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Money and the Natural Rate of Interest: Structural Estimates for the United States and the Euro Area*

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Abstract

We examine the role of money in three environments: the New Keynesian model with separable utility and static money demand; a nonseparable utility variant with habit formation; and a version with adjustment costs for holding real balances. The last two variants imply *forward-looking* behavior of real money balances, with forecasts of future interest rates entering current portfolio decisions. We conduct a structural econometric analysis of the U.S. and euro area economies. FIML estimates confirm the forward-looking character of money demand. A consequence is that real money balances are valuable in anticipating future variations in the natural interest rate.

Key Words: Money, natural rate, New Keynesian models.

JEL Classification Numbers: E51; E52.

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1 Introduction

The growing use of sticky-price optimizing models, or a "New Keynesian" framework, in macroeconomics has simultaneously reaffirmed the relevance of monetary policy actions for the behavior of output and inflation, and downplayed the importance of monetary aggregates. The baseline version of the New Keynesian model, with household preferences separable across time and arguments, does generate a standard money demand function. But much work with New Keynesian models, exemplified by Rotemberg and Woodford (1997), uses the fact that the IS function, Phillips curve, and interest-rate policy rule contain no money term as grounds for not referring to money or the money demand function in the analysis at all. And insofar as money has an indicator role in this New Keynesian baseline, it is as a noisy indicator of current output (see e.g. Dotsey and Hornstein, 2003).[1] The money stock then becomes one of many candidates as indicators of current economic activity—hardly a role that conveys great significance to money in macroeconomic analysis.

One modification to the New Keynesian model that restores an explicit role for money is to drop the assumption that household preferences are separable across consumption and real money balances. As shown in Andrés, López-Salido and Vallés (2006), Ireland (2004), Woodford (2003) and below, relaxing this assumption does introduce terms involving real balances into the model's IS and Phillips curve (or marginal cost) equations. But plausible calibrations do not seem to generate a sizable role for this channel (McCallum, 2000; Woodford, 2003), while econometric estimates so far provide even less empirical support (see Ireland, 2004, for the U.S., and Andrés, López-Salido and Vallés, 2006, for the euro area).

Nelson (2002) argues that neither the separable nor the nonseparable preference specification conveys on money the role stressed for it in the monetarist literature. That literature, as discussed in Artis (1993), Meltzer (2001), and references therein, rests on two propositions: first, that yields beside the short-term interest rate enter both the IS and the money demand functions; and second, that the money stock therefore provides information about determinants of aggregate demand beyond short-term real interest rates. This

[1] Coenen, Levin, and Wieland (2005), using an empirical model with some optimizing features, similarly limit money's value to its indicating fluctuations in today's GDP. This is also implicitly the role of money considered by King and Lin (2005), since their model's money demand function is of the standard (static, two-argument) form.

perspective transforms the central issue from being whether money appears explicitly in the IS and Phillips curve equations, to whether money serves as a good proxy for movements in asset prices that do appear directly in the economy's IS equation, some of which may be difficult to observe directly. Nelson argues that a first step in capturing these ideas is to add to the New Keynesian model a forward-looking dimension to money demand, arising from portfolio adjustment costs. In this environment, it is not money's role as a static indicator of output, but instead the *interest-elastic* and *forward-looking* character of real money balances that conveys on money an important role as an indicator.

The different perspectives on money suggested by the three model settings—the standard New Keynesian model with separable utility and static money demand; the nonseparable utility variant; and the New Keynesian model modified to allow for dynamics in money demand—are brought out in Table 1. The departures from the baseline model both add grounds for looking at money, but do so in different ways.

In this paper, we distinguish between the alternative views of the role of money in the transmission mechanism by conducting a structural econometric analysis of the U.S. and euro area economies. The dynamic stochastic general equilibrium model that we estimate delivers each model variant described in Table 1 as a special case. A key result is that our maximum likelihood estimates confirm the forward-looking character of money demand. Using our estimated model, we are able to demonstrate the enhanced ability of money to capture the transmission mechanism of monetary policy when money demand has a forward-looking element. In particular, we show that the value of money as a proxy for variations in the natural interest rate and the real interest-rate gap is increased.

By focusing on the forward-looking character of money demand, we overturn much conventional wisdom about the limited informational value of monetary aggregates. For example, Romer and Romer (1990, pp. 167, 169) conjecture that since "quantities—either of money or of loans—can be adjusted only slowly... interest-rate movements generally precede movements in financial aggregates." But in an optimizing general equilibrium model, the existence of adjustment costs in the holding of real balances actually makes money *precede* interest-rate movements rather than the reverse. This reflects the fact that adjustment costs make it optimal for agents to allow their forecasts of future interest rates to affect today's portfolio decision. In addition, a recent critique of the role of money by Woodford (2007) dismisses the possibility that money is a good candidate for information on the flexible-price economy because "money demand depends on the actual level of transactions in the economy, *not* on how that level of activity compares to the 'natural rate.'" (Woodford, 2007, p. 32). With our generalization of money demand, this judgment no longer applies. The general money demand function arising from our analysis includes expected future levels of output and interest rates as additional arguments, and so (with the wearing-off of price stickiness over time) real money balances will be informative about current expectations of future natural rates and indirectly of the current natural rate.

A relatively small portion of the study of money's place in the transmission mechanism has been in the context of optimizing models estimated by systems methods. The investigations of the role of money by Nelson (2002), Dotsey and Hornstein (2003), and Woodford (2003), for example, use calibrated models. Smets and Wouters (2003), in estimating a DSGE model of the euro area by Bayesian maximum likelihood, exclude money from the list of variables modeled.[2] A considerable amount of econometric work has been done on the role of money in the euro area, as discussed by Issing *et al* (2001), but this work is typically either explicitly reduced form or has relied on postulated behavioral relationships that lack microfoundations (e.g., IS-LM systems without proper account for forward-looking behavior, or with lagged terms not traced explicitly to private sector optimization).

Work that does meet our joint criteria of using DSGE modelling, estimating by systems methods, and putting money in the likelihood, includes Ireland (2003, 2004) and Andrés, López-Salido and Vallés (2006).[3] Relative to these studies, the present paper estimates a model sufficiently general to distinguish between all three model settings described in Table 1, not just the separable and nonseparable preference specifications. In addition, we carry out an analysis of the dynamic relationship between money and the natural rate, and the consequent usefulness of money to monetary policy.

Our model is laid out in Section 2. Section 3 includes some analytical results on the relation between money and the natural rate. Section 4 presents our empirical results. We find considerable support for the forward-looking

[2]Important recent papers for the U.S. that do include money among the variables of interest, though using a different estimations procedure from maximum likelihood, are Christiano, Eichenbaum, and Evans (2005), and Altig, Christiano, Eichenbaum, and Linde (2005).

[3]Another example is Bergin (2003), but the model he estimates is not suited to the study of interest-rate policy rules.

money demand variant of the model, and in Section 5 we show how this specification improves the value of money as a proxy for the natural rate of interest. Section 6 considers robustness results, while Section 7 concludes.

2 A Sticky-Price Model with Money

The model has many features commonly used in sticky-price versions of the New Keynesian model, but is closest to Andrés, López-Salido and Vallés (2006), Ireland (2004), and Nelson (2002). The economy consists of a representative household, a continuum of producing firms indexed by $j \in [0,1]$ and a monetary authority. We abstract from capital accumulation. The model has certain symmetry properties that allow us to focus on the behavior of a representative goods-producing firm.

2.1 Households

2.1.1 The Nonseparability Effect

The representative household of the economy maximizes the following expected stream of utility:

$$\max_{C_t,N_t,M_t,B_t} E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[\Psi \left(\frac{C_t}{C_{t-1}^h}, \frac{M_t}{e_t P_t} \right) - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

where C_t is the CES aggregator of the quantities of the different goods consumed, i.e. $C_t = \left(\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$.[4] The variables M_t/P_t and N_t represent real balances and hours, respectively; a_t is a preference shock, and e_t is a shock to the household's demand for real balances. The parameter $\beta \in (0,1)$ is a discount factor, $\varphi \geq 0$ represents the inverse of the Frisch labor supply elasticity, and h allows for the presence of (internal) habit formation.

We allow for *nonseparability* across consumption and real balances in preferences, as well as for habit formation in consumption. Intra-temporal nonseparability makes it possible to test the relevance of an explicit money-balances term in the equations determining supply and demand decisions.

[4] Accordingly, $P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}}$ is the aggregate price index that is consistent with the first-order conditions of the producing firms that face the differentiated demand, with $P_t(j)$ the price of good j.

This is the main influence of money emphasized in recent studies. Habits have been emphasized by Fuhrer (2000) and Christiano, Eichenbaum, and Evans (2005), among others, as an important component of the monetary transmission mechanism that helps to account for the gradual response of output to monetary policy shocks. The dynamic interaction between nominal and real variables is further enriched by the presence of intertemporal non-separability that generates a battery of cross-equation restrictions. Finally, the marginal utility of consumption is a function of real money holdings, but is independent of preferences over leisure. In addition, the postulated separability between the consumption/real balances basket and hours implies that aggregate spending relations are not altered by respecification of the firm's problem (see Driscoll, 2000).

2.1.2 The Direct Effect

As noted above, empirical evidence is generally unfavorable for the position that money enters the IS equation through a nonseparability channel. This finding has sometimes been characterized as decisive evidence against the role of money in aggregate demand determination in New Keynesian models. Such a characterization, however, overlooks the fact that the core monetarist literature did not claim that money entered the IS equation. In this regard, it is useful to keep in mind that more than 40 years ago Milton Friedman observed that he did not oppose describing aggregate demand developments in terms of interest rates, as this was "purely a semantic question of how one wants to describe the channels"; what was important was that "if there are changes in the stock of money there will be changes in interest rates" (Friedman, 1964).

Along these lines, Nelson (2002) has elaborated on the idea that a key link between real balances and real aggregate demand occurs not via the nonseparability channel, but through "direct effects" that are not well captured by short term real interest rates. In this framework, money is serving as an index for yields besides the short rate (in his application, the real long-term rate) that are relevant for aggregate demand (this builds on Meltzer, 2001, and the references therein). Nelson (2002) captures this idea by simply allowing for portfolio adjustment costs. Formally,

$$\max_{C_t,N_t,M_t,B_t} E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[\Psi\left(\frac{C_t}{C_{t-1}^h}, \frac{M}{e_t P_t} \right) - \frac{N_t^{1+\varphi}}{1+\varphi} \right] - G(\bullet) \qquad (2)$$

with

$$G(\bullet)=\frac{d}{2}\left\{\exp(c\left\{\frac{\frac{M_t}{P_t}}{\frac{M_{t-1}}{P_{t-1}}}-1\right\})+\exp(-c\left\{\frac{\frac{M_t}{P_t}}{\frac{M_{t-1}}{P_{t-1}}}-1\right\})-2\right\} \tag{3}$$

where $c > 0$, $d > 0$. This functional form for portfolio adjustment costs is that of Christiano and Gust (1999), modified to apply to real balances and applied to a model without "limited participation" features. An advantage of this portfolio adjustment cost specification is that for a wide range of c and d values, the portfolio adjustment costs incurred to carry out typical monetary transactions are trivial when converted into units of resources surrendered by the representative agent—see Chari, Christiano, and Eichenbaum (1995, p. 1369). Yet, at the same time, these costs imply substantial effects on money demand dynamics. The effects on dynamics, moreover, are supported by many existing empirical findings regarding money demand. First, as we discuss in detail below, the money demand dynamics imply that expectations of interest rates matter for money demand, which is indirectly supported by empirical studies that find that the nominal long-term interest rate matters in the money demand function. Second, most empirical work on money demand finds that the lagged dependent variable enters positively in the money demand function, a result also supported by this specification. Third, and relatedly, work on money in business cycle models frequently distinguishes between a long-run interest elasticity of money demand supported by studies of long runs of data and a more moderate short-run elasticity to be used in business-cycle work (see e.g. Khan, King and Wolman, 2003, and Altig, Christiano, Eichenbaum, and Linde, 2005). Our money demand specification justifies this distinction between short-run and long-run elasticities, as the demand function's income and interest elasticities now refer to money's reaction to long-term averages of output and interest rates.

As noted above, we specify portfolio adjustment costs in terms of real rather than nominal balances. A forward-looking money demand term would also appear if we instead placed nominal balances in the cost function.[5] But specifying costs in terms of real balances, besides its algebraic convenience,

[5] The money demand specification would also be little changed if we placed portfolio adjustment costs in the budget constraint rather than the utility function: placing it directly in utility allows the costs and services from money (the latter manifested by the usual money-in-the-utility-function term) to be treated more symmetrically; and is also algebraically more convenient.

captures the notion that portfolio adjustment costs are not necessarily literal transaction costs, but instead reflect the convenience of maintaining–other things equal–a certain amount of purchasing power in the form of money, in line with Friedman and Schwartz's (1982, p. 24) notion that money delivers extra services as a "temporary abode of purchasing power" and Modigliani's (1944, p. 51) view of money as a "reserve against contingencies."

The budget constraint each period is:

$$\frac{M_{t-1}+B_{t-1}+W_tN_t+T_t+D_t}{P_t} = C_t+\frac{B_t/r_t+M_t}{P_t} \tag{4}$$

Households enter period t with money holdings M_{t-1} and bonds B_{t-1}. At the beginning of the period, they receive lump-sum nominal transfers T_t, labor income W_tN_t, where W_t denotes the nominal wage, and a nominal dividend D_t from the firms. They use some of these funds to purchase new one-period securities at nominal cost B_t/r_t, where r_t denotes the gross nominal interest rate between t and $t+1$. The household carries M_t units of money into the period $t+1$.

2.2 Firm Behavior and Price Setting

The production function for firm j is

$$Y_t(j) = z_t N_t(j)^{1-\alpha} \tag{5}$$

where $Y_t(j)$ is output, $N_t(j)$ represents the number of work-hours hired from the household (i.e. $N_t = \int_0^1 N_t(j)\, dj$), z_t is a common technology shock and $(1-\alpha)$ represents the elasticity of output with respect to hours. Letting $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$, the market-clearing condition implies $Y_t = C_t$.

The representative firm sells its output in a monopolistically competitive market and sets nominal prices on a staggered basis, as in Calvo (1983). Each firm has with probability $1-\theta$ an opportunity to reset its price in any given period, irrespective of the time elapsed since the last adjustment. Thus, each period a measure $1-\theta$ of producers reset their prices to maximize their stream of expected profits. Therefore, θ^k will be the probability that the price set at time t will still hold at time $t+k$. Notice that, if there were no constraints on the adjustment of prices, the typical firm would set a price according to the rule $P_t(j) = (\frac{\varepsilon}{\varepsilon-1})MC_t(j)$, where $MC_t(j) = \frac{W_t}{\frac{\partial Y_t(j)}{\partial N_t(j)}}$ is the nominal marginal cost and $\frac{\varepsilon}{\varepsilon-1}$ is the steady-state price markup.

This framework implies that the inflation rate is a wholly forward-looking variable. Much recent research has, however, highlighted the importance of allowing for a hybrid specification in which a portion of inflation dynamics is explained by a backward-looking element, thereby accounting for the inertia in inflation patterns. Thus, following Christiano, Eichenbaum and Evans (2005), we allow for some degree of indexation. Those firms that do not set the optimal price at time t will adjust prices to lagged inflation: $P_{t+i}(j) = P_{t+i-1}(j)\left(\frac{P_{t+i-1}}{P_{t+i-2}}\right)^{\kappa} = P_{t+i-1}(j)\left(\pi_{t+i-1}\right)^{\kappa}$, where κ is a parameter that indicates the degree of non-optimizers' price adjustment whose extreme values imply no indexation ($\kappa = 0$) or full indexation ($\kappa = 1$). The aggregate price level evolves as follows:

$$P_t = \left[\theta\left(P_{t\text{-}1}\left(\pi_{t\text{-}1}\right)^{\kappa}\right)^{(1\text{-}\varepsilon)} + (1\text{-}\theta)\left(P_t^f\right)^{(1\text{-}\varepsilon)}\right]^{\frac{1}{(1\text{-}\varepsilon)}} \tag{6}$$

2.3 Central Bank Reaction Function

We assume that the central bank sets the nominal interest rate following a general augmented Taylor-type interest rate rule. In particular, the nominal rate responds not only to the interest rate in the previous period and to deviations of output and inflation from their steady-state values, but also to nominal money growth:

$$\begin{aligned}\ln(r_t/r) &= \rho_r \ln(r_{t-1}/r) + (1\text{-}\rho_r)\rho_\pi \ln(\pi_t/\pi) \\ &\quad + (1-\rho_r)\rho_y \ln(y_t/y) + (1-\rho_r)\rho_\mu \ln(\mu_t/\mu) + \varepsilon_{r_t}\end{aligned}$$

where the innovation ε_{r_t} is normally distributed with standard deviation σ_r; and $\mu_t = M_t/M_{t-1}$ is the rate of money growth. An interest-rate rule that depends on money growth (or the change in real balances) might be rationalized, as in Svensson (1999), as part of an optimal reaction function when money-growth variability appears in the central bank's loss function. Alternatively, the response to money might be rationalized by money's usefulness in forecasting inflation.

2.4 Equilibrium

The symmetric equilibrium can be loglinearized to yield the following set of equations:[6]

$$\begin{aligned}\widehat{y}_t &= \frac{\phi_1}{\phi_1+\phi_2}\widehat{y}_{t-1}+\frac{\beta\phi_1+\phi_2}{\phi_1+\phi_2}E_t\widehat{y}_{t+1}\text{-}\frac{1}{\phi_1+\phi_2}\left[\widehat{r}_t\text{-}E_t\widehat{\pi}_{t+1}\right]\text{-}\frac{\beta\phi_1}{\phi_1+\phi_2}E_t\widehat{y}_{t+2}\\ &+\frac{\psi_2}{\psi_1}\frac{1}{(1\text{-}\beta h)}\left(\frac{1}{\phi_1+\phi_2}\right)\widehat{m}_t\text{-}\frac{\psi_2}{\psi_1}\frac{1}{(1\text{-}\beta h)}\left(\frac{1+\beta h}{\phi_1+\phi_2}\right)E_t\widehat{m}_{t+1}\\ &+\frac{\psi_2}{\psi_1}\frac{1}{(1\text{-}\beta h)}\left(\frac{\beta h}{\phi_1+\phi_2}\right)E_t\widehat{m}_{t+2}\text{-}\frac{\psi_2}{\psi_1}\left(\frac{1\text{-}\beta h\rho_e}{1\text{-}\beta h}\right)\left(\frac{1\text{-}\rho_e}{\phi_1+\phi_2}\right)\widehat{e}_t\\ &+\left(\frac{1\text{-}\beta h\rho_a}{1\text{-}\beta h}\right)\left(\frac{1\text{-}\rho_a}{\phi_1+\phi_2}\right)\widehat{a}_t\end{aligned} \tag{7}$$

$$\widehat{\pi}_t\text{-}\kappa\widehat{\pi}_{t\text{-}1}=\beta(E_t\{\widehat{\pi}_{t+1}\}\text{-}\kappa\widehat{\pi}_t)+\lambda\widehat{mc}_t \tag{8}$$

$$\begin{aligned}\widehat{mc}_t &= (\chi+\phi_2)\,\widehat{y}_t\text{-}\phi_1\widehat{y}_{t\text{-}1}\text{-}\beta\phi_1E_t\widehat{y}_{t+1}\text{-}\frac{\psi_2}{\psi_1}\frac{1}{(1\text{-}\beta h)}\widehat{m}_t+\frac{\psi_2}{\psi_1}\frac{\beta h}{(1\text{-}\beta h)}E_t\widehat{m}_{t+1}\\ &+\frac{\psi_2}{\psi_1}\frac{(1\text{-}\beta h\rho_e)}{(1\text{-}\beta h)}\widehat{e}_t\text{-}\frac{\beta h(1\text{-}\rho_a)}{(1\text{-}\beta h)}\widehat{a}_t\text{-}\,(1+\chi)\,\widehat{z}_t\end{aligned} \tag{9}$$

$$\widehat{r}_t=\rho_r\widehat{r}_{t\text{-}1}+(1\text{-}\rho_r)\rho_y\widehat{y}_t+(1\text{-}\rho_r)\rho_\pi\widehat{\pi}_t+(1\text{-}\rho_r)\rho_\mu\;\widehat{\mu}_t+\varepsilon_{r_t} \tag{10}$$

$$\widehat{\mu}_t=\widehat{m}_t\text{-}\widehat{m}_{t\text{-}1}+\widehat{\pi}_t \tag{11}$$

$$\widehat{a}_t=\rho_a\widehat{a}_{t\text{-}1}+\varepsilon_{a_t} \tag{12}$$

$$\widehat{e}_t=\rho_e\widehat{e}_{t\text{-}1}+\varepsilon_{e_t} \tag{13}$$

$$\widehat{z}_t=\rho_z\widehat{z}_{t\text{-}1}+\varepsilon_{z_t} \tag{14}$$

where $\widehat{m}_t, \widehat{mc}_t$ represent (log-deviations of) real balances and real marginal costs, respectively; and the following relationships hold between structural parameters, the steady-state (upper-barred variables), and the composite parameters of equations (7)-(9),

$\psi_1=\left(\frac{-\Psi_1}{(\overline{Y})^{(1-h)}\Psi_{11}}\right)$ $\qquad \phi_1=\frac{(\psi_1^{-1}-1)h}{1-\beta h}$

$\psi_2=\left(\frac{-\Psi_{12}}{(\overline{Y})^{(1-h)}\Psi_{11}}\right)\left(\frac{\overline{m}}{\overline{e}}\right)$ $\qquad \phi_2=\frac{\psi_1^{-1}+(\psi_1^{-1}\text{-}1)\beta h^2\text{-}\beta h}{1\text{-}\beta h}$

$\lambda=(1\text{-}\theta)(1\text{-}\beta\theta)\xi$ $\qquad \chi=\frac{\varphi+\alpha}{1\text{-}\alpha}$

$\xi=\frac{(1\text{-}\alpha)}{1+\alpha(\varepsilon\text{-}1)}\theta^{-1}$

[6] The symbol $\widehat{\ }$ denotes percentage deviations of a variable from its steady-state value.

Equation (7) arises from the household's optimal intertemporal allocation of wealth. The case of nonseparability across consumption and real balances makes the marginal utility of consumption a function of the amount of real balances optimally demanded by the households. The presence of habits makes the marginal utility of consumption also dependent on lags of output and further leads of money and output. Therefore, in equilibrium, output will depend on current and expected real balances after accounting for the money demand shock. Notice that as $h \to 0$, expression (7) approaches the usual Euler equation for consumption under time-separable preferences. The real-balances term will disappear from the aggregate demand equation under the parameter restriction $\psi_2 = 0$, i.e. as long as the cross-derivative between consumption and real balances is zero in the utility function. As we discuss in the next section, however, a strong indicator role for money, not captured by standard money demand specifications, may prevail even if the restriction $\psi_2 = 0$ holds.

Aggregate demand also depends upon the present discounted value of current and future real short-term interest rates; so the sensitivity of output to interest-rate movements depends upon the coefficient ψ_1, which is inversely related to the households' degree of risk aversion.

The supply side of the model is characterized by two equations: first, a New Keynesian Phillips curve, (8), which allows both prior and expected future inflation, as well as real marginal cost, to matter for current inflation; and second, a relationship between real marginal cost, detrended output, real balances, and the technology shock, equation (9). Notice that, if we assume that all new prices (p_t^*) are set on a profit-maximizing basis, i.e. $\omega = 0$, then inflation becomes a purely forward-looking variable. Moreover, the assumption of decreasing returns to labor implies that the link between output and inflation depends not only on the degree of nominal rigidities, but also the elasticity of output with respect to employment $(1 - \alpha)$, and the labor supply elasticity (φ) through the coefficient χ. The nonseparability in preferences across real balances and consumption implies a direct influence of the former variable on marginal cost and so on inflation. In the presence of habits, real marginal cost also depends on leads and lags of output, money balances and the preference shock a_t. To close the model, we specify AR(1) processes for the aggregate demand shock (12), the money demand shock (13) and the technology shock (14), with innovations ε_{a_t}, ε_{e_t} and ε_{z_t} respectively, as well as a money demand equation, which we now discuss.

2.5 Money Demand

The model is completed with a specification of money demand behavior. The specification of portfolio adjustment costs determines the form of the money demand relationship. The model without adjustment costs implies that the money demand equation is as follows:

$$\begin{aligned} \widehat{m}_t &= \gamma_1 \widehat{y}_t - \gamma_2 \widehat{r}_t + \left[\gamma_2 (r-1)(h\phi_2 - \phi_1) - h\gamma_1\right] \widehat{y}_{t\text{-}1} \\ &\quad - \left[\gamma_2 (r\text{-}1)\beta\phi_1\right] E_t \widehat{y}_{t+1} + \frac{\psi_2}{\psi_1} \frac{(r\text{-}1)\beta h \gamma_2}{(1\text{-}\beta h)} E_t \widehat{m}_{t+1} \\ &\quad - \frac{(r\text{-}1)\beta h (1\text{-}\rho_a)}{(1\text{-}\beta h)} \gamma_2 \widehat{a}_t + \left[1 - (r-1)\gamma_2 \left(\frac{\psi_2}{\psi_1} \frac{\beta h \rho_e}{(1-\beta h)} + 1\right)\right] \widehat{e}_t \end{aligned} \qquad (15)$$

where $\gamma_1 = \left(r \frac{(\overline{Y})^{(1-h)}}{\overline{m}} \frac{\psi_2}{\psi_1} + (r-1)\frac{1}{\psi_1}\right)\gamma_2$ and $\gamma_2 = \frac{r}{(r-1)} \frac{\overline{e}}{\overline{m}} \left(\frac{\Psi_2}{(r-1)\overline{e}\Psi_{12} - r\Psi_{22}}\right)$.

Expressions (15), (10) and (11) describe the money market. Equation (15) is a generalized money demand equation, where the coefficients γ_1 and γ_2 are the long-run real-income and interest-rate response parameters. Again the presence of habits in the utility function generates a dynamic equation in which money demand depends also on future output and real balances as well as on the preference shock a_t. Equation (11) is an identity connecting nominal money growth, real balances, and inflation.

As noted above, allowing for nonseparability across real balances and consumption gives real balances an explicit role in both the output and inflation equilibrium relationships.[7] Finally, note that equation (15) can be solved forward such that m_t is a function of the present discounted value of future nominal interest rates (see also the next section). This underlies the so-called "direct effect," whereby money variations reflect determinants of aggregate demand other than the current short-term interest rate. To establish the role of money, we must separately identify such an effect from

[7] A reduced-form equation that has been proposed in the literature to look at the inflation-forecasting properties of monetary aggregates is the P^* model; see e.g. Orphanides and Porter (2000). Svensson (2000) argues that the P^* model provides some basis for emphasizing the real balances gap (i.e. the difference between the current level of real balances and its long-run equilibrium level). The present setup provides a sound microfoundation for the presence of a sort of "real balances deviation," $\widehat{m}_t - \widehat{e}_t$, in inflation dynamics. Notwithstanding this, this model imposes cross-parameter restrictions that should be tested in order to assess the empirical relevance of this term; and in contrast to the P^* approach, the role of money specified here is integrated into a standard Phillips curve framework, where inflation depends on real marginal cost.

the "real balance effect" or, more precisely, "nonseparability effect" related to the cross-derivative of the marginal utility of consumption and real balances. In order to do that, we need to consider a specification with portfolio adjustment costs.

To that end, if we consider the specification of preferences given by equations (2) and (3) we obtain an alternative money demand equation which allows us to identify both effects separately:

$$\begin{aligned}(1+\delta_0(1+\beta))\widehat{m}_t &= \gamma_1\widehat{y}_t\text{-}\gamma_2\widehat{r}_t+\left[\gamma_2(r\text{-}1)(h\phi_2\text{-}\phi_1)\text{-}h\gamma_1\right]\widehat{y}_{t-1}\text{-}\left[\gamma_2(r\text{-}1)\beta\phi_1\right]E_t\widehat{y}_{t+1} \\ &\quad +\delta_0\widehat{m}_{t\text{-}1}+\left[\frac{\psi_2}{\psi_1}\frac{(r\text{-}1)\beta h\gamma_2}{(1\text{-}\beta h)}+\delta_0\beta\right]E_t\widehat{m}_{t+1}\text{-}\frac{(r\text{-}1)\beta h(1\text{-}\rho_a)}{(1\text{-}\beta h)}\gamma_2\widehat{a}_t \\ &\quad +\left[1-(r-1)\gamma_2\left(\frac{\psi_2}{\psi_1}\frac{\beta h\rho_e}{(1\text{-}\beta h)}+1\right)\right]\widehat{e}_t \qquad (16)\end{aligned}$$

where $\delta_0 = -\frac{c^2 d}{\Psi_{22}\overline{m}^2} > 0$. The two channels are captured through the coefficients on past and expected future real balances. In particular, under no portfolio adjustment costs, i.e. $d \to 0$, then $\delta_0 \to 0$, the behavior of current real money balances does not depend on lagged real balances. In addition, even if there is no nonseparability effect, i.e. $\psi_2 \to 0$, expected future real balances still matter for current values of that variable. Finally, note that it is not possible to separately identify the parameters d and c. We therefore normalize $c = 1$, allowing us to estimate the coefficient on adjustment cost d.

3 Money and the Natural Rate of Interest

In Wicksell's (1898) original outline of the link between price-level behavior and the spread between real and natural interest rates, he emphasized the connection of money creation with this spread. That is, to keep actual rates steady in the face of a real shock that raises the natural rate, the monetary authority must create additional money. In standard New Keynesian models, this connection is present, but because real money demand is a static function of current output and the policy instrument (the short-term nominal interest rate), all information about the natural rate contained in real money balances comes via the coefficients on these two variables. The remaining variation in real balances simply reflects money demand shocks that devalue the usefulness of money as an indicator.

When real money demand is forward-looking, however, the information in real balances about the natural rate is increased. If real money is registering weakness or strength that is hard to account for in the behavior of current income and the short-term nominal interest rate, that may be a signal of changes in current or expected future values of the natural real interest rate. We explore this property in our estimated model, but to provide intuition, in this section we briefly consider a version of the model with white noise IS and money demand shocks, and portfolio adjustment costs like those in equation (2), but no other source of nonseparability in utility. Then the money demand condition (16) may be written:

$$\widehat{m}_t = \mu_y \widehat{y}_t + \mu_r \widehat{r}_t + \mu_1 \widehat{m}_{t-1} + \beta \mu_1 \; E_t \widehat{m}_{t+1} + e'_t \tag{17}$$

where $\mu_1 \equiv \frac{\delta_0}{(1+\delta_0(1+\beta))}$, $\mu_y \equiv \frac{\gamma_1}{(1+\delta_0(1+\beta))}$, $\mu_r \equiv -\frac{\gamma_2}{(1+\delta_0(1+\beta))}$, and $e'_t = \frac{1}{(1+\delta_0(1+\beta))} e_t$. Notice that the long-run income elasticity and interest-rate semi-elasticity of money demand correspond to γ_1 and γ_2, respectively. The IS equation becomes:

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - \sigma \widehat{rr}_t + \widehat{\nu}_t \tag{18}$$

where $\widehat{rr}_t = [\widehat{r}_t - E_t \widehat{\pi}_{t+1}]$, $\widehat{\nu}_t = (1-\rho_a)\sigma \widehat{a}_t$ and $\sigma \equiv \frac{1}{\phi_2}$.

For future reference, we note also that equation (18) implies:

$$\widehat{y}^*_t = -\sigma \sum_{i=0}^{\infty} \widehat{rr}^*_{t+i} + \sigma \widehat{a}_t \tag{19}$$

and

$$\widehat{y}_t - \widehat{y}^*_t = -\sigma \sum_{i=0}^{\infty} (\widehat{rr}_{t+i} - \widehat{rr}^*_{t+i}) - \rho_a \sigma \widehat{a}_t \tag{20}$$

where $\widehat{rr}^*_t$ and $\widehat{y}^*_t$ are the natural levels of the short-term real interest rate and output, respectively.

3.1 Forward-Looking Money Demand Equation

Solving condition (17) using the methods described in Sargent (1987), we obtain:

$$(1\text{-}\psi L)\widehat{m}_t = \left(\frac{\psi}{\mu_1}\right) \sum_{i=0}^{\infty} (\beta\psi)^i E_t \{\mu_y \widehat{y}_{t+i} + \mu_r \widehat{r}_{t+i}\} + e'_{t+i} \tag{21}$$

where L is the lag operator, ψ is a stable root ($0 < \psi < 1$), and μ_1 is a function of δ_0, i.e a function of c in equation (3). Representation (21) establishes that real money demand is a function of its lagged value and the expected stream of output and nominal interest rates, as well as the white-noise money demand shock e'_t.

We now write this expression in a manner that separates the forward-looking terms from the current and lagged variables:

$$\widehat{m}_t{=}\psi\widehat{m}_{t-1}{+}b_0\widehat{y}_t{+}c_0\widehat{r}_t{+}\sum_{i=1}^{\infty}b_iE_t\widehat{y}_{t+i}{+}\sum_{i=1}^{\infty}c_iE_t\widehat{r}_{t+i}{+}e'_t \tag{22}$$

where $b_0{=}\psi\frac{\gamma_1}{\delta_0}$, $c_0{=}{-}\psi\frac{\gamma_2}{\delta_0}$, and b_i and c_i coefficients are defined in conformity with equation (21). Condition (22) can, in turn, be decomposed using equations (19), (20), and the Fisher relation $\widehat{rr}_t{=}[\widehat{r}_t{-}E_t\widehat{\pi}_{t+1}]$ as:

$$\begin{aligned}\widehat{m}_t \;=\; & \psi\widehat{m}_{t-1}{+}b_0\widehat{y}_t{+}c_0\widehat{r}_t{+}\sum_{i=1}^{\infty}b_iE_t\widehat{y}^*_{t+i}{+}\sum_{i=1}^{\infty}c_iE_t\widehat{rr}^*_{t+i}{+} \\ & \sum_{i=1}^{\infty}b_iE_t\{\widehat{y}_{t+i}{-}\widehat{y}^*_{t+i}\}{+}\sum_{i=1}^{\infty}c_iE_t\{\widehat{rr}_{t+i}{-}\widehat{rr}^*_{t+i}\}{+}\sum_{i=1}^{\infty}c_iE_t\widehat{\pi}_{t+1+i}{+}e'_t\end{aligned} \tag{23}$$

The above expression casts the forward-looking variables in terms of natural output levels, natural real interest rates, output gaps, real interest-rate gaps, and expected future inflation rates. This way of looking at our generalization of money demand indicates that the new specification overturns Woodford's (2007) critique of money. According to that critique, money is uninformative about natural-rate values because money demand depends only on current actual real GDP, not the natural levels of output or interest rates. This critique is no longer valid when there are portfolio adjustment costs.

Further restrictions on this condition can be obtained by an explicit specification of price-setting behavior. We demonstrate this here with two examples: one-period price setting and Calvo price setting.

3.2 Example 1: One-period-ahead price setting

Consider first the simple specification of price adjustment, used by Obstfeld and Rogoff (1996) and many others, where nominal prices must be set one period in advance but are then free to adjust. This specification implies that real variables are always expected to revert to their flexible-price (natural) values from next period onward: for $i > 0$, $E_t\widehat{y}_{t+i} = E_t\widehat{y}^*_{t+i}$ and $E_t\widehat{rr}_{t+i} = E_t\widehat{rr}^*_{t+i}$. The money demand expression may then be written as:

$$\widehat{m}_t{=}\psi\widehat{m}_{t\text{-}1}{+}b_0\widehat{y}_t{+}c_0\widehat{r}_t{+}\sum_{i=1}^{\infty}d_iE_t\widehat{rr}^*_{t+i}{+}\sum_{i=1}^{\infty}c_iE_t\widehat{\pi}_{t+1+i}{+}e'_t \tag{24}$$

where $d_i{=}c_i - \sigma(\sum_{i=1}^{\infty} b_i)$. Money demand thus contains valuable information beyond that recorded by its responses to current income and the current nominal rate: it varies in reaction to movements in expected future natural real rates, as well as expected future inflation.

3.3 Example 2: Calvo price setting

The basic version of Calvo price setting implies:

$$\widehat{\pi}_t{=}\beta E_t\widehat{\pi}_{t+1}{+}\alpha(\widehat{y}_t - \widehat{y}^*_t) \tag{25}$$

Solving this forward and substituting in equation (20), we have:

$$\widehat{\pi}_t{=}{-}\sigma\alpha\sum_{i=0}^{\infty}\beta^iE_t\{\sum_{j=0}^{\infty}(\widehat{rr}_{t+j} - \widehat{rr}^*_{t+j})\} \tag{26}$$

or

$$\widehat{\pi}_t{=} - \sigma\alpha\sum_{i=0}^{\infty}\phi_i(\widehat{rr}_{t+i} - \widehat{rr}^*_{t+i}) \tag{27}$$

where $\phi_i{=}{-}\sigma\alpha\sum_{j=0}^{i}\beta^j$. This expression implies that the money demand condition (23) may be written as:

$$\widehat{m}_t{=}\psi\widehat{m}_{t\text{-}1}{+}b_0\widehat{y}_t{+}c_0\widehat{r}_t + \sum_{i=1}^{\infty}d_iE_t\widehat{rr}^*_{t+i} + \sum_{i=1}^{\infty}f_iE_t\{\widehat{rr}_{t+i} - \widehat{rr}^*_{t+i}\}{+}e'_t \tag{28}$$

where d_i is defined as above, and $f_i{=}{-}\sigma[\sum_{j=1}^{\infty} b_j]{+}c_i$, for $i = 1$, and $f_i = -\sigma[\sum_{j=i}^{\infty} b_j] + c_i - c_{i-1}\alpha\sigma\sum_{j=0}^{i-2}\beta^j$ for $i > 1$. Equation (28) reveals that *all* of the variation in real balances not arising from its "conventional" determinants (i.e. current real income, the current short interest rate, lagged balances and the money demand shock) is associated with movements in expected future real-rate gaps or expected future natural real interest rates.[8] We note that the relationship between real money balances and the natural rate is quite complex, not only because of the dynamics involved, but also because the natural rate enters with both negative and positive coefficients in the expression.

This perspective on the money demand relationship highlights three advantages of our estimation of our structural model by full-information methods. First, standard estimated money demand functions neglect forward-looking behavior. The resulting specification error overlooks the information about the natural rate in money demand, instead attributing the associated variation in real balances to money demand shocks, lagged adjustment, and responses to current income and the nominal interest rate. Our approach instead isolates the forward-looking component of money demand, and so offers the prospect of consistent estimation of the money demand parameters. Second, by specifying the shock processes and policy behavior explicitly, and so the implied path of the expectations terms that appear in agents' optimality conditions, we are able to extract natural-rate estimates from the other unobservable determinants of money demand. Third, other empirical estimates of natural rate and real-rate gap series using systems methods, whether with *ad hoc* models (e.g., Laubach and Williams, 2003) or DSGE models (e.g., Smets and Wouters, 2003), sacrifice information on the natural rate by not including real money balances in the set of variables modelled. Our systems estimates, by contrast, include money in the likelihood function, and so exploit the valuable information in money suggested by equation (28).

4 Empirical Evidence

The maximum likelihood estimation procedure follows Hansen and Sargent (1997) and later applications can be found in Kim (2000) and Ireland (2001, 2004). The procedure involves expressing the stationary solution of the model state-space form and estimating the model's parameters using a recursive Kalman filter algorithm (see Ireland, 2003, for details).

[8] When we generalize (28) for the case of serially correlated IS shocks and habit formation in preferences, additional lagged variables and the current IS shock appear, but expected future values of the natural rate continue to appear prominently.

4.1 Baseline Estimates

This section presents our parameter estimates for each economy. The loglinearized optimizing model that we estimate refers to deviations of variables from their steady-state values (or steady-state growth paths in the case of output and real money), rather than describing actual levels of variables. For each economy studied, following Ireland (2003, 2004), we detrend output and real balances separately prior to estimation. Inflation and nominal interest rates also exhibit a (downward) trend over our sample; nevertheless, we continue to use the (demeaned) levels of these variables in estimation, on the grounds that the trends may be reduced or eliminated when these variables are cast as linear combinations (e.g. as a real interest rate). We consider this issue further in Section 6.

In Tables 2 and 3 we present the results of the parameter estimates for the unrestricted (nonseparable preferences) models of Section 2 (equations (7)-(14), and (16)). For comparability, we use narrow measures of money: domestic monetary base for the U.S. and M1 for the euro area. The sample periods used are also similar: 1979:3 to 2003:3 for the U.S., 1980:1 to 2004:4 for the euro area.[9]

The main result concerns the effect of money on output and inflation that may be captured by either ψ_2 and/or by δ_0. The null of $\psi_2 = 0$ cannot be rejected for either economy. This implies separability of utility across consumption and real balances, so that a role for money does not arise from explicit terms involving money in either the IS equation or the Phillips curve. This result is consistent with those obtained by Ireland (2004) and Andrés, López-Salido and Vallés (2006). Money, however, seems to have a different kind of "direct effect," i.e., an important forward-looking element, as the strongly significant value of δ_0 obtained with all specifications indicates. Adjustment costs are thus important for the dynamics of real balances. As discussed above, this forward-looking element of money demand confers on money considerable importance as an indicator of the determinants of aggregate demand.

We find strong evidence of habit formation in the two economies. The reported h values for the U.S. are fixed at 0.95; we obtained similar estimates in

[9] Our data for the United States consist of series downloaded from the Federal Reserve Bank of St. Louis FRED database as well as an updated version of the Anderson-Rasche (2000) money base series, adjusted for temporary increases in the base during the millennium transition and September 2001.

unrestricted estimation, but encountered convergence problems with allowing h to be estimated freely.[10] The interest-rate elasticity of the IS function (ψ_1) is significantly positive, suggesting an intertemporal elasticity of substitution slightly below (but not significantly different from) one for the euro area and somewhat lower for the U.S..

Since money does not seem to be relevant when allowed to appear explicitly in the IS and Phillips curves, its relation to output and prices is established by the money demand and the policy-rule equations. We find moderate money-growth responses in the estimated interest-rate policy rules. For money demand, the interest-rate semi-elasticity is large and significant in the euro area, and is fixed at a similar value for the U.S. after unrestricted estimation suggested values of that order. We obtain low estimated income elasticities of money demand. This may reflect the fact that we use detrended data, and therefore sacrifice information from the levels of the data. In addition, Lucas (1988) argues that more plausible money demand estimates arise from fixing the income elasticity at unity. To explore the implications of this restriction for the hypotheses of interest to us, we have reestimated the model removing the theoretical restrictions regarding the income elasticity of money demand and imposing a unit elasticity instead (i.e., $\gamma_1 = 1$).[11] These appear in column 2 in Tables 2 and 3. Although the other estimated parameters change somewhat, imposing this value does not affect the results regarding the role of money in the model: nonseparability in preferences can be safely suppressed; whereas the dynamic (forward-looking) component of money demand remains highly significant.

The estimates for the supply side of the economy reveal the importance of the forward-looking component of inflation and the low degree of indexation: κ is zero in both economies. This general pattern is one that is not consistent with the estimated Phillips curves obtained by other methods (Fuhrer, 1997, Galí, Gertler, and López-Salido, 2001) in which a strong role for lagged inflation in the Phillips curve has been found. Nevertheless, our results are in line with recent microeconomic evidence (see e.g. the findings of the Inflation Persistence Network discussed in Angeloni *et al*, 2005). One way to rationalize the apparent lack of indexation is that our model implies

[10] Giannoni and Woodford (2005) also obtain values of h above 0.9 for the U.S., for a sample period similar to ours.

[11] Thus these estimates restrict the income elasticity, but remove the cross-parameter restriction regarding income and interest elasticities of money demand. The result is a higher overall likelihood value despite the income-elasticity restriction.

a strong autoregressive pattern for the stochastic term in the Phillips curve (i.e., $\frac{\psi_2}{\psi_1}\frac{(1\text{-}\beta h\rho_e)}{(1\text{-}\beta h)}\widehat{e}_t - \frac{\beta h(1\text{-}\rho_a)}{(1\text{-}\beta h)}\widehat{a}_t - (1{+}\chi)\,\widehat{z}_t$). Additionally, the presence of habits in preferences changes the dynamic pattern of the marginal cost variable, which now depends on leads and lags of output.

An interesting difference arises across economies in the estimated slope parameter for the Phillips curve, λ. Although not very precise, the point estimates suggest that nominal rigidity is more important in the United States ($\lambda = 0.09$) than in the euro area (0.6). In addition, it is possible to back out (from the parameter χ) the value of the elasticity of labor supply from the estimates. The implied elasticity is somewhat lower for the U.S. than for the euro area.

The estimated interest-rate rules also display many similarities across economies. There is significant interest-rate smoothing of similar magnitude (around 0.8), and the interest-rate response to output (ρ_y) is modest but significant in both economies (around 0.15). The response of the nominal rate to the inflation rate is well above 1.0. Finally, money growth is present in both estimated policy rules. This term may be approximating either genuine money targeting by the central bank during the sample, or a way of targeting future inflation, by responding to information beyond that contained in current π_t.

Summing up, all the models have reasonable point estimates for most structural parameters. Our estimates of money demand elasticities are less satisfactory, possibly reflecting the use of detrended data in estimation. The main hypothesis of interest are not affected by the imposition of more conventional values for the money demand elasticities. The estimated values of the intertemporal elasticity of substitution in private spending appear reasonable. Both economies exhibit strong habit formation in preferences, while labor supply is highly elastic. The Phillips curve estimates suggest a very low degree of "backward" or "dynamic" indexation, and the euro area displays less nominal stickiness (i.e., higher implied probabilities of price adjustment) than the U.S. The estimated policy rules indicate strong long-run responses to inflation and a high degree of interest-rate smoothing. The money demand shock and both real shocks display strong inertia.

5 Dynamics of Money and the Natural Rate

In this section we examine the dynamics of money and the natural rate of interest in our estimated models. There are two shocks that drive real variables in the flexible-price economy: the IS (preference) shock and the technology shock. As stressed in Section 3, when money demand is forward-looking, some variation in real balances, given current income and the nominal interest rate, will reflect portfolio responses to those real shocks (either aggregate demand or technology). Because these real shocks are the determinants of the natural rate of interest, the portion of real balance variation arising from these shocks will reveal information about natural real rate behavior.[12] We investigate the implied relationship between the natural rate and the real money stock by examining key moments and impulse responses of the model. We thus aim to illustrate how the value of money is increased in our estimated models, relative to the New Keynesian baseline, by the specification of money demand dynamics for which we have found empirical support.

From equation (28) it can be seen that real money fluctuations are correlated with the natural rate, given the other determinants of money demand. Two factors therefore drive the response of the real balances to any real shock: first, the response of the natural rate to the shock, and second, the policy response to the shock, as recorded in how actual rates in the next few quarters change relative to their natural value. We now analyze how these two terms behave in response to each of the real shocks considered in our analysis: the IS shock and the technology shock.

5.1 IS Shocks

In standard sticky-price models, one can conjecture that the reduced-form relationship between real balances and the IS shock is negative. This is based on the presumption that in response to a positive IS shock, both the

[12] The natural real rate of interest corresponds to the short-term real interest rate that would prevail when the Calvo probability approaches 1.0, i.e., when all prices are flexible (and all firms are forward-looking). The natural-rate process will be invariant to the monetary policy rule, but will be a (possibly dynamic) function of the two real shocks in the model. In the present application, obtaining a natural-rate series entails evaluating our model with parameters describing preferences and production at their estimated values, solving the model under flexible prices, and obtaining a Wold-style representation of the natural rate. The natural-rate estimates are then generated by a finite-order approximation of the Wold representation (see Neiss and Nelson, 2003, for details).

natural rate and potential output shift up. If policymakers then partially accommodate the shock, allowing real rates to follow the natural rate to a limited extent, there will tend to be increases in output, the nominal rate, and expected inflation, and the emergence of a negative real rate gap (i.e., actual rates below their natural levels). Under that scenario, the negative real balances/natural rate relation emerges. Money demand fundamentally depends upon the expected path of nominal rates, and provided expected future values of the natural rate move in the same direction as the nominal rate in response to the IS shock, a negative relationship between real balances and the natural rate will emerge in the data.

Figures 1 and 2 report the responses to an IS shock in each economy (considering the case of a unitary income elasticity of money demand). The above conjecture is confirmed by the impulse response of real balances and the natural rate in the U.S. and euro area. An IS shock drives up the natural rate, while real balances move down, so exhibiting an inverse relationship with the natural rate. The negative relationship is amplified by monetary policy, which raises the nominal interest rate in response to the (temporary) increase in real GDP.

Despite this apparent confirmation of our intuition regarding the effect of an IS shock, it should be noted that the signs of the response of the natural rate to the IS shock are sensitive to the degree of habit formation in the model. Figure 3 plots the impact effect of the IS shock on the natural rate in our model as a function of the degree of habit formation (i.e., the parameter h), with all the other parameters of the model held at their estimated values. As can be seen, when habits become very powerful (corresponding to h well above 0.90), households become so stubborn about maintaining their consumption at its previous level that they need to be induced by lower real interest rates to consume a larger quantity of output.

Even as the impact effect of the natural interest rate varies in sign, potential output consistently exhibits a positive response to IS shocks. Indeed, the condition for this positive response is simply that h is strictly positive. Since the IS shock does not enter the production function directly, it must affect the labor-leisure choice to affect potential output. When $h = 0$, the marginal utilities of consumption and leisure are raised by equal percentages by an IS shock, neutralizing the effect of the shock on labor supply.[13] But with h

[13]This represents a difference from the framework of Amato and Laubach (2004). There, IS shocks affect only the consumption term in the utility function and so stimulate labor

positive, the marginal utility of consumption is raised by more than the marginal utility of leisure, so labor supply increases to permit a path of higher consumption today and in coming periods. Accordingly, IS shocks raise potential output under habit formation (and reduce real marginal cost—see equation (9)).

5.2 Technology Shocks

In Figures 4 and 5, we plot the response of key model variables to the technology shock. In response to the shock, actual and potential output rise, and the natural rate of interest falls. The reduction in the natural rate is less pronounced in the euro area. The natural rate will decline if the constraint on consumption implied by the level of potential output is relaxed more today than in the future. In this case, thanks to greater output supply, the entire path of consumption can be higher than previously, but because the productivity shock wears off over time, potential output is raised more in the immediate few quarters than in the later quarters, so the natural rate declines.

Real balances exhibit an inverse relationship with the natural rate, but tend to register their peak response well after the natural rate has started returning to its steady-state value. For both the economies studied here, this reflects differences between the response of the nominal rate and the natural rate of interest to the technology shock. The shock initially raises potential output relative to actual GDP because nominal rates respond positively to output, restraining the extent to which real aggregate demand can expand with the increased potential. This produces a reduction in inflation, which leads, via interest-rate smoothing, to a protracted fall in nominal rates, and so a protracted rise in real balances.

The figures highlight that the forward-looking character of money demand enriches the relationship between real balances and the natural rate. To examine this further, we have computed some second moment statistics. Table 4 gives partial correlations between the real money stock and the natural interest rate for each of our estimated models. The partial correlations are the correlations between the two series holding constant the two determinants of money demand—current output and the nominal interest rate—that appear in the standard model without any forward-looking components

supply even when $h = 0$.

of money demand.[14] The correlations are an outcome of the interaction of model structure, policy rule, and shock processes in the model. The natural rate depends on the IS and technology shocks, while the real money stock depends on these shocks plus two shocks that do not matter for the natural rate, i.e., the monetary policy and money demand shocks. Despite the noise created in the real money series, it nevertheless has a negative correlation with the natural rate in all three estimated models. It is important to emphasize that, in the absence of habit formation, these partial correlations *are identically zero* when money demand is described by the New Keynesian benchmark of Table 1, which has no forward-looking money demand (either from habit formation or from adjustment costs).[15] Nonzero correlations will reflect the increased value of money as an indicator of real shocks, imparted by the combination of portfolio adjustment costs and habit formation. These two features create forward-looking money demand dynamics that make the partial correlation negative in the two economies.

As the preceding discussion indicates, the impulse responses and model correlations are inherently a function of both the estimated policy rule and the structure of private sector behavior. The forward-looking character of money demand is part of the structure of the model, and would prevail across different policy rules. This structural feature should also be taken into account in an analysis of optimal policy in our model. How the forward-looking nature of money demand impacts the welfare analysis of an optimizing model such as ours is beyond the scope of this paper, but is an important area for future research.

5.3 Natural Rate Realizations

By drawing on the estimated technology shock and IS shock series implied by our model, and the model-consistent expressions for the natural interest rate, it is possible to generate realizations for estimates of the natural rate.

[14] The correlations are computed using expressions for the analytical moments from the models' VAR representations.

[15] Any variation in real balances in such a benchmark that is not recorded in the current nominal interest rate and output is uninteresting noise. Habit formation helps bring the partial correlation away from zero by introducing forward-looking terms into the money demand function. Another reason why habit formation makes the correlation nonzero is that it puts an IS shock term into the money demand equation, but this term does not seem to be quantitatively relevant at our parameter values.

In Figure 6 we plot the resulting natural-rate series for the U.S. (using the parameter estimates obtained under the restriction of unit income elasticity of money demand).[16]

While, in the model, the natural interest rate is stationary, the realization of the series is near-nonstationary, varying from around 10% annualized in the early 1980s to less than 1% in 2003. This seems to reflect two interrelated factors: (1) the strain imposed on our model by the need to account for the downward in-sample trend in inflation—a difficult task because our model's structure presumes that inflation is stationary; (2) the high level of persistence of both real shocks. The latter seems to be contributing to the smoothness of the realized series more than it does to its downward trend. This is confirmed when we generate the natural-rate realization implied by the alternative parameterization of Amato and Laubach (2004), who assume white noise real shocks. This series is also depicted in the figure. Their specification, like ours, produces a downward trend in the empirical natural rate, but their natural-rate series is even more volatile than ours because of the wider swings around the trend. The variability of both natural-rate realizations, however, underscores Orphanides and Williams' (2002) emphasis on the amount of uncertainty in the natural rate.

6 Sensitivity analysis

In this section we consider the robustness of our estimates for the United States to alternative assumptions about the trends in the data and the definition of money.

6.1. Results using detrended inflation and interest-rate data

Our baseline estimates were produced from a likelihood in which data for output and real balances (both in log per capita units) appeared in detrended form, but in which the interest rate and inflation entered in demeaned levels.[17] Underlying these choices of data transformation was the assumption of stationarity of interest rates and inflation. This assumption is common in empirical work with DSGE models. But it may not be a good approximation

[16] The shock series are backed out from our observed data using the Kalman filter. Because the shock series have zero mean, the steady-state value of the natural rate is added back to the generated natural-rate series.

[17] Strictly speaking, they appear as demeaned logs of gross rates; this is equivalent for practical purposes to using demeaned levels of the net rates.

on sample periods—notably our U.S. sample of 1979 to 2003—which have featured, on average, disinflation. Here we consider estimates for the United States that allow for a trend in inflation and interest rates.

One option (see Ireland, 2007) is to treat the monetary authorities' inflation target as an unobserved variable that puts a common trend into both inflation and nominal interest rates. Instead, we have made a trend adjustment by separately linearly detrending nominal interest rates and inflation and putting these detrended variables in the likelihood function in place of the original levels series. While this approach does not allow us to trace the inflation target to fundamental shocks in the manner of Ireland (2007), it is somewhat more eclectic about the source of the interest-rate trend than Ireland's approach: we implicitly allow the real-rate component of the nominal interest rate to have a trend, instead of attributing the downward trend in interest rates entirely to the fall in the expected-inflation component.

Results are reported in Table 5, both with an unrestricted income elasticity of money demand and with an imposed income elasticity of unity. The estimates are similar to those we obtained with the original levels of inflation and interest rates in the likelihood. For our purposes, the most important result is that the key portfolio adjustment parameter δ_0 remains very sizable. Imposing a unit income elasticity restriction has the same effect it did previously, reducing the portfolio adjustment parameter by about a half but still leaving it significantly positive. The interest-rate responses to inflation are somewhat reduced, perhaps reflecting loss of information about this parameter from the detrending, but it remains sizable (around 1.7 to 1.9, compared to 2.2 previously). Preference, production, and price-adjustment parameter estimates are little changed, and there continues to be no support for utility nonseparability or for dynamic indexation of price contracts.

6.2. Results using U.S. M2

Our baseline results for the United States used a narrow money concept (the domestic monetary base). Here we report estimates using the M2 definition of money.[18] In line with our baseline estimates using narrow money, we enter M2 into the likelihood in logged, real per-capita, detrended form. Because M2 has a sizable own rate of return, owing to its interest-bearing deposit component one would expect its interest elasticity with respect to market interest rates to be smaller than in the case of narrow money. There-

[18] We use seasonally adjusted M2 data from the FRED database with an adjustment in 1983 Q1 for the introduction of money market deposit accounts.

fore, we estimate unrestrictedly rather than using the same value we used with the M0 estimates.

Results are reported in the final column of Table 5. Most of the structural estimates are similar to the estimates obtained using narrow money. The main differences that emerge are a larger role for IS shocks, which are both more persistent and more variable, and a somewhat higher Phillips curve slope. As in the estimates using narrow money, there continues to be support for separability of utility and absence of price indexation: the point estimates of ψ_2 and κ are zero. The inflation response in the interest-rate rule is smaller, with an increased share of the policy reaction to nominal variables taking the form of a response to money growth. The estimated policy response of about 0.5 to M2 growth, complementing a larger direct response to inflation, is in line with Ireland's (2001, Table 1) estimate of the U.S. policy rule after 1979.

Confirming our previous results with narrow money, δ_0 is positive and significant, which supports the position that adjustment costs are an important factor determining dynamics of real money balances. The other money demand parameter estimates indicate a low unrestricted income elasticity, as we found with narrow money. As before, this low elasticity most likely reflects our use of detrended money data. Experiments with an M2 specification that imposed γ_1 =1.0 indicated that δ_0 remained significant under this restriction. The M2 interest semielasticity estimate γ_2 in Table 5 is lower than the 2.0 value we used when estimating narrow money demand. The low value presumably reflects the effect of the own rate on deposits in reducing the aggregate elasticity of M2 demand, as discussed above.

7 Conclusions

In this paper, we have looked at the role of money in a general framework that encompasses three competing environments: the baseline New Keynesian model with separable utility and static money demand; nonseparable utility between consumption and real balances, along with habit formation; and the model modified to allow for adjustment costs for holding real balances. The last two variants imply a forward-looking character of real money balances that conveys on money an important role as a monetary policy indicator. The standard New Keynesian baseline is a restrictive special case in which money is less informative. We distinguished between these alternative settings by conducting a structural econometric analysis for the United States and the euro area. Our likelihood estimates confirmed the forward-looking character of money demand. A major source of this forward-looking behavior is the existence of portfolio adjustment costs.

We illustrated how the value of money is increased in our estimated models, relative to the New Keynesian baseline, by the specification of money demand dynamics for which we have found empirical support. We concentrated on the links between money and the natural rate, and demonstrated that money can have value as an indicator of future variations in the natural rate, even when inflation dynamics are viewed through a "neo-Wicksellian framework" of the type advocated by Woodford (2003).

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Table 1. Money's role in the transmission mechanism[a]

	New Keynesian model Baseline	Nonseparable utility	Forward-looking money demand
Is the money demand equation needed to obtain inflation and output paths?	NO	YES	NO
Do output and inflation have non-zero impulse responses to money demand shocks?	NO	NO	NO
Does money contain information about real shocks not present in scale variable?	NO	NO	YES

(*a*) In all cases, interest-rate rule assumed to have no response to money.

Table 2. Maximum likelihood estimates, Euro area, M1

Estimated Parameters	Unrestricted Estimates (1)	Unit Elasticity (2)
β	0.9902	0.9902
	(0.0027)	(0.0026)
ψ_1	0.9073	0.8992
	(0.0474)	(0.0504)
ψ_2	0.0000	0.0047
	(0.0202)	(0.0201)
h	0.8907	0.9064
	(0.0277)	(0.0272)
δ_0	3.2920	2.9558
	(0.5775)	(0.2625)
γ_1	0.0527	1.0000
	(0.0226)	(–––)
γ_2	2.5323	3.1893
	(0.7861)	(0.8247)
κ	0.0000	0.0000
	(0.0007)	(0.1125)
χ	0.4677	0.7448
	(0.4040)	(0.4784)
λ	0.6148	0.3892
	(0.5297)	(0.0951)
ρ_r	0.7185	0.7451
	(0.0731)	(0.0389)
ρ_y	0.1818	0.1951
	(0.0448)	(0.0396)
ρ_π	1.8653	1.8550
	(0.1273)	(0.1383)
ρ_μ	0.1537	0.1653
	(0.0635)	(0.0676)
ρ_a	0.9859	0.9839
	(0.0168)	(0.0187)
ρ_e	0.9842	0.9852
	(0.0176)	(0.0162)
ρ_z	0.9796	0.9781
	(0.0163)	(0.0176)
σ_a	0.0772	0.0715
σ_e	0.0210	0.0202
σ_z	0.0041	0.0041
σ_r	0.0016	0.0015
Log-Likelihood	1704.19	1704.36

Table 3. Maximum likelihood estimates, U.S., M0

Estimated Parameters	Unrestricted Estimates	Unit Elasticity
	(2)	(2)
β	0.99	0.99
	(–––)	(–––)
ψ_1	0.7252	0.5152
	(0.075)	(0.229)
ψ_2	0.0000	0.0000
	(0.019)	(0.060)
h	0.9500	0.9500
	(–––)	(–––)
δ_0	8.3406	4.4229
	(2.090)	(0.418)
γ_1	0.0486	1.0000
	(0.010)	(–––)
γ_2	2.0000	2.0000
	(–––)	(–––)
κ	0.0000	0.0000
	(–––)	(–––)
χ	1.0228	1.3082
	(0.360)	(0.800)
λ	0.0926	0.0255
	(0.087)	(0.030)
ρ_r	0.7262	0.7874
	(0.074)	(0.053)
ρ_y	0.1435	0.1890
	(0.069)	(0.070)
ρ_π	1.8966	1.7664
	(0.251)	(0.266)
ρ_μ	0.1603	0.2209
	(0.116)	(0.148)
ρ_a	0.8350	0.7003
	(0.103)	(0.169)
ρ_e	0.8940	0.9419
	(0.056)	(0.043)
ρ_z	0.9489	0.9283
	(0.031)	(0.039)
σ_a	0.0132	0.0147
σ_e	0.0410	0.0277
σ_z	0.0079	0.0100
σ_r	0.0027	0.0026
Log-Likelihood	1561.83	1562.91

Table 4. **Partial correlations of real money stock and natural rate, estimated models**
Holding current output and nominal interest rate constant

Euro area	–0.144
U.S.	–0.176

Table 5. Additional results for the United States

Estimated Parameters	U.S., M0, using detrended inflation and interest-rate data:		U.S., using M2 money definition
	γ_1 unrestricted	$\gamma_1 = 1$	
β	0.985 (—)	0.985 (—)	0.990 (—)
ψ_1	0.808 (0.003)	0.594 (0.181)	0.831 (0.070)
ψ_2	0.000 (0.024)	0.000 (0.044)	0.000 (0.086)
h	0.950 (—)	0.950 (—)	0.950 (—)
δ_0	10.229 (1.145)	4.929 (1.669)	5.909 (1.603)
γ_1	0.1029 (0.039)	1.0 (—)	0.023 (0.012)
γ_2	2.0 (—)	2.0 (—)	1.080 (0.560)
κ	0.0 (—)	0.0 (—)	0.0 (—)
χ	0.9057 (0.572)	0.970 (0.871)	0.811 (0.329)
λ	0.0607 (0.070)	0.049 (0.041)	0.166 (0.081)
ρ_r	0.7777 (0.066)	0.7823 (0.072)	0.634 (0.060)
ρ_y	0.1257 (0.074)	0.1313 (0.087)	0.089 (0.042)
ρ_π	2.2397 (0.7074)	2.1730 (1.146)	1.378 (0.212)
ρ_μ	0.2059 (0.172)	0.2297 (0.264)	0.530 (0.191)
ρ_a	0.6228 (0.156)	0.6211 (0.129)	0.879 (0.055)
ρ_e	0.8772 (0.061)	0.9226 (0.047)	0.967 (0.029)
ρ_z	0.8911 (0.078)	0.8825 (0.082)	0.961 (0.023)
σ_a	0.0093	0.0097	0.013
σ_e	0.0494	0.0308	0.009
σ_z	0.0110	0.0116	0.007
σ_r	0.0027	0.0027	0.003
Log-likelihood	1568.07	1569.02	1678.70

Figure 1. Responses to IS shock: Euro area

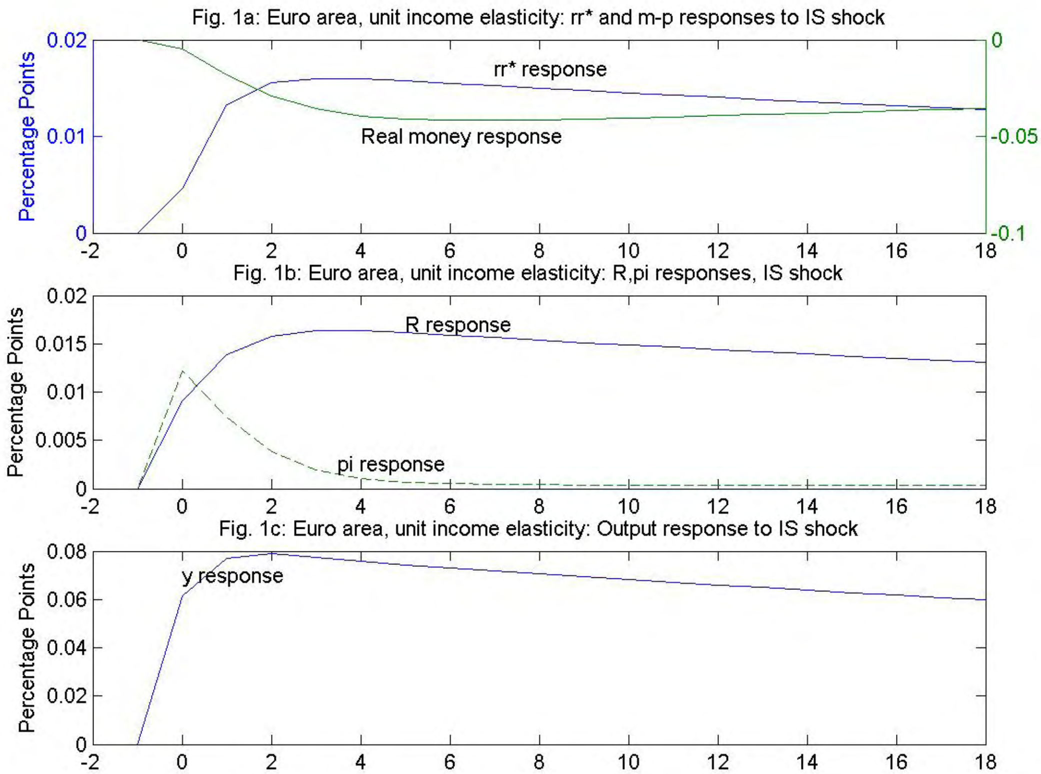

Figure 2. Responses to IS shock: United States

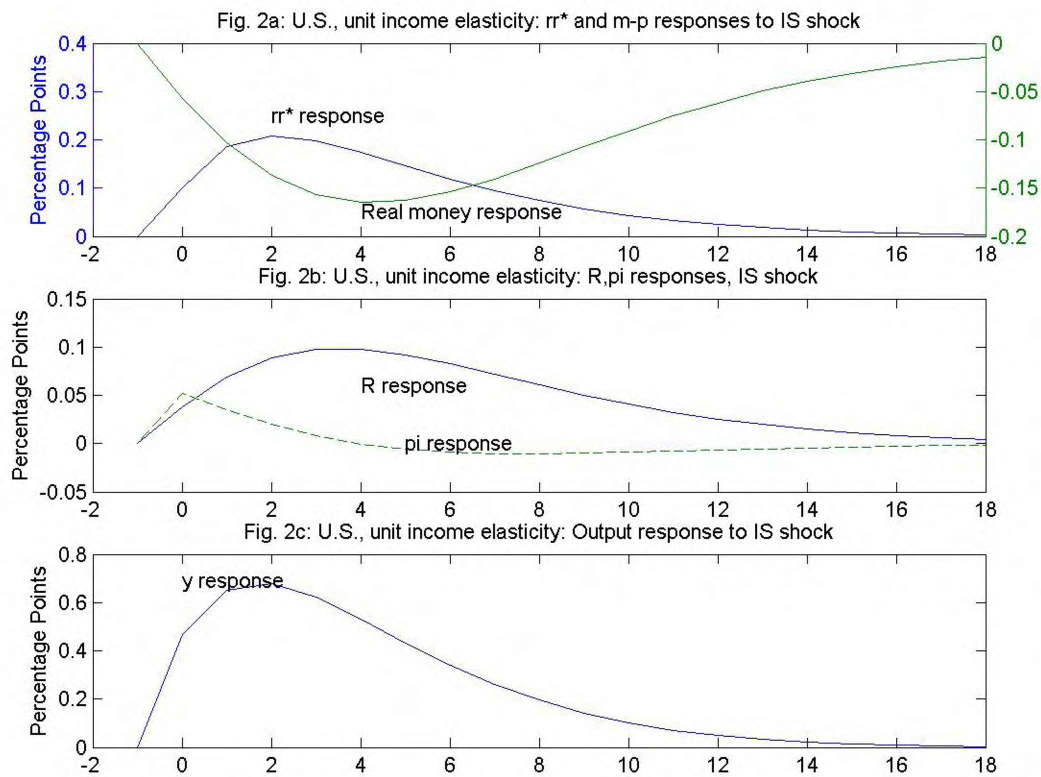

Figure 3. Habit formation and response of natural rate to IS shocks

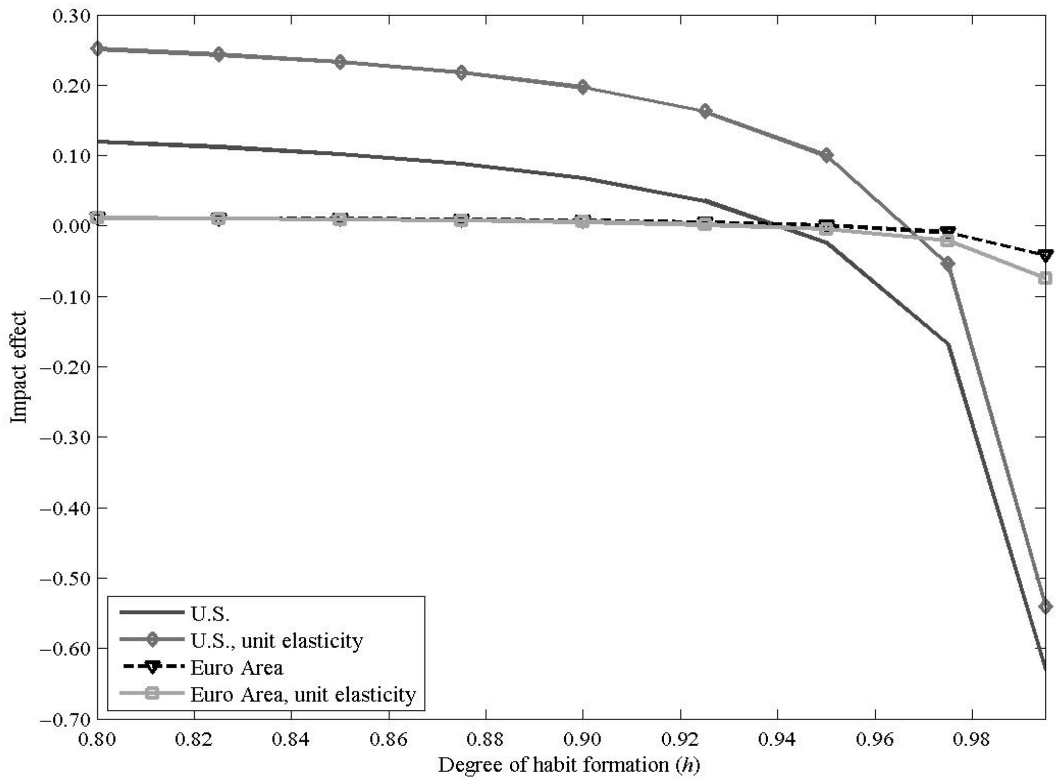

Figure 4. Responses to technology shock: Euro area

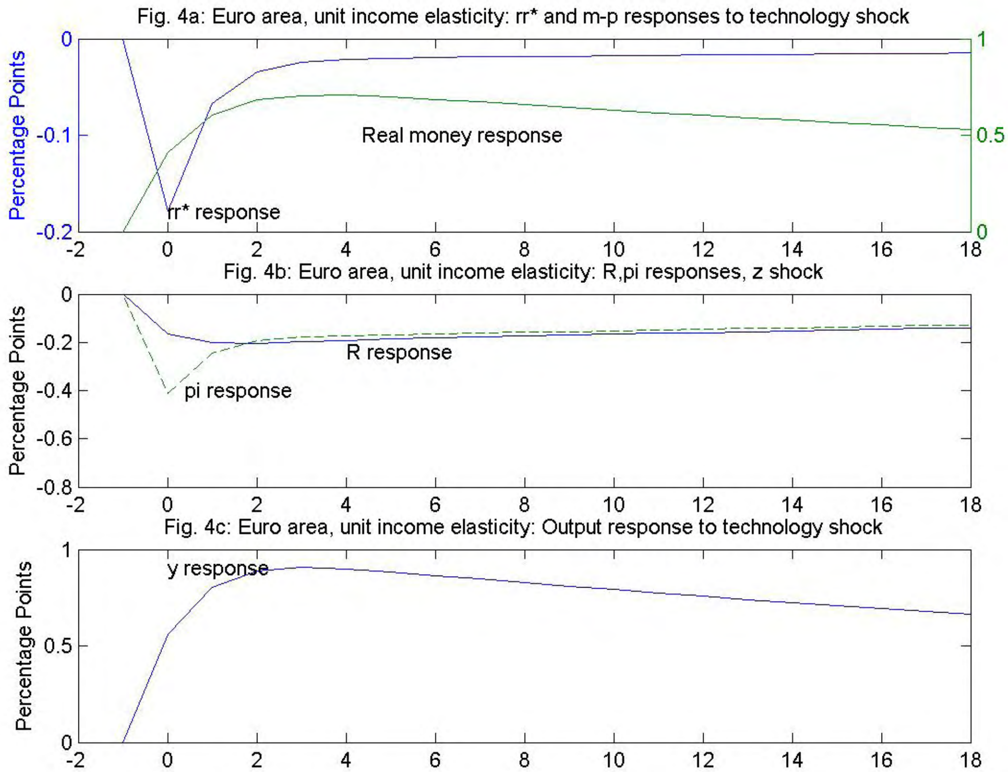

Figure 5. Responses to technology shock: United States

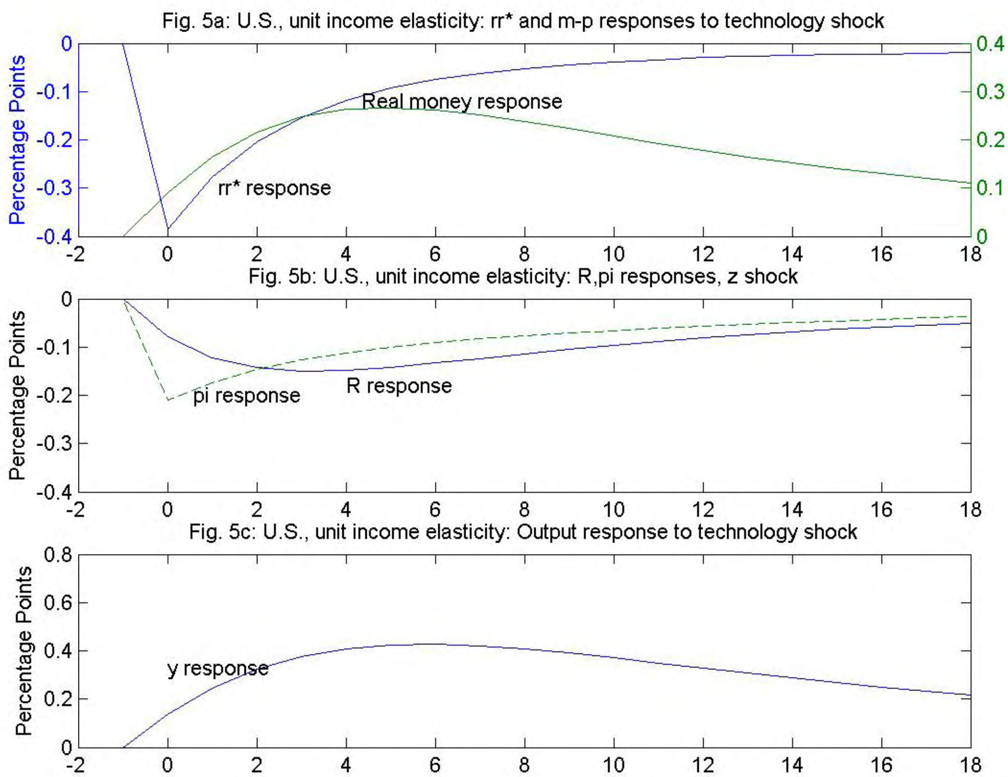

Figure 6. Realizations of the natural interest rate: United States