent to 44.0 percent, and that the average time-to-degree increased by more than 4 months. Their conclusion is that the increase is most likely being driven by congestion in the college process due to inadequate institutional resources. Another possibility not tested in their paper is that increases in the availability of financial aid encourage students to remain in school longer. While time-to-degree is affected in all of the experiments that we run, the effect is quite small. However, our model is benchmarked towards the end of the period highlighted in their study. It is possible that any increase in time-to-degree caused by the introduction of financial aid has already occurred.

Dynamic general equilibrium models are arguably the most well suited for studying national policy initiatives that have aggregate effects, although the empirical econometric approach is by far the most popular. This is because aggregate effects in-turn impact the response to the policy itself (e.g. national student loan program). Heckman, Lochner, and Taber (1998) point out that most empirical studies neglect the general equilibrium effects on wages and taxes. Thus, it is misleading to extrapolate the results from a local policy change to the national level. Using a general equilibrium overlapping generations model, Heckman, Lochner, and Taber (1998) find that neglecting the general equilibrium effects on wages and taxes overestimates the enrollment response to a tuition subsidy by more than ten times. Their model allows for the decomposition of welfare effects for students affected by the policy. Those induced into college after the tuition subsidy or those that stay in college after the change are better off, but those that would not go to college with or without the subsidy or those that do not enroll because of the policy are worse off. This is because taxes must be raised to finance the subsidy and this reduces after-tax wages.

Our paper is most closely related to the work of Caucutt and Kumar (2003), Akyol and Athreya (2005), and Gallipoli, Meghir, and Violante (2006). However, there are significant differences between the objectives of these papers and our own. Caucutt and Kumar (2003) and Akyol and Athreya (2005) use overlapping generations models to study the effect current policies have on inequality, as well as to rationalize the level of higher education subsidies found in the U.S. and other developed countries. Both studies conclude that increasing higher education subsidies beyond current levels contribute little to increasing welfare. Caucutt and Kumar (2003) also find quantitatively important efficiency effects depending on the type of policy instituted by the government. Gallipoli, Meghir, and Violante (2006) examine the education process beginning in high school and ending in college. Their focus is on addressing the impact of one specific type of education policy, tuition subsidies, has on economic inequality.

While the relationship between inequality and education subsidies is important, our objective is to understand the consumer mechanisms at work that determines the effectiveness of various policies. By formulating college as a complex, multi-period investment we are able to delve deeper into understanding the trade-offs of many types of policy proposals, not just one. Understanding how different policies affect the enrollment and completion decisions of students is essential for drawing conclusion of how and why education subsidies affect economic equity.

The remainder of the paper is organized as follows. In section 2 we provide a general and detailed description of the model. A stationary equilibrium for the economy is defined in section 3. Section 4 reviews the parametrization of the benchmark economy. The estimation

of the benchmark economy as well as an evaluation of the model is presented in section 5. A discussion of our policy experiments can be found in section 6. Section 7 concludes.

## 2 Economic Environment

### 2.1 General Description

The economy is populated by overlapping generations of individuals that are economically active up to period  $J$  at which time they enter retirement. At the beginning of the first period of life each individual draws an innate ability and asset position from a joint distribution. With this information individuals decide to enroll in college or enter the work force as a full-time high school educated worker. The option to enroll in college is only available during the first period of life. To graduate from college a student must successfully complete a fixed minimum number of credits within three periods.

After enrolling in college a new student decides on the number of credits to register for and the amount of effort to exert in turning registered credits into completed credits. Students fund their purchase of registered credits and per-period consumption by drawing on four resources: labor income earned from endogenously supplying labor, student loans, initial assets, and government provided grants. The total cost of obtaining an education is a function of the number of credits registered for.

At the beginning of the second period each college student draws a new college ability from a conditional distribution. Upon learning their new college ability each student decides to drop out of college and enter the work force as a full-time worker or continue their education. Dropping out is a nonreversible decisions and the return to a partial education is uncertain. Students that decide to continue in college face the same problem as first period students, but particular students in the second period may differ in their college ability, the number of credits they have completed, and their current asset position. There is no more uncertainty over ability after the second period.

Students that have satisfied the minimum college degree requirement in two periods begin the third period as college educated workers. Students that have not completed the required minimum number of credits face the same problem as an agent beginning the second period. After making their dropout/continuation decision students choose registered credits and consumption expenditures, as well as how much to borrow and work. Should a student fail to complete their degree by the end of the third period they are effectively a dropout.

Upon entering the labor market by either forgoing college, dropping out, or graduating, workers choose how much labor to supply at the given education and age specific wage rate, how much to consume, and tomorrow's asset position. Earnings are subject to nondistor-

tionary taxation. We assume that the repayment of student loans begins immediately after leaving school and that only a fraction of debt incurred in school may be rolled-over each period. Thus, because no agents in our model begin life with a negative asset position, those individuals that never attend college are subject to a strict borrowing constraint. Extending the credit limits in this manner allows us to summarize the idea that more skilled agents usually face looser credit constraints without having to endogenize borrowing constraints. A similar approach can be found in Akyol and Athreya (2005).

At each date there is a single output good produced in the economy using a constant returns to scale production technology that is a function of aggregate capital and labor. Aggregate labor is comprised of age and education specific labor inputs. The government runs a balanced budget tax and transfer educational grant program. Our analysis only focuses on a stationary equilibrium where all the aggregates and prices are time invariant.

## 2.2 Demographics

The economy is populated by overlapping generations of individuals that are indexed by their age,  $j \in \mathcal{J} = \{1, 2, \dots, J\}$ . Each agent is economically active until age  $J - 1$ , after which they enter retirement at age  $J$ . Consumers are considered "young" from birth up to age  $j_o$ , and thereafter until retirement they are characterized as "old." There is no survival uncertainty.<sup>3</sup> For convenience the total measure of agents in the economy is normalized to unity. We assume that each newborn population grows relative to the previous generation at a constant rate  $\eta$  each period. The cohort shares  $\{\mu_j\}_{j=1}^J$  are computed as  $\mu_j = \mu_{j-1}/(1+\eta)$ , where  $\sum_{j=1}^J \mu_j = 1$ .

## 2.3 Firms

Firms operate a constant returns to scale technology to produce the only output good used for consumption and capital. While production depends on aggregate capital  $K$  and labor  $N$  in the standard Cobb-Douglas fashion, it also depends on two CES sub-aggregates of high school educated labor  $H$  and college educated labor  $G$ . We modify the production function used by Card and Lemieux (2001) to the study changes in the skill premium across age groups by incorporating capital as a factor of production.<sup>4</sup> Specifically, output is determined

---

<sup>3</sup>The survival probabilities for individuals of age 65 and less are sufficiently close to one that we may abstract from modelling mortality risk and the structure of annuity markets.

<sup>4</sup>They argue that this form of production function is consistent with two observations: The first one is that the gap in average earnings between workers with a college degree and those with only high school diploma rose about 25 percent in the mid 1970's to a 40 percent in 1998. The second one is that most of the rise can be attributed to the increase in the college wage premium of the younger cohorts.



according to:

$$Y = f(K, N) = AK^\alpha N^{1-\alpha}, \quad (1)$$

and the aggregator for labor inputs is defined by

$$N = (A_H H^\rho + A_G G^\rho)^{\frac{1}{\rho}}, \quad (2)$$

where  $A_H$  and  $A_G$  represent the technology efficiency parameters of high school and college graduates, respectively. The labor input from high school and college graduates is computed using CES sub-aggregators that satisfy

$$H = \left( \sum_j \nu_j H_j^\varphi \right)^{1/\varphi}, \quad (3)$$

$$G = \left( \sum_j \psi_j G_j^\varphi \right)^{1/\varphi}, \quad (4)$$

where  $\nu_j$  and  $\psi_j$  are the efficiency parameters for age group  $j$  high school educated workers  $H_j$  and college educated workers  $G_j$ , respectively. The parameters  $\rho$  and  $\varphi$  are functions of the elasticity of substitution between high school and college workers  $\sigma_E$ , and between different aged workers within education groups  $\sigma_A$ , respectively. Specifically, the relationships are  $\rho = 1 - 1/\sigma_E$  and  $\varphi = 1 - 1/\sigma_A$ . Because we only model two general age groups, young and old, the high school and college labor aggregating functions simplify to:

$$H = (\nu_o H_o^\varphi + \nu_y H_y^\varphi)^{1/\varphi}, \quad (5)$$

$$G = (\psi_o G_o^\varphi + \psi_y G_y^\varphi)^{1/\varphi}. \quad (6)$$

In equations (5) and (6) subscripts refer to the two age groups from which labor is hired. Perfect competition requires workers and capital to be paid their marginal products. The implied equilibrium factor prices are:

$$w_o^h = \nu_o (1 - \alpha) A A_H \left( \frac{K}{N} \right)^\alpha \left( \frac{N}{H} \right)^{1-\rho} \left( \frac{H}{H_o} \right)^{1-\varphi}, \quad (7)$$

$$w_y^h = \nu_y (1 - \alpha) A A_H \left( \frac{K}{N} \right)^\alpha \left( \frac{N}{H} \right)^{1-\rho} \left( \frac{H}{H_y} \right)^{1-\varphi}, \quad (8)$$

$$w_o^g = \psi_o (1 - \alpha) A A_G \left( \frac{K}{N} \right)^\alpha \left( \frac{N}{G} \right)^{1-\rho} \left( \frac{G}{G_o} \right)^{1-\varphi}, \quad (9)$$

$$w_y^g = \psi_y (1 - \alpha) A A_G \left( \frac{K}{N} \right)^\alpha \left( \frac{N}{G} \right)^{1-\rho} \left( \frac{G}{G_y} \right)^{1-\varphi}, \quad (10)$$

$$r = \alpha A K^{\alpha-1} N^{1-\alpha} - \delta.$$

To distinguish between the wages of workers with different education levels the superscripts  $h$  and  $g$  in equations (7) – (10) are used to identify high school educated workers and college educated workers, respectively.

Since we explicitly model the college dropout decision we must to assign a wage rate for the students pursuing this option. Kane and Rouse (1995) find that on average those that attended two year colleges earned approximately 10 percent more than those with just a high school education. To capture this partial return to completing some higher education the wages of college dropouts are modeled as a linear combination of high school educated workers and college educated workers:

$$w_i^d = \chi w_i^h + (1 - \chi) w_i^g, \quad i = o, y, \quad (11)$$

where  $\chi \in (0, 1)$  dictates the return to partial education.

## 2.4 Consumers

Consumers preferences are defined over consumption  $c$ , leisure  $l$ , and retirement assets  $a_J$  according to the following expected, discounted utility function:

$$E \left\{ \sum_{j=1}^{J-1} \beta^{j-1} u(c, l) + \phi(a_J) \right\},$$

where  $\beta$  is the subjective discount factor and the function  $\phi(\cdot)$  is the agent's value function upon retirement. Because there is no uncertainty after the final period, or more generally that all uncertainty is *iid*, the use of a terminal value function is valid.<sup>5</sup> The partial derivatives of the utility function  $u : \mathbb{R}_2 \rightarrow \mathbb{R}$  satisfy  $u_i > 0$ ,  $u_{ii} < 0$ , and  $u_{ij} > 0$  and are consistent with the Inada conditions. The retirement value function  $\phi : \mathbb{R} \rightarrow \mathbb{R}$  is  $C^2$  and strictly concave. Specific functional forms for the per-period utility function and retirement value function are discussed in the parameterization section.

Upon first entering the economy new high school graduates are differentiated by their initial asset position and innate ability  $(a_0, \theta_h)$  which are drawn from a joint probability distribution  $\Omega(a_0, \theta_h)$ . The manner in which initial assets and ability are determined is an

---

<sup>5</sup>See Merton (1971).

extremely important feature of the model. In abstracting away from the pre-college portion of a student's life we have neglected important socioeconomic influences that invariably determine the college preparedness of an agent, as well as the financial resources available to potentially college bound students. For example, wealthier families may be able to invest more heavily in their child's secondary education which leads to a correlation between family wealth and college preparedness. Restuccia and Urruria (2004) use a quantitative model of intergenerational human capital transmission and find that approximately one-half of the intergenerational correlation in earning is accounted for by the parents investment in early education. In addition, wealthier families may offer more financial support to their child to go to college. The potential correlation between wealth and ability, and then wealth and financial support implies a correlation between a student's college financial resources and their ability. The joint probability distribution allows us account for this correlation which effectively summarizes the socioeconomic influences prior to college. The estimation of this distribution is discussed in depth when we present the parameterization of the benchmark economy later in the paper.

In the first period of life newborns are offered the opportunity to enroll in college or enter the labor market with a high school education. As a result of this decision we can classify each agent as being in one of two categories: student, or a full-time worker. We present the problem of the college student first followed by the problem of the worker.

## 2.5 College Student Problem

College is modeled as a multi-period risky investment that requires a student to successfully complete a minimum of credits  $\underline{x}$  within three periods to graduate. Students progress through college by combining their ability  $\theta \in \Theta$ , effort  $e$ , and registered credits  $\tilde{x}$  using an education technology,  $Q(\theta, e, \tilde{x})$ . The education technology is a non-linear function dictating the production of completed credits  $x$  according to:

$$x = Q(\theta, e, \tilde{x}) = \theta \tilde{x} e^\gamma, \quad 0 < \gamma < 1.$$

Some features of this technology deserve special attention since our approach of modelling schooling decisions through college credits and not human capital is non standard. We choose to model progression through college in terms of credits instead of human capital in order to more accurately incorporate the cost of education into the model using empirical data. The specified technology is multiplicative in ability, registered credits, and effort. In addition, the marginal returns to investment in education are constant in the first two factors and diminishing in effort. The multiplicative structure implies that students with higher ability

are more productive at the margin in terms of completing all college credits, and it is not just a scaling factor in the level of produced credits. Students can affect the production of completed credits by choosing the number of registered credits and/or supplying more effort. For example, a student with low ability  $\theta_i < \theta_j$  can choose to register for a large number of credits  $\tilde{x}_i > \tilde{x}_j$  and obtain the same return (in terms of completed credits) as that of student with higher ability, but the cost in terms of tuition will be higher. The assumption that higher-ability types are more productive is common in the human capital literature, see Becker (1993). An alternative mechanism for low ability students is to increase the time effort in school, but it has a utility cost since an increase in effort reduces the time available for leisure and work. However, the education technology exhibits diminishing returns to effort following the work of Ben-Porath (1967).<sup>6</sup> Despite the apparent differences, the college credit function is a version of the frequently used human capital accumulation equation, where the stock of human capital is replaced with the agent's credit stock. As mentioned earlier, allowing the labor supply of college students to be determined endogenously addresses a previously neglected interaction with the student's choice of debt. It also serves another important function related to the riskiness of college. In the presence of uncertainty over the ability to complete college students may choose to hedge the risk by substituting labor income for debt. This further increases the chances of failure as time spent working may be drawn away from school. Students from the lower end of the asset distribution are particularly vulnerable because we correlate ability with initial assets. The structure of the model allows us to exploit the recursive nature of the consumer's problem. In addition, we break the agent's optimization problem into distinct time periods in order to make explicit how the agent's information set and trade-offs change. Each agent has a total of five state variables: assets  $a$ , current ability  $\theta$ , completed college credits  $x$ , age  $j$ , and education indicator  $s$ . The education indicator state  $s$  lies in the set  $\mathcal{S} = \{h, d, c, g\}$  where  $h$  refers to a high school educated worker,  $d$  a college dropout,  $c$  an enrolled college student, and  $g$  a college graduate. Let  $v_j^s(a, x, \theta)$  be the value function of an age  $j$  agent with education level  $s$ , assets  $a$ , completed college credits  $x$ , and schooling ability  $\theta$ .<sup>7</sup>

**First Period of College:** Given initial assets and ability, an agent that decides to enroll in college must choose consumption  $c$ , registered credits  $\tilde{x}$ , effort  $e$ , leisure  $l$ , labor supply  $n$ , and tomorrow's asset position  $a'$ . A freshman student has an initial endowment of college

---

<sup>6</sup>Ben-Porath assumes that the human capital technology exhibits diminishing returns in effort and the stock of human capital,  $f(h, e, \theta) = \theta(he)^\gamma$ . The curvature of the production function allows to characterize interior solutions and also bounds the stock of human capital. In our model, we formalize the acquisition of education through credits that are bounded by the minimum number of credits required to graduate.

<sup>7</sup>Writing the value function as  $v_j^s(a, x, \theta)$  rather than  $v(a, x, \theta, s, j)$  keeps the notation compact and saves space.

credits  $x = 0$ . The first period college problem may be written as:

$$v_1^c(a, x, \theta_h) = \max_{c, \tilde{x}, a', e, l, n} \left\{ u(c, l) + \beta E_{\theta_c, w_y^d} \max [v_2^c(a', x', \theta'_c), v_2^d(a', x', \theta'_c)] \right\} \quad (12)$$

subject to

$$\begin{aligned} c + \mathcal{T}\tilde{x} + a' &\leq w_y^h n + a + y \\ x' &= \theta_h \tilde{x} e^\gamma \\ a' &\geq a_{2c} \\ l + e + n &= 1 \\ \tilde{x} &> 0, \quad x' \leq \tilde{x} \end{aligned}$$

The total education expenditure depends on the number of registered credits  $\tilde{x}$  and the per-credit price  $\mathcal{T}$ . In order to finance their education, students may draw on their initial assets and three additional resources. First, students may work while in school earning a young high school graduates wage  $w_y^h$ . Second, the government provides all students with a per-period college grant  $y$ . Students also have access to the financial market where they are permitted to take a negative position in the only financial asset,  $a' \in \mathcal{A}$  up to the borrowing constraint  $a_{2c}$ . We allow the per-period loan limit to vary in each period of college as indicated by the time indexing. Each agent has a time endowment normalized to the unity. During college years this endowment can be allocated between work, effort in school, and leisure. The last two constraints simply states that students must register for positive credits, and that completed credits may not be greater than registered credits.

The continuation value functions for a first-year student depend whether the students continues with their education in the following period  $v_2^c(\cdot)$ , or drops from school and joins the labor force as a full-time worker  $v_2^d(\cdot)$ . The expectation in the continuation value is the result of two sources of risk associated with obtaining an education. First, we assume that after the first period of college each student's college ability  $\theta_c$  is randomly drawn from the conditional distribution  $\Pi(\theta_h, \theta_c)$ . Once the agent's college ability is determined there is no further uncertainty over ability.<sup>8</sup> Second, should a student choose to dropout they receive a high school graduate's wage with probability  $p$  and a college dropout's wage with probability  $(1 - p)$ . The uncertainty over wages enables us to easily incorporate the documented partial return to college. Thus, the expectation in the value function is with respect to next periods college ability  $\theta_c$  and the wage a dropout will receive  $w_y^d$ .

---

<sup>8</sup>As we discuss in greater detail when we outline our parameterization of the model, we estimate the conditional probability distribution to match empirical data that indicates that successful high school students are more likely to be successful college students.

**Second Period of College:** At the beginning of the period each student draws a new college ability type  $\theta_c \sim \Pi(\theta_h \mid \theta_c)$ . After learning their new ability the student decides to dropout or continue on with college. The second period college problem is similar to that of the first period. However, the borrowing constraint in the second period is relaxed with respect to the previous period. In addition, the student now has to weigh the option of completing enough credits to graduate at the end of the period. The student now solves:

$$v_2^c(a, x, \theta_c) = \max_{c, \tilde{x}, a', e, l, n} \left\{ u(c, l) + \beta E_{w_y^d} \max [v_3^c(a', x', \theta'_c), v_3^d(a', x', \theta'_c), v_3^g(a', x', \theta'_c)] \right\} \quad (13)$$

subject to

$$\begin{aligned} c + \mathcal{T}\tilde{x} + a' &\leq w_y^h n + y + (1 + r) a \\ x' &= x + \theta_c \tilde{x} e^\gamma \\ a' &\geq a_{3c} \\ l + e + n &= 1 \\ \tilde{x} &> 0, \quad x' \leq \tilde{x}, \quad x' \geq \underline{x}, \end{aligned}$$

where  $v_3^g(a', x', \theta'_c)$  is the value of entering the labor market in the third period as a college graduate. The law of motion for completed credits now includes the stock of completed credits from the previous college year  $x$ . To satisfy the graduation requirement a college student must complete  $x' \geq \underline{x}$  college credits. Note that the production function of credits depends only on the realized value of college ability and is therefore independent of past abilities. A college student is always allowed to borrow as least as much in the second period as in the third period. This assumption allows the agent to at least roll over the previous periods debt if  $a_{3c} = a_{2c}$ , and increase accumulated student loan debt if  $a_{3c} < a_{2c}$ . If the credit constraint were not to be relaxed a college student at the borrowing limit during the first-period of college would be forced to repay the principal and accrued interest  $(1 + r) a_{2c}$  in the third period, while only relying on labor income and grants to fund their education. Because all ability uncertainty is resolved before the student makes any decisions, the expectation operator is only defined over the wage rate of dropouts.

**Third Period of College:** Students that extend their time in school into the third period solve a slightly different problem than in the second period. Should a student not be able to complete  $\underline{x}$  credits in the final period they are automatically classified as dropouts as there is no further college periods. As in the second period, we allow the borrowing constraint to change although we do not require that it allow for an increased level of debt.<sup>9</sup>

---

<sup>9</sup>When we estimate the benchmark economy we specify  $a_{4c} < a_{3c} < a_{2c}$  so the agent may continually

The problem in the final period of college is

$$v_3^c(a, x, \theta_c) = \max_{c, \tilde{x}, a', e, l, n} \left\{ u(c, l) + \beta E_{w_y^d} \max [v_4^d(a', x', \theta_c), v_4^g(a', x', \theta_c)] \right\} \quad (14)$$

subject to

$$\begin{aligned} c + \mathcal{T}\tilde{x} + a' &\leq w_y^h n + y + (1 + r) a \\ x' &= x + \theta_c \tilde{x} e^\gamma \\ l + e + n &= 1 \\ a' &\geq a_{4c} \\ \tilde{x} &> 0, \quad x' \leq \tilde{x}, \quad x' \geq \underline{x} \end{aligned}$$

## 2.6 College Enrollment Decision

A newborn high school graduate with innate ability  $\theta_h$ , initial assets  $a_0$ , and no college credits ( $x = 0$ ) will choose to go to college when the expected discounted utility of doing so is as least as great as the utility gain from entering the workforce as a high school educated worker. This cut-off may be summarized in terms of the agent's value function under each scenario:

$$v_1^c(a, 0, \theta_h) \geq v_1^h(a, 0, \theta_h) \quad (15)$$

To compute the initial value functions it is necessary to solve the model using backward recursion from the last period followed by the workers problem. We turn into these problems next.

## 2.7 Workers

All workers solve the same general problem regardless of their path to the workforce: forgoing college ( $s = h$ ), dropping out ( $s = d$ ), or graduating ( $s = g$ ). After leaving school the laws of motion for credits and ability for college students are trivially  $x' = x$  and  $\theta' = \theta$ , respectively, and all the relevant educational information is summarized by the college status  $s$ , age  $j$ , and asset position  $a$ . Workers choose consumption, tomorrow's asset position, and how much labor to supply at the given education and age specific wage rate. All income is subject to a lump-sum tax  $\tau$ . The problem of a worker in the period immediately preceding retirement is complicated by our use of a terminal value function to model post-retirement. We present the problem of workers aged  $j < J - 1$  first and postpone the aged  $J - 1$  worker's problem

---

increase borrowing while in school. However in our policy experiments we investigate how restricted debt in the third period of college affects time-to-degree.

to the next section. For ages  $j < J - 1$  the worker's wage rate is age dependent

$$w_j^s = \begin{cases} w_y^s & \text{if } j < j_o \\ w_o^s & \text{if } j_o \leq j < J - 1 \end{cases}$$

Notice that this specification differs from the standard formulation where the profile of earning changes over the life-cycle according to some hump-shaped profile of exogenously specified efficiency units of labor. In the current specification the age and education heterogeneity, as well as the evolution of the asset distribution are responsible for changes in the labor supply. Full-time workers allocate their time endowment between leisure and work as effort in school is no longer required. The worker's optimization problem may be written as:

$$v_j^s(a) = \max_{c, l, n, a'} \{u(c, 1 - n) + \beta v_{j+1}^s(a')\} \quad (16)$$

subject to

$$\begin{aligned} c + a' &\leq w_j^s n + (1 + r) a - \tau, \\ a' &\geq \min[0, \kappa a], \quad \kappa \in (0, 1). \end{aligned}$$

Our borrowing constraint is nonstandard and requires some discussion due to the restrictions we impose on student loan repayment. We assume that repayment of student loans begins immediately after leaving school and that only a fraction,  $\kappa \in (0, 1)$ , of outstanding loans may be rolled-over each period. This prevents us from adding an additional state variable while simultaneously approximating the repayment time period currently placed on many student loans.<sup>10</sup> Agents are not permitted to hold negative assets beyond what they enter the workforce with in the form of student loans. Thus, tomorrow's asset decision must satisfy  $a' \geq \min[0, \kappa a]$ . Since all agents begin life with a non-negative asset position, it is clear that forgoing college results in a hard borrowing constraint. This specification is equivalent to an education dependent borrowing constraint where  $a'_s \geq \kappa a$  when  $s = d, g$  and  $a'_h \geq 0$ .

---

<sup>10</sup>Under the federal student loan program the standard repayment option for Stafford loans is 10 years. Matching this repayment length exactly would require adding the number of repayment periods remaining as a state variable.



## 2.8 Retirement

Compulsory retirement occurs at age  $J$ . Because agents have utility defined over terminal assets the period  $J-1$  worker problem is slightly different than the standard worker problem. The problem in the period immediately preceding retirement is:

$$v_{J-1}^s(a) = \max_{c,l,n,a_{J+1}} \{u(c,l) + \phi(a_J)\}, \quad (17)$$

subject to  $a_J > 0$  and the old worker's budget constraint. Here,  $\phi(\cdot)$  determines the value retirees place on assets. This allows us to abstract away from post retirement behavior which we feel is appropriate as we are concerned with behavior extremely early in the economic life-cycle. This is a convenient adaptation of the method used in Roussanov (2004) and Akyol and Athreya (2006).

## 2.9 Government

The government runs a tax and transfer education grant program. All workers not in college are taxed a lump-sum tax  $\tau$  which is redistributed to college students in the form of grants  $y$ . Our balanced budget assumption implies that in equilibrium the government's tax revenue must equal total grant expenditures. The lump-sum tax that balances the education budget can be written as:

$$\tau = y \frac{\int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=c} \times \mathcal{J}} \mu_j \Gamma(da \times d\theta \times dx \times ds \times dj)}{\int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s \neq c} \times \mathcal{J}} \mu_j \Gamma(da \times d\theta \times dx \times ds \times dj)}, \quad (18)$$

where  $\Gamma(\cdot)$  represents the measure of households over the state space. The government budget constraint needs to be modified when we consider tuition subsidies, or merit based programs. However, we defer these discussions to the results section.

It can be argued that compared with a marginal income tax, our assumption of a lump-sum tax may not accurately capture the distortionary effect taxes have on the incentive to pursue a college education. However, given that only a small mass of the population is receiving grants, the per-capita tax burden in this economy is likely not to have a significant affect on the return to education. In a subsequent paper we plan to investigate this proposal by examining the optimal tax instrument to finance a publicly provided higher education subsidy program.

## 2.10 College Sector

There is an extensive literature on the supply side of education. The objective of the paper is to focus on the demand side by specifying a simple college sector that produces the credits. We assume a competitive education sector with constant returns to scale, or linear cost structure. Free entry in the sector ensures that profits will be zero and the price per credit equals the marginal cost of producing credits. The advantage of this formulation is that allows to parameterize the cost of college education as fraction of average income and it simplifies an already complex model.

## 3 Stationary Equilibrium

To define the notion of stationary equilibrium it is useful to introduce some additional notation. For an individual of a given age  $j \in \mathcal{J} = (1, 2, \dots, J) \subset I$  and education status  $s \in \mathcal{S} = (h, d, g, \mathfrak{c})$ , the relevant state vector in the recursive representation is denoted by  $\Lambda_j^s = (a, x, \theta)$ . Let  $a^s \in \mathcal{A}^s \equiv \mathcal{A}$ ,  $\theta \in \Theta$ ,  $x \in \mathcal{X} \subset I$ . Notice that the set of asset holding is conditioned by the education status as a result of the education specific borrowing constraint. We also define  $\Lambda = (a, x, \theta, s, j)$  to be the state vector including the education status and age, and  $\Gamma(\Lambda)$  represents the distribution of individuals over the entire state space.

A *stationary recursive equilibrium* for this economy is a collection of: (i) individual value functions  $\{v_j^s(\Lambda_j^s), \phi(\Lambda_j^s)\}$ , (ii) individual decision rules for college students ( $s = \mathfrak{c}, d, g$  and  $j = 1, 2, 3$ ) that include consumption, loan holdings, labor supply, effort, registered credits, and education choices  $\{c_j^s(\Lambda_j^s), a_{j+1}^s(\Lambda_j^s), n_j^s(\Lambda_j^s), e_j^s(\Lambda_j^s), \tilde{x}_j^s(\Lambda_j^s), s_j(\Lambda_j^s)\}$ , (iii) individual decision rules for workers and retirees ( $s = h, d, g$  and  $j = 1, \dots, J$ ) that include consumption, asset holdings loan holdings, and labor supply  $\{c_j^s(\Lambda_j^s), a_{j+1}^s(\Lambda_j^s), n_j^s(\Lambda_j^s)\}$ , (iv) college enrollment decision  $I_1^c(a, 0, \theta_h)$ , (v) aggregate capital and labor inputs  $\{K, H_y, H_o, G_y, G_o\}$ , (vi) price vector  $\{r, w_y^g, w_o^g, w_y^h, w_o^h, w_y^d, w_o^d\}$ , (vii) education policy  $\pi = \{\tau, y\}$  and (viii) a stationary population distribution  $\{\mu_j\}$  and an invariant distribution  $\Gamma(\Lambda)$  of individuals over the entire state space such that:

1. Given prices  $\{r, w_y^g, w_o^g, w_y^h, w_o^h, w_y^d, w_o^d\}$  and tax and grant policy  $\pi$ , the individual decision rules  $\{c_j^s(\Lambda_j^s), a_{j+1}^s(\Lambda_j^s), l_j^s(\Lambda_j^s), e_j^s(\Lambda_j^s), \tilde{x}_j^s(\Lambda_j^s), s_j(\Lambda_j^s)\}$  solve the respective education problem (13), (14), and (15) when  $s = \mathfrak{c}, d, g$  and  $j = 1, 2, 3$ . For workers, the decision rules  $\{c_j^s(\Lambda_j^s), a_{j+1}^s(\Lambda_j^s), l_j^s(\Lambda_j^s)\}$  solve problems (17) and (18) when  $s = h, d, g$  and  $j = 1, \dots, J$ . And the college enrollment decision  $I_1^c(a, 0, \theta_h)$  solves problem (16).
2. Given prices  $\{r, w_y^g, w_o^g, w_y^h, w_o^h, w_y^d, w_o^d\}$ , the representative firms chooses optimally

factors of production and prices are set to the marginal products according to (7), (8), (9), (10), (11), and (12).

3. The labor market for each educational level clears:

$$\begin{aligned}
H_y &= \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=h,c} \times \mathcal{J}_{j < j_o}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda) + \chi \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=d} \times \mathcal{J}_{j < j_o}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda), \\
H_o &= \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=h} \times \mathcal{J}_{j < j_o < J}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda) + \chi \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=d} \times \mathcal{J}_{j < j_o < J}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda), \\
G_y &= \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=g} \times \mathcal{J}_{j < j_o}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda) + (1 - \chi) \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=d} \times \mathcal{J}_{j < j_o}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda), \\
G_o &= \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=g} \times \mathcal{J}_{j < j_o < J}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda) + (1 - \chi) \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=d} \times \mathcal{J}_{j < j_o < J}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda).
\end{aligned}$$

where  $d\Gamma(\Lambda) \equiv \Gamma(da \times d\theta \times dx \times ds \times dj)$ .

4. The asset market clears:

$$K' = \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S} \times \mathcal{J}} \mu_j a_{j+1}^s(\Lambda_j^s) \Gamma(d\Lambda).$$

5. The goods market clears:

$$\int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S} \times \mathcal{J}} \mu_j c_j^s(\Lambda_j^s) d\Gamma(\Lambda) + K' - (1 - \delta)K = f(K, N),$$

6. The government budget constraint is satisfied:

$$\tau \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s \neq c} \times \mathcal{J}} \mu_j \Gamma(da \times d\theta \times dx \times ds \times dj) = y \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=c} \times \mathcal{J}} \mu_j \Gamma(da \times d\theta \times dx \times ds \times dj),$$

7. Letting  $T : \mathcal{M} \rightarrow \mathcal{M}$  be an operator which maps the set of distributions into itself. Aggregation requires  $\Gamma'(\Lambda') = T(\Gamma)$ , and  $T$  be consistent with individual decisions. We restrict the solution to equilibria that satisfy

$$T(\Gamma) = \Gamma$$

There is a two remarks about the definition of equilibrium. First, the labor market

conditions are slightly more complex due to the existence of college dropouts. Recall that there is uncertainty over the exact wage a college dropout will receive; a fraction will receive the high school wage while the rest will receive the dropout wage. The labor supply of dropouts earning the high school wage is aggregated into the high school labor supply. The aggregation of the labor supply for the dropout wage earners is carried out in-line with how the dropout wage is determined. Because we calculate the college dropout's wages as a linear combination of the wages of high school educated workers and college educated workers we must aggregate a fraction of their labor into both education group's labor supply. We weight the labor supply of college dropouts according to the fraction of the wage which is determined by high school and college education workers. Second, market clearing in the asset market is determined at the point where the quantity of capital demanded by firms is equal to net resources provided by households in the form of savings. While the loan programs are usually funded by the government, this form of lending is equivalent to the issue of government debt that is then purchased by the households. The total magnitude of debt to issue should be equivalent to the aggregate level of outstanding college debt.

## 4 Benchmark Economy

To solve the benchmark economy we must first specify our demographic assumptions, pick functional forms for the per-period utility function and retirement value function, assign initial assets and ability, pin down all parameters associated with the education process and aggregate production function, as well as estimate the joint ability and asset distribution and conditional ability distribution. The time period that we choose to use in benchmarking our model is crucial. Beginning with the 1993-94 school year the federal student loan program changed in three significant ways due to the 1992 Reauthorization of the Higher Education Act (HEA92): the federal need based formula changed to allow less needy students to qualify for need-based aid; there was a widespread increase in the availability of unsubsidized student loans; and the nominal aggregate student loan limits increased approximately 33 percent for dependent students and 23 percent for independent students. Data availability limitations force us to benchmark our model to the pre-HEA92 period of the early 1990s.

We parameterize the benchmark economy in three steps. First, a number of common parameters in the model are taken directly from the literature. Second, we estimate production efficiency parameters, initial assets, borrowing limits, and the ability transition matrix using data from the pre-HEA92 period. Third, given the parameters we found in the previous two steps, we choose the remaining parameters so that our model replicates the economic and educational environment as close as possible to that of the early 1990s in the U.S. while at

the same time respecting the market clearing conditions.

## 4.1 Demographics

A period in this model is two years. Agents begin life at age 18. They are considered young until age 36 ( $j_o$ ) at which time they become old until they enter retirement at age 66 ( $J$ ). The population growth rate  $\eta$  is set to an annual rate of 1.20 percent.

## 4.2 Preferences

Preferences come from the CRRA family of utility functions. Speciality, the per-period utility function is

$$u(c, l) = \frac{(c^\lambda l^{1-\lambda})^{1-\sigma}}{1-\sigma},$$

and the retirement value function is of the form

$$\phi(a_J) = \beta_R \frac{(a_J)^{1-\sigma}}{1-\sigma}.$$

The per-period utility function was chosen to allow for consumption and leisure to be complements, a potentially important feature of the college experience. In-line with preference parameter values found in the life-cycle literature we set the risk aversion parameter  $\sigma = 2.0$ , the utility weight of consumption is chosen to be  $\lambda = 0.33$ , and the agent's subjective discount factor is  $\beta = 0.98$ . The remaining parameters,  $\beta_R$ , is determined in the estimation of the benchmark economy.

## 4.3 Initial Ability, Initial Assets, and the Ability Transition Matrix

Black, Devereux, and Salvanes (2003) contend that the correlation between educational attainment of parents and their children are most likely due to inherited ability and family characteristics such as the resources to invest in education when the child is young. As we discussed in the model description section above, the initial asset and ability state pair  $(a_0, \theta_h)$  essentially summarize all socioeconomic characteristics of the agent that were determined prior to making the college/work decision. Thus, the parametrization of the joint probability distribution  $\Omega(a_0, \theta_h)$  should be in-line with the empirical facts on the relationship between initial assets of young agents and their schooling ability. Previous work by Cameron and Heckman (1998) and Black, Devereux, and Salvanes (2003) suggests

that long-term family resources have a strong influential affect on a student’s ability as measured by standardized testing at the end of their high school years. In addition, Keane and Wolpin (2001) find that parental transfers are a monotonically increasing function of the parent’s education. Given the positive relationship between parental education and parental wealth, this implies that our parameterization of the probability distribution  $\Omega(a_0, \theta_h)$  should capture a correlation between initial assets available to agents and their innate high school ability.

To completely characterize the ability learning process we must also estimate the ability transition matrix  $\Pi(\theta_h, \theta_c)$ . It would be inappropriate to simply assume that one’s college ability is independent of their high school ability. A more reasonable assumption is that better performing high school students are more likely to perform well in college. That is, there is persistence in ability going from high school to college. Such an assumption does not give us any guidance on how persistent ability is however. We therefore free ourselves from making arbitrary assumption about  $\Pi(\theta_h, \theta_c)$  by estimating the ability transition matrix using data on high school and college GPAs.

We employ the use of the 1993 National Postsecondary Student Aid Survey (NPSAS:93) and High School and Beyond Sophomore Cohort (HS&B) data to carry out our estimation of the joint probability distribution  $\Omega(a_0, \theta_h)$  using a two-step procedure that naturally correlates initial assets and innate ability. Neither data set contains a sufficiently complete record of family income, parental contributions, and high school ability to allow us to estimate  $\Omega(a_0, \theta_h)$  with one set of data. However the HS&B data does contains high school and college GPA data, along with other control variables that enables us to estimate the ability transition matrix  $\Pi(\theta_h, \theta_c)$  with a single data set.

The probability distribution  $\Omega(a_0, \theta_h)$  is estimated as follows. In the first step we use the NPSAS:93 data and partition family income into quintiles. For each income quintile, initial assets are estimated as the discounted average four year family contribution for students attending four-year institutions. In the second step, we take the HS&B data and partition family socioeconomic status into quintiles. A kernel density of cumulative high school GPAs, normalized to lie in the unit interval, is then estimated for each socioeconomic quintile. Under the assumption that income and socioeconomic status are sufficiently correlated, this procedure provides us with an estimate of the distribution of initial ability (as measured by high school GPA) and initial assets (as measured by family contributions). Therefore, we have naturally correlated assets and ability to match their empirical counterparts. Because we normalize our measure of mass to one, the kernel density estimate provides us with the joint probabilities for  $\Omega(a_0, \theta_h)$ . We transform these estimates of initial assets into model units by expressing them as a fraction of the wage of young college graduates in the

benchmark model according to the corresponding ratio calculated using annual wage data from the March CPS supplements.

The estimation of the ability transition matrix is more straight forward. Using the HS&B data we normalize high school and college GPA to lie in the unit interval, and then partition them into quintiles. We then estimate an ordered probit to obtain the probability of moving from high school ability quintile  $q_i$  to college ability quintile  $q_j$ . Our probit model controls for numerous personal and institutional characteristics affecting college GPA, including: selectivity of college, type of college (public or private), degree expectations, race, high school region, mother and father’s education, and income. Finally, we average the predicted transition probabilities across individuals in each respective high school GPA quintile. The probability transition matrix values are

$$\Pi(\theta_h | \theta_c) = \begin{bmatrix} 0.41 & 0.24 & 0.18 & 0.12 & 0.05 \\ 0.29 & 0.23 & 0.21 & 0.17 & 0.10 \\ 0.19 & 0.20 & 0.23 & 0.22 & 0.16 \\ 0.12 & 0.16 & 0.22 & 0.25 & 0.25 \\ 0.05 & 0.10 & 0.17 & 0.26 & 0.42 \end{bmatrix}.$$

where  $\pi_{ij}$  represents the probability of a newborn in the  $i$ th ability quintile drawing a college ability in the  $j$ th quintile.

## 4.4 College

On average, a bachelor’s degree in the U.S. requires a student to complete 120 credit hours. For computational purposes, we scale down the credit choice set so that one model credit corresponds to 10 credit hours. Thus, the graduation credit requirement is  $\underline{x} = 12$ . The returns to effort in terms of credits produces is dictated by the curvature of the credit production technology. Because our credit accumulation function is analogous to the human capital accumulation function we set  $\gamma = 0.70$ , a standard parameterization in the literature. The remaining college parameters are related to the financial aspect of higher education.

In the 1992-93 school year, tuition and fees represented only 40 percent of the estimated total annual cost at public 4-year institutions.<sup>11</sup> It is important to accurately measure the total direct cost of college as the influence of credit constraints may be biased downward otherwise. Therefore, our measure of tuition is average tuition, fees, and room and board charges at public four-year institutions during the 1992-3 school year as reported by College Board (2006) measured on a per-credit basis consistent with our model units. This measure

---

<sup>11</sup>Tuition and fees were \$2,334 compared to total charges of \$5,834.

of tuition more completely reflects the total direct cost of attending college than simply using tuition. As with assets, we express the per-credit cost of college  $\mathcal{T}$  as a fraction of young college graduates' wage.<sup>12</sup> We find that the per-credit cost of college is approximately 7 percent of a young college graduate's wage calculated using the 1994 March CPS supplement. In the benchmark estimation we set  $\mathcal{T} = 0.07w_y^g$ .

Per-period grants  $y$  are estimated using the 1992-93 National Postsecondary Student Aid Study. Our measure of grants is designed to account for the wide range of financial aid available to students in addition to student loans. Grants are computed as the average sum of all federal, state, institutional, and other grants and scholarships. As a percent of a young college graduate's wage, we find total grants to be 5.18 percent. Therefore, in the benchmark estimation  $y = 0.0518w_y^g$  per period.

**Table 1: 1992-1993 Stafford Loan Annual Loan Limits**

Class Level	Loan Limit (Data)	Period	Loan Limit (Model)
First & Second Year	\$2,625	First	$-0.17w_y^c$
Third & Fourth Year	\$2,625	Second	$-0.42w_y^c$
Fifth Year	\$4,000	Third	$-0.67w_y^c$

Per-period borrowing constraints are chosen to match as closely as possible the Federal Stafford Loan Program as it existed during the 1992-93 academic school year. Since our model of college corresponds to a maximum of six years, but students may only participate in the federal loan program for up to five years, we restrict the amount a student may borrow if they take three periods. As with tuition and grants we must convert the empirical loan limits into model units. Again, this is done by expressing benchmark loan limits as a fraction of the young college graduates wage. In table 1 we present the loan limits used in the model along with their empirical counterparts.

Under the federal program students repayment does not begin until after leaving school. To mimic this feature of the student loan program we have adjusted the loan limits to allow for cumulative debt  $a$  to be rolled over each period. The final student loan parameter relates to the repayment of debt after leaving school. The standard repayment plan under the federal program is ten years which can be approximated by setting the fraction of debt that maybe rolled over each period after entering the workforce  $\kappa$  to be 0.50.

<sup>12</sup>To account for the fact that we have scaled the 120 credit hour degree requirement down by twelve, we also adjust the per-credit cost of college before expressing in terms of wages.



## 4.5 Production Function

The production function has a total of ten parameters. In addition, we must specify the depreciation rate which we choose as  $\delta = 0.06$ . Capital's share of income  $\alpha$  is set to 0.36. We normalize the aggregate productivity parameter  $A$  to unity. The elasticity of substitution between high school and college workers  $\sigma_E$ , and the elasticity of substitution between young and old workers  $\sigma_A$  pin down the parameters  $\rho$  and  $\varphi$ . Card and Lemieux (2001) estimate the elasticity of substitution between high school and college educated workers to be about 2.5, and the elasticity of substitution between young and old workers to be around 5. Using these estimates and the fact that  $\rho = 1 - 1/\sigma_E$  and  $\varphi = 1 - 1/\sigma_A$ , we set  $\rho = 0.60$  and  $\varphi = 0.80$ .

The remaining productivity parameters  $(\nu_y, \nu_o, \psi_y, \psi_o, A_H, A_G)$  were estimated as follows. First note that we can invert the equilibrium wage equations and form the following ratios

$$\frac{A_C}{A_H} = \frac{w_C}{w_H} \left( \frac{G}{H} \right)^{1-\rho} \quad (19)$$

$$\frac{\psi_o}{\psi_y} = \frac{w_{C_o}}{w_{C_y}} \left( \frac{G_o}{G_y} \right)^{1-\varphi} \quad (20)$$

$$\frac{\nu_o}{\nu_y} = \frac{w_{H_o}}{w_{H_y}} \left( \frac{H_o}{H_y} \right)^{1-\varphi}. \quad (21)$$

The ratio of productivity units determines the the various age and skill premiums in the economy. Because the premiums are relative we can set  $\psi_y = \nu_y = A_H = 1$ . Thus, given our choices for  $\rho$  and  $\varphi$ , and values for average wages and aggregate labor supply which we calculate from the 1994 March CPS supplement using the method of Card and Lemieux (2001), equations(19) – (21) provide us with estimates for the three remaining parameters. These were found to be  $\nu_o = 1.29$ ,  $\psi_o = 1.68$ , and  $A_C = 1.37$ .

## 5 Estimation and Model Evaluation

Using a minimum distance approach we estimate the model's three remaining parameters: the relative importance of retirement wealth  $\beta_R$ , and the two parameters relating to the college dropout wage,  $\chi$  and  $p$ . The parameters are chosen so that the following three aggregate economic statistics produced in the baseline economy comes as close as possible to those of the educational environment of the early 1990s in the U.S. while at the same time respecting the market clearing conditions.

**Enrollment and dropout rates:** The first two statistics that we target are the college enrollment rate and college dropout rate. Enrollment and dropout rates are nontrivial to

calculate because of the many ways to define them. Our model is best used to study first-time college students considering a four-year college path. To keep our targets in-line with our model, we choose to target the enrollment and dropout rates corresponding to these students. According to the BLS, 62 percent of recent high school graduates enrolled in college (broadly defined) in 1993. Nearly two-thirds (64 percent) of these students enrolled in 4-year institutions. Thus, we choose the enrollment rate target to be 40 percent. Gladieux and Perna’s (2005) use the Beginning Postsecondary Students data (BPS) to estimate the college dropout rate for first time students. The authors classify anyone who has not graduated by the end of the sixth year of the study as a dropout. In reality, a good portion of those students still in school after six years will eventually graduate. Using the tables provided in their paper a range of 24%-39% can be placed on the real dropout rate depending on the graduation/dropout assumption of students still in school at the end of the study. We chose a benchmark dropout target rate of 26%, well within in the range of the real dropout rate.

**Time-to-Degree:** The third target is time-to-degree at four-year colleges and universities. We rely on the recent empirical work of Bound, Lovenheim, and Turner (2006) that focuses on this often neglected topic. Similar to the problem with accurately calculating dropout rates, time-to-degree estimates suffer from the data’s failure to track every entering student until they complete their degree. Bound, Lovenheim, and Turner (2006) use the National Educational Longitudinal Study of 1988 to estimate the time-to-degree for the high school class of 1992 that first enrolled at non-top 50 4-year institutions at 5.23 years. This value is most likely lower than the actually time to degree because they condition on having received a B.A. within eight years of entering. However this horizon is sufficiently long that their estimate should not be far from the true mean. Thus, the benchmark time-to-degree target is set to 5.23 years.

In table 2 we summarize the benchmark estimation results. The top panel shows how well the model performs compared to our chosen targets, while the middle panel presents the corresponding parameter estimates.

**Table 2: Estimation of the Model (Annualized Values)**

<b>Statistic</b>	<b>Target</b>	<b>Model</b>
Enrollment Rate	40.0%	39.6%
Dropout Rate	26.0%	27.8%
Time-to-Degree	5.2 years	5.4 years

  

<b>Variable</b>	<b>Parameter</b>	<b>Value</b>
Return to partial education	$\chi$	0.86
Prob. of return to partial education	$p$	0.92
Continuation value factor	$\beta_R$	3.68

Overall, the model performs well relative to the educational statistics calculated from the data. In the baseline economy the enrollment rate is 39.6 percent, whereas the empirical counterpart at four-year institutions is 40.0 percent. The model’s predicted dropout rate is slightly higher than the target in the data 27.8 percent versus 26.0 percent. However, the obtained figure is within the range of estimates of Gladieux and Perna’s (2005). With respect to our last estimation target we find that the model is consistent with an average time-to-degree of within 2 months and 12 days of that calculated by Bound, Lovenheim, and Turner (2006). To put this difference in perspective, consider that the benchmark time-to-degree is within the terms of a regular academic quarter of that found in Bound, Lovenheim, and Turner (2006). The parameters controlling the dropout wage rate reveal that the college dropout wages is determined as a 86/14 percent weighted average of the high school wage and college wage, respectively. However, the model’s predicted probability of receiving the dropout wage is only 8 percent. These two parameters combined are consistent with the fact that the return to partial education is much closer to that of high school graduates wage than to a college graduates wage.

Since we use a production function with heterogenous labor inputs it is interesting to explore the implied wage premiums that result from the parameterized baseline model. Wage rates, or more specifically wage premium, are a key factor determining enrollment and dropout rates. The wage rates in the model do not represent an agent’s net return to education. Calculating the return to education is complicated by other factors that vary across individuals such as the initial assets and the agent’s financing choice. For individuals that have sufficient resources at hand the cost of education is lower than those that need to use student loans which require interest payments. With this in mind, table 3 below compares the following skill and age premiums produced by the model with those found in the data: college to high school skill premium  $w^g/w^h$ , the college age premium  $w_o^c/w_y^c$ , and

the high school age premium  $w_o^h/w_c^h$ . It is important to be aware that the wage premiums are the result of solving for the equilibrium and are not targeted in the estimation procedure.

**Table 3: Wage Premiums by Age**

Wage Premium	Ages	Data	Model
College Skill Premium ( $w^g/w^h$ )	18-65	1.87	1.82
College Age Premium ( $w_o^g/w_y^g$ )	24-65	1.55	1.52
High School Age Premium ( $w_o^h/w_c^h$ )	18-35	1.27	1.25

Data source: Current Population Survey (March Supplements)

The wage premiums implied by the model are consistent with their empirical counterparts calculated using the March CPS. For example the model predicts a college skill premium of 1.82 compared to an empirical skill premium of 1.87.<sup>13</sup> The college age premium is determined by the ratio of wages of college graduates that are between ages 36 and 65 and those between ages 24 and 35. This feature captures the upward slopping profile of earnings over the life-cycle. This upward trend in wages over time within education groups is also found in high school graduates as indicated by a high school age premium of 1.27 in the model. The model’s capacity to replicate certain features from the data without directly targeting them is certainly important if we are to use this model for policy analysis.<sup>14</sup>

## 6 Education Policy

In this section we explore the impact of various education policies on the incentives to enroll in college, extend time in school, and drop out. Predicting the affects of education policy in our model is complicated by several features. First, the structure of the financial aid package offered interacts with each agent’s existing resources, borrowing constraints, ability, and time endowment which must be allocated optimally among effort required, labor supply, and leisure. Second, we model college as a multi-period lumpy investment. The restriction on students that credits may only be chosen in a discrete fashion prevents them from adjusting their investment decision in a continuous manner in response to policies. In turn, this requires that policy interventions be of sufficient size to affect behavior across the various behavioral margins. Third, the correlation between ability and financial resources is likely to impact

<sup>13</sup>Because there are two age specific labor inputs for each education level, the skill premium is the ratio average college wage  $w^g$  to average high school wage  $w^h$ . These were calculated as  $w^g = \frac{dY}{dN} \frac{dN}{dG}$  and  $w^h = \frac{dY}{dN} \frac{dN}{dH}$ , respectively.

<sup>14</sup>For computational reasons these parameters have not been estimated. The cost of solving the model with the degree of accuracy needed for policy analysis is relatively large. Given the model success of replicating the wage premiums we view the approach as the only viable alternative.

the behavior of lower income students who are also from the bottom end of the ability distribution. At the same time there general equilibrium effects in the labor markets that encourage targeted students to enter the labor market without obtaining a college education.

It is our contention that the complexity of the model is what makes it ideal for studying the role of education policy intervention. The majority of our analysis is focused on policies that affect the cost of tuition and change the size of government provided grants. In contrast with Gallipoli, Meghir, and Violante (2006) who assume a single arbitrary change in the size the education program, we consider the impact of tuition and grant policies across a range education program sizes. The advantage of this approach is that allows to determine the effectiveness of each program to impact the decisions of college students across the various behavioral margins.

## 6.1 Tuition

In this section we turn our attention to comparing three different tuition programs: a pure subsidy program, a merit based tuition subsidy, and flat tuition rate policy. Under the pure subsidy program the per credit tuition price faced by students in each period of college is  $\mathcal{T}^* = (1 - \mu) \mathcal{T}$ , where  $\mu$  is the per credit subsidy. The remaining cost of subsidizing tuition  $\mu \mathcal{T}$  is financed by tax revenue collected from workers. Instead of specifying the subsidy rate explicitly we fix the aggregate education budget at a percent above the zero subsidy baseline education budget and then calculate the resulting per credit subsidy. This allows us to precisely control the education budget under different increases. Because we do not allow for government debt all increases in government expenditures as a result of an increase in educational spending must be financed through current tax revenue. In order to gauge the magnitude of increase in the education budget that would be needed to significantly alter college behavior we implement the pure subsidy program under a 20 percent, 100 percent, and 150 percent increase of the baseline budget.

The merit based tuition programs condition the tuition subsidy on the completion of at least  $x_\mu$  credits in the first year in college. Students that meet or exceed this merit based qualification requirement receive a *one time* tuition subsidy in the second period. For brevity the merit based program is carried out under only a doubling of the education budget. To qualify for the subsidy students are required to complete 50 credits (or 5 model credits) in the first period of college. Meeting the first period credit requirement places the student on a path to graduate in under 5 years. Formally, the merit based tuition function that

determines the cost of each education credit in the second period only can be represented by

$$\mathcal{T}(x) = \begin{cases} (1 - \mu) \mathcal{T} & \text{if } x \geq x_\mu \\ \mathcal{T} & \text{if } x < x_\mu \end{cases}$$

Our last tuition experiment is the introduction of a flat tuition rate equal to the product of the baseline per credit tuition price and graduation credit requirement,  $\mathcal{T}\underline{x}$ . Thus, the flat tuition assumes that the cost of education is independent of the number of credits registered for, but still equal to the total cost under per credit pricing and a four year college path. In table 5 below, we present the results of the three different tuition policy experiments.

**Table 5: Tuition Based Policies**

<b>Education Statistic</b>	<b>Tuition Subsidy Program by Size</b>				<b>Merit</b>	<b>Flat</b>
	<b>Baseline</b>	<b>20%</b>	<b>100%</b>	<b>150%</b>	<b>Based (100%)</b>	<b>Tuition</b>
Enrollment Rate	39.62%	41.33%	44.01%	46.71%	40.35%	31.23%
Dropout Rate	27.84%	28.47%	28.73%	29.31%	23.88%	15.04%
Time-to-Degree (years)	5.39	5.37	5.61	5.73	5.35	4.78
Subsidy ( $\mu$ )	0%	4.94%	24.03%	32.69%	67.50%	0%
Expenditures/GDP	1.62%	1.91%	3.14%	3.89%	3.09	1.20%
<b>Labor Market</b>						
Fraction Skilled Labor	28.14%	28.95%	30.69%	32.27%	29.59%	26.35%
College Skill Premium ( $w^g/w^h$ )	1.82	1.79	1.73	1.67	1.77	1.89
College Age Premium ( $w_o^g/w_y^g$ )	1.52	1.52	1.51	1.50	1.52	1.53
H.S. Age Premium ( $w_o^h/w_c^h$ )	1.25	1.25	1.24	1.21	1.25	1.25

The aggregate effects on education point towards an adverse selection problem resulting from implementing uniform tuition subsidies as it does not appear possible to simultaneously increase enrollment and reduce dropouts. While lowering the cost of school enables some of the poorer students to enroll and eventually complete their degree, it also encourages less well prepared students to attempt college stemming from the correlation between wealth and ability. The result is an improvement in enrollment, but a deterioration in the completion rate. Notice though that time-to-degree is not an monotonically increasing function of subsidy expenditures. Comparing the baseline economy with the 20 percent tuition subsidy experiment we see a relatively flat response in time-to-degree. While tuition is subsidized, the subsidy is too small to prevent dropping out and extending time in school. Thus, only

students that graduated in the baseline economy graduate when spending is increased moderately by 20 percent, and their decision to prolong school is minimally impacted. Further expenditure increases cause time-to-degree to increase in part because a fraction of students that were on the margin of dropping out are now able to complete their degree. In addition, as mentioned previously, newly enrolled students from the lower end of the ability distribution will require longer than average to finish college. An increase in time spent in college equal to one semester results from a 150 percent increase in the baseline education budget.

Turning to the labor market we see improvements with respect to the composition of the labor force and wage inequality. Not surprisingly the 150 percent budget increase generates the largest decrease in the skill premium equal to 8.2 percent versus a decrease of 5 percent with a 100 percent increase, and only 1.65 percent resulting from a 20 percent spending increase. The within education group age premium are relatively flat across subsidy experiment although a slight compression in wages occurs for high school educated workers under the 150 percent expenditure increase. As more young college eligible agents enroll in college there must be a general equilibrium effect incentivizing some students to enter the labor market without a higher education.

Conditional on enrolling in college, the government has no ability in our model to directly observe ability. As we have seen this leads tuition subsidies to simultaneously increase the enrollment rate and dropout rate. While the skilled labor force increases, indicating that the enrollment effect dominates the dropout effect, it may be possible to screen students requiring financial aid and positively influence the completion rate. One popular method of doing this is by offering merit based aid. The potential downside of such a policy is that the correlation between assets and ability makes it unlikely that for a given increase in spending, a merit based policy will be able to solicit the same enrollment response as uniform tuition subsidies while at the same time improving attrition. Looking at the results from a 100 percent expenditure increase directed into merit based subsidies we see this to indeed be the case. Relative to the benchmark economy enrollment does rise although the increase is modest. More importantly is the significant decrease in college dropouts; a 14.2 percent when compared to the benchmark .

An alternative to merit based aid that allows for partial screening of students by ability is the introduction of a flat tuition pricing strategy. In this experiment we assume that the cost of education is a flat tuition fee independent of the number of credits registered  $\bar{T}$ . Ignoring class congestion, this type of credit pricing implicitly subsidizes individuals that have incentives to proceed through college quickly (wealthy, high ability agents), from those that require more time (poorer, low ability). As a result we would expect to see a reduction in enrollment, time in school, and the dropout from the introduction of a fixed cost to

enrollment. In-line with this reasoning, the model predicts a dramatic reduction in the aggregate enrollment rate, the number of college dropouts, and the time to degree compared to the baseline pricing policy. The results suggest that instituting a flat tuition rate equal to cost for a normal four year student would reduce enrollment by 21.2 percent. Due to the correlation between financial resources and ability the students that do enroll are better off financially and in terms of ability. The interaction between the two leads this type of pricing strategy to be very effective in generating completed degrees, and reducing dropouts time-to-degree. Specifically, time-to-degree is reduced 11.2 percent while the number of dropouts are nearly cut in half. Despite the increase in the graduation rate, the reduced fraction students enrolling results in increased wage inequality as indicated by the skill premium.

An important caveat relating to the flat tuition policy as implemented in our model must be discussed. It appears as though instituting a flat tuition pricing strategy reduces aggregate educational expenditures as a percent of GDP. However, in all actuality we would expect there to be the need for institutional subsidies in order to induce universities to adopt such a policy. Thus, focus should be given more towards the affect such a policy has on the behavior of student than it does on budget or behavior of universities.

The experiments suggest that if the objective of education policy is to increase enrollment, uniform tuition subsidies seem to be moderately effective depending on the amount of resources allocated to education. Tuition subsidies change the relative price of education and as a result students consume more education credits. The downside of the policy is that since on net more marginal ability students choose to participate the number of dropouts and time to degree increases. Merit based programs appear to provide better incentives to complete college, although the affect on enrollment and time-to-degree is small. The main reason is that less able students do not benefit from the merit based tuition reduction. As a result, the program only benefits a subset of the student population that is capable of completing the minimum number of credits. A flat rate tuition policy would be most effective if the objective is to reduce the number of college dropouts and time-to-degree. Unfortunately, instituting such a policy would have severe negative implications for enrollment and wage inequality. The model suggests that an education policy that simultaneously wants to increase enrollment and reduce the number of dropouts and time to degree has to combine tuition subsidies for a self-selected groups of students with flat tuition for the remaining. This pricing strategy would eliminate the apparent trade-off between enrollment and dropout rates of more simple education policies.



## 6.2 Grants

Grants and scholarship are a popular way of providing students with alternatives to working or borrowing while in school. The two main types of grants and scholarships are need based and merit based. Regardless of the type of grant, they differ in one fundamental way from tuition based policies. As we discussed previously, tuition subsidies change the relative price of education and in turn generate both a substitution and income effect. On the other hand, a change in size of grants available to students is only associated with a income effect. As a result, grants do not necessarily provide the same incentives to tilt more of ones budget towards direct educational expenditures and away from leisure. While an individual subject to an increase in financial resources can afford to purchase more credits they are also able to consume more leisure. But an increase in leisure lessens the amount available for work and effort. Thus it is unclear if grants have the features necessary to improve upon tuition subsidies. In the case of need based grants, they may be an effective tool for increasing enrollment by allowing poor students to enroll, take a few credits and then allocate the majority of their time to work and leisure. But just as grants may create large enrollment incentives, so too may they result in a large number of dropouts due to the correlation of financial assets and schooling ability. Compared to need based grants we would expect merit based grants to carry with them a more moderate enrollment response, and hopefully an improvement in college completion.

Similar to the previous section on tuition policies, we employ the use of our model to explore the consequences of instituting a uniform increase in grant spending as well as towards a merit based program. Under the uniform grant policy all students experience an increase in their per period grant. For comparability to the tuition subsidy experiments we increase the educational budget 20 percent, 100 percent, and 150 percent, and then solve for the corresponding new per period grant that makes the aggregate increase attainable. Again, every increase in educational spending is financed by an increase in the lump-sum tax charged to workers. For simplicity we only institute the merit grant program under a 100 percent increase. Merit based grants are only provided during the second period. In order to receive the merit based grant each student complete  $x_\mu$  credits by the end of the first period. Students that fail to achieve the minimum number of credits only receive the benchmark grant. The results of the various policy experiments are summarized in table 9.

**Table 9: Grant Based Policies**

Education Statistic	Grant Program by Size				Merit Grant (100%)
	Baseline	20%	100%	150%	
Enrollment Rate	39.62%	41.05%	51.42%	55.53%	39.65%
Dropout Rate	27.84%	29.10%	39.93%	42.88%	22.70%
Time-to-Degree (years)	5.39	5.46	5.58	5.80	5.63
Grants (% of 4-year college cost)	13.15%	15.09%	21.79%	25.02%	55.71%
Expenditures/GDP	1.62%	1.93%	3.16%	3.92%	3.19%
<b>Labor Market</b>					
Fraction Skilled Labor	28.14%	28.60%	30.26	30.94%	29.29%
College Skill Premium ( $w^g/w^h$ )	1.82	1.80	1.74	1.72	1.76
College Age Premium ( $w_o^g/w_y^g$ )	1.52	1.51	1.51	1.51	1.51
High School Age Premium ( $w_o^h/w_c^h$ )	1.25	1.25	1.24	1.24	1.24

By comparing the response to uniform grants to that of tuition based policies we find that grants face the same general trade-offs as tuition subsidies. The increase in grant spending has a positive effect on the enrollment rate, but it also increases the dropout rate and the time to degree. Immediately though we see a much greater response in enrollment and dropout behavior to large increases in spending directed towards grants than to tuition subsidies. While the enrollment responses with uniform grants are similar to those of a tuition subsidy under a 20 percent aggregate expenditure, a doubling or more in education spending leads to approximately a 17 to 19 percent increase in enrollment over that seen with tuition subsidies. The same holds true in terms of drop out behavior. Uniform grants increased the dropout rate 39 to 46 percent more than tuition subsidies do under the two largest budget increases. While we do not model the interaction between skill acquisition and employment risk in this paper, Gladieux and Perna's (2005) document higher rates of unemployment for college dropouts. Thus, uniform grants may be even more detrimental when the employment of college dropouts is considered.

The benefit of increased enrollment does not appear to translate in vast improvements in the skill composition of the labor force or wage inequality. When compared to tuition subsidies, grants are marginally worse along these two dimensions. In addition, the relative budgetary cost of implementing a broad based grant program increases (in terms of GDP). Only if the goal of public policy is to target enrollment should uniform grants alone be encouraged over uniform tuition subsidies.

Turning attention towards the merit based grant program we see that relative to the benchmark, the enrollment response is quite flat while college completion is significantly

improved upon. The improvement in competition comes at the small cost of extending the average time needed to finish school by less than 3 months. As the result of more students graduating the fraction of skilled workers increases. Wage inequality between skilled and unskilled workers is reduced due to the general equilibrium effects of more college graduates. When compared to the merit based tuition program we find similar results along most dimensions. The tuition program performs marginally better with respect to enrollment and total cost while the grant program improves slightly upon the dropout rate. Notice however that the relative price effect of merit based tuition subsidies forces students to direct more expenditures towards college credits and appears to explain the improvement in time-to-degree relative to the merit based grant program.

### **6.3 Loan Limit**

The existence and magnitude of borrowing constraints has been a point of contention for some time. Carneiro and Heckman (2002) contend that at most 8 percent of the U.S. population is credit constrained when it comes to post-secondary education. In the absence of unanimous agreement, the most common approach taken by researchers has been to make assumptions about borrowing constraints and proceed. For example, Caucutt and Kumar (2003) assume that all borrowing for human capital investment is prohibited while Akyol and Athreya (2005) always allow agents to borrow enough to cover their education. Recently Keane and Wolpin (2001) and Keane (2002) have suggested that borrowing limits interact in an important way with labor supply. If students are allowed to work in addition to borrow than any tightening of education related loan limits work primarily through the labor-supply margin and not the enrollment margin. We investigate this conclusion further by tightening loan limits with and without allowing agents to work while in school. This extends the work of Keane and Wolpin (2001) who only allow students three work options: no work, part-time work, and full-time work. As in Keane (2002), agents in our model are permitted to continuously adjust their labor supply. However in Keane (2002) there is no heterogeneity amongst individuals.

**Table 10: Loan Limits and Labor Supply**

Loan Limit Reduction:	Baseline	15%	15%	40%	40%	60%	60%
Work in College:	✓	✓		✓		✓	
<b>Education Statistic</b>							
Enrollment Rate	39.62%	39.43%	39.12%	38.71%	38.31%	37.73%	33.94%
Dropout Rate	27.8%	28.50%	25.83%	29.31%	26.27%	29.77%	26.83%
Time-to-Degree (years)	5.39	5.45	5.25	5.60	5.24	5.61	5.21

In table 10 we present the results from reducing the baseline loan limits with and without allowing students to work. While enrollment does fall somewhat when borrowing limits are tightened, it is only when loan limits are reduced by over half of their baseline amount *and* the work option is removed do we see a significant enrollment effect. Removing the work option and reducing borrowing limits by 60 percents leads to a nearly 6 percentage point (14 percent) decline in enrollment from the baseline. Allowing the agent to work to finance his education in face of such a drastic reduction in available credit mitigates the enrollment response. Students must commit more time to work and as a results we see an increase in time to degree and the dropout rate.

Reducing the borrowing constraint by anything less than 60 percent only marginally impacts enrollment. This holds whether individuals are permitted to work or not. Interestingly, when students do not have the option to work we see a decrease in both the dropout rate and the time needed to complete college. Students borrow more to cover the lost labor income, but now since their time is only allocated between school and leisure they are able to commit more time to school.

The results are interesting and compare to those of Keane (2002). They suggest that ignoring the labor supply of college students when studying borrowing constraints can lead to erroneous results, especially when the focus is on the severity of credit constraints. Nominal aggregate loan limits under the federal student loan program increased approximately 33 percent in the early 1990s. The failure to index the loan limits to inflation and the rise in tuition since then has resulted in real loan limits below those of the early 1990s. While enrollment has not suffered we do know that more and more students are working to finance their education and are taking longer to complete their schooling. The labor supply/loan limit interaction may be able to explain at least part of this phenomenon.

## 7 Conclusions

In this paper we develop a quantitative theory of college education which is embedded within the context of general equilibrium overlapping generations economy. We depart from the standard human capital literature and model college as a multi-period risky investment with endogenous enrollment, time-to-degree, and dropout behavior. The tuition expenditures required to complete college can be funded using federal grants, student loans, and working while in college. We use the model to test the effectiveness of three distinct education policies: tuition subsidies (broad based, merit based, and flat tuition), grant subsidies (broad based and merit based), and loan limit restrictions (with and without endogenous in-school labor supply). Our model predicts that broad based tuition subsidies and grants increase college enrollment. However, due to the correlation between ability and financial resources most of these new students are from the lower end of the ability distribution and eventually dropout or take longer than average to complete college. Merit based education policies counteract this adverse selection problem but at the cost of a muted enrollment response. We find that tuition programs perform marginally better with respect to enrollment, time to degree, and total cost while grant based programs improves slightly upon dropouts. The final policy experiment highlights an important interaction between borrowing constraints and the labor supply of college students. The baseline model is consistent with the findings of Cameron and Heckman (1998, 1999) and Keane and Wolpin (2001) that find short term liquidity constraints play no significant role in college attendance decisions. Nevertheless, a significant decrease in enrollment is found to occur only when borrowing constraints are severely tighten and the option to work while in school is removed. This result suggests that previous models that have ignored the student's labor supply when analyzing borrowing constraints may be lacking and insufficient for understanding the impact of education policy.

In a situation where the government has no information about student ability or college performance, we find that a significant adverse selection problem that prevents broad-based education policies (tuition subsidies and grant) from simultaneously increasing enrollment and reducing the number of dropouts and time to degree. However, there may exist merit based programs that would eliminate the apparent trade-off between enrollment and dropout rates of the uniform education policies. We leave the study of all these policies for future research.

## References

- [1] Akyol, Ahmet and Kartik Athreya 2005. Risky higher education and subsidies, *Journal of Economic Dynamics and Control*.

- [2] -, (2006). "Unsecured Credit and Self Employment," Working paper, Dept of Economics, York University.
- [3] Altonji, Joseph (1991). "The Demand for and Return to Education when Education Outcomes are Uncertain," NBER working paper 3714.
- [4] Becker, Gary, 1964. Human Capital Investment
- [5] Ben-Porath, Yoram, 1967. The production of human capital and the life cycle of earnings, *The Journal of Political Economy*.
- [6] Black, Sandra E., Devereux, Paul J, and Kjell G. Salvanes (2003). "Why the Apple Doesn't Fall Far: Understanding the Intergenerational Transmission of Human capital," NBER Working paper 10066.
- [7] Cameron S.V. and James, J. Heckman (1998). "Life Cycle Schooling Educational Selection," *Journal of Political Economy* 106. pp: 262-333.
- [8] -, (1999). "The Dynamics of Educational Attainment for Blacks, Hispanics and Whites," NBER working paper 7249.
- [9] Caucutt, Elizabeth and Krishna Kumar, 2003. Higher education subsidies and heterogeneity: A dynamic analysis, *Journal of Economic Dynamics and Control*.
- [10] Comay, Melnik, and Pollatschek (1973) "The Option Value of Education and the Optimal Path for Investment in Human Capital," *International Economic Review* 14(2).
- [11] Gallipoli, Giovanni, Meghir, Costas, and Giovanni Violante (2006). "Education Decisions and Policy Interventions: A General Equilibrium Evaluation," working paper NYU.
- [12] Heckman, James J, Lochner, Lance, and Christopher Taber (1998). "General-Equilibrium Treatment Effects: A Study of Tuition Policy," *The American Economic Review Papers and Proceedings* 88:2.
- [13] Kane, Thomas and Cecilia Rouse (1995). "Labor Market Returns to Two - and Four Year - College," *The American Economic Review*, 85(3).
- [14] Keane, Michael P and Kenneth I Wolpin (2001). "The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment," *International Economic Review*.

- [15] Michael P. Keane, (2002). "Financial Aid, Borrowing Constraints, and College Attendance: Evidence from Structural Estimates," *American Economic Review*, vol. 92(2), pages 293-297.
- [16] Levhari, David and Yoram Weiss, 1974. "The effect of risk on the investment in human capital", *The American Economic Review*.
- [17] Lucas, Robert E, Jr. 1988. "On the Mechanics of Economic Development. *Journal of Monetary Economics*, Vol 22, p 3-42.
- [18] Mankiw, Charles. 1989. "Schooling as Experimentation: A Reappraisal of the Postsecondary Dropout Phenomenon," *Economics of Education Review*, 8(4).
- [19] Merton, Robert C., 1971. Optimum consumption and portfolio rules in a continuous-time model, *Journal of Economic Theory*.
- [20] Restuccia, Diego, and Carlos Urrutia (2004). "Intergenerational Persistence of Earnings: The Role of Early and College Education," *American Economic Review*, Vol94(5), pp. 1354-1378.
- [21] Roussanov, Nikolai. 2004. "Human Capital Investment and Portfolio Choice over the Life-Cycle," Working paper, University of Pennsylvania.
- [22] Uzawa, Hirofumi. 1965. "Optimal Technical Change in an Aggregative Model of Economic Growth," *International Economic Review*, Vol6, pp. 18-31.

## 8 Appendix

### 8.1 Computational Procedures

The computation of the student problem is very complex as it is not concave. As a result, the first-order conditions cannot be used. To avoid any problem we have opted for the discretization of the two continuous state variables: ability, and student loans/financial assets. We have found that a uniform distribution over ability coupled with our kernel density estimates for the mass of agents approximates the true ability distribution extremely well. The asset grid is not equally spaced. We have added more grid points when the grid in assets is negative and near the borrowing constraints. We use the recursive structure of the problem to solve the model backwards from the terminal condition and construct the value function and the optimal decision rules.

The complexity of the computation also increases because we have to solve the consumer problem and calculate the equilibrium many times to guarantee that markets clear and the model statistics are consistent with the chosen targets. Since the model has to clear six markets we place more weight on the market clearing conditions than on the parameterization targets. The equilibrium and model statistics are solved using nonlinear least squares. The objective function to minimize has two distinct components: the model equilibrium conditions and the parameter values that best fit the data. Let  $\Theta$  be the vector of model parameters and  $\bar{p}(\Theta)$  equilibrium prices that depend on the parameter values, the error minimization problem solves

$$L(\Theta) = \min_{\Theta} \left\{ \sum_{k=1, \dots, 6} \gamma_k \left( \frac{\bar{p}_{j+1}^i(\Theta_{j+1})}{\bar{p}_j^i(\Theta_j)} - 1 \right)^2 + \sum_N \alpha_n \left( \frac{\bar{F}_n}{\bar{F}_n(\Theta)} - 1 \right)^2 \right\}.$$

where  $\bar{p}_{j+1}^i(\Theta_{j+1})$  represents the equilibrium price calculated with parameters  $\Theta_{j+1}$  in iteration  $j + 1$ , and  $\bar{F}_n(\Theta)$  represents the model statistics that need to match their counter part in the data  $\bar{F}_n$ .

The indirect inference procedure proceeds as follows:

- Guess a vector of parameters  $\Theta$  and a vector of equilibrium prices  $\bar{p}(\Theta)$
- Solve the household's problem to obtain the value function and decision rules.
- Given the policy functions, calculate the implied invariant distribution  $\Gamma(\Lambda)$ , the implied aggregates  $\{\bar{F}_n\}_{n=1}^N$  and equilibrium prices  $\{\bar{p}_k(\Theta)\}_{k=1}^6$ .
- Calculate  $L(\Theta)$ , and find the estimator of  $\hat{\Theta}$  and the implied equilibrium prices  $\hat{p}(\Theta)$  that solves minimize the objective function.