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Imperfect Competition and Indeterminacy of Aggregate Output*

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Abstract

This paper shows imperfect competition can lead to indeterminacy in aggregate output in a standard DSGE model with imperfect competition. Indeterminacy arises in the model from the composition of aggregate output. In sharp contrast to the indeterminacy literature pioneered by Benhabib and Farmer [3] and Gali [19], indeterminacy in our model is global; hence it is more robust to structural parameters. In addition, sunspots in our model can be autocorrelated. The paper provides a justification for exogenous variations in desired markups, which play an important role as a source of cost-push shocks in the monetary policy literature. Our model outperforms a standard RBC model driven by technology shocks in several dimensions, including the volatility of labor market and the hump-shaped output dynamics.

Keywords: Indeterminacy, Global Sunspots, Self-fulfilling Expectations, Procyclical Productivity, Marginal Costs, Counter-Cyclical Markup, Cost-Push Shocks, Hump-Shaped Impulse Responses.

JEL codes: E31, E32.

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1 Introduction

A well-accepted set of stylized facts of the business cycle includes: 1) aggregate consumption, investment, employment, marginal cost and productivity comove with aggregate output; 2) the detrended components in these aggregate quantities are highly persistent; 3) the impulse responses of output (as well as other variables) are hump-shaped; and 4) consumption and productivity are less volatile than output, employment is roughly as volatile as output, and investment is more volatile than output during a cycle.[1] As shown by Kydland and Prescott [38], many of these stylized facts can be explained by technology shocks in a perfectly competitive general equilibrium model. However, standard RBC models driven by technology shocks fail to explain all of the aforementioned stylized facts. For example, under technology shocks, the relative volatility of labor to output in a standard model is too small to match the U.S. data (e.g., see Prescott [41]). Even with the assumption of an infinitely large labor supply elasticity (e.g., Hansen's [28] indivisible labor), employment is still not volatile enough relative to output to match the U.S. data. In addition, as pointed out by Bils [7], technology shocks cannot explain why the marginal cost is procyclical (or why the markup is counter-cyclical), which is an empirical regularity well-documented in the literature.[2]

Conventional Keynesian wisdom argues the level of aggregate output is essentially indeterminate (i.e., the supply curve is flat), hence autonomous changes in aggregate demand (e.g., due to animal spirits) are the main driving force of the business cycle. A major challenge to this wisdom, however, is to model autonomous shifts in expectations as an independent source of shocks in general equilibrium with rational agents. To do so, the ability to model extrinsic uncertainty (uncertainty not related to the fundamentals) is key. The seminal works of Shell [48], Azariadis [2], and Cass and Shell [9], among others, opened up this possibility and provided the first breakthrough in meeting the challenge. Using dynamic equilibrium models, these works show that economic fluctuations can be driven by agents' self-fulfilling expectations without changes in the fundamentals, such as preferences, technologies, and endowments. These important works, however, fall short in confronting the time series data of the business cycle. It was not until the seminal work of Kydland and Prescott [38] that quantitative models of the business cycle which can be calibrated to confront time series data became available. Benhabib and Farmer [3], Farmer and Guo [16], and Gali [19] are among the first to show the possibility of generating quantitative predictions of the business cycle within the framework of Kydland and Prescott using shifts in agents' expectations as a key driving force.[3]

[1] See, e.g., Kydland and Prescott [38], Prescott [41], Hansen [28], Bils [7], Cogley and Nason [12], and Rotemberg and Woodford [42, 45, 46], among others.

[2] See, e.g., Bils [7], Martins, Scarpetta, and Pilat [39], and Rotemberg and Woodford [42, 46], among others.

[3] For the earlier literature along this line of research, see Farmer and Woodford [17] and Woodford [55, 56, 57],

However, this new generation of expectations-driven business-cycle models typically relies on local indeterminacy of the steady state to generate fluctuations driven by sunspots.[4] Since many structural parameters jointly determine the eigenvalues of a dynamic model near the steady state, the reliance on local indeterminacy imposes severe restrictions on the structures of the economy. If such restrictions are not satisfied, equilibria with expectation-driven fluctuations are not possible. For example, indeterminacy in the Benhabib-Farmer type of models requires the equilibrium locus of the labor demand curve to be upward sloping and steeper than the labor supply curve.[5] For these reasons, slight modifications of the model (such as changes in the time discounting factor, the rate of capital depreciation, capital's share in total income, or allowing for adjustment costs in labor or investment) can easily eliminate indeterminacy, hence insulating the models from fluctuations driven by self-fulfilling expectations.[6]

This paper provides a model of expectations-driven business cycles that does not rely on the local indeterminacy of the steady state. Sunspots equilibria in our model are less dependent on parameters in the utility function and production technologies. Hence, our model makes expectations-driven fluctuations a more robust feature of dynamic general equilibrium economies. The model is a standard general equilibrium model featuring the Dixit-Stiglitz [13] type of imperfect competition. It is closely related to the model of Gali [19]. Similar to Gali, we generate expectation-driven fluctuations via indeterminacy in the composition of aggregate output. However, two key features distinguish our model from Gali [19]. First, unlike Gali [19], sunspots equilibria in our model do not hinge on local indeterminacy of the steady state, hence they are robust to the topological properties of the steady state (i.e., sink versus saddle). Second, shocks to expectations in our model can follow any stationary $ARMA(p,q)$ processes, in contrast to the class of models based on Gali [19] and Benhabib and Farmer [3], where shocks to expectations are confined to *i.i.d.* processes.[7] When expectation shocks are restricted to be *i.i.d.* processes, Schmitt-Grohe [25] shows that such shocks cannot explain the hump-shaped impulse response pattern of the U.S. business cycle documented by Cogley and Nason [12]. Our model overcomes this shortcoming of the current generation of dynamic-stochastic-general-equilibrium (DSGE) sunspots models.[8]

The motivation for focusing on the composition of aggregate output derives from the classical idea of Kalecki [34] and Pigou [40] where changes in the elasticities of demand can explain the

among others.

[4]Recent literature along this line includes Benhabib and Farmer [4], Weder [51], Wen [52], Harrison and Weder [29], and Jaimovich [32], among many others.

[5]See Aiyagari [1], Schmitt-Grohe [24] and Wen [54] for more discussions on this issue.

[6]See, e.g., Georges [21], Wen [53], Kim [35], and Herrendorf and Valentinyi [30].

[7]However, see Wang and Wen [49] for modifications.

[8]Benhabib and Wen [6] show that serially correlated fundamental shocks to preferences or government spending in a Benhabib-Farmer type model can explain the hump-shaped impulse response pattern of the U.S. business cycle. Consistent with Schimitt-Grohe [25], they also show that sunspots shocks in their model are not able to resolve the problem because of the *i.i.d.* restriction on the sunspots variable.

procyclical movements in marginal costs, a feature standard sticky-wage Keynesian models cannot explain.[9] Given that movements in the marginal cost (or markup) are also closely related to movements in measured productivity (see, e.g., Hall [26, 27]), the link between shifts in demand elasticity and the business cycle is an important channel to exploit. Pioneering work along this line includes Gali [19, 20] and Rotemberg and Woodford [42, 43, 44, 46].[10] We model the composition of aggregate output in the simplest possible way by assuming output can be produced by more than one alternative technologies, which use the same types of intermediate inputs except the elasticities of substitutions among the inputs differ. This difference in the elasticities of substitution leads to a difference in the elasticity of demand for intermediate goods. Given that the intermediate goods have the same unit cost in the Dixit-Stiglitz economy, final good producers are indifferent regarding which technology is used. However, since the suppliers of intermediate inputs are monopolistic-competitive firms, the markup differs across different technologies due to the difference in the elasticity of demand. Thus an autonomous change in the composition of aggregate output can lead to changes in the elasticities of demand and the competitiveness among intermediate good firms. This in turn can lead to counter-cyclical markup and fluctuations in aggregate output.[11]

We show sunspots shocks to the composition of aggregate output can explain all of the aforementioned business cycle facts simultaneously. In particular, besides being able to explain procyclical marginal cost (counter-cyclical markup), the model also explains procyclical productivity under constant returns to scale technologies. With respect to the persistence and volatilities of consumption, investment, employment and output series, our model performs at least as well as a standard RBC model driven by technology shocks. In some aspects, especially with respect to hump-shaped output dynamics and employment volatility, our model outperforms standard RBC models. Since our model can generate persistent but trend-reverting comovements in output, consumption, investment, and employment under sunspots shocks, it automatically explains the forecastable comovements puzzle raised by Rotemberg and Woodford [45] against RBC models.[12]

[9] See Bils [7] for detailed discussions.

[10] Also see Chatterjee et al. [10], among others.

[11] The link between counter-cyclical markup and output fluctuations has also been exploited recently by Jaimovich [32] using a DSGE imperfect competition model. Jaimovich shows firm entry and exit are an important magnification mechanism for technology shocks and can also lead to local indeterminacy of the steady state under imperfect competition. Since firm entry and exit can change the competitiveness among monopolistic firms, Jaimovich is able to explain the counter-cyclical markup and procyclical productivity simultaneously. Our approach differs from Jaimovich because indeterminacy in Jaimovich's model is local, instead of global, hence sunspots equilibria in his model are sensitive to structural parameters in the utility function and intermediate-good production technologies, which jointly determine the eigenvalues of the model near the steady state. Further, our model can generate hump-shaped impulse responses under sunspots shocks whereas Jaimovich's model can not, making his model incapable of addressing the criticism raised by Schmitt-Grohe [25] against sunspots-driven business cycle models. Recently, Dos Santas Ferreira and Dufourt [14] also provide a business cycle model in which endogenous markup fluctuations are the main driving force. In their model, sunspots-driven fluctuations occur due to the indeterminacy of the number of firms in a dynamic entry model with imperfect Cournotian competition and increasing returns to scale. Similar to our model, they do not rely on the sink property of the equilibrium to generate indeterminacy. However, the structure of their model is fundamentally different from ours. Furthermore, our model can generate hump-shaped impulse responses under AR(1) sunspots shocks while their model cannot.

[12] See Benhabib and Wen [6] for more discussions on this issue.

The rest of the paper is organized as follows. Section 2 presents a highly stylized model to convey the general ideas and to derive the basic results. A key assumption of the stylized model is that firms can switch without cost from one technology to another in order to minimize production costs. Section 3 develops a more general model with switching costs and shows the relationship between the general model and the stylized model. Section 4 utilizes the models to estimate sunspots shocks and Section 5 concludes the paper.

2 A Stylized Model

2.1 Firms

There is a single final good in the economy. The good is produced by using a continuum of intermediate goods. Two similar technologies are available for producing the final good:

$$Y = \left[\int_0^1 Y(i)^{\frac{\sigma_k - 1}{\sigma_k}} di \right]^{\frac{\sigma_k}{\sigma_k - 1}}, \qquad k = \{h, l\}, \sigma_h > \sigma_l; \tag{1}$$

where σ_k is the elasticity of substitution among intermediate goods, h denotes high elasticity, and l denotes low elasticity.[13] Let $P(i)$ denotes the price of intermediate good, $Y(i)$. The minimum cost for producing one unit of the final good by using technology $k = \{h, l\}$ is given by

$$c_k = \left[\int_0^1 P(i)^{1-\sigma_k} di \right]^{\frac{1}{1-\sigma_k}}. \tag{2}$$

We normalize the price of the final good to be one. Clearly, if $c_h < c_l$ (or $c_l < c_h$), then only the h-technology (or the l-technology) will be used. If $c_h = c_l$, then both technologies can be used in arbitrary proportion. Under imperfect competition, this latter property of the model can be utilized to generate equilibrium indeterminacy in the marginal cost of production and composition of aggregate output.[14]

The demand function for intermediate good i in period t is given by

$$Y_t(i) = \left[\theta_t P_t(i)^{-\sigma_h} + (1 - \theta_t) P_t(i)^{-\sigma_l} \right] Y_t \tag{3}$$

[13] The model can be easily extended to a continuum of technologies. The assumption that there are more than one aggregation technologies available to produce the same good is based on the intuition that the same set of inputs can be combined in different ways to yield the same kind of output. In other words, there are more than one way to skin a cat. For example, Ferguson [18] found substantial variations in the elasticity of substitution between factor inputs across similar 4-digit industries, suggesting that similar goods can be produced using similar inputs with different elasticity of substitution. Our assumption is analogous to this empirical finding.

[14] It will be shown shortly the equilibrium is unique under perfect competition. Indeterminacy of output arises only under imperfect competition.

where θ_t is the fraction of the final good produced using the high-elasticity (h) technology in period t. Clearly,

$$\theta_t = \begin{cases} 1 & \text{if } c_{h,t} < c_{l,t} \\ (0,1) & \text{if } c_{h,t} = c_{l,t} \\ 0 & \text{if } c_{h,t} > c_{l,t} \end{cases}. \tag{4}$$

The production technology for intermediate goods is given by the Cobb-Douglas function,

$$Y(i) = \left[e(i)K(i)\right]^{\alpha} N(i)^{1-\alpha}, \tag{5}$$

where $e(i) \in [0,1]$ is the rate of capital utilization for firm i. Following Greenwood et al. [23], we assume that a higher rate of capital utilization leads to a higher rate of capital depreciation:

$$\delta(i) = \frac{\delta_0}{v} e(i)^v, \qquad v > 1. \tag{6}$$

This feature is not important for indeterminacy but is needed for generating strongly procyclical productivity.

Intermediate firms have monopoly power over the supply of intermediate goods, but are competitive in the factor markets for capital and labor. Let W denote the real wage of labor, and $R + \delta(i)$ denote the user's cost of capital for firm i (where R is the real interest rate). Cost minimization,

$$\min \left\{ WN(i) + (R + \delta(i))K(i) \right\}, \tag{7}$$

subject to

$$\left[e(i)K(i)\right]^{\alpha} N(i)^{1-\alpha} \geq Y(i), \tag{8}$$

implies the following demand functions for capital service and labor inputs:

$$W = (1-\alpha)\phi(i)\frac{Y(i)}{N(i)}, \tag{9}$$

$$R + \delta(i) = \alpha\phi(i)\frac{Y(i)}{K(i)}, \tag{10}$$

$$e(i)^v = \alpha\phi(i)\frac{Y(i)}{K(i)}; \tag{11}$$

where $\phi(i)$ denotes the Lagrangian multiplier for the constraint (8) and is also the real marginal cost of firm i. The above equations imply $e(i)^v = \frac{v}{v-1}R$, suggesting that all firms will choose the same rate of capital utilization (hence the rate of capital depreciation is the same across firms).

Equation (11) can be used to derive a reduced-form production function at the optimal level of capital utilization:

$$Y(i) = (\alpha\phi(i))^{\frac{\alpha}{v-\alpha}} K(i)^{\alpha\frac{v-1}{v-\alpha}} N(i)^{(1-\alpha)\frac{v}{v-\alpha}}. \tag{12}$$

Combining the factor demand functions for labor and capital with the reduced-form production function gives

$$\phi(i) = \left(\frac{W}{1-\alpha}\right)^{1-\alpha} \left(\frac{R}{\alpha}\frac{v/(v-1)}{e}\right)^{\alpha}. \tag{13}$$

This implies that the marginal cost is also the same across all firms.

Each monopolist firm chooses prices to maximize profits by solving

$$\max\,(P(i)-\phi)\,Y(i) \tag{14}$$

subject to (3). We assume that intermediate good firms have perfect information for aggregate demand when setting prices.[15] The optimal monopolistic price is determined by the relationship

$$Y(i) = (P(i)-\phi)\left[\sigma_h \theta P(i)^{-\sigma_h-1} + \sigma_l(1-\theta)P(i)^{-\sigma_l-1}\right] Y. \tag{15}$$

2.2 Households

There is a representative household in the economy, whose objective is to choose the paths of consumption $\{c_t\}$, capital stock $\{k_{t+1}\}$, and supply of labor hours $\{n_t\}$ to solve

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t) \tag{16}$$

subject to

$$c_t + k_{t+1} = W_t n_t + (1+R_t)\,k_t + D_t, \tag{17}$$

where $1-n$ denotes leisure time, and D denotes real profits distributed from firms. The period utility function, $u(c, 1-n)$, is concave in consumption and leisure. The first order conditions for the household are given by $W_t = -u_n/u_c$ and $u_c(c_t, n_t) = \beta E_t\left[u_c(c_{t+1}, n_{t+1})\,(1+R_{t+1})\right]$.

2.3 General Equilibrium

A general equilibrium is defined as the set of prices and quantities, $\{W, R, \phi, P(i), Y(i),\ N(i), K(i), c, k, n, e\}$, such that firms maximize profits and the household maximizes utility subject to

[15] In the case of imperfect information, firms can only set prices based on expected aggregate demand. This can lead to an additional source of sunspot equilibria in the model. See Wang and Wen [50] for the analysis of sunspots equilibria under imperfect information.

their respective technological and budget constraints, and all markets clear:

$$n_t = N_t \equiv \int N_t(i)di \tag{18}$$

$$k_t = K_t \equiv \int K_t(i)di \tag{19}$$

$$c_t + K_{t+1} - (1-\delta_t)K_t = Y_t. \tag{20}$$

We analyze symmetric equilibria where all intermediate good firms choose the same prices and produce the same equilibrium quantities.

In a symmetric equilibrium, we have $P(i) = 1$ and $Y(i) = Y$ for all i. Equation (15) then implies

$$\phi_t = \frac{(\sigma_h - 1)\,\theta_t + (\sigma_l - 1)\,(1-\theta_t)}{\sigma_h\theta_t + \sigma_l(1-\theta_t)}. \tag{21}$$

It also follows that $c_h = c_l = 1$ for all t. Consequently, θ_t is indeterminate. This indeterminacy of the composition of the final good provides the basis for expectations-driven fluctuations in the model. Define $\sigma_t \equiv \sigma_h\theta_t + \sigma_l(1-\theta_t)$. Equation (21) can then be written as

$$\phi_t = \frac{\sigma_t - 1}{\sigma_t}. \tag{22}$$

Thus, a shift in the composition of aggregate output (e.g., an increase in θ) translates into a higher aggregate elasticity of substitution among intermediate goods (σ_t), which in turn translates into a higher marginal cost (or a lower markup) and a higher level of output due to more intensive competition among the firms.

Notice that if competition is perfect, then prices must equal marginal costs. Hence in equilibrium the marginal cost is given by $\phi = 1$, which is independent of θ_t. In this case, changes in θ_t have no effect on the economic activities of the model. In other words, sunspots do not matter under perfect competition.

Also notice that the model is observationally equivalent to models of cost-push shocks in which $\sigma_h = \sigma_l = \sigma$ and the elasticity parameter (σ) is assumed to be an exogenous random variable. A large body of the existing sticky-price literature shows that cost-push shocks induced by random changes in σ are important for monetary policy analysis and for explaining the business cycle (see, e.g., Clarida, Gali, and Gertler [11], Ireland [31], and Smets and Wouters [47]). However, this literature has not yet provided justifications as to why the technology parameter σ can change randomly. Our model can be viewed as providing an interpretation for this *ad hoc* assumption of random changes in σ.

2.4 Equilibrium Dynamics

In general equilibrium, the first order conditions of the model can be summarized by the following equations:

$$\phi_t = \frac{\sigma_t - 1}{\sigma_t} \tag{23}$$

$$W_t = (1-\alpha)\phi_t \frac{Y_t}{N_t} \tag{24}$$

$$\frac{v}{v-1} R_t = \alpha\phi_t \frac{Y_t}{K_t} \tag{25}$$

$$\frac{1}{v} e_t^v = \frac{1}{v-1} R_t \tag{26}$$

$$Y_t = (\alpha\phi_t)^{\frac{\alpha}{v-\alpha}} K_t{}^{\alpha\frac{v-1}{v-\alpha}} N_t{}^{(1-\alpha)\frac{v}{v-\alpha}}. \tag{27}$$

$$W_t = -\frac{u_n(c_t, N_t)}{u_c(c_t, N_t)} \tag{28}$$

$$u_c(c_t, N_t) = \beta E_t \left[u_c(c_{t+1}, N_{t+1})(R_{t+1} + 1) \right] \tag{29}$$

$$c_t + K_{t+1} - (1 - \frac{1}{v} e_t^v) K_t = Y_t, \tag{30}$$

plus a standard transversality condition, $\lim_{T\to\infty} \beta^T u_c(c_T, N_T) K_{T+1} = 0$. These eight equations plus the transversality condition together determine the paths of eight aggregate variables $\{\phi_t, W_t, R_t, Y_t, N_t, e_t, c_t, K_{t+1}\}_{t=0}^{\infty}$ in general equilibrium, given the initial value of the capital stock, K_0.

Notice that the marginal cost (ϕ_t) can be viewed as an exogenous forcing variable in the model since it is determined entirely by the sunspots variable σ_t in Equation (23). Given the sequence of $\{\phi_t\}_{t=0}^{\infty}$, the rest of the equations (24)-(30) are very similar to a standard neoclassical growth model or RBC model. Thus, equilibrium indeterminacy in our model hinges only on the composition of aggregate output, θ_t, not on any other structural parameters associated with the utility functions or the production technologies. Also, indeterminacy in our model is global, instead of local, hence it is independent of the topological properties of the steady state. This is in sharp contrast to the existing indeterminacy literature along the line of Benhabib and Farmer [3] and Gali [19].

From the reduced-form production function, it is worth noting that the marginal cost acts like a technology shock in a standard RBC model. Thus our model is similar to standard RBC models driven by technology shocks except that in our model, not only is productivity procyclical, but so is the marginal cost. In addition, our model yields better predictions regarding the volatility of employment relative to output than a standard RBC model driven by technology shocks. The importance of explaining employment volatility is emphasized by Prescott [41] and Hansen [28].

The model can be solved by log-linearization around a steady state. Let the period-utility function be given by

$$u(c,n) = \log(c) + a\log(1-n) \tag{31}$$

and the time period be a quarter. The model has a continuum of steady states determined by the value of the marginal cost, $\phi = 1 - \frac{1}{\sigma} = 1 - \frac{1}{\sigma_h\theta + \sigma_l(1-\theta)}$, where $\theta \in [0,1]$. For each possible value of θ, there is a corresponding value of ϕ and hence a corresponding steady state. We linearize the model around $\theta = \frac{1}{2}$ and set $\sigma_h = 12$ and $\sigma_l = 8$ so that the average markup is 10% in the steady state. The results are not sensitive to the choice of these parameter values. As a benchmark, we assume that the log of θ_t follows a stationary $AR(1)$ process, $\log\theta_t = \rho_\theta \log\theta_{t-1} + \varepsilon_t$, where $\rho_\theta = 0.9$.[16] Following the existing RBC literature, let the time discount factor $\beta = 0.99$, the capital's income share $\alpha = 0.35$, the steady-state rate of capital depreciation $\delta = 0.025$ and the constant δ_0 is chosen so that the steady-state capital utilization rate equals that in the data (which imply $v \simeq 1.4$). The steady state hours worked per week is 35 (implying $\bar{n} \simeq 0.2$). The quantitative results are robust to small changes in these parameter values.

In what follows, the total factor productivity (TFP) is measured by the Solow residual, $Y/(K^\alpha N^{1-\alpha})$. We adopt this conventional measure because in the absence of technology shocks, the true measure of the TFP under variable capacity utilization, $Y/(e^\alpha K^\alpha N^{1-\alpha})$, is constant in our model.

Figure 1 shows the impulse responses of output, consumption, investment, hours worked, markup, and the conventional measure of total factor productivity (TFP) to a sunspot shock to the composition of aggregate output (θ). The size of the shock is normalized so that on impact the marginal cost increases by one percent. It is evident from Figure 1 that the model can generate persistent movements in output, consumption, investment, employment, and total factor productivity. Except for the markup, all of these variables are procyclical, consistent with the data. On impact, consumption increases less than output, while investment increases more than output, consistent with the typical volatility orders among consumption, output, and investment. Also, the dynamic path of consumption indicates a strong smoothing motive, similar to that in a RBC model driven by $AR(1)$ technology shocks. Employment is nearly as volatile as output, and the conventional measure of the total factor productivity (TFP) is strongly procyclical. In addition, the markup (measured as price over marginal cost, ϕ^{-1}) is strongly counter-cyclical, consistent with

[16] Notice that there are no restrictions on the distribution of θ_t in the model. We choose an $AR(1)$ process so as to compare it with an $AR(1)$ technology shock process typically assumed in the RBC literature. Hump-shaped impulse responses for output can be generated in the model if an AR(2) process is assumed.

the data (see, e.g., Bils [7], and Rotemberg and Woodford [46]). Thus, qualitatively speaking, the model explains the U.S. data well and is comfortably comparable to a standard RBC model driven by technology shocks.[17]

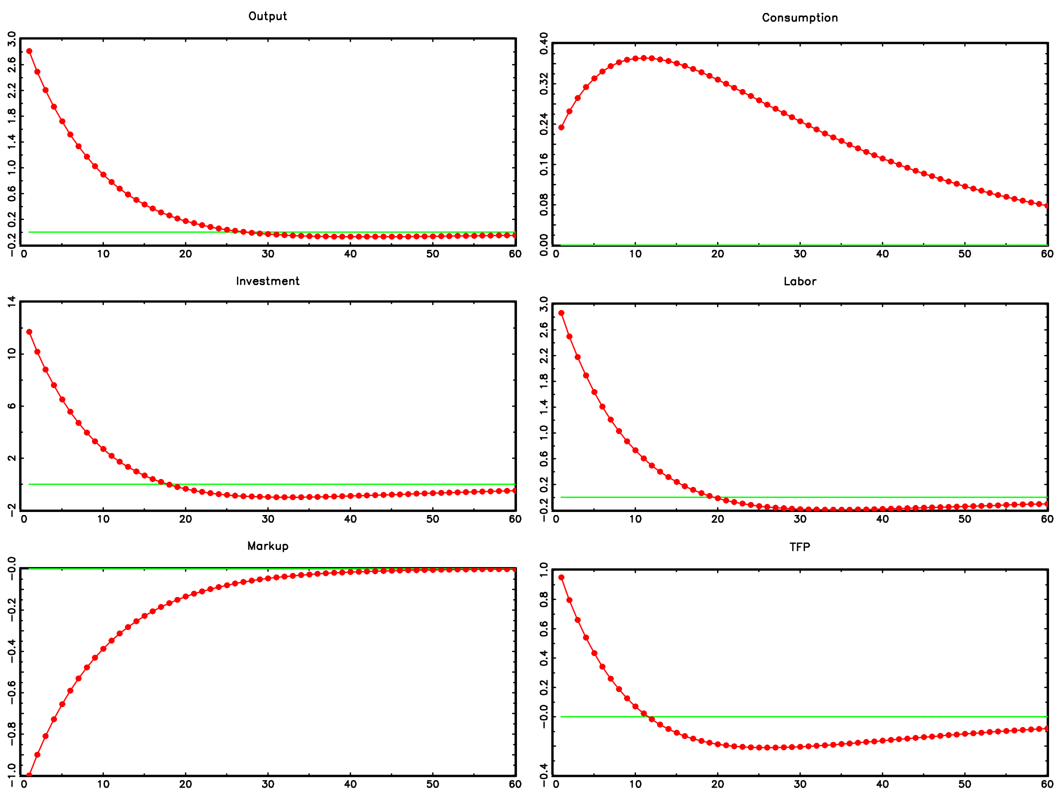

Figure 1. Impulse Responses to a Sunspot Shock.

Table 1 reports the standard set of moments of business cycles often cited in the literature. The U.S. data include real GDP (Y), real total consumption (C), real total investment (I), total aggregate hours worked per week (N), and a measure of the markup. The capital stock (K) is deduced from investment using the definition, $K_{t+1} = (1 - \delta_t)K_t + I_t$, where $\delta_t = \frac{\delta_0}{v} e_t^v$ with a steady-state value $\bar{\delta} = 0.025$.[18] The initial capital stock, K_0, is chosen such that the constructed capital stock series has a linear growth trend in log terms, which implies a stationary output-to-

[17] The fact that imperfectly competitive models driven by sunspots shocks can explain procyclical TFP and countercyclical markup has been pointed out by Jaimovich [32].

[18] The national income data are from the Bureau of Economic Analysis (1947:1 - 2005:4). The employment data are from the Bureau of Labor Statistics (1947:01 - 2005:12), and the data on capacity utilization is from the Federal Reserve Bank of St. Louis. Since the data on capacity utilization starts in 1967:01, all other data are truncated to the same starting period. Quarterly data are converted from monthly data using the third month of each quarter. Following Rotemberg and Woodford [46], the measure of markup is the ratio of total labor income to nominal GDP, where labor's income is the sum of compensation of employees and proprietors' income. A common convention since Johnson [33] has been to allocate two-thirds of the income of proprietors to labor income, and one-third to capital income. The difference this makes is small for the sample rage we consider. See Krueger [37] for more discussions on this issue.

capital ratio for the entire sample.[19] The total hours worked (N) is constructed by multiplying total non-farm employment and the average hours worked per week. All data are logged and HP filtered when computing Table 1.

Table 1. Standard Moments

Variable (x)		y	c	i	n	-ϕ	TFP
U.S. Data	σ_x/σ_y	1	0.52	3.33	1.01	0.32	0.54
	$cor(x_t, y_t)$	1	0.83	0.92	0.88	-0.16	0.75
	$cor(x_t, x_{t-1})$	0.87	0.90	0.92	0.92	0.70	0.75
RBC	σ_x/σ_y	1	0.37	3.48	0.67	0.00	0.51
	$cor(x_t, y_t)$	1	0.59	0.97	0.94	0.00	0.95
	$cor(x_t, x_{t-1})$	0.88	0.99	0.86	0.86	0.00	0.86
Model 1	σ_x/σ_y	1	0.33	3.93	0.97	0.39	0.33
	$cor(x_t, y_t)$	1	0.56	0.97	0.98	-0.98	0.76
	$cor(x_t, x_{t-1})$	0.88	0.99	0.86	0.87	0.90	0.87
Model 2	σ_x/σ_y	1	0.38	4.02	0.95	0.41	0.34
	$cor(x_t, y_t)$	1	0.61	0.97	0.98	-0.98	0.71
	$cor(x_t, x_{t-1})$	0.96	0.99	0.96	0.96	0.97	0.96

The performance of the stylized model (labeled Model 1 in the third panel) relative to a standard real-business-cycle model with variable capital utilization (labeled RBC in the second panel) in explaining the U.S. data can be summarized as follows.[20] In both models, the variance of the shocks can be chosen so that the predicted variance of output matches the data exactly. Hence, with respect to output, the only thing that matters is the persistence (autocorrelation), which both models predict well. With respect to consumption, the two models yield similar predictions and both models underpredict its volatility and correlation with output. With respect to investment, both models can explain its excess volatility relative to output, with the RBC model closer to the data. With respect to labor's volatility, our model is very close to the data whereas the RBC model substantially under predicts it. With respect to markup, the RBC model is obviously inconsistent with the data since the markup is zero in the RBC model; where as our model does a reasonably good job explaining its volatility and persistence, although it overpredicts its countercyclicality. Finally, with respect to the TFP, our model and the RBC model generate similar predictions that are qualitatively consistent with the data, with our model underpredicting the volatility of TFP

[19] Experiments with different values of K_0 show that setting $K_{1967:1} = 10000 + I_{1967:1}$ gives a very good result in terms of balanced growth.

[20] RBC in the table denotes a standard RBC model with variable capacity utilization (see, e.g., King and Rebelo [36]) with the same parameter values as in our model (i.e., $\alpha = 0.35, \delta = 0.025, v = 1.4, \beta = 0.99, \gamma = 0.$). The $AR(1)$ coefficient for technology shocks is 0.9. Note that the marginal cost is constant ($\phi_t = 1$) in the RBC model.

relative to output by a larger margin than the RBC model. However, the RBC model overpredicts the correlation between TFP and output by a large margin whereas our model matches the data almost exactly. Overall, it is fair to say that our model is at least as good if not better than the RBC model in explaining the U.S. business cycle.

It is well known that in the U.S. economy hours worked are as volatile as output and labor productivity is strongly procyclical. The RBC theory relies on technology shocks to explain the procyclical labor productivity, consequently overpredicting its correlation with output and failing to explain the highly volatile hours worked relative to output. This problem is resolved in our model under sunspots shocks to marginal costs. Notice that under technology shocks our model yields almost identical equilibrium dynamics to those of the RBC model. Hence the reason our model can generate larger volatility of hours relative to output than the RBC model lies completely in the source of shocks. To understand this, consider output without capacity utilization, $Y = K^{\alpha}N^{1-\alpha}$. Notice the marginal cost, ϕ, does not appear in the production function but does appear in the aggregate labor demand function in Equation (24), $W = (1-\alpha)\phi\frac{Y}{N}$. The fact that the marginal cost is directly related to sunspots shocks and that it appears in the labor demand while not affecting directly the production function implies that almost all the movements in output are due to direct movements in hours worked and not to productivity changes as it would be in the case of technology shocks. Hence, output is less correlated with productivity and hours worked are more volatile relative to output under sunspots shocks than under technology shocks.[21]

However, similar to standard RBC models, the stylized model is not able to generate hump-shaped impulse responses unless sunspots shocks are themselves hump-shaped. This suggests that the model is not able to address the criticisms raised by Cogley and Nason [12], Rotemberg and Woodford [45], and Schmitt-Grohe [25]. In addition, the assumption that firms can switch between different technologies without cost is not realistic. In the next section we present a more general model to overcome these shortcomings.

3 A Dynamic Entry Model

A firm's ability to switch costlessly between different technologies in each period is a key to generating global indeterminacy in the stylized model. In reality, switching technologies may be costly. This section considers a dynamic entry model with switching costs. In the model, firms can choose which technology to adopt for production upon entry, but after entry firms opt to stick to the

[21] When the leisure function is linear in hours worked (Hansen's [28] indivisible labor), our model predicts $\frac{\sigma_n}{\sigma_y} = 1.05$ and the RBC model predicts $\frac{\sigma_n}{\sigma_y} = 0.89$, which is still lower than the data. King and Rebelo [36] show that the RBC model can generate labor volatility similar to output under indivisible labor and capacity utilization. Their results, however, are obtained under the assumption that the depreciation elasticity v is an independent parameter and is very close to one (i.e., $v = 1.01$). But v is not a free parameter and is determined by the steady state relationship, $v = \frac{1-\beta(1-\delta)}{\beta\delta} + 1$. Suppose $\beta = 0.99$ and $\delta = 0.025$, we have $v = 1.4$.

chosen technology forever due to costs of switching technologies. The source of sunspots in this model lies in the fraction of new entrants choosing different technologies. Clearly, this dynamic entry model is reduced to the stylized model if firms can survive for only one period after entry and must reenter the market in every subsequent period. Hence, the assumption of zero switching costs in the stylized model is innocuous. Most importantly, when firms can survive for more than one period after entry, the dynamic responses of aggregate output to sunspots shocks can become hump-shaped. The dynamic entry model builds on the model of Ghironi and Melitz [22].

3.1 Firms

A final good is produced from intermediate goods by the technology

$$Y_t = M_t^{\frac{1}{1-\varepsilon}} \left[\int_0^{M_t} Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{32}$$

where M_t is the number of intermediate-good firms in period t and the term $M^{\frac{1}{1-\varepsilon}}$ is a normalization factor that eliminates increasing returns to specialization.[22] The demand function for intermediate good i is $Y(i) = \frac{1}{M} P(i)^{-\epsilon} Y$ and the price index for the final good is $P = M^{\frac{1}{1-\epsilon}} \left(\int_0^M P(i)^{1-\epsilon} di \right)^{\frac{1}{\epsilon-1}}$. The final good price is normalized to one, $P = 1$.

All intermediate goods are produced using the same set of materials $X(j), j \in [0,1]$. There are two alternative technologies available for producing intermediate goods:

$$Y(i) = \left[\int_0^1 X_i(j)^{\frac{\sigma_k - 1}{\sigma_k}} dj \right]^{\frac{\sigma_k}{\sigma_k - 1}}, \qquad k = \{h, l\}, \sigma_h > \sigma_l; \tag{33}$$

where σ_k is the elasticity of substitution among materials, h denotes high elasticity, and l denotes low elasticity. Let $P_x(j)$ denote the price of material good $X(j)$. The minimum cost for producing one unit of the intermediate good by using technology $k = \{h, l\}$ is given by $c_k(i) = \left[\int_0^1 P_x(j)^{1-\sigma_k} dj \right]^{\frac{1}{1-\sigma_k}}$. Since all intermediate good firms use the same set of material, we have $c_k(i) = c_k$ for all $i \in [0, M_t]$ and the aggregate demand for material good j by all intermediate good firms in period t is given by

$$X(j) = P_x(j)^{-\sigma_h} \int_0^{\theta M} \frac{Y(i)}{c_h^{-\sigma_h}} di + P_x(j)^{-\sigma_l} \int_{\theta M}^{M} \frac{Y(i)}{c_l^{-\sigma_l}} di, \tag{34}$$

[22] This normalization is in order to show that our results of global indeterminacy do not depend on non-convexities such as increasing returns to scale. However, our results hold regardless of this normalization.

where θ_t is the fraction of the intermediate good firms choosing the high-elasticity (h) technology in period t, which includes both new entrants and incumbent firms.[23]

In each period, there are infinite potential entrants of intermediate good firms. These entrants are forward looking, and correctly anticipate their future expected profits in every period (the pre-entry profit is equal to post-entry average profit) as well as the probability ω (in every period) of incurring the exit-inducing shock. The exogenous exit shock occurs at the end of each period (after production and entry).[24] An entrant can decide which technology to adopt for production upon entry. Since in equilibrium the unit cost of producing intermediate goods is the same for both technologies, once a technology is chosen upon entry, incumbent firms will have no incentive to switch if there are switching cost, which we assume to be the case. In order to enter, each intermediate good firm must also pay a fixed cost Φ in terms of final good, implying the representative household (as owner of firms) pays for the entry cost.[25] The value of an intermediate good firm i is determined by the discounted stream of profits:

$$V_t(i) = E_t \sum_{s=0}^{\infty} \beta^s (1-\omega)^s \frac{u'(c_{t+s})}{u'(c_t)} \pi(i)_{t+s}, \tag{35}$$

where β is the household's time discounting factor, $u'(c)$ is the household's marginal utility of consumption, $1-\omega$ is the probability of survival after production (which is uniform across firms and time), and π is the period profit of the firm. Free entry implies that the value of the last firm who enters must equal the entry cost: $V_t = \Phi$.[26] Notice that if the probability of dying out equals one, the free entry condition becomes $V(i) = \pi(i) = \Phi$. Profit maximization implies a constant markup, $P(i) = \frac{\varepsilon}{\varepsilon-1} c_k$. The period profit is therefore $\pi(i) = (P(i) - c_k) Y(i) = \frac{1}{\varepsilon} Y(i)$.

Suppose the number of new entrants in period t is m_t, and among them a fraction s_t adopt the h-technology and another fraction $1-s_t$ adopt the l-technology. Suppose by the end of period $t-1$ there is a measure of $M_{h,t-1}$ firms using the h type technology and $M_{l,t-1}$ using the l type technology, we then have $\theta_t = \frac{M_{h,t}}{M_{h,t}+M_{l,t}}$. The laws of motion for the number of firms are given respectively by

[23] The demand function is derived from $\min \int_0^1 P_x(j) X(j) dj$ subject to $\left[\int_0^1 X(j)^{\frac{\sigma_k-1}{\sigma_k}} dj\right]^{\frac{\sigma_k}{\sigma_k-1}} \geq Y(i)$. The Lagrangian multiplier for the constraint is $c_k(i)$.

[24] Notice that the period profit of a monopoly firm is always positive, hence firms will never want to exit after entry. Thus, in order to have a stationary time series of the number of firms M_t in equilibrium, we follow Ghironi and Melitz [22] by assuming each incumbent firm has a constant probability ω of dying out after production in each period.

[25] It makes no difference for model dynamics if the fixed cost is paid in terms of intermediate output by firms. Using the final good to pay for the fixed cost, however, can simplify the relationship between aggregate labor share and the sunspots shocks, which makes it easier to estimate the sunspots shocks from data.

[26] Following Ghironi and Melitz [22], we assume that the variance of the shocks are small so that there is always a positive measure of new entrants to ensure this equality to hold.

$$M_{h,t} = (1-\omega)M_{h,t-1} + s_t m_t, \tag{36}$$

$$M_{l,t} = (1-\omega)M_{l,t-1} + (1-s_t)\, m_t. \tag{37}$$

The production technology for material goods is given by the Cobb-Douglas function,

$$X(i) = [e(i)K(i)]^{\alpha}\, N(i)^{1-\alpha}, \tag{38}$$

where $e(i) \in [0,1]$ is the rate of capital utilization for firm i and it is related to capital depreciation according to $\delta(i) = \frac{\delta_0}{v} e(i)^v, v > 1$. Material good firms also have monopoly power over the supply of the material goods, but are competitive in the factor markets for capital and labor. Let W denote the real wage of labor, and $R + \delta(i)$ denote the user's cost of capital for firm i (where R is the real interest rate). Cost minimization by the material good firms implies the following demand functions for inputs: $W = (1-\alpha)\phi(i)\frac{X(i)}{N(i)}$, $R + \delta(i) = \alpha\phi(i)\frac{X(i)}{K(i)}$, $e(i)^v = \alpha\phi(i)\frac{X(i)}{K(i)}$; where $\phi(i)$ denotes the material good producer's marginal cost. The above equations imply $e(i)^v = \frac{v}{v-1}R$, suggesting that all material good firms will choose the same rates of capital utilization (hence the rate of capital depreciation is the same across firms). The reduced-form production function at the optimal level of capital utilization is given by $X(i) = (\alpha\phi(i))^{\frac{\alpha}{v-\alpha}}\, K(i)^{\alpha\frac{v-1}{v-\alpha}} N(i)^{(1-\alpha)\frac{v}{v-\alpha}}$. Combining the factor demand functions for labor and capital with the reduced-form production function gives $\phi(i) = \left(\frac{W}{1-\alpha}\right)^{1-\alpha} \left(\frac{R}{\alpha}\frac{v/(v-1)}{e}\right)^{\alpha}$. This implies that the marginal cost is also the same across all material good producers.

Each material good firm chooses prices to maximize profits by solving

$$\max\, (P_x(i) - \phi)\, X(i) \tag{39}$$

subject to (33). The optimal monopolistic price is determined by the relationship

$$X(j) = [P_x(j) - \phi] \left[\sigma_h P_x(j)^{-\sigma_h - 1} \int_0^{\theta M} \frac{Y(i)}{c_h^{-\sigma_h}} di + \sigma_l P_x(j)^{-\sigma_l - 1} \int_{\theta M}^{M} \frac{Y(i)}{c_l^{-\sigma_l}} di\right]. \tag{40}$$

Household: The household's problem is identical to the stylized model. Hence the first order conditions are the same: $W_t = -u_n/u_c$ and $u_c(c_t, n_t) = \beta E_t\,[u_c(c_{t+1}, n_{t+1})\,(1 + R_{t+1})]$.

3.2 General Equilibrium

A general equilibrium is defined as the set of prices and quantities, $\{W, R, \phi, P(i), P_x(j), Y(i), X(j), N(j), K(j), e, M, c, k, n\}$, such that firms maximize profits and the household maximizes utility subject to their respective technological and budget constraints, and all markets clear:

$$n_t = N_t \equiv \int_0^1 N_t(j)dj \tag{41}$$

$$k_t = K_t \equiv \int_0^1 K_t(j)dj \tag{42}$$

$$X(j) = \int_0^{M_t} X_i(j)di \quad \text{for all } j \tag{43}$$

$$c_t + K_{t+1} - (1-\delta_t)K_t + m_t\Phi = Y_t. \tag{44}$$

We analyze symmetric equilibria where all intermediate good firms choose the same prices and produce the same equilibrium quantities and all material good firms choose the same prices and produce the same equilibrium quantities.

In a symmetric equilibrium, $P(i) = 1$, $P_x(j) = c_h = c_l = \frac{\varepsilon-1}{\varepsilon}$ and $X(j) = MY(i) = Y$ for all i and j. Equation (40) then implies

$$\phi_t = \frac{\varepsilon-1}{\varepsilon}\frac{(\sigma_h-1)\,\theta_t + (\sigma_l-1)\,(1-\theta_t)}{\sigma_h\theta_t + \sigma_l(1-\theta_t)}. \tag{45}$$

This equation is equivalent to Equation (21) if $\varepsilon \to \infty$. However, θ_t is no longer a sunspot variable here, instead s_t (the fraction of new entrants which adopt the h-type technology) is the source of sunspots. This is so because in the dynamic entry model new entrants are indifferent between the h-type and the l-type technology regardless of switching costs.

3.3 Equilibrium Dynamics

In a symmetric general equilibrium, the first order conditions of the model can be summarized by the following equations:

$$\Phi = \pi_t + \beta(1-\omega)E_t\frac{u_{c,t+1}}{u_{c,t}}\Phi \tag{46}$$

$$\pi_t = \frac{1}{\varepsilon}\frac{Y_t}{M_t} \tag{47}$$

$$\phi_t = \frac{\varepsilon-1}{\varepsilon}\frac{(\sigma_h-1)\,\theta_t + (\sigma_l-1)\,(1-\theta_t)}{\sigma_h\theta_t + \sigma_l(1-\theta_t)} \tag{48}$$

$$W_t = (1-\alpha)\phi_t\frac{Y_t}{N_t} \tag{49}$$

$$\frac{v}{v-1}R_t = \alpha\phi_t\frac{Y_t}{K_t} \tag{50}$$

$$\frac{1}{v}e_t^v = \frac{1}{v-1}R_t \tag{51}$$

$$Y_t = (\alpha\phi_t)^{\frac{\alpha}{v-\alpha}} K_t{}^{\alpha\frac{v-1}{v-\alpha}} N_t{}^{(1-\alpha)\frac{v}{v-\alpha}}. \tag{52}$$

$$W_t = -\frac{u_n(c_t, N_t)}{u_c(c_t, N_t)} \tag{53}$$

$$u_c(c_t, N_t) = \beta E_t\left[u_c(c_{t+1}, N_{t+1})(R_{t+1}+1)\right] \tag{54}$$

$$c_t + K_{t+1} - (1 - \frac{1}{v}e_t^v)K_t + m_t\Phi = Y_t \tag{55}$$

$$M_t = (1-\omega)M_{t-1} + m_t \tag{56}$$

$$\theta_t M_t = (1-\omega)\theta_{t-1}M_{t-1} + s_t m_t; \tag{57}$$

where the first equation is a recursive form of the value function in (35) after substituting out V_t by the zero profit condition for new entry, $V_t = \Phi$. These twelve equations plus standard transversality condition together determine the equilibrium paths of twelve aggregate variables $\{\pi_t, \phi_t, W_t, R_t,\ Y_t, N_t,\ e_t, c_t, K_{t+1}, M_t, \theta_t, m_t\}_{t=0}^{\infty}$ in general equilibrium, given the initial values of the capital stock, K_0, the total number of firms, M_0, the fraction of new entrants who adopt the h-type technology, θ_0, and the stochastic process of sunspots $\{s_t\}_{t=1}^{\infty}$.

Notice that the model is reduced to the stylized model if firms can survive for only one period after entry (i.e., the probability of dying out is one, $\omega = 1$) and the intermediate goods market is perfectly competitive (i.e., $\varepsilon = \infty$ and $\pi = \Phi = 0$). In this case, Equations (56) and (57) imply $\theta_t = s_t$, and the remaining eight equations (48)-(55) are thus equivalent to the eight equations (23)-(30) in the stylized model.

Calibration: Similar to the stylized model, the dynamic entry model has a continuum of steady states determined by the value of $s \in [0, 1]$. For each possible value of s, there is a corresponding steady state. We linearize the model around $s = \frac{1}{2}$ and set $\sigma_h = 12$ and $\sigma_l = 8$ so that the average markup is 10% in the steady state, which is also the markup assumed in the intermediate good sector ($\varepsilon = 10$). The results are not sensitive to these parameter values. The period-utility function is the same as before and the parameters $\{\beta, \alpha, \delta, v, \gamma\}$ are set to the same values as in the stylized model. Again, the quantitative results are robust to small changes in these parameter values. In the steady state, the total share of fixed entry costs to output is $\frac{m\Phi}{Y} = \frac{1}{\varepsilon}\frac{\omega}{1-\beta(1-\omega)} \simeq \frac{1}{\varepsilon}$ (since β is close to one), which is close to the steady-state markup in the economy.

The persistence or endogenous propagation mechanism of the model under sunspots shocks depends on the value of ω. As a benchmark, we assume the dying out probability $\omega = \{0.5, 0.1\}$, implying that a firm can survive for two periods on average in the first case and for ten periods in the second case. The persistence of the model increases with the length of the survival periods of the firm.[27] Another source of persistence comes from the serial correlation properties of the sunspots shocks. This is exogenously imposed from outside on the model. As before, we assume sunspots follow a stationary $AR(1)$ process, $\log s_t = \rho_s \log s_{t-1} + \varepsilon_t$, where $\rho_s = 0.9$.[28]

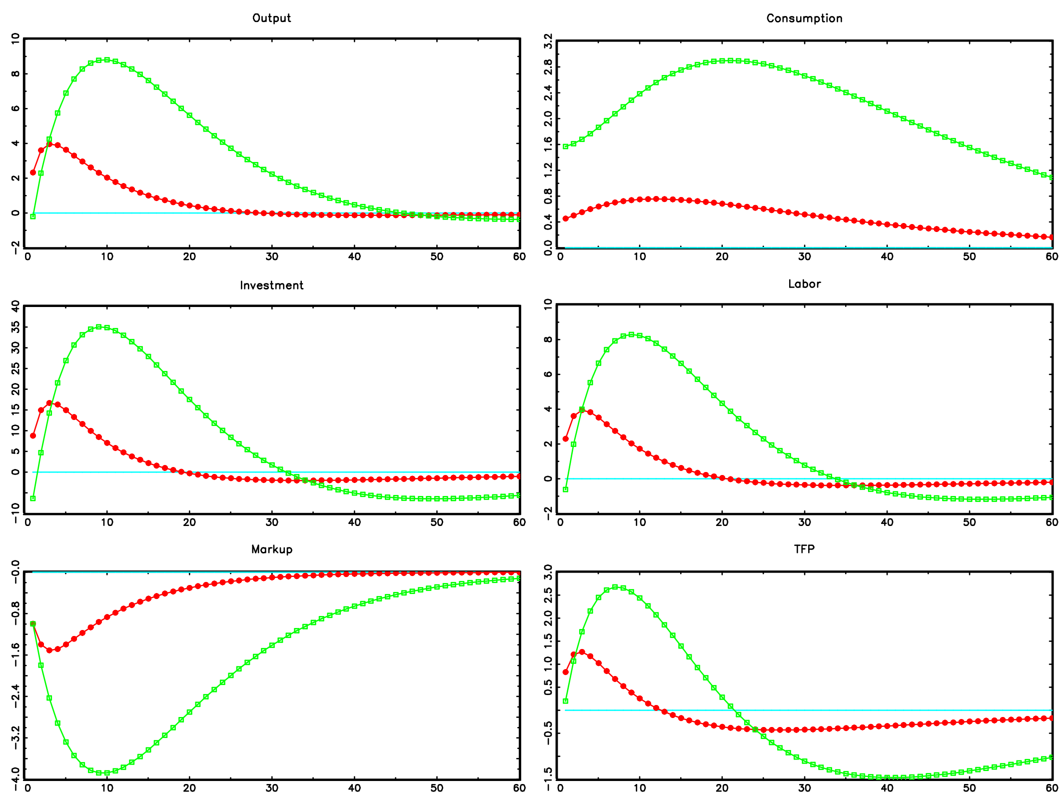

Figure 2. Impulse Responses to a Sunspot Shock ($\bullet\bullet\bullet$ symbols indicate $\omega = 0.5$; $\diamond\diamond\diamond$ symbols indicate $\omega = 0.1$).

Figure 2 shows the impulse responses of output, consumption, investment, hours worked, markup and total factor productivity (TFP) to a sunspot shock. The dynamic responses are highly persistent and hump-shaped, in stark contrast with the stylized model. However, similar to the stylized model, the dynamic entry model continues to explain the basic features of the business cycle well. Except for the markup, all variables are procyclical, consistent with the data.

[27] Ghironi and Melitz [22] assumes $\omega = 0.025$, implying that a firm can survive for 40 periods on average.

[28] A large value of ρ is not necessary for generating hump-shaped impulse responses. The value of ρ can be very small as long as the value of $1 - \omega$ is large enough. The reason is that the dynamics of the marginal cost follow a $AR(2)$ process in the model, with its two roots dictated by only two parameters: $1 - \omega$ and ρ. Any combinations of the two parameters with the property $1 + \rho - \omega = const$ yield similar impulse response patterns. For example, the pair $\{1 - \omega = 0.5, \rho = 0.9\}$ and the pair $\{1 - \omega = 0.9, \rho = 0.5\}$ generate similar hump-shaped impulse responses.

On impact, consumption increases less than output, while investment increases more than output, consistent with the typical volatility orders among consumption, output, and investment. Also, Employment is nearly as volatile as output, and the TFP is strongly procyclical.[29] The bottom panel in Table 1 shows the dynamic entry model (Model 2) generates similar predictions to the stylized model for the selected moments reported therein. Hence the hump-shaped dynamics are obtained without deteriorating the model's performance in other dimensions.

The reason the dynamic entry model can generate a richer propagation mechanism than the stylized model is due to the longer survival period of firms after entry. A longer survival period implies inertia in the composition of incumbent firms adopting different technologies, which in turn implies persistence in the marginal cost. Persistence movement in the marginal cost translates into persistent movements in output and other variables. Interestingly, such persistence does not exist under technology shocks because technology shocks do not affect the composition of firms, hence they have no effects on the propagation mechanism of the model even though they can affect the equilibrium number of firms, M_t. Therefore, the richer propagation mechanism of the dynamic entry model will manifest only under sunspots shocks, not under technology shocks.

As Cogley and Nason [12] point out, the growth rates of GDP and other variables in the U.S. are serially correlated, which standard RBC models driven by $AR(1)$ technology shocks fail to predict. Table 2 shows the dynamic entry model is able to generate serial correlation in growth rate as found in the U.S. economy.[30] Since the model can generate persistent, trend-reverting, hump-shaped positive comovements among output, consumption, investment, and employment under sunspots shocks, it automatically explains the forecastable comovements puzzle raised by Rotemberg and Woodford [45] against RBC models and by Schmitt-Grohe [25] against traditional sunspots models.[31]

[29] Notice the initial negative response of hours worked (as well as investment) to a sunspot shock when $\omega = 0.1$. This negative response is due to the fact that the impulse response of the marginal cost is hump-shaped: the smaller the value of ω, the more sustained the hump. When the initial hump becomes persistent enough, agents anticipate continuous future increases in the marginal cost (ϕ) after the initial shock to the composition of firms. This indicates a higher degree of competition among firms and higher real wages in the future, suggesting greater future returns to both working and investment. Hence, households opt to intertemporally substitute future leisure with current leisure by working less and future consumption with current consumption by saving less. The magnitude of the negative effect on hours worked depends on the elasticity of labor supply (γ) and the parameters controlling the persistence of the initial increases in the marginal cost. For example, the negative effect increases if $\gamma \to 0$ and $\omega \to 0$ or $\rho \to 1$. See the next section for discussions on the dynamic properties of the marginal cost under sunspots shocks.

[30] The parameters for persistence in the dynamic entry model are set at $\omega = 0.5, \rho = 0.9$. The results are similar if $\omega = 0.1, \rho = 0.5$. The model's predictions are qualitatively consistent with the data. The model can better match the data quantitatively if $\{\omega, \rho\}$ are allowed to be estimated using the Method of Moments.

[31] See Benhabib and Wen [6] for more discussions on this issue.

Table 2. Autocorrelation of Growth Rates*

	Δy	Δc	Δi	Δn
U.S. Data	0.27 (0.08)	0.39 (0.08)	0.53 (0.07)	0.59 (0.07)
RBC	-0.06	0.09	-0.07	-0.07
Our Model	0.53	0.18	0.60	0.54

* Numbers in parentheses are standard errors.

4 Estimating Sunspots

Sunspots shocks in our models are related to marginal cost. In the stylized model, sunspots are equivalent to shocks to the marginal cost. In the dynamic entry model, the source of sunspots is the composition of new entrants choosing technology types, namely, s_t. To derive the relationship between sunspots and marginal cost in the dynamic entry mode, consider log-linearized equations around the steady state. Using circumflex to denote variables in the log-linearized system, Equation (48) implies

$$\hat{\phi}_t = \theta \left[\frac{\sigma_h - \sigma_l}{\bar{\sigma} - 1} - \frac{\sigma_h - \sigma_l}{\bar{\sigma}} \right] \hat{\theta}_t, \tag{58}$$

where $\bar{\sigma} = \frac{1}{2}(\sigma_h + \sigma_l)$. Equations (56) and (57) imply

$$\hat{\theta}_t = (1 - \omega)\hat{\theta}_{t-1} + \omega \hat{s}_t. \tag{59}$$

Combining these two equations gives

$$\hat{\phi}_t = (1 - \omega)\hat{\phi}_{t-1} + \eta \hat{s}_t, \tag{60}$$

where $\eta = \frac{\omega}{\frac{\sigma_h - \sigma_l}{\bar{\sigma} - 1} - \frac{\sigma_h - \sigma_l}{\bar{\sigma}}}$. Since η is a scale parameter, we can re-define sunspots as $\eta \hat{s}_t$. Given marginal cost, sunspots shocks can be estimated using Equation (60).

The above equations also reveal why the dynamic entry model has richer propagation mechanism than the stylized model: the change in the marginal cost due to a change in the composition of firms (s_t) is serially autocorrelated because firms chose to stick to their chosen technology and can survive for more than one period after entry. The degree of the serial correlation depends precisely on the probability of survival in each period. Also notice that, since technology shocks do not affect the composition of firms (s_t), they do not generate persistent changes in the marginal cost. Hence, technology shocks are not able to generate hump-shaped output dynamics in the model.

As discussed by Rotemberg and Woodford [46], there is no simple way to measure the marginal cost since it is not directly observable and the relationship between marginal cost and other macro variables depends on the production technology and the labor market structure assumed in the models. In special cases, such as Cobb-Douglas production function, the marginal cost becomes proportional to labor's share. Hence we can use labor's share to estimate sunspots. But even in this case, the measure of the marginal cost is not unique since it can also be linked to capital's share and capital's share may behave very differently from labor's share, let alone the fact that labor's share itself is also difficult to measure precisely (see Krueger [37]). Hence the empirical exercise conducted here is only meant to be suggestive and should not be taken as definitive.[32]

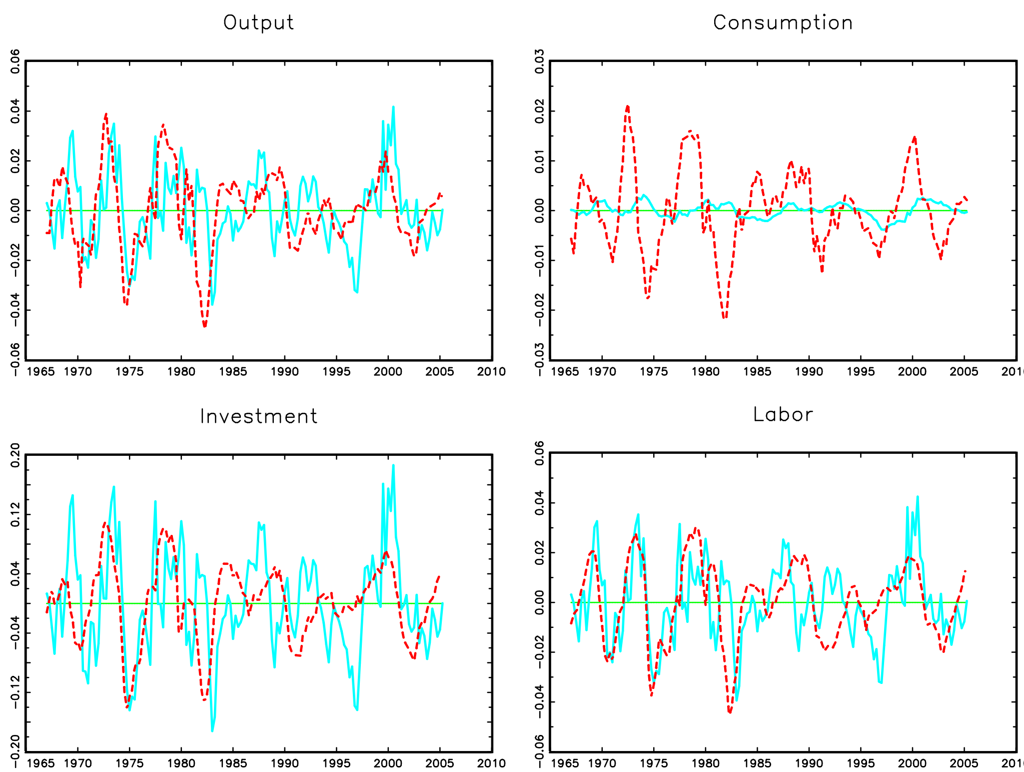

Figure 3. Time Series Simulation under Estimated Sunspots Shocks (dashed lines represent U.S. data, solid lines represent model).

Figure 3 shows the model-generated time series of output, consumption, investment, and hours under the estimated sunspots shocks based on labor's share.[33] The solid lines represent the model and dashed lines the U.S. data. The volatilities of output, investment and hours worked of the

[32] Labor's share in this paper is measured as the ratio of labor's income to GDP, where labor's income is the sum of compensation of employees and proprietors' income. This gives $1 - \alpha = 0.65$. If we follow the convention by allocating only two-thirds of the income of proprietors to labor income, and one-third to capital income, the results are not affected in a significant way.

[33] The value of ω is estimated from the persistence of labor's share as in Equation (60), which gives $\omega = 0.67$. The results do not change significantly if ω varies within a 2-standard error band. Also, the predictions of the model are not sensitive to $\{\sigma_h, \sigma_l\}$ because these two parameters enter the dynamic system only via the scale parameter η in Equation (60). Since we can redefine $\eta s_t = \hat{\phi}_t - (1 - \omega)\hat{\phi}_{t-1}$ as the measured sunspots process, this measure is independent of $\{\sigma_h, \sigma_l\}$.

U.S. economy are matched quite well by the model, but the predicted volatility of consumption by the model is too smooth relative to the data. In particular, the model is able to explain 97% of output volatility in the Data. The standard deviation of output for the U.S. economy is 0.0155, this value is 0.0150 for the model. However, the correlation between model and data is low. For example, with respect to output, the correlation is 0.2. In general, the model generated time series tend to lag the data. This is because labor's share lags the business cycle, as noticed by Rotemberg and Woodford [46]. Although the match is far from perfect, it is encouraging. It is worth noting that if technology shocks are estimated as a Solow residual by taking into account variable capacity utilization, then RBC models driven by the estimated technology shocks do not match the U.S. time series data as well as our model because the estimated Solow residual tend to be uncorrelated with aggregate output (see Burnside et al., [8]).

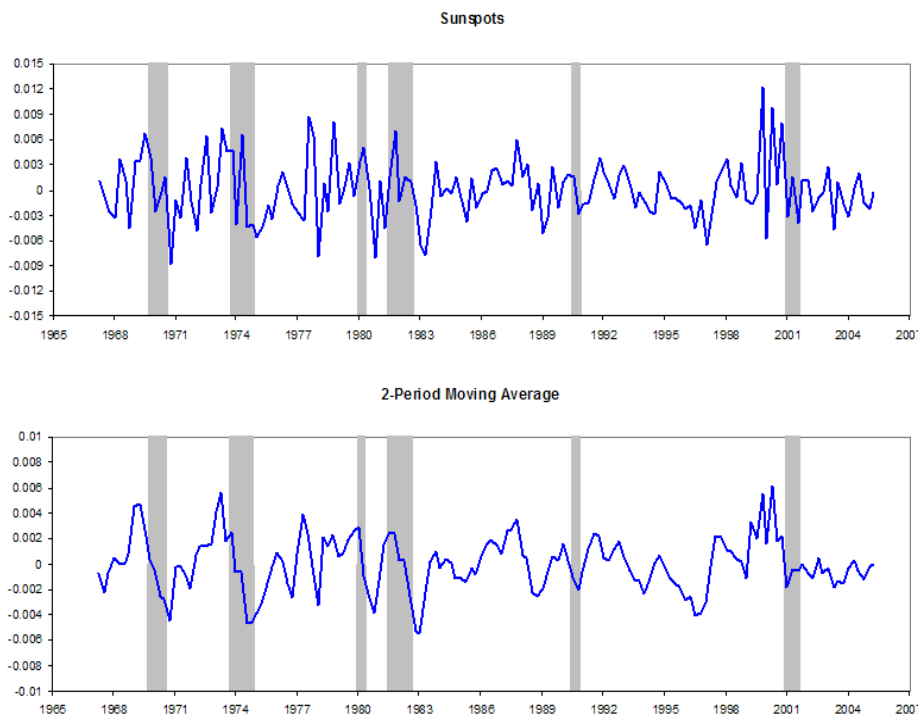

Figure 4. Estimated Sunspots Process.

The top window in Figure 4 shows the estimated sunspots process, along with the NBER recession dates. The process is very noise (serial correlation = 0.05). For this reason, we also plot a 2-period moving average of the sunspots at the bottom of Figure 4. This enables one to see better the underlining correlation between sunspots and business cycles. As the bottom window shows, recessions in the U.S. economy usually correspond to decreases in sunspots. Since sunspots in our model reflect the composition of output and the degree of competitiveness in the economy, a decrease in sunspots implies a lower output due to a declined competitiveness.

5 Conclusion

This paper offers a simple DSGE model of indeterminacy in aggregate output. Indeterminacy in the model is global and independent of the eigenvalues near the steady state, hence it is robust to the topological properties of the steady state and the associated parameters in the utility function and production technologies. Thus our work extends the existing local-indeterminacy literature pioneered by Benhabib and Farmer [3] and Gali [19] by making indeterminacy a more robust feature of DSGE models. Sunspots shocks in our model are not restricted to *i.i.d.* processes and can be estimated from measures of marginal costs. We show that the model outperforms a standard RBC model in explaining some key features of the U.S. business cycle, especially the hump-shaped output dynamics and the relative volatility of hours worked with respect to output. The paper also provides a justification for exogenous variations in marginal costs, which play an important role as a source of cost-push shocks in the monetary policy literature.

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