Where's the Beef? The Trivial Dynamics of Real Business Cycle Models

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Abstract: The extremely weak propagation mechanisms of real business cycle (RBC) models are well acknowledged, and some effort has been devoted to improving the models on this dimension. This paper builds on these efforts to provide an explicit explanation of why various existing RBC models do not replicate real world business cycles, and discusses modifications necessary to bring real business cycle theory into closer conformity with the data. (JEL E13, E32)

Key Words: Business Cycle Modeling, Propagation Mechanism.
1. Introduction

Despite success in matching several characteristics of the data, one of the most salient shortcomings of real business cycle (RBC) theory lies in its predictions for the dynamics of the growth of U.S. output. The standard RBC model predicts that output growth is essentially a white noise process, but the actual autocovariance generating function of the growth rate of post-war U.S. GNP has a striking periodic pattern at business cycle frequencies (see, e.g., Watson (1993)). Hence, though standard RBC models can produce persistence in the level of output, they lack the propagation mechanism necessary to generate movements around business cycle frequencies in output growth. More bluntly, business cycles do not really exist in current real-business-cycle model economies. This paper provides a very simple explanation for why this is the case.

Early in the development of RBC theory, it was recognized that persistent technology shocks were required for an equilibrium business cycle model to generate a serial correlation pattern in output that matched observed data (e.g., King-Plosser-Rebelo, 1988). A later realization, based on experience with such models, was that the intrinsic real business cycle model dynamics were essentially the same as the extrinsic dynamics generated by exogenous shocks. For example, upon simulating economies with and without "time to build" features, Rouwenhorst (1991) found that these features had much less to do with model-generated fluctuations than did the nature of the technology shock process driving the model. Using a standard RBC model, Campbell (1994) studied the macroeconomic effects of the elasticity of intertemporal substitution and the persistence of shocks. While the weak propagation mechanism of the model was not his focus, he did note that the interest rate in the model was too smooth, and that this was due to the highly persistent nature of the capital stock.

In an more extensive simulation study, Cogley and Nason (1995) focussed specifically on the weak propagation mechanism of RBC models. In particular, they studied several different types of RBC models which included capital adjustment costs, time-to-build, employment adjustment costs, etc. They found that the models could not produce the hump-shaped impulse response patterns and volatilities
observed in U.S. data. They went on to conclude that RBC models must rely on impulse dynamics to replicate observed output dynamics.

The regularities observed by Rouwenhorst, Campbell, and Cogley and Nason, have a simple explanation: The reason that standard RBC models cannot propagate shocks is that capital is the only endogenous state variable, and it is smooth. Because capital is the only state variable, the decision rules for output, investment, and employment possess at most an ARMA(1,1) structure. The smoothness of the capital stock causes the AR and MA parameters to be nearly the same -- a "pole-zero" cancellation which converts the ARMA(1,1) into an ARMA(0,0) in terms of the technology shock. Therefore, the model dynamics mimic those of the input. In what follows, I expand on this theme by describing the fundamental empirical problem faced by RBC models and outlining the feature of a prototypical RBC model which leads to this empirical failure, and then discussing conditions necessary to enrich the propagation mechanisms of RBC models.

2. A Simple Example

As is well known, quarterly aggregate output growth in the post-war U.S. is significantly positively autocorrelated at least for the first 2-3 lags, and negatively autocorrelated at longer lags. This is a clear indication for the existence of dynamic multiplier effects and cycles, because it implies that the impulse responses of the level of output should display a hump-shaped pattern with a distinctive peak at about the 3rd or 4th quarter.¹

To understand why various standard RBC models fail to generate persistent and cyclical propagation mechanisms in responding to shocks, consider a simple RBC model with fixed labor. Assume: (1) The momentary utility function of a representative agent at time t is given by a concave

¹The hump-shaped pattern of impulse response function of output is also preserved and shared by other variables in a multivariate VAR for output, consumption, investment and employment. The univariate case of output is chosen simply as an illustration of the point.
function, \( u(c_t, l - N) \), where \( c_t \) is consumption, \( N \) is hours worked and is fixed for the moment; (2) The production function is given by \( y_t = A k_t^\alpha N^{1-\alpha} \), where \( y_t \) is output, \( A_t \) is a technology shock, \( k_t \) is capital input; (3) The investment technology is given by \( i_t = k_{t+1} - (1-\delta)k_t \), where \( \delta \) is the rate of depreciation for capital; (4) There is no government sector and the economy is closed, so, \( y_t = c_t + i_t \).

To maximize expected lifetime utility, the representative agent attempts to smooth his or her consumption path. In this simple framework, consumption smoothing is achieved by allowing investment to absorb most of the impact of exogenous shocks in each period. This feature also enables the model to replicate the observed relative volatility ordering among consumption, output, and investment. To see why, denote by \( \%x_t \) the percentage deviation from steady state for variable \( x_t \). The income identity \( (y_t = c_t + i_t) \) then implies \( \%y_t = (1-s)\%c_t + s\%i_t \), where \( s \in (0,1) \) is the steady-state investment-output ratio. Hence, the percentage deviation of output from the steady state is a convex combination of the percentage deviations of consumption and investment. Consumption smoothing implies \( \%c_t < \%y_t \), which then in turn implies \( \%i_t > \%y_t \). Hence, the volatility ordering for the 3 variables is \( \%i_t > \%y_t > \%c_t \).

But, strictly speaking, \( \%x_t \) is not the volatility measure adopted in RBC literature. It measures only the log level deviations of \( x_t \) from its trend (or mean), not the variance of such deviations. In order for the above conclusion to hold, we need the following more rigorous arguments:

Denoting \( \sigma_x \) as the standard deviation of variable \( \%x_t \), the income identity then implies

\[
(1-s)^2 \sigma^2_c = \sigma_y^2 + s^2 \sigma_i^2 - 2s \sigma_{yi}
\]

Consumption smoothing implies choosing the covariance between investment and output to solve (approximately):

\[
\min \sigma^2_c = \frac{1}{(1-s)^2(\sigma_y^2 + s^2 \sigma_i^2 - 2s \sigma_{yi})}
\]

subject to the constraint that the covariance of output and investment be positive semi-definite, i.e., \( |\sigma_{yi}| \leq \sigma_y \sigma_i \). By inspection, the solution sets \( \sigma_{yi} = \sigma_y \sigma_i \). This implies \((1-s)\sigma_c = |\sigma_y \sigma_i| \). Since this equality must hold for all \( s \in [0,1] \), we must have \( \sigma_{cy} \sigma_{cy} \) and, consequently, \((1-s)\sigma_c + \sigma_{cy} = \sigma_y \). Hence, \( \sigma_{cy} \sigma_{yi} \implies \sigma_{cy} \sigma_y \). For example, suppose the steady-state investment-output ratio is 0.3 and the relative standard deviation of consumption to output is 0.6. Then the relative standard deviation of investment to output is about 2. It is also easy to check that the inequality, \( \sigma_{cy} \sigma_{cy} \), is satisfied. However, as the persistence of technology
The propagation mechanism of this simple RBC model can be represented by the following transmission chain: a positive technology shock increases output, which in turn increases investment, which augments the next period's capital stock, which in turn increases the next period's output, and so on. Schematically,

\[ A_t \rightarrow y_t \rightarrow i_t \rightarrow k_{t+1} \rightarrow y_{t+1} \rightarrow i_{t+1} \rightarrow k_{t+2} \rightarrow \ldots \]

However, this propagation chain is extremely weak even in the case in which all changes in output (around the steady state) are matched by changes in investment. To see this, log-linearize the production function, \( y_t = A_t k_t^\alpha \), the income identity, \( y_t = c_t + i_t \), and the law of motion for the capital stock, \( k_{t+1} = i_t + (1-\delta)k_t \), around the steady state:

\[ \% y_t = \% A_t + \alpha \% k_t, \]

\[ (1-s)\%c_t + s\%i_t = \%y_t, \]

\[ \%k_{t+1} = \delta \%i_t + (1-\delta)\%k_t. \]

Given the specified production technology, in equilibrium a one percent increase in total productivity \((A_t)\) will induce a one percent increase in output \(y_t\). Supposing that the steady state investment-output ratio is approximately 0.3 - a value consistent with U.S. data - and that consumption growth is kept constant, then there will be at most about a 3.3% increase in investment \((1/0.3 \times 1\%)\). The next-period increment in the capital stock will then be approximately \((3.3\delta)\%\). Since the percentage increment in shock increases, the above static minimization problem becomes less accurate in approximating the consumer's dynamic programming problem. Because consumption responds to permanent changes in income, a more persistent technology shock induces more volatile consumption. Hence, the optimal solution will no longer be to minimize the variance of consumption.

Nevertheless, the same conclusion still holds, we now should have an inequality instead of an equality: \((1-s)\%c_t + s\%i_t \geq \%y_t\), which derives from the observation that \(0 \leq \%y_t \leq \%c_t \leq \%i_t\). In such a case, the same relative volatility ratio of 2 for investment would now imply a higher relative volatility ratio than 0.6 for consumption, or vice versa. However, in order to have \(\%c_t \leq \%y_t\) at the first place, that the sources of shocks are in the supply side is crucial for RBC models.
output equals the share of capital times the percentage increment in the capital stock, the output increment in period $t+1$ will be about $\rho\% + \alpha(3.3\delta)\%$, where $\rho$ measures the persistence of the technology shock. Using empirically plausible values for $\alpha$ (0.4) and $\delta$ (0.025), the increment in output in the second quarter will be only about 0.033% above its steady state, implying that the percentage increase in output in the next period attributable to last period's investment (i.e., the newly constructed capital) is relatively small, and so is the new investment which follows. Hence, in the absence of employment growth or persistence in the technology shock, the initial 1% increase in output and 3.3% increase in investment are quickly damped down to approximately 0.033% and 0.11% respectively after only one period. In other words, the impulse responses of $y_t$ and $i_t$ shrink by a factor of 30 after only one period!

The bottom line is that this model economy lacks an internal mechanism to propagate shocks. As a result, the growth rates of output and investment are negatively serially correlated, and their impulse responses mimic those of technology shocks. Hence, many variations based on this prototypical RBC model do not propagate shocks either. For example, incorporating endogenous labor choice into the model can help only to amplify shocks at the initial period, but not to propagate shocks for the following periods. Although there is now an additional margin along which the agent can respond in the impact period by adjusting labor supply, employment will soon return to its original steady state: since the capital stock has changed so little during the course of time, so must the capital-labor ratio. Therefore, variable employment does not help to propagate technology shocks over time.

Several other variants of this model have been explored; however, the new features of these models fail to resolve the basic problem. For example, assuming non-time-separable preferences in the form of Kydland and Prescott (1982) will not help to enrich the model's propagating mechanism either. The increased degree of intertemporal substitution of leisure will generate even more negatively autocorrelated employment growth, which will only worsen the situation discussed above.

The mechanism of time-to-build (Kydland-Prescott 1982), which assumes that investment takes
more than one period to become productive capital, does not resolve the problem: time-to-build does not in any way induce output or investment growth in the second period, but only postpones new capital formation. (The output increment may even drop more in the second period, as a result of delayed capital formation.)

If the utilization rate of capital is variable, as in the model of Greenwood et al. (1988), then the current technology shock can be further amplified, due to the additional slackness of productive capacity that can be exploited. However, similar to the role of endogenous labor choice, capital utilization can help only in amplifying shocks, not in propagating shocks, because the role of capital utilization rate is essentially like that of hours worked. Their contributions to the persistence of output responses are thus very similar: in the absence of a new technology shock, both of them will quickly fall back to their steady state level. Because the capital stock has changed little, the equilibrium capital-labor ratio and capital utilization ratio will show minimal change. Consequently, there is little contribution to output growth after the first period.

It then follows that the richer propagating mechanism existing in the factor hoarding model of Burnside and Eichenbaum (1994) is purely due to labor hoarding behavior, not to variable capital utilization. The reason that labor hoarding behavior can enrich the propagation mechanism of a standard RBC model is the following. Since employment needs to be determined one period in advance, every variable in the model except employment will respond to a technology shock in the same way as in the standard model for the first period. The crucial difference lies in the period after the shock. If the technology shock is highly persistent, then employment will increase in the second period after the

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3 This statement is equivalent to that employment decisions at date t are made based on information at date t-1.
shock in order to exploit the expected technology shock residual in the future. Such a release of suppressed employment may trigger even greater positive responses from the economy in the second period than in the first period, especially if the elasticity of labor supply is high enough, or if there exist additional sources of slackness in the economy due to a low rate of capital utilization or intensity of labor effort. Thus, labor hoarding functions as a delayed push in transmitting the technology shock for one additional period. This is, however, not substantial enough to allow the model to fully replicate the dynamics of U.S. output growth.

To summarize, the reason that various existing RBC models do not significantly propagate business cycle shocks is that they rely exclusively on capital accumulation as the propagation mechanism, and capital is an extremely poor medium for the propagation of business cycles. A necessary step toward the construction of real business cycle propagation mechanism is thus to incorporate additional sources of persistent dynamics into the RBC framework. For this, we turn to the next section.

3. Richer Propagation Mechanisms

In this section, I use equilibrium decision rules to discuss the conditions necessary to enrich the propagation mechanisms of a standard equilibrium business cycle model. The linearized decision rules for output (y) and capital (k) in a typical RBC model driven by a technology shock (A) can be expressed

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4 If technology shock does not persist, then the labor hoarding model will behave just like the standard model in terms of the transmission mechanism, because employment will be unresponsive to a temporary technology shock and hence be constant over time.

5 As mentioned earlier, the impulse response of post war U.S. GNP displays a smooth hump-shaped pattern and reaches its peak at about the 4th quarter. But the impulse response of output implied by labor hoarding models peaks right at the second quarter. This indicates that the propagation of shocks in this model is still not rich enough.
as:

\[
\hat{y}_t = \pi_{yk} \hat{k}_t + \pi_{ya} \hat{A}_t
\]

\[
\hat{k}_{t+1} = \pi_{kk} \hat{k}_t + \pi_{ka} \hat{A}_t
\]

(2)

where hat variables denote log-linearized variables around the steady state (which also correspond to the transitory components of variables when the Blanchard-Quah (1989) VAR decomposition is applied). The coefficients can be interpreted as elasticities.

The excessive smoothness of the capital stock implies that the autoregressive coefficient \(\pi_{kk}\) in the capital equation is close to one and the technology impact parameter \(\pi_{ka}\) is close to zero. Since output is very responsive to technology shocks and relatively not responsive to changes in capital stock, \(\pi_{ka}\) is very large in comparison to \(\pi_{yk}\) (this follows from the production function: the capital elasticity of output is at most 0.4, capital's share; the technology elasticity of output is at least one, since employment helps amplifying the impact of technology shocks on output).\(^6\) This particular pattern of parameter value has important implications, which can be seen as follows.

Solving for capital in (2) and substituting into output in (2) produces

\[
(1 - \pi_{kk} L) \hat{y}_t = \pi_{ya} \hat{A}_t + (\pi_{yk} \pi_{ka} - \pi_{kk} \pi_{ya}) \hat{A}_{t-1}
\]

(3)

where \(L\) is the lag operator. Since both \(\pi_{yk}\) and \(\pi_{ka}\) are extremely small relative to \(\pi_{kk}\) and \(\pi_{ya}\), this expression can be approximated as

\[
(1 - \pi_{kk} L) \hat{y}_t = \pi_{ya} (1 - \pi_{kk} L) \hat{A}_t.
\]

(4)

Pole-zero cancellation on both sides of the equation then gives

\[
\hat{y}_t = \pi_{ya} \hat{A}_t.
\]

(5)

Therefore, the dynamics of output in a typical RBC model essentially reflect that of the technology

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\(^6\) For example, in the standard RBC model of King-Plosser-Rebelo (1988), \(\pi_{kk}=0.953, \pi_{ka}=0.137, \pi_{ya}=1.608\), and \(\pi_{yk}=0.249\).
shock. First differencing equation (5), we get

\[ \Delta \hat{y}_t = \pi_{yu} \Delta \hat{A}_t. \quad (6) \]

Since the technology shock is nearly a random walk process, the growth rate of output in a standard RBC model is therefore essentially white noise. This accounts for the typical spectral shape of output growth in the RBC model found by Watson (1993).

From equation (5), it follows that the best hope to enrich the dynamics of output in a typical RBC model lies in the introduction of additional endogenous state variables.\(^7\) To see why, suppose \(x_t\) is another state variable in the time \(t\) state space \(\Omega_t\), so that

\[ \hat{y}_t = \pi_{yu} \hat{A}_t + \pi_{yx} \hat{x}_t. \quad (7) \]

\(^7\)Cogley and Nason (1993, 1995) also noticed the kind of pole-zero cancellation in a particular RBC model when numerical expressions for the ARMA structure of output were derived. But they didn't explain why the poles and zeros are similar, i.e., what leads to the pole-zero cancellation and why this could happen. Hence, in order to check whether or not the pole-zero cancellation takes place, they need to derive numerical ARMA expressions for each different RBC model studied. This exercise is not different from impulse response simulations. But from the discussion given above, we see that it is not needed; since a general ARMA structure and the implied pole-zero cancellation can be derived simply based on the fact that capital is the only state variable and is very smooth.

However, if capital is fully depreciated after one period, then it becomes identical to investment flow and is hence very volatile. In this case, consumption, investment, output, and capital all share the same persistence parameter, which is the capital's share in the production function (0.4). This is so because the optimal decision rule for consumption becomes proportional to total income if there is no stock of wealth. The internal propagation mechanism of the model is still weak under such circumstances. This has been pointed out by Blanchard and Fischer (1990).

\[^8\] The introduction of an additional shock variable will not resolve the problem. For example, Cogley and Nason (1995) discuss a two-shock RBC model in which one shock is a permanent technology shock and the other is a stationary government spending shock. In such a case, the trend stationary (transitory) component of output, \(\hat{y}_t\), will still be approximately a linear function of the government spending shock. For this reason, when the Blanchard-Quah (1989) decomposition is applied to the model, Cogley and Nason find that the transitory component of output behaves essentially like the government spending shock.
Since $x_t$ is an endogenous variable, it must be a function of the state space at date $t-1$, i.e., $x_t = f(\Omega_{t-1})$. The simplest case would be that

$$\dot{x}_t = \pi_{xa} \hat{A}_{t-1}. \tag{8}$$

Then the growth rate for output becomes a MA(2) process:

$$\Delta \hat{y}_t = \pi_{ya} \Delta A_t + \pi_{yx} \pi_{xa} \Delta A_{t-1}. \tag{9}$$

which means that technology shock can be propagated for an additional period of time if both $\pi_{yx}$ and $\pi_{xa}$ are significantly greater than zero. And this is exactly the dynamic behavior of output growth implied by the labor hoarding model of Burnside et al (1993) and the factor hoarding model of Burnside and Eichenbaum (1994). The corresponding state variable for $x_t$ in these models is the employment level $N_t$, which (because of labor hoarding) is determined based on information at date $t-1$.

Another simple case for $x_t = f(\Omega_{t-1})$ is:

$$\dot{x}_t = \pi_{xx} \dot{x}_{t-1} + \pi_{xa} \hat{A}_{t-1}. \tag{10}$$

so that $x_t$ is an infinite distributed lag of technology $A_t$. An example of such a state variable can be found in Cooper and Haltiwanger (1993). In their model, there exists a dynamic macroeconomic complementarity in the form of a lagged productive externality:

$$y_t = \bar{y}_{t-1} \phi f (A_t, k_t, N_t), \phi > 0 \tag{11}$$

where aggregate output has an external effect on the individual sector's productivity with a one period lag. This production function implies that the additional state variable $x_t$ is $y_{t-1}$, and the linearized decision rule for output can be expressed as:

$$\dot{y}_t = \pi_{yy} \dot{y}_{t-1} + \pi_{ya} \hat{A}_t. \tag{12}$$

So the lagged productive externality helps to propagate shocks over time. To see the implication for the

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9To avoid the kind of pole-zero cancellation which occurred in the case of capital, such state variable $x_t$ needs to be very volatile (large $\pi_{xa}$).
spectral shape of output growth, suppose that $\pi_{yy}$ is sufficiently high (say, 0.9 because of a large dynamic externality). Then only mild persistence in technology shock (say, $\rho=0.4$) is required for the model to reproduce the hump-shaped impulse response function of the level of output and the spectral peak of output growth at business cycle frequencies. More specifically, letting $\epsilon$ denote the innovation of technology and operating on both sides of (12) with $(1-0.4L)$, we have

$$\hat{y}_t = 1.3 \hat{y}_{t-1} - 0.36 \hat{y}_{t-2} + \epsilon_t,$$

(13)

(where $\pi_{ya}$ is normalized to one). A one standard deviation in the shock term leads to a sequence of output responses as $\{1, 1.30, 1.33, 1.26, 1.16, 1.05, 0.95, ...\}$, which peaks at the third quarter. The spectral density function of output growth can be found as

$$f_y(e^{-i\omega}) = \frac{1}{2\pi} \left| \frac{1 - e^{-i\omega}}{1 - 1.3 e^{-i\omega} + 0.36 e^{-2i\omega}} \right|^2 \sigma_\epsilon^2,$$

(14)

which has a distinctive peak at the business cycle frequency $\omega=0.1\pi$, corresponding to a 20-quarter (5-year) cycle.

Another example illustrates the case in which the additional state variable is an infinite distributed lag of the technology shock. In this example, the dynamics of output are enriched by allowing habit formation in the leisure-labor choice:

$$u(c_t, N_t) = \log(c_t) - \frac{\sigma}{1-\gamma} (N_t - \rho N_{t-1})^{1-\gamma}, \rho \in (0,1), \gamma < 0$$

(15)

where the utility ($u$) of a representative agent depends positively on consumption ($c$) and negatively on a distributed lag of labor supply ($N$). Due to habit formation ($\rho>0$), higher labor supply in the past period induces higher labor supply in the current period. The decision rules for output and labor can be expressed as:

$$\hat{y}_t = \pi_{yn} \hat{N}_{t-1} + \pi_{ya} \hat{A}_t$$

$$\hat{N}_t = \pi_{mn} \hat{N}_{t-1} + \pi_{na} \hat{A}_t$$

(16)

In this case, the corresponding state variable for $x_t$ is $N_{t+1}$, and the growth rate of output follows an
ARMA process:\(^{10}\)

$$
\Delta \hat{y}_t = \pi_{nn} \Delta \hat{y}_{t-1} + \pi_{ya} \Delta A_t + (\pi_{yn} \pi_{na} - \pi_{nn} \pi_{ya}) \Delta \hat{A}_{t-1}.
$$

(17)

Since the distributed lag of leisure in the utility function can be of any order with a wide range of parameter values, the implied dynamics for output growth can mimic many ARMA(p,q) process.

4. Conclusion

A satisfactory theory of business cycles must be able to account for the salient periodic transmission mechanisms observed in aggregate U.S. data. This remains a major challenge to RBC theory, because the transmission mechanism of standard real business cycle models, the aggregate capital stock, does not permit shocks to output to persist any longer than the technology shocks which induced them. Some recent work has produced additional sources of endogenous dynamics, but it remains to be seen whether combinations of these sources will produce sufficient persistence.

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\(^{10}\) Since employment is quite volatile, similar pole-zero cancellation as in the case of capital does not happen here. Habit formation on consumption, on the other hand, does not work. This is the case because consumption is very smooth, hence shocks cannot be significantly propagated through the autocorrelation properties of consumption.
References


