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Wavelet: A New Tool for Business Cycle Analysis*

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Abstract: One basic problem in business-cycle studies is how to deal with nonstationary time series. The market economy is an evolutionary system. Economic time series therefore contain stochastic components that are necessarily time dependent. Traditional methods of business cycle analysis, such as the correlation analysis and the spectral analysis, cannot capture such historical information because they do not take the time-varying characteristics of the business cycles into consideration. In this paper, we introduce and apply a new technique to the studies of the business cycle: the wavelet-based time-frequency analysis that has recently been developed in the field of signal processing. This new method allows us to characterize and understand not only the timing of shocks that trigger the business cycle, but also situations where the frequency of the business cycle shifts in time. Our empirical analyses show that 1973 marks a new era for the evolution of the business cycle.

Keywords: Business cycle, wavelets, time-frequency analysis, non-stationary time series, and spectrum.

JEL Classification: C10, E32.

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I. Introduction

The business cycle, one of the most puzzling phenomena in capitalistic, free-market economies, has long been the central focus of macroeconomic researches. The biggest challenge to researchers in this field is to capture business cycle patterns that vary in nature across time. Current studies of business cycles are based almost exclusively on the assumption of covariance-stationarity. The market economy is an evolutionary dynamic system, however. New methods of financial intermediation are continuously introduced and developed; government fiscal policies and monetary targets are constantly shifting; input-output relations across industries are subject to frequent changes due to improvements in production technologies and market organizations. Therefore, economic time series contain stochastic components that are necessarily time dependent.

The analysis of nonstationary time series cannot be accomplished by classical time domain representations such as correlation methods, or by frequency domain representations based on the Fourier transform (Boashash, 1987). To analyze business cycles that evolve over time, we need to develop a concept of time-frequency distribution that takes into account jointly and simultaneously the information of time and frequency.

In this paper, we introduce and apply a new technique of time series analysis to business cycle studies that is recently developed in the field of signal processing in engineering: a joint time-frequency distribution based on the wavelet transform. The wavelet transform is a powerful tool for analyzing nonstationary time series. The joint time-frequency presentation enables us to capture the evolutionary aspects of the spectral distribution of the business cycle across time.

Although time-frequency analysis has its origin almost 50 years ago (Gabor, 1946; Ville, 1948), significant advances occurred only in the last 15 years or so. Recently, time-frequency representations have become an extremely powerful tool for analyzing nonstationary time series in many fields, such as engineering, medical sciences, biology, geology, and astronomy, to name just a few. A number of articles have also been published to deal with applications in economics and finance (Most notably, James B. Ramsey, 1996).[1]

To help understand this new methodology, we compare the wavelet-based time-frequency analysis to a traditional approach based on the short-time Fourier transform. We show that the wavelet transform has many advantages over the traditional approach in that the wavelet transform has a beautiful property: its window size adjusts itself optimally to longer basis functions at low frequencies and to shorter basis functions at high frequencies. Consequently, it has sharp frequency resolution for low frequency movements and sharp time resolution for high frequency movements. Thus, the new method is capable of capturing simultaneously the time-varying nature of low frequency cycles and the frequency distribution of sudden and abrupt shocks in the original time series. Applying this new method to post war US data, we are able to show that 1973 marks a new era for the evolution of the business cycle since World War II.

The rest of the paper is organized as follows. Section II describes the short-time Fourier transform. Section III describes the wavelet transform. Section IV explains the implementation of the wavelet transform when applying to actual data. Section V uses artificial time series to demonstrate the advantages of wavelet transform over the short-time Fourier transform. Section VI applies the wavelet based time-frequency analysis to economic data. Finally, Section VII concludes the paper.

II. Short-Time Fourier Transform and Spectrogram

Fourier analysis is a mathematical tool for studying the cyclical nature of a time series in the frequency domain. Under Fourier transform, however, the time information of a time series is completely lost. When one looks at the spectral density function of a time series, no information regarding the time when a particular cycle emerges or disappears

[1] Also see Ramsey and Zhang, 1996, 1997; Ramsey and Thomson, 1998; Ramsey, Usikov and Zaslavsky, 1995; Ramsey and Lampart, 1998a, 1998b; Chen, 1996; and Hong and Lee, 1998.

is revealed. For time series in which the time information is not important but the frequency content is of primary interest, this limitation is of little consequence. Fourier analysis is thus useful for analyzing periodic and stationary signals whose moments do not change over time. However, many interesting and important time series are not stationary and need to be analyzed in both time and frequency domain simultaneously.

Performing the mapping of a one-dimensional time series into a two-dimensional space of time and frequency is thus needed in order to extract relevant time-frequency information. A classical time-frequency representation, called the short-time Fourier transform (STFT), has been extensively used in the past for analyzing nonstationary time series since its introduction by Gabor (Gabor, 1946). The basic idea of STFT is to break a time series $x(t)$ into many smaller sub-samples and apply Fourier transform to each sub-sample. Specifically, the time series is multiplied by a window function $h(t)$ centered at a time point n, and the spectrum of the windowed time series, $x(t)h^*(t-n)$, is calculated by

$$STFT_x(n,\omega) = \sum_{t=1}^{N} x(t)h^*(t-n)e^{-i\omega t}, \qquad (1)$$

where ω is the angular frequency and * denotes the complex conjugation. Because the window $h(t-n)$ effectively suppresses data signals outside the window, the STFT produces a sequence of 'local' spectrum of the time series $x(t)$. This intuitive approach has interesting consequences. For example, there exists a one-to-one correspondence of the STFT with the original signal and an exact inverse transform exists.

To obtain the energy (spectral density) distribution of the time series, the most familiar representation is the spectrogram. The spectrogram of a time series $x(t)$ is defined as the squared magnitudes of the STFT:

$$SPEC_x(n,\omega) = \left| \sum_{t=1}^{N} x(t)h^*(t-n)e^{-i\omega t} \right|^2. \qquad (2)$$

A crucial feature inherent in the STFT method is that the length of the window can be selected arbitrarily, but is fixed exogenously once the selection is made. To enhance the time information, therefore, one must choose a short window; and to enhance the frequency resolution, one must choose a long window, which means that the time information (nonstationarities) occurring within the window interval is suppressed. Hence, a longer window implies the loss of information along the time dimension, and a shorter window implies the loss of information along the frequency dimension. The length of the window is therefore the main issue involved in practice.

III. Wavelet Transform and Scalogram

The Fourier transform breaks down a time series into constituent sinusoids of different frequencies. Since a sine wave function has a specific frequency but infinite duration in time, it is perfectly localized in frequency but not localized in time. The wavelet transform, on the other hand, breaks down a time series into shifted and scaled versions of a mother wavelet function that has limited spectral band and limited duration in time. One major advantage afforded by wavelet transform is thus the ability to perform natural local analysis of a time series in the sense that the length of wavelets (windows) varies endogenously in an optimal way. It stretches into a long wavelet function to measure the low frequency movements; and it compresses into a short wavelet function to measure the high frequency movements. In order to capture abrupt changes, for example, one would like to have very short basis functions (narrow windows). At the same time, in order to isolate slow and persistent movements, one would like to have very long basis functions (wide windows). This is exactly what can be achieved with the wavelet transforms.

The wavelet transform is defined as the convolution of a signal $x(t)$ with a wavelet function $\Psi(t)$, called mother wavelet, shifted in time by a location parameter n, and dilated by a scale parameter a, as shown by the following equation

$$WT_x(n,a) = \frac{1}{\sqrt{|a|}} \sum_{t=1}^{N} x(t) \Psi^* \left(\frac{t-n}{a} \right), \tag{3}$$

where $\Psi^*(.)$ is the complex conjugate of the basic wavelet function $\Psi(t)$, the parameter a is the scaling factor that controls the length of the wavelet; and n is the time location at where the wavelet is centered. Scaling a wavelet simply means stretching or compressing it. The scale factor a is hence inversely related to the frequency of the wavelet.

The squared modulus of the wavelet transform, called scalogram, is defined as

$$SCAL_x(n,a) = \left| \frac{1}{\sqrt{|a|}} \sum_{t=1}^{N} x(t) \Psi^* \left(\frac{t-n}{a} \right) \right|^2 . \qquad (4)$$

The scalogram characterizes the distribution of the energy (spectral density) of a time series across the two-dimensional time-scale plane. Thus, the wavelet transform of a time series depends on two parameters: scale (or frequency) and time. This leads to a so-called time-scale representation that provides a tool for the analysis of nonstationary signals (Rioul and Vetterli, 1991; Daubechies, 1990).

There are several types of wavelet functions available, such as Morlet, Mexican hat, Haar, Shannon, Daubechies wavelet function, etc. The choice of the wavelet function depends on the application. With respect to time and frequency localization, the Haar and Shannon wavelets take opposite extremes. Having compact support in time, the Haar wavelet has poor decay in frequency, whereas the Shannon wavelet has compact support in frequency with poor decay in time. Other wavelets typically fall in the middle of these two extremes. In fact, having exponential decay in both the time and frequency domain, the Morlet wavelet has optimal joint time-frequency concentration (Teolis, 1964). The wavelet that is used for analysis of economics fluctuations in this paper is Morlet wavelet, which is a modulated Gaussian function with exponential decay property. It is defined as

$$\Psi(t) = \exp(-0.5(t/a)^2)\exp(i2\pi ft), \qquad (5)$$

where a is the scaling factor that controls the length of the wavelet and f is the modulation (frequency) parameter. The scale parameter a and the frequency parameter f are related to each other by the relationship:

$$a = f_0 / f, \qquad (6)$$

where f_0 is a free parameter controlling the basic shape of the wavelet. As a decreases, the wavelet function is compressed, implying a waveform of higher frequency.

IV. Implementations

In STFT, a window function h is chosen. This window function is first placed at the beginning of the time series and the Fourier transform is performed. Then the window is shifted to a new location and another Fourier transform is computed. This procedure continues until the end of the time series is reached. The spectrogram is computed accordingly as the squared modulus of the short-time Fourier transform.

The wavelet transform is implemented in a similar manner. The time series is multiplied by a wavelet function, similar to the window function in the STFT, and the wavelet transform is computed according to equation (3) for different values of the scale parameter (a) at different time location (n). Suppose $x(t)$ is the time series to be analyzed. The mother wavelet is chosen to serve as a prototype for all wavelets in the process. All the wavelets (window functions) that are used subsequently are the stretched (or compressed) and shifted versions of the mother wavelet. The computation starts with a value of the scaling factor $a = a_1$, and the wavelet is placed at the beginning of the time series. Since the wavelet function has only finite time duration (it takes zero values outside the wavelet), it serves just like a window in the STFT. The constant $1/\sqrt{a_1}$ is for normalization purpose so that the transformed signal will have the same energy at every scale. Next, with the same scale $a = a_1$, the wavelet function is shifted to the next sample point, and the wavelet transform is computed again. This procedure is repeated until the wavelet reaches the end of the time series. The result is a sequence of numbers corresponding to the scale $a = a_1$.

Next, the scale factor is changed to $a = a_2$, and the whole procedure described above is repeated. When the process is completed for all desired values of a, the result is an energy (density) distribution of the original time series along the two-dimensional time-frequency space.

V. Applications to Test Signals

To show the effects and the advantages of wavelet-based time-frequency analysis over the traditional STFT based time-frequency analysis, we present spectrograms and scalograms of two test signals. The signals are of length 512 points each. The STFT uses a Hanning window, and the scalogram is obtained with the Morlet wavelet. The horizontal axis is time and the vertical axis is frequency in both spectrograms and scalograms respectively.

The first test signal showing in Figure 1.*a* (top window) is composed of sine waves whose frequency shifts periodically across time in the low frequency region. In the middle of the sample, however, there is a sharp transitory white noise impulse. The power spectrum of the test signal is shown in the left window of Figure 1.*a* (the frequency axis is normalized by 2π). It is seen there that the power spectrum is completely silent about the time-varying nature of the cycle and about the white noise impulse. Instead, it shows that there are simultaneously several major cycles contained in the low frequency region.

The central window in Figure 1.*a*, however, shows how remarkably the scalogram captures not only the time-varying nature of the low frequency cycle, but also the exact timing of the white noise impulse at time location 256. Notice that the frequency of the shifting cycle is highly localized along the frequency dimension on one hand, and the timing of the frequency shift is also highly localized along the time dimension on the other hand.

As a comparison, the spectrogram based on STFT is shown in Figure 1.*b* and Figure 1.*c*. We see there that the spectrogram either gives an imprecise frequency localization of the time-varying low frequency cycle when the window size is small enough to adequately capture the timing of the high frequency impulse (Figure 1.*b*), or misses the white noise impulse entirely when the window size is large enough to capture adequately the frequency location of the time-varying low frequency cycle in the original time series (Figure 1.*c*).

The second test signal used is composed of two parts: the 1st part is a time-varying low frequency sinusoidal cycle, and the 2nd part is a constant high frequency cycle with 6 sample points gap in the middle of the series. The time series is shown in the top window in Figure 2.*a*, and the power spectrum is shown in the left window in Figure 2.*a*.

The central window in Picture 2.*a* shows that the scalogram is able to capture not only the frequency location of the time-varying low frequency cycle, but also the exact timing of the missing signals presented in the constant high frequency cycle. There is no energy (spectral density) distribution in the middle of the scalogram due to the 6 missing data points in the high frequency cycle (notice the sharp breaking edges in the middle of the scalogram).

STFT, on the other hand, is unable to simultaneously capture all the information adequately. With a short window (Figure 2.*b*), the time information with respect to the exact timing of the missing data points is captured, but the frequency location of the low frequency cycle is not localized at all along the frequency axis. With a large window (picture *c*), on the other hand, the frequency locations of the cycles are well localized along the frequency axis, but the exact location and timing of the 6 missing data points are not very well captured or localized along the time axis. This is so because both the time and the frequency resolutions of STFT are fixed once the window length is fixed. In contrast, scalogram allows good frequency resolution at low frequencies and good time resolution at high frequencies.

VI. Application to Economic Data

Since Second World War, the US economy has experienced several important institutional changes. These institutional changes have likely had important impact on the structure of the US economy. The US economy has also experienced several unprecedented shocks that may also have brought deep structural adjustment to the economy. The oil price shock during the early 70s, for example, could have resulted in a fundamental reorganization of the input-output structure in the economy, especially with regard to the energy-intensive industries.

It is then of great interest to investigate whether these changes have also brought fundamental changes to the nature of the US business cycle. In particular, it is of great interest to know whether the old business cycles observed by economists almost half century ago are still alive, and whether new business cycles have emerged during those years of social changes and economic development.

Applying the wavelet-based time-frequency transform to the growth rate of real GDP (1960:1 - 1996:3), we find that the US business cycle has the following defining features:

1) Business cycles through out the sample period are concentrated mostly in the frequency region below 10 quarters per cycle. They are triggered mostly by external shocks.

2) Business cycles become far more active during the 70s and 80s after the oil price shocks in the early 70s. The two most active business cycles occurred around 1974 and 1983, both are triggered apparently by external impulses. The periodicity of the two cycles is about 6 years per cycle.

3) There exist business cycles that are not triggered by any external shocks to GDP, such as the 1991 business cycle. On the other hand, strong external shocks to GDP do not necessarily trigger business cycles, such as the shocks during 1977-1978.

Figure 3 shows the contour of energy (power spectrum) distribution of the US GDP growth across time and frequency. The time series (top window) reveals very little about the frequency location of the cycles, while the spectrum (left window) reveals nothing about the timing of the different cycles. The scalogram (center), however, shows that there have been three major business cycles since 1960. The first occurred in 1961, triggered by a sharp external impulse during that year. The 1961 cycle has a frequency of 0.1 cycles per quarter (or 10 quarters per cycle) and is short lived (it lasted about one year). The second major cycle took place in 1973, apparently triggered by two impulses during 1972 and 1973, and was greatly intensified by another impulse near 1975. This business cycle lasted about 3-4 years and peaked at the frequency of about 0.04 cycles per quarter (or 25 quarters per cycle). The third major cycle occurred during 1982-1984, apparently triggered by a shock in 1982. This cycle lasted about 3 years and peaked also at a frequency similar to the 1973 cycle. The 1973 cycle and the 1982 cycle dominated all other business cycles since 1960. Notice that the cycle in 1991 is very mild compared to the three major cycles mentioned above. It is apparently not triggered by any external shocks to GDP. The scalogram also reveals that a major shock around 1977-1978 did not trigger any business cycle around that time. In addition, there is a short-lived business cycle in 1966 triggered by an external impulse that is not obvious or noticeable, however, in the original time series (see top window in Figure 3).

We think that these findings are of great importance to the business cycle theory. They not only help us identify the important historical shocks that triggered the business cycle, but also provide important information regarding the evolution of the business cycle across time. If the business cycle is unstable over time, for example, then there is the need for finding a common propagation mechanism to explain that instability. Without exception, existing real business cycle models all predict a stable business cycle with the same characteristic frequencies. But the scalogram shows otherwise: business cycles come and go; they emerge at different frequencies and at different times; they are not at all alike.

VII. Conclusions

A new technique of time series analysis based on joint time-frequency representation was proposed. Two popular time-frequency approaches, the short-time Fourier transform and the wavelet transform were compared. Our analyses showed that the wavelet-based time-frequency analysis is superior to the Fourier transform based time-frequency analysis. Applying the wavelet-based analysis to economic data, we found that business cycles in the US have not been stable over time. In particular, business cycles became far more active since the oil price crisis in the early 70s.

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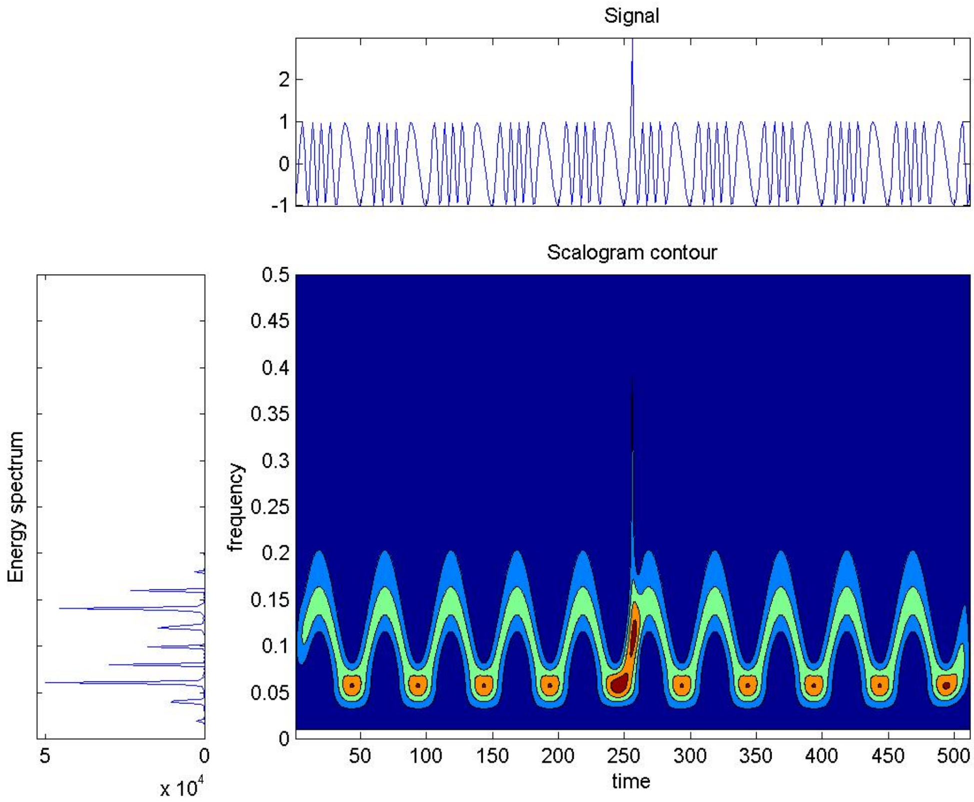

Figure 1.*a*: Scalogram contour with signal (top) and spectrum (left).

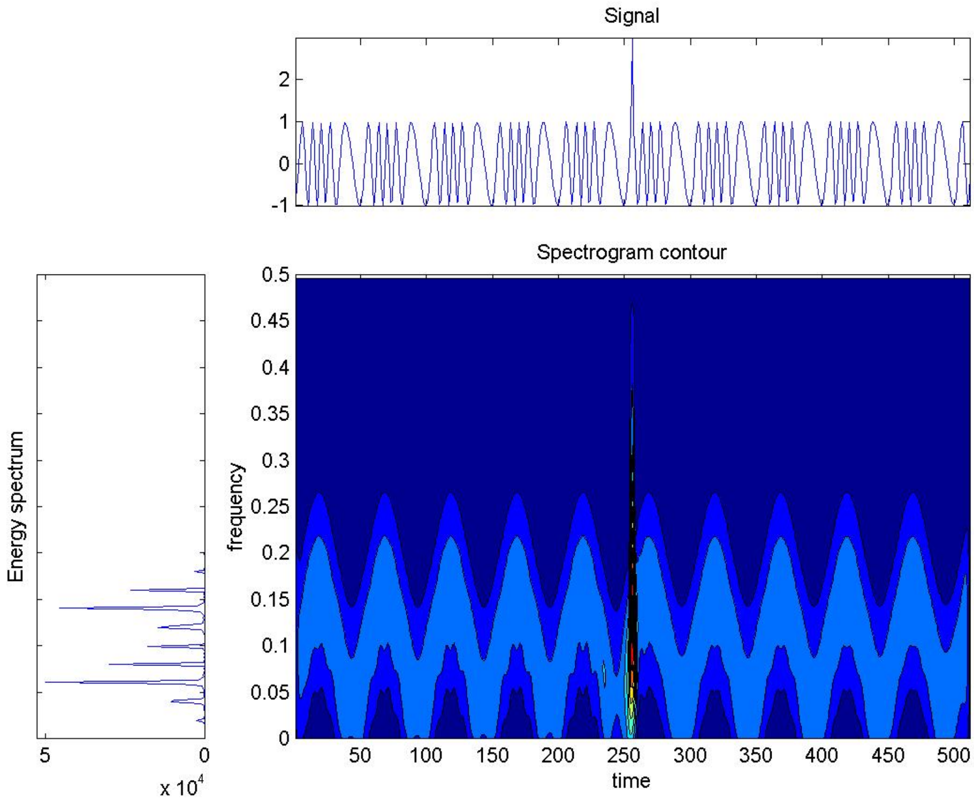

Figure 1.*b*: Spectrogram contour with signal (top) and spectrum (left) (window = 7).

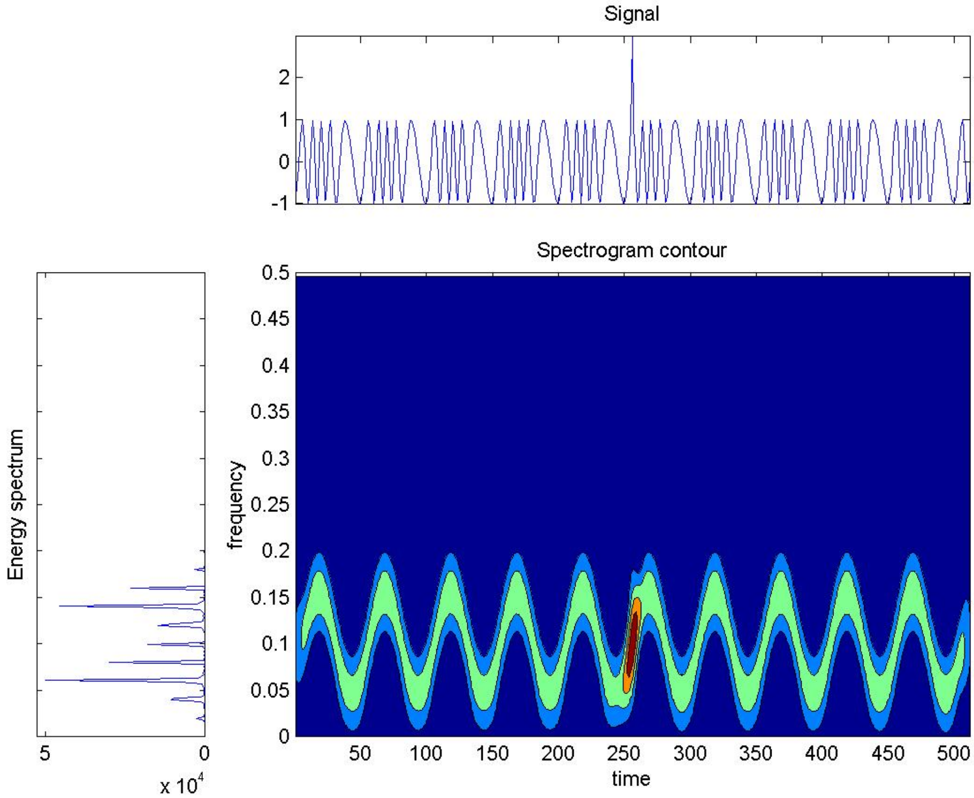

Figure 1.*c*: Spectrogram contour with signal (top) and spectrum (left) (window = 19).

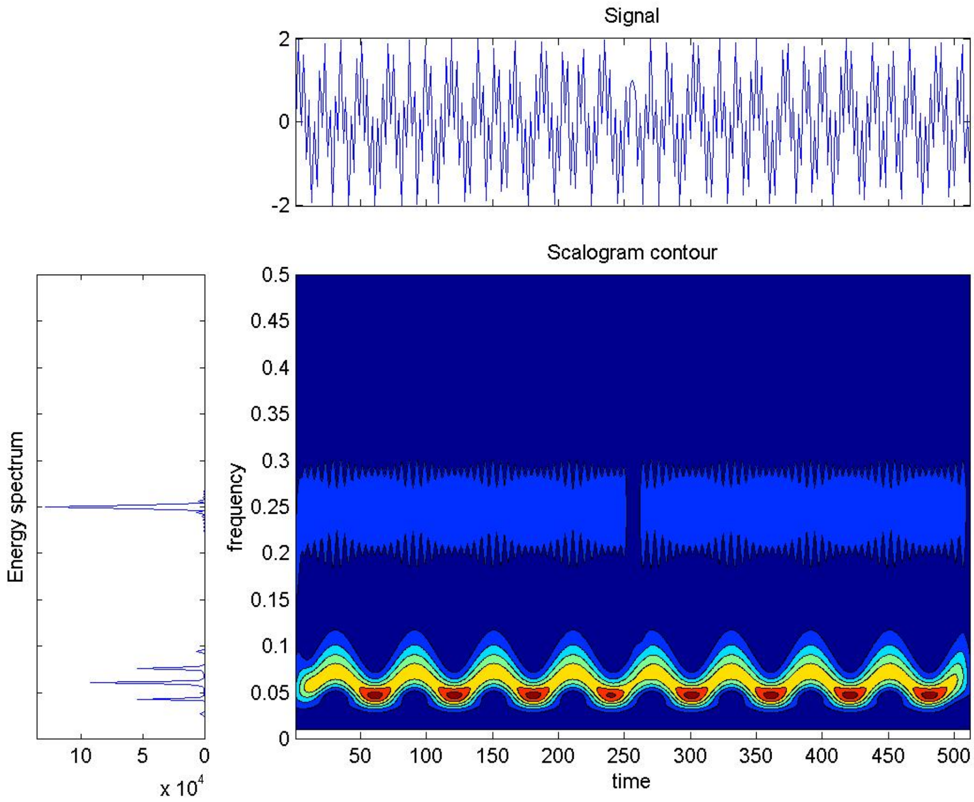

Figure 2.*a*: Scalogram contour with signal (top) and spectrum (left).

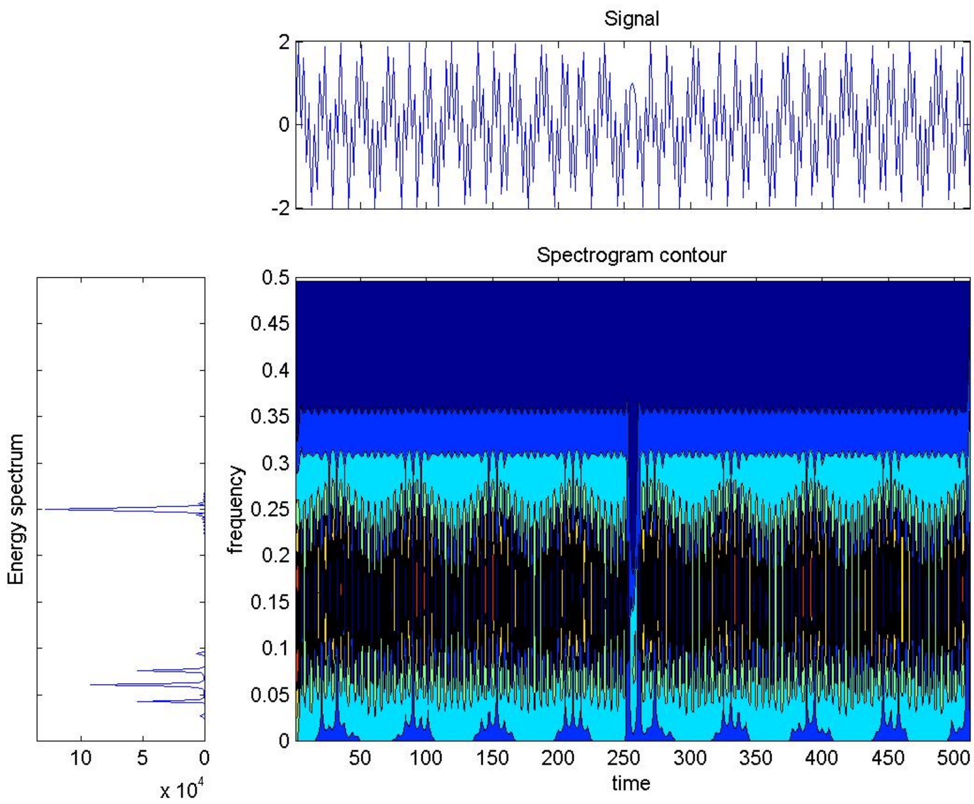

Figure 2.*b*: Spectrogram contour with signal (top) and spectrum (left) (window = 7).

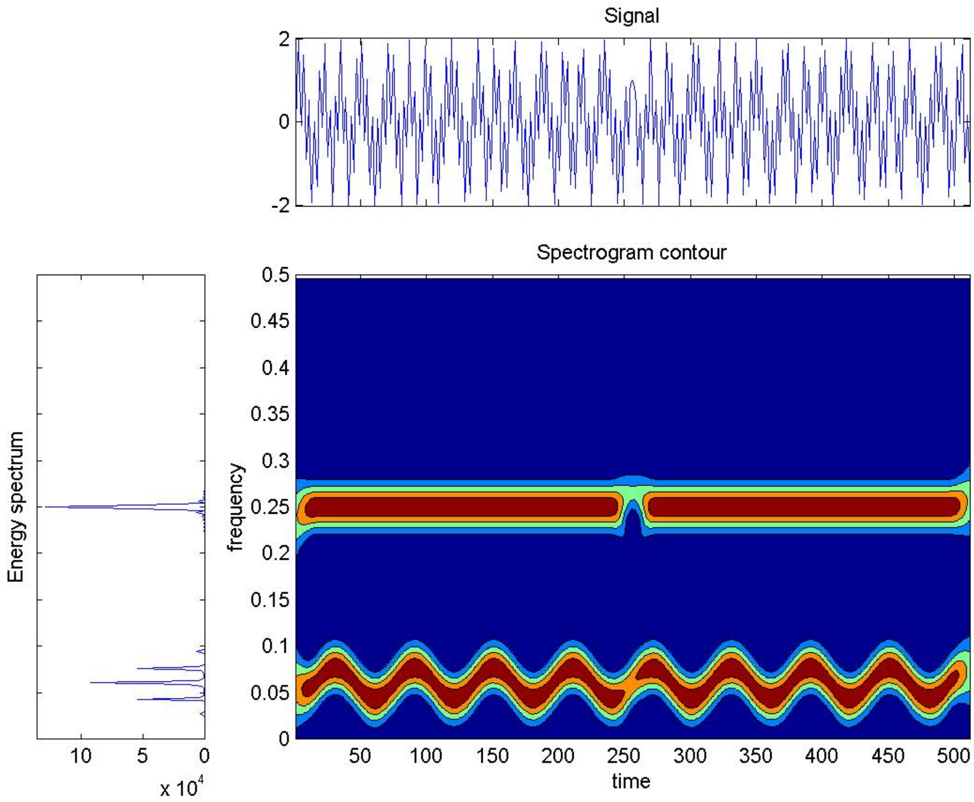

Figure 2.*c*: Spectrogram contour with signal (top) and spectrum (left) (window = 35).

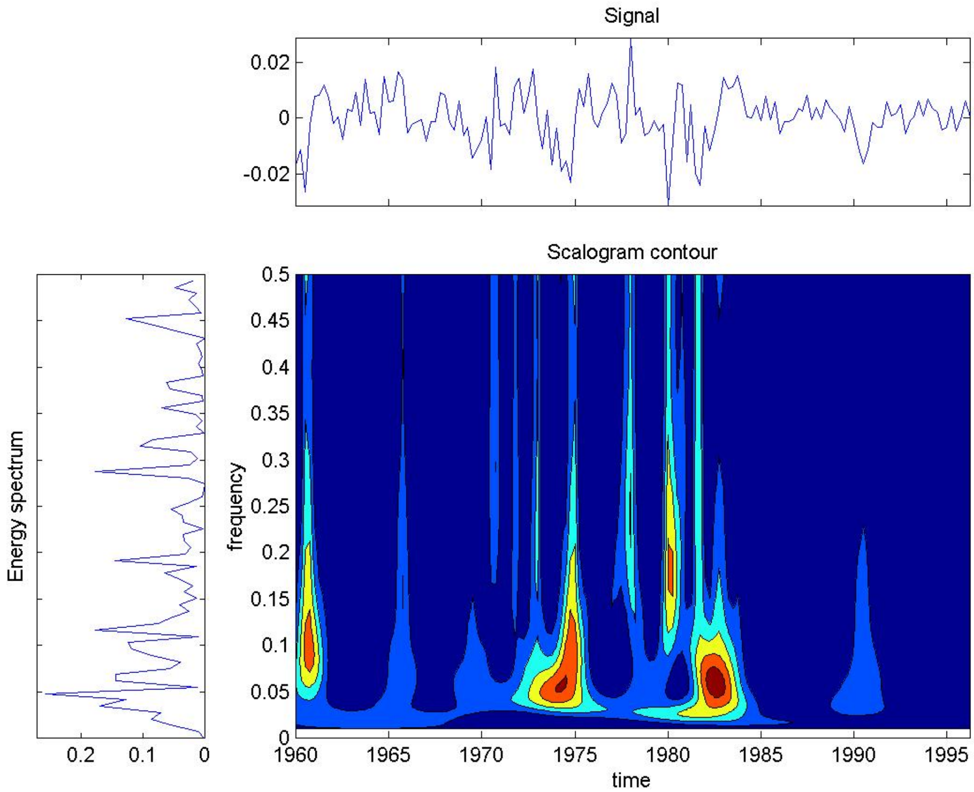

Figure 3: Scalogram contour with time series (top) and spectrum (left).
U.S. GDP growth rate (1960:1 - 1996:3).