Volatility, Growth, and Welfare

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Abstract

This paper constructs an endogenous growth model driven by self-fulfilling expectation shocks to explain the stylized fact that the average growth rate of GDP is related negatively to volatility and positively to capacity utilization. The implied welfare gain from further stabilizing the U.S. economy is about a quarter of annual consumption, which is consistent in order of magnitude with estimates based on the empirical studies of Ramey and Ramey (1995) and Alvarez and Jermann (2004). Hence, policies designed to reduce fluctuations can generate large welfare gains because smaller fluctuations are associated with permanently higher rates of growth.

Keywords: Endogenous Growth, Welfare Cost of Business Cycle, Stabilization Policy, Sunspots, Imperfect Competition, Coordination Failures.

JEL codes: E12, E32, O40.

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1 Introduction

Ramey and Ramey (1995) present empirical evidence of a negative relationship between business cycle volatility and long-run growth: countries with lower output volatility tend to grow faster over time. This negative relationship has also been validated by other empirical studies.[1] The same negative relationship between the business cycle and long-run growth also exists in the U.S. time series. In particular, for the post-war period, the rate of capacity utilization is positively correlated with the mean growth rate and negatively with its standard deviation (see Figure 1). This suggests that, on average, periods of higher capacity utilization are also periods of faster growth and smaller fluctuations.[2]
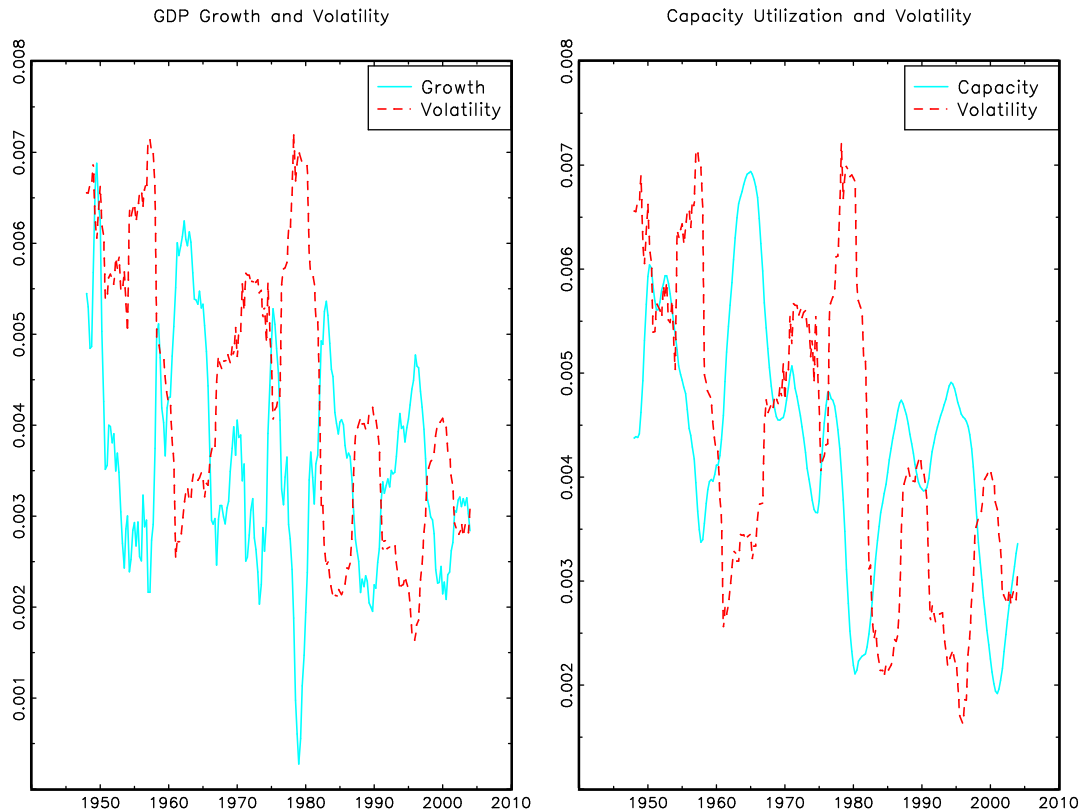
Figure 1. U.S. Time Series (1948:1 - 2007:4).

More specifically, Figure 1 shows movements of the U.S. manufacturing sector's capacity utilization rate, the real GDP growth rate, as well as the standard deviation of GDP growth based on

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[1]See, e.g., Aghion, Angeletos, Banerjee, and Manova (2005), Easterly, Islam, and Stiglitz (2000), Hnatkovska and Loayza (2004), Kroft and Lloyd-Ellis (2002), Martin and Rogers (2000), and Mobarak (2005), among others. Barlevy (2004a) documents a negative relationship between growth and volatility using time series data from the U.S.

[2]Barlevy (2004a) also documents a negative relationship between the mean GDP growth rate and its volatility based on U.S. data.

a 16-quarter rolling-window moving average.[3] It is clear from the graph that the average growth rate and the capacity utilization rate are both negatively related to the standard deviation of GDP growth. This stylized fact has important welfare and policy implications; it suggests that stabilization policies may be able to increase the average growth rate by raising the average capacity utilization rate, or vise versa.

Standard RBC models that treat the sources of long-run growth and short-tun fluctuations as orthogonal components of total factor productivity (TFP) cannot explain such stylized facts. This paper proposes an explanation for the stylized facts based on an endogenous growth model featuring sunspot shocks driven by firms' self-fulfilling expectations about aggregate demand. A novel feature of our model is that volatilities are endogenous, instead of coming from exogenous shocks to preferences or technology. Endogenous volatility stems from time-varying markups, which provide the crucial link among capacity utilization, mean growth, and volatility in our model. The motivation for focusing on time-varying markups as a fundamental source of fluctuations derives from the argument that movements in the marginal cost (or markup) are closely related to movements in measured TFP (see, e.g., Hall, 1986, 1988); hence, the link between shifts in marginal costs and the business cycle is an important channel to exploit. Pioneering work along this line includes Gali (1994,1996) and Rotemberg and Woodford (1991, 1992, 1995, 1999).[4]

To generate a time-varying markup, we use the mechanism analyzed by Ng (1980, 1992), Cooper and John (1988), and especially Wang and Wen (2007).[5] In these papers firms face demand externalities and set prices simultaneously without knowing the true marginal costs (because the equilibrium marginal costs depend on other firms' output levels). Hence, they may face extrinsic uncertainty regarding the equilibrium outcome of aggregate demand and marginal costs. Due to the strategic complementarity among firms' actions, which arises from imperfect substitutability of firms' output in the goods market, the extrinsic uncertainty can be self-fulfilling. Consequently, the economy is subject to coordination failures and endogenous fluctuations.[6]

The key distortion in a standard New Keynesian monopolistic competition model is a constant markup over marginal costs. However, when marginal costs become stochastic due to extrinsic uncertainty, there exists a further distortion which leads to larger average markups. This addi-

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[3]The definition of the variables are as follows: the average 4-year capacity utilization rate is $\bar{u}_t = \frac{1}{16} \sum_{j=0}^{15} u_{t+j}$, the average 4-year GDP growth rate is $\bar{g}_t = \frac{1}{16} \sum_{j=0}^{15} g_{t+j}$, and the standard deviation of GDP growth is $\sigma_t = \left[\frac{1}{16} \sum_{j=0}^{15} (g_{t+j} - \bar{g}_t)^2 \right]^{\frac{1}{2}}$. A rolling window is used in the estimation. The sample range is 1948:1 - 2007:4. The capacity series u is based on the U.S. manufacturing sector and is normalized so that it has similar magnitude as the other series in the graph. The results are robust for window sizes longer than 3 years.

[4]Also see Chatterjee, Cooper, and Ravikumar (1993), among others.

[5]Also see Svensson (1986), Ireland (1996), Carlstrom and Fuerst (1998a, 1998b), Adão, Correia, and Teles (2003), among others.

[6]See Wang and Wen (2007) for detailed analysis of this mechanism of endogenous fluctuations in standard DSGE models with imperfect competition.

tional distortion arises because monopoly price is a constant markup over a weighted average of the marginal costs (as in standard New Keynesian sticky price models). However, because a higher marginal cost is associated with a higher aggregate demand, high-cost states will receive disproportionately higher weights under expected profit maximization, where the weights themselves depend endogenously on expected demand. This nonlinearity and unequal distribution of weights generates an additional distortion between prices and average marginal costs, which would not exist if the marginal cost were constant.

This additional distortion due to stochastic marginal cost leads to lower average output because firms opt to charge higher prices with higher average markups during periods of higher demand. In an endogenous growth model, a lower output level translates into lower investment and lower economic growth. Hence, the larger the variability of the marginal cost, the lower will be the average growth rate. This explains the negative correlation between volatility and growth. Because production capacity (the capital stock) is predetermined by past investment, lower output in any period also means a lower rate of capacity utilization.

Since volatility and growth is negatively related in the model, the welfare cost of business cycles can be several orders larger than that calculated by Lucas (1987, 2003). In addition, because expectations-driven fluctuations in the model are inefficient, the welfare gain from eliminating such fluctuations by stabilization policies is astonishingly large.[7]

Our paper is related to a large literature that studies the link between the business cycle and growth. Important works in this literature include King and Rebelo (1993), Stadler (1990), Acemoglu and Zilibotti (1997), Jones, Manuelli, and Stacchetti, (2000), Francois and Lloyd-Ellis (2003), Barlevy (2004a), Jones, Manuelli, Siu, and Stacchetti (2005), and Aghion, Angeletos, Banerjee, and Manova (2005), among many others. However, the main body of this literature focuses on complete-market economies with exogenous business cycles driven by shocks from outside the economy. Consequently, the policy implications of this literature differ fundamentally from ours. When fluctuations are driven by exogenous shocks (such as shocks to TFP) in a complete-market economy, little scope exists for stabilization policies because fluctuations are optimal responses to such shocks. Thus, there is no gain from stabilizing the economy even if the welfare gain from eliminating the exogenous shocks is large (see, e.g., Barlevy, 2004a, 2004b). In contrast, in our model fluctuations are endogenously driven by coordination failures and self-fulfilling expectations about aggregate demand; business cycles are thus intrinsically inefficient.[8]

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[7]Yellen and Akerlof (2006) emphasize that the long-run Phillips curve may not always and everywhere be vertical; hence, stabilization policies can have potentially large welfare effects.

[8]For comprehensive literature reviews on the issue of welfare cost of business cycles and the benefits of stabilization, see Lucas (2003) and the literature therein. For previous works that evaluate welfare cost by linking endogenous growth to exogenous fluctuations, see Blackburn and Pelloni (2005), de Hek (1999), Epaulard and Pommeret (2003), Jones, Manuelli, Siu, and Stacchetti (2005), and Krebs (2003), among others.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 calibrates the model and examines its dynamic properties. Section 4 discusses the welfare cost of business cycles. Section 5 concludes the paper.

2 The Model

2.1 Firms

There is a final good in the economy, which is produced by using intermediate goods according to the Dixit-Stiglitz technology:

$$
Y = \left(\int_0^1 y(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{1}
$$

where $\epsilon > 1$ measures the elasticity of substitution among intermediate goods $y(i)$. The price of the final good is normalized to one and the price of the intermediate good i is denoted $p(i)$. The final good producers behave competitively; profit maximization in the final good sector yields the demand function for intermediate goods,

$$
y(i) = p(i)^{-\epsilon} Y. \tag{2}
$$

Substituting this into the production function yields the aggregate price index, $\int_0^1 p(i)^{1-\epsilon} di = 1$.

The economy has a continuum of monopolistic intermediate good producers of measure one, each producing a single differentiated good $y(i)$. Intermediate goods are produced by capital (k) only. The production function for intermediate goods is identical across firms and is given by:

$$
y(i) = Au(i)k(i), \tag{3}
$$

where A denotes the level of technology common to all firms and $u(i)$ the rate of capacity utilization for firm i. Intermediate good producers are assumed to be price takers in the input market. Let r denote the market interest rate, and let $\delta(i)$ denote the rate of capital depreciation for firm i. Following Greenwood et al. (1988), the rate of capital depreciation is assumed to depend on its usage rate:

$$
\delta(i) = \frac{\alpha}{1+\theta} u(i)^{1+\theta}, \quad \theta > 0. \tag{4}
$$

Hence the user's cost of capital facing firm i is $r + \delta(i)$.[9]

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[9]Since the capital stock is fixed in period t, in order for sunspot shocks to affect aggregate output, a second production factor (such as capacity utilization or hours worked) that can adjust instantaneously is needed. The importance of capacity utilization in understanding business cycles and growth has been emphasized by Greenwood et al. (1988), King and Rebelo (1999), Wen (1998), and Chatterjee (2005), among others. For an alternative approach with fixed capacity utilization but variable labor demand, see our working paper (Wang and Wen, 2006).

The cost function of an intermediate firm can be found by minimizing $[r + \delta(i)] k(i)$ subject to $Au(i)k(i) \geqslant y(i)$. Denote $\phi(i)$ as the Lagrangian multiplier for the above constraint, which is also the marginal cost. Cost minimization yields the following relationship:

$$r + \delta(i) = \phi(i)Au(i) \tag{5}$$

$$\alpha u(i)^\theta = \phi(i)A. \tag{6}$$

These first-order conditions imply $\delta(i) = \frac{1}{1+\theta}\alpha^{-\frac{1}{\theta}} (\phi(i)A)^{\frac{\theta+1}{\theta}}$ and

$$r = \theta\delta(i) = \frac{\theta}{1+\theta} (\alpha)^{-\frac{1}{\theta}} (\phi(i)A)^{\frac{\theta+1}{\theta}}. \tag{7}$$

The above equation shows that both the depreciation rate and the marginal cost are the same across firms. The intuition is that the production technology has constant returns to scale and firms face the same market interest rate; thus, the marginal cost ϕ must be the same across all firms. Consequently, the optimal rates of capital utilization and depreciation are also the same across firms.

By the downward sloping demand curve (2), there exist demand externalities across firms; namely, how much a firm should produce depends not only on its own price $(p(i))$ but also on the aggregate demand (Y). However, firms must each choose a price one period in advance without knowing the aggregate economic conditions (such as aggregate demand or aggregate marginal cost) that may prevail in period t. Yet these aggregate economic conditions depend crucially on the actions of the other firms over which an individual firm has no influence. Thus, each individual firm must form expectations for the level of aggregate demand (Y) when setting prices. Such expectations can become self-fulfilling, as explained by Ng (1980, 1992), Cooper and John (1988), and Wang and Wen (2007).

Without loss of generality, assume there are no fundamental shocks in the economy; then the only type of uncertainty, if any, is *extrinsic uncertainty*—in the language of Cass and Shell (1983) (i.e., sunspots). An intermediate good firm's objective function is then to solve

$$\max_{p_t(i)} E_{t-1} [(p_t(i) - \phi_t) y_t(i)] \tag{8}$$

subject to the demand function (2).[10] The optimal price is given by

$$p_t(i) = \frac{\epsilon}{\epsilon - 1} \frac{E_{t-1}(\phi_t Y_t)}{E_{t-1}Y_t}. \tag{9}$$

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[10]Modifying the firm's profit function to include the marginal utility of income, $Eu'(c) [(p(i) - \phi)y(i)]$, where $u'(c)$ is household's marginal utility, has no effect on the existence of sunspot equilibria in the model, although the particular distribution of the sunspot process may differ (see, e.g., Wang and Wen, 2007).

Assuming firms are rational and have the same information sets, then they all set the same prices in a symmetric equilibrium. Thus, $p(i) = p = 1$ and

$$E_{t-1}(\phi_t Y_t) = \frac{\epsilon - 1}{\epsilon} E_{t-1}(Y_t). \tag{10}$$

In the limiting case where $\epsilon \to \infty$, the model converges to a perfectly competitive economy. Notice that without the information lag, equation (10) implies the marginal cost is constant, $\phi_t = \frac{\epsilon-1}{\epsilon}$. In this case, the equilibrium is unique, as in standard Dixit-Stiglitz models.

Equation (10) permits multiple sunspot equilibria where the marginal cost (ϕ) and aggregate output (Y) follow stochastic processes and such equilibria are not mere randomizations over fundamental equilibria (note the fundamental equilibrium is always uniquely given by $\phi = \frac{\epsilon-1}{\epsilon}$). The intuition is as follows. Without the information lag, firms know the marginal cost; hence, they set prices as a markup over the marginal cost, $p_t(i) = \frac{\epsilon}{\epsilon-1}\phi_t$. Since $p(i) = 1$ in a symmetric equilibrium (by normalization), the equilibrium marginal cost equals $\frac{\epsilon-1}{\epsilon}$, implying a $\frac{1}{\epsilon}\%$ markup. Because the marginal cost determines the real interest rate and the household income, the other aggregate variables are hence all pined down uniquely in general equilibrium.

However, if the marginal cost is not known to firms at the time of choosing prices $p_t(i)$, monopolistic prices can depend only on expected marginal cost, which is a function of aggregate demand, $\phi(Y)$. Because of demand externalities across firms, such expectations can be self-fulfilling so that there exist multiple equilibrium paths of the marginal cost.

Intuitively, a higher output level leads to higher marginal costs, i.e., $\phi'_Y \geq 0$.[11] Equation (10) can be rewritten as

$$\left[E_{t-1}\phi_t - \frac{\epsilon - 1}{\epsilon}\right] E_{t-1}Y_t = -cov\left(\phi_t, Y_t\right), \tag{11}$$

where the covariance term $cov(\phi, Y) \geq 0$ because $\phi'_Y \geq 0$. Therefore, we have the following restriction:

$$E_{t-1}\phi_t \leq \frac{\epsilon - 1}{\epsilon}, \tag{12}$$

where the equality holds if and only if the marginal cost is constant (as in a standard New Keynesian model). Most importantly, the more variable the marginal cost, the lower is its expected value.

The intuition for understanding the inequality in equation (12) is as follows. With uncertainty in aggregate demand and the marginal cost, monopoly price is a constant markup over a weighted

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[11]This positive relationship between output level and marginal costs can be seen from equations (3) and (6). The sign is a straightforward consequence of assuming the depreciation rate to be convex in the utilization rate so that higher production is associated with a higher marginal cost.

average of marginal costs across different states, as equation (9) implies

$$p_t(i) = \frac{\epsilon}{\epsilon - 1} \sum_s \omega(s)\phi(s), \tag{13}$$

where $\omega(s) = \frac{q(s)Y(s)}{\sum q(s)Y(s)}$ and $q(s)$ denotes probability of state s. So $p_t(i)$ is a linear function of a weighted average of marginal costs across states, with marginal costs above average ($\phi(s) > E\phi$) receiving a weight $q(s)\frac{Y(s)}{EY}$—i.e., higher than the probability $q(s)$. In fact if $\phi(s) > E\phi$, then $Y(s) > EY$ (given $\phi'_Y > 0$), i.e., $\frac{q(s)Y(s)}{\sum q(s)Y(s)} > q(s)$.

This nonlinear and unequal distribution of weights leads to a higher monopoly price or markup. When the price is normalized to unity, a higher monopoly price translates into a lower average marginal cost ϕ and thus a lower level of output (aggregate demand). This explains the inequality in equation (12), which is the key to delivering the negative relationship between volatility and mean growth in the model.

2.2 Households

We close the model by adding a standard household sector. There is a continuum of infinitely lived identical households of measure one. The representative household chooses paths of consumption ($\{C_t\}_{t=0}^\infty$) and capital holdings ($\{K_{t+1}\}_{t=0}^\infty$) to solve

$$\max E_0 \sum_{t=0}^\infty \beta^t \log(C_t) \tag{14}$$

subject to $K_0 > 0$ and the budget constraint,

$$C_t + K_{t+1} = (1 + r_t)K_t + D_t, \tag{15}$$

where D_t denotes real profits distributed from intermediate good firms. The first-order condition is given by $\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}}(1 + r_{t+1})$, plus the transversality condition, $\lim_{T\to\infty} \beta^T \frac{K_{T+1}}{C_T} = 0$.

2.3 Symmetric Rational Expectations Equilibria

In this paper we restrict attention to symmetric equilibria where $y(i) = Y$ and $k(i) = K$ for all $i \in [0, 1]$. The equilibrium conditions in this economy can be summarized by the following equations:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \left(1 + \frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}}\left(\phi_{t+1}A\right)^{\frac{\theta+1}{\theta}}\right), \tag{16}$$

$$C_t + K_{t+1} = Y_t + (1 - \delta_t)K_t = \left[1 + \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \phi_t^{\frac{1}{\theta}} \left(1 - \frac{\phi_t}{1+\theta} \right) \right] K_t, \tag{17}$$

$$E_{t-1}\phi_t^{\frac{1+\theta}{\theta}} = \frac{\epsilon - 1}{\epsilon} E_{t-1}\phi_t^{\frac{1}{\theta}}; \tag{18}$$

where the last equation is derived from equations (3), (6), and (10).[12] These three equations, in conjunction with a transversality condition, fully determine the equilibrium paths of the marginal cost, consumption, and the capital stock. In particular, given any path of the marginal cost (ϕ) as specified by equation (18), equations (16) and (17) fully determine the paths of consumption and the capital stock.

Notice that equation (18) implies $E\phi^{\frac{1}{\theta}}(E\phi - \frac{\epsilon-1}{\epsilon}) = -cov(\phi^{\frac{1}{\theta}}, \phi) \le 0$; hence, any stochastic process $\{\phi_t\}_{t=0}^{\infty}$ satisfying $E\phi \le \frac{\epsilon-1}{\epsilon}$ and $cov(\phi^{\frac{1}{\theta}}, \phi) = E\phi^{\frac{1}{\theta}}(\frac{\epsilon-1}{\epsilon} - E\phi)$ constitutes a rational expectations equilibrium path for the marginal cost.[13] The fundamental equilibrium (in the absence of extrinsic uncertainty or sunspots) corresponds to the case where $cov(\phi^{\frac{1}{\theta}}, \phi) = 0$ and $\phi = \frac{\epsilon-1}{\epsilon}$, and it is clearly unique.[14] But there also exist multiple sunspot equilibria. As an example for constructing such sunspot equilibria, consider the process

$$\phi_t = \frac{\epsilon - 1}{\epsilon}\varepsilon_t, \tag{19}$$

where ε denotes sunspot shocks. Equation (18) implies

$$E_{t-1}\varepsilon_t^{\frac{1+\theta}{\theta}} = E_{t-1}\varepsilon_t^{\frac{1}{\theta}}. \tag{20}$$

Clearly, any random variable satisfying the distribution

$$E_{t-1}\varepsilon_t \in [0,1], \quad cov(\varepsilon_t^{\frac{1}{\theta}}, \varepsilon_t) = E_{t-1}\varepsilon_t^{\frac{1}{\theta}} \left(1 - E_{t-1}\varepsilon_t \right), \tag{21}$$

constitutes an equilibrium. This paper restricts attention to *i.i.d.* sunspot shocks with mean $E\varepsilon = \bar{\varepsilon} \in [0,1]$.

Defining a balanced growth path in the model as an equilibrium path along which consumption, the capital stock, and output all grow at the same expected rate, we have the following propositions:

Proposition 1 *For any and each i.i.d. sunspot shock process, there always exists a balanced growth path along which the stochastic growth rates of consumption and capital are both given by* $\ln[s(1+\chi_t)]$,

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[12]Note that K_t is a state variable known to firms in the end of period $t-1$.

[13]To avoid complex values, the condition $E\phi \ge 0$ must be imposed.

[14]Notice that the uniqueness occurs regardless of fundamental shocks. For example, suppose the technology A is a stochastic process; then, in the fundamental equilibrium, we still have $\phi = \frac{\epsilon-1}{\epsilon}$.

and the growth rate of output is given by $\ln \left[\left(\frac{\phi_t}{\phi_{t-1}} \right)^{1/\theta} s(1 + \chi_t) \right]$, where $\chi_t \equiv \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \phi_t^{\frac{1}{\theta}} \left(1 - \frac{\phi_t}{1+\theta} \right)$

and $s \equiv \beta E_t \frac{1+r_{t+1}}{1+\chi_{t+1}}$.

Proof. Since ϕ_t is *i.i.d.*, any function of ϕ_t is also *i.i.d.* An educated guess of the equilibrium paths of consumption and the capital stock is given by

$$C_t = (1 - s)(1 + \chi_t)K_t, \tag{22}$$

$$K_{t+1} = s(1 + \chi_t)K_t, \tag{23}$$

where $s = \beta E_t \frac{1+r_{t+1}}{1+\chi_{t+1}}$ denotes the optimal rate of savings, which is a constant under the *i.i.d.* assumption and is derived from the intertemporal Euler equation

$$\frac{1}{(1 - s)(1 + \chi_t)K_t} = \beta E_t \frac{1 + r_{t+1}}{(1 + \chi_{t+1})(1 - s)s(1 + \chi_t)K_t}. \tag{24}$$

Using equations (22) and (23), it can be shown that $\frac{C_{t+1}}{C_t} = s(1 + \chi_{t+1})$. Hence, the balanced growth rates of consumption and capital are both given by $g_t = \ln[s(1 + \chi_t)]$. The growth rate of output is given by $g_{yt} = \ln \frac{u_t K_t}{u_{t-1} K_{t-1}} = \frac{1}{\theta} \left(\ln \phi_t - \ln \phi_{t-1} \right) + \ln[s(1 + \chi_{t-1})]$, which has the same (unconditional) expected value as g. ∎

Proposition 2 *In the absence of extrinsic uncertainty, the model has a unique balanced growth path with its growth rate determined by*

$$g^* = \ln s(1 + \chi) = \ln \left[\beta \left(1 + \frac{\theta}{1 + \theta} \alpha^{-\frac{1}{\theta}} \left(\frac{\epsilon - 1}{\epsilon} A \right)^{\frac{\theta+1}{\theta}} \right) \right]. \tag{25}$$

Proof. In the absence of extrinsic uncertainty, Equation (10) implies the marginal cost is constant, $\phi = \frac{\epsilon-1}{\epsilon}$. Hence, r and χ are all constant. Consequently, the fundamental (no-sunspot) growth rate in the economy is uniquely determined by $\ln \beta(1 + r)$. ∎

Proposition 3 *If the following condition,*

$$\epsilon > \frac{1 + \theta}{\theta} + 2\alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}, \tag{26}$$

is satisfied, the mean growth rate of a stochastic growth path is strictly less than the deterministic growth rate without uncertainty $(\phi_t = \frac{\epsilon-1}{\epsilon})$, i.e., $E[s(1 + \chi_t)] < \beta(1 + r)$.

Proof. See the **Appendix**. ∎

As an example, consider the limiting case where $\epsilon = \infty$. In this case, the deterministic (gross) growth rate is given by $g^* = \beta(1+r) = \beta \left(1 + \frac{\theta}{1+\theta} \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \right)$, and the price equation (18) becomes

$$E\phi_t^{\frac{1}{\theta}+1} = E\phi_t^{\frac{1}{\theta}}. \tag{27}$$

Since we restrict our attention to the interval, $0 \leq \phi \leq 1$, the only distribution that can satisfy the above relationship for the marginal cost is the binary distribution, $\phi_t = \{0, 1\}$ with probability $\{1-p, p\}$. Under this distribution, we have $r_t = \chi_t$; hence, $s = \beta E \frac{1+r_{t+1}}{1+\chi_{t+1}} = \beta$ and $E\chi_t = p\frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}$. The mean (gross) growth rate is hence given by

$$\bar{g} = \beta \left(1 + p\frac{\theta}{1+\theta} \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \right), \tag{28}$$

which is strictly less than the deterministic (gross) growth rate g^* for any $p \in (0, 1)$. In this limiting case, the condition $\epsilon > \frac{1+\theta}{\theta} + 2A^{\frac{1}{\theta}+1}$ is trivially satisfied.

Notice that if the source of uncertainty comes from shocks to the aggregate technology (A), the relationship between volatility and growth is then strictly positive. However, if a sufficiently large adjustment cost of investment is assumed, then this relationship can become negative (see Barlevy, 2004a).

3   Model Simulation

Let the time period be a year and the time discounting rate $\beta = 0.98$.[15] For the U.S. economy, the markup is approximately $10\% \sim 20\%$, which implies that $\phi = 0.9 \sim 0.8$ or $\epsilon = 10 \sim 6$. Let $\theta = 0.6$ and the annual rate of depreciation $\delta = 10\%$. Hence, Equation (7) implies $r = \theta\delta = 6\%$ in the deterministic economy without sunspots.[16] Since $\alpha u^\theta = \phi A$ and $\delta = \frac{1}{1+\theta} \alpha^{-\frac{1}{\theta}} (\phi A)^{\frac{\theta+1}{\theta}}$, these two equations can help pin down the values of $\{\alpha, A\}$ once the value of the utilization rate (u) is given. Let $u = \phi = 0.9$ in the deterministic economy (which implies $\epsilon = 10$); then the above two relationships imply $\alpha = 0.18938$ and $A = 0.19753$.

Given these values, the condition required in Proposition 3, $\epsilon > \frac{1+\theta}{\theta} + 2\alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}$ (≈ 3.1), is more than satisfied. It can be shown easily that this condition is still satisfied under other plausible parameter configurations, such as when the annual real interest rate in the deterministic equilibrium

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[15]This implies that $\beta = 0.995$ in a quarterly model.

[16]The average interest rate in the model can be significantly lower under the influence of sunspots than it is in the deterministic equilibrium.

is as low as 1.5%. Figure 2 shows the regions of parameter values where the condition in Proposition 3 is satisfied. Specifically, each downward sloping curve represents a different combination of structural parameters (δ, ϕ) that defines the left-hand side of equation (26), the horizontal lines correspond to different values of ϵ; so any point lying above the downward sloping curve(s) satisfies the restriction in equation (26). In particular, given the elasticity parameter ϵ, the higher the interest rate, the easier the condition can be satisfied. For example, when $\delta = 0.1$ and $\phi = 0.9$ (implying $\epsilon = 10$), the condition is satisfied for $r > 1.2\%$; when $\delta = 0.1$ and $\phi = 0.8$ (implying $\epsilon \approx 6$), the condition is satisfied for $r > 2.4\%$.
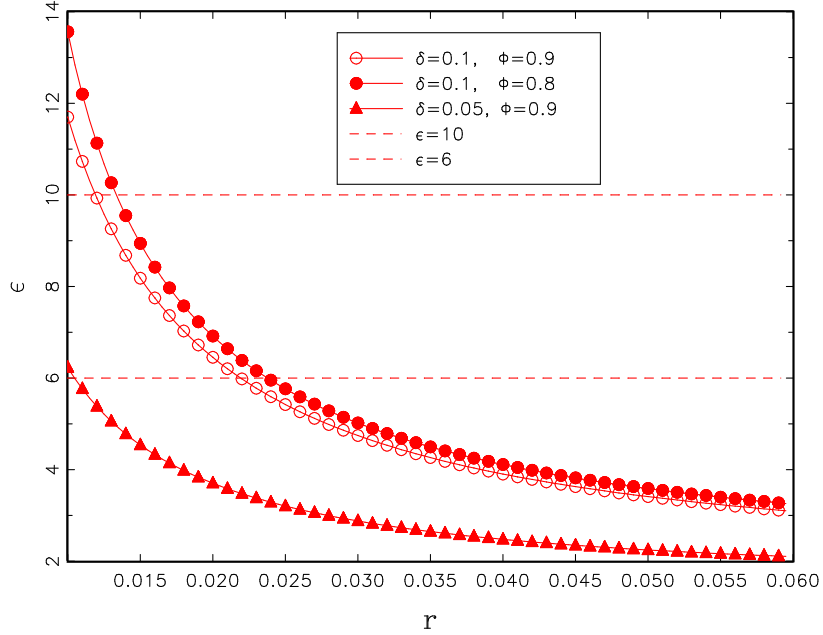

Figure 2. Parameter Region for $\epsilon > \frac{1+\theta}{\theta} + 2\alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}$.

Based on the calibrated parameter values, the deterministic growth rate is given by $\ln s(1+\chi) = \ln \beta(1+r) \simeq 0.0381$; in other words, the fundamental growth rate is about 4% a year. To compute the mean growth rate of a stochastic growth path, we generate a time series for $\phi_t = \frac{\epsilon-1}{\epsilon}\varepsilon_t$, where the sunspot shock (ε) has the log-normal distribution $\ln \varepsilon \sim N(\mu, \sigma^2)$ with

$$e^{\frac{\sigma^2}{\theta}} E\varepsilon_t = 1. \tag{29}$$

Notice that this distribution satisfies equation (20) and the condition $0 < E\varepsilon_t < 1$.

Based on these calibrated parameter values, Table 1 shows the statistical relationship between growth volatility (σ_g) and the mean growth rate ($\bar{g}$) as the standard deviation of sunspot shocks (σ) increases. The statistics reported in the table are estimates based on simulated time series

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with a sample size of 10^7. The first row is the standard deviation of sunspots (σ), the second row the implied average marginal cost ($\bar{\phi}$), the third row the implied average growth rate ($\bar{g}$), and the last row the implied standard deviation of the growth rate (σ_g). The table shows that, as the standard deviation of the sunspot shock (σ) increases, the average markup ($1 - \bar{\phi}$) and the standard deviation of the stochastic growth rate (σ_g) also increase, while the mean growth rate of the economy ($\bar{g}$) decreases. The same pattern of results can also be confirmed under a uniform distribution of sunspot shocks (see Table 2).[17] This prediction of a negative relationship between volatility and growth is consistent with the empirical regularity documented by Ramey and Ramey (1995) in cross-country data.

Table 1. Predicted Volatility and Growth (Log-Normal Distribution)

σ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\bar{\phi}$	0.90	0.89	0.84	0.77	0.69	0.59	0.49	0.40	0.31	0.23	0.17
$\bar{g}(\%)$	3.81	3.68	3.32	2.83	2.26	1.5	0.6	-0.1	-1.0	-1.2	-1.9
σ_g	0	0.003	0.01	0.02	0.03	0.05	0.07	0.09	0.15	0.20	0.30

Table 2. Predicted Volatility and Growth (Uniform Distribution)

σ	0	0.029	0.058	0.087	0.12	0.14	0.17	0.23	0.29	0.35	0.40
$\bar{\phi}$	0.9	0.899	0.895	0.888	0.880	0.868	0.853	0.815	0.763	0.693	0.619
$\bar{g}(\%)$	3.81	3.79	3.77	3.70	3.63	3.54	3.41	3.10	2.69	2.17	1.70
σ_g	0	0.001	0.002	0.003	0.004	0.005	0.007	0.011	0.016	0.022	0.026

Figure 3 shows simulations of the stochastic growth paths of consumption and the implied log consumption levels for each of the distributions considered above. In particular, the simulation under the log-normal distribution is presented in the first row windows (A and B), and the simulation based on uniform distribution is presented in the second row windows (C and D). The growth rate series are graphed in the left column windows (A and C) and the log consumption level series are graphed in the right column windows (B and D). In windows A and C, which show the growth series, the solid horizontal line represents the deterministic growth rate in the model in the absence of sunspot shocks, the solid random time series represents the growth rate of the model under the influence of sunspots, and the dashed time series represents the annual consumption growth of the U.S. economy for the period 1947-2005. Clearly, the model is able to generate growth volatility

[17]Even with the large sample size, the standard deviation of the growth rate (σ_g) is quite large for the log-normal distribution when σ is close to 1, suggesting that the estimated mean growth rate can have large standard errors if the volatility is very high. Despite this, the tendency for the mean growth rate to decline as the growth volatility increases is clear. When a uniform distribution is assumed instead for sunspot shocks, the standard error of the growth rate (σ_g) is much smaller and the mean growth rate is more tightly estimated even with large values of σ, which makes the negative relationship between volatility and growth even clearer (see Table 2). Note that, under the uniform distribution, the growth rate of the model is always positive when the parameters of the distribution (mean and variance) of sunspot shocks satisfy equation (20).

similar to the U.S. data. Since the mean growth rate in the model under a particular sunspot process is lower than that of the U.S. data, the implied consumption level (Window B or D) is stochastically dominated by the U.S. consumption level.

Notice that, because of the negative relationship between volatility and mean growth, the model-generated time series of growth rate tends to have a lower mean than the U.S. data. However, a mean growth rate similar to or above the actual U.S. data can also be generated from the model by choosing the structural parameters (such as θ) to yield a higher deterministic growth rate in the model.
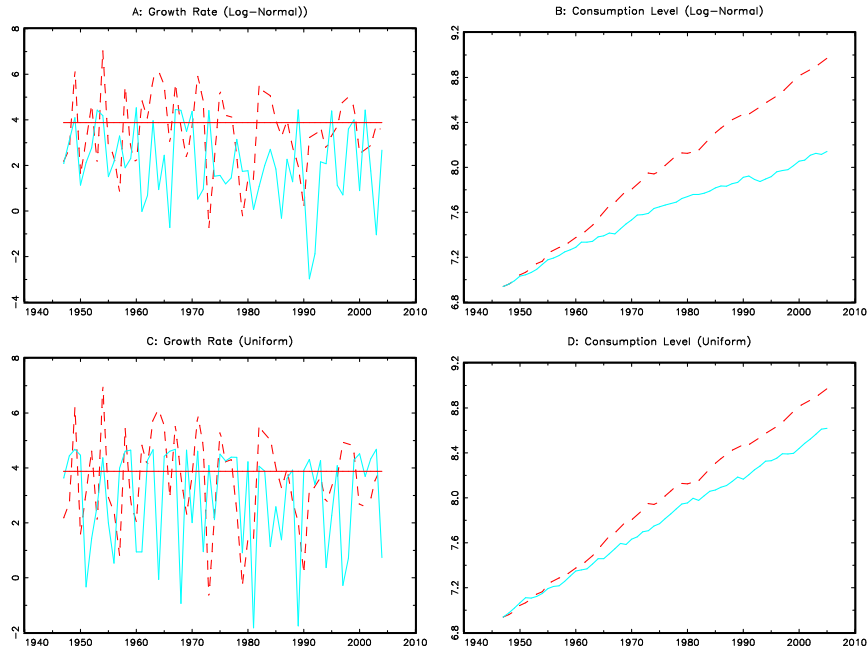

Figure 3. Volatility and Growth (—— Model; - - - Data).

As suggested by Windows B and D, along a lower consumption growth path due to a higher volatility, the loss in consumption is irreversible (unrecoverable) even if the mean growth rate recovers to the previous level because of a decrease in volatility. That is, a large and ever-increasing gap in consumption levels can be caused by business cycles (i.e., a higher volatility) alone. This property is striking and in sharp contrast to the Okun's gap and the random walk characterization of aggregate output. The lesson: When growth is endogenous, fluctuations in output can affect not only the consumption level permanently, but also its long-run growth rate.

4 Welfare Cost of Fluctuations

4.1 The Lucas Calculation

The Lucas calculation of the cost of business cycles is based on a simple yet fundamental assumption: Volatility and growth are unrelated. Given this dichotomy and the fact that the aggregate consumption series is smooth, Lucas (1987 and 2003) concludes that the welfare cost of fluctuations is trivial in terms of consumption.

Suppose a representative consumer is endowed with the stochastic consumption stream

$$c_t = Ae^{ut}e^{-(1/2)\sigma^2}\varepsilon_t, \tag{30}$$

where u is a deterministic growth rate and $\ln(\varepsilon_t)$ is a normally distributed random variable with zero mean and variance σ^2. Hence, $e^{-(1/2)\sigma^2}E\varepsilon_t = 1$, which is the normalization on the mean of ε_t taken by Lucas (1987). The preference over consumption is assumed to be $E\sum_{t=0}^{\infty}\left(\beta^t\frac{c_t^{1-\gamma}}{1-\gamma}\right)$. The welfare gain can be computed as a λ percentage increase in consumption one would get by eliminating all the volatility (without affecting the mean growth rate u), namely:

$$E\sum_{t=0}^{\infty}\left[\beta^t\frac{((1+\lambda)c_t)^{1-\gamma}}{1-\gamma}\right] = \sum_{t=0}^{\infty}\left[\beta^t\frac{\left(Ae^{ut}\right)^{1-\gamma}}{1-\gamma}\right], \tag{31}$$

which implies a welfare cost of $\lambda \approx \frac{\gamma}{2}\sigma^2$. The annual U.S. real consumption growth in the period of 1947-2005 is about 3.5% with a standard deviation of 0.0165. Assuming log utility ($\gamma = 1$), the welfare cost is estimated to be $\lambda \approx \frac{1}{2}(0.0165)^2 \approx 0.014\%$. This is less than 1.5¢ for every \$100 of annual consumption.[18]

4.2 Calculation based on Hall's (1978) Random Walk

A crucial feature of the Lucas calculation is that random shocks to consumption have no permanent effect on the consumption level. According to the permanent income theory, however, consumption follows a random walk; hence, transitory shocks can have permanent effects (Hall, 1978). Adopting the random walk framework, the consumption path can be described by

$$c_t = c_{t-1}\left(e^{u-\frac{\sigma^2}{2}}\varepsilon_t\right), \tag{32}$$

where u is a drift term in the random walk specification of log consumption, which determines the average growth rate of consumption. This characterization of consumption is also an implication of

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[18]Of course, a higher γ can increase the estimation. Micro evidence suggests that $\gamma \in [1,4]$. But even with $\gamma = 100$, the annual cost of the business cycle is still less than 1.5 percent of consumption.

the RBC theory where technology shocks following random walks. The welfare cost of fluctuations can then be computed as the solution (λ) to equation (31) based on the random-walk consumption in (32). Again assuming log utility ($\gamma = 1$) and $\ln \varepsilon_t \sim N(0, \sigma^2)$, we have

$$\lambda \approx \frac{\sigma^2}{2} \frac{\beta}{(1 - \beta)}. \tag{33}$$

Notice that the welfare measure under the random walk assumption is a multiplier ($\frac{\beta}{1-\beta}$) times the welfare measure of Lucas. This is the result obtained by Obstfeld (1994a).[19] This multiplier exists because a one dollar increase in consumption today is translated into a $\sum_{t=1}^{\infty} \beta^t = \frac{\beta}{1-\beta}$ dollar increase in life-time consumption. Letting $\beta = 0.98$ and $\sigma = 0.0165$, we get $\lambda \approx 0.67\%$, which is about 48 times larger than the welfare gain under the Lucas specification of the consumption path. However, it is still small in absolute magnitude.

4.3 Calculation based on Ramey and Ramey (1995)

According to the empirical studies of Ramey and Ramey, volatility and growth are negatively related. Hence eliminating volatility should increase the growth rate, which implies a large welfare cost of business cycles, consistent with Lucas's (1987) analysis on the welfare effect of long-run growth. But Lucas did not relate business cycle to growth; hence, he failed to appreciate the welfare cost of fluctuations. To illustrate this, consider a counterfactual experiment where completely removing uncertainty can increase the growth rate by π percent from u to $u(1 + \pi)$. Then equation (31) becomes

$$E \sum_{t=0}^{\infty} \left[\beta^t \frac{((1 + \lambda)c_t)^{1-\gamma}}{1 - \gamma} \right] = \sum_{t=0}^{\infty} \left[\beta^t \frac{\left(A e^{u(1+\pi)t} \right)^{1-\gamma}}{1 - \gamma} \right]. \tag{34}$$

Under the random-walk consumption path (32) and log utility specification ($\gamma = 1$), equation (28) implies

$$\lambda \approx \left(\frac{\sigma^2}{2} + \pi u \right) \frac{\beta}{1 - \beta}. \tag{35}$$

According to Ramey and Ramey (1995, p.1141), one standard deviation of the volatility in the growth rate of output translates into about one-third of a percentage point of the mean growth rate. Applying this estimate to consumption, it means that by decreasing the consumption volatility from $\sigma = 0.0165$ to zero, the gain in growth rate is about $\frac{0.0165}{3} = 0.55\%$, which is about 16% of the current mean consumption growth rate for the U.S. economy ($u = 3.5\%$). This implies that

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[19] Also see Reis (2009) for a more general ARMA specification of the consumption process.

$\pi = 16\%$ and $\pi u = 0.55\%$. Assuming $\beta = 0.98$, we have $\lambda \approx 27\%$. This is an enormous welfare gain: more than one quarter of total annual consumption.[20]

4.4 Calculation Based on Our Model

To facilitate the analysis, we assume that there are no sunspots in period 0 and sunspots appear only form period 1 and beyond. Consumption in our endogenous growth model follows the path $c_t = c_{t-1}[s(1 + \chi_t)]$. Notice that since sunspot shocks are *i.i.d.*, we have $E_0 g(\chi_1) = E_0 g(\chi_2) = \cdots = E_0 g(\chi_t)$ for all $t > 0$. Hence, the expected life-time utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t(1 + \lambda)) = \frac{\ln(1 + \lambda)}{1 - \beta} + \frac{\ln c_0}{1 - \beta} + \frac{\beta \left(\ln s + E \ln(1 + \chi_t)\right)}{(1 - \beta)^2}, \tag{36}$$

where

$$c_0 = (1 - s)(1 + \chi_0)k_0 \tag{37}$$

$$s = \beta E_t \frac{1 + r_{t+1}}{1 + \chi_{t+1}}. \tag{38}$$

In the absence of uncertainty, the model implies $\phi = \frac{\epsilon - 1}{\epsilon}$, so the fundamental growth rate of consumption is given by $\ln \beta(1 + r) = 3.81\%$. The life-time utility of the deterministic consumption path is given by

$$\sum_{t=0}^{\infty} \beta^t \ln \left(c_0^* (1.0381)^t\right) = \frac{\ln c_0^*}{1 - \beta} + \frac{\beta \ln(1.0381)}{(1 - \beta)^2}, \tag{39}$$

where

$$c_0^* = (1 - s^*)(1 + \chi_0)k_0 \tag{40}$$

$$s^* = \beta \frac{1 + \bar{r}}{1 + \bar{\chi}}. \tag{41}$$

Notice that, in general $c_0 \neq c_0^*$ and $s \neq s^*$ because the average saving rate s is affected by sunspots.

Comparing the two expressions in (36) and (39) gives the welfare cost:

$$\lambda \approx \frac{\beta}{1 - \beta} [0.0381 - E \ln s(1 + \chi_t)] + \ln \frac{1 - s^*}{1 - s}. \tag{42}$$

Notice that the welfare cost (or gain) is the multiplier ($\frac{\beta}{1-\beta}$) times the difference between the maximum sustainable growth rate under full information (i.e., without sunspots) and the mean of the stochastic growth rate under the influence of sunspots, adjusted by the difference in saving

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[20]Interestingly, this estimate is close to the estimate obtained by Alvarez and Jermann (2004) using a nonparametric asset-pricing approach.

rates due to uncertainty ($\ln \frac{1-s^*}{1-s} \approx s - s^*$). As Proposition 3 shows, the mean growth rate of a stochastic growth path is strictly less than the fundamental growth rate; hence, the first term on the RHS of equation (42) is always positive. Furthermore, as Table 1 and Table 2 both show, when the volatility of sunspot shocks increases in the model, the mean of the stochastic growth rate, $E \ln s(1 + \chi)$, decreases, which increases the value of λ. On the other hand, since uncertainty raises s because of precautionary saving motives, the second term on the RHS of equation (42) is also positive, further increasing the welfare costs of the business cycle by decreasing the initial consumption level c_0 under uncertainty.

For example, under the assumption of a log-normal distribution (Table 1), a standard deviation of 0.2 for sunspot shocks (ε_t) implies a stochastic consumption growth path with mean 3.32 and a standard deviation $\sigma_g = 0.01$ (along with an average markup of $100 \times (1 - \bar{\phi}) = 16\%$), not too far from the U.S. data. The expected saving rate in the deterministic case is $s^* = 0.96384$ and that under uncertainty is $s = 0.96511$, higher than without uncertainty. Substituting $E \ln s(1 + \chi) = 3.32\%$ and $\ln \frac{1-s^*}{1-s} = 3.58\%$ into equation (42) implies $\lambda \approx 27\%$. Under the assumption of a uniform distribution (Table 2), a standard deviation of 0.17 for sunspot shocks implies a stochastic growth path with mean 3.41 and a standard deviation $\sigma_g = 0.007$ (along with an average markup of 15%). The saving rate in the deterministic case is $s^* = 0.96384$ and that under uncertainty is $s = 0.96472$. Substituting this information into equation (42) implies $\lambda \approx 22\%$. The welfare cost would be much higher (e.g., $\lambda = 40\%$) if we increase σ_g from 0.007 to 0.011.

Thus, based on our endogenous growth model, the welfare cost of business cycles with volatility and mean consumption growth similar to the U.S. data is around $22\% \sim 27\%$ of annual consumption (or even higher). Such estimates are quite close in the order of magnitude to those based on Ramey and Ramey's empirical studies as well as to those obtained by Alvarez and Jermann (2004) based on a nonparametric asset-pricing approach.

Notice that uncertainty implicitly enters equation (42) to affect the welfare measure through two channels. First, the expected value, $E \ln (1 + \chi_t)$, is negatively related to the variance of χ_t because $\ln (1 + \chi_t)$ is strictly concave in χ_t. For example, the Lucas (1987) model features $E \ln \varepsilon_t = -\frac{1}{2}\sigma^2$ under the assumption of log-normal distribution for ε_t. This is the direct channel emphasized by Obstfeld (1994b) Epaulard and Pommeret (2003). Second, in our model the mean growth $(1 + \bar{\chi})$ and average saving rate s are endogenously tied to uncertainty. In particular, higher uncertainty leads to lower average growth through the marginal cost (see equation (12)). Hence, in our simulation analyses we cannot perform standard "mean-preserving spread" experiments for sunspot shocks as in the Lucas (1987) model where the mean growth (μ) and uncertainty (σ^2) are independent of each other.

4.5 Robustness Analyses

The above results may be sensitive to our assumptions of the parameter values for a number of reasons. First, the time discounting factor β significantly affects the multiplier in the welfare measure. Second, in the previous calibration we fix the value of θ and then pick the corresponding mean growth rate and volatility in Table 1 (or Table 2) to compute welfare costs. An alternative calibration strategy is to fix the mean growth rate and the volatility of growth using U.S. data, and then deduce the implied value of θ and other parameters in the model.

Consider the effects of β first. Suppose the value of β reduces from 0.98 to 0.96 in our previous model and continue to assume lognormal distribution of sunspots with standard deviation $\sigma = 0.2$. The implied welfare cost becomes $\lambda = 26.3\%$, very similar to the original estimate. The reason is as follows. With a lower value of β, although the multiplier $\frac{\beta}{1-\beta}$ has reduced by half from 49 to 24, the implied gap between the maximum growth and the mean growth, $\ln \beta (1+r) - E \ln s(1+\chi)$, happens to be doubled, which almost exactly cancels the reduction in the multiplier. However, this near complete cancellation is not a general feature of the model.

Now consider an alternative calibration of θ. Let the growth rate volatility $\sigma_g = 0.0165$ and the mean growth rate $E \ln s(1+\chi) = 3.57\%$, as in the U.S. economy. Suppose $\beta = 0.98$ first. These parameter values imply $\theta = 0.715$, the volatility of sunspots $\sigma = 0.32$, and the maximum growth rate $\ln \beta(1+r) = 4.54\%$. Under this alternative calibration, the implied average markup is 22% and the welfare cost $\lambda = 55\%$ according to equation (42). This increase in the welfare costs originates mostly from the fact that the gap between the maximum growth and the mean growth is nearly doubled. However, under this alternative calibration, changing the value of β will significantly change the implied welfare costs. For example, if we reduce the value of β from 0.98 to 0.96 while maintaining $\theta = 0.715$, the implied welfare cost becomes 39% while the average markup remains essentially unchanged.

In addition, the negative correlation between volatility and mean growth may depend on the coefficient of risk aversion in the utility function. Intuitively, with a lower intertemporal elasticity of substitution in consumption, volatility may induce more saving than with log utility. The (higher) positive effect of volatility on growth through this saving channel could partially offset the negative effect through higher markups and reduced production. For example, suppose we use a more general CRRA utility function (as in equation (31)) with $\gamma = 5$. To ensure that the implied deterministic growth rate remains the same as in the case of $\gamma = 1$, we must recalibrate the value of the productivity coefficient from $A = 0.19753$ to $A = 0.3706$. The results are reported in Table 3 for log-normal distribution.

Table 3. Predicted Volatility and Growth (Log-Normal Distribution)

	$\gamma = 1$ $(A = 0.19753)$					$\gamma = 5$ $(A = 0.3706)$				
σ	0	0.1	0.2	0.3	0.4	0	0.1	0.2	0.3	0.4
$\bar{g}(\%)$	3.81	3.68	3.32	2.83	2.26	3.81	3.72	3.60	3.50	3.32
σ_g	0	0.003	0.01	0.02	0.03	0	0.01	0.026	0.043	0.057

When $\gamma = 5$, the negative correlation between mean growth ($\bar{g}$) and growth volatility (σ_g) still exists but is weaker than the case with $\gamma = 1$. Namely, when $\gamma = 5$, as the volatility of output growth increases (with σ), the mean growth rate declines at a significantly slower pace than in the case of $\gamma = 1$. This suggests that the implied welfare cost of fluctuations will also be significantly smaller with $\gamma = 5$ than with log utility, everything else equal except the value of A. As argued by Obstfeld (1994b), the negative impact of uncertainty on welfare can work through two channels: (i) A direct channel that uncertainty increases the volatility of consumption, and (ii) an indirect channel that uncertainty reduces the mean growth rate. Since the welfare effect under the direct channel is small (as effectively demonstrated by Lucas, 1987), the indirect channel dominates the direct channel in our endogenous growth model. Since a higher value of γ weakens the second (indirect) channel in our model through a precautionary saving effect, the large welfare cost due to sunspots is consequently reduced.

However, this result is subject to caveats because it depends on how to re-calibrate the model when γ changes. Holding all other parameters constant, a change in γ would change the deterministic growth rate g^*. To make the results comparable, we can either keep the same deterministic growth rate g^* by changing the productivity parameter A, or keep the same value of $A = 0.19753$ by allowing the deterministic growth rate to change. These two calibration methods give very different results. In Table 3, a higher value of γ also implies a higher productivity A for given growth volatility σ_g. This factor has played a role in weakening the negative relationship between volatility and growth.

As a second example, suppose we keep $A = 0.19753$ while increasing γ to 5, then the implied deterministic growth rate would decrease from $g^* = 3.81\%$ to a substantially lower value of $g^* = 0.66\%$. The results are reported in Table 4, which shows that the mean growth rate is substantially lower for each possible value of the sunspot volatility.

Table 4. Predicted Volatility and Growth (Log-Normal Distribution)

	$\gamma = 1$ $(A = 0.19753)$					$\gamma = 5$ $(A = 0.19753)$				
σ	0	0.1	0.2	0.3	0.4	0	0.1	0.2	0.3	0.4
$\bar{g}(\%)$	3.81	3.68	3.32	2.83	2.26	0.657	0.638	0.589	0.512	0.405
σ_g	0	0.003	0.01	0.02	0.03	0	0.002	0.005	0.009	0.012

The intuition behind is that a higher value of γ implies not only a higher degree of risk aversion but also a lower intertemporal elasticity of substitution between current and future consumption.

The first effect tends to increase the saving rate because of a stronger precautionary saving motive, while the second effect tends to decrease the saving rate because of a higher cost of intertemporal substitution. In the second case, the negative relationship between volatility and growth may become even stronger instead of weaker. For example, Table 4 shows that when the volatility of output growth increases from $\sigma_g = 0$ to $\sigma_g = 0.01$, the mean growth rate is reduced by 13% (i.e., $1 - \frac{3.32}{3.81} = 0.13$) when $\gamma = 1$ while it is reduced by more than 22% (i.e., $1 - \frac{0.512}{0.657} = 0.22$) when $\gamma = 5$. The implied welfare costs based on Table 3 would be much smaller when $\gamma = 5$ but would be similar in magnitude based on Table 4.

5   Conclusion

This paper shows that market imperfections (i.e., imperfect competition and self-fulfilling speculations under imperfect information) may lead to endogenous fluctuations in firms' markups and aggregate demand, which can directly translate into fluctuations in output growth and adversely affect its mean growth rate. Consequently, the welfare cost of the business cycle and the associated gain from stabilization can be extremely large. In particular, our analysis suggests that the welfare gain from further stabilizing the U.S. economy can be several orders larger than that calculated by Lucas (1987)—more than a quarter of annual consumption. This estimate is very close in order of magnitude to that implied by the empirical findings of Ramey and Ramey (1995).

However, our main estimates are based on log utility function with a relatively low degree of risk aversion. Adopting a more general recursive utility function with separate parameter values for the degree of risk aversion and the intertemporal elasticity of substitution (as in Epaulard and Pommeret, 2003) may change the estimated welfare costs of business cycles and the negative relationship between growth and volatility. Intuitively, with endogenous growth, a higher degree of risk aversion can induce higher mean growth through stronger precautionary savings, which partially offsets the negative effects of sunspots on the mean growth (due to firms pricing behaviors). On the other hand, a lower intertemporal elasticity of substitution can induce lower mean growth through weaker saving motives, which can strengthen the negative effects of sunspots on the mean growth. More detailed analyses along these lines are left to future works.

In addition, since our results are based on a highly stylized model, the implied welfare cost of fluctuations could also be overstated when labor market dynamics and labor's cost share are not included in the analysis. Given the importance of understanding the link between growth and the business cycle, the development of a more comprehensive framework is needed in future work.

Appendix: Proof of Proposition 3.

Proof. The key in the proof is to show that $E\phi_t^{\frac{1}{\theta}} \le \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}}$ and that the average growth rate is a strictly increasing function of $E\phi_t^{\frac{1}{\theta}}$ under certain conditions. In such a case, the maximum growth rate is achieved when $\phi_t = \frac{\epsilon-1}{\epsilon}$.

The growth rate of the model is given by

$$g_t = s(1 + \chi_t), \tag{43}$$

where $\chi_t = \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \phi_t^{\frac{1}{\theta}}(1 - \frac{\phi_t}{1+\theta})$, $s = \beta E \frac{1+r_t}{1+\chi_t}$, and $r_t = \frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}}(\phi_t A)^{\frac{1+\theta}{\theta}}$. The monopolistic price follows the rule, $E\phi_t^{\frac{1+\theta}{\theta}} = \frac{\epsilon-1}{\epsilon}E\phi_t^{\frac{1}{\theta}}$. Since $Ex^{1+\theta} \ge (Ex)^{1+\theta}$, we have $\frac{\epsilon-1}{\epsilon}E\phi_t^{\frac{1}{\theta}} = E\phi_t^{\frac{1+\theta}{\theta}} \ge \left(E\phi_t^{\frac{1}{\theta}}\right)^{1+\theta}$. It follows that

$$E\phi_t^{\frac{1}{\theta}} \le \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}}. \tag{44}$$

Given the definition of χ_t, we have

$$
\begin{aligned}
E\chi_t &= \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \left(E\phi_t^{\frac{1}{\theta}} - \frac{1}{1+\theta}E\phi_t^{\frac{1+\theta}{\theta}}\right) \tag{45}\\
&= \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} E\phi_t^{\frac{1}{\theta}}\left(1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right) \\
&\le \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}} \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}}\left(1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right).
\end{aligned}
$$

Notice that s can be approximated as

$$
\begin{aligned}
s &\simeq \beta E(1 + r_t - \chi_t) \tag{46}\\
&= \beta E\left(1 + \frac{\theta}{1+\theta}\alpha^{-\frac{1}{\theta}}(\phi_t A)^{\frac{1+\theta}{\theta}} - \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}\phi_t^{\frac{1}{\theta}}\left(1 - \frac{\phi_t}{1+\theta}\right)\right) \\
&= \beta\left(1 + \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}\left(E\phi_t^{\frac{\theta+1}{\theta}} - E\phi_t^{\frac{1}{\theta}}\right)\right) \\
&= \beta\left(1 - \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}\frac{1}{\epsilon}E\phi_t^{\frac{1}{\theta}}\right).
\end{aligned}
$$

Denoting $\bar{x} \equiv E\phi_t^{\frac{1}{\theta}}$, the mean growth rate is then given by

$$
\begin{aligned}
\bar{g} &= s\left(1 + E\chi_t\right) \tag{47}\\
&= \beta\left[1 - \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}\frac{1}{\epsilon}\bar{x}\right]\left[1 + \alpha^{-\frac{1}{\theta}} A^{\frac{1+\theta}{\theta}}\left(1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)\bar{x}\right].
\end{aligned}
$$

Differentiating with respect to $\bar{x}$, it can be shown that if the condition

$$\bar{x} < \frac{\epsilon - \left(1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1}}{2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}} \tag{48}$$

is satisfied, then $\bar{g}$ is a strictly increasing function of $\bar{x}$. Since $\bar{x} \le \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}}$ according to (44); hence, the maximum growth rate will be achieved by the certainty equilibrium where $\phi_t = \frac{\epsilon-1}{\epsilon}$, provided that Condition (48) holds.

But a sufficient condition for (48) to hold is the condition,

$$\left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}} < \frac{\epsilon - \left(1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1}}{2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}}. \tag{49}$$

Notice that $\left(1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1} \le \frac{1+\theta}{\theta}$ since $\epsilon \le \infty$, we have the following inequality for the right-hand side of (49):

$$\frac{\epsilon - \left(1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1}}{2\alpha^{-1/\theta}A^{(1+\theta)/\theta}} \ge \frac{\epsilon - \frac{1+\theta}{\theta}}{2\alpha^{-1/\theta}A^{(1+\theta)/\theta}}. \tag{50}$$

If the right-hand side of the above equation is greater than one, namely, if

$$\epsilon > \frac{1+\theta}{\theta} + 2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}, \tag{51}$$

we then have

$$\frac{\epsilon - \left(1 - \frac{\epsilon-1}{\epsilon}\frac{1}{1+\theta}\right)^{-1}}{2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}} \ge \frac{\epsilon - (1+\theta)/\theta}{2\alpha^{-\frac{1}{\theta}}A^{\frac{1+\theta}{\theta}}} > 1 \ge \left(\frac{\epsilon-1}{\epsilon}\right)^{\frac{1}{\theta}}. \tag{52}$$

Hence, (51) is a sufficient condition for the inequality (48) to hold. ∎

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