Evaluating State Tax Revenue Variability: A Portfolio Approach

Authors	Thomas A. Garrett
Working Paper Number	2006-008A
Creation Date	January 2006
Citable Link	https://doi.org/10.20955/wp.2006.008
Suggested Citation	Garrett, T.A., 2006; Evaluating State Tax Revenue Variability: A Portfolio Approach, Federal Reserve Bank of St. Louis Working Paper 2006-008. URL https://doi.org/10.20955/wp.2006.008

Published In	Applied Economics Letters
Publisher Link	https://doi.org/10.1080/13504850601018403

Evaluating State Tax Revenue Variability: A Portfolio Approach

Thomas A. Garrett

Federal Reserve Bank of St. Louis
Research Division
PO Box 442
St. Louis, MO 63166

JEL Codes: H71, G11

Keywords: state revenue, variability, portfolio, taxes

Abstract

State revenue variability is evaluated using a volatility model rooted in portfolio theory. The model evaluates how closely a state's revenue portfolio is constructed to minimize variability in total state tax revenue. The model complements parametric methods of revenue variability.

Corresponding Author: Thomas A. Garrett, Federal Reserve Bank of St. Louis, Research Division, PO Box 442, St. Louis, Missouri 63166-0442
Phone: (314) 444-8601
Fax: (314) 444-8731
Email: garrett@stls.frb.org

Evaluating State Tax Revenue Variability: A Portfolio Approach

I. Introduction

Each state has a portfolio of taxes from which revenues are obtained. Although personal income taxes, corporate income taxes, sales, and excise taxes are common to most states, the degree to which each state relies on each revenue source varies dramatically.[1] The tax mix of a state is influenced by economic conditions, voter preferences, political behavior and the incentives of state legislators, and industry mix (Besley and Case, 2003; Crain, 2003).

Parametric estimation of tax revenue variability is traditionally done by regressing the percentage change in the tax base (or revenue) on the percentage change income (Williams et al., 1973; Holcombe and Sobel, 1996, 1997). Comparing the elasticity of each tax allows a determination of which tax exhibits greater variability over the business cycle. Generally, corporate income taxes experience the greatest short-run variability, followed by personal income taxes, sales taxes, and excise taxes.[2]

This note suggests an alternative approach to examining state revenue variability through the use of a volatility model based on portfolio theory (Markowitz, 1952).[3] Unlike traditional parametric methods, the portfolio approach provides a unique avenue to determine a tax's share of total tax revenue that minimizes the overall variability in total state tax revenue given a state's portfolio of tax revenue sources. Furthermore, by

[1] In 2000, for example, over 61 percent of tax revenue in Washington came from the state's sales tax compared to only 15 percent in Vermont. Nearly 70 percent of California's total tax revenue was obtained through the personal income tax, whereas Tennessee only generated 2.5 percent of its total tax revenue from the personal income tax (only interest and dividends are taxed). Excise taxes accounted for 32 percent of total tax revenue in New Hampshire, but California obtained only 7.5 percent of its total tax revenue from excise taxes.
[2] The exemption of food from the sales tax increases the variability of sales tax revenue.
[3] Szakmary and Szakmary (1995) use the volatility model to answer the question of whether lottery revenues stabilize or destabilize total state revenue.

comparing the actual revenue share of a tax with the variance minimizing share, the model provides an answer to the question of how well a state's portfolio is constructed to minimize the variance in total state tax revenue. This is an interesting exercise given that revenue minimization is not usually the goal of governments when they establish tax rates and tax bases. Economists and budget officials should find the model and its empirical applications useful, as both can be used in conjunction with parametric techniques for analyzing revenue variability. Employing both methods can provide a more complete picture of tax revenue variability.

II. A Model of Revenue Variability

A state has several taxes from which it generates total tax revenue. The following model, based on portfolio theory, assumes a single source of tax revenue is evaluated with the sum of all other tax revenue. The percentage change in total state tax revenue (*R*) is a weighted average of these two tax revenue sources and can be written as:

$$R_t = w_{t-1} \cdot T_t + (1 - w_{t-1}) \cdot O_t \qquad (1)$$

where *T* is the percentage change in revenue from the tax of interest, *O* is the percentage change in tax revenue from all other tax sources, and w_{t-1} is the tax's share of total state tax revenue in period *t-1*.

The variance of *R*, denoted as V_R, measures the variability in total state tax revenue. Using the standard formula for computing variance and assuming *w* is constant over time, V_R can be expressed as:

$$V_R = w^2 V_T + (1-w)^2 V_O + 2w(1-w) \cdot \text{cov}(T, O) \qquad (2)$$

where V_T and V_O denote the variance in the percentage change in tax revenue from the specific tax and the variance in the percentage change in revenue from all other taxes, respectively.

Differentiating equation (2) with respect to w gives:

$$\frac{\delta V_R}{\delta w} = w \cdot V_T + (w-1) \cdot V_O + (1-2w) \cdot \text{cov}(T,O) \tag{3}$$

Let w^* denote the tax's share of total tax revenue that minimizes the variance in total state tax revenue. Setting the derivative in (3) equal to zero and solving for w^* yields:

$$w^* = \frac{V_O - \text{cov}(T,O)}{V_T + V_O - 2\,\text{cov}(T,O)} \tag{4}$$

Or, by convention

$$w^* = \frac{V_O - \text{cov}(T,O)}{V(T-O)} \tag{5}$$

Second order conditions confirm w^* is a minimum:

$$\frac{\partial^2 V_R}{\partial w^2} = V_T + V_O - 2\,\text{cov}(T,O) \equiv V(T-O) > 0 \tag{6}$$

The expression in (5) reveals that w^* is a function of the variance of T and O and the covariance between T and O. Furthermore, the correlation between T and O also determines w^* since $\text{cov}(T,O) = \rho_{T,O} \cdot \sigma_T \cdot \sigma_O$. A time series of state tax revenue data can be used to compute V_T, V_O, cov(T,O) and the variable of interest, w^*.[4]

[4] Note that w^* can fall outside the range $0 \leq w^* \leq 1$. The conditions under which this will occur are discussed later in the paper.

III. Application

Annual revenue data from personal income taxes, corporate income taxes, general sales taxes, and excise (selective sales) taxes were obtained over the period 1977 to 2000 for a sampling of U.S. states.[5] A fifth category of tax revenue, termed "other tax revenue" was computed by subtracting the summed revenue from the four taxes from actual total state tax revenue.[6] The value of w^* for each tax is computed using equation (5). How closely the share of each tax minimizes the variance of total state tax revenue can be determined by comparing the variance minimizing value of w^* with the actual value of w. Because w is not constant over time, w^* is compared to both w_{1999} and to the mean value of w over the sample period. The results of this exercise appear in Table 1.

[Table 1 about here]

The data in Table 1 reveal that the actual tax revenue shares in some states are very close to the minimizing shares, but in some states there is a relatively large discrepancy between the actual shares and the variance minimizing shares. For example, the actual shares of tax revenue in Arkansas, Iowa, Louisiana and West Virginia are generally within several percentage points of the variance minimizing shares. On the other hand, the actual sales tax revenue share in Alabama is nearly 60 percentage points lower than the revenue minimizing share.

Generalizing across the sample of states, actual shares of personal and corporate income tax revenue are closer to the variance minimizing shares than are actual shares of

---

[5] State tax revenue data are available from the U.S. Census, *State Government Finances*. The excise tax category consists of taxes on alcohol, motor fuels, tobacco, amusements, pari-mutuels, public utilities, insurance premiums, and other select taxes.

[6] On average, the revenue from the "other" category accounts for 10 percent of total state tax revenue. However, states such as Delaware, Montana, and New Hampshire have a significantly higher percentage of total tax revenue coming from "other" taxes (about 25 percent, on average).

sales and excise revenue. In fact, the actual shares of excise tax revenue and, to some degree sales tax revenue, are below their variance minimizing shares for many states. One possible reason for this finding is the changing tax mix that has occurred in many states over the past several decades. Specifically, the share of total tax revenue from sales and excise taxes has decreased over the past 25 years while the share of total tax revenue from personal income taxes has increased. Furthermore, sales and excise taxes tend to be less volatile in response to business cycle changes than income taxes (Holcombe and Sobel, 1996, 1997). Thus, states are becoming more reliant on more volatile tax revenues. This suggests that states should rely more heavily on excise and sales taxes (i.e. w for excise and sales taxes should be higher) in order to minimize the variance in total tax revenue.[7]

In the case of California, the variance minimizing shares of personal income and corporate income tax revenue are negative since there is no inherent constraint on $0 \leq w^* \leq 1$ in equation (5). It can be shown that, for a specific tax, $w^*<0$ when the positive correlation between revenue from the specific tax and all other tax revenue is high and the standard deviation of the percentage change in revenue for the tax of interest is relatively large.[8] The fact that $0 \leq w \leq 1$ suggests that specific revenue sources in some states, such as California, cannot not be structured to minimize tax total revenue variability given the existing tax mix and individual shares of tax revenue.

---

[7] This may result in a tradeoff between growth and variability. Because excise taxes, unlike income and sales taxes, are not indexed to inflation, frequent rate increases would be required to maintain a real level of excise tax revenue. In addition, the trend from a goods producing economy to a service economy does not bode well for growing sales tax revenue since services are less likely to be taxed than durable and non-durables. Thus, while sales and excise taxes are less volatile, they may also have a lower growth potential than income taxes. See Holcombe and Sobel (1997) for a discussion on tax revenue growth and variability.

[8] Consider the case of $w^*<0$. This implies that $V_O < cov(T,0)$ in equation (5). Rewriting $cov(T,O)$ as $\rho_{T,O} \cdot \sigma_T \cdot \sigma_O$, it can be shown from $V_O < \rho_{T,O} \cdot \sigma_T \cdot \sigma_O$ that $w^*<0$ when $\sigma_T > \sigma_O / \rho_{T,O}$ and $\rho_{T,O} > 0$. Now consider the case of $w^*>1$. This implies that $V_O - cov(T,0) > V(T-O)$ in equation (5). Rewriting $V(T-O)$ as $V_T + V_O - 2cov(T,O)$ and using $cov(T,O) = \rho_{T,O} \cdot \sigma_T \cdot \sigma_O$, it can be shown that $w^*>1$ when $\sigma_T < \sigma_O \cdot \rho_{T,O}$ and $\rho_{T,O} > 0$.

IV. Discussion

This note has extended a model of portfolio theory to an analysis of state tax revenue variability. The model established the conditions in which the revenue share of a certain tax will minimize the variance in total state tax revenue. Applying actual tax revenue data for a sample of U.S. states revealed that some states have a tax mix that is relatively close to the mix that minimizes total tax revenue variability. The model also revealed that actual excise tax and sales tax revenue shares are below their revenue minimizing levels, thus reflecting states' growing reliance on more volatile taxes.

The model presented in this note serves as a useful complement to parametric techniques that have been used to estimate tax variability. The parametric approach provides evidence on how revenue from each tax responds to business cycle changes, and the portfolio approach can determine how the percentage change in revenue from a specific tax then affects the variability in total state tax revenue by examining how the variance minimizing share of each tax has changed. Furthermore, because the estimate of w^* is a function of the sample period used, future work could examine the stability of w^* over various periods of the business cycle.

An extension of the model could allow for the analysis of multiple taxes rather than considering a specific tax and all other tax revenue. In this case, equation (1) would contain, say, percent changes in sales tax revenue, excise tax revenue, income tax revenue, and corporate income tax revenue. The covariance between each pair of tax sources would be obtained, thus providing disaggregated information on the relationship between individual tax sources. Such an extension would therefore provide a more detailed analysis of how changes in the tax mix influence total tax revenue variability.

Table 1 – Actual and Variance Minimizing Tax Shares – Selected States

		Alabama	Arkansas	Arizona	California	Connecticut	Iowa
Sales Tax	w_{1999}	0.2734	0.3477	0.4388	0.3134	0.3344	0.3381
	mean w	0.2795	0.3626	0.4432	0.3296	0.3943	0.3024
	w^*	0.8880	0.3523	0.0728	0.8573	0.6713	0.2377
Excise Tax	W_{1999}	0.2454	0.1358	0.1306	0.0779	0.1666	0.1484
	mean w	0.2764	0.1860	0.1471	0.0926	0.2219	0.1539
	w^*	0.2990	0.3865	0.1504	0.5619	0.5806	0.1844
Personal Income Tax	w_{1999}	0.3162	0.3111	0.2782	0.4246	0.3751	0.3523
	mean w	0.2658	0.2905	0.2290	0.3609	0.1761	0.3643
	w^*	0.0050	0.3173	0.4489	-0.0422	0.0393	0.3868
Corporate Income Tax	w_{1999}	0.0386	0.0460	0.0723	0.0754	0.0493	0.0482
	mean w	0.0505	0.0630	0.0602	0.1130	0.1148	0.0590
	w^*	0.0103	0.1060	0.0382	-0.0131	0.0393	0.0423

		Illinois	Louisiana	New York	Ohio	Vermont	West Virginia
Sales Tax	w_{1999}	0.2805	0.3083	0.2059	0.3231	0.1481	0.2717
	mean w	0.3159	0.3095	0.2087	0.3166	0.1830	0.3818
	w^*	0.5155	0.2758	0.6930	0.1298	0.5878	0.2255
Excise Tax	w_{1999}	0.1908	0.2600	0.1253	0.1547	0.1740	0.2636
	mean w	0.1945	0.1993	0.1359	0.2047	0.2850	0.2135
	w^*	0.3908	0.2469	0.2001	0.5371	0.9249	0.2858
Personal Income Tax	w_{1999}	0.3417	0.2366	0.5317	0.3956	0.2762	0.2786
	mean w	0.3094	0.1623	0.4917	0.3170	0.3308	0.2414
	w^*	0.2137	0.1024	0.2822	0.1106	0.7076	0.1251
Corporate Income Tax	w_{1999}	0.0992	0.0441	0.0746	0.0413	0.0358	0.0797
	mean w	0.0857	0.0725	0.0875	0.0640	0.0594	0.0609
	w^*	0.0550	0.0023	0.1405	0.0788	0.2193	0.0619

Note: w_{1999} is revenue as a share of total state tax revenue in 1999, the mean of w is the average share of total revenue over the sample period 1977-2000, and w^* is the share of tax revenue that minimizes the variance of total state tax revenue based on equation (5). Per equation (1), each specific source of tax revenue is examined individually, with all remaining taxes summed into the "other tax revenue" category (not shown).

References

Besley, T. and A. Case, 2003, Political Institutions and Policy Choices: Evidence from the United States, Journal of Economic Literature, 41, 7-73.

Crain, M., 2003, Volatile States: Institutions, Policy, and the Performance of American State Economies, (University of Michigan Press, Ann Arbor).

Holcombe, R. and R. Sobel, 1996, Measuring the Growth and Variability of Tax Bases over the Business Cycle, National Tax Journal, 49, 535-552.

Holcombe, R. and R. Sobel, 1997, Growth and Variability in State Tax Revenue (Greenwood Press, Westport, Connecticut).

Markowitz, H., 1952, Portfolio Selection, Journal of Finance, 7, 77-91.

Szakmary, A. and C. Szakmary, 1995, State Lotteries as a Source of Revenue: A Re-Examination, Southern Economic Journal, 64, 1167-1181.

Williams, W., R. Anderson, D. Froehle, and K. Lamb, 1973, The Stability, Growth, and Stabilizing Influence of State Taxes, National Tax Journal, 26, 267-274.