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Monetary Policy, Judgment and Near-Rational Exuberance*

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Abstract

We study how the use of judgement or "add-factors" in macroeconomic forecasting may disturb the set of equilibrium outcomes when agents learn using recursive methods. We examine the possibility of a new phenomenon, which we call *exuberance equilibria*, in the New Keynesian monetary policy framework. Inclusion of judgement in forecasts can lead to self-fulfilling fluctuations in a subset of the determinacy region. We study how policymakers can minimize the risk of exuberance equilibria.

JEL codes: E520, E610.

Key words: Learning, expectations, excess volatility, bounded rationality, monetary policy.

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1 Introduction

1.1 Judgement variables in forecasting

Inflation targeting has become commonplace among the world's central banks. One feature of inflation targeting is that forecasts concerning the expected future path of the economy are often published or otherwise clearly communicated, in part to guide private sector expectations concerning likely economic developments contingent on the expected path of monetary policy. The forecast represents the central bank's best guess about the future path of the economy, given all relevant information available at the time the forecast is made. More generally, in all industrialized economies there is a forecasting community which discusses and assimilates incoming data. This community uses macroeconometric models, and the forecasts that are regularly published and discussed by the community guide the expectations of private sector and government decision-makers. In this way there is a widely understood consensus forecast for industrialized economies which in principle corresponds to the rational expectations of macroeconomic theories.

It is well-established that forecasting in practice nearly always means the use of judgement in addition to best-effort statistical analysis.[1] In the forecasting community this is known as "add-factoring" the forecast. Forecasters are well aware of the deficiencies of their models, and that important economic effects may not be captured well in the econometric analysis. The forecasters therefore naturally make post-estimation adjustments to their forecasts. Surely in many cases this judgemental adjustment is helpful. Svensson (2003, 2005), in particular, formally shows how to solve for optimal monetary policy when policymakers explicitly incorporate judgement terms that affect the forecasts of key variables, and shows that this can improve economic per-

[1] A forthright discussion of how prominently judgement enters into actual macroeconomic forecasting is contained in Reifschneider, Stockton, and Wilcox (1997). As they state, "... [econometric] models are rarely, if ever, used at the Federal Reserve without at least the *potential* for intervention based on judgement. Instead, [the approach at the Federal Reserve] involves a mix of strictly algorithmic methods ("science") and judgement guided by information not available to the model ("art") (p. 2, italics in original).

formance. An important policy channel emphasized in this analysis is the influence of the judgementally adjusted forecasts on private-sector expectations. Jansson and Vredin (2001) and Svensson and Tetlow (2005) provide empirical analyses of the impact of judgement on forecasting by the Bank of Sweden and the US Board of Governors, respectively.

However, the judgemental adjustment is also almost surely mistaken in some cases as well. When unique qualitative events occur that are thought to somehow have an affect on the economy but for which there is no reliable past experience, any adjustment that might be made contains a certain amount of guesswork.[2] In some cases, forecast add-factor adjustments might be made when in fact the event in question will have negligible fundamental impact on the economy. What is the effect of the judgement in these cases, considering that the judgementally-adjusted forecast, if believed, will affect private sector behavior and hence alter actual economic outcomes? The goal of this paper is to explore this topic.

1.2 What we do

We investigate the extent to which judgemental adjustment may lead to the possibility of self-fulfilling fluctuations. For expositional simplicity, we focus on the extreme case where the judgement variable is not intrinsically related to economic fundamentals at all. Thus our results come from a situation where the forecasting judgement being added is, fundamentally speaking, not useful in forecasting the variables of interest. This is not the most realistic case since much judgemental adjustment is in reality likely to be quite sound. But for the purposes of this paper we are most interested in the inevitable "guesswork" component of judgement which is unrelated to fundamentals, and its impact on the economy. We stress this assumption is not essential.[3]

[2] Examples of these types of events in the U.S. include the Cuban Missile Crisis, wage and price controls, Hurricane Katrina, the Y2K millenium bug, the savings and loan crisis, and the September 11th, 2001 terrorist attacks.

[3] Bullard, Evans, and Honkapohja (2006) show that self-fulfilling fluctuations can occur in cases where judgement is related to unobserved fundamentals.

We study systems with well-defined rational expectations equilibria. We replace rational expectations with adaptive learning using the methodology of Evans and Honkapohja (2001). We then investigate the equilibrium dynamics of the system if the econometric models of the agents are supplemented with judgement. We define the concept of an *exuberance equilibrium* by imposing three requirements. The first is that the perceived evolution of the economy corresponds to the actual evolution by imposing a rational expectations equilibrium with limited information, or more specifically the consistent expectations equilibrium (CEE) concept, as developed by Sargent (1991), Marcet and Sargent (1995) and Hommes and Sorger (1998). Secondly, we require individual rationality in individual agents' choice to include the judgement variable in their forecasting model, given that all other agents are using the judgement variable and hence causing it to influence the actual dynamics of the macroeconomy. Finally, we require learnability (*a.k.a.* expectational stability). When all three of these requirements are met, we say that an exuberance equilibrium exists. In our exuberance equilibria, all agents would be better off if the judgement variable were not being used, but as it is being used, no agent wishes to discontinue its use.

In this paper we present the exuberance equilibrium concept without extensive details in order to focus on the monetary policy application and to draw out some possible policy implications. The reader is referred to our companion paper, Bullard, Evans, and Honkapohja (2006) for a more detailed treatment of the equilibrium concept.

1.3 Main findings

We apply our framework to the canonical New Keynesian model of Woodford (2003) and Clarida, Gali, and Gertler (1999). We show that exuberance equilibria may exhibit considerable volatility relative to the underlying fundamental rational expectations equilibrium in which judgement does not play a role. Numerically, we show that exuberance is a clear possibility even in the case where the underlying rational expectations equilibrium is determinate. Thus an interesting and novel finding is the possibility of "sunspot-like" equilibria, but without requiring that the underlying rational expectations equilibrium of the model is indeterminate.[4]

Our findings suggest a new danger for policy makers: Choosing policy to induce both determinacy and learnability may not be sufficient, because the policy maker must also avoid the prospect of exuberance equilibria.[5] We show how policy may be designed to avoid this danger. More specifically, in the cases we study, policymakers must be more aggressive than the requirements for determinacy and learnability alone would indicate in order to avoid the possibility of exuberance equilibria.

2 Economies with judgement

2.1 Overview

Our results depend on the idea that agents participating in macroeconomic systems are learning using recursive algorithms, and that the systems under learning eventually converge. In many cases, as discussed extensively in Evans and Honkapohja (2001), this convergence would be to a rational expectations equilibrium. The crucial aspect for the present paper is that once agents have their macroeconometric forecast from their regression model, the forecast is then judgementally adjusted.

To fix ideas, consider an economy which may be described by

$$y_t = \beta y^e_{t+1} + u_t, \tag{1}$$

where y_t is a vector of the economy's state variables, β is a conformable matrix, and u_t is a vector of stochastic noise terms. For convenience we have dropped any constants in this equation. The term y^e_{t+1} represents the possibly

[4] Indeterminacy and sunspot equilibria are distinct concepts, as discussed in Benhabib and Farmer (1999). We consider only linear models, for which the existence of stationary sunspot equilibria requires indeterminacy—see for example Propositions 2 and 3 of Chiappori and Guesnerie (1991).

[5] For discussions of determinacy and learnability as *desiderata* for the evaluation of monetary policy rules, see Bullard and Mitra (2002) and Evans and Honkapohja (2003a). For a survey see Evans and Honkapohja (2003b) and Bullard (2006).

non-rational expectation of private sector agents. The novel feature of this paper is that we allow judgement, ξ_t, to be added to the macroeconometric forecast, $E_t^\star y_{t+1}$,

$$y_{t+1}^e = E_t^\star y_{t+1} + \xi_t. \tag{2}$$

We stress that if the judgement vector is null, the model corresponds to a version of systems analyzed extensively in Evans and Honkapohja (2001), and that the conditions for convergence of learning to rational expectations equilibrium in that case are well-established.

2.2 The nature of judgemental adjustment

We first discuss how we model the judgemental add-factor. We view this as an attempt to allow for the impact of occasional unique events. Let η_t represent "news" about qualitative events judged to have significant impact on the economy, where η_t measures that part of the anticipated impact on y_{t+1} that is believed not to be reflected in $E_t^\star y_{t+1}$. The forecasted future impact of this news is given by the derivative matrix

$$\frac{\partial y_{t+1+j}}{\partial \eta_t} = \psi_{t,j}, \text{ for } j = 1, 2, 3, \ldots,$$

Since we are here concerned with the judgemental adjustment, $\psi_{t,j}\eta_t$ measures the judgemental forecaster's view about the extent to which this news about qualitative events will fail to be reflected over time in the econometric forecast.

We think of η_t as pertaining to "unique" events and it has two components: (i) the expected effect of new qualitative events and (ii) new information about recent qualitative events that still have an impact on the economy. Since η_t represents news we assume it to be a martingale difference sequence (which for convenience we will take to be white noise). It might often take the value zero.

The future impact $\psi_{t,j}$ of η_t could in general have a complex time profile that reflects specific features of the unique qualitative events. For *analytical* *simplicity only* we make the assumption

$$\psi_{t,j} = \rho^j$$

for all t, j. Here ρ is a conformable matrix with roots inside the unit circle. Then

$$\xi_t = \sum_{j=0}^{\infty} \psi_{t-j,j} \eta_{t-j} = \sum_{j=0}^{\infty} \rho^j \eta_{t-j} = (I - \rho L)^{-1} \eta_t,$$

where L is the lag operator, and the total judgemental adjustment in y^e_{t+1} satisfies

$$(I - \rho L)\, \xi_t = \eta_t \tag{3}$$

or equivalently $\xi_t = \rho \xi_{t-1} + \eta_t$. Thus the expected effects of the judgemental variables on y_{t+1} can be summarized as $\rho \xi_{t-1}$, the expected impact of past news, plus η_t, the impact of current news.

While the VAR(1) form of ξ_t is convenient for our analysis, the judgemental forecasters would resist any attempt by the econometricians to reduce it to a measurable variable since they would not think it appropriate to treat past qualitative events as similar to current qualitative events, that is, they would regard it as a mistake to treat the judgement variable as a useful econometric time series. The view that the judgement variable ξ_t captures unique features added to forecasts is consistent with Svensson (2005), who also treats the judgemental term as appropriately included as an adjustment to forecasts rather than as a variable to be incorporated into the econometric model. Our analysis differs from Svensson (2005) in that we focus on the implications of erroneous judgement. Specifically, we assume that u_t and η_t evolve independently, so that the judgement variable in fact has no fundamental effect on the economy described by equation (1). This is obviously an important and extreme assumption but it is also the one that we think is the most interesting for the purpose of illustrating our main points, as it is the starkest case.[6]

[6] In Bullard, Evans, and Honkapohja (2006) we show that no substantive changes to our results are introduced when η_t and u_t are correlated.

2.3 Econometric forecasts

We now turn to the nature of the macroeconometric forecast. The hallmark of the recursive learning literature is the assignment of a *perceived law of motion* to the agents, so that we can view them as using recursive algorithms to update their forecasts of the future based on actual data produced by the system in which they operate.[7] A key aspect of this assignment is to keep the perceived law of motion (at least approximately) consistent with the actual law of motion of the system, which will be generated by the interaction of equation (1) with the agents' expectations formation process.[8] The econometric forecast will be generated by a time-series model for the endogenous variables y_t.

Suppose the econometric time-series model, in moving average form, is

$$y_t = \theta(L) v_t, \tag{4}$$

where $\theta(L) = \theta_0 + \theta_1 L + ...$ is a square summable matrix lag polynomial. Then

$$E_t^{\star} y_{t+1} = \frac{\theta(L) - \theta_0}{L} v_t. \tag{5}$$

is the minimum mean square error forecast based on this perceived law of motion. We call (5) the *econometric forecast.* It is based on the econometric model, the perceived law of motion, alone, and is the traditional description of the expectations formation process both under rational expectations and in the learning literature.

2.4 Exuberance equilibrium

2.4.1 Overview

Since expectations in the economy are being formed via equation (2), and since these expectations affect the evolution of the economy's state through

[7]Our analysis differs from but is related to the literature in finance on strategic professional forecasting, see e.g. Ottaviani and Sørensen (2004) and the references therein.

[8]In line with this literature, the econometric forecasts are based on reduced form models. It would also be of interest to examine the questions we study in the context of econometric forecasts based on structural models.

equation (1), we deduce an *actual law of motion* for this system. Combining (1), (2), (3), and (5) and solving for y_t gives the actual law of motion

$$y_t = \left(I - \beta L^{-1}\left(I - \theta_0\theta(L)^{-1}\right)\right)^{-1}\left(\beta(I - \rho L)^{-1}\eta_t + u_t\right). \tag{6}$$

judgement naturally influences the evolution of the state because it influences the views of economic actors concerning the future. The critical question is then whether there are conditions under which the agents would continue to use the add-factored forecast (2) when the economy is evolving according to equation (6). That is, could the agents come to perceive that the judgement variable is in fact useful in forecasting the state variable, even though by construction there is no fundamental relationship? Our main purpose in this paper is to answer this question.

In order to guide our thinking, we define the concept of an *exuberance equilibrium* and seek to understand the conditions under which such an equilibrium would exist. An exuberance equilibrium is one in which the evolution of the judgement variable influences actual economic outcomes, even though there may be no fundamental impact of the judgement factor. Our concept has three key components, all of which are discussed in detail in the subsections below. The first is that the econometric forecast should be consistent with the data generated by the model. In some sense, the econometric model should not be falsifiable. To impose this condition, we use the CEE concept. The second component is that each individual agent in the economy should conclude that it is in their interests to judgementally adjust their forecast, given that all other agents are making a similar judgemental adjustment. The third component is that the stationary outcome is stable in the learning process being used by the agents. Thus, given the model with judgement (1), (3), (5), and (2), an *exuberance equilibrium* exists if (*i*) a CEE exists, (*ii*) individual agents rationally decide to include the (non-trivial) judgement variable in their forecasts given that all other agents are judgementally adjusting their forecasts, and (*iii*) the CEE is learnable.

Are there conditions under which an exuberance equilibrium could exist? There are, and we argue that the conditions are in fact worrisomely plausible.

In order to obtain some intuition, we turn to an analysis of each of the above conditions.

2.4.2 Consistent expectations

The core idea of a CEE is that the econometric forecasters should see no difference between their perceived law of motion for how the economy evolves and the actual data from the economy. One way to develop conditions under which such an outcome may occur is to require that the autocovariances of the perceived law of motion correspond exactly to the autocovariances of the actual law of motion.[9]

It is possible to obtain an analytic expression for the CEE in the univariate case.[10] The procedure for the univariate case is difficult to generalize to a multivariate setting. However, it is straightforward to show how to obtain approximate CEE based on agents using a VAR(p) PLM. Estimation of VARs is in practice the standard forecasting tool in multivariate settings. We will show that while a VAR(p) process cannot deliver an exact CEE, for large values of p it will deliver close approximations.

The PLM is therefore specified as

$$y_t = \sum_{i=1}^{p} b_i y_{t-i} + v_t, \tag{7}$$

where y_t, v_t are $n \times 1$ vectors, the b_i are $n \times n$ matrices and $Ey'_{t-i}v_t = 0$ for $i = 1, \ldots, p$. This leads to econometric forecasts $E_t^{\star} y_{t+1} = \sum_{i=0}^{p-1} b_{i+1} y_{t-i}$, and to the judgementally adjusted forecasts

$$y_{t+1}^e = \sum_{i=0}^{p-1} b_{i+1} y_{t-i} + \xi_t.$$

The ALM is thus

$$y_t = (I - \beta b_1)^{-1} \left\{ \sum_{i=1}^{p-1} \beta b_{i+1} y_{t-i} + \beta \xi_t + u_t \right\}. \tag{8}$$

[9] See Hommes and Sorger (1998), Hommes, Sorger, and Wagener (2004) and Branch and McGough (2005). A closely related idea is developed in Marcet and Sargent (1995).

[10] See Bullard, Evans and Honkapohja (2006).

It is easily verified that the ALM is a VARMA(p,1) process and this is the sense in which the VAR(p) PLM can only give an approximate CEE.

Let $b = (b_1, \ldots, b_p)$ and let $P[y_t \,|Y_{t-1}] = T(b)'Y_{t-1}$ be the linear projection of y_t on Y_{t-1} where $Y'_{t-1} = (y'_{t-1}, \ldots, y'_{t-p})$. Using standard results on linear projections,

$$T(b) = \left(Ey_t Y'_{t-1}\right)(EY_{t-1}Y'_{t-1})^{-1}. \tag{9}$$

An approximate CEE is defined as a value $\bar{b}$ that satisfies the equation $\bar{b} = T(\bar{b})$. We require also that all roots of $\det(I - \sum_{i=1}^{p} b_i L^i) = 0$ lie outside the unit circle so that y_t is a stationary process. In an approximate CEE, for each variable the forecast errors v_t of the econometric forecasters have the property that they are orthogonal to Y'_{t-1}. It follows that the agents are "getting right" all of the first p autocovariances of the y_t process. For a stationary process the autocovariances $Ey_t y_{t-j} \to 0$ as $j \to \infty$ and thus stationary fixed points $\bar{b}$ deliver approximate CEE in the sense that as p becomes large the econometric forecasters neglect only high order autocovariances that are vanishingly small.

2.4.3 Learnability

Since we have made an assumption that the econometricians in the model are learning using recursive algorithms, we also need to impose learnability of any proposed equilibrium as a condition for plausibility. We study the stability of the system under learning following the literature on least squares learning in which the economic agents making forecasts are assumed to employ econometric models with parameters updated over time as new data becomes available.[11] The standard way to analyze systems under learning is to employ results on recursive algorithms such as recursive least squares. In many applications it can be shown that there is convergence to rational expectations equilibrium, provided the equilibrium satisfies a stability condition.

[11] See Evans and Honkapohja (2001) for analysis of adaptive learning in macroeconomics and references to the literature.

For the case at hand we verify the learnability condition numerically as a by-product of our computation of the approximate CEE, as we now describe. To compute $T(b)$ we write the system in first order form

$$z_t = Bz_{t-1} + D \begin{pmatrix} u_t \\ \eta_t \end{pmatrix}$$

with $z_t = (Y_t', \xi_t)'$. The relevant values for $\left(Ey_t Y_{t-1}'\right)$ and $(EY_{t-1}Y_{t-1}')$ can be obtained from the equation

$$\mathrm{vec}(\mathrm{Var}(z_t)) = [I - B \otimes B]^{-1}\,\mathrm{vec}(D\left[\mathrm{Var}\begin{pmatrix} u_t \\ \eta_t \end{pmatrix}\right] D').$$

Here $\mathrm{Var}\,(z_t)$ is the covariance matrix of z_t, $\mathrm{vec}(K)$ is the vectorization of a matrix K and $\otimes$ is the Kronecker product. The equilibrium $\bar{b}$ can then be calculated by the *E-stability algorithm*

$$b_s = b_{s-1} + \gamma(T(b_{s-1}) - b_{s-1}), \qquad (10)$$

where γ is chosen to be a small positive constant.

This procedure will automatically give us learnable equilibria in the following sense. The econometricians are estimating a VAR(p) PLM for y_t and are assumed to update their parameter estimates over time using recursive least squares (RLS). As previously explained, the decision makers add their judgemental adjustment to the econometricians' forecast and, together with the variable u_t, the current value of y_t is determined. The vector $T(b)$ denotes the true coefficients projection for given forecast coefficients b. Under RLS learning it can be shown that the estimates b_t at time t on average move in the direction $T(b_t)$. Equation (10) describes this adjustment in notional time s. Using the techniques of Evans and Honkapohja (2001), it can be shown that RLS learning converges locally to $\bar{b}$ if it is a locally asymptotically stable fixed point of (10), for sufficiently small $\gamma > 0$. Formal details of the RLS algorithm and learning are outlined in Appendix A.

2.4.4 Incentives to include judgement

Finally, we also require the condition concerning the incentives to include judgement. When all agents in the model are making use of the judgementally adjusted forecast described in equation (2), they induce an actual law of motion for the system, which is described by equation (6). An individual agent may nevertheless decide that it is possible to make more efficient forecasts by simply ignoring the judgemental adjustment. If this is possible, then it is not individually rational for all agents to use the add-factored forecast. We check this individual forecast efficiency condition by comparing the variance of the forecast error for the judgemental forecast (2) to the variance of the forecast error with judgement not included, the econometric forecast (5), under the condition that all other agents are using the judgementally adjusted forecast and thus are inducing the actual law of motion (6).

In other words, we need conditions that the covariance matrix of $y_{t+1}^{e} - y_{t+1}$ is in some sense smaller than the covariance matrix of $E_t^{\star} y_{t+1} - y_{t+1}$.[12] Denote the covariance matrix without judgement as $\mathcal{M}(0)$ and with judgement as $\mathcal{M}(1)$. We will usually interpret the incentive to include judgement condition to mean that the element by element comparison of the matrices along the diagonal are all smaller for $\mathcal{M}(1)$. That is, we require that

$$\mathcal{M}(0)_i - \mathcal{M}(1)_i > 0$$

for all diagonal elements i. By setting up the model in first-order state space form, and including in the state the forecast errors with and without judgement, it is straightforward to compute $\mathcal{M}(0) - \mathcal{M}(1)$ and test numerically for the existence of exuberance equilibria.

When an approximate CEE is stable under learning and satisfies the incentives condition to include judgement, then we refer to it as an *approximate exuberance equilibrium.*

We sometimes refer to alternative versions of the incentive condition as a method of categorizing our results. If the individual rationality condition is met in the sense that the difference between the two covariance matrices is a positive definite matrix, in conjunction with the other two requirements, we say that a *strong exuberance equilibrium* exists. If some diagonal elements

[12] Since y_{t+1}^{e} and $E_t^{\star} y_{t+1}$ have the same mean as y_{t+1} the variance of the forecast error is the same as the mean squared error.

of the difference between the two covariance matrices are positive, while others are negative, when all other conditions are met, this means that the agents may or may not come to the conclusion that including the judgemental adjustment is valuable. We will refer to this case as *indefinite*. Another possibility is that the diagonal elements of the difference between the two covariance matrices are all negative when all other conditions are met. In this case the agents would most likely conclude that the inclusion of judgement was not valuable. We call this case one of *non-exuberance*. Finally, to be complete, the difference could be a negative definite matrix in which case we say that there is *strong non-exuberance.*

2.4.5 Remark on model averaging

In our exuberance equilibrium concept, we allow atomistic agents to contemplate deviations from the judgementally adjusted forecast. These deviations involve ignoring the judgemental component of the forecast entirely. One might wish to consider the possibility of merely "downweighting" the judgemental forecast to create a new forecast which is a weighted average of the pure econometric forecast and the judgementally adjusted forecast. In some circumstances, this downweighting may produce a lower MSE forecast than the judgementally adjusted forecast. If all agents pursued this option, the best-response aspect of the exuberance equilibrium described in Section 2.4.4 would break down. This is the main reason why we call our exuberance equilibria "near-rational." However, although our equilibrium is in this sense not fully rational, we think that in practice it is very plausible.

As discussed earlier in the paper, we think of our model as being a stark case, one in which the judgement we analyze is all noise. But we also acknowledge that much real-world judgemental adjustment is likely to be quite sound. We want to think of the agents in our model as operating in an economy where most judgemental adjustment is in fact sound. For these cases, downweighting a judgement that is being used economywide will lead to worse forecasts in a MSE sense. This provides one rationale for why the agents in the model do not consider downweighting the judgemental forecast.[13]

We also think that it would be difficult to describe a plausible real-time process that could discover the value of downweighting in an actual economy. The contemplated concept is that when an event such as 9/11 occurs and judgemental adjustments are made to the forecast, agents in the economy downweight the judgemental forecast somewhat on the basis that this will produce better long-run forecasting performance. For this to be verified in the data, the agents need the track record of the judgemental forecaster over a period of many quarters. The judgements would have been made over a variety of unique events, ranging from the Cuban Missile Crisis to Hurricane Katrina. We do not think most participants in the economy would be willing to average across forecast performance for such disparate events. Even if they were willing to do so, it is possible to show that it can take very large samples of data to detect any deviation from full rationality of the judgementally adjusted forecast.[14]

Finally, possible downweighting would still leave the judgement influential for the state variables of the system. The excess volatility associated with the exuberance equilibrium driven solely by noisy judgement would decline over time through a learning process if we allowed downweighting. But if we allow for some of the judgement to be sound, then the value of relying on judgemental forecasting would then reassert itself and agents would again be susceptible to poor judgements that influence actual outcomes.

3 Exuberance and monetary policy

3.1 A New Keynesian model

We now study examples of exuberance equilibria in a New Keynesian macroeconomic model suggested by Woodford (2003) and Clarida, Gali, and Gertler

[13] We note that Svensson (2005) and Svensson and Tetlow (2005) do not allow for model averaging when they model the inclusion of judgement..

[14] See Bullard, Evans, and Honkapohja (2006).

(1999). We use a simple, three-equation version given by

$$x_t = x^e_{t+1} - \sigma^{-1}\left[r_t - \pi^e_{t+1}\right] + \tilde{u}_{x,t}, \tag{11}$$

$$\pi_t = \kappa x_t + \delta \pi^e_{t+1} + \tilde{u}_{\pi,t}, \tag{12}$$

$$r_t = \varphi_\pi \pi_t + \varphi_x x_t. \tag{13}$$

In these equations, x_t is the output gap, π_t is the deviation of inflation from target, and r_t is the deviation of the nominal interest rate from the value that is consistent with inflation at target and output at potential. All variables are expressed in percentage point terms and the steady state is normalized to zero. The terms $\tilde{u}_{x,t}$ and $\tilde{u}_{\pi,t}$ represent stochastic disturbances to the economy. The parameter σ^{-1} is related to the elasticity of intertemporal substitution in consumption of a representative household. The parameter κ is related to the degree of price stickiness in the economy, and δ is the discount factor of a representative household.[15] The third equation describes the Taylor-type policy rule in use by the policy authority, in which the parameters φ_π and φ_x are assumed to be positive. In the formulation (11)-(13), only private sector expectations affect the economy.

Substituting (13) into (11) and writing the system in matrix form gives (1) where $y_t = [x_t, \pi_t]'$, $u_t = C\tilde{u}_t$, $\tilde{u}_t = [\tilde{u}_{x,t}, \tilde{u}_{\pi,t}]'$ with covariance matrix Σ_u,

$$\beta = \frac{1}{\sigma + \varphi_x + \kappa\varphi_\pi}\begin{bmatrix} \sigma & 1 - \delta\varphi_\pi \\ \kappa\sigma & \kappa + \delta\left(\sigma + \varphi_x\right) \end{bmatrix},$$

and

$$C = \frac{1}{\sigma + \varphi_x + \kappa\varphi_\pi}\begin{bmatrix} \sigma & -\varphi_\pi \\ \kappa\sigma & \sigma + \varphi_x \end{bmatrix}.$$

3.2 Results

3.2.1 A Taylor-type monetary policy rule

We now illustrate the possibility of approximate exuberance equilibria in the New Keynesian model. We use Woodford's (2003) calibration $\sigma = 0.157$,

[15]This formulation of the model is based upon individual Euler equations under (identical) private sector expectations. Other models of bounded rationality are possible, see, for instance, Preston (2005) for a formulation in which long-horizon expectations directly affect individual behavior.

$\kappa = 0.024$, and $\delta = 0.99$. For the exuberance variable we assume the matrix describing the degree of serial correlation is $\rho = \text{diag}(0.99, 0.95)$ and $\Sigma_\eta = \text{diag}(0.0035, 0.0035)$.[16] The variances of the fundamental shocks are assumed to be $\Sigma_{\tilde{u}} = \text{diag}(1.1, 0.03)$. We have not calibrated these shocks except to choose values that, in the exuberance equilibrium, roughly match U.S. inflation and output-gap variances measured in percent.

The policy parameters φ_π and φ_x can be varied and we are interested in values of φ_π and φ_x that might be consistent with exuberance equilibrium. Consider $\varphi_\pi = 1.05$ and $\varphi_x = 0.05$. These values satisfy the Taylor principle and deliver a determinate and learnable rational expectations equilibrium in Bullard and Mitra (2002). Suppose that econometricians estimate a VAR(3). We calculated the approximate CEE and found that the output variance is approximately 2.54 and the inflation variance is approximately 6.14. The matrix describing the condition for individual rationality, $\mathcal{M}(0) - \mathcal{M}(1)$, is positive definite, hence the CEE is strongly exuberant. The exuberance equilibrium exhibits excess volatility. In fact, the ratio of the output-gap standard deviation in the exuberance equilibrium to its standard deviation in the fundamental rational expectations equilibrium is about 1.5 and for the standard deviation of inflation the corresponding ratio is almost 16!

A change in the Taylor-rule coefficients can diminish the likelihood of exuberance equilibria. When φ_π is increased to 1.1 the equilibrium is no longer strongly exuberant but it does remain exuberant. However, if φ_π is increased to 1.5 and φ_x is increased to 0.1, the possibility of an exuberance equilibrium is eliminated. In this sense, a more aggressive policy tends to reduce the likelihood of an exuberance equilibrium.

We next analyze the idea that more aggressive policy is less likely to be associated with the existence of exuberance equilibrium more systematically. For this, we calculate the conditions for exuberance equilibrium using the calibration given above but allowing the Taylor rule coefficients to vary. The results are given in Figure 1, where $\varphi_\pi \in (0, 1.25)$ and $\varphi_x \in (0, 0.25)$ at selected grid points. The open squares indicate the points where determinacy

[16] Somewhat lower values of the ρ parameters delivered qualitatively similar results.

and learnability of the rational expectations equilibrium hold for this model.[17] The Figure displays the points at which exuberance equilibria exist. These points tend to be for values of φ_x less than about 0.08, and for values of φ_π up to 1.25. Again, these exuberance equilibria exist in the region associated with determinacy, and therefore can arise in parameter regions where sunspot equilibria are ruled out.

3.2.2 A forward-looking monetary policy rule

It is also of interest to investigate an alternative Taylor-type interest rate rule,

$$r_t = \varphi_\pi \pi^e_{t+1} + \varphi_x x^e_{t+1}, \tag{14}$$

in which policymakers react to forecasts of future values of the inflation deviation and the output gap. Interest-rate rules depending on expectations of future inflation and the output gap have been discussed extensively in the monetary policy literature and are subject to various interpretations. Here we are assuming that the monetary authorities form forecasts in the same way as the private sector, that is, by constructing an econometric forecast to which they consider adding the same judgement variable. We might hope that by reacting aggressively enough to expectations such a rule would diminish the likelihood of exuberance equilibria. With the policy rule (14) the reduced form system is the same except that the coefficient matrices become

$$\beta = \begin{bmatrix} 1-\sigma^{-1}\varphi_x & \sigma^{-1}(1-\varphi_\pi) \\ \kappa(1-\sigma^{-1}\varphi_x) & \delta+\kappa\sigma^{-1}(1-\varphi_\pi) \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & 0 \\ \kappa & 1 \end{bmatrix}.$$

Using the same calibration, we calculate whether the conditions for exuberance equilibria hold for $\varphi_\pi \in (0, 3.5)$ and $\varphi_x \in (0, 0.35)$ at selected grid points. The results are plotted in Figure 2.[18] The open squares again indicate the points where determinacy and learnability of the rational expectations

[17] The blank area to the left in this figure is associated with indeterminacy of rational expectations equilibrium.

[18] There is a subtlety in this example due to the fact that the central bank has non-negligible macroeconomic effects. We assume that in comparing the performance of forecasts with and without judgement they compare forecasts to actual, realized, data.

equilibrium hold for this model.[19] As with the standard Taylor-type rule, the Figure indicates that exuberance equilibria exist near the point $(1, 0)$. More aggressive policy delivers non-exuberance. In particular, very small values of φ_x are sufficient to yield non-exuberance if φ_π is greater than (approximately) 1.8. We conclude that by following an explicit policy of reacting against the deviations of expectations from the values justified by the fundamental shocks, monetary authorities enhance the stability of the economy.

3.2.3 Optimal monetary policy rules

Finally, we discuss optimal discretionary policy as in Evans and Honkapohja (2003). They assign a standard quadratic objective to the policymaker with weight α on output gap variance. They write the resulting optimal policy as a Taylor-type rule in the expected output gap and the expected inflation deviation, along with reactions to fundamental shocks in the economy. Their policy rule delivers determinacy, and the unique stationary rational expectations equilibrium is stable under least squares learning for all values of structural parameters and the policy weight. We can denote this optimal policy rule as

$$r_t = \varphi_\pi^\star \pi_{t+1}^e + \varphi_x^\star x_{t+1}^e + \varphi_{u,x}^\star \tilde{u}_{x,t} + \varphi_{u,\pi}^\star \tilde{u}_{\pi,t}. \tag{15}$$

where the optimal values $\varphi_x^\star = \varphi_{u,x}^\star = \sigma$, and the matrices β and C become

$$\beta = \begin{bmatrix} 0 & \sigma^{-1}\left(1-\varphi_\pi^\star\right) \\ 0 & \delta + \kappa\sigma^{-1}\left(1-\varphi_\pi^\star\right) \end{bmatrix}, \text{ and } C = \begin{bmatrix} 0 & -\sigma^{-1}\varphi_u^\star \\ 0 & 1-\sigma^{-1}\kappa\varphi_u^\star \end{bmatrix}.$$

We have $\varphi_{u,\pi}^\star = \delta^{-1}\left(\varphi_\pi^\star - 1\right)$, where $\varphi_\pi^\star$ depends on α. A small policy weight on output gap variability $\alpha \to 0$ (an inflation hawk), is associated with an optimal value $\varphi_\pi^\star = 1 + \sigma\delta\kappa^{-1} \approx 7.47$. A large weight on output gap variability, $\alpha \to \infty$, (an inflation dove), is associated with an optimal value $\varphi_\pi^\star \to 1$. Thus we can calculate whether exuberance equilibria exist for all

[19]For the forward-looking rule, indeterminacy of the fundamental rational expectations equilibrium occurs not only in the blank area to the left in the figure, but also in the blank area toward the top of the figure.

possible values of the policymaker weight α by choosing values for $\varphi_\pi^\star \in (1, 7.47)$.

The results of this calculation[20] indicate that for values of $\varphi_\pi^\star \in (1, \bar{\varphi}_\pi)$ the equilibrium is in the indefinite region. For values $\varphi_\pi^\star \in (\bar{\varphi}_\pi, 7.47)$, the equilibrium is non-exuberant. The cutoff value is $\bar{\varphi}_\pi \approx 1.557$. Thus standard optimal policy calculations alone are not enough to ensure non-exuberance. To move into the non-exuberance region, policymakers must have a sufficiently small weight on output gap variability. The policy weight value associated with $\varphi_\pi^\star = 1.557$ is quite low, approximately $\alpha \approx 0.00612$. More weight than this on output gap variance implies a value for $\varphi_\pi^\star$ that is too low, in the sense that it places the equilibrium in the indefinite region.[21]

4 Conclusions and possible extensions

We have shown how the use of judgement or "add-factors" may cause a type of self-fulfilling fluctuation—which we call exuberance equilibria—to occur in a standard New Keynesian model of monetary policy. The excess volatility we isolate occurs in a subset of the determinacy region of the economy, a portion of the parameter space that has typically been viewed as desirable in the literature. We have also shown how more aggressive monetary policy rules, ones which specify stronger reactions to economic events, can mitigate or eliminate the possibility of exuberance equilibria.

Since macroeconometric models are at best crude approximations to economic reality, there is a clear rationale for judgemental adjustment. And, indeed, most judgement is likely to be sound. Yet, judgemental adjustments are unlikely to be perfect and may at times be far off the fundamental realities of the macroeconomy. We have highlighted the difficulties which may

[20] The exact optimal policy rule would create perfect multicollinearity in this system. To avoid this complication, we set $\varphi_x = 1.01\sigma$, slightly higher than the optimal value.

[21] If we assume that the policymaker has the same preferences as the representative household, we obtain a value of $\alpha \approx .00313$ at the calibrated values of Woodford (2003). (This is calculated as $\kappa/\theta = 0.024/7.67$, where θ is the parameter controlling the price elasticity of demand.) The value of $\varphi_\pi^\star$ for any specified α is $1 + \kappa\delta\sigma\left(\alpha + \kappa^2\right)$. This would suggest an optimal value of $\varphi_\pi^\star \approx 2.0$, large enough to imply non-exuberance.

arise when judgement of this latter type enters an economy with strong expectational feedback.

In this paper, we have focussed on a situation in which there is substantial agreement in the economy about the relevant judgemental adjustments. This is a restrictive assumption, and differences of opinion certainly exist in actual economies about what sorts of judgemental adjustments should be made. Allowing for differences in judgements would probably make the conditions for exuberance equilibrium more difficult to achieve. On the other hand, this could create new phenomena, such as momentum effects, arising when a large fraction of agents begins to agree in their judgements. Another interesting case may occur when differences of opinion pit the central bank against financial markets, raising questions about the nature of equilibrium and the optimal reaction of the monetary authority. We think it would be interesting to address these issues in future research.

A Appendix: Recursive learning

Econometricians estimate the PLM

$$y_t = \sum_{i=1}^{p} b_i y_{t-i} + v_t$$

using recursive least squares. Let $b_t = (b_{1,t}, ..., b_{p,t})$ denote the parameter estimates at time t and let $Y'_{t-1} = (y'_{t-1}, ..., y'_{t-p})$ be the vector of state variables. The RLS algorithm is

$$\begin{aligned} b'_t &= b'_{t-1} + t^{-1} R_t^{-1} Y_{t-1}(y_t - b_{t-1} Y_{t-1})' \\ R_t &= R_{t-1} + t^{-1}(Y_{t-1} Y'_{t-1} - R_{t-1}), \end{aligned}$$

Here R_t is an estimate of the matrix of second moments of Y_{t-1} and the first equation is just the recursive form of the multivariate least squares formula. Note that assumptions about timing are as follows. At the end of period $t-1$ econometricians update their parameter estimates to b_{t-1} using data up to $t-1$. At time t econometricians use these parameter estimates and observed

Y_t to make their forecast $E_t^{\star} y_{t+1}$.[22] At the end of time t econometricians update the parameters to b_t. For further discussion of RLS learning see Chapters 2, 8 and 10 of Evans and Honkapohja (2001).

The question of interest is whether $\lim_{t\to\infty} b_t \to \bar{b}$, where $\bar{b} = (\bar{b}_1, ..., \bar{b}_p)$ denotes the approximate CEE . In this case $\bar{b}$ is said to be locally learnable. It can be shown that the asymptotic dynamics of (b_t, R_t) are governed by an associated differential differential equation and that, in particular, the asymptotic dynamics of b_t are governed by

$$\frac{db}{d\tau} = \left[Ey_t(b)Y_{t-1}(b)'\right]\left[EY_t(b)Y_{t-1}(b)'\right]^{-1} - b = T(b) - b.$$

Here τ denotes notional or virtual time, $y_t(b)$ is the stationary stochastic process given by (8) for fixed b and $Y_{t-1}(b)' = (y_{t-1}(b)', ..., y_{t-p}(b)')$. Numerically, convergence can be verified using the E-stability algorithm (10), which can also be used to compute the approximate CEE.

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[22] Note that y_t and $E_t^* y_{t+1}$ are simultaneously determined. Alternative information assumptions could be made but would not affect our main results.

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Figure 1

Existence of Exuberance Equilibria

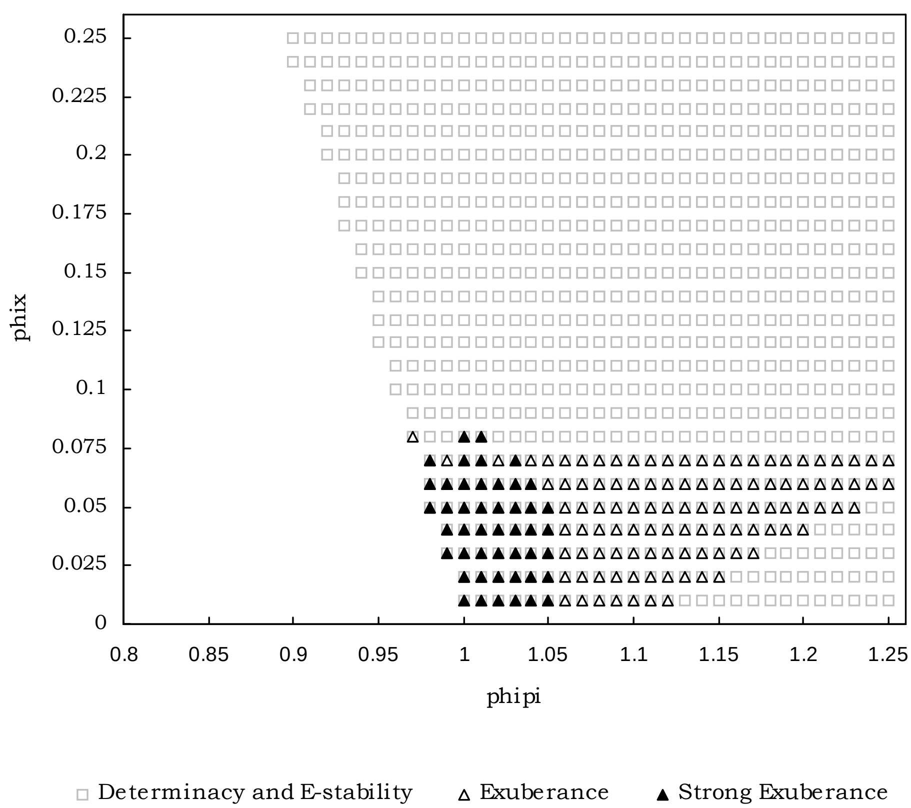

Figure 1: Exuberance equilibria in the New Keynesian model. Open boxes indicate points where the REE is determinate. Triangles indicate points where exuberance equilibria exist.

Figure 2

Exuberance with Forward-Looking Rules

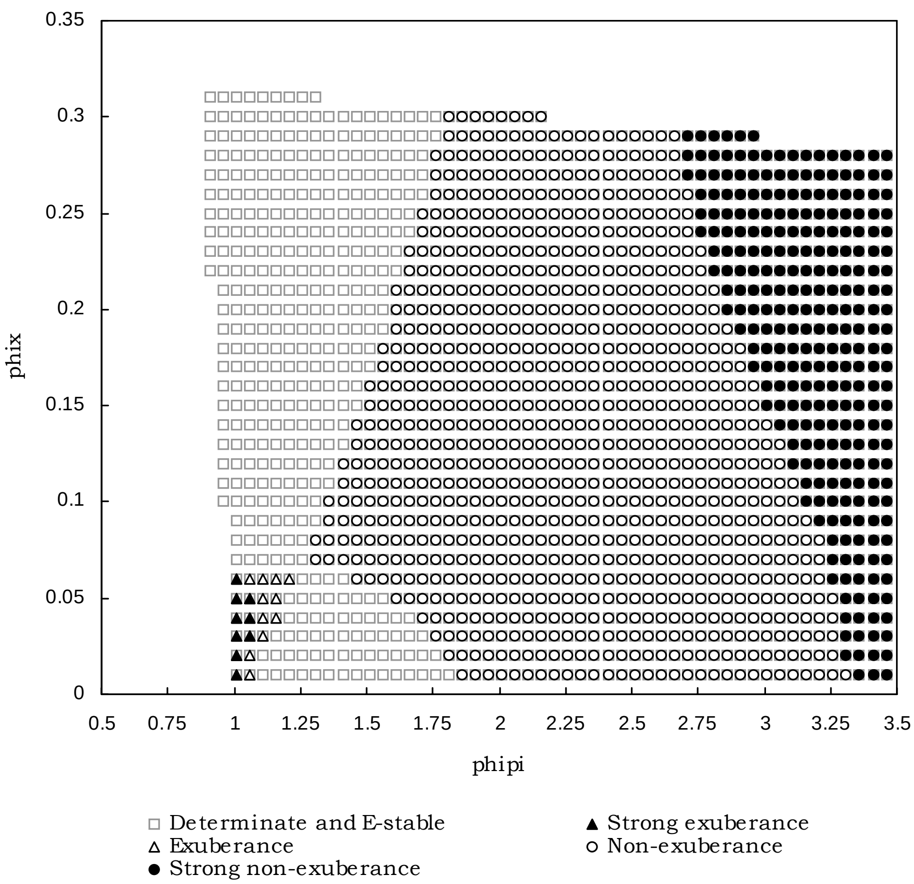

Figure 2: Sufficiently aggressive policy is again associated with non-exuberance when the policy rule is forward-looking.