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Abstract

We extend Lubik and Schorfheide's (2004) likelihood-based estimation of dynamic stochastic general equilibrium (DSGE) models under indeterminacy to encompass a sample period including both determinacy and indeterminacy by implementing the change-point methodology (Chib, 1998). The most striking finding about the indeterminacy regime, which is estimated to coincide with the Great Inflation of the 1970s, is that it exhibits the price puzzle, in that the inflation rate rises immediately and in a sustained manner following a positive interest rate shock. Thus, the price puzzle might have been a genuine phenomenon under indeterminacy, rather than a false finding to be excised through specification search and parameter restrictions.

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I. Introduction

In this paper, we extend Lubik and Schorfheide (2004)'s [denoted LS (2004)] likelihood-based estimation of monetary dynamic stochastic general equilibrium (DSGE) models under indeterminacy to encompass a sample period including both determinacy and indeterminacy regimes where the transitions are treated as a change-point process. This approach lets the data speak as to when the indeterminacy regime took place in order to characterize the indeterminacy-specific parameters without including extraneous observations from the 1960s, for example. In the context of the monetary DSGE model we study, indeterminacy results for certain parameter values of the monetary policy rule. Indeterminacy refers to the inability to calculate a unique rational expectations forecast error given a set of structural shocks; in other words, there is room for self-fulfilling expectations and responses to extrinsic sunspot variables.

Our estimates suggest that an indeterminacy regime held between 1972 and 1982 in the United States. A key feature that we find in the indeterminacy regime is an impulse response function for inflation to an interest rate shock that exhibits the price puzzle. The price puzzle occurs when an interest rate shock is followed immediately by a sustained increase in the inflation rate. Because we know of no previous likelihood-based estimated DSGE model that has implied the price puzzle, we discuss the price puzzle in some detail.[1]

A fundamental tenet of monetary policymaking and inflation control is that a surprise increase in the short-term interest rate will lower the path of the price level from what it otherwise would have been. Thus, it has been disconcerting that impulse response functions from many identified vector autoregressive models (VARs) have exhibited the price puzzle, as first illustrated in Sims (1992). Hansen (2004) showed that it is not easy to explain away the price puzzle through the inclusion of additional variables, such as commodity prices. The commodity price "fix" does not work for all sub-sample periods, especially the pre-1980 period.

The conventional response to the price puzzle is that empirical findings of the price puzzle are necessarily false readings and a sure sign of a specification problem in the empirical model that generated such a result. Until now estimated dynamic stochastic general equilibrium (DSGE) models have not implied a price puzzle (Castelnuovo and Surico, 2006). Partly to avoid the price puzzle, Uhlig (2006) introduced a way to ensure that identified VARs do not contradict

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[1]Christiano et al. (2005) introduce staggered wage contracts and other nominal rigidities and obtain an impulse response such that the immediate response to a positive interest rate shock is higher prices, but the effect only lasts about 10 periods before a permanent decrease in prices takes hold.

the consensual view of how the economy responds to shocks: posit 'sensible' sign restrictions on impulse response functions and make sure that the only admissible parameter values in Bayesian estimation are ones that obey those sign restrictions.

An alternative view of the price puzzle is that it could be a genuine phenomenon. That is, there could be circumstances in which positive interest rate shocks lead to increases in the inflation rate. In particular, an economy under indeterminacy could exhibit the price puzzle. Lubik and Schorfheide (2003), denoted LS (2003), presented the necessary tools for likelihood-based estimation of a DSGE model under indeterminacy. Thus, only recently have macroeconomists been able to estimate a DSGE model under both determinacy and indeterminacy and calculate posterior odds that a given sample period pertained to indeterminacy [LS (2004)]. The empirical results in LS (2004) hint at the possibility of a relationship between indeterminacy and the price puzzle: "According to our pre-Volcker estimates under Prior 1, an increase in the nominal rate can have [eventually] a slightly inflationary effect...indeterminacy can alter the propagation of fundamental shocks" [LS (2004), p. 207]. It is important to note here that the LS (2004) impulse response did not show the price puzzle, although it suggests the possibility. To exhibit the price puzzle, the impact response of the interest rate shock on inflation has to be positive.

Precisely because the model estimated by LS (2004) does not exhibit the price puzzle, Castelnuovo and Surico (2006) took the model and parameter estimates from LS (2004), simulated data, estimated structural VARs and showed that VAR-identified impulse response functions displayed the price puzzle. Castelnuovo and Surico's exercise was aimed at strengthening the case that the price puzzle was a false finding.

We show that, by using a different (and we suggest a reasonable) diffuse prior on the indeterminacy-specific parameters that govern how the dynamic response of the economy to fundamental shocks differs under indeterminacy, that the estimated monetary DSGE model generates the price puzzle. Thus, we suggest that the price puzzle is not necessarily a false finding that pertains only to mis-specified VARs. We cannot, of course, rule out the possibility that the finding of indeterminacy is a result of model mis-specification, as LS (2004) also noted. Beyer and Farmer (2007) argue that if a model is lacking autoregressive dynamics that are present in the data, then one could improve the fit by falsely choosing parameter values from the indeterminacy region, since indeterminacy can enrich the model's dynamic response to fundamental shocks.[2] One argument we have against this interpretation of our results, however,

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[2]Beyer and Farmer (2007) provide a simple example where a determinate model with richer dynamics is

is that if the DSGE model we use were fundamentally lacking in dynamics, then it would be odd that the estimated indeterminacy regime pertains to less than a quarter of a 45-year sample period.
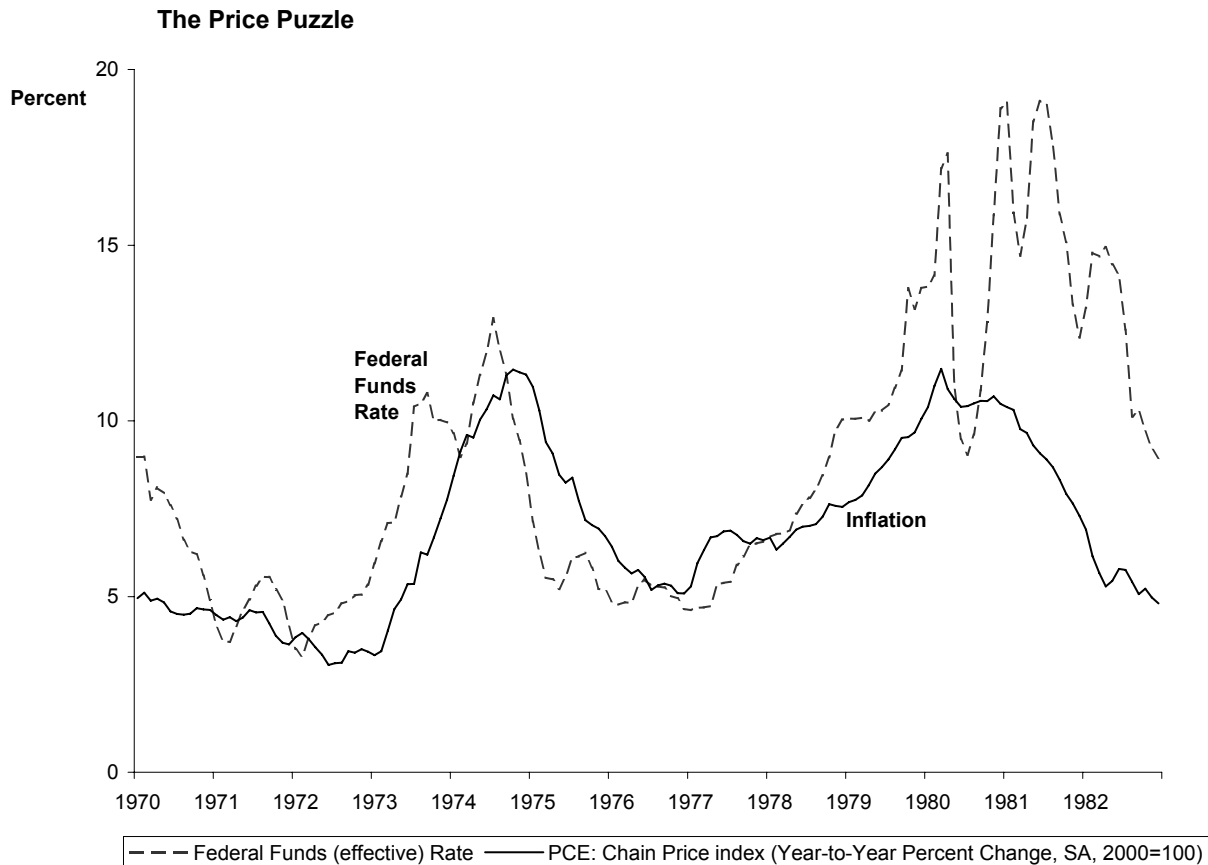
The Price Puzzle

Figure 1. The tendency of the federal funds rate to precede changes in inflation in the same direction during the 1970s.

To see why the Great Inflation of the 1970s is associated with the price puzzle in U.S. data, it is sufficient to look at a plot of the federal funds rate and inflation. Figure 1 highlights the tendency of the fed funds rate to precede movements in inflation in the same direction between the early 1970s and early 1980s. Accompanying Figure 1, we also provide a preview of our results in Figure 2. Our estimate of the indeterminacy period corresponds almost exactly with the 1972 to 1982 period indicated in Figure 1. The draws of the determinacy regimes make clear in Figure 2 that the transition from the first determinacy regime to indeterminacy took place in a relatively narrow window between late 1971 and early 1973. The timing of the transition from indeterminacy back to a determinate regime is even more clear-cut: between late 1981 and late 1982. Consequently, the subset of data used to characterize the economy under indeterminacy

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observationally equivalent to a model with indeterminacy.
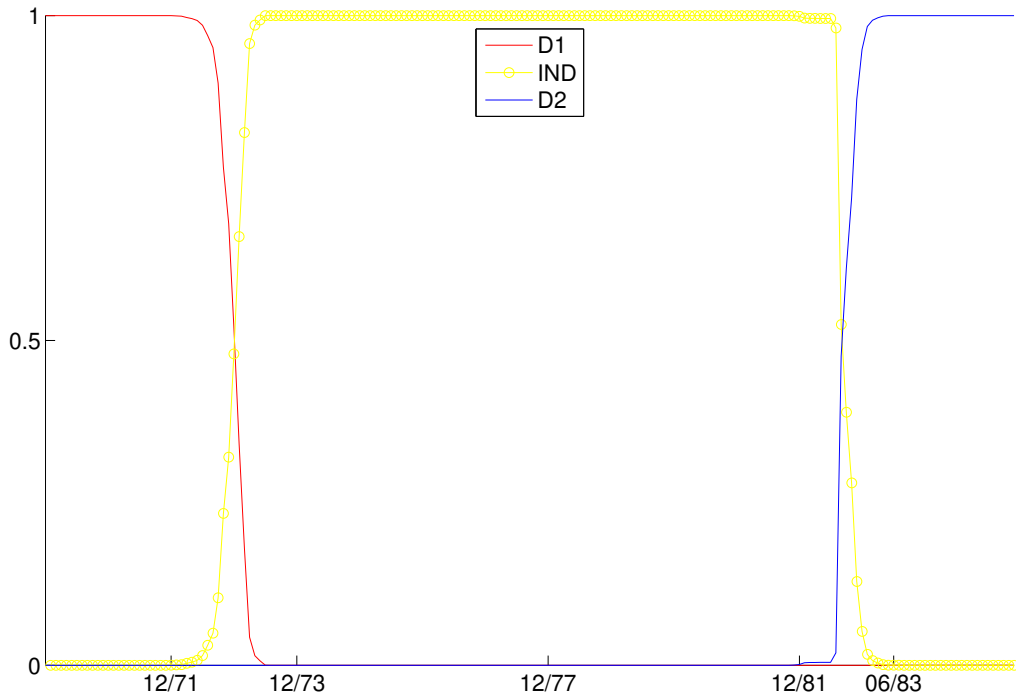
Figure 2. Posterior probability of regimes: solid red line denotes the first determinacy regime $D1$, dash-dot yellow indicates indeterminacy regime Ind, and solid blue line marks the second determinacy regime $D2$.

is well defined and does not include extraneous observations from the 1960s, as samples that use a pre-Volcker break do, such as LS (2004). In the empirical results section, we will further characterize the indeterminacy regime with respect to the price puzzle in the form of impulse response functions.

With this set of regime dates, our article provides a new perspective on the Great Inflation of the 1970s, which is useful because the Great Inflation poses the same sort of challenge for monetary policy that the Great Depression did for market economies: How did things go so wrong for such a long time?[3] Nelson (2005) and Romer and Romer (2002) lay the blame for the Great Inflation on the Monetary Neglect Hypothesis, which is that monetary policymakers attributed inflation to nonmonetary cost-push factors, such as commodity prices and wage settlements. This explanation struggles to cover what was essentially a decade-long problem, however, especially since it requires policymakers to ignore well-understood Monetarist doctrine that inflation is a monetary phenomenon. Nelson (2005) suggests that misperceptions regarding potential output during the 1970s, documented in Orphanides (2003), could have served as a

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[3]Cole and Ohanian (2002) study the ability of standard macroeconomic models to simulate a Great Depression event.

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propagation mechanism for monetary policy neglect and mistakes in order to explain the length of the Great Inflation.

We propose an explanation for the duration of the Great Inflation that is based on indeterminacy. As LS (2003) note, one can view indeterminacy as a circumstance that triggers belief shocks that alter agent forecasts in a manner consistent with rational expectations. Our finding of the price puzzle under indeterminacy is consistent with the notion that, under indeterminacy, people viewed an interest rate shock as simply another cost-push shock, so the belief was that inflation ought to rise. For this interpretation, the wording of Lubik and Schorfheide is crucial: indeterminacy *triggers* revisions to beliefs such that the economy responds differently to fundamental shocks and not just sunspots.

Nelson (2005), however, dismisses indeterminacy as a factor in explaining the Great Inflation based on the the contention that it would be a strange coincidence if sunspot shocks occurred at the same time with the same sign across many countries.[4] In our explanation, indeterminacy plays a crucial role, but we do not claim that sunspot shocks directly account for high inflation. Instead, we suggest that the way indeterminacy might alter beliefs and the economy's response to fundamental shocks is largely an empirical issue that we set out to investigate.

Our indeterminacy explanation for the duration of the Great Inflation is meant to be complementary to explanations based on learning. Sargent et al. (2005) assume that monetary policymakers knew the structure of the economy and the workings of the inflation process, but persistent misperceptions (even in a learning context) regarding the current parameter values led policymakers to pursue inflationary policies inadvertently. Cogley and Sargent (2005) and Bullard and Eusepi (2005) view the Great Inflation as the outcome of a learning process about structural change. Primiceri (2005) explains the duration of the Great Inflation by focusing on a backward-looking Keynesian model in which policymakers would gauge that disinflation would be very costly.

The remainder of the paper consists of: a presentation of the DSGE model with change points; a discussion of estimation of likelihood-based models under indeterminacy, as developed in LS (2003), with an extension to regime change points; our use of a diffuse prior, which we show leads to the price puzzle—in fact, we show that a restrictive prior is necessary to avoid the price puzzle under indeterminacy; finally, a discussion of the results vis-a-vis the price puzzle. Section IV E discusses the drawbacks of sub-sample estimation, which LS (2004) used as an

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[4]Bullard and Singh (2006) discuss worldwide transmission of endogenous volatility due to indeterminacy.

alternative to regime changes.

II. A Monetary DSGE Model

We added a time-varying inflation target to the standard New Keynesian monetary model (Woodford, 2003) because we did not want to slant the evidence in favor of indeterminacy by forcing the inflation target to remain constant. The Woodford model is widely considered a benchmark in the monetary DSGE literature and therefore serves as a good starting point for the incorporation of determinacy change-points. With the time-varying target in place, we take the log-linearized version of the Woodford model and express the state variables as deviations from their steady state levels:

$$
\begin{aligned}
\widetilde{y}_t &= E_t\widetilde{y}_{t+1} - \tau(\tilde{R}_t - E_t\tilde{\pi}_{t+1}) + g_t & (1)\\
\tilde{\pi}_t &= \beta E_t\tilde{\pi}_{t+1} + \kappa(\widetilde{y}_t - z_t)\\
\tilde{R}_t &= \rho_R\tilde{R}_{t-1} + \\
&\quad + (1 - \rho_R)[\tilde{\pi}_t^T + \psi_1(\tilde{\pi}_t - \tilde{\pi}_t^T) + \psi_2(\widetilde{y}_t - z_t)] + \varepsilon_{R,t}\\
\tilde{\pi}_t^T &= \rho_\pi\tilde{\pi}_{t-1}^T + \varepsilon_{\pi,t}\\
g_t &= \rho_g g_{t-1} + \varepsilon_{g,t}\\
z_t &= \rho_z z_{t-1} + \varepsilon_{z,t}
\end{aligned}
$$

where y is detrended log output, π is inflation, R is the nominal federal funds interest rate, and tilde denotes percentage deviation from the corresponding steady-state value.

The first two structural equations in the model emerge from general equilibrium theory: the first equation is the intertemporal Euler (I-S) equation derived from the household's optimization problem, where $\tau > 0$ denotes the intertemporal substitution elasticity and the second equation is the expectational Phillips curve with slope κ obtained from production sector's profit maximization condition, where β denotes the household's discount factor. The third equation represents a Taylor-type monetary policy rule with interest-rate smoothing. The coefficients ψ_1 and ψ_2 reflect the strength of Fed's reaction to inflation and the output gap, respectively. The fourth equation describes the AR(1) dynamics of the inflation target π_t^T.[5] The last two equations describe the autoregressive demand shock process g and technology shock process z. The

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[5]We will assume that inflation target follows AR(1) process with autocorrelation coefficient $\rho_\pi = 0.85$ and standard deviation $\sigma_\pi = 0.05$ to avoid concerns related to the identification of these parameters.

fundamental shock $\boldsymbol{\varepsilon}$ consists of the monetary policy shock ε_R, demand shock ε_g, technology shock ε_z, and inflation target shock ε_π.

Let π^* and r^* denote annualized steady-state values for π and $R - \pi$, respectively. Let parameter vector $\boldsymbol{\theta} = \{\tau, \beta, k, \{\psi_1^{(s_t=i)}, \psi_2^{(s_t=i)}\}_{i=0}^2, \rho_\pi, \rho_R, \rho_g, \rho_z\}$. We then have a vector of rational expectations forecast errors

$$\boldsymbol{\eta}_t = [\widetilde{y}_t - E_{t-1}\widetilde{y}_t, \tilde{\pi}_t - E_{t-1}\tilde{\pi}_t]',$$

the vector of observed macro data

$$\mathbf{y}_t = [\widetilde{y}_t, \pi_t, R_t]',$$

and the state vector

$$\boldsymbol{\alpha}_t = [\widetilde{y}_t, \widetilde{\pi}_t, \widetilde{R}_t, \widetilde{\pi}_t^T, E_t[y_{t+1}], E_t[\pi_{t+1}], g_t, z_t]'$$

III.   A General DSGE Model with Multiple Change Points

Most linearized rational expectations models, including eq. (1) can be written in the following state-space form:

$$
\begin{aligned}
\mathbf{y}_t &= \mathbf{A}(\boldsymbol{\theta}) + \mathbf{B}(\boldsymbol{\theta})\boldsymbol{\alpha}_t \\
\boldsymbol{\Gamma}_0(\boldsymbol{\theta})\boldsymbol{\alpha}_t &= \boldsymbol{\Gamma}_1(\boldsymbol{\theta})\boldsymbol{\alpha}_{t-1} + \boldsymbol{\Psi}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t + \boldsymbol{\Pi}(\boldsymbol{\theta})\boldsymbol{\eta}_t,
\end{aligned}
\tag{2}
$$

where $\mathbf{y}_t$ is a vector of macro data observed without measurement noise; coefficient matrices $\mathbf{A}(\boldsymbol{\theta}), \mathbf{B}(\boldsymbol{\theta}), \boldsymbol{\Gamma}_0(\boldsymbol{\theta}), \boldsymbol{\Gamma}_1(\boldsymbol{\theta}), \boldsymbol{\Psi}(\boldsymbol{\theta}), \boldsymbol{\Pi}(\boldsymbol{\theta})$ are functions of the underlying DSGE model's parameter vector $\boldsymbol{\theta}$; the state vector $\boldsymbol{\alpha}_t$ contains a combination of observed and latent elements, such as endogenous variables and their expectations; $\boldsymbol{\varepsilon}_t$ is a normally distributed vector of fundamental shocks; and $\boldsymbol{\eta}_t$ is a vector of rational expectations forecast errors.

The model specification in eq. (2) with time-invariant coefficients is at odds, however, with the recent empirical findings by Clarida et al. (2000), LS (2004), and Beyer and Farmer (2007), which show strong evidence of switches between active and passive monetary policy rules by the Federal Reserve. In this model, passive monetary policy induces indeterminacy, whereupon

multiple equilibria exist and the macro dynamics can differ substantively from the corresponding determinacy regime.

In this paper, we consider a multiple change-point model for time-varying determinacy regimes, denoted by a regime indicator variable s_t, which enables us to estimate most model parameters using data from the full-sample period. Let us assume that the sample starts with determinacy $D1$ and the determinacy state variable $s_0 = 0$. Then, at some time point t_1 there is one structural break to indeterminacy Ind: $s_{t_1} = 1$, and then at time t_2 there is a second break to a new determinacy $D2$: $s_{t_2} = 2$. We assume that the determinacy transitions cannot occur in the reverse direction, i.e. transitions from $s_t = 2$ to $s_{t+1} = 1$ are impossible. A standard Markov switching model, in contrast, does not rule out a return to previous states. In a change-point model, the Markov state transition probability matrix has the following unidirectional form:

$$\mathbf{P} = \begin{pmatrix} p_{00} & 1 - p_{00} & 0 \\ 0 & p_{11} & 1 - p_{11} \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

where $p_{ij} = Pr(s_t = j | s_{t-1} = i)$.

Consider the monetary DSGE model where the only regime-dependent parameters are the coefficients in a contemporaneous Taylor-type monetary policy rule:

$$R_t = \psi_1^{(s_t)} \pi_t + \psi_2^{(s_t)} y_t + \varepsilon_{R,t} \tag{4}$$

where the federal funds rate R_t is set according to the level of contemporaneous inflation π_t and the output gap y_t, and is subject to a monetary policy shock $\varepsilon_{R,t}$. The regime-dependent response coefficients $\psi_1^{(s_t)}$ and $\psi_2^{(s_t)}$ are selected by monetary policymakers. The econometrician's inferences about the timing of the regimes, which transition across time from $\psi_j^{(i-1)}$ to $\psi_j^{(i)}$, $i, j = 1, 2$, are central to our inference problem. As with most regime-switching models, we take switches to be exogenous events. In the DSGE context, the transition dates occur when the econometrician finds evidence that agents in the economy began forming their expectations based on a different monetary policy rule. Due to the nature of these determinacy regimes, our change-point model does not suffer from the "label switching" problem because here the states are identified not simply by labels but by indeterminacy versus determinacy.

The linearized DSGE model in (2) can be extended to allow for regime-dependent coefficients:

$$\mathbf{y}_t = \mathbf{A} + \mathbf{B}\boldsymbol{\alpha}_t \tag{5}$$

$$\boldsymbol{\Gamma}_0(\boldsymbol{\theta}(s_t))\boldsymbol{\alpha}_t = \boldsymbol{\Gamma}_1(\boldsymbol{\theta}(s_t))\boldsymbol{\alpha}_{t-1} + \boldsymbol{\Psi}(\boldsymbol{\theta}(s_t))\boldsymbol{\varepsilon}_t + \boldsymbol{\Pi}(\boldsymbol{\theta}(s_t))\boldsymbol{\eta}_t,$$

where the dependence of coefficient matrices in the first (measurement) equation on a state-dependent parameter vector $\boldsymbol{\theta}(s_t)$ is suppressed in order to emphasize the fact that both $\mathbf{A}$ and $\mathbf{B}$ depend only on the state-invariant portion of the parameter vector.

If macroeconomic agents faced an inference problem of endogenously learning about the regimes in this dynamic environment, then rational expectations formation would no longer be linear and the solution method we propose below would no longer apply. Therefore, it is important to point out that in this paper, when forming rational expectations about the saddle path of the economy, agents accept regime changes as completely exogenous events and assume that the current regime will last indefinitely.[6] It is only the econometrician who is assumed to have to infer the timing of the breaks. Recent work on DSGE models with Markov switching parameters has derived valid forward-looking solutions (Dueker et al. (2006), Davig and Leeper (2007), Davig and Leeper (2006), Farmer et al. (2006)) but the model solution methods to date have not been fast enough to be used in the estimation of DSGE models, so we leave this topic for future work.

The model of eq. (5) represents a set of simultaneous equations. Thus, an econometrician must "solve" the DSGE model, i.e. find a way to express $\boldsymbol{\eta}_t$ as a function of $\boldsymbol{\varepsilon}_t$. Sims (2002) provided precise necessary and sufficient conditions to distinguish between three possible cases, depending on the parameters in the transition equation: a) such a function might be a deterministic one-to-one map, which would correspond to determinacy; b) there might be multiple solutions (indeterminacy), in which case LS (2003) suggest a simple (linear) model for $\boldsymbol{\eta}_t$ which uniquely determines it as a function of both structural (fundamental) shocks $\boldsymbol{\varepsilon}_t$ and a sunspot shock; c) no solution.

We solve the linearized DSGE model in (5) and put it in Gaussian state-space form

$$\mathbf{y}_t = \mathbf{A} + \mathbf{B}\boldsymbol{\alpha}_t \tag{6}$$

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[6]This assumption is not unique to the change-point model. Any sequence of sub-sample estimates, including LS (2004), requires the same assumption implicitly.

$$\boldsymbol{\alpha}_t = \boldsymbol{\Gamma}_1^*(\boldsymbol{\theta}(s_t))\boldsymbol{\alpha}_{t-1} + \boldsymbol{\Gamma}_2^*(\boldsymbol{\theta}(s_t), \tilde{\mathbf{M}})\boldsymbol{\varepsilon}_t,$$

using the Generalized Schur (QZ) decomposition of $(\boldsymbol{\Gamma}_0(\boldsymbol{\theta}(s_t)), \boldsymbol{\Gamma}_1(\boldsymbol{\theta}(s_t)))$ to avoid possible problems with inverting $\boldsymbol{\Gamma}_0(\boldsymbol{\theta}(s_t))$; the column space spanning conditions in Sims (2002) are used to rule out "no solution" parameter configurations. Then, following LS (2003), we apply the singular value decomposition to the matrix $\mathbf{Q}_2\boldsymbol{\Pi}$ ($\mathbf{Q}$ from the QZ decomposition, where $\mathbf{Q}_2$ uses only the rows of $\mathbf{Q}$ corresponding to unstable eigenvalues and $\boldsymbol{\Pi}$ from eq. (2)) . Proposition 1 from LS (2004) shows how to solve for $\boldsymbol{\eta}_t$ as a function of shocks $\boldsymbol{\varepsilon}_t$ (augmented by sunspot shocks under indeterminacy) and coefficients $\tilde{\mathbf{M}}$, which control the additional impact the fundamental shocks have on $\boldsymbol{\alpha}_t$ under indeterminacy. Therefore, the transition equation coefficient matrices $\boldsymbol{\Gamma}_1^*(\boldsymbol{\theta}(s_t)), \boldsymbol{\Gamma}_2^*(\boldsymbol{\theta}(s_t), \tilde{\mathbf{M}})$ come from the output of a numerical (non-analytic) 'DSGEsolve' function of the original model parameters $\boldsymbol{\theta}(s_t)$, coefficients $\tilde{\mathbf{M}}$, and determinacy state s_t at each time t.[7]

One of the key features of the model solution under indeterminacy is that the forecast errors depend on parameters in a vector $\tilde{\mathbf{M}}$, which consists of additional free parameters that can be used to fit the data. The forecast errors under indeterminacy take the following form (see LS (2003), LS (2004) and Appendix A for definitions of these matrices):

$$\mathbf{y} - E[\mathbf{y}] = (-\mathbf{V}_1\mathbf{D}_{11}^{-1}\mathbf{U}_1'\mathbf{Q}_2\boldsymbol{\Psi} + \mathbf{V}_2\tilde{\mathbf{M}})\boldsymbol{\varepsilon} + \mathbf{V}_2\boldsymbol{\zeta} \tag{7}$$

Because the singular value decomposition results in $\mathbf{V}$ having orthonormal rows, the effects on the forecast errors $\boldsymbol{\eta}$ from fundamental shocks $\boldsymbol{\varepsilon}$ are orthogonal to the effects from the sunspot shock, $\boldsymbol{\zeta}$ when $\tilde{\mathbf{M}} = 0$. The rational expectations solution does not require orthogonality between the effects of fundamental and sunspot shocks (the partial derivatives), however.[8] Thus the parameters in $\tilde{\mathbf{M}}$ enter the model under indeterminacy and control the cross product between the effects on forecast errors between the fundamental shocks and the sunspot shock. The prior that one chooses for $\tilde{\mathbf{M}}$ reflects prior beliefs about the cross product between the effects of fundamental and sunspot shocks on forecast errors. As we discuss below, the value of M_r, in particular, is critical in leading to the price puzzle or not under indeterminacy.

As mentioned above and in greater detail in Appendix A, the solution of the DSGE model

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[7]See Appendix A for details.

[8]Note that the effects of the shocks on the forecast errors are partial derivatives of eq. 7 and are not random variables. It is therefore appropriate to discuss whether these effects consist of orthogonal matrices but not to refer to them as having correlations because they are not random variables.

involves a singular value decomposition. The singular value decomposition is subject to a sign normalization, but under determinacy the sign normalization does not matter because $\mathbf{V}_1 \mathbf{D}_{11}^{-1} \mathbf{U}_1'$ from eq. (7) is invariant to the sign normalization. Under indeterminacy, the sign normalization matters because $\mathbf{V}_2$ is *not* invariant to the sign normalization. This normalization can present a problem if, for small changes in the parameter values, the singular value decomposition procedure switches from one sign normalization to the other. If such a sign switch takes place, it could be accommodated by a corresponding change in the sign of the $\tilde{\mathbf{M}}$ coefficients. We conjecture that a consistent normalization to prevent sign flipping on the sunspot shock was implicitly accomplished in LS (2004) when the prior for $\tilde{\mathbf{M}}$ was centered at a point that would imply continuity of the impulse response function at the boundary between the determinacy and indeterminacy regions of the parameter space. But this prior also goes a long way towards ruling out the price puzzle in the indeterminacy regime because, for the price puzzle to hold under indeterminacy, the sign of the impulse response on impact has to differ from that in the determinacy regime.[9]

We chose to focus on a prior centered at $\tilde{\mathbf{M}} = 0$. LS (2003, p. 279) note that at this value, the prior belief espoused is that the effect on the forecast errors of the fundamental shocks is orthogonal to the effect on the forecast error of the the sunspot shock. Given this mean, we nevertheless made the prior quite diffuse for parameters related to the sunspot shock and indeterminacy $\tilde{\mathbf{M}} = \{M_r, M_g, M_z, M_\pi\}$ (see Table 2). We accomplish sign identification in the singular value decomposition by explicitly requiring that the last element of row vector $\underset{1 \times 2}{\mathbf{V}_2}$ is negative. If it is positive, both elements of $\mathbf{V}_2$ are forced to change sign, thereby forcing the last element always to be negative.

With $\tilde{\mathbf{M}}$ in the parameter vector, let the vector $\mathbf{s}_n = \{s_t \in \{0, 1, 2\}\}_{t=1}^n$ and the matrix $\boldsymbol{\alpha}_{1:n} = (\boldsymbol{\alpha}_1, ..., \boldsymbol{\alpha}_n)$ respectively denote determinacy regime states and state variable vector from the State-Space representation, where n is the number of observations. Let $\boldsymbol{\Theta} = \{\boldsymbol{\theta}(s_t = i), \tilde{\mathbf{M}}\}_{i=0}^2$ be the collection of model parameters across all determinacy states and use $p(\cdot)$ to denote a discrete mass function, while $\pi(\cdot)$ denotes a density function of some random variable

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[9]We might consider two alternative priors for $\tilde{\mathbf{M}}$ that have intuitive appeal: i) that the effects on forecast errors from fundamental and sunspot shocks are orthogonal ($\tilde{\mathbf{M}} = 0$); ii) continuity of impulse response functions at the boundary between determinacy and indeterminacy. LS (2004) present a simple example that compares a univariate model under determinacy and indeterminacy. In their example, however, there is no unstable root under indeterminacy, so there is no value of $\tilde{\mathbf{M}}$ that will orthogonalize the effects on the forecast error of the fundamental shock and the sunspot shock—except in the trivial case where the fundamental shock is eliminated altogether. Hence their example helps motivate the 'continuity' prior without fomenting discussion of the 'orthogonal effects' prior.

with one or more continuous components. In order to estimate our model in (5) given the data $\mathbf{Y}_n = (\mathbf{y}_1, ..., \mathbf{y}_n)$, our objective is to make draws from the posterior density $\pi(\boldsymbol{\Theta}, \mathbf{P}, \mathbf{s}_n | \mathbf{Y}_n)$, which we accomplish by making draws from the density $\pi(\boldsymbol{\Theta}, \mathbf{P}, \mathbf{s}_n, \boldsymbol{\alpha}_{1:n} | \mathbf{Y}_n)$ augmented with the state variables $\boldsymbol{\alpha}_{1:n}$.

Because of the need to infer determinacy regimes, estimation of this model is much more tractable in a Bayesian Markov Chain Monte Carlos (MCMC) framework. The MCMC algorithm consists of principal two blocks, detailed in Section B, which can be summarized as follows:

MCMC Algorithm for a General Linear DSGE Model:

1. Initialize $\boldsymbol{\Theta}$ and $\mathbf{s}_n$

2. Sample $\mathbf{P}, \boldsymbol{\Theta}$ and $\boldsymbol{\alpha}_{1:n}$ from $\mathbf{P}, \boldsymbol{\Theta}, \boldsymbol{\alpha}_{1:n} | \mathbf{Y}_n, \mathbf{s}_n$ by drawing

(a) $\mathbf{P}$ from $\mathbf{P} | \mathbf{Y}_n, \mathbf{s}_n$

(b) $\boldsymbol{\Theta}$ from $\boldsymbol{\Theta} | \mathbf{Y}_n, \mathbf{s}_n$ (c) $\boldsymbol{\alpha}_{1:n}$ from $\boldsymbol{\alpha}_{1:n} | \mathbf{Y}_n, \boldsymbol{\Theta}, \mathbf{s}_n$

3. Sample $\mathbf{s}_n$ from $\mathbf{s}_n | \mathbf{Y}_n, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:n}$

The first block in step 2 utilizes the method of composition (Chib, 2001) to produce three sub-blocks. In Step 2a we sample the unknown elements of the transition probability matrix $\mathbf{P}$ using conjugate prior-posterior update. In Step 2b we draw parameters $\boldsymbol{\Theta}$ via tailored Metropolis-Hastings (MH), where the target density is found by integrating out the states $\{\boldsymbol{\alpha}_t\}_{t=0}^{T}$ using the Kalman Filter given determinacy states $\mathbf{s}_n$. Every time t the algorithm requires coefficient matrices of the transition equation $\boldsymbol{\Gamma}_1^*(\boldsymbol{\theta}(s_t))$ and $\boldsymbol{\Gamma}_2^*(\boldsymbol{\theta}(s_t), \tilde{\mathbf{M}})$, a call to the 'DSGEsolve' function is made. The computational burden can be substantially reduced by recognizing the fact that within each Kalman filter loop these matrices remain constant until the determinacy state transition takes place. Similar handling of $\boldsymbol{\Gamma}_1^*(\boldsymbol{\theta}(s_t))$ and $\boldsymbol{\Gamma}_2^*(\boldsymbol{\theta}(s_t), \tilde{\mathbf{M}})$ applies to the remaining steps in the algorithm. In Step 2c we sample the states using one-period smoothing. Conditioning on the states $\boldsymbol{\alpha}_{1:n}$ in Step 3 reduces our second block to sampling determinacy states $\mathbf{s}_n$ in a regression with multiple change points. The sampler is iterated J times with 10 percent burn-in.

The resulting Gaussian state-space model with change points in the transition equation

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coefficients has the following form:[10]

$$\mathbf{y}_t = \mathbf{A} + \mathbf{B}\boldsymbol{\alpha}_t \tag{8}$$

$$\boldsymbol{\alpha}_t = \boldsymbol{\Gamma}_1^*(\boldsymbol{\theta}(s_t))\boldsymbol{\alpha}_{t-1} + \boldsymbol{\Gamma}_2^*(\boldsymbol{\theta}(s_t), \tilde{\mathbf{M}})\boldsymbol{\varepsilon}_t$$

where in the case of determinacy, $\boldsymbol{\varepsilon}_t$ is a 4x1 vector of fundamental exogenous shocks:

$$\boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{R,t} \\ \varepsilon_{g,t} \\ \varepsilon_{z,t} \\ \varepsilon_{\pi^*,t} \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega}^D), \quad \boldsymbol{\Omega}^D = \begin{pmatrix} \sigma_R^2 & 0 & 0 & 0 \\ 0 & \sigma_g^2 & \rho_{gz}\sigma_z\sigma_g & 0 \\ 0 & \rho_{gz}\sigma_z\sigma_g & \sigma_z^2 & 0 \\ 0 & 0 & 0 & \sigma_\pi^2 \end{pmatrix}$$

In the case of one-degree indeterminacy, we will assume that η_t can be expressed as a linear combination of the four-dimensional exogenous shock and one-dimensional sunspot shock. Then, $\boldsymbol{\varepsilon}_t$ is a 5x1 vector of fundamental shocks and the sunspot shock ζ_t, which is uncorrelated with the fundamental shocks by assumption:

$$\boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{R,t} \\ \varepsilon_{g,t} \\ \varepsilon_{z,t} \\ \varepsilon_{\pi^*,t} \\ \zeta_t \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Omega}), \quad \boldsymbol{\Omega} = \begin{pmatrix} \sigma_R^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_g^2 & \rho_{gz}\sigma_z\sigma_g & 0 & 0 \\ 0 & \rho_{gz}\sigma_z\sigma_g & \sigma_z^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\pi^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_\zeta^2 \end{pmatrix}$$

In order to cast the model in (8) in the same form under both determinacy and indeterminacy, we make $\boldsymbol{\Gamma}_2^*(\boldsymbol{\theta}(s_t), \tilde{\mathbf{M}})$ 8x5 for all states by filling in the last column with zeros in case of determinacy.

IV.   The estimated DSGE model of the U.S. economy with multiple change-points

We fit the linearized monetary DSGE model described in eq. (1) to U.S. monthly data on output, inflation and a short-term nominal interest rate from February 1959 to April 2005. For

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[10]Note that the first element of the state vector s_t is an observed data point. The Kalman filter can still be used for a linear Gaussian state-space model even if some elements of the state vector are observed (Harvey (1981) p.109). In that case, Kalman filter will produce an updated forecast of the state vector with observed elements exactly matching the data and zeros in the rows and columns corresponding to such elements in the covariance matrix. The remaining seven elements are latent, although the second and the third elements are latent only up to the constants π^* and r^*.

monthly data, industrial production was used as the output series. The model calls for output to be expressed in terms of an output gap and we follow many previous studies by using the Hodrick and Prescott (1997) filter to use a stationary measure of an output gap. Inflation is measured as the annualized log-difference in the Personal Consumption Expenditures price index. The monthly average of the daily effective federal funds rate is the short-term interest rate. In order to check for robustness of our results we used both quarterly and monthly data sets. Our quarterly data $\mathbf{Y}_n$ is given by 3x1 vector $\mathbf{y}_t$, where $t = 1959Q2, ..., 2004Q3$, while we obtained all the reported results using monthly data $\mathbf{Y}_n$ given by 3x1 vector $\mathbf{y}_t$, where $t = 1959.02, ..., 2005.04$.[11] The specific parameters in $\mathbf{\Theta}$ are

$$\mathbf{\Theta} = \{\tau, \beta, k, \{\psi_1^{(s_t=i)}, \psi_2^{(s_t=i)}\}_{i=0}^2, \rho_R, \rho_g, \rho_z, \pi^*, r^*, M_r, M_g, M_z, M_\pi, \sigma_R, \sigma_g, \sigma_z, \sigma_\varsigma\}$$

A. *Prior*

The empirical DSGE literature is prone to the "dilemma of absurd parameter estimates" (An and Schorfheide, 2005). When fit to the data, DSGE likelihood functions often peak in regions of the parameter space that are at odds with our prior perception of the workings of the economy and other data (financial markets, etc.). This undesirable feature of contemporary DSGE models has contributed to the wide application of Bayesian methods in this field and provided some researchers with justification for using a prior to shape the posterior. In this paper we take the issue of prior sensitivity quite seriously. While a certain amount of prior information must enter the posterior density, many papers fail to strike a proper balance because they impose low prior density in directions that the likelihood function would favor. Many Bayesian DSGE papers have over a third of the key structural model parameters with a posterior mean outside or close to the boundary of the 90 percent prior probability interval. In these cases, prior-sensitivity analysis would reveal substantial changes in parameter estimates.

We strive for a less-restrictive prior that still delivers reasonable parameter estimates. While we have used the same functional form as LS (2004) to specify the prior for most parameters in θ and $\mathbf{\Omega}$ that appear in both papers, our objective is to come up with a diffuse prior for all parameters, centered at values consistent with the literature. A detailed description of the prior used in our paper appears in Table 2. All variables are assumed to have independent prior densities. Gamma prior for the intertemporal substitution elasticity $\tau > 0$ is identical to LS (2004) and is consistent with empirical findings in other papers. For regime-dependent

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[11]Sims and Zha (2006) also used monthly data for approximately the same period.

parameters the prior support must lie in the parameter space consistent with the corresponding determinacy regime. Therefore, $\psi_1^{(1)}$ prior has mean 0.4 and standard deviation 0.2, which is sufficiently bounded away from 1 from the left. The prior for both $\psi_1^{(0)}$ and $\psi_1^{(2)}$ has mean 2.3 and standard deviations 1.5 and 0.6, respectively. The Beta prior density for all autocorrelation coefficients ρ_R, ρ_g, ρ_z is an obvious choice given the natural restriction of their support to the unit interval. Because monthly data tends to exhibit more autocorrelation than quarterly data, we shifted prior densities of these parameters slightly upward, compared to the LS (2004) prior, keeping standard deviations the same or slightly larger than in their paper. In order to minimize the prior's impact on the posterior of Phillips curve slope κ, the prior standard deviation is set to 1.4. The mean of the Gamma prior for κ must be larger than its standard deviation for the moments to exist. Then, a reasonable choice for the prior mean is 1.5 due to the fact that previous studies find that the Phillips curve is not very steep. The priors for steady-state inflation rate π^* and the real interest rate r^* are exactly the same as in LS (2004). For the variances of shocks, we used an inverse gamma prior, as is common for variances. As discussed above, estimation of DSGE models under indeterminacy is still a relatively new area of research, so there is little prior information about the magnitude or even the signs of the $\{M_r, M_g, M_z, M_\pi\}$ coefficients. As a result, we assumed a diffuse normal prior centered at zero for all elements of $\tilde{\mathbf{M}}$ to have a prior that the effects on forecast errors from the sunspot shock are orthogonal to the effects from structural shocks.

The joint prior of elements of the transition probability matrix is given by

$$\pi(\mathbf{P}) \propto \prod_{i=1}^{2} p_{ii}^{(a-1)}(1 - p_{ii})^{(b-1)}$$

where $a \gg b$ are selected to yield persistent regimes. We used $a = 8$, $b = 0.1$ resulting in expected regime duration of 81 months, but extensive experimentation showed a low degree sensitivity of our results to the prior specification of these parameters.

V.   Empirical results with a focus on the price puzzle

Application of a Tailored Metropolis-Hastings (M-H) algorithm (as opposed to Random Walk MH) in the sampling parameters $\boldsymbol{\theta}$ resulted in extremely efficient MCMC draws of this 23-parameter block, with autocorrelations among parameter draws that die out rapidly, leading to low inefficiency factors shown in Table 2 in the range between 1.5 and 4.7. See Appendix

B for a description of the Tailored M-H algorithm. Despite the wide application of Random Walk MH (RW-MH) in estimated DSGE models, our results show that, compared to Tailored MH, RW-MH takes longer to converge such that, even conditional on the determinacy states $\mathbf{s}_n$, the RW-MH produces substantially higher inefficiency factors ranging between 92.5 and 169.5. Therefore we argue in favor of using Tailored MH for estimating DSGE models.

A. *Prior-posterior updates and impulse responses*

The prior-posterior updates displayed in Figures (3), (4) and (5) are characteristic of a diffuse prior updated with an informative likelihood, resulting in a posterior that is not driven by the prior.[12] Among the $\tilde{\mathbf{M}}$ coefficients, M_r and M_z have particularly informative posteriors. In robustness checks, it is worth noting that when we move the prior mean for all $\tilde{\mathbf{M}}$ coefficients to +/-3 (with correspondingly larger standard deviations), the posterior means are still very close to where we found them when the prior mean is set to zero. The parameter estimates are in Table 2, but one cannot glean from the reported $\tilde{\mathbf{M}}$ coefficients the overall impact on the economy of indeterminacy.

Impulse response functions fill this role.[13] Figures 6 through 8 provide impulse responses for the estimated monthly DSGE model. While nothing in the responses to shocks under determinacy goes against intuition, it is interesting to note that, under determinacy, the first-order autoregressive structure of equation (6) results mostly in monotonic impulse responses to shocks. Under indeterminacy, in contrast, the extra response to structural shocks implied by the $\tilde{\mathbf{M}}$ coefficients allows for non-monotonic impulse responses with more persistent hump shapes. See the response of inflation and the interest rate to an inflation target shock in Figure 7, for example. As noted in LS (2004) and in Beyer and Farmer (2007), higher-order autocorrelation in the data is a consequence of indeterminacy, and it is higher-order autocorrelation that makes non-monotonic, hump-shaped impulse responses more prevalent in this model specification. Beyer and Farmer (2004) note that, when using a mis-specified model that incorrectly limits the order of serial correlation in the state vector, it is possible to bias the coefficients from such a model toward indeterminacy. Our finding of a relatively short period of indeterminacy that accounts for less than a quarter of the entire sample period indicates that the Woodford (2003) model we use is not so lacking in appropriate dynamics as to lead to finding indeterminacy throughout

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[12]Kernel smoothing was used to produce smoothed plots of marginal posterior densities using MCMC draws after burn-in.

[13]See Appendix C for the details on constructing the impulse response functions.

the sample period. The shift in dynamics under indeterminacy stems from two sources: the system has one more stable eigenvalue under indeterminacy than under determinacy and, as noted above, the $\tilde{\mathbf{M}}$ coefficients impart extra response to structural shocks that is absent under determinacy. But, we find that in fitting the data these extra dynamics are only relevant for the 1972-82 period.

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B. *Price puzzle and impulse responses*

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The key finding in our estimates of how the economy behaved under indeterminacy is that a monetary policy shock was associated with the price puzzle. As highlighted in Figure 6, the impulse response function of inflation to a monetary policy (interest rate) shock during the indeterminacy period was decidedly positive on impact and with persistence, according to the 90 percent probability intervals. Moreover, unlike the corresponding impulse response in LS (2004), the point estimate of the inflation response is never negative, even for the first few months. One interpretation of the price puzzle, then, is that under indeterminacy people understood that monetary policy would not be sufficiently active to drive inflation toward the target rate. In this context, an interest rate shock became yet another source of cost-push inflation, with no countervailing force stemming from a monetary policy rule designed to control inflation.

To investigate the difference between our impulse response function that implied the price puzzle and the impulse response from LS (2004) that did not, we recalculated impulse responses at our parameter values *except* we used values for $\tilde{\mathbf{M}}$ that correspond to continuity at the determinacy boundary. The graph in the center of Figure 9 shows how the price puzzle is eliminated at this value of $\tilde{\mathbf{M}}$ because the impact response is negative, just as it is in LS (2004). Figure 9 also demonstrates that the 'continuity' value of $\tilde{\mathbf{M}}$ (given the values of the other parameters from Table 1) does more than necessary to eliminate the price puzzle by making the impact response of a positive interest rate shock on inflation considerably more negative than seen under either determinacy regime. LS (2004) avoid this overcompensation by the continuity value of $\tilde{\mathbf{M}}$ through the use of a conditional prior. That is, the estimated values of the other parameters affect the prior mean of $\tilde{\mathbf{M}}$. One could argue that the use of a conditional prior makes the values of the other parameters, such as the slope of the Phillips curve, less structural.

The large, negative impact response of a monetary policy shock on inflation in Figure 9 results because the values of $\tilde{\mathbf{M}}$ that make the impulse responses continuous at the determinacy boundary are somewhat extreme, with $M_r = 4.57, M_g = -91.29, M_z = 1.07, M_\pi = -0.0445$, in com-

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parison with the posterior mean values from Table 2, where $M_r = -0.465, M_g = -0.198, M_z = -0.19, M_\pi = -0.23$. The value of M_r, in particular, plays a key role in the impact response of a monetary policy shock. It is natural to ask whether a more moderate value for M_r would suffice to remove the price puzzle under indeterminacy (again holding the other parameters at their posterior means). It turns out that in order to shut down the price puzzle by having the impact response of a monetary policy shock on inflation not be positive under indeterminacy, a necessary condition is $M_r > 0.057$. In other words, the impact due to non-zero M_r on the inflation forecast error from an interest rate shock under indeterminacy must be sufficiently disinflationary, which is in the same direction that the sunspot shock pushes inflation. But, this sign of M_r differs from the sign of the posterior mean value we find for M_r, which is -0.465. Moreover, the value of M_r needed to remove the price puzzle under indeterminacy is close to the upper end of the 90 percent probability interval from Table 2: (-0.973,0.0611). Thus, we conclude that the price puzzle is a robust feature of the indeterminacy regime.

Nevertheless, because our finding of the price puzzle hinges largely on the fact that the prior we use differs from the continuity prior of LS (2004), we study the boundary region between determinacy and indeterminacy in some detail. Impulse responses for parameters near the boundary between determinacy and indeterminacy are particularly interesting for understanding the difference between these two regimes, despite the fact that our parameter estimates for each regime are many standard deviations away from this boundary.[14] Fixing all parameters at their estimated values, we vary ψ_1 to investigate the dynamics of the inflation impulse response to the monetary policy shock (see Table 1). An interesting result is that as ψ_1 approaches 1 from the left (indeterminacy), the impulse response monotonically increases, whereas when ψ_1 approaches 1 from the right (determinacy), the impulse response monotonically decreases.[15] The discontinuity in the impulse responses at the boundary between the determinacy and indeterminacy still appears even if all elements of $\tilde{\mathbf{M}}$ are set to zero (see asterisk-marked entries in Table 1). The source of this additional type of discontinuity is that the number of unstable eigenvalues goes down, so the model has richer dynamics under indeterminacy. LS (2004) centered the prior (both prior 1 and prior 2) of the indeterminacy coefficients $\tilde{\mathbf{M}}$ at a point that would undo the discontinuity that would otherwise obtain under indeterminacy.

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[14]In the Woodford model a parameter vector corresponds to determinacy as long as $\psi_1 > 1 - \frac{\beta\psi_2}{\kappa}(\frac{1}{\beta} - 1) \simeq 1$.

[15]Obviously, this analysis is not a rigorous proof and is presented for motivational purposes only. In order to approach this question more rigorously, one would have to study the relationship between the elements in impulse response matrices as a function of $\boldsymbol{\theta}$ using multivariate calculus techniques. Unfortunately, $\boldsymbol{\Gamma}_1^*, \boldsymbol{\Gamma}_2^*$ do not have a closed-form solution as a function of $\boldsymbol{\theta}$, which tremendously complicates any such functional analysis.

ψ_1	3.64	1.50	1.20	1.03	1.02	0.98*	0.90*	0.49*	0.98	0.90	0.49
IR	-0.342	-0.914	-1.140	-1.324	-1.337	0.643	0.641	0.626	0.859	0.852	0.816

Table 1: This table depicts the impact response on inflation from a monetary policy shock for various values of ψ_1 holding all other parameters fixed at the estimated values. In order to emphasize that the discontinuity of impulse responses on the boundary between determinacy and indeterminacy ($\psi_1 \simeq 1$) is driven not only by non-zero values of $\tilde{\mathbf{M}}$ but also by an extra stable eigenvalue, the asterisk denotes the case when all elements of $\tilde{\mathbf{M}}$ are fixed to be zero.

C. *Variance decompositions*

Figures 10, 11 and 12 present variance decompositions under all three regimes for output, inflation and the federal funds rate for horizons up to 25 months. For output, the variance decompositions differ little across determinacy regimes because for all three regimes demand and technology shocks account for almost all of the forecast error variance. In contrast, inflation has a much different variance decomposition under indeterminacy, as shown in Figure 11. Inflation target shocks become much less important under indeterminacy because they largely can be ignored when the Federal Reserve is passive about achieving any target. Interestingly, monetary policy shocks play only a small role in inflation's forecast variance at short horizons under indeterminacy, but the share due to monetary policy shocks becomes greater at longer horizons. Under indeterminacy, the sunspot shock contributes greatly to the variance of inflation, especially at short horizons. For the federal funds rate, as seen in Figure 12, the variance decomposition is also much different under indeterminacy. Monetary policy shocks account for a much greater portion of interest rate variance at all horizons under indeterminacy, whereas demand and technology shocks account for less. The effect of the sunspot shock is largely confined to inflation, as the share of interest rate variance due to the sunspot shock is quite small.

D. *Time-varying inflation target*

Our empirical estimates also provide results on how the time-varying inflation target behaved across the sample period. Figure 13 *a*) plots twelve-month moving averages of the deviation of monthly inflation from its long-run target (which is estimated to be 2.89 percent at an annual rate in Table 2) and the posterior mean of the time-varying inflation target. During the indeterminacy period, the inferred inflation target remained fairly low, in contrast to other studies with time-varying inflation targets that impose determinacy throughout the 1970s, such

as Ireland (2005). In our model the persistently high inflation of the 1970s is consistent with a relatively low inflation target. When monetary policy was too passive to drive inflation toward any particular target, the model inferred a target rate of inflation near the unconditional mean. During the Great Inflation, indeterminacy helps hold up the likelihood function because a high inflation target would have a very low probability density, given the low unconditional mean of the inflation target. In general, our results fit the story that in the Great Inflation the Federal Reserve either countenanced a high inflation target (a determinacy explanation) or adopted a passive monetary policy rule in which the target rate of inflation had little meaning (the indeterminacy explanation).

Figure 13 b) plots the the 90 percent probability interval for the time-varying inflation target. It is perhaps surprising to see how closely the width of the probability interval matches the policy determinacy regime periods. For this reason, we describe U.S. monetary policy as "less guided" in the period of policy indeterminacy because it is simply harder to discern the target rate of inflation, without even considering whether the monetary policy rule is doing what is necessary to achieve the target. It is also interesting to note that the width of the probability interval for the inflation target is somewhat wider in the second, post-1980 determinacy period. The difference is in the two Taylor rules. Table 2 shows that the Taylor rule response coefficient for inflation decreased in the second determinacy period compared with the 1960s ($\psi_1^{(2)} < \psi_1^{(0)}$), while the output gap response remained approximately the same in the second determinacy period compared with the 1960s ($\psi_2^{(2)} \simeq \psi_2^{(0)}$). These results show that the Federal Reserve has made considerable progress in clarifying its inflation objective since the 1970s, but there is room for further improvement.

Figure 14 reports the time-varying inflation target implied by the indeterminate policy rule during the 1970s and a counterfactual inflation target that the determinate rules would have implied. The fact that the inflation target required for determinacy in the 1970s is many standard deviations above its mean indicates why the econometric inferences greatly favor indeterminacy in the 1970s. The counterfactual analysis also shows why the public did not return to form expectations based on a determinate policy rule until macroeconomic conditions were such that the determinate Taylor rule with a low inflation target was consistent with current interest rates.

E. *Consequences of sub-sample estimation*

One benefit of the change-point model is that we have only one set of estimates of the deep structural parameters (τ, κ, β, r^*, etc.). Thus when we compare impulse response functions and variance decompositions across the determinacy and indeterminacy regimes, we do not have to worry about whether the differences are coming from the regimes or from conflicting estimates of the other structural parameters—the Keating (1990) critique.

In Table 3 we present a side-by-side comparison of sub-sample parameter estimates and the full-sample estimates obtained using our change-point method. Note how the estimates of the Phillips curve slope κ go from 2.29 in pre-Volcker sub-sample to 4.54 in Volcker-Greenspan sub-sample. The steady state rate of inflation π^* is estimated to be 4.89 at the Pre-Volcker period, while considering only Post-1982 sub-sample results in π^* of 2.36. If we add the volatile 1979-1982 period, this value goes up to 2.86. Although other parameters, including the intertemporal substitution elasticity τ, do not change as dramatically as κ and π^*, jointly these changes could still have significant influence on the implied dynamics of the economy. This sub-sample instability is also found in recursive estimates of DSGE models (Canova, 2005).

Figure 15 highlights the impact that discrepancies in sub-sample estimates have by comparing impulse responses implied by three different sets of parameter estimates: the solid line corresponds to the impulse responses implied by the full-sample estimates, while the two dotted lines correspond to Pre-Volcker and Volcker-Greenspan sub-samples. By fixing monetary policy response coefficients ψ_1 and ψ_2 to the corresponding values from the second determinacy regime $D2$, we ensure that the entire difference in impulse responses is accounted for by the discrepancies in the sub-sample estimates of the deep structural parameters, and not the changes in the monetary policy. In Figure 16 we conduct a similar exercise by comparing the impulse responses implied by full-sample with the Pre-Volcker period estimates, but this time ψ_1 and ψ_2 are fixed at their indeterminacy values, as estimated in our change-point model. The differences in impulse responses under indeterminacy regime are even more pronounced than in Figure 15 under determinacy.

VI. Conclusions

We extend Lubik and Schorfheide's (2004) likelihood-based estimation of dynamic stochastic general equilibrium (DSGE) models under indeterminacy to include time-varying monetary policy regimes which encompass sample periods of both determinacy and indeterminacy. With our change-point approach, we are able to date the indeterminacy regime to lie between 1972 and 1982. Only when actual inflation approached the neighborhood of the unconditional mean of the inflation target in 1982 did the inferred regime return to determinacy.

In contrast to LS (2004), we investigated a prior on indeterminacy-specific parameters that postulates that the effect of the sunspot shock on forecast errors is orthogonal to the effect of fundamental shocks, as opposed to centering the prior where continuity of impulse response functions at the boundary between determinacy and indeterminacy. The latter prior essentially rules out the price puzzle under indeterminacy by forcing the impact response of inflation to an interest rate shock to be negative, as it is under determinacy. The orthogonal effects prior, in contrast, allows the indeterminacy-specific parameters to take whatever sign helps fit the data, and those values imply the price puzzle under indeterminacy.

Thus, in contrast to a large and growing literature that treats the price puzzle as a false finding to be excised through additional specification search and parameter restrictions, our results suggest that the price puzzle could be a genuine phenomenon under the right circumstances: a passive monetary policy that induces indeterminacy. In this way, our results bolster the finding from identified VARs estimated over sub-samples that the price puzzle is limited to samples that focus largely on the 1970s. As the first estimated DSGE model to not only generate the price puzzle but also estimate the time period when it occurred (which turns out to coincide closely with the Great Inflation of the 1970s), we would suggest that the price puzzle is perhaps not a false finding to be 'resolved' but possibly a consequence of indeterminacy.

Given the stark differences between the behavior of the economy under indeterminacy in the 1970s and determinacy in other time periods, our methodology for incorporating determinacy regime shifts in estimated DSGE would prove useful in any monetary DSGE model where the sample period includes the 1970s.

A. Coefficient matrices and singular value decomposition

The model in (1) can be re-written in the form of equation (2) where the coefficient matrices
are:

$$
\mathbf{A} \;=\; \begin{pmatrix} 0 \\ \pi^* \\ r^* + \pi^* \end{pmatrix},
$$

$$
\mathbf{B} \;=\; \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},
$$

$$
\boldsymbol{\Gamma}_0(\boldsymbol{\theta}(s_t)) \;=\; \begin{pmatrix}
0 & 0 & \tau & 0 & -1 & -\tau & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & -\beta & 0 & \kappa \\
0 & 0 & 1 & (1-\rho_R)(\psi_1^{(s_t)}-1) & 0 & 0 & 0 & (1-\rho_R)\psi_2^{(s_t)} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
$$

$$
\boldsymbol{\Gamma}_1(\boldsymbol{\theta}(s_t)) \;=\; \begin{pmatrix}
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \kappa & -1 & 0 & 0 \\
0 & 0 & \rho_R & 0 & (1-\rho_R)\psi_2^{(s_t)} & (1-\rho_R)\psi_1^{(s_t)} & 0 & 0 \\
0 & 0 & 0 & \rho_\pi & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \rho_g & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_z \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix},
$$

$$
\boldsymbol{\Psi}(\boldsymbol{\theta}(s_t)) \;=\;
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix},
\quad
\boldsymbol{\Pi}(\boldsymbol{\theta}(s_t)) =
\begin{pmatrix}
-1 & 0 \\
\kappa & -1 \\
(1-\rho_R)\psi_2^{(s_t)} & (1-\rho_R)\psi_1^{(s_t)} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 1
\end{pmatrix}
$$

where the data on annualized inflation π_t and the interest rate R_t can be expressed as the corresponding steady state values plus 12 times their demeaned monthly counterparts. Note that the output measure in vector $\mathbf{y}_t$ does not require removal of a mean because we use Hodrick-Prescott filtered data, which is mean-zero by construction.

In order to estimate our DSGE model following the MCMC algorithm in section III, we need to solve our linearized DSGE model in the form of equation (6) conditional on s_t at each time t. We now consider details of the 'DSGEsolve' function, where the state dependence is implicit and it is suppressed in the interest of clarity. Following Sims (2002) we apply generalized Schur QZ decomposition[16] $(\boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1) = (\mathbf{Q}'\boldsymbol{\Lambda}\mathbf{Z}', \mathbf{Q}'\boldsymbol{\Omega}\mathbf{Z}')$ to partition the resulting system into two parts: 1) the collection of non-explosive components denoted by subscript 1 and corresponding to generalized eigenvalues (ratio of the diagonal elements of $\boldsymbol{\Omega}$ over the corresponding diagonal elements of $\boldsymbol{\Lambda}$) that are less than one; 2) explosive components denoted by subscript 2 otherwise. Then, we use "solution uniqueness" and "stability" conditions worked out in Sims (2002) to write in the partitioned matrix form:

$$
\begin{pmatrix} \mathbf{I} & -\boldsymbol{\Phi} \\ \mathbf{0} & \boldsymbol{\Lambda}_{22}^{-1} \end{pmatrix}
\begin{pmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12} \\ \mathbf{0} & \boldsymbol{\Lambda}_{22} \end{pmatrix}
\begin{pmatrix} \mathbf{w}_1(t) \\ \mathbf{w}_2(t) \end{pmatrix}
=
\begin{pmatrix} \mathbf{I} & -\boldsymbol{\Phi} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}
\begin{pmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} \\ \mathbf{0} & \boldsymbol{\Omega}_{22} \end{pmatrix}
\begin{pmatrix} \mathbf{w}_1(t-1) \\ \mathbf{w}_2(t-1) \end{pmatrix}
$$

$$
+ \begin{pmatrix} \mathbf{I} & -\boldsymbol{\Phi} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}
\begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{pmatrix}
\Psi \varepsilon_t
\tag{9}
$$

where $\mathbf{w}(t) = \mathbf{Z}'\boldsymbol{\alpha}_t$, $\boldsymbol{\Phi} = \mathbf{Q}_1\boldsymbol{\Pi}(\mathbf{Q}_2\boldsymbol{\Pi})^{-1}$ and $\mathbf{I}$, $\mathbf{0}$ denote identity and zero matrices respectively with dimensionality easily deduced from the above equation.

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[16]See Golub and Van Loan (1996)

Next, as in LS(2004), we apply the Singular Value Decomposition to $\mathbf{Q}_2\mathbf{\Pi}$:

$$\mathbf{Q}_2\mathbf{\Pi} \;\; = \;\; \left[\begin{array}{cc} \mathbf{U}_1 & \mathbf{U}_2 \end{array} \right] \left[\begin{array}{cc} \mathbf{D}_{11} & 0 \\ 0 & 0 \end{array} \right] \left[\begin{array}{c} \mathbf{V}_1' \\ \mathbf{V}_2' \end{array} \right] \tag{10}$$

Then, the "solution" of the DSGE model in (1) is the transition equation of the SSM (6):

$$\boldsymbol{\alpha}_t \;\; = \;\; \mathbf{\Gamma}_1^*\boldsymbol{\alpha}_{t-1} + \mathbf{\Gamma}_2^*\boldsymbol{\varepsilon}_t \tag{11}$$

where from equation (9) in the partitioned matrix form we have[17]

$$\underset{8\times8}{\mathbf{\Gamma}_1^*} = \underset{8\times8}{\mathbf{Z}} \left[\begin{array}{cc} \left[\begin{array}{cc} \underset{6\times6}{\mathbf{I}} & \underset{6\times2}{-\mathbf{\Phi}} \end{array} \right] \underset{8\times8}{\mathbf{\Lambda}} \\ \underset{2\times6}{\mathbf{0}} \quad \underset{2\times2}{\mathbf{I}} \end{array} \right]^{-1} \left[\begin{array}{cc} \left[\begin{array}{cc} \underset{6\times6}{\mathbf{I}} & \underset{6\times2}{-\mathbf{\Phi}} \end{array} \right] \underset{8\times8}{\mathbf{\Omega}} \\ \underset{2\times8}{\mathbf{0}} \end{array} \right] \underset{8\times8}{\mathbf{Z}}\,'$$

Finally, from LS (2003) and LS (2004) Proposition 1 we can solve for $\boldsymbol{\eta}_t$ as a function of shocks and coefficients $\tilde{\mathbf{M}} = \{M_r, M_g, M_z, M_\pi\}$, which control the additional impact each fundamental shock has on $\boldsymbol{\eta}_t$ under indeterminacy to find:

$$\underset{8\times5}{\mathbf{\Gamma}_2^*} = \left[\begin{array}{cc} \mathbf{\Gamma}_0^{-1}(\mathbf{\Psi} - \mathbf{\Pi}\mathbf{V}_1\mathbf{D}_{11}^{-1}\underset{8\times4}{\mathbf{U}_1'\mathbf{Q}_2\mathbf{\Psi}} + \mathbf{\Pi}\mathbf{V}_2\tilde{\mathbf{M}}) & \underset{8\times1}{\mathbf{\Gamma}_0^{-1}\mathbf{\Pi}\mathbf{V}_2} \end{array} \right]$$

which under determinacy reduces to

$$\underset{8\times4}{\mathbf{\Gamma}_2^*} = \left[\begin{array}{c} \mathbf{\Gamma}_0^{-1}(\mathbf{\Psi} - \mathbf{\Pi}\mathbf{V}_1\mathbf{D}_{11}^{-1}\mathbf{U}_1'\mathbf{Q}_2\mathbf{\Psi}) \end{array} \right]$$

It is important to point out that the singular value decomposition is unique only up to a sign normalization. Therefore, some kind of sign identification restrictions on $\mathbf{V}_2$ are necessary to ensure that under indeterminacy sign flipping of the sunspot shock does not occur.

B.   Detailed MCMC Algorithm

This paper follows the multiple change-point estimation approach originally introduced in Chib (1998) based on results in Chib (1996). The centerpiece of this method is a transformation in terms of a latent discrete state variable that indicates the regime from which a particular observation has been drawn. In other words, instead of using a single-move sampler to draw

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[17]Matrix dimensions are provided for the case of determinacy in this model.

times t_1 and t_2 at which the structural break occurred, we will draw determinacy states $\mathbf{s}_n = \{s_t \in \{0,1,2\}\}_{t=1}^n$ in a multi-move sampler, which is by far more efficient than a single-move, because it groups highly correlated elements of a Markov Chain in one block drastically reducing autocorrelation of the draws.

Our objective is to draw from the posterior $\pi(\boldsymbol{\Theta}, \mathbf{P}, \mathbf{s}_n | \mathbf{Y}_n)$ which can be accomplished by simulating the following full conditional distributions:

1. $\pi(\boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:n} | \mathbf{s}_n, Y_n) = \pi(\mathbf{P} | \mathbf{s}_n) \pi(\boldsymbol{\Theta} | \mathbf{Y}_n, \mathbf{s}_n, \mathbf{P}) \pi(\boldsymbol{\alpha}_{1:n} | \boldsymbol{\Theta}, \mathbf{P}, \mathbf{s}_n, \mathbf{Y}_n)$

2. $p(\mathbf{s}_n | \mathbf{Y}_n, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:n}) = \prod_{t=1}^{n-1} p(s_t | \mathbf{Y}_n, \mathbf{s}^{t+1}, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:n})$,

where we adapt the notation similar to Chib (1998): $\mathbf{s}_t = (s_1, ..., s_t)$, $\mathbf{s}^{t+1} = (s_{t+1}, ..., s_n)$, $\boldsymbol{\alpha}_{1:n} = (\boldsymbol{\alpha}_1, ..., \boldsymbol{\alpha}_n)$, $\mathbf{Y}_t = (\mathbf{y}_1, ..., \mathbf{y}_t)$ and use $p(\cdot)$ to denote a discrete mass function, while $\pi(\cdot)$ denotes a density function of some random variable with one or more continuous components. To avoid ambiguity we would like to emphasize the difference between the latent state of determinacy s_t and the latent state variable $\boldsymbol{\alpha}_t$ from the state-space model in equation (6).

Full details of each step of the MCMC algorithm follow:

1. Initialize $\boldsymbol{\Theta}$ and $\mathbf{s}_n$

2. The first block is sampled using method of composition in three parts. Sample $\mathbf{P}, \boldsymbol{\Theta}$ and $\boldsymbol{\alpha}_{1:n}$ from $\mathbf{P}, \boldsymbol{\Theta}, \boldsymbol{\alpha}_{1:n} | \mathbf{Y}_n, \mathbf{s}_n$ by drawing

   (a) $\mathbf{P}$ from $\mathbf{P} | \mathbf{Y}_n, \mathbf{s}_n$ using a conjugate update. Let n_{ii} be the number of one-state transitions from state i to state i (i.e. staying put). Then, a Bernoulli likelihood $p(\mathbf{s}_n | p_{ii})$ multiplied by the Beta prior $\pi(p_{ii})$ (given in subsection A) results in

   $p_{ii} | \mathbf{s}_n \sim Beta(a + n_{ii}, b + 1)$, $i = 0, 1$.

   (b) $\boldsymbol{\Theta}$ from $\boldsymbol{\Theta} | \mathbf{Y}_n, \mathbf{s}_n$ using the Kalman filter to find the target density and a Tailored Metropolis-Hastings algorithm to sample from it.

   For convenience of notation, let $t_0 = 0$, t_1 and t_2 be the time points of the first and second structural breaks respectively, and $t_3 = n$. Let $\pi(\boldsymbol{\Theta})$ denote the prior given in Table 2. As discussed above, the likelihood function will differ across states. There-fore, conditional on state s_t at each time t we will call 'DSGEsolve' function to cast it

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in the multivariate normal form of equation (6). In other words, in the implementation of all the remaining steps of the algorithm, $\mathbf{\Gamma}_1^*(\boldsymbol{\theta}(s_t))$ and $\mathbf{\Gamma}_2^*(\boldsymbol{\theta}(s_t), \tilde{\mathbf{M}})$ are obtained each time by calling the 'DSGEsolve' function. Obviously, during implementation of this algorithm, these coefficients need to be computed only three times when looping over $t = 1, ..., n$, because $[\mathbf{\Gamma}_1^*(\boldsymbol{\theta}(s_t)) \quad \mathbf{\Gamma}_2^*(\boldsymbol{\theta}(s_t), \tilde{\mathbf{M}})] = [\mathbf{\Gamma}_1^*(\boldsymbol{\theta}(s_{t-1})) \quad \mathbf{\Gamma}_2^*(\boldsymbol{\theta}(s_{t-1}), \tilde{\mathbf{M}})]$ as long as $s_t = s_{t-1}$.

By letting $f^{(\mathbf{s}_t)}$ denote the Gaussian density function given all the states up to time t, the density $\pi(\boldsymbol{\Theta}|\mathbf{Y}_n, \mathbf{s}_n, \mathbf{P})$ could be sampled using the usual Kalman filter recursion formula:

$$\pi(\boldsymbol{\Theta}|\mathbf{Y}_n, \mathbf{s}_n, \mathbf{P}) \propto f(\mathbf{Y}_n|\boldsymbol{\Theta}, \mathbf{s}_n)\pi(\boldsymbol{\Theta}) = \tag{12}$$
$$\prod_{i=1}^{3} \prod_{t=t_{i-1}+1}^{t_i} f^{(\mathbf{s}_t)}(\mathbf{y}_t|\mathbf{Y}_{t-1}, \boldsymbol{\Theta}, \mathbf{s}_t)\pi(\boldsymbol{\Theta})$$

where

$$f^{(\mathbf{s}_t)}(\mathbf{y}_t|\mathbf{Y}_{t-1}, \boldsymbol{\Theta}, \mathbf{s}_t) = \mathcal{N}(\mathbf{y}_t|\mathbf{A} + \mathbf{B}\boldsymbol{\alpha}_{t|t-1}^{(\mathbf{s}_t)}, \mathbf{f}_{t|t-1}^{(\mathbf{s}_t)}) \tag{13}$$

such that the log-likelihood term is proportional to:

$$\begin{aligned} log(f(\mathbf{Y}_n|\boldsymbol{\Theta}, \mathbf{s}_n)) \quad \propto \quad & -\sum_{i=1}^{3} \sum_{t=t_{i-1}+1}^{t_i} [log((det(\mathbf{f}_{t|t-1}^{(\mathbf{s}_t)})) \\ & + \quad (\mathbf{y}_t - \mathbf{A} - \mathbf{B}\boldsymbol{\alpha}_{t|t-1}^{(\mathbf{s}_t)})'(\mathbf{f}_{t|t-1}^{(\mathbf{s}_t)})^{-1}(\mathbf{y}_t - \mathbf{A} - \mathbf{B}\boldsymbol{\alpha}_{t|t-1}^{(\mathbf{s}_t)})] \end{aligned}$$

where, suppressing the dependence on the states $\mathbf{s}_t$ for transparency, the details of Kalman filter updates are as follows:

1) state forecast mean $\boldsymbol{\alpha}_{t|t-1} = \mathbf{\Gamma}_1^*\boldsymbol{\alpha}_{t-1|t-1}$

2) state forecast variance $\mathbf{P}_{t|t-1} = \mathbf{\Gamma}_1^*\mathbf{P}_{t-1|t-1}(\mathbf{\Gamma}_1^*)' + \mathbf{\Gamma}_2^*\boldsymbol{\Omega}(\mathbf{\Gamma}_2^*)'$

3) data forecast variance $\mathbf{f}_{t|t-1} = \mathbf{B}\mathbf{P}_{t|t-1}\mathbf{B}' + \mathbf{0}$

4) Kalman gain $\mathbf{K}_t = \mathbf{P}_{t|t-1}\mathbf{B}'\mathbf{f}_{t|t-1}^{-1}$

5) update state mean $\boldsymbol{\alpha}_{t|t} = \boldsymbol{\alpha}_{t|t-1} + \mathbf{K}_t(\mathbf{y}_t - \mathbf{A} - \mathbf{B}\boldsymbol{\alpha}_{t|t-1})$

6) update state variance $\mathbf{P}_{t|t} = (\mathbf{I} - \mathbf{K}_t\mathbf{B})\mathbf{P}_{t|t-1}$

The above equations require an initialization[18] of $\boldsymbol{\alpha}_{t=0|t=0}$ and $\mathbf{P}_{t=0|t=0}$, which are

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[18]See Harvey (1988)

set equal to their steady-state values.

Let $\mathbf{m} = \{\mathbf{\Theta}\}$. The density $g(\mathbf{m}) = \pi(\mathbf{m}|\mathbf{Y}_n, \mathbf{s}_n, \mathbf{P})$ found in equation (12) serves as a target density for the Tailored Metropolis-Hastings (MH) algorithm originally proposed by Chib and Greenberg (1994).[19] First, using numerical optimization (a GSL implementation of BFGS algorithm), we find $\mathbf{m}^* = \arg\max_{\mathbf{m}} \ln(g(\mathbf{m}))$. Then, we compute the Fisher information matrix $\mathbf{V} = \{-\partial^2 \ln(g(\mathbf{m}))/\partial\mathbf{m}\partial\mathbf{m}'\}^{-1}$ evaluated for $\mathbf{m} = \mathbf{m}^*$. The proposal density is based on $(\mathbf{m}^*, \mathbf{V})$ and is specified as a multivariate t-distribution with ξ degrees of freedom serving as a tuning parameter. This step is completed by sampling a proposal value $\mathbf{m}'$ from $f_T(\cdot|\mathbf{m}^*, \mathbf{V}, \xi)$ and accepting it with probability

$$\alpha(\mathbf{m}, \mathbf{m}'|\mathbf{Y}_n, \mathbf{s}_n, \mathbf{P}) = \min\left\{\frac{g(\mathbf{m}')}{g(\mathbf{m})}\frac{f_T(\mathbf{m}|\mathbf{m}^*, \mathbf{V}, \xi)}{f_T(\mathbf{m}'|\mathbf{m}^*, \mathbf{V}, \xi)}, 1\right\}$$

The current value of $\mathbf{m}$ is retained if the proposed value is rejected. We have discovered that a constant proposal density found using the above procedure for some reasonable fixed $\mathbf{s}_n$ produces a good acceptance rate (76%) and reliable mixing results (see inefficiency factors in Table 2). This observation has substantially reduced total MCMC computing time and allowed us to make 10,000 draws in about 10 minutes with low inefficiency factors.

(c) $\boldsymbol{\alpha}_{1:n}$ from $\boldsymbol{\alpha}_{1:n}|\mathbf{Y}_n, \mathbf{\Theta}, \mathbf{s}_n$. Conditional on $\mathbf{s}_n$, we are faced with a simple linear Gaussian state-space model with time-varying coefficients. The standard approach is to draw $\boldsymbol{\alpha}_{1:n}$ using one-period smoothing, which amounts to adding two more steps to the Kalman filter procedure above:

7) $\mathbf{M}_t = \mathbf{P}_{t|t}(\mathbf{\Gamma}_1^*)'(\mathbf{P}_{t+1|t})^{-1}$

8) $\mathbf{P}_{t|t+1} = \mathbf{P}_{t|t} - \mathbf{M}_t\mathbf{P}_{t+1|t}(\mathbf{M}_t)'$

and then sampling the states backwards starting with

$\boldsymbol{\alpha}_n \sim \mathcal{N}(\boldsymbol{\alpha}_{n|n}, \mathbf{P}_{n|n})$

followed by ($\forall t = n - 1, ...1$)

$\boldsymbol{\alpha}_t \sim \mathcal{N}(\boldsymbol{\alpha}_{t|t+1}, \mathbf{P}_{t|t+1})$, where $\boldsymbol{\alpha}_{t|t+1} = \boldsymbol{\alpha}_{t|t} + \mathbf{M}_t(\boldsymbol{\alpha}_{t+1} - \mathbf{\Gamma}_1^*\boldsymbol{\alpha}_{t|t})$.

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[19]See Chib (2001) for discussion of various MH algorithms and their tuning

However, several technical complications occur as a result of degeneracy of the state space model in (6). In particular, in case of (in)determinacy, one can sample only one (two) element(s) of the state vector and, computing three more elements from the observed data points, solve for the remaining four (three) elements such that the integrity of the transition equation of the SSM was preserved at all times. Let us define a matrix $\mathbf{C}$ that horizontally concatenates the last two columns of the matrix $\mathbf{\Gamma}_1^*$ with the matrix $\mathbf{\Gamma}_2^*$. The structure of our SSM model is such that in the case of determinacy the $rank(\mathbf{C})$ is always equal to 4 and for indeterminacy it is 5. The idea is to find some weighting matrix $\mathbf{W}$ which would allow us to use the first three elements of the state vector, which are observed given π^* and r^*, and the sampled fourth (and fifth) element(s) of $\boldsymbol{\alpha}_{t+1}$ to find the remaining four (three) elements in case of (in)determinacy:

$$\boldsymbol{\alpha}_{t+1}(5:8) = \mathbf{\Gamma}_1^*(5:8,:)\boldsymbol{\alpha}_t + \mathbf{W}[\boldsymbol{\alpha}_{t+1}(1:4) - \mathbf{\Gamma}_1^*(1:4,:)\boldsymbol{\alpha}_t]$$

Let us define $\mathbf{C}_1 = \mathbf{C}(1:4,:)$ to be the first four rows of $\mathbf{C}$ matrix (five for indeterminacy). Then the matrix W that we are looking for must satisfy

$$\mathbf{C} = \begin{pmatrix} \mathbf{C}_1 \\ \mathbf{W}\mathbf{C}_1 \end{pmatrix} = \begin{pmatrix} \mathbf{I} \\ \mathbf{W} \end{pmatrix} \mathbf{C}_1$$

where $\mathbf{I}$ is an identity matrix. Therefore $\mathbf{W}$ could be found from

$$\begin{pmatrix} \mathbf{I} \\ \mathbf{W} \end{pmatrix} = (\mathbf{C}\mathbf{C}_1')(\mathbf{C}_1\mathbf{C}_1')^{-1}$$

Breakpoints between regimes also required some special treatment. All additional technical details are available from authors upon request.

3. Following Chib (1998), in the second block we sample $\mathbf{s}_n$ from $\mathbf{s}_n|\mathbf{Y}_n, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:n}$ by drawing s_t backwards from time $t = n-1$ conditional on $\mathbf{s}^{t+1}$. Chib (1996) has shown that

$$p(s_t|\mathbf{Y}_n, \mathbf{s}^{t+1}, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:n}) \propto p(s_t|\mathbf{Y}_t, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:n})p(s_{t+1}|s_t, \mathbf{P}, \boldsymbol{\alpha}_{1:n})$$

where $p(s_{t+1}|s_t, \mathbf{P}, \boldsymbol{\alpha}_{1:n}) = p_{s_t s_{t+1}}$

Starting with $t = 1$, Chib (1998) utilizes a recursive forward calculation to find the mass function $p(s_t|\mathbf{Y}_t, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:n}) = p(s_t|\mathbf{Y}_t, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:t-1})$ ($\forall t = 1, ..., n$) by recursively transforming $p(s_{t-1}|\mathbf{Y}_{t-1}, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:t-2})$ through:

$$p(s_t = k|\mathbf{Y}_t, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:t-1}) = \frac{p(s_t = k|\mathbf{Y}_{t-1}, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:t-2}) f^{(s_t=k)}(\mathbf{y}_t|\boldsymbol{\Theta}, s_t = k, \boldsymbol{\alpha}_{1:t-1})}{\sum_{l=1}^{m} p(s_t = l|\mathbf{Y}_{t-1}, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:t-2}) f^{(s_t=l)}(\mathbf{y}_t|\boldsymbol{\Theta}, s_t = l, \boldsymbol{\alpha}_{1:t-1})}$$

where $p(s_t = k|\mathbf{Y}_{t-1}, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:t-2}) = \sum_{l=k-1}^{k} p_{lk} \times p(s_{t-1} = l|\mathbf{Y}_{t-1}, \boldsymbol{\Theta}, \mathbf{P}, \boldsymbol{\alpha}_{1:t-2})$ and $f^{(s_t=l)}(\mathbf{y}_t|\boldsymbol{\Theta}, s_t = l, \boldsymbol{\alpha}_{1:t-1}) =$

$\mathcal{N}(\mathbf{y}_t|\mathbf{A} + \mathbf{B}(\boldsymbol{\Gamma}_1^*(\boldsymbol{\theta}(s_t = l))\boldsymbol{\alpha}_{t-1}), \mathbf{B}\boldsymbol{\Gamma}_2^*(\boldsymbol{\theta}(s_t = l), \tilde{\mathbf{M}})\boldsymbol{\Omega}(\boldsymbol{\Gamma}_2^*(\boldsymbol{\theta}(s_t = l), \tilde{\mathbf{M}}))'\mathbf{B}')$ using the fact that $\boldsymbol{\alpha}_{1:n}$ is treated as known in this block, which is effectively reduced to a linear regression with change point coefficients model where $\boldsymbol{\alpha}_{1:n}$ serve as explanatory variables.

C.   Computation of Impulse Response Functions

Consider the transition equation of the SSM (6):

$$\boldsymbol{\alpha}_t = \boldsymbol{\Gamma}_1^*(\boldsymbol{\theta}(s_t))\boldsymbol{\alpha}_{t-1} + \boldsymbol{\Gamma}_2^*(\boldsymbol{\theta}(s_t), \tilde{\mathbf{M}})\boldsymbol{\varepsilon}_t$$

Figures 6, 7, and 8 display Impulse Responses $\mathbf{IR}_j(s_t = i)$ to one-standard-deviation shocks $0 \leq j \leq 24$ periods ahead given determinacy state $s_t = i$ ($i = 0, 1, 2$), which are calculated using the following formula:

$$\mathbf{IR}_j(s_t = i) = \frac{\partial \boldsymbol{\alpha}_{t+j}}{\partial \boldsymbol{\varepsilon}_t} = [\boldsymbol{\Gamma}_1^*(\boldsymbol{\theta}(s_t = i))]^j \boldsymbol{\Gamma}_2^*(\boldsymbol{\theta}(s_t = i), \tilde{\mathbf{M}}) Cholesky(\boldsymbol{\Omega})$$

D.   Computation of Variance Decomposition

Let $\mathbf{ir}_j^l(s_t = i)$ denote l'th column of j-period ahead Impulse Response $\mathbf{IR}_j(s_t = i)$ given determinacy state i. Then, the contribution of shock l to the mean squared error of the j-period ahead state variable forecast is

$$\mathbf{ir}_0^l(s_t = i)(\mathbf{ir}_0^l(s_t = i))' + \mathbf{ir}_1^l(s_t = i)(\mathbf{ir}_1^l(s_t = i))' + \cdots + \mathbf{ir}_{j-1}^l(s_t = i)(\mathbf{ir}_{j-1}^l(s_t = i))'$$

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MCMC Parameter Estimates

Model	Posterior			Ineff.	Prior		
Parameter	Mean	St. Dev.	90% Prob. Int.	Factor	Density	Mean	St. Dev.
τ	0.4696	0.1004	(0.3064, 0.6399)	1.5	gamma	0.53	0.14
β	0.9918	0.0041	(0.9847, 0.9981)	2.7	gamma	0.99	0.005
κ	2.4762	0.5683	(1.5443, 3.4345)	3.2	gamma	1.50	1.40
ρ_R	0.8688	0.0137	(0.8457, 0.8911)	3.0	beta	0.60	0.20
ρ_g	0.9866	0.0055	(0.9775, 0.9956)	4.3	beta	0.80	0.15
ρ_π	0.85	fixed					
ρ_z	0.9683	0.0065	(0.9576, 0.9794)	3.7	beta	0.80	0.15
π^*	2.8942	0.3186	(2.3618, 3.4143)	2.4	normal	4.00	2.00
r^*	2.4579	0.4368	(1.7294, 3.1572)	2.5	normal	2.00	1.00
σ_R	0.0590	0.0029	(0.0542, 0.0638)	2.7	invgamma	0.10	0.10
σ_z	0.7602	0.0235	(0.7215, 0.7982)	4.0	invgamma	0.80	0.30
σ_g	0.0271	0.0047	(0.0190, 0.0348)	3.8	invgamma	0.04	0.03
σ_π	0.05	fixed					
σ_ζ	0.1727	0.0131	(0.1507, 0.1944)	4.4	invgamma	0.25	0.13
ρ_{gz}	0.9243	0.0307	(0.8727, 0.9747)	4.7	normal	0.30	0.70
M_r	-0.4653	0.3146	(-0.973, 0.0611)	3.1	normal	0.00	1.00
M_g	-0.1977	0.9998	(-1.869, 1.4313)	2.5	normal	0.00	1.00
M_z	-0.1902	0.0585	(-0.286, -0.097)	2.6	normal	0.00	1.00
M_π	-0.2288	0.9831	(-1.840, 1.3747)	3.2	normal	0.00	1.00
$\psi_1^{(0)}$	3.9317	0.4934	(3.1201, 4.7842)	3.6	normal	2.30	1.50
$\psi_1^{(1)}$	0.4985	0.1429	(0.2652, 0.7321)	4.6	normal	0.40	0.20
$\psi_1^{(2)}$	2.3724	0.2222	(2.0101, 2.7400)	3.0	normal	2.30	0.60
$\psi_2^{(0)}$	0.3088	0.1014	(0.1416, 0.4821)	4.3	normal	0.30	0.12
$\psi_2^{(1)}$	0.2967	0.0991	(0.1351, 0.4626)	1.8	normal	0.30	0.12
$\psi_2^{(2)}$	0.3144	0.0987	(0.1534, 0.4760)	2.2	normal	0.30	0.12

Table 2: Monthly data parameter estimates using 10,000 MH iterations and 10% burn-in
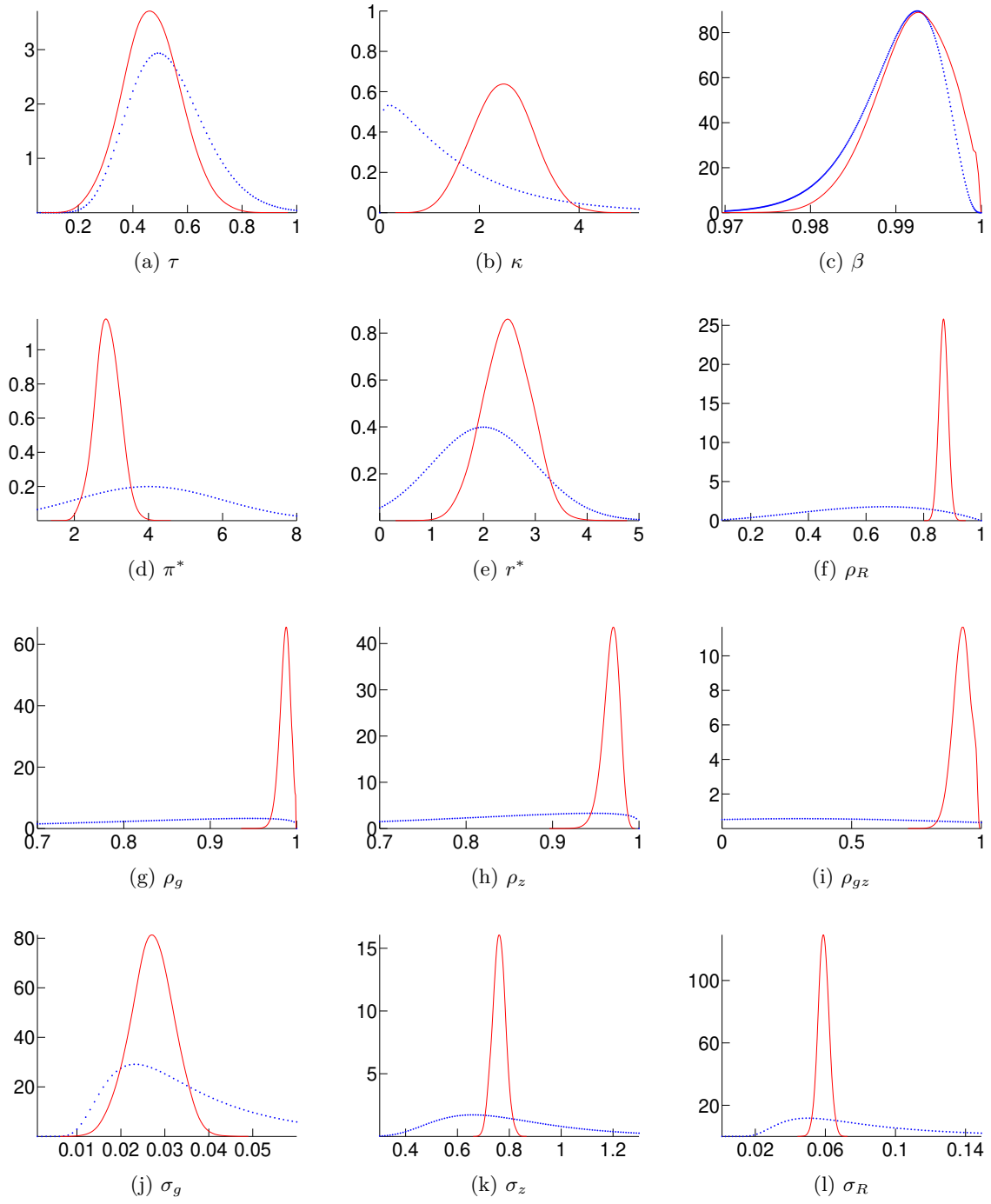
Figure 3. Prior-Posterior Update: Dotted blue line depicts the prior and solid red line the posterior of selected regime-<u>in</u>dependent parameters
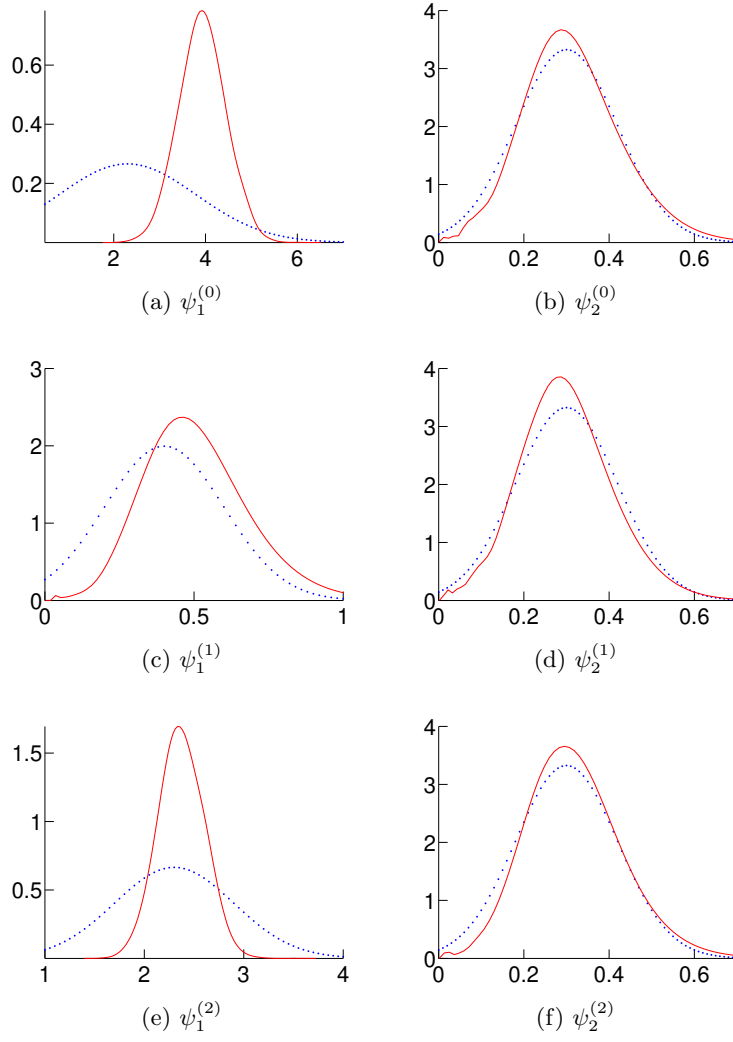
Figure 4. Prior-Posterior Update: Dotted blue line depicts the prior and solid red line the posterior of regime-dependent parameters

(a) M_r

(b) M_g

(c) M_z

(d) M_π

Figure 5. Prior-Posterior Update: Dotted blue line depicts the prior and solid red line the posterior of indeterminacy region-determined parameters

Figure 6. Impulse Responses: mean (solid) and 90% probability interval (dash).

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Impulse Response of Output to Technology Shock in First Determinacy Regime

Impulse Response of Output to Technology Shock in Indeterminacy Regime

Impulse Response of Output to Technology Shock in Second Determinacy Regime

Impulse Response of Inflation to Technology Shock in First Determinacy Regime

Impulse Response of Inflation to Technology Shock in Indeterminacy Regime

Impulse Response of Inflation to Technology Shock in Second Determinacy Regime

Impulse Response of Interest Rate to Technology Shock in First Determinacy Regime

Impulse Response of Interest Rate to Technology Shock in Indeterminacy Regime

Impulse Response of Interest Rate to Technology Shock in Second Determinacy Regime

(a) technology shock

Impulse Response of Output to Inflation Taget Shock in First Determinacy Regime

Impulse Response of Output to Inflation Target Shock in Indeterminacy Regime

Impulse Response of Output to Inflation Target Shock in Second Determinacy Regime

Impulse Response of Inflation to Inflation Target Shock in First Determinacy Regime

Impulse Response of Inflation to Inflation Target Shock in Indeterminacy Regime

Impulse Response of Inflation to Inflation Target Shock in Second Determinacy Regime

Impulse Response of Interest Rate to Inflation Target Shock in First Determinacy Regime

Impulse Response of Interest Rate to Inflation Target Shock in Indeterminacy Regime

Impulse Response of Interest Rate to Inflation Target Shock in Second Determinacy Regime

(b) inflation target shock

Figure 7. Impulse Responses: mean (solid) and 90% probability interval (dash).

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Impulse Response of Output to Sunspot
Shock in Indeterminacy Regime

Impulse Response of Inflation to Sunspot
Shock in Indeterminacy Regime

Impulse Response of Interest Rate to
Sunspot Shock in Indeterminacy Regime

Figure 8. Impulse Responses to sunspot shock: mean (solid) and 90% probability interval (dash).

Impulse Response of Inflation to Monetary
Policy Shock in First Determinacy Regime

Impulse Response of Inflation to Monetary
Policy Shock in Indeterminacy Regime

Impulse Response of Inflation to Monetary
Policy Shock in Second Determinacy Regime

Figure 9. Impulse Responses obtained by forcing the "continuity" restriction on M, which precludes the price puzzle

Percent of Variance Due to Monetary Policy Shock in First Determinacy Regime

Percent of Variance Due to to Monetary Policy Shock in Indeterminacy Regime

Percent of Variance Due to Monetary Policy Shock in Second Determinacy Regime

Percent of Variance Due to Demand Shock in First Determinacy Regime

Percent of Variance Due to Demand Shock in Indeterminacy Regime

Percent of Variance Due to Demand Shock in Second Determinacy Regime

Percent of Variance Due to Technology Shock in First Determinacy Regime

Percent of Variance Due to Technology Shock in Indeterminacy Regime

Percent of Variance Due to Technology Shock in Second Determinacy Regime

Percent of Variance Due to Inflation Taget Shock in First Determinacy Regime

Percent of Variance Due to Inflation Target Shock in Indeterminacy Regime

Percent of Variance Due to Inflation Target Shock in Second Determinacy Regime

Percent of Variance Due to Sunspot Shock in Indeterminacy Regime

Figure 10. Variance Decomposition: percent of variance in output with 90% probability interval (dash)

Percent of Variance Due to Monetary Policy Shock in First Determinacy Regime

Percent of Variance Due to Monetary Policy Shock in Indeterminacy Regime

Percent of Variance Due to Monetary Policy Shock in Second Determinacy Regime

Percent of Variance Due to Demand Shock in First Determinacy Regime

Percent of Variance Due to Demand Shock in Indeterminacy Regime

Percent of Variance Due to Demand Shock in Second Determinacy Regime

Percent of Variance Due to Technology Shock in First Determinacy Regime

Percent of Variance Due to Technology Shock in Indeterminacy Regime

Percent of Variance Due to Technology Shock in Second Determinacy Regime

Percent of Variance Due to Inflation Target Shock in First Determinacy Regime

Percent of Variance Due to Inflation Target Shock in Indeterminacy Regime

Percent of Variance Due to Inflation Target Shock in Second Determinacy Regime

Percent of Variance Due to Sunspot Shock in Indeterminacy Regime

Figure 11. Variance Decomposition: percent of variance in inflation with 90% probability interval (dash)

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Percent of Variance Due to
Monetary Policy Shock in First
Determinacy Regime

Percent of Variance Due to
Monetary Policy Shock in
Indeterminacy Regime

Percent of Variance Due to
Monetary Policy Shock in Second
Determinacy Regime

Percent of Variance Due to
Demand Shock in First
Determinacy Regime

Percent of Variance Due to
Demand Shock in Indeterminacy
Regime

Percent of Variance Due to
Demand Shock in Second
Determinacy Regime

Percent of Variance Due to
Technology Shock in First
Determinacy Regime

Percent of Variance Due to
Technology Shock in
Indeterminacy Regime

Percent of Variance Due to
Technology Shock in Second
Determinacy Regime

Percent of Variance Due to Inflation
Target Shock in First Determinacy
Regime

Percent of Variance Due to
Inflation Target Shock in
Indeterminacy Regime

Percent of Variance Due to
Inflation Target Shock in Second
Determinacy Regime

Percent of Variance Due to
Sunspot Shock in Indeterminacy
Regime

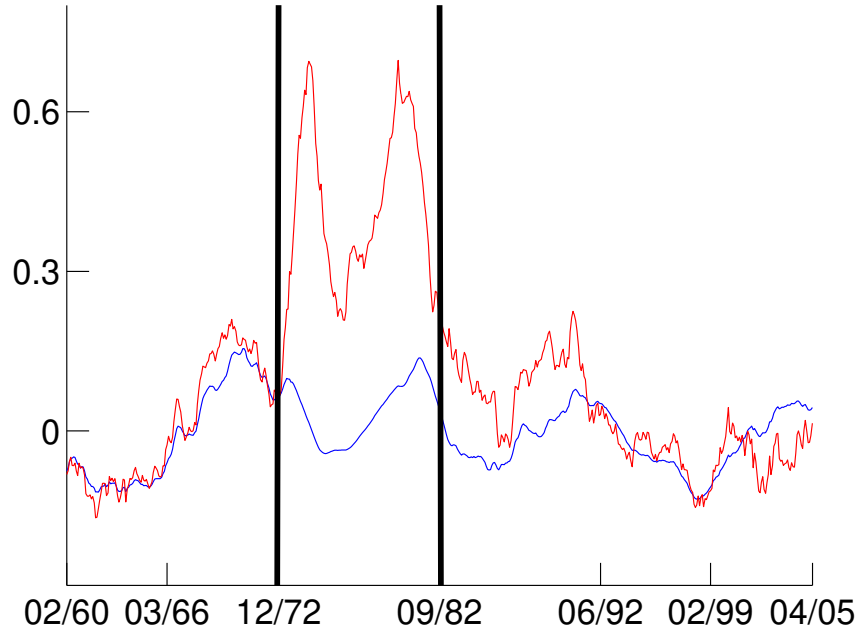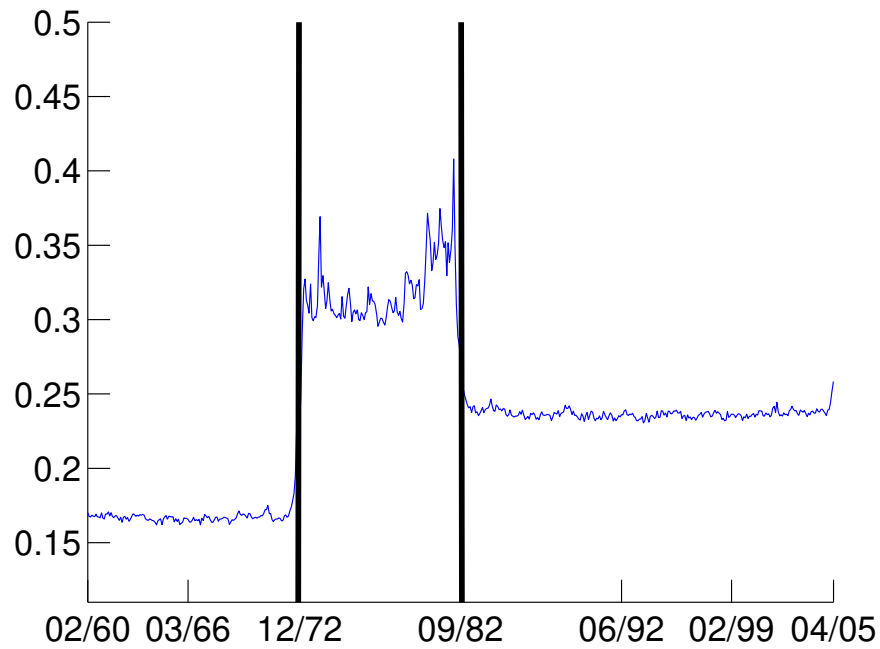**Figure 12**. Variance Decomposition: percent of variance in interest rate with 90% probability interval (dash)

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(a) twelve-month moving average

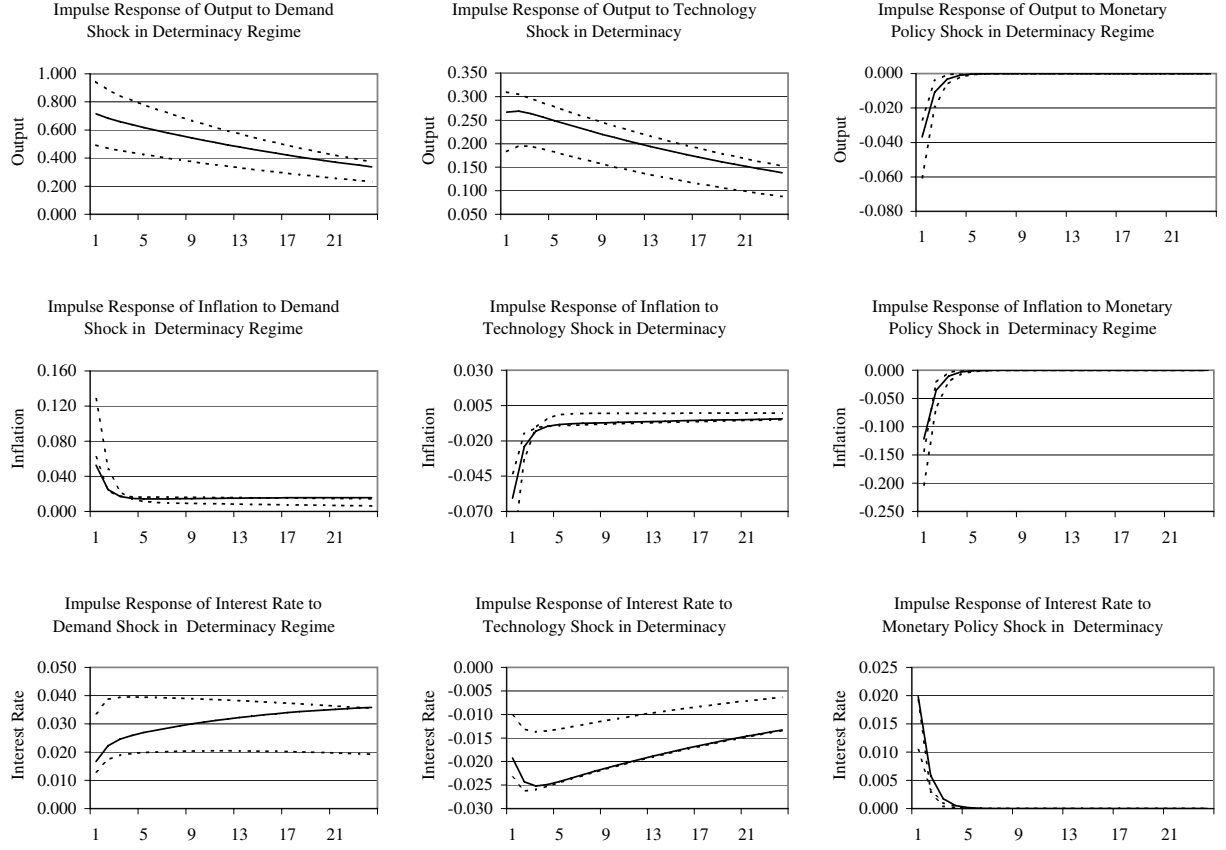

(b) 90% probability interval width

Figure 13. Inflation Target: graph *a*) depicts the evolution of the estimated demeaned monthly inflation target (blue line) and demeaned monthly actual inflation (red line); graph *b*) shows the width of the estimated inflation target 90% probability interval; bold black vertical lines on both graphs denote the estimated borders of indeterminacy regime
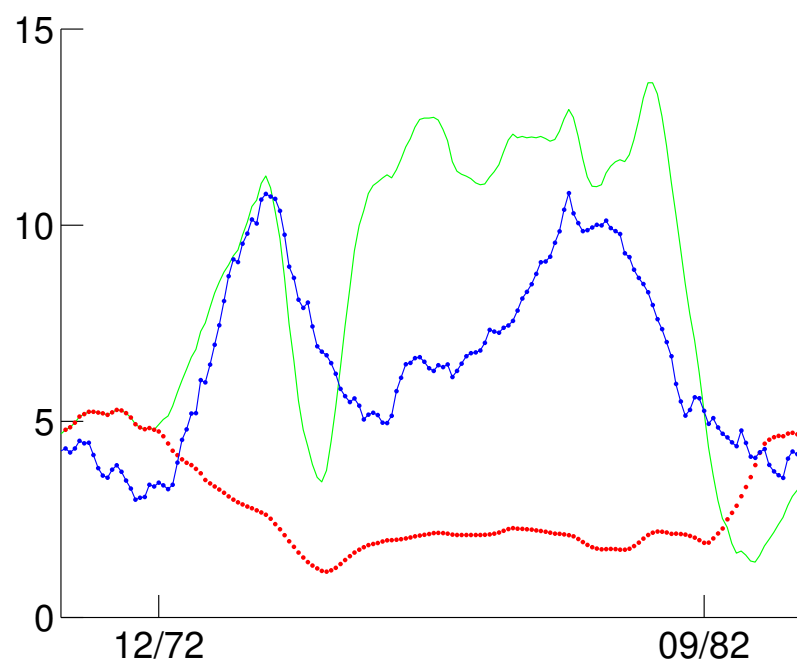
Figure 14. Counterfactual analysis: twelve-month moving averages of the actual annual inflation (dash-dot blue), inferred mean inflation target from the full model (dotted red), and counterfactual mean inflation target inferred by imposing D1 regime (solid green).

Parameter	Change-Point	Pre-Volcker	Post-1982	Volcker-Greenspan
τ	0.4696	0.5464	0.4417	0.5548
β	0.9918	0.9925	0.9925	0.9925
κ	2.4762	2.2935	1.8509	4.5362
ρ_R	0.8688	0.9585	0.9456	0.7750
ρ_g	0.9866	0.9742	0.9844	0.9835
ρ_z	0.9683	0.9623	0.9650	0.9690
π^*	2.8942	4.8933	2.3626	2.8581
r^*	2.4579	1.1889	1.7959	2.2070
σ_r	0.0590	0.0311	0.0264	0.0973
σ_z	0.7602	0.9330	0.5034	0.5780
σ_g	0.0271	0.0387	0.0183	0.0278
σ_ζ	0.1727	0.1326	-	-
ρ_{gz}	0.9243	0.9735	0.9300	0.8374
$M_{\zeta,r}$	-0.4653	-0.0139	-	-
$M_{\zeta,g}$	-0.1977	-0.0128	-	-
$M_{\zeta,z}$	-0.1902	-0.2528	-	-
$M_{\zeta,\pi}$	-0.2288	-0.5825	-	-
$\psi_1^{(0)}$	3.9317	-	-	-
$\psi_1^{(1)}$	0.4985	0.4078	-	-
$\psi_1^{(2)}$	2.3724	-	2.7410	2.6123
$\psi_2^{(0)}$	0.3088	-	-	-
$\psi_2^{(1)}$	0.2967	0.2982	-	-
$\psi_2^{(2)}$	0.3144	-	0.3191	0.2957

Table 3: Monthly data parameter estimates using the entire period 1959:02-2005:04 and three sub-samples: 1959:02-1979:06, 1982:10-2005:04, 1979:07-2005:04, corresponding to Pre-Volcker, Post-1982, and Volcker-Greenspan periods respectively. Following the sub-sample findings of LS (2004), Pre-Volcker period was estimated by imposing indeterminacy, while both Post-1982 and Volcker-Greenspan periods were estimated for determinacy regime.

Impulse Response of Output to Demand
Shock in Determinacy Regime

Impulse Response of Output to Technology
Shock in Determinacy

Impulse Response of Output to Monetary
Policy Shock in Determinacy Regime

Impulse Response of Inflation to Demand
Shock in Determinacy Regime

Impulse Response of Inflation to
Technology Shock in Determinacy

Impulse Response of Inflation to Monetary
Policy Shock in Determinacy Regime

Impulse Response of Interest Rate to
Demand Shock in Determinacy Regime

Impulse Response of Interest Rate to
Technology Shock in Determinacy

Impulse Response of Interest Rate to
Monetary Policy Shock in Determinacy

Figure 15. Impulse Responses under Determinacy: the solid line corresponds to the impulse responses generated from parameter estimates found using the entire data set while dotted lines correspond to parameter estimates obtained from Pre-Volcker and Volcker-Greenspan sub-samples. The response coefficients of the monetary policy were fixed at the level of change-point estimates for regime $D2$ to ensure that the only difference in impulse responses comes from the discrepancy in the estimates of the deep structural parameters across these sub-samples.

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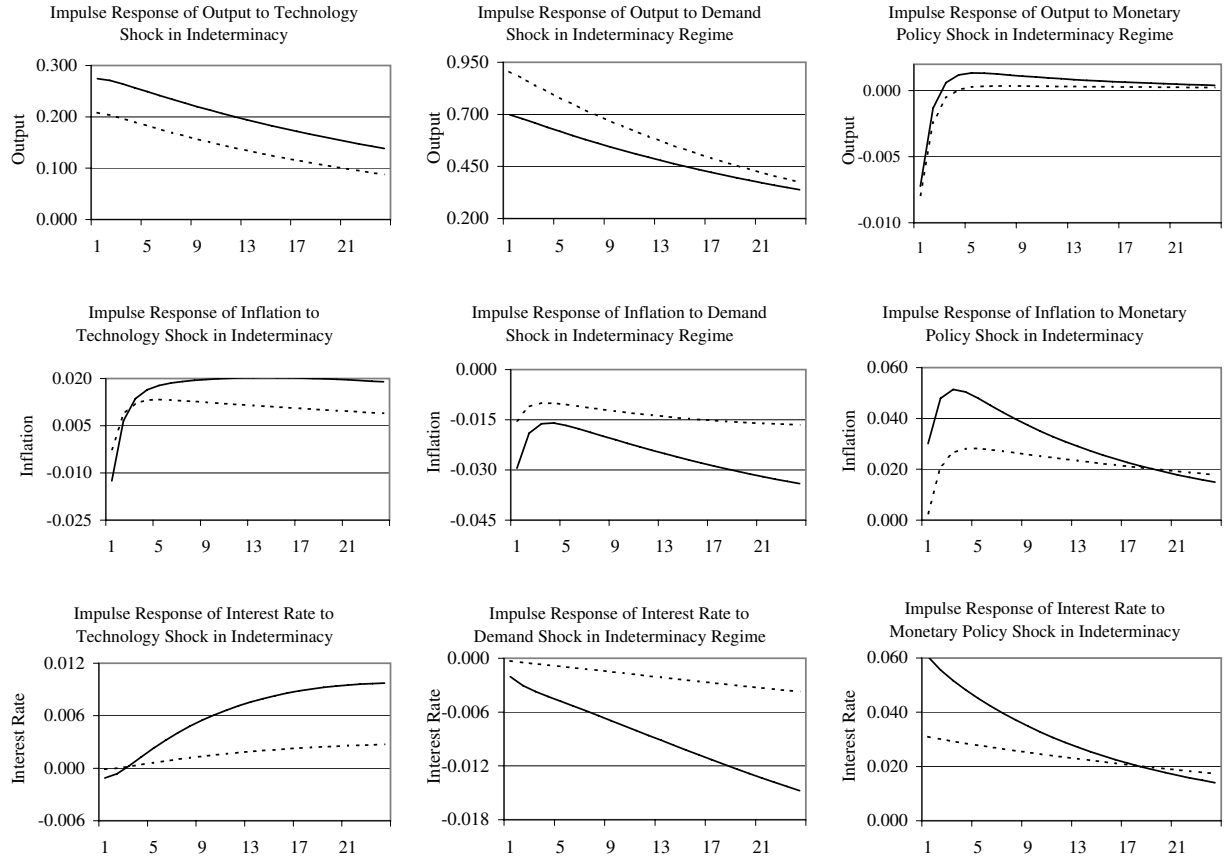
Figure 16. Impulse Responses under Indeterminacy: the solid line corresponds to the impulse responses generated from parameter estimates found using the entire data set while the dotted line corresponds to parameter estimates obtained from the Pre-Volcker sub-sample. The response coefficients of the monetary policy were fixed at the level of change-point estimates for regime IND to ensure that the only difference in impulse responses comes from the discrepancy in the estimates of the deep structural parameters between the sub-sample and full-sample inference.

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