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Abstract

Labor hoarding is a widely believed empirical behavior of firms and a prominent explanation for procyclical labor productivity. Conventional wisdom attributes labor hoarding to labor adjustment costs. This paper argues that the conventional wisdom is inadequate for understanding labor hoarding because it ignores the role of inventories. Since idle labor can be used to produce inventories, why do firms hoard labor when inventory is an option? Using a dynamic rational expectations model of profit-maximizing firms facing demand uncertainty, this paper studies the dynamic interactions between labor hoarding and inventory accumulation. Closed-form decision rules for labor and inventory decisions are derived. The analysis shows that labor adjustment costs alone are far from sufficient for explaining labor hoarding.

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1 Introduction

Labor hoarding (or excess labor) is a widely believed empirical fact about firms' behavior of coping with demand uncertainty (see, e.g., Clark 1973, Fay and Medoff 1985, and Fair 1969, 1985). It is also the single most important concept in the business-cycle literature as an explanation for procyclical labor productivity (e.g., see Bernanke and Parkinson 1991, Dornbusch and Fischer 1981, Miller 1971, Rotemberg and Summers 1990, and Summers 1986, among many others). The conventional wisdom for labor hoarding is based on labor adjustment costs. Namely, due to various types of adjustment costs of labor (e.g., search and training costs), firms opt not to lay off workers when demand is temporarily low, because firing workers may be more costly than hoarding them (in addition to search and training costs, there is also the opportunity cost of losing sales when demand suddenly picks up). Consequently, firms opt to incur idle labor during recessions, which makes labor productivity appear to be procyclical (see, e.g., Okun 1962, Miller 1971, Becker 1975, and Fay and Medoff 1985, among others).

This paper argues that such arguments are incomplete and insufficient for explaining labor hoarding because they ignore the role of inventories. Holding excess supplies of finished goods as inventories is another way to cope with demand shocks, one that also avoids the adjustment costs of labor. For example, when demand is temporarily low, firms can accumulate inventories by producing at full capacity and using inventories to meet with possibly higher future demand, rather than reducing production by hoarding labor and using the hoarded labor to meet with possibly higher future demand. Either way can obviate the need for a firm to adjust its labor stock. Considering that most manufactured goods are storable at low cost and that inventories are far more liquid than production factors in serving final demand, hoarding labor is perhaps a more costly way to cope with demand uncertainty than holding inventories.

Despite the intimate relationship between inventories and labor hoarding, the vast bulk of the labor hoarding literature has neglected inventories; likewise, the vast bulk of the inventory literature has neglected labor hoarding. As a consequence, the mechanism of labor hoarding has not been well understood. This unsatisfactory situation has been highlighted recently by Galeotti, Maccini and Schiantarelli (2005). They show that estimating firms' Euler equations without taking into account the interactions between inventory decisions and labor decisions can lead to serious misidentifications. Their empirical work suggests that the cross-equation restrictions imposed by firms' inventories and labor decisions are extremely important for correctly identifying key parameters of standard optimization models of firms.

It is well documented in the empirical literature that the inventories-to-sales ratio is counter-

cyclical (e.g., see Bils and Kahn, 2000) and that the labor utilization rate is procyclical (e.g., see Fay and Medoff 1985 and Fair 1985), suggesting that times when inventories are high relative to sales are also times when firms hoard labor. The question therefore is: When firms can use inventories of finished goods to buffer demand shocks, why is it also necessary to keep inventories of labor? In other words, what are precisely the gains of labor hoarding relative to inventory holding? This seems to be a fundamental question for understanding labor hoarding, yet it is seldom addressed by either the labor-hoarding literature or the inventory literature.

Topel (1982) is an important exception. Realizing the close link between inventories and excess labor, Topel uses a dynamic optimization model to study the interactions between labor decisions and inventory decisions. His analysis suggests that inventories are substitutable for labor hoarding, hence firms with high costs of labor hoarding tend to rely more on inventories to buffer demand shocks. Topel also argues that the prediction is consistent with the data. However, Topel's analysis is based on a deterministic model and his empirical analysis relies on the assumption of demand uncertainty. Furthermore, Topel does not directly address the necessary and sufficient conditions for labor hoarding. Since closed-form decisions rules are not available in Topel's model, many empirical implications of his model, such as the impulse responses of inventories and labor hoarding to demand shocks, cannot be directly tested.

This paper builds on Topel's pioneering work by taking demand uncertainty into explicit consideration. Using a version of Topel's model, I am able to obtain closed-form decision rules for inventory and labor hoarding. The analysis reveals that inventory dynamics can be altered by labor hoarding in an important way, and vice versa, when both inventories and labor decisions are jointly taken into consideration. Such results confirm the econometric analyses of Topel (1982) and Galeotti, Maccini and Schiantarelli (2005). Further and more importantly, it is shown that labor adjustment costs alone are not sufficient for inducing labor hoarding behavior. In addition to the adjustment costs, more conditions are required in order for firms to hoard labor in equilibrium. These conditions include:

1) The present value of gains from hoarding labor (e.g., postponing current production costs and avoiding future hiring costs) must exceed that of holding inventories. Otherwise, firms will use only inventories to cope with demand uncertainty.[1]

(2) There must exist an information technology that allows the firm to update and revise its expectations about demand after the labor hiring decision has been made but before the inventory decision is made. Otherwise, firms will never hoard labor; instead, they will use inventories alone to buffer demand shocks even if hoarding labor is an option and is less costly than holding inventories.

When production is instantaneous, condition (1), in conjunction with labor adjustment costs,

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[1]This implies that a lower cost of holding inventories relative to hoarding labor is not a necessary condition for inventories to exist. In other words, it may still be optimal for firms to hold inventories in addition to hoarding labor even if hoarding labor is less costly than holding inventories.

is sufficient for inducing labor hoarding. However, if production takes time, then condition (2) is also a necessary condition for labor hoarding to be optimal. Condition (1) looks trivial from a theoretical view point: it states the trade off between inventories and labor hoarding in present value terms. However, this condition may be difficult to justify empirically, since common sense suggests that hoarding labor may be more expensive than holding inventories. Labor is one of the most important cost factors in production besides capital, whereas keeping inventories requires only depreciation and cheap space in a warehouse.[2]

Condition (2) is subtle and less obvious. In reality, time is always a crucial factor in economic activities. For example, it takes time to search for and train workers. Due to this, firms' hiring decisions must be made before their production decisions are made. Also, it takes time to produce and deliver goods. So the production decisions have to be made before demand uncertainty is fully resolved. The existing literature has shown that time lags in production/delivery are the key for explaining many long-standing puzzles of the inventory behavior (see, e.g., Kahn 1987, 1992, and Wen 2005) because they give rise to a liquidity advantage of inventories over production factors in dealing with demand shocks. This makes hoarding labor less attractive than holding inventories. Thus, planning for zero labor hoarding at the time of labor hiring is always optimal from the point of view of cost minimization. However, imagine that after the labor hiring decisions but before the production (capacity utilization) decisions are made, there is new information indicating that demand is lower than expected. The firm can either stay the course and continue to produce at full capacity (anticipating more inventory accumulation), or reduce capacity utilization (labor hoarding) and incur less inventories. If hoarding labor is cheaper than holding inventories, the second strategy is clearly optimal since liquidity is no longer a concern. That is, the acquisition of new information warrants the need to reconsider the planned inventory level, and the readjustment of the planned inventory level is made possible by utilizing the option of labor hoarding. Hence, labor hoarding can serve as a technological option to exercise in order to reduce inventory costs despite the fact that labor is not as liquid as inventories in buffering demand shocks.

The bottom line is that, in an economy with time-to-search and time-to-produce, even high costs of inventories (in addition to adjustment costs of labor) do not sufficiently justify labor hoarding. An information technology that enables the firm to update and extract new information about demand changes is also needed. Without the information technology or the information updating, labor hoarding is never optimal even if it is less costly than holding inventories. On the other hand, even if there is information updating about demand after the hiring decisions but before the production decisions are made, such that readjustment of planned inventory levels is warranted, a

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[2]Labor costs account for about 60% of total production costs for a typical firm in the U.S. Assuming that the markup is 20% and that half of this markup is used to cover the inventory holding and depreciation costs, then for every dollar of revenue, about 60 cents are used to cover the labor costs and 10 cents are used to cover the inventory holding costs.

firm may still find it optimal not to hoard labor if holding inventories is less costly than hoarding labor. Hence both condition (1) and condition (2) are necessary for the coexistence of inventories and labor hoarding. In the absence of either condition, firms will hoard finished goods inventories but not labor in spite of labor adjustment costs.

In economies with time-to-produce, when both conditions (1) and (2) can be met (in addition to labor adjustment costs), profit-seeking firms do have incentive to enhance supply flexibility by holding not only goods inventories but also excess supplies of labor in reserve, so as to fully guard against demand uncertainty. This reconfirms Blinder's (1982) conjecture and Topel's (1982) analysis regarding firms' strategic behavior under demand uncertainty. That is, inventories of labor are partial substitutes for inventories of goods as a means of coping with demand shocks.[3]

The availability of closed-form decision rules makes the model attractive in serving as a convenient and simple framework for further econometric and empirical analysis in related empirical issues. The model also offers a micro-foundation for understanding aggregate employment and labor productivity movements. It is shown that as a consequence of labor hoarding, measured labor productivity is procyclical despite non-increasing returns to scale. Interestingly, the model does not require the assumption of unobservable labor effort as an additional production factor in order to explain procyclical labor productivity. Unobservable effort is a necessary condition for other types of labor hoarding models to explain the procyclical output-labor ratio (e.g., see Rotemberg and Summers 1990, and Burnside, Eichenbaum and Rebelo 1993).[4]

Hopefully, the theoretical analyses in this paper can promote and stimulate further empirical work on inventories and labor hoarding. For example, how to correctly measure and compare labor hoarding costs and inventory holding costs of a particular firm? Is it true that hoarding labor is necessarily cheaper than holding inventories? Can we use the observed extent of labor hoarding and inventory level of a firm to infer the firm's hidden costs of labor hoarding and inventory holding, as well as the firm's information-processing structure about expected demand? Do firms that have larger inventory fluctuations tend to have smaller labor productivity fluctuations? In other words, how to measure the elasticity of substitution between inventories and hoarded labor?

The rest of the paper is organized as follows. Section 2 studies a simple two-period model with *i.i.d* demand shocks and fixed output prices to gain intuition. Section 3 extends the two-period model to an infinite horizon with serially correlated demand shocks and endogenous output prices. Section 4 concludes the paper.

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[3]This understanding of firms' strategic behavior has also long been held by other economists. See, for example, Miller (1971) and Becker (1975), among others.

[4]Fay and Medoff (1985) argue that the effort margin is a less important contributor to labor hoarding.

2 A Two-Period Model

Assume that the firm's demand is given by $\theta_t = \gamma + \varepsilon_t$, where γ is a positive constant and ε is an *i.i.d.* random variable with zero mean and support $[-\gamma, \gamma]$, so that the realized demand is always non-negative. Let $F(\varepsilon)$ be the *c.d.f* of ε, $F(\bar{\varepsilon}) = \Pr[\varepsilon \leq \bar{\varepsilon}]$. The maximum amount the firm can sell in period t is its inventory as of the end of the previous period, denoted by s_{t-1}, plus whatever it produces during period t (y_t). Assuming that the goods price p is sufficiently high, we have $\tau_t = \min\{\theta_t, y_t + s_{t-1}\}$, where τ_t denotes actual sales in period t. To allow for the possibility of inventories, assume that the production decision must be made before demand uncertainty is fully resolved. In particular, the production decision is based on an imperfect signal about demand, $x_t = \varepsilon_t + v_t$, where the noise term v_t is othorgonal to ε_t with zero mean and variance σ_v^2.

For simplicity, assume that labor is the only factor of production and the production technology has constant returns to scale, so

$$y_t = L_t. \tag{2.1}$$

It is useful to distinguish between workers on line (L) and workers on reserve (R). Workers on line are those who are engaged in production. Workers on reserve are those who are on the payroll but do not produce output. The wage rate paid to workers on line each period is w per worker ($w < p$ is a known constant), and the wage rate paid to workers on reserve each period is a fraction $(\alpha < 1)$ of w.[5] The total stock of workers on payroll is denoted $W_t \equiv L_t + R_t$. Since in each period the firm can either hire or fire workers to adjust the firm's labor stock, the law of motion for the stock of workers is given by $W_t = W_{t-1} + N_t$, or

$$L_t + R_t = L_{t-1} + R_{t-1} + N_t, \tag{2.2}$$

where N is a flow variable denoting new hiring (or firing). The right hand side, $(L_{t-1} + R_{t-1} + N_t)$, is hence the total stock of workers available for work at the beginning of period t.

It is assumed that decisions for N_t (hiring) need to be made one period in advance. Hence adjusting the labor stock later in response to better information about demand changes is impossible. This feature of labor as a quasi-fixed factor (Oi 1962) gives rise to an incentive for labor hoarding in this model. But this feature alone, as will be shown shortly, is not sufficient for inducing labor hoarding behavior. Assume that the costs involved in hiring a worker are given by c (e.g., search and job training costs).[6] The profits in period t are simply revenue minus costs, $p\tau_t - w(L_t + \alpha R_t) - cN_t$.

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[5] Costs are usually higher when producing than when not producing. For example, producing goods requires both labor and materials, hence not producing can save both labor costs and the material costs. Since no production factor other than labor is assumed in the production function, the assumption $\alpha < 1$ captures the idea that production costs are higher when producing than when not producing.

[6] For simplicity, the costs of firing workers are not modeled. However, adding such a feature does not change the implications of the model, since this merely reinforces the adjustment costs of labor.

The sequence of decision making in the model is as follows. The firm decides first how many workers to hire based on information available at the beginning of period $t-1$. A new signal about demand, $x_t = \varepsilon_t + v_t$, is then observed towards the end of period $t-1$, based on which the firm decides how much output to produce by choosing the utilization rate of labor (i.e., the number of workers on line as well as the number of workers on reserve). The true demand shock, ε_t, is revealed at the beginning of period t, based on which the firm makes decisions on sales and inventory accumulations.

Define information sets as follows: $\Omega_0 \subset \Omega_1 \subset \Omega_2$, where Ω_0 is the initial information set in the beginning of period $t-1$ when decisions for N_t are made, $\Omega_1 = \Omega_0 \cup x_t$ is the updated information set for production decisions at the end of period $t-1$, and $\Omega_2 = \Omega_1 \cup \varepsilon_t$ is the further updated information set for sales and inventory decisions in period t. The timing of events is illustrated below:
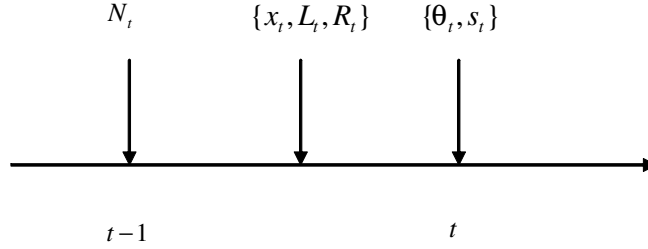
Since there are only two periods and the shocks are *i.i.d*, we can set the initial values $\{L_{t-1}, R_{t-1},\ s_{t-1}\} = 0$. In this case, both inventories and labor stock lose their potential future values due to the lack of durability. To ensure that inventories may have positive values, we assume that inventories can be sold at a price lower than the regular output price p. The firm's problem is to solve

$$\max E \left\{ p\tau_t + \delta p s_t - w\left[L_t + \alpha R_t\right] - cN_t \right\}$$

subject to

$$\tau_t + s_t = L_t \tag{2.3}$$

$$L_t + R_t = N_t \tag{2.4}$$

and two non-negativity constraints on inventory stock and hoarded labor, $s_t \geq 0$ and $R_t \geq 0$. Note that $\delta \in (0,1)$ in the profit function indicates that end-of-period inventories sell at the equivalent of a low price.

Denoting $\left\{ \lambda_t^s, \lambda_t^R, \pi_t^s, \pi_t^R \right\}$ as the Lagrangian multipliers for constraints (2.3), (2.4) and the two non-negativity constraints, respectively, the first order conditions for $\{N, L, R, s, \tau\}$ are given by

$$c = E\left[\lambda_t^R | \Omega_0\right] \tag{2.5}$$

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$$w + \lambda_t^R = E\left[\lambda_t^s | \Omega_1\right] \tag{2.6}$$

$$\alpha w + \lambda_t^R = \pi_t^R \tag{2.7}$$

$$\lambda_t^s = \delta p + \pi_t^s \tag{2.8}$$

$$\lambda_t^s = p, \quad \text{if } \theta_t > y_t \tag{2.9}$$

plus two complementary slackness conditions, $s_t \pi_t^s = 0$ and $R_t \pi_t^R = 0$.

2.1 Analysis

The key of the analysis is obtaining the expected values of goods (λ^s) and labor (λ^R), so that (2.5) and (2.6) can be used to solve for the decision rules of hiring and labor utilization. By the linear projection theory (signal extraction), we have

$$E[\theta_t | \Omega_1] = E[\theta_t | x_t] = \gamma + \xi x_t, \tag{2.10}$$

where $\xi = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_v^2}$. Denoting the forecast error for demand at the end of period $t - 1$ as $\mu_t \equiv \theta_t - E[\theta_t | x_t]$, we have

$$\mu_t = \varepsilon_t - \xi x_t. \tag{2.11}$$

Namely, the forecast error for ε_t based on information after x_t is observed is simply the difference between the realized demand innovation ε and the projection of ε_t on x_t (signal extraction). If there is no noise $(\sigma_v^2 = 0)$, then $\xi = 1$ and the forecast error is zero. On the other hand, if x contains no information about ε $(\sigma_v^2 = \infty)$, then $\xi = 0$ and the forecast error is the same as $\theta_t - E[\theta_t | \Omega_0] = \varepsilon_t$. Denote the cumulative probability density function $(c.d.f)$ of μ as $G(\mu)$.

The equilibrium of the model is characterized by threshold strategies. Namely, the firm uses optimal cut-off values for the forecast errors and the signal to determine whether to stockout or not in the goods and the labor markets. To proceed, consider period $t's$ decision after observing ε_t. Given that the forecast error is $\mu_t = \varepsilon_t - \xi x_t$, there are two possible cases to consider:

Case A: $\mu_t < \mu_t^*.$[7] The forecast error for demand turns out to be low due to a low realization of ε_t. In this case, the realized demand is low $(\theta_t < y_t)$, the actual sales are thus determined by $\tau_t = \theta_t$ and the firm opts to incur inventories, $s_t > 0$. Hence, $\pi_t^s = 0$ and $\lambda_t^s = \delta p.$[8] The condition $s_t > 0$, implies $y_t - \theta_t = y_t - (E[\theta_t | \Omega_1] + \mu_t) > 0$, or equivalently,

$$\mu_t < y_t - E[\theta_t | \Omega_1] \equiv \mu_t^*. \tag{2.12}$$

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[7] μ^* is an optimal cut-off value to be determined endogenously later.
[8] The marginal case with equality, $\mu_t = \mu_t^*$, is included under case B to be considered later.

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In other words, the cut-off value of μ_t is determined by $\mu_t^* \equiv y_t - E\left[\theta_t|\Omega_1\right]$.

Case B: $\mu_t \geq \mu_t^*$. In this case, supply falls short of demand $(\theta_t \geq y_t)$. Hence, $\tau_t = y_t, s_t = 0, \pi_t^s > 0$, and the marginal value of goods $\lambda_t^s = p$.

The above analysis suggests that the probability of case A and case B depends endogenously on the choice of the output level before the realization of demand, since the optimal cut-off value (μ_t^*) is determined by y_t. In other words, the probability of a stockout in inventories in period t is determined by production decisions made at the end of period $t - 1$.

The value of output is given by λ_t^s. The expected value of output based on signal x is given by

$$E\left[\lambda_t^s|\Omega_1\right] = \delta p \int_{\mu_t < \mu_t^*} dG(\mu) + p \int_{\mu_t \geq \mu_t^*} dG(\mu) \in (\delta, p). \tag{2.13}$$

Therefore, the first-order condition with respect to L_t (2.6) becomes:

$$w + \lambda_t^R = \delta p \int_{\mu_t < \mu_t^* \equiv y_t - E[\theta_t|\Omega_1]} dG(\mu) + p \int_{\mu_t \geq \mu_t^* \equiv y_t - E[\theta_t|\Omega_1]} dG(\mu), \tag{2.14}$$

which determines the optimal production level (y_t) based on information set Ω_1. The left-hand side (LHS) of equation (2.14) is the marginal cost of output known by the end of $t - 1$, which has two components: the wage cost for putting a worker on line (w), and the opportunity cost for losing slackness in the labor resource constraint (λ_t^R), which reflects the tightness of labor resources for the firm. Hence, the optimal output level depends on the shadow value of labor input (λ_t^R) at time $t - 1$.

To determine the value of λ^R, we again have two cases to consider:

Case C: $x_t < x_t^*$.[9] The received signal at the end of period $t-1$ is low, indicating a low demand in period t. In this case, optimal demand for workers on line should be low because expected future demand is low. Hence it is optimal to hoard labor $(R_t > 0)$. Therefore, $\pi_t^R = 0$ and $\lambda_t^R = -\alpha w$, which indicates that the marginal value of labor is given by its hoarding costs, $-\alpha w$. Equation (2.14) implies

$$w(1 - \alpha) = \delta p \int_{\mu_t < \mu_t^*} dG(\mu) + p \int_{\mu_t \geq \mu_t^*} dG(\mu) \tag{2.15}$$

$$= \delta p G(\mu_t^*) + p\left[1 - G(\mu_t^*)\right].$$

This equation suggests that, conditioned on low signals for demand $(x_t < x_t^*)$, the optimal cut-off

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[9]x^* is an optimal cut-off value of the signal to be determined endogenously.

value for μ_t is a constant, $\mu_t^* = \bar{\mu}$, which solves the equation,

$$G(\bar{\mu}) = \frac{p - (1 - \alpha)w}{p(1 - \delta)}. \tag{2.16}$$

Given that $\mu_t^* \equiv y_t - E[\theta_t | \Omega_1]$, the optimal output level (or the optimal number of workers on line) is thus determined by

$$y_t = \bar{\mu} + E[\theta_t | \Omega_1] \tag{2.17}$$

$$= \bar{\mu} + \gamma + \xi x_t.$$

That is, when the signal for demand is low, the optimal output level is determined by a constant inventory target $(\bar{\mu})$ plus the expected demand $(\gamma + \xi x_t)$ based on the most up-to-date information available at the end of period $t - 1$.

Since $G(\bar{\mu}) \leq 1$, we must require that the value of hoarding labor be greater than the value of holding inventories,

$$(1 - \alpha)w \geq \delta p; \tag{2.18}$$

where $(1 - \alpha)w$ is the value gained (or the opportunity cost saved) by putting a worker on reserve instead of on line. The requirement (2.18) amounts to ensuring that the cost of labor hoarding be sufficiently below the cost of holding inventories in order for case C $(R_t > 0)$ to be optimal. To see this more clearly, suppose $(1 - \alpha)w < \delta p$ (hoarding labor is less beneficial relative to holding inventories). Then the RHS of equation (2.15) always exceeds the LHS, hence we must require $\lambda_t^R > -\alpha w$, or equivalently $\pi_t^R > 0$, to hold. This implies that $R_t = 0$ must hold (by the complementary slackness condition), suggesting that it is optimal not to hoard labor if the cost of hoarding labor exceeds the cost of holding inventories. Hence labor hoarding and inventories do not coexist if hoarding costs are too high relative to inventory costs. In the special case where $\delta = 0$ (i.e., inventories have no market value), a much less restrictive condition, $\alpha < 1$, is required in order for labor hoarding to be optimal (i.e., it requires the cost of production to be higher than the cost of no production or inaction).[10]

Case D: $x_t \geq x_t^*$. In this case, demand for workers on line is high because the observed signal x suggests a high future demand for goods. Hence, hoarding labor is not optimal, suggesting $R_t = 0$ and $y_t = N_t$ (producing at full capacity).

The optimal decision rule for production is thus given by

$$y_t = \begin{cases} \bar{\mu} + \gamma + \xi x_t & \text{if } x_t \leq x_t^* \\ \\ N_t & \text{if } x_t > x_t^* \end{cases}. \tag{2.19}$$

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[10]See footnote 5.

To determine the optimal cut-off point x_t^*, note that under case C, $y_t = \bar{\mu} + \gamma + \xi x_t$, hence the condition $R_t > 0$ implies $0 < N_t - y_t = N_t - \bar{\mu} - \gamma - \xi x_t$, or

$$x_t < \frac{1}{\xi} [N_t - \bar{\mu} - \gamma] \equiv x_t^*, \tag{2.20}$$

which defines the cut-off point for x_t.

Clearly, the probability of case C and case D is determined endogenously by the cut-off value, $x_t^* \equiv \frac{1}{\xi} [N_t - \bar{\mu} - \gamma]$, which in turn is determined by the quality of information (ξ), the average demand (γ), the target inventory-production level ($\bar{\mu}$), and most importantly, the level of labor stock (N) that is determined by the firm's hiring choices made in the past (at the beginning of period $t - 1$).

Define the *c.d.f* of x as $Z(x)$. To determine the optimal level of hiring (N_t) based on information set Ω_0, equation (2.5) implies

$$c = E\left[\lambda_t^R | \Omega_0\right] \tag{2.21}$$

$$= -\alpha w \int_{x_t < x_t^*} dZ(x) + \int_{x_t \geq x_t^*} \left(E\left[\lambda_t^s | \Omega_1\right] - w\right) dZ(x)$$

where

$$E\left[\lambda_t^s | \Omega_1\right] = \delta p \int_{\mu_t < y_t - E[\theta_t | \Omega_1]} dG(\varepsilon) + p \int_{\mu_t \geq y_t - E[\theta_t | \Omega_1]} dG(\varepsilon), \tag{2.13$'$}$$

according to equation (2.13) presented before. Recall that under case D, $\lambda^R = -\delta w + \pi^R > -\delta w$ and $y_t = N_t$. Hence, conditioned on $x_t \geq x_t^*$, we have

$$y_t - E\left[\theta_t | \Omega_1\right] = N_t - E\left[\theta_t | \Omega_1\right] \tag{2.22}$$

$$= \bar{\mu} + \xi(x_t^* - x_t),$$

where the second equality is obtained by using the definition of the cut-off point, $x_t^* \equiv \frac{1}{\xi} [N_t - \bar{\mu} - \gamma]$,

which implies $N_t \equiv \xi x_t^* + \bar{\mu} + \gamma$. Therefore, we have

$$\int_{x_t \geq x_t^*} E\left[\lambda_t^s | \Omega_1\right] dZ(x) \tag{2.23}$$

$$= \delta p \int_{x_t \geq x_t^*} \left(\int_{\mu_t < \bar{\mu} + \xi(x_t^* - x_t)} dG(\mu)\right) dZ(x) + p \int_{x_t \geq x_t^*} \left(\int_{\mu_t \geq \bar{\mu} + \xi(x_t^* - x_t)} dG(\mu)\right) dZ(x)$$

$$= \delta p \int_{x_t \geq x_t^*} G\left(\bar{\mu} + \xi(x_t^* - x_t)\right) dZ(x) + p \int_{x_t \geq x_t^*} \left[1 - G\left(\bar{\mu} + \xi(x_t^* - x_t)\right)\right] dZ(x)$$

$$\equiv \Pi\left(x_t^*\right).$$

Thus the condition (2.21) that solves for the optimal level of N_t is reduced to

$$c = -\alpha w Z(x_t^*) - w\left[1 - Z(x_t^*)\right] + \Pi(x_t^*), \tag{2.24}$$

where the RHS of (2.24) depends on x^* only. This suggests that the target for excess labor is a constant, $x_t^* = \bar{x}$; so $N_t = \xi\bar{x} + \bar{\mu} + \gamma$ is the optimal decision rule for hiring.

2.2   Results

The decision rules of the model are summarized by:

$$N_t = \xi\bar{x} + \bar{\mu} + \gamma \tag{2.25}$$

$$y_t = \begin{cases} \bar{\mu} + \gamma + \xi x_t & \text{if } x_t < \bar{x} \\ \\ \bar{\mu} + \gamma + \xi\bar{x} & \text{if } x_t \geq \bar{x} \end{cases} \tag{2.26}$$

$$R_t = \begin{cases} \xi\left(\bar{x} - x_t\right) & \text{if } x_t < \bar{x} \\ \\ 0 & \text{if } x_t \geq \bar{x} \end{cases} \tag{2.27}$$

$$s_t = \begin{cases} \bar{\mu} + \xi x_t - \varepsilon_t & \text{if } \varepsilon_t < \bar{\mu} + \xi x_t \text{ and } x_t < \bar{x} \\ \\ \bar{\mu} + \xi\bar{x} - \varepsilon_t & \text{if } \varepsilon_t < \bar{\mu} + \xi x_t \text{ and } x_t \geq \bar{x} \\ \\ 0 & \text{if } \varepsilon_t \geq \bar{\mu} + \xi x_t \end{cases} \tag{2.28}$$

where the identity $\mu_t = \varepsilon_t - \xi x_t$ has been utilized throughout the derivations. Finally, the measured productivity is given by the ratio of output to total labor stock:

$$\frac{y_t}{L_t + R_t} = \begin{cases} \frac{\bar{\mu} + \gamma + \xi x_t}{\bar{\mu} + \gamma + \xi \bar{x}} & \text{if } x_t < \bar{x} \\ \\ 1 & \text{if } x_t \geq \bar{x} \end{cases}. \tag{2.29}$$

Proposition 1 *The optimal volume of labor hoarding is zero if the firm does not have the information technology to update information about future demand after the decision for hiring but before the decision for production is made.*

Proof. No information updating is equivalent to the case that the signal x contains no useful information about demand, $x_t = v_t$. Hence $\xi = 0$. The decision rule for labor hoarding (2.27) implies that the planned level of labor hoarding is zero. The intuition is that there is no need to revise production plans (which are made at the time of hiring) if the signal x is not informative. Hence, N_t always determines the optimal output level y_t. ∎

Proposition 2 *The optimal volume of inventories is zero if the signal provides perfect information about demand.*

Proof. Perfect information about demand implies that $x_t = \varepsilon_t$; namely, the information about future demand is fully revealed by the end of period $t - 1$, so that $\mu_t = 0$ and $\xi = 1$. In this case, $G(\mu)$ is a degenerate step function and the support of μ_t is a single point at $\mu_t = 0$. Since the optimal cut-off point μ_t^* must be contained in the support of the distribution $G(\mu)$, case A ($s_t > 0$) is ruled out and the only possible solution is the marginal case with $\mu_t^* = \bar{\mu} = 0$ and $s_t = 0$. The intuition of this proposition is that inventories no longer provide insurance against the possibility of stockout when the firm has perfect information about demand at the time of capacity utilization choice. ∎

Thus, two crucial conditions must be met in order to observe the coexistence of labor hoarding and inventories. They are: (1) There cannot be perfect information about demand at the time when the production decision is made; otherwise the firm does not hold inventories; (2) there is an information technology that enables the firm to update and revise expectations about future demand after the hiring decision but before the capacity utilization decision for labor is made; otherwise the firm holds inventories but does not hoard labor. In addition to the two conditions, there is a third condition (stated previously) for the coexistence of inventories and labor hoarding, which is that the potential gains from labor hoarding must be greater than those from inventory holding, $(1 - \alpha) w > \delta p$; otherwise the firm holds inventories but does not hoard labor, everything else equal.

Proposition 3 *The measured productivity of labor is procyclical as long as there is labor hoarding.*

Proof. The covariance between labor productivity and output is given by $cov\left(y, \frac{y}{L+R}\right) = \frac{\pi}{\bar{\mu}+\gamma+\xi\bar{x}}\sigma_y^2$ > 0, where π is the probability that demand is low (i.e., $\pi \equiv \Pr\left[x_t < \bar{x}\right]$). $\blacksquare$

Since the major goal of this section is to gain intuition, the discussion of the more important and interesting implications of the model is postponed until the more general model is presented below.

3 Infinite Horizon

The above analysis may suffer from several shortcomings due to its simplicity. First, the expected future values of labor and inventories are not taken into consideration in the two-period model. This may lead to biased decision rules in favor of labor hoarding or inventories. Second, the assumption of *i.i.d* demand shocks implies that the optimal labor stock is a constant, and this may be driving the procyclical productivity in the two-period model. When demand shocks are serially correlated, optimal hiring should depend on past demand shocks. This may alter the procyclical productivity result obtained above. Third, output price is fixed exogenously. This assumption may lead to biased decision rules in favor of holding inventories since if the output price is endogenous, firms may opt not to hold inventories when price can be adjusted downward to equate demand and supply. This section relaxes these assumptions and shows that the main insights obtained in the two-period model are robust.

Assume that the firm's inverse demand function is given by $p_t = p(\tau_t, \theta_t)$, where τ denotes sales and θ denotes demand shocks. Denote the firm's revenue function as $r(\tau_t, \theta_t) = \tau_t p(\tau_t, \theta_t)$ with $r'_\tau(\tau) > 0$ and $r''_\tau(\tau) < 0$. Assume that demand shocks follow the process $\theta_t = \gamma + \rho\theta_{t-1} + \varepsilon_t$, where $\rho \in (0,1)$ measures the degree of serial correlation and ε is an *i.i.d.* random variable with zero mean and support $[-\gamma, \gamma]$. Continue to denote $F(\varepsilon)$ as the *c.d.f* of ε. Since inventories and labor are durable, the law of motion for the goods market is modified to $\tau_t + s_t = (1-\delta)s_{t-1} + y_t$, and the law of motion for the labor market is modified to $L_t + R_t = L_{t-1} + R_{t-1} + N_t$. Other than these changes, the structure of the model is the same as before.

Define information sets as follows: $\Omega_{t-1} \subset \Omega_{xt} \subset \Omega_t$, where Ω_{t-1} is the information set in the beginning of period $t-1$ when decisions for N_t are made, $\Omega_{xt} = \Omega_{t-1} \cup x_t$ is the updated information set for production (labor utilization) decisions at the end of period $t-1$, and $\Omega_t = \Omega_{xt} \cup \varepsilon_t$ is the further updated information set for sales and inventory decisions in period t. The timing of events is the same as illustrated in the figure in section 2.

The firm's problem is to solve

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$$\max E \sum_{t=0}^{\infty} \beta^t \left\{ r(\tau_t) - w\left[L_t + \alpha R_t\right] - cN_t \right\}$$

subject to

$$\tau_t + s_t = (1 - \delta)s_{t-1} + L_t \qquad (3.1)$$

$$L_t + R_t = L_{t-1} + R_{t-1} + N_t \qquad (3.2)$$

and $s_t \geq 0, R_t \geq 0$; where $\beta \in (0,1)$ is the discount factor (the inverse of the interest rate) and $\delta \in (0,1)$ is the depreciation rate of inventories.

Denoting $\left\{\lambda_t^s, \lambda_t^R, \pi_t^s, \pi_t^R\right\}$ as the Lagrangian multipliers for constraints (3.1), (3.2) and the two non-negativity constraints respectively, the first order conditions for $\{N, L, R, s, \tau\}$ are given by

$$c = E\left[\lambda_t^R | \Omega_{t-1}\right] \qquad (3.3)$$

$$w + \lambda_t^R = \beta E\left[\lambda_{t+1}^R | \Omega_{xt}\right] + E\left[\lambda_t^s | \Omega_{xt}\right] \qquad (3.4)$$

$$\alpha w + \lambda_t^R = \beta E\left[\lambda_{t+1}^R | \Omega_{xt}\right] + \pi_t^R \qquad (3.5)$$

$$\lambda_t^s = (1 - \delta)\beta E\left[\lambda_{t+1}^s | \Omega_t\right] + \pi_t^s \qquad (3.6)$$

$$\lambda_t^s = r'(\tau_t, \theta_t); \qquad (3.7)$$

plus two complementary slackness conditions:

$$s_t \pi_t^s = 0 \qquad (3.8)$$

$$R_t \pi_t^R = 0. \qquad (3.9)$$

Compared with the two-period model, note here that $E\left[\lambda_{t+1}^R | \Omega_{xt}\right]$ in equations (3.4) and (3.5) is the expected future value of labor that must be taken into account due to labor's durability, and $(1 - \delta)E\left[\lambda_{t+1}^s | \Omega_t\right]$ in equation (3.6) is the expected future value of inventory that must be taken into account due to inventory's durability.

3.1 Analysis

The procedure to obtain the decision rules of the model is similar to that in the two-period model by first finding the expected shadow values of goods (λ^s) and labor (λ^R). By signal extraction, we have $E[\theta_t | \Omega_{xt}] = E[\theta_t | \theta_{t-1}, x_t] = \gamma + \rho \theta_{t-1} + \xi x_t$, where $\xi = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_v^2}$. Denoting the forecast error

for demand at the end of period $t-1$ as $\mu_t \equiv \theta_t - E[\theta_t|\Omega_{xt}]$, we have $\mu_t = \varepsilon_t - \xi x_t$. Denote the cumulative probability density function of μ as $G(\mu)$.

Since $c = E[\lambda_t^R|\Omega_{t-1}]$ according to (3.3), by the law of iterated expectations, we have $E[\lambda_{t+1}^R|\Omega_{xt}]$ $= E[E[\lambda_{t+1}^R|\Omega_t]|\Omega_{xt}] = c$. Hence equations (3.4) and (3.5) can be simplified to

$$w + \lambda_t^R = \beta c + E[\lambda_t^s|\Omega_{xt}] \tag{3.4'}$$

$$\alpha w + \lambda_t^R = \beta c + \pi_t^R \tag{3.5'}$$

Taking expectations on both sides of (3.4') with respect to Ω_{t-1}, we have $w + E[\lambda_t^R|\Omega_{t-1}] = \beta c + E[\lambda_t^s|\Omega_{t-1}]$, which implies

$$E[\lambda_t^s|\Omega_{t-1}] = w + (1-\beta)c. \tag{3.10}$$

Hence, equation (3.6) can be simplified to

$$\lambda_t^s = (1-\delta)\beta[w + (1-\beta)c] + \pi_t^s. \tag{3.6'}$$

The interpretation of (3.10) is straightforward. In equilibrium, the expected value of producing one unit of inventory based on information available at the time of hiring, $E[\lambda_t^s|\Omega_{t-1}]$, equals its marginal cost, which has two components: the cost of putting a worker on line for production (w) and the shared average cost of hiring a worker $((1-\beta)c)$. The hiring cost must be divided by the number of periods (with discounting) for which the worker is included in the labor stock (since $(1-\beta)c = \frac{c}{1+\beta+\beta^2+...}$). Alternatively, in order to produce one unit of inventory, the firm needs to incur the hiring cost (c) in addition to the wage cost of putting the worker on line (w). Due to durability, the firm also gets to save future hiring cost with discounted value of $-\beta c$. Thus, $(1-\delta)\beta[w + (1-\beta)c]$ in equation (3.6') is simply the value of inventory after depreciation. Since the shadow value of goods could be higher than its inventory value when demand is high, equation (3.6') is adjusted by the slackness multiplier $\pi^s \geq 0$. Hence, (3.6') implies that in case the current demand is low $(\pi_t^s = 0)$, the value of a good is its inventory value; and in case the current demand is high $(\pi_t^s > 0)$, the value of a good is its inventory value plus a markup. The average value of a good is given by (3.10) based on information Ω_{t-1}, which has to exceed its inventory value.[11]

Now consider period $t's$ decision after observing ε_t. Given that the *ex post* forecast error is $\mu_t = \varepsilon_t - \xi x_t$, there are two possible cases to consider:

Case A: $\mu_t < \mu_t^*$. In this case, the realized demand is low and the firm opts to incur inventories, $s_t > 0$. Hence, $\pi_t^s = 0$ and $\lambda_t^s = (1-\delta)\beta[w + (1-\beta)c]$ according to (3.6').

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[11] The expected value of goods will be different when the information set changes from Ω_{t-1} to Ω_{xt}, as (3.4') shows.

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In order to obtain closed-form solutions, assume a linear inverse demand function of the form $p = \theta - \frac{1}{2}\tau$, so that $r(\tau, \theta) = \theta\tau - \frac{1}{2}\tau^2$. Thus, marginal revenue is $\theta - \tau$. Equation (3.7) then implies that the optimal quantity of sales is given by

$$\tau_t = \theta_t - \eta, \tag{3.11}$$

where $\eta \equiv (1 - \delta)\beta[w + (1 - \beta)c]$ denotes the value of inventory. Namely, in the case that demand is low, the marginal revenue of output is simply its inventory value. The condition $s_t > 0$ implies that $0 < y_t + (1 - \delta)s_{t-1} - \tau_t = y_t + (1 - \delta)s_{t-1} + \eta - (E[\theta_t|\Omega_{xt}] + \mu_t)$, or equivalently,

$$\mu_t \quad < \quad y_t + (1 - \delta)s_{t-1} + \eta - E[\theta_t|\Omega_{xt}] \tag{3.12}$$

$$\equiv \quad \mu_t^*.$$

In other words, the cut-off value of μ_t is determined by $\mu_t^* \equiv y_t + (1 - \delta)s_{t-1} + \eta - E[\theta_t|\Omega_{xt}]$.

Case B: $\mu_t \geq \mu_t^*$. In this case, supply falls short of actual demand due to a large forecast error. Hence, the firm opts to stockout ($s_t = 0, \pi_t^s \geq 0$) and the optimal sales are given by the maximum supply, $\tau_t = y_t + (1 - \delta)s_{t-1}$. The shadow value of goods increases in this case since $\lambda_t^s = \eta + \pi_t^s \geq \eta$. Equation (3.7) then becomes

$$\lambda_t^s \quad = \quad \theta_t - y_t - (1 - \delta)s_{t-1} \tag{3.13}$$

$$= \quad \mu_t - \mu_t^* + \eta,$$

where the second equality is obtained by using the definition of μ_t^*. Hence, the markup for output price $(\lambda^s - \eta)$ when demand is high is measured by the unexpected forecast error, $\mu_t - \mu_t^* \geq 0$.

The probability of case A and case B again depends endogenously on the choice of the output level before the realization of demand (ε), since μ_t^* is determined by y_t. Based on case A and case B, the expected value of goods after seeing the signal x is given by

$$E[\lambda_t^s|\Omega_{xt}] = \int_{\mu_t < \mu_t^*} \eta \, dG(\mu) + \int_{\mu_t \geq \mu_t^*} [\mu_t - \mu_t^* + \eta] \, dG(\mu). \tag{3.14}$$

Therefore, the first-order condition with respect to L_t (equation 3.4′) becomes:

$$w + \lambda_t^R = \beta c + \int_{\mu_t < \mu_t^*} \eta \, dG(\mu) + \int_{\mu_t \geq \mu_t^*} [\mu_t - \mu_t^* + \eta] \, dG(\mu), \tag{3.4″}$$

which determines the optimal production level (y_t) based on information set Ω_{xt}.

Equation (3.4″) shows that the optimal output level depends also on the shadow value of labor (λ_t^R) because it is part of the marginal cost of production. However, λ^R depends on the tightness of the labor resource available to the firm, which in turn depends on the existing labor stock $(L_{t-1} + R_{t-1} + N_t)$ that is determined earlier before signal x is observed. Thus, to determine λ^R, we have two possible cases to consider:

Case C: $x_t < x_t^*$. The received signal at the end of period $t-1$ is low, indicating a low demand in period t. In this case, optimal demand for workers on line should be low because expected future demand is low. Hence it is optimal to hoard labor $(R_t > 0)$. Therefore, $\pi_t^R = 0$ and $\lambda_t^R = \beta c - \alpha w$ according to (3.5′), which suggests that the value of labor hoarding is given by the discounted value of gain by avoiding next-period hiring (βc) minus the current hoarding costs, αw. Substituting out λ^R in equation (3.4″) gives

$$(1-\alpha)w \;=\; \eta \int_{\mu_t < \mu_t^*} dG(\mu) + \int_{\mu_t \geq \mu_t^*} [\mu_t - \mu_t^* + \eta] \, dG(\mu) \tag{3.15}$$

$$\equiv\; \Gamma(\mu_t^*).$$

Equation (3.15) suggests that, conditioned on a low signal for demand $(x_t < x_t^*)$, the optimal cut-off value for μ_t is constant:

$$\mu_t^* = \bar{\mu}, \tag{3.16}$$

where $\bar{\mu}$ solves the equation, $\Gamma(\bar{\mu}) = (1-\alpha)w$, as defined in (3.15). The optimal output level (or the optimal number of workers on line) is thus determined by $y_t = \bar{\mu} + E\left[\theta_t | \Omega_{xt}\right] - (1-\delta)s_{t-1} - \eta$ according to (3.12).

Since the RHS of equation (3.15) is the average of two terms with the second term greater than the first, in order for an interior solution of $\bar{\mu}$ to exist, we must require $(1-\alpha)w > \eta \equiv (1-\delta)\beta\left[w + (1-\beta)c\right]$, where the LHS $((1-\alpha)w)$ is the value of labor hoarding $(=$ savings for production costs by putting one less worker on line), and the RHS (η) is the discounted present value of inventory after depreciation. Notice that this value is proportional to $[w + (1-\beta)c]$, which is the marginal cost of producing inventory. This requirement amounts to ensuring that the cost of labor hoarding be sufficiently low relative to the cost of holding inventories in order for case C $(R_t > 0)$ to be optimal. Suppose this condition does not hold. Then the RHS of equation (3.15) always exceeds the LHS, hence we must require $\pi_t^R > 0$ in equation (3.4″). This implies $R_t = 0$. In the special case of $\beta = 1$, the above requirement becomes $\alpha < \delta$, implying that the wage cost of hoarding a worker cannot exceed the value lost by holding inventory. Otherwise, the firm is better off producing inventory using the hoarded labor.

Case D: $x_t \geq x_t^*$. In this case, demand for workers on line is high because the observed signal x indicates a high demand. Hence, hoarding labor is not optimal, suggesting $R_t = 0$ and $y_t = L_{t-1} + R_{t-1} + N_t$ (producing at full capacity).

Again, the probability of case C and case D depends on the cut-off value x_t^*. To determine x_t^*, note that under case C, $y_t = \bar{\mu} + \gamma + \rho\theta_{t-1} + \xi x_t - (1-\delta)s_{t-1} - \eta$, hence the condition $R_t > 0$ implies $L_{t-1} + R_{t-1} + N_t - y_t > 0$ or $L_{t-1} + R_{t-1} + N_t - \bar{\mu} - \gamma - \rho\theta_{t-1} - \xi x_t + (1-\delta)s_{t-1} + \eta > 0$, or equivalently,

$$x_t \quad < \quad \frac{1}{\xi}[L_{t-1} + R_{t-1} + N_t - \bar{\mu} - \gamma - \rho\theta_{t-1} + (1-\delta)s_{t-1} + \eta] \tag{3.17}$$

$$\equiv \quad x_t^*.$$

The cut-off value for signal x is hence determined by (3.17). Clearly, the probability of case C and case D depends endogenously on the choice of x_t^*, which in turn depends on the hiring decisions made earlier (N_t) before x_t is observed.

3.2   Results

Proposition 4 *The optimal cut-off value (x_t^*) is a constant, $x_t^* = \bar{x}$. Hence, the optimal decision rule for hiring is given by*

$$N_t = \xi\bar{x} + \bar{\mu} + \gamma - \eta - L_{t-1} - R_{t-1} - (1-\delta)s_{t-1} + \rho\theta_{t-1}. \tag{3.18}$$

Proof. Define the *c.d.f* of x as $Z(x)$. The optimal level of hiring (N_t) based on information set Ω_{t-1} is given by equation (3.3),

$$c \quad = \quad E\left[\lambda_t^R | \Omega_{t-1}\right] \tag{3.19}$$

$$= \quad (\beta c - \alpha w)\int_{x_t < x_t^*} dZ(x) + \int_{x_t \geq x_t^*} (\beta c - w + E[\lambda_t^s|\Omega_{xt}])\, dZ(x)$$

where the first term is given by case A and the second term uses (3.4′). Recall that

$$E[\lambda_t^s|\Omega_{xt}] = \int_{\mu_t < \mu_t^*} \eta dG(\mu) + \int_{\mu_t \geq \mu_t^*} [\mu_t - \mu_t^* + \eta]\, dG(\mu) \tag{3.13′}$$

according to (3.14) where $\mu_t^* \equiv y_t + (1-\delta)s_{t-1} + \eta - E[\theta_t|\Omega_{xt}]$. Also recall that under case D, $y_t = L_{t-1} + R_{t-1} + N_t$. Hence, conditioned on $x_t \geq x_t^*$, the cut-off value in (3.13′) is

$$\mu_t^* \quad \equiv \quad L_{t-1} + R_{t-1} + N_t + (1-\delta)s_{t-1} + \eta - E[\theta_t|\Omega_{xt}] \tag{3.20}$$

$$\equiv \quad \bar{\mu} + \xi(x_t^* - x_t),$$

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where the second equality is obtained by using the definition for x_t^* in (3.17). Therefore, we have

$$\int_{x_t \geq x_t^*} E\left[\lambda_t^s | \Omega_{xt}\right] dZ(x) \tag{3.21}$$

$$= \int_{x_t \geq x_t^*} \left[\begin{array}{c} \left(\int_{\mu_t < \bar{\mu} + \xi(x_t^* - x_t)} \eta dG(\mu)\right) \\ + \left(\int_{\mu_t \geq \bar{\mu} + \xi(x_t^* - x_t)} \left[\mu_t - \bar{\mu} - \xi(x_t^* - x_t) + \eta\right] dG(\mu)\right) \end{array} \right] dZ(x)$$

$$\equiv \Pi(x_t^*).$$

Given that $\{x_t, \mu_t\}$ are $i.i.d$, the above expression is a function of x_t^* only and hence can be denoted as $\Pi(x_t^*)$. Thus condition (3.19) that solves for the optimal level of N_t is reduced to

$$c = (\beta c - \alpha w)Z(x_t^*) + (\beta c - w)\left[1 - Z(x_t^*)\right] + \Pi(x_t^*). \tag{3.22}$$

This suggests that a constant, $x_t^* = \bar{x}$, is the solution for (3.22). Using the definition for x_t^* in (3.17), one obtains the decision rule for hiring. ∎

The decision rules of the model can be summarized as follows:

$$N_t = \xi\bar{x} + \bar{\mu} + \gamma - \eta - L_{t-1} - R_{t-1} - (1-\delta)s_{t-1} + \rho\theta_{t-1} \qquad (3.23)$$

$$y_t = \begin{cases} \bar{\mu} + \gamma + \rho\theta_{t-1} + \xi x_t - (1-\delta)s_{t-1} - \eta & \text{if } x_t < \bar{x} \\ \bar{\mu} + \gamma + \rho\theta_{t-1} + \xi\bar{x} - (1-\delta)s_{t-1} - \eta & \text{if } x_t \geq \bar{x} \end{cases} \qquad (3.24)$$

$$\tau_t = \begin{cases} \theta_t - \eta & \text{if } \varepsilon_t < \bar{\mu} + \xi x_t \\ \bar{\mu} + \gamma + \rho\theta_{t-1} + \xi x_t - \eta & \text{if } \varepsilon_t \geq \bar{\mu} + \xi x_t \text{ and } x_t < \bar{x} \\ \bar{\mu} + \gamma + \rho\theta_{t-1} + \xi\bar{x} - \eta & \text{if } \varepsilon_t \geq \bar{\mu} + \xi x_t \text{ and } x_t \geq \bar{x} \end{cases} \qquad (3.25)$$

$$R_t = \begin{cases} \xi(\bar{x} - x_t) & \text{if } x_t < \bar{x} \\ 0 & \text{if } x_t \geq \bar{x} \end{cases} \qquad (3.26)$$

$$s_t = \begin{cases} \bar{\mu} + \xi x_t - \varepsilon_t & \text{if } \varepsilon_t < \bar{\mu} + \xi x_t \text{ and } x_t < \bar{x} \\ \bar{\mu} + \xi\bar{x} - \varepsilon_t & \text{if } \varepsilon_t < \bar{\mu} + \xi x_t \text{ and } x_t \geq \bar{x} \\ 0 & \text{if } \varepsilon_t \geq \bar{\mu} + \xi x_t \end{cases} \qquad (3.27)$$

$$\frac{y_t}{L_t + R_t} = \begin{cases} \frac{\xi x_t + \bar{\mu} + \gamma + \rho\theta_{t-1} - (1-\delta)s_{t-1} - \eta}{\xi\bar{x} + \bar{\mu} + \gamma + \rho\theta_{t-1} - (1-\delta)s_{t-1} - \eta} & \text{if } x_t < \bar{x} \\ 1 & \text{if } x_t \geq \bar{x} \end{cases} \qquad (3.28)$$

Notice that the unconditional mean of inventories is given by $(1 - \pi_s)\bar{\mu}$, where $\pi_s \equiv \Pr[\varepsilon_t < \bar{\mu} + \xi x_t]$ $= \Pr[\mu_t < \bar{\mu}]$. The unconditional mean of hoarded labor is given by $(1 - \pi_R)\xi\bar{x}$, where $\pi_R \equiv \Pr[x_t < \bar{x}]$. Hence we can interpret $\bar{\mu}$ as the target level of inventories and $\xi\bar{x}$ as the target level of labor hoarding. These target levels are strictly positive (except in degenerate cases) given the non-negativity constraints on the inventory stock and the hoarded labor stock, reflecting the precautionary motive for preventing stockout. These target levels become zero only in the limiting case where demand uncertainty is completely eliminated.

The dynamic interactions between inventories and labor hoarding are highlighted by the decision rules (3.26) and (3.27). Consider the case of $x_t < \bar{x}$. The decision rules for inventories and labor

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hoarding in this case are given by

$$\begin{cases} R_t = \xi(\bar{x} - x_t), \\ \\ s_t = \bar{\mu} + \xi x_t - \varepsilon_t. \end{cases} \tag{3.29}$$

Upon receiving a large signal x_t, which indicates a high demand, the firm opts to reduce labor hoarding and increase production so as to build up inventories to meet the anticipated demand increase. In fact, ξ units of reduction in labor hoarding is associated with ξ units of increase in the inventory stock. This strategic behavior of using hoarded labor to cope with anticipated rises in demand helps the firm to maintain a relatively stable level of inventory stock. Also note that one unit increase in demand (ε_t) directly reduces the inventory stock by one unit. But with the help of hoarded labor, the net change in the inventory stock is less than one unit (i.e., $|\xi - 1| < 1$). The reason that with a one unit anticipated increase in demand (x_t), production rises by less than one unit ($\xi < 1$), is that the signal contains noise. If the noise is zero ($\xi = 1$), the firm would be capable of using labor hoarding alone to completely meet anticipated changes in demand without using inventories. On the other hand, if the signal contains no information about demand ($\xi = 0$), the firm does not use the labor hoarding margin; it uses only inventories to buffer demand shocks.

Proposition 5 *The optimal volume of labor hoarding is zero if the firm does not have the information technology to update information about future demand (or equivalently, if $x_t = v_t$ and $\xi = 0$) after the decision for hiring but before the decision for production is made.*

Proof. See equation (3.26). ∎

Proposition 6 *The optimal volume of inventories is zero if there is perfect information about demand (i.e., $x_t = \varepsilon_t$ and $\xi = 1$) at the time of making production decisions.*

Proof. If $\mu_t = 0$, case A ($\mu_t < \mu_t^*, s_t > 0$) has zero probability since $G(\mu)$ is a degenerate step function and the support of μ collapses into a single point at $\mu = 0$. Hence the only sensible solution is the marginal case with $s_t = 0$ and $\mu_t^* = \bar{\mu} = 0$ (see equation 3.27). ∎

There is now a fast growing literature addressing the fact of a less volatile U.S. economy since the mid-1980s (e.g., see McConnell and Perez-Quiros 2000, and Stock and Watson 2002). This literature finds that the less volatile U.S. economy is mostly attributable to a less volatile inventory. One popular explanation for the inventory volatility reduction is that improved information technology and inventory management reduce inventory fluctuation by enhancing firms' ability to forecast demand (McConnell and Perez-Quiros, 2000). This information technology hypothesis is perfectly consistent with the above proposition. That is, an improved information technology reduces the

need for using inventories to buffer demand shocks, which reduces the variability of inventories, leading to less volatile output production.

Proposition 7 *The measured productivity of labor is procyclical as long as there is labor hoarding.*

Proof. Denote $\pi \equiv \Pr\left[x_t < \bar{x}\right]$. Denote y_{1t} as the log output when $x_t < \bar{x}$ and y_{2t} as the log output when $x_t \geq \bar{x}$. Notice that y_{1t} has larger variance than y_{2t} due to the presence of x_t. Also notice that the correlation between y_{1t} and y_{2t} is less than one because the corresponding x_t in y_{2t} is the constant $\bar{x}$. Then conditioned on $\pi > 0$ (i.e., there is labor hoarding), the correlation between log output and log productivity is given by

$$corr\left(y_{1t}, y_{1t} - y_{2t}\right) = 1 - corr\left(y_{1t}, y_{2t}\right) > 0.$$

∎

Proposition 8 *An increase in the hiring (firing) cost of labor (c) raises the optimal target level of both inventories ($\bar{\mu}$) and labor hoarding ($\xi\bar{x}$).*

Proof. Equation (3.15) can be rewritten as

$$(1 - \alpha)w = \eta + \int_{\mu_t \geq \mu_t^*} \left[\mu_t - \mu_t^*\right] dG(\mu).$$

Since the value of inventory, $\eta = (1-\delta)\beta\left[w + (1-\beta)c\right]$, increases with c, μ^* must increase in order to keep the above equation unchanged on both sides after an increase in c.

Equation (3.22) can be rearranged as

$$w + (1 - \beta)c = (1 - \alpha)wZ(x^*) + \Pi(x^*).$$

Differentiating both sides with respect to c gives

$$1 - \beta = (1 - \alpha)wz(x^*)\frac{dx^*}{dc} + \underbrace{\frac{\partial \Pi}{\partial x^*}\frac{dx^*}{dc}}_{(-)} + \underbrace{\frac{\partial \Pi}{\partial \mu^*}\frac{d\mu^*}{dc}}_{(-)},$$

where $z()$ is the $p.d.f$ of x^*. Note that $(1 - \alpha)wz(x^*) > 0$. Since it can be shown using the definition of $\Pi(x^*)$ in (3.21) that $\frac{\partial \Pi}{\partial \mu^*} < 0$, and $\frac{\partial \Pi}{\partial x^*}$ and $\frac{dx^*}{dc}$ have the opposite signs, the last two terms on the RHS of the above equation are negative (it was shown earlier that $\frac{d\mu^*}{dc} > 0$). Since the LHS is positive, it must be the case that $\frac{dx^*}{dc} > 0$ for the RHS of the above equation to be positive also. ∎

The previous two propositions not only reconfirm the conventional wisdom that the degree of labor hoarding depends positively on the size of the adjustment costs of the labor stock (e.g., hiring

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costs) since hoarding labor can help obviate or reduce the need for adjusting a firm's labor stock, but also reveals that similar effects of avoiding or reducing labor adjustment can also be achieved by inventories. This shows the substitutability between these two forms of strategic behavior of the firm in coping with demand uncertainty. It is also easy to see that as the cost of labor hoarding (α) increases, the optimal level of labor hoarding decreases while that of inventories increases. These results are consistent with Topel's (1982) analysis.

Proposition 9 *The target-level of inventory stock ($\bar{\mu}$) depends positively on the variance of demand (σ_ε^2).*

Proof. Given that $\mu_t = \varepsilon_t - \xi x_t = (1 - \xi)\varepsilon_t - \xi v_t$, we have $\sigma_\mu^2 = (1 - \xi)^2 \sigma_\varepsilon^2 + \xi^2 \sigma_v^2$. Recall that $\xi = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_v^2}$. Hence, it can be shown that $\frac{\partial \sigma_\mu^2}{\partial \sigma_\varepsilon^2} = (1 - \xi)^2 > 0$. Namely, the variance of the forecast error increases as the variance of demand increases. On the other hand, equation (3.15) indicates that $\Gamma(\mu^*)$ is monotonically decreasing in μ^* since the second term exceeds the first on the RHS. Now, consider an increase in the variance of μ_t that preserves the mean ($E\mu_t = 0$). Since a mean-preserving spread increases the weight of the tail of the distribution, the right hand side of (3.15) increases (since $\mu^* \geq 0$). Thus μ^* must also increase in order to keep the right-hand side of (3.15) unchanged. Therefore, a higher variance of ε will induce a higher value of $\mu^* = \bar{\mu}$. ∎

Proposition 10 *The target-level of labor hoarding ($\xi\bar{x}$) depends positively on the variance of demand (σ_ε^2).*

Proof. Given that $x_t = \varepsilon_t + v_t$, we have $\sigma_x^2 = \sigma_\varepsilon^2 + \sigma_v^2$. Hence, an increase in the variance of ε is associated with an increase in the variance of x. Equation (3.19) indicates that the RHS is monotonically decreasing in μ^* since the first term on the RHS is less than c (the LHS) and the second term must therefore be greater than c. Now, consider an increase in the variance of x that preserves the mean ($Ex_t = 0$). Since a mean-preserving spread increases the weight of the tail of the distribution, the right hand side of (3.19) increases (since $x^* \geq 0$). Thus x^* must also increase in order to keep the right-hand side of (3.19) unchanged. Therefore, a higher variance of ε is associated with a higher value of $x^* = \bar{x}$. It is easy to see that ξ increases with σ_ε^2 but decreases with σ_v^2, hence $\xi\bar{x}$ increases with the variance of ε. ∎

Proposition 11 *The variance of labor hoarding increases with the variance of demand (ε), and the increase is more than the increase (if any) in the variance of inventories.*

Proof. By equations (3.26) and (3.27), in the case that there is labor hoarding ($x_t < \bar{x}$), the variance of R_t is given by $\sigma_R^2 = \xi^2(\sigma_\varepsilon^2 + \sigma_v^2)$, which increases with σ_ε^2; and the variance of s_t is given

by $\sigma_s^2 = (1 - \xi)^2 \sigma_\varepsilon^2 + \xi^2 \sigma_v^2$, which does not respond to changes in σ_ε^2 since

$$
\begin{aligned}
\frac{\partial \sigma_s^2}{\partial \sigma_\varepsilon^2} &= -2(1 - \xi)\sigma_\varepsilon^2 \frac{\partial \xi}{\partial \sigma_\varepsilon^2} + 2\xi\sigma_v^2 \frac{\partial \xi}{\partial \sigma_\varepsilon^2} \\
&= -2\left[(1 - \xi)\sigma_\varepsilon^2 + \xi\sigma_v^2\right] \frac{\partial \xi}{\partial \sigma_\varepsilon^2} \\
&= 0.
\end{aligned}
$$

Call this case the case α. On the other hand, in the case that there is no labor hoarding $(x_t \geq \bar{x})$, the variance of R_t is zero and the variance of s_t is $\pi\xi^2\sigma_\varepsilon^2$ (where $\pi \equiv \Pr[\mu_t < \bar{\mu}]$), which increases with σ_ε^2. Call this case the case β. Since an increase in σ_ε^2 makes case α (labor hoarding) more likely and case β less likely (i.e., $\bar{x}$ increases with σ_ε^2 implies $\Pr[x_t < \bar{x}]$ increases with σ_ε^2), we conclude that the variance of labor hoarding (σ_R^2) increases and the increase is more than the increase (if any) in the variance of inventories (σ_s^2). ∎

The intuition behind this result is that hoarded labor neutralizes the impact of demand shocks on inventories. The existence of inventories depends positively on the size of the forecast error. However, the size of the forecast error depends negatively on the variance of the true demand relative to noise, since the smaller the noise relative to true demand, the larger is the value of ξ and the better is the quality of the information, hence the more responsive is labor hoarding to demand shocks. A larger response in labor hoarding leads to less movement in inventories (see equation 3.29).[12] Thus, not only is the increase in the variance of inventories less, but it is even possible for the variance of inventories to decrease as the variance of demand increases. This result appears to contradict the predictions of the existing inventory literature (e.g., see Kahn 1987, and Maccini and Zabel 1996). The reason, though, is simple. In the inventory literature, inventories exist because of production lags, and labor hoarding does not factor into the analysis. Hence the variance of inventories increases with demand uncertainty because a higher uncertainty is associated with a larger amount of unwanted inventories when the demand is low. Here, inventories exist not because of production lags *per se* but because of imperfect information. A larger variance of demand reduces the relative size of noise and improves the quality of information, inducing the firm to adjust production more fully by utilizing hoarded labor. This raises the variance of labor hoarding. Since larger adjustments in production (or labor hoarding) reduce the impact of demand shocks on inventories, this may lead to less variable inventories. In other words, as the variance of demand increases, the burden of adjustment falls largely upon labor utilization (labor hoarding) instead of upon inventories. This would not be the case without the option of labor hoarding.

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[12]Mathematically, we have $\frac{\partial R_t}{\partial \varepsilon_t} = -\xi$ and $\frac{\partial s_t}{\partial \varepsilon_t} = \xi - 1$ in the case of labor hoarding $(x_t < \bar{x})$. Thus, a higher value of ξ (due to a high value of σ_ε^2) increases the response of labor hoarding but reduces the response of inventories.

Hence, labor hoarding can alter the dynamics of inventories in an important way. The analysis confirms Blinder's (1982) conjecture regarding firms' strategic behavior under demand uncertainty. That is, inventories of labor are partial substitutes for inventories of goods as a means to cope with demand shocks.

4   Conclusion

This paper provides a simple model to study firms' optimal decisions of labor hoarding and their dynamic interactions with inventory decisions. It shows that the conventional wisdom which explains the existence of labor hoarding is inadequate. With the option of inventories, labor adjustment costs alone do not sufficiently justify labor hoarding because firms can produce inventories by using hoarded labor. Other conditions must also be met in order for labor hoarding to be optimal. These conditions include lower costs of hoarding labor than holding inventories, and an information-updating technology that makes labor hoarding both necessary and optimal as information about demand unfolds. In the absence of either of these conditions, firms will opt to use inventories to cope with demand uncertainty despite the fact that labor hoarding is an option and that labor is a quasi-fixed factor (due to adjustment costs).

While the required information technology for information updating appears to be reasonable, the relative cost structure of labor hoarding versus inventories, however, is an empirical question that has to be further investigated by reference to the data. Therefore, this paper not only provides a simple and convenient framework for studying labor hoarding, but also raises new empirical questions for future studies. These empirical questions include (to name just a few): How to correctly measure and compare labor hoarding costs and inventory holding costs for any particular firm? What is the elasticity of substitution between inventories and hoarded labor? How to assess a firm's information structure about future demand? What would be the consequence of estimating an inventory model without taking into account firms' labor decisions, or vice versa? These important questions and related issues regarding inventories and labor hoarding have only begun to gain attention from the profession very recently (see Galeotti, Maccini and Schiantarelli, 2005).

It is now a well-known stylized fact that the U.S. economy has become less volatile since the mid-1980s (e.g., see McConnell and Perez-Quiros 2000, and Stock and Watson 2002). One popular explanation for the volatility reduction is that improved information technology and inventory management reduce inventory fluctuation by enhancing firms' ability to forecast demand (McConnell and Perez-Quiros, 2000). The model presented in this paper is perfectly consistent with this information-technology hypothesis. In addition, the model also offers a second alternative explanation for the volatility reduction in GDP. That is, the reduction in adjustment costs of labor

due to improved labor market competition can also reduce the need for using inventories and labor hoarding to cope with demand shocks. Based on this alternative explanation, not only inventories but also employment can become less volatile after labor market conditions are improved. This may also explain the less volatile U.S. economy since the mid-1980s. Exploring further and empirically testing this plausible alternative explanation is a potentially fruitful project worth pursuing in the future.

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