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International Asset Allocation under Regime Switching, Skew and Kurtosis Preferences*

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Abstract

This paper proposes a new tractable approach to solving asset allocation problems in situations with a large number of risky assets which pose problems for standard approaches. Investor preferences are assumed to be defined over moments of the wealth distribution such as its mean, variance, skew and kurtosis. Time-variations in investment opportunities are represented by a flexible regime switching process. In the context of a four-moment international CAPM specification that relates stock returns in five regions to returns on a global market portfolio, we find evidence of distinct bull and bear states. Ignoring regimes, an unhedged US investor's optimal portfolio is strongly diversified internationally. The presence of regimes in the return distribution leads to a large increase in the investor's optimal holdings of US stocks as does the introduction of skew and kurtosis preferences. Our paper therefore offers an explanation of the strong home bias observed in US investors' asset allocation based on regime switching and skew and kurtosis preferences.

Key words: International Asset Allocation, Regime Switching, Skew and Kurtosis Preferences, Home Bias.

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Abstract

This paper proposes a new tractable approach to solving asset allocation problems in situations with a large number of risky assets which pose problems for standard approaches. Investor preferences are assumed to be defined over moments of the wealth distribution such as its mean, variance, skew and kurtosis. Time-variations in investment opportunities are represented by a flexible regime switching process. In the context of a four-moment international CAPM specification that relates stock returns in five regions to returns on a global market portfolio, we find evidence of distinct bull and bear states. Ignoring regimes, an unhedged US investor's optimal portfolio is strongly diversified internationally. The presence of regimes in the return distribution leads to a large increase in the investor's optimal holdings of US stocks as does the introduction of skew and kurtosis preferences. Our paper therefore offers an explanation of the strong home bias observed in US investors' asset allocation based on regime switching and skew and kurtosis preferences.

1. Introduction

Despite the increased integration of international capital markets, investors continue to hold equity portfolios that are largely dominated by domestic assets. According to Thomas, Warnock and Wongswan (2004), by the end of 2003 US investors held only 14% of their equity portfolios in foreign stocks at a time when such stocks accounted for 54% of the world market capitalization.¹ This evidence is poorly understood: Calculations reported by Lewis (1999) suggest that a US investor with mean-variance preferences should hold upwards of 40% in foreign stocks or, equivalently, only 60% in US stocks.

Potential explanations for the home bias include barriers to international investment and transaction costs (Black (1990), Stulz (1981)); hedging demand for stocks that have lower correlations with domestic state variables such as inflation risk or non-traded assets (Adler and Dumas (1983), Serrat (2001)); information asymmetries and higher estimation uncertainty for foreign than domestic stocks (Gehrig (1993), Brennan and Cao (1997)) and political/country risk (Erb et al. (1996)).²

As pointed out by Lewis (1999) and Karolyi and Stulz (2002), the first of these explanations is weakened by the fact that barriers to international investment have come down significantly over the last thirty years and by the large size of gross investment flows. Yet there is little evidence that US investors' holdings of foreign stocks have been increasing over the last decade where this share has fluctuated around 10-15%. The second explanation is weakened by the magnitude by which foreign stocks should be correlated more strongly with domestic risk factors as compared with domestic stocks. In fact, correlations with deviations from purchasing power parity can exacerbate the home bias puzzle (Cooper and Kaplanis (1994)) as can the strong positive correlation between domestic stock returns and returns on human capital (Baxter and Jermann (1997)). It is also not clear that estimation uncertainty provides a good explanation.³ Finally, political risk seems to apply more to emerging and developing financial markets and is a less obvious explanation of investors' limited diversification among stable developed economies. Observations such as these lead Lewis (1999, p. 589) to conclude that "Two decades of research on equity home bias have yet to provide a definitive answer as to why domestic investors do not invest more heavily in foreign assets."

This paper proposes a new explanation for the home bias. We modify the standard international CAPM (ICAPM) specification that assumes mean-variance preferences over a time-invariant distribution of local stock returns in two ways. First, we allow investor preferences to depend not only on the first two moments of returns but also on third and fourth moments such as skew and kurtosis. This turns out to be important because the co-skew and co-kurtosis properties of US stocks with the world market portfolio make these stocks attractive to domestic investors. Our approach follows recent papers such as Harvey and Siddique (2000), Dittmar (2002) and Harvey, Liechty, Liechty and Muller (2004) that emphasize the need to consider moments beyond the mean and variance in portfolio choice and asset pricing applications.

Second, we model local stock returns in the context of a four-moment ICAPM with regimes that track

¹Similar home biases are present in other countries, see French and Poterba (1991) and Tesar and Werner (1994).

²Behavioral explanations (e.g. 'patriotism' or a generic preference for 'familiarity') have been proposed by Coval and Moskowitz (1999) and Morse and Shive (2003). Uppal and Wang (2003) provide theoretical foundations based on ambiguity aversion. Other papers have explored the effects of heterogeneity in the quality of corporate governance (investor protection) on international portfolio diversification, e.g. Dahlquist et al. (2004).

³When investors have strong beliefs that the world ex-US portfolio has a zero alpha, Pastor (2000) finds that US investors' home bias can be explained in a CAPM context where the US domestic market is the benchmark portfolio and the world ex-US portfolio is an additional asset. However, in the more common setting used in international finance where the world portfolio is the benchmark, the smallest allocation to non-US stocks generated in his model is 30 percent.

time-variations in the volatility, skew and kurtosis of the world market portfolio. In addition, we allow the world price of covariance, co-skew and co-kurtosis risk to vary across regimes. Empirical evidence suggests that returns on stocks and other financial assets can be captured by this class of models.⁴ The regime switching model accurately approximates the return distribution and captures volatility clustering, return correlations that strengthen in down markets, outliers that occur simultaneously in several markets, fat tails and skewness. We find evidence of two regimes in the joint distribution of international stock returns: A bear state with high volatility and low mean returns and a bull state with high mean returns and low volatility. Variations in the skew and kurtosis of the world market portfolio are also linked to uncertainty induced by regime switches. The uncertainty surrounding a switch from a bull to a bear state takes the form of an increased probability of large negative returns (high kurtosis and large negative skew). When exiting from the bear state to the bull state, the kurtosis again goes up—reflecting the increased uncertainty associated with a regime shift—while the volatility and skew decline to their normal levels.

Both modifications of the standard model are needed to explain the home country bias. Regimes in the distribution of international equity returns generate skew and kurtosis and therefore affect the asset allocation of a mean-variance investor differently from that of an investor whose objectives depend on higher moments of returns. This is significant since the single state model is severely misspecified and fails to capture basic features of international stock market returns.

Our sample estimates suggest that a US mean-variance investor with access to the US, UK, European, Japanese and Pacific stock markets should hold only 30 percent in domestic stocks. The presence of bull and bear states raises this investor’s weight on US stocks to 50 percent. Introducing both skew and kurtosis preferences and bull and bear states further increases the weight on US stocks to 70 percent of the equity portfolio.

Accounting for a relatively large set of risky assets as we do in our analysis creates problems for standard techniques. An additional contribution of our paper is therefore to propose a new tractable approach to optimal asset allocation that is both convenient to use and offers new insights into asset allocation problems in the presence of regime switching. When coupled with a utility specification that incorporates skew and kurtosis preferences, the otherwise complicated numerical problem of optimal asset allocation is reduced to that of solving for the roots of a low-order polynomial. The ability of our approach to solve the portfolio selection problem in the presence of multiple risky assets is important since gains from international asset allocation can be quite sensitive to the number of included assets.⁵

Four papers are closely related to ours. Ang and Bekaert (2002) consider bivariate and trivariate regime switching models that capture asymmetric correlations in volatile and stable markets and characterize a US investor’s optimal asset allocation under power utility. Our analysis extends Ang and Bekaert’s to include a wider set of stock markets and employs a moment-based utility specification that offers advantages both computationally and in terms of the economic intuition for how results change relative to the case with mean-variance preferences. Furthermore, we work with a model that has a straightforward interpretation as a time-varying version of the ICAPM in which the types (co-skewness and co-kurtosis in addition to

⁴See, e.g., Ang and Bekaert (2002), Ang and Chen (2002), Bekaert and Harvey (1995), Engel and Hamilton (1990), Guidolin and Timmermann (2006), Gray (1996), Perez-Quiros and Timmermann (2000) and Whitelaw (2001).

⁵For example, using all-equity portfolios and power utility with a coefficient of risk aversion of five, Ang and Bekaert (2002) find that the null of no international diversification cannot be rejected for a US investor who also considers UK stocks. However, this hypothesis is strongly rejected when the US investor has access to both UK and German stocks.

covariance risk), quantities and prices of risk are allowed to depend on an underlying state variable that has an intuitive interpretation in terms of bull and bear states in international equity markets.

Harvey, Liechty, Liechty and Muller (2004) propose a Bayesian framework for portfolio choice based on (second and third-order) Taylor expansions of an underlying expected utility function. They assume that the distribution of asset returns is a multivariate skewed normal. In their application to an international diversification problem, they find that under third-moment preferences, roughly 50 percent of the equity portfolio should be invested in US stocks. A different but related approach is proposed by Das and Uppal (2004) who use a multivariate jump-diffusion model in which jumps affect several markets simultaneously. This captures the stylized fact that large declines occur simultaneously across international stock markets. Correlated jumps provide an alternative to capturing the existence of (unconditional) skew and fat tails in the empirical distribution of asset returns. In fact, Das and Uppal find that under levels of (relative) risk aversion similar to the ones employed in our paper, it can be optimal to limit the extent of international portfolio diversification. While our model shares some intuition with this approach, the bull and bear states identified by our regime switching model are quite different and do not identify isolated outliers or jumps.⁶

Dittmar (2002) investigates the asset pricing implications of a single-state four-moment CAPM and finds that it offers considerable explanatory power for the cross-section of US stock returns. The resulting pricing kernel is a polynomial function in aggregate wealth. Like Dittmar's, our approach approximates the unknown marginal utility function by means of a Taylor series expansion of the utility function. However, differently from Dittmar, we allow the quantity and price of risk to follow a regime switching process and explore the international portfolio choice implications of the model.

The plan of the paper is as follows. Section 2 describes the return process in the context of an ICAPM extended to account for higher order moments, time-varying returns and regime switching and reports empirical results for this model. Section 3 sets up the optimal asset allocation problem for an investor with a polynomial utility function over terminal wealth when asset returns follow a regime switching process. Section 4 describes the solution to the optimal asset allocation problem, while Section 5 reports extensions and robustness checks. Section 6 concludes. Appendices provide technical details.

2. A Four-Moment ICAPM with Regime Switching in Asset Returns

Our assumptions about the return process build on extensive work in asset pricing based on the no-arbitrage stochastic discount factor model for (gross) returns on an arbitrary asset (i) R_{t+1}^i :

$$E[R_{t+1}^i m_{t+1} | \mathcal{F}_t] = 1 \quad i = 1, \dots, h. \quad (1)$$

Here $E[. | \mathcal{F}_t]$ is the conditional expectation given information available at time t , \mathcal{F}_t , and m_{t+1} is the investor's intertemporal marginal rate of substitution between current and future consumption or—under restrictions established by Brown and Gibbons (1985)—current and future wealth.

The two-moment CAPM follows from this equation when the pricing kernel, m_{t+1} , is linear in the returns on an aggregate wealth portfolio. Harvey (1991) shows that when countries are viewed as equity portfolios in a globally integrated market, differences across country portfolios' expected returns should be driven by

⁶Other papers have considered international asset pricing models under regime switching (Bekaert and Harvey (1995)) and the effects of non-normalities and higher order moments on international portfolio choice (Bekaert et al. (1998)).

their conditional covariances with returns on a world market portfolio, R_{t+1}^W :

$$E[R_{t+1}^i | \mathcal{F}_t] - R_t^f = \frac{E[R_{t+1}^W | \mathcal{F}_t] - R_t^f}{Var[R_{t+1}^W | \mathcal{F}_t]} Cov[R_{t+1}^i, R_{t+1}^W | \mathcal{F}_t]. \quad (2)$$

Here both equity returns, R_{t+1}^i , and the conditionally risk free return, R_t^f , are expressed in the same currency (e.g. US dollars).

The two-moment CAPM can be extended to account for higher order terms such as $Cov[R_{t+1}^i, (R_{t+1}^W)^2 | \mathcal{F}_t]$ and $Cov[R_{t+1}^i, (R_{t+1}^W)^3 | \mathcal{F}_t]$ that track the conditional co-skew or co-kurtosis between the aggregate (world) portfolio and local portfolio returns. Such terms follow from a nonlinear model for the pricing kernel that depends on higher order powers of returns on the world market portfolio. Consistent with this, and building on Harvey and Siddique (2000) and Dittmar (2002), suppose that the pricing kernel can be approximated through a third-order Taylor series expansion of the marginal utility of returns on aggregate wealth:

$$m_{t+1} = g_{0t} + g_{1t}R_{t+1}^W + g_{2t}(R_{t+1}^W)^2 + g_{3t}(R_{t+1}^W)^3, \quad (3)$$

where $g_{jt} = U^{j+1}/U'$ is the ratio of derivatives of the utility function (where $U^{(1)} \equiv U'$ is the first derivative, etc.) evaluated at current wealth. Assuming positive marginal utility ($U' > 0$), risk aversion ($U'' < 0$), decreasing absolute risk aversion ($U''' > 0$) and decreasing absolute prudence ($U'''' < 0$), it follows that $g_{1t} < 0$, $g_{2t} > 0$ and $g_{3t} < 0$.

Since (1) implies

$$E[R_{t+1}^i | \mathcal{F}_t] = \frac{1}{E[m_{t+1} | \mathcal{F}_t]} - \frac{Cov[R_{t+1}^i, m_{t+1} | \mathcal{F}_t]}{E[m_{t+1} | \mathcal{F}_t]},$$

the cubic pricing kernel (3) gives rise to a four-moment asset pricing model:

$$E[R_{t+1}^i | \mathcal{F}_t] - R_t^f = \gamma_{1t}Cov(R_{t+1}^i, R_{t+1}^W | \mathcal{F}_t) + \gamma_{2t}Cov(R_{t+1}^i, (R_{t+1}^W)^2 | \mathcal{F}_t) + \gamma_{3t}Cov(R_{t+1}^i, (R_{t+1}^W)^3 | \mathcal{F}_t), \quad (4)$$

where $\gamma_{jt} = -g_{jt}R_t^f$ ($j = 1, 2, 3$) so $\gamma_{1t} > 0$, $\gamma_{2t} < 0$ and $\gamma_{3t} > 0$, assuming that a conditionally riskfree asset exists. This means that covariance and co-kurtosis risk earn positive risk premia while co-skew risk earns a negative risk premium. The positive premium on co-kurtosis risk suggests that the standard CAPM covariance premium carries over to ‘large’ returns. Co-skew earns a negative risk premium since an asset with a high return during times when the world portfolio is highly volatile is desirable to risk averse investors.

There are good reasons to be skeptical about the exact validity of (4). On theoretical grounds, a reason for the failure of the CAPM to hold exactly in an international context is that it requires the world market portfolio to be perfectly correlated with world consumption (Stulz (1981)). Furthermore, Bekaert and Harvey (1995) show that limited international capital market integration means that terms such as $Var[R_{t+1}^i | \mathcal{F}_t]$ will affect the risk premium. On empirical grounds, conditional CAPM specifications have been tested extensively for international stock portfolios and found to have significant limitations. Harvey (1991) reports that not all of the dynamic behavior of country returns is captured by a two-moment model and interprets this as evidence of either incomplete market integration, the existence of other priced sources of risk or model misspecification. The four-moment CAPM also ignores the presence of persistent ‘regimes’ documented for stock returns in the papers cited earlier.

2.1. Regime Switches

To allow for conditional time-variations in the return process and the possibility of misspecification biases, we extend the four-moment CAPM as follows. First, consistent with (3) and (4) we assume that returns on the world market portfolio depend not only on the conditional variance, $Var[R_{t+1}^W|\mathcal{F}_t]$, but also on the conditional skew, $Sk[R_{t+1}^W|\mathcal{F}_t]$, and kurtosis, $K[R_{t+1}^W|\mathcal{F}_t]$ of this portfolio.⁷ Furthermore, to use a flexible representation without imposing too much structure, the price of risk associated with these moments is allowed to depend on a latent state variable, S_{t+1} , that is assumed to follow a Markov process but is otherwise not restricted. In turn this state-dependence carries over to the price of the risk factors appearing in the equations for returns on the individual stock market portfolios, denoted by $\gamma_{1,S_{t+1}}$ (covariance risk), $\gamma_{2,S_{t+1}}$ (co-skew risk) and $\gamma_{3,S_{t+1}}$ (co-kurtosis risk). Finally, consistent with empirical evidence in the literature (Harvey (1989) and Ferson and Harvey (1991)) we allow for predictability of returns on the world market portfolio through a vector of instruments, \mathbf{z}_{t+1} , assumed to follow some autoregressive process. Defining excess returns on the h individual country portfolios, $x_{t+1}^i = R_{t+1}^i - R_t^f$ ($i = 1, \dots, h$) and the world portfolio, $x_{t+1}^W = R_{t+1}^W - R_t^f$, our model is

$$\begin{aligned} x_{t+1}^i &= \alpha_{S_{t+1}}^i + \gamma_{1,S_{t+1}} Cov[x_{t+1}^i, x_{t+1}^W|\mathcal{F}_t] + \gamma_{2,S_{t+1}} Cov[x_{t+1}^i, (x_{t+1}^W)^2|\mathcal{F}_t] + \gamma_{3,S_{t+1}} Cov[x_{t+1}^i, (x_{t+1}^W)^3|\mathcal{F}_t] \\ &\quad + \mathbf{b}_{S_{t+1}}^i \mathbf{z}_t + \eta_{t+1}^i \\ x_{t+1}^W &= \alpha_{S_{t+1}}^W + \gamma_{1,S_{t+1}} Var[x_{t+1}^W|\mathcal{F}_t] + \gamma_{2,S_{t+1}} Sk[x_{t+1}^W|\mathcal{F}_t] + \gamma_{3,S_{t+1}} K[x_{t+1}^W|\mathcal{F}_t] + \mathbf{b}_{S_{t+1}}^W \mathbf{z}_t + \eta_{t+1}^W \\ \mathbf{z}_{t+1} &= \boldsymbol{\mu}_{z,S_{t+1}} + \mathbf{B}_{zS_{t+1}} \mathbf{z}_t + \boldsymbol{\eta}_{t+1}^Z. \end{aligned} \tag{5}$$

Consistent with the restrictions implied by the four-moment ICAPM, the risk premia $\gamma_{j,S_{t+1}}$ ($j = 1, 2, 3$) are common across the individual assets and the world market portfolio. However, we allow for asset-specific intercepts, $\alpha_{S_{t+1}}^i$, that capture other types of misspecification. The innovations $\boldsymbol{\eta}_{t+1} \equiv [\eta_{t+1}^1 \dots \eta_{t+1}^h \eta_{t+1}^W (\boldsymbol{\eta}_{t+1}^Z)'] \sim N(\mathbf{0}, \boldsymbol{\Omega}_{S_{t+1}})$ can have a state-dependent covariance matrix, while the predictor variables, \mathbf{z}_{t+1} , follow a first order autoregressive process with state-dependent parameters, $\mathbf{B}_{zS_{t+1}}$. This is consistent with the persistence in commonly used predictor variables.

To complete the model we assume that the state variable, S_t , follows a k -state Markov process with constant transition probability matrix, \mathbf{P} :

$$\mathbf{P}[i, j] = \Pr(s_t = j | s_{t-1} = i) = p_{ij}, \quad i, j = 1, \dots, k. \tag{6}$$

Our model can thus be viewed as a time-varying version of the multi-beta latent variable model of Ferson (1990) where both the risk premia and the amount of risk depend on a latent first-order Markov state variable.

There are several advantages to modelling returns in this way. Conditional on knowing the state next period, S_{t+1} , the return distribution is Gaussian. However, since future states are not known in advance, the return distribution is a mixture of normals with weights reflecting the current state probabilities. Such mixtures of normals provide a flexible representation that can be used to approximate many distributions (Harvey and Zhou (1993)). They can accommodate mild serial correlation in returns—documented for returns on the world market portfolio by Harvey (1991)—and volatility clustering since they allow the first

⁷In what follows, conditional skewness and kurtosis are defined as $Sk[R_{t+1}^W|\mathcal{F}_t] \equiv E[(R_{t+1}^W - E(R_{t+1}^W|\mathcal{F}_t))^3|\mathcal{F}_t]$ and $K[R_{t+1}^W|\mathcal{F}_t] \equiv E[(R_{t+1}^W - E(R_{t+1}^W|\mathcal{F}_t))^4|\mathcal{F}_t]$, respectively.

and second moments to vary as a function of the underlying state probabilities (Timmermann (2000)). Finally, multivariate regime switching models allow return correlations across markets to vary with the underlying regime, consistent with the evidence of asymmetric correlations in Longin and Solnik (2001) and Ang and Chen (2002).

To gain intuition for (5), consider the special case with a single state where the price of risk is constant and—because the innovations $\boldsymbol{\eta}_{t+1} \sim N(\mathbf{0}, \boldsymbol{\Omega})$ are drawn from a time-invariant distribution—the higher moment terms $Cov[x_{t+1}^i, (x_{t+1}^W)^2 | \mathcal{F}_t]$, $Cov[x_{t+1}^i, (x_{t+1}^W)^3 | \mathcal{F}_t]$, $Sk[x_{t+1}^W | \mathcal{F}_t]$, and $K[x_{t+1}^W | \mathcal{F}_t]$ are constant and hence do not explain variations in returns:

$$\begin{aligned} x_{t+1}^i &= \alpha^i + \gamma_1 Cov[x_{t+1}^i, x_{t+1}^W | \mathcal{F}_t] + \mathbf{b}^i \mathbf{z}_t + \eta_{t+1}^i \\ x_{t+1}^W &= \alpha^W + \gamma_1 Var[x_{t+1}^W | \mathcal{F}_t] + \mathbf{b}^W \mathbf{z}_t + \eta_{t+1}^W \\ \mathbf{z}_{t+1} &= \boldsymbol{\mu}_z + \mathbf{B}_z \mathbf{z}_t + \boldsymbol{\eta}_{t+1}^Z. \end{aligned} \tag{7}$$

This is an extended version of the ICAPM in which instruments (\mathbf{z}_t) are allowed to predict the risk premia and alphas are not restricted to be zero ex-ante. When the restrictions $\alpha^i = \alpha^W = 0$ and $b^i = b^W = 0$ are imposed on all return equations, (7) simplifies to the standard ICAPM in which $\gamma_1 = E[x_{t+1}^W | \mathcal{F}_t] / Var[x_{t+1}^W | \mathcal{F}_t]$ so

$$E[x_{t+1}^i | \mathcal{F}_t] = \frac{Cov[x_{t+1}^i, x_{t+1}^W | \mathcal{F}_t]}{Var[x_{t+1}^W | \mathcal{F}_t]} E[x_{t+1}^W | \mathcal{F}_t] \equiv \beta_t^i \cdot E[x_{t+1}^W | \mathcal{F}_t].$$

2.2. Moments of Returns

Our asset pricing model (5) depends on moments of returns on the world market portfolio in addition to the covariances, co-skew and co-kurtosis between returns on the local and global market portfolios. Estimating the skew and kurtosis of asset returns is difficult (Harvey and Siddique (2000)). However, our mixture model allows us to obtain precise conditional estimates in a flexible manner as it captures skew and kurtosis as a function of the mean, variance and persistence parameters of the underlying states. Furthermore, as we next show, when the world price of covariance, co-skew and co-kurtosis risk is identical across all markets, the model implies a tight set of restrictions across asset returns.

Letting $\mathbf{y}_{t+1} = (\mathbf{x}_{t+1}', x_{t+1}^W, \mathbf{z}_{t+1}')'$ be a vector of excess returns and predictor variables with intercepts $\boldsymbol{\mu}_{S_{t+1}} = (\alpha_{S_{t+1}}^1, \dots, \alpha_{S_{t+1}}^h, \alpha_{S_{t+1}}^W, \boldsymbol{\mu}_{z_{S_{t+1}}}')$, we can collect the conditional moments of returns and the world price of co-moment risk in the matrices \mathbf{M}_{S_t} and $\boldsymbol{\Upsilon}_{S_{t+1}}$ as follows

$$\begin{aligned} \mathbf{M}_{S_t} &\equiv \left(\left[\begin{array}{ccc} Cov[\mathbf{x}_{t+1}, x_{t+1}^W | \mathcal{F}_t] & Cov[\mathbf{x}_{t+1}, (x_{t+1}^W)^2 | \mathcal{F}_t] & Cov[\mathbf{x}_{t+1}, (x_{t+1}^W)^3 | \mathcal{F}_t] \\ Var[x_{t+1}^W | \mathcal{F}_t] & Sk[x_{t+1}^W | \mathcal{F}_t] & K[x_{t+1}^W | \mathcal{F}_t] \end{array} \right] \otimes \boldsymbol{\iota}_3' \right) \odot (\boldsymbol{\iota}_3' \otimes \mathbf{I}) \\ &\quad \mathbf{O} \\ \boldsymbol{\Upsilon}_{S_{t+1}} &\equiv \begin{bmatrix} \gamma_{1,S_{t+1}}^1 & \dots & \gamma_{1,S_{t+1}}^h & \gamma_{1,S_{t+1}}^W & 0 & \dots & 0 \\ \gamma_{2,S_{t+1}}^1 & \dots & \gamma_{2,S_{t+1}}^h & \gamma_{2,S_{t+1}}^W & 0 & \dots & 0 \\ \gamma_{3,S_{t+1}}^1 & \dots & \gamma_{3,S_{t+1}}^h & \gamma_{3,S_{t+1}}^W & 0 & \dots & 0 \end{bmatrix}. \end{aligned}$$

where $\boldsymbol{\iota}_3$ is a 3×1 vector of ones and \mathbf{J} is a matrix that selects the co-moments of excess returns:

$$\mathbf{J} \equiv \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

We can then write (5) more compactly as

$$\mathbf{y}_{t+1} = \boldsymbol{\mu}_{S_{t+1}} + \mathbf{M}_{S_t} \text{vec}(\boldsymbol{\Upsilon}_{S_{t+1}}) + \mathbf{B}_{S_{t+1}} \mathbf{y}_t + \boldsymbol{\eta}_{t+1}. \quad (8)$$

Here $\mathbf{B}_{S_{t+1}}$ captures autoregressive terms in state S_{t+1} and also collects the coefficients $\mathbf{b}_{S_{t+1}}^i$ and $\mathbf{b}_{S_{t+1}}^W$ that measure the impact of the lagged instruments \mathbf{z}_t on the risk premia; $\boldsymbol{\eta}_{t+1} \sim N(\mathbf{0}, \boldsymbol{\Omega}_{S_{t+1}})$ is the vector of heteroskedastic innovations.

To characterize the moments of returns on the world market portfolio and the co-moments with local market returns, note that mean returns can be computed from

$$\bar{\mathbf{y}}_{t+1} \equiv E[\mathbf{y}_{t+1} | \mathcal{F}_t] = \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \tilde{\boldsymbol{\mu}}_l + \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \mathbf{A}_l \mathbf{y}_t, \quad (9)$$

where $\boldsymbol{\pi}_t$ is the vector of state probabilities, \mathbf{e}_l is a vector of zeros with a one in the l -th position so $(\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l)$ is the ex-ante probability of being in state S_{t+1} at time $t+1$ given information at time t , \mathcal{F}_t , and $\tilde{\boldsymbol{\mu}}_l \equiv \boldsymbol{\mu}_l + \mathbf{M}_{S_t} \text{vec}(\boldsymbol{\Upsilon}_l)$.

Because $\tilde{\boldsymbol{\mu}}_l$ involves higher order moments of the world market portfolio such as $\mathbf{M}_{S_t} \text{vec}(\boldsymbol{\Upsilon}_l)$ as well as higher order co-moments between individual portfolio returns and returns on the global market portfolio, the (conditional) mean returns $E[\mathbf{y}_{t+1} | \mathcal{F}_t]$ enter the right-hand side of (8). For instance, computing $\text{Cov}[\mathbf{x}_{t+1}, x_{t+1}^W | \mathcal{F}_t]$ requires knowledge of the first h elements of $E[\mathbf{y}_{t+1} | \mathcal{F}_t]$. Appendix B explains our iterative estimation procedure used to solve the associated nonlinear optimization problem.

The conditional variance, skew and kurtosis of returns on the world market portfolio, x_{t+1}^W , can now be computed as follows:

$$\begin{aligned} \text{Var}[x_{t+1}^W | \mathcal{F}_t] &= \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \left[(\tilde{\mu}_l^W - \mathbf{e}'_{h+1} \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_{h+1} \mathbf{A}_l - \bar{\alpha}_{h+1}) \mathbf{y}_t)^2 \right] + \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \text{Var}[\eta_{t+1}^W | S_{t+1} = l] \\ \text{Sk}[x_{t+1}^W | \mathcal{F}_t] &= \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \left[(\tilde{\mu}_l^W - \mathbf{e}'_{h+1} \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_{h+1} \mathbf{A}_l - \bar{\alpha}_{h+1}) \mathbf{y}_t)^3 \right] \\ &\quad + 3 \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \left[(\tilde{\mu}_l^W - \mathbf{e}'_{h+1} \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_{h+1} \mathbf{A}_l - \bar{\alpha}_{h+1}) \mathbf{y}_t) \text{Var}[\eta_{t+1}^W | S_{t+1} = l] \right] \\ K[x_{t+1}^W | \mathcal{F}_t] &= \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \left[(\tilde{\mu}_l^W - \mathbf{e}'_{h+1} \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_{h+1} \mathbf{A}_l - \bar{\alpha}_{h+1}) \mathbf{y}_t)^4 \right] \\ &\quad + 6 \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \left[(\tilde{\mu}_l^W - \mathbf{e}'_{h+1} \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_{h+1} \mathbf{A}_l - \bar{\alpha}_{h+1}) \mathbf{y}_t)^2 \text{Var}[\eta_{t+1}^W | S_{t+1} = l] \right]. \end{aligned} \quad (10)$$

Clearly the skew and kurtosis are functions of the mean and variance parameters $\{\tilde{\mu}_{i,l}, \dots, \tilde{\mu}_{h,l}, \mathbf{A}_l, \mathbf{\Omega}_l\}_{l=1}^k$, state probabilities, $\boldsymbol{\pi}_t$, and the mean of the VAR coefficients, $\bar{\alpha}_j \equiv \mathbf{e}'_j \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \mathbf{A}_l$. Hence, no new parameters are introduced to capture the higher moments of the return distribution. Such model-based estimates are typically determined with considerably more accuracy than estimates of the third and fourth moments obtained directly from realized returns which tend to be very sensitive to outliers.

Similarly, the covariance between country returns, x_{t+1}^i , and the world market return, x_{t+1}^W , is

$$\begin{aligned} Cov[x_{t+1}^i, x_{t+1}^W | \mathcal{F}_t] &= \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \left[(\tilde{\mu}_{i,l} - \mathbf{e}'_i \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_i \mathbf{A}_l - \bar{\alpha}_i) \mathbf{y}_t) (\tilde{\mu}_l^W - \mathbf{e}'_{h+1} \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_{h+1} \mathbf{A}_l - \bar{\alpha}_{h+1}) \mathbf{y}_t) \right] \\ &\quad + \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) Cov[\eta_{t+1}^i, \eta_{t+1}^W | S_{t+1} = l], \end{aligned} \quad (11)$$

Given estimates of the parameters and state probabilities, $Cov[x_{t+1}^i, x_{t+1}^W | \mathcal{F}_t, S_t]$ can easily be calculated.

Finally, the co-skew and co-kurtosis between local market returns and the world market return is

$$\begin{aligned} Cov[x_{t+1}^i, (x_{t+1}^W)^2 | \mathcal{F}_t] &= \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \left[(\tilde{\mu}_{i,l} - \mathbf{e}'_i \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_i \mathbf{A}_l - \bar{\alpha}_i) \mathbf{y}_t) (\tilde{\mu}_l^W - \mathbf{e}'_{h+1} \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_{h+1} \mathbf{A}_l - \bar{\alpha}_{h+1}) \mathbf{y}_t)^2 \right] \\ &\quad + \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \left[(\tilde{\mu}_{i,l} - \mathbf{e}'_i \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_i \mathbf{A}_l - \bar{\alpha}_i) \mathbf{y}_t) Var[\eta_{t+1}^W | S_{t+1} = l] \right] \\ &\quad + 2 \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \left[(\tilde{\mu}_l^W - \mathbf{e}'_{h+1} \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_{h+1} \mathbf{A}_l - \bar{\alpha}_{h+1}) \mathbf{y}_t) Cov[\eta_{t+1}^i, \eta_{t+1}^W | S_{t+1} = l] \right] \end{aligned}$$

and

$$\begin{aligned} Cov[x_{t+1}^i, (x_{t+1}^W)^3 | \mathcal{F}_t] &= \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \left[(\tilde{\mu}_{i,l} - \mathbf{e}'_i \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_i \mathbf{A}_l - \bar{\alpha}_i) \mathbf{y}_t) (\tilde{\mu}_l^W - \mathbf{e}'_{h+1} \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_{h+1} \mathbf{A}_l - \bar{\alpha}_{h+1}) \mathbf{y}_t)^3 \right] \\ &\quad + 3 \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \left[(\tilde{\mu}_{i,l} - \mathbf{e}'_i \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_i \mathbf{A}_l - \bar{\alpha}_i) \mathbf{y}_t) (\tilde{\mu}_l^W - \mathbf{e}'_{h+1} \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_{h+1} \mathbf{A}_l - \bar{\alpha}_{h+1}) \mathbf{y}_t) Var[\eta_{t+1}^W | S_{t+1} = l] \right] \\ &\quad + 3 \sum_{l=1}^k (\boldsymbol{\pi}'_t \mathbf{P} \mathbf{e}_l) \left[(\tilde{\mu}_l^W - \mathbf{e}'_{h+1} \bar{\mathbf{y}}_{t+1} + (\mathbf{e}'_{h+1} \mathbf{A}_l - \bar{\alpha}_{h+1}) \mathbf{y}_t)^2 Cov[\eta_{t+1}^i, \eta_{t+1}^W | S_{t+1} = l] \right]. \end{aligned}$$

Terms such as $(\tilde{\mu}_{i,l} - \mathbf{e}'_i \bar{\mathbf{y}}_{t+1}) (\tilde{\mu}_l^W - \mathbf{e}'_{h+1} \bar{\mathbf{y}}_{t+1})$ show the deviations of the state-specific mean from the overall mean and do not arise in single-state models.

2.3. Data

In addition to the world market portfolio, our analysis incorporates the largest international stock markets, namely the United States, Japan, the United Kingdom, the Pacific region (ex-Japan), and continental Europe. More markets could be included but parameter estimation errors are likely to become increasingly important when more markets are included so we do not go beyond five equity portfolios in addition to the world market portfolio ($h = 6$).⁸

⁸ At the end of 2005 these markets represented roughly 97% of the world equity market capitalization.

Following common practice, we consider returns from the perspective of an unhedged US investor and examine excess returns in US dollars on Morgan Stanley Capital International (MSCI) indices.⁹ The risk-free rate is measured by the 30-day US T-bill rate provided by the Center for Research in Security Prices. Our data are monthly and cover the sample period 1975:01 - 2005:12, a total of 372 observations. Returns are continuously compounded and adjusted for dividends and other non-cash payments to shareholders. A number of studies have documented the leading role of US monetary policy and the US interest rate as a predictor of returns across international equity markets.¹⁰ Consistent with the analysis in Ang and Bekaert (2002), we therefore include the short US T-bill rate as a predictor variable. Again our framework allows more variables to be included at the cost of having to estimate more parameters.

Table 1 reports summary statistics for the international stock returns, the world market portfolio and the US T-bill rate. Mean returns are positive and lie in a range between 0.37 and 0.75 percent per month. Return volatilities vary from four to seven percent per month. Comparing the performance across stock markets, US stock returns are characterized by a fairly high mean and low volatility. Returns in all but one market (Japan) are strongly non-normal with skews and fat tails as also found by Harvey and Zhou (1993) and Das and Uppal (2004). The strong rejection of normality suggests that a flexible model is required to accommodate skews and fat tails in the return distribution. While the short US interest rate is highly persistent, there is little evidence of serial correlation in excess stock returns. However, many of the excess return series display strong evidence of time-varying volatility.

2.4. Empirical Results

As a benchmark Panel A of Table 2 reports parameter estimates for the standard single-state two-moment CAPM (7). Alphas are positive in five regions and economically large but imprecisely estimated and statistically insignificant. Our model's failure to capture returns in Japan is consistent with the strong rejections for Japan in the two-moment CAPM tests reported in Harvey (1991) and is perhaps to be expected in view of the gradual liberalization of financial markets in Japan during the 1980s and the analysis in Bekaert and Harvey (1995). The negative coefficients on the lagged T-bill rate are also consistent with the literature. At 5.3, the estimated world price of covariance risk, γ_1 , is positive and significant as expected.

Next consider the model with two states, estimates of which are shown in Panel B of Table 2. In the first state the regression coefficients on the lagged T-bill rate were found to be insignificant for all stock markets and hence we impose that these coefficients are zero. In the second state the coefficients on the T-bill rate are large and negative and most are significant. Notice also that short-term rates are more persistent and volatile in the bear state, so clearly short-term rates help to identify the state.

Alpha estimates are negative in state 1 but positive in state 2 for all portfolios. The alphas in the two states may appear to be quite large in economic terms.¹¹ However, as they measure returns conditional on being in a particular state and the state is never known in advance, they are not directly comparable to the corresponding estimates from the single state model. To account for this, we simulated 50,000 returns from the two-state model over a 12-month horizon, allowing for regime shifts and uncertainty about future

⁹This is consistent with other authors' finding that US investors predominantly hold large and liquid foreign stocks such as those that dominate the MSCI indices (Thomas, Warnock and Wongswan (2004)).

¹⁰See Obstfeld and Rogoff (1995) for the micro foundations of such models and Kim (2001) for empirical evidence.

¹¹Furthermore, the alphas in the two states are sufficiently precisely estimated that the hypothesis that they are equal to zero is very strongly rejected by a likelihood ratio test.

states. Measured this way, the 12-month alphas starting from the first and second states are 0.06 and 0.70 for the US, while those for Japan are -0.45 and 0.86. The world portfolio generates alphas of -0.13 and 0.70, starting from the first and second state, respectively. Hence, although the individual state alphas appear to be quite large conditional on knowing the true state, in many regards they imply weaker evidence of mispricing than the single-state model which assumes that non-zero alphas are constant and constitute evidence of permanent model misspecification or mispricing.

Volatility is highest in the first state for five of the equity portfolios, the one exception being the UK.¹² Note that to reduce the number of parameters, the model reported in Table 2 assumes that the correlations between country-specific innovations is the same in the two states.¹³ However, as we shall see below, this does *not* imply that the correlations between country returns ($Cor(x_{t+1}^i, x_{t+1}^j)$) are the same in the two states since state-dependence in both the alphas and in the $b_{S_{t+1}}^i$ and $b_{S_{t+1}}^W$ coefficients generate time-variations in return correlations.

The persistence of the first state (0.90) is considerably lower than that of the second state (0.94) and so the average duration of the first state (ten months) is far shorter than that of the second one (20 months). In steady state one-third and two-thirds of the time is spent in the states one and two, respectively. These findings show that neither of the states identifies isolated ‘outliers’ or jumps—a feature distinguishing our model from that proposed by Das and Uppal (2004).

The economic interpretation suggested by these findings is that state one is a bear state with low mean returns and relatively high volatility, while state two is a bull state with higher mean returns and more modest volatility. Figure 1 shows that the two states are generally well identified with state probabilities near zero or one most of the time. Returns were associated with the bear state during a three-year period between 1979 and 1982 and again during shorter spells in 1984, 1987, 1990/1991 and 2002. These periods coincide with global recessions (the early 1980s, 1990s and 2002 recessions) and occasions with high return volatility such as October 1987.

Figure 2 plots the time series of expected returns for the stock portfolios in excess of the US T-bill rate. Periods where the bear state is most likely are shown as gray areas. Clearly the bear state is associated with systematically lower mean excess returns across all markets (in addition to higher volatility, see Table 2). Mean excess returns are always positive for the US portfolio and it is very rare that the expected excess return is negative for any of the other markets.

Figure 3 shows that consistent with previous studies (Ang and Bekaert (2002), Longin and Solnik (1995, 2001) and Karolyi and Stulz (1999)), return correlations are higher in the bear state than in the full sample. Pairwise correlations between US stock returns and returns in Japan, Pacific ex-Japan, UK and Europe in the bear (bull) states are 0.39 (0.27), 0.65 (0.47), 0.67 (0.48) and 0.59 (0.45) and are thus systematically higher in the bear state. This happens despite the fact that correlations between return innovations are identical in the two states. In part this is due to the higher volatility of the common world market return in the bear state. Furthermore, since mean returns are different in the two states, return correlations also depend on the extent of the co-variation between these parameters.

Turning to the risk premia, the premium on covariance with returns on the world market portfolio (γ_1)

¹²The finding for the UK is due to two outliers in January and February of 1975 with monthly excess returns of 44 and 23 percent. If excluded from the data, the volatility in the first state is highest also for the UK.

¹³This restriction is supported by the data: a likelihood ratio test of the restriction that correlations do not depend on the state, i.e. $Cov(\eta_{t+1}^i, \eta_{t+1}^j) = Cor(\eta_{t+1}^i, \eta_{t+1}^j)\sigma_{S_{t+1}}^i\sigma_{S_{t+1}}^j$, produces a p -value of 0.11 and is not rejected.

is positive in both states but, at 15.9, is much higher in the bull state than in the bear state for which an estimate of 9.5 is obtained. The number reported by Harvey (1991) for the subset of G7 countries is 11.5 and hence lies between these two values. Consistent with the large difference between the covariance risk premium in the bull and bear state, Harvey rejects that the world price of risk is constant.

A similar conclusion holds for the co-kurtosis premium (γ_3) which is positive and insignificant in the bear state but positive and significant in the bull state. The estimates of γ_3 can be compared to the price of covariance risk, γ_2 , by scaling them by the ratio of the world market kurtosis to its variance so the units are the same. This yields a co-kurtosis risk premium of 1.7 and 12.3 in the bear and bull state, respectively, and a steady state average of 8.7. As expected, the co-skew premium (γ_2) is negative in both states although it is only significant (and by far largest) in the bull state. When converted to the same units as the covariance risk premium, the estimates are -1.1 and -3.1 in the bear and bull state, respectively, while the steady state average is -2.4.

We conclude from this analysis that all coefficients have the expected sign and are economically meaningful: Investors dislike risk in the form of higher volatility or fatter tails but like positively skewed return distributions. Furthermore, both the co-skew and co-kurtosis risk premia appear to be important in economic terms as they are of the same order of magnitude as the covariance risk premium.

A final way to interpret the two states is through the time-variation in the conditional moments of the world market portfolio. To this end, Figure 4 shows the volatility, skew and kurtosis implied by our model estimates, computed using (10). Large changes in the conditional skew and kurtosis turn out to be linked to regime switches. Preceding a shift to the bear state, the kurtosis of the world market portfolio rises while its skew becomes large and negative and volatility is low. Uncertainty surrounding shifts from a bull to a bear state therefore takes the form of an increased probability of large negative returns. Once in the bear state, the kurtosis gets very low and the skew close to zero, while world market volatility is much higher than normal. Hence the return distribution within the bear state is more dispersed, although closer to symmetric. Finally, when exiting from the bear state to the bull state, the kurtosis again rises—reflecting the increased uncertainty associated with a regime shift—while volatility and skew decline to their normal levels. These large variations in the volatility, skew and kurtosis of world market returns means that our model is able to capture the correlated extremes across local markets found to be an important feature of stock returns in Harvey et al. (2004).

2.4.1. Are Two States Needed?

A question that naturally arises in the empirical analysis is whether regimes are really present in the distribution of international stock market returns. To answer this we computed the specification test suggested by Davies (1977), which very strongly rejected the single-state specification. A more extensive analysis of the number of regimes confirmed the presence of two states in the joint return distribution.¹⁴

Furthermore, to see whether the two-state model does a better job at accounting for the characteristics of returns on the international stock market portfolios, Table 3 reports a set of specification tests for the standardized residuals from the single state and two-state models. Such diagnostics are similar to the ones reported for the international CAPM regression residuals by Harvey and Zhou (1993). Like them, we find

¹⁴Regime switching models have parameters that are unidentified under the null hypothesis of a single state. Standard critical values are therefore invalid in the hypothesis test. Details of the analysis are available on request.

that, with the exception of Japan, the single-state model is strongly rejected and fails to capture even the most basic properties of the international returns data. In contrast, the two-state model performs far better and is either not rejected for most of the portfolios or reduce the diagnostic test statistics very considerably. Hence the evidence of misspecification is far weaker for the two-state model.¹⁵

3. The Investor's Asset Allocation Problem

We next turn to the investor's asset allocation problem. Consistent with the analysis in the previous section, we assume that investor preferences depend on higher order moments of asset returns and allow regimes to be present in the return process.

3.1. Preferences over Moments of the Wealth Distribution

Suppose that the investor's utility function $U(W_{t+T}; \boldsymbol{\theta})$ only depends on wealth at time $t + T$, W_{t+T} , and a set of shape parameters, $\boldsymbol{\theta}$, where t is the current time and T is the investment horizon. Consider an m -th order Taylor series expansion of U around some wealth level v_T :

$$U(W_{t+T}; \boldsymbol{\theta}) = \sum_{n=0}^m \frac{1}{n!} U^{(n)}(v_T; \boldsymbol{\theta}) (W_{t+T} - v_T)^n + \varsigma_m, \quad (12)$$

where the remainder ς_m is of order $o((W_{t+T} - v_T)^m)$ and $U^{(0)}(v_T; \boldsymbol{\theta}) = U(v_T; \boldsymbol{\theta})$. $U^{(n)}(\cdot)$ denotes the n -th derivative of the utility function with respect to terminal wealth. Provided that (i) the Taylor series converges; (ii) the distribution of wealth is uniquely determined by its moments; and (iii) the order of sums and integrals can be exchanged, the expansion in (12) extends to the expected utility functional:

$$E_t[U(W_{t+T}; \boldsymbol{\theta})] = \sum_{n=0}^m \frac{1}{n!} U^{(n)}(v_T; \boldsymbol{\theta}) E_t[(W_{t+T} - v_T)^n] + E_t[\varsigma_m],$$

where $E_t[\cdot]$ is short for $E[\cdot | \mathcal{F}_t]$. We thus have

$$E_t[U(W_{t+T}; \boldsymbol{\theta})] \approx \hat{E}_t[U^m(W_{t+T}; \boldsymbol{\theta})] = \sum_{n=0}^m \frac{1}{n!} U^{(n)}(v_T; \boldsymbol{\theta}) E_t[(W_{t+T} - v_T)^n]. \quad (13)$$

While the approximation improves as m gets larger, many classes of Von-Neumann Morgenstern expected utility functions can be well approximated using a relatively small value of m and a function of the form:

$$\hat{E}_t[U^m(W_{t+T}; \boldsymbol{\theta})] = \sum_{n=0}^m \kappa_n E_t[(W_{t+T} - v_T)^n], \quad (14)$$

with $\kappa_0 > 0$, and κ_n positive (negative) if n is odd (even).

3.2. Solution to the Asset Allocation Problem

We next characterize the solution to the investor's asset allocation problem when preferences are defined over moments of terminal wealth while, consistent with the analysis in Section 2, returns follow a regime

¹⁵We also tested our model against a "pure" regime switching four-moment ICAPM which corresponds to (7) with $\alpha^i = \alpha^W = 0$ so only the risk premia and the amount of risk differ across states. A likelihood-ratio test of these restrictions produced a p -value of essentially zero.

switching process. Following most papers on portfolio choice (e.g., Ang and Bekaert (2002) and Das and Uppal (2004)), we assume a partial equilibrium framework that treats returns as exogenous.

The investor maximizes expected utility by choosing among h risky assets whose continuously compounded excess returns are given by the vector $\mathbf{x}_t^s \equiv (x_t^1 \ x_t^2 \ \dots \ x_t^h)'$. Portfolio weights are collected in the vector $\boldsymbol{\omega}_t^1 \equiv (\omega_t^1 \ \omega_t^2 \ \dots \ \omega_t^h)'$ while $(1 - \boldsymbol{\omega}_t' \boldsymbol{\iota}_h)$ is invested in a short-term interest-bearing bond. The portfolio selection problem solved by a buy-and-hold investor with unit initial wealth then becomes

$$\begin{aligned} \max_{\boldsymbol{\omega}_t} \quad & E_t [U(W_{t+T}(\boldsymbol{\omega}_t); \boldsymbol{\theta})] \\ \text{s.t. } W_{t+T}(\boldsymbol{\omega}_t) = & \left\{ (1 - \boldsymbol{\omega}_t' \boldsymbol{\iota}_h) \exp(R_{t+T}^b) + \boldsymbol{\omega}_t' \exp(\mathbf{R}_{t+T}^s) \right\}, \end{aligned} \quad (15)$$

where $\mathbf{R}_{t+T}^s \equiv (\mathbf{x}_{t+1}^s + r_{t+1}^b) + (\mathbf{x}_{t+2}^s + r_{t+2}^b) + \dots + (\mathbf{x}_{t+T}^s + r_{t+T}^b)$ is the vector of continuously compounded equity returns over the T -period investment horizon while $R_{t+T}^b \equiv r_{t+1}^b + r_{t+2}^b + \dots + r_{t+T}^b$ is the continuously compounded return on the bond investment. Accordingly, $\exp(\mathbf{R}_{t+T}^s)$ is a vector of cumulated returns. Short-selling can be imposed through the constraint $\omega_t^i \in [0, 1]$ for $i = 1, 2, \dots, h$.

To gain intuition we first study the problem under the simplifying assumption of a single risky asset ($h = 1$), a risk-free asset paying a constant return r_f and a regime switching process with two states:

$$\begin{aligned} x_{t+1} &= \mu_{S_{t+1}} + \sigma_{S_{t+1}} \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, 1), \\ \Pr(S_{t+1} = i | S_t = i) &= p_{ii}, \quad i = 1, 2 \end{aligned} \quad (16)$$

This specification is consistent with the ICAPM analysis in section 2 since the conditional moment information from (5) can be folded into $\{\mu_{S_{t+1}}, \sigma_{S_{t+1}}\}$ as described in Section 2.

With a single risky asset (stocks) and initial wealth set at unity, the wealth process is

$$W_{t+T} = \{(1 - \omega_t) \exp(Tr_f) + \omega_t \exp(R_{t+T})\} \quad (17)$$

where R_{t+T} is the continuously compounded stock return over the T periods and ω_t is the stock holding. For a given value of ω_t , the only unknown component in (17) is the cumulated return, $\exp(R_{t+T})$. Under the assumption of two states, $k = 2$, the n th non-central moment of the cumulated returns is given by

$$\begin{aligned} M_{t+T}^{(n)} &= E[(\exp(r_{t+1} + \dots + r_{t+T}))^n | \mathcal{F}_t] \\ &= \sum_{S_{t+T}=1}^2 E[(\exp(r_{t+1} + \dots + r_{t+T}))^n | S_{t+T}, \mathcal{F}_t] \Pr(S_{t+T} | \mathcal{F}_t) \\ &\equiv M_{1t+T}^{(n)} + M_{2t+T}^{(n)}, \end{aligned} \quad (18)$$

where $r_t \equiv x_t + r_f$. Using properties of the moment generating function of a log-normal random variable, each of these conditional moments $M_{it+1}^{(n)}$ ($i = 1, 2$) satisfies recursions

$$\begin{aligned} M_{it+T}^{(n)} &= E[\exp(n(r_{t+1} + \dots + r_{t+T-1})) | S_{t+T}] E[\exp(nr_{t+T}) | S_{t+T}, \mathcal{F}_t] \Pr(S_{t+T} | \mathcal{F}_t) \\ &= \left(M_{it+T-1}^{(n)} p_{ii} + M_{-i, t+T-1}^{(n)} (1 - p_{-i-i}) \right) \exp\left(n\mu_i + \frac{n^2}{2}\sigma_i^2\right), \quad (i = 1, 2) \end{aligned}$$

where we used the notation $-i$ for the converse of state i , i.e. $-i = 2$ when $i = 1$ and vice versa. In more compact notation we have

$$\begin{aligned} M_{1t+1}^{(n)} &= \xi_1^{(n)} M_{1t}^{(n)} + \beta_1^{(n)} M_{2t}^{(n)} \\ M_{2t+1}^{(n)} &= \xi_2^{(n)} M_{1t}^{(n)} + \beta_2^{(n)} M_{2t}^{(n)}, \end{aligned} \quad (19)$$

where

$$\begin{aligned}\xi_1^{(n)} &= p_{11} \exp\left(n\mu_1 + \frac{n^2}{2}\sigma_1^2\right), & \beta_1^{(n)} &= (1 - p_{22}) \exp\left(n\mu_1 + \frac{n^2}{2}\sigma_1^2\right), \\ \xi_2^{(n)} &= (1 - p_{11}) \exp\left(n\mu_2 + \frac{n^2}{2}\sigma_2^2\right), & \beta_2^{(n)} &= p_{22} \exp\left(n\mu_2 + \frac{n^2}{2}\sigma_2^2\right).\end{aligned}$$

Equation (19) can be reduced to a set of second order difference equations:

$$M_{it+2}^{(n)} = (\xi_1^{(n)} + \beta_2^{(n)})M_{it+1}^{(n)} + (\xi_2^{(n)}\beta_1^{(n)} - \beta_2^{(n)}\alpha_1^{(n)})M_{it}^{(n)}, \quad (i = 1, 2). \quad (20)$$

Collecting the two regime-dependent moments in the vector $\boldsymbol{\vartheta}_{it+T}^{(n)} \equiv (M_{it+T}^{(n)} \ M_{it+T-1}^{(n)})'$, equation (20) can be written in companion form:

$$\boldsymbol{\vartheta}_{it+T}^{(n)} = \begin{bmatrix} \xi_1^{(n)} + \beta_2^{(n)} & \xi_2^{(n)}\beta_1^{(n)} - \beta_2^{(n)}\xi^{(n)} \\ 1 & 0 \end{bmatrix} \boldsymbol{\vartheta}_{it+T-1}^{(n)} \equiv \mathbf{A}^{(n)} \boldsymbol{\vartheta}_{it+T-1}^{(n)}.$$

The elements of $\mathbf{A}^{(n)}$ only depend on the mean and variance parameters of the two states $(\mu_1, \sigma_1^2, \mu_2, \sigma_2^2)$ and the state transitions, (p_{11}, p_{22}) . Substituting backwards, we get the i th conditional moment:

$$\boldsymbol{\vartheta}_{it+T}^{(n)} = \left(\mathbf{A}^{(n)}\right)^T \boldsymbol{\vartheta}_{it}^{(n)}.$$

Applying similar principles at $T = 1, 2$ and letting $\pi_{1t} = \Pr(S_t = 1 | \mathcal{F}_t)$, the initial conditions used in determining the n th moment of cumulated returns are as follows:

$$\begin{aligned}M_{1t+1}^{(n)} &= (\pi_{1t}p_{11} + (1 - \pi_{1t})(1 - p_{22})) \exp\left(n\mu_1 + \frac{n^2}{2}\sigma_1^2\right), \\ M_{1t+2}^{(n)} &= p_{11} (\pi_{1t}p_{11} + (1 - \pi_{1t})(1 - p_{22})) \exp\left(2n\mu_1 + n^2\sigma_1^2\right) + \\ &\quad + (1 - p_{22}) (\pi_{1t}(1 - p_{11}) + (1 - \pi_{1t})p_{22}) \exp\left(n(\mu_1 + \mu_2) + \frac{n^2}{2}(\sigma_1^2 + \sigma_2^2)\right), \\ M_{2t+1}^{(n)} &= (\pi_{1t}(1 - p_{11}) + (1 - \pi_{1t})p_{22}) \exp\left(n\mu_2 + \frac{n^2}{2}\sigma_2^2\right), \\ M_{2t+2}^{(n)} &= p_{22} (\pi_{1t}(1 - p_{11}) + (1 - \pi_{1t})p_{22}) \exp\left(2n\mu_2 + n^2\sigma_2^2\right) + \\ &\quad + (1 - p_{11}) (\pi_{1t}p_{11} + (1 - \pi_{1t})(1 - p_{22})) \exp\left(n(\mu_1 + \mu_2) + \frac{n^2}{2}(\sigma_1^2 + \sigma_2^2)\right).\end{aligned} \quad (21)$$

Finally, using (18) we get an equation for the n th moment of the cumulated return:

$$M_{t+T}^{(n)} = M_{1t+T}^{(n)} + M_{2t+T}^{(n)} = \mathbf{e}_1' \boldsymbol{\vartheta}_{1t+T}^{(n)} + \mathbf{e}_2' \boldsymbol{\vartheta}_{2t+T}^{(n)} = \mathbf{e}_1' \left(\mathbf{A}^{(n)}\right)^T \boldsymbol{\vartheta}_{1t}^{(n)} + \mathbf{e}_2' \left(\mathbf{A}^{(n)}\right)^T \boldsymbol{\vartheta}_{2t}^{(n)}, \quad (22)$$

where \mathbf{e}_i is a 2×1 vector of zeros except for unity in the i th place.

Having obtained the moments of the cumulated return process, it is simple to compute the expected utility for any m th order polynomial representation by using (14) and (17):

$$\begin{aligned}\hat{E}_t[U^m(W_{t+T}; \boldsymbol{\theta})] &= \sum_{n=0}^m \kappa_n \sum_{j=0}^n (-1)^{n-j} v_T^{n-j} \binom{n}{j} E_t[W_{t+T}^j] \\ &= \sum_{n=0}^m \kappa_n \sum_{j=0}^n (-1)^{n-j} v_T^{n-j} \binom{n}{j} \sum_{i=0}^j \binom{j}{i} \omega_t^i M_{t+T}^i ((1 - \omega_t) \exp(Tr_f))^{j-i}.\end{aligned} \quad (23)$$

The first order condition is obtained by differentiating this equation with respect to ω_t :

$$\sum_{n=0}^m \kappa_n \sum_{j=0}^n (-1)^{n-j} v_T^{n-j} \binom{n}{j} \sum_{i=1}^j \binom{j}{i} \omega_t^{i-1} (1 - \omega_t)^{j-i-1} M_{t+T}^i \exp((j-i)Tr_f) (i - j\omega_t) = 0.$$

The solution takes the form of the roots of an $m - 1$ order polynomial in ω_t , which are easily obtained. The optimal solution for ω_t corresponds to the root for which (23) has the highest value.

From this analysis it is clear that the optimal asset allocation depends on the following factors:

1. The current state probabilities $(\pi_t, 1 - \pi_t)$ which determine the moments of future returns.
2. State transition probabilities (p_{11}, p_{22}) which affect the speed of mean reversion in the investment opportunity set towards its steady state.
3. Differences between mean parameters (μ_1, μ_2) and variance parameters (σ_1, σ_2) (and more generally covariance parameters) across states. For example, skew in the return distribution can only be induced provided that $\mu_1 \neq \mu_2$, c.f. Timmermann (2000).
4. The number of moments of the wealth distribution that matters for preferences, m , in addition to the weights on the various moments.
5. The investment horizon, T .

3.3. General Results

In many applications \mathbf{r}_{t+1} is a vector of returns on a multi-asset portfolio. The number of states, k , may also exceed two. For generality, we assume the following process for a vector of $h + 1$ excess returns:¹⁶

$$\mathbf{x}_{t+1} = \tilde{\boldsymbol{\mu}}_{S_{t+1}} + \sum_{j=1}^p \mathbf{B}_{j,S_{t+1}} \mathbf{x}_{t-j} + \boldsymbol{\varepsilon}_{t+1}, \quad (24)$$

where $\tilde{\boldsymbol{\mu}}_{S_{t+1}} = (\mu_{s_{t+1}}^1, \dots, \mu_{s_{t+1}}^{h+1})'$ is a vector of conditional means in state S_{t+1} (possibly used to “fold in” all components of the mean in state S_{t+1}), $\mathbf{B}_{j,S_{t+1}}$ is a matrix of autoregressive coefficients associated with the j th lag in state S_{t+1} , and $\boldsymbol{\varepsilon}_{t+1} = (\varepsilon_{t+1}^1, \dots, \varepsilon_{t+1}^{h+1})' \sim N(\mathbf{0}, \boldsymbol{\Omega}_{S_{t+1}})$ is a vector of zero-mean return innovations with state-dependent covariance matrix $\boldsymbol{\Omega}_{S_{t+1}}$.

With $h + 1$ risky assets (the last of which can be taken to represent the risky returns on a short-term bond, $x_{t+i}^b = r_{t+i}^b$) and k states, the wealth process becomes

$$W_{t+T} = \boldsymbol{\omega}'_t \exp \left[\sum_{i=1}^T (\mathbf{x}_{t+i} + r_{t+i}^b) \right] + (1 - \boldsymbol{\omega}'_t \boldsymbol{\iota}_h) \exp \left[\sum_{i=1}^T r_{t+i}^b \right].$$

We next present a simple and convenient recursive procedure for evaluating the expected utility associated with a vector of portfolio weights, $\boldsymbol{\omega}_t$, of relatively high dimension:

¹⁶This equation is more convenient than (5) but is fully consistent with the earlier setup if the last elements of the return vector, \mathbf{r}_{t+1} , are used to capture the predictor variables \mathbf{z}_{t+1} (themselves asset returns). Furthermore, the four-moment ICAPM factors are easily folded into the intercept by defining $\tilde{\boldsymbol{\mu}}_{S_{t+1}} \equiv \boldsymbol{\mu}_{S_{t+1}} + \mathbf{M}_{S_t} \text{vec}(\boldsymbol{\Upsilon}_{S_{t+1}})$.

Proposition 1. *Under the regime-switching return process (24) and m -moment preferences (14), the expected utility associated with the portfolio weights ω_t is given by*

$$\begin{aligned}\hat{E}_t[U^m(W_{t+T})] &= \sum_{n=0}^m \kappa_n \sum_{j=0}^n (-1)^{n-j} v_T^{n-j} {}_nC_j E_t[W_{t+T}^j] \\ &= \sum_{n=0}^m \kappa_n \sum_{j=0}^n (-1)^{n-j} v_T^{n-j} \binom{n}{j} \sum_{i=0}^j \binom{j}{i} E_t \left[(\omega'_t \exp(\mathbf{R}_{t+T}^s))^i \right] ((1 - \omega'_t \mathbf{t}_h) \exp(Tr^f))^{j-i}.\end{aligned}$$

The n th moment of the cumulated return on the risky asset portfolio is

$$E_t \left[(\omega'_t \exp(\mathbf{R}_{t+T}^s))^n \right] = \sum_{n_1=0}^n \cdots \sum_{n_h=0}^n \lambda(n_1, n_2, \dots, n_h) \left(\prod_{i=1}^h \omega_i^{n_i} \right) M_{t+T}^{(n)}(n_1, \dots, n_h),$$

where $\sum_{i=1}^h n_i = n$, $0 \leq n_i \leq n$ ($i = 1, \dots, h$),

$$\lambda(n_1, n_2, \dots, n_h) \equiv \frac{n!}{n_1! n_2! \dots n_h!}.$$

and $M_{t+T}^{(n)}(n_1, \dots, n_h)$ can be evaluated recursively, using (A4) in the Appendix.

Appendix A proves this result. The solution is in closed-form in the sense that it reduces the expected utility calculation to a finite number of steps each of which can be solved by elementary operations.

It is useful to compare our method to existing alternatives. Classic results on optimal asset allocation have been derived for special cases such as power utility with constant investment opportunities or under logarithmic utility (Merton (1969) and Samuelson (1969)). For general preferences there is no closed-form solution to (15), but given its economic importance it is not surprising that a variety of solution approaches have been suggested. Recent papers that solve (15) under predictability of returns include Ang and Bekaert (2002), Brandt (1999), Brennan, Schwarz and Lagnado (1997), Campbell and Viceira (1999, 2001). These papers generally use approximate solutions or numerical techniques such as quadrature (Ang and Bekaert (2002)) or Monte Carlo simulations (Detemple, Garcia and Rindisbacher (2003)) to characterize optimal portfolio weights. Quadrature methods may not be very precise when the underlying asset return distributions are strongly non-normal. They also have the problem that the number of quadrature points increases exponentially with the number of assets. Monte Carlo methods can also be computationally expensive to use as they rely on discretization of the state space and use grid methods.¹⁷ Although existing methods have clearly yielded important insights into the solution of (15), they are therefore not particularly well-suited to our analysis of international asset allocation which involves a large number of portfolios.

4. International Portfolio Holdings

We next consider empirically the optimal international asset allocation under regime switching and four-moment preferences. The weights on the first four moments of the wealth distribution are determined to ensure that our results can be compared to those in the existing literature. Most studies on optimal asset

¹⁷In continuous time, closed-form solutions can be obtained under less severe restrictions. For instance Kim and Omberg (1996) work with preferences in the HARA class defined over final wealth and assume that the single risky asset return is mean-reverting.

allocation use power utility so we calibrate our coefficients to the benchmark

$$U(W_{t+T}; \theta) = \frac{W_{t+T}^{1-\theta}}{1-\theta}, \quad \theta > 0. \quad (25)$$

For a given coefficient of relative risk aversion, θ , (25) serves as a guide in setting values of $\{\kappa_n\}_{n=0}^m$ in (14) but should otherwise not be viewed as an attempt to approximate results under power utility. Expanding the powers of $(W_{t+T} - v_T)$ and taking expectations, we obtain the following expression for the four-moment preference function:

$$\hat{E}_t[U^4(W_{t+T}; \theta)] = \kappa_{0,T}(\theta) + \kappa_{1,T}(\theta)E_t[W_{t+T}] + \kappa_{2,T}(\theta)E_t[W_{t+T}^2] + \kappa_{3,T}(\theta)E_t[W_{t+T}^3] + \kappa_{4,T}(\theta)E_t[W_{t+T}^4], \quad (26)$$

where¹⁸

$$\begin{aligned} \kappa_{0,T}(\theta) &= v_T^{1-\theta} \left[(1-\theta)^{-1} - 1 - \frac{1}{2}\theta - \frac{1}{6}\theta(\theta+1) - \frac{1}{24}\theta(\theta+1)(\theta+2) \right] \\ \kappa_{1,T}(\theta) &= \frac{1}{6}v_T^{-\theta} [6 + 6\theta + 3\theta(\theta+1) + \theta(\theta+1)(\theta+2)] > 0 \\ \kappa_{2,T}(\theta) &= -\frac{1}{4}\theta v_T^{-(1+\theta)} [2 + 2(\theta+1) + (\theta+1)(\theta+2)] < 0 \\ \kappa_{3,T}(\theta) &= \frac{1}{6}\theta(\theta+1)(\theta+3)v_T^{-(2+\theta)} > 0 \\ \kappa_{4,T}(\theta) &= -\frac{1}{24}\theta(\theta+1)(\theta+2)v_T^{-(3+\theta)} < 0. \end{aligned}$$

Expected utility from final wealth increases in $E_t[W_{t+T}]$ and $E_t[W_{t+T}^3]$, so higher expected returns and more right-skewed distributions lead to higher expected utility. Conversely, expected utility is a decreasing function of the second and fourth moments of the terminal wealth distribution. Our benchmark results assume that $\theta = 2$, a coefficient of relative risk aversion compatible with much empirical evidence. Later we allow this coefficient to assume different values.¹⁹

A solution to the optimal asset allocation problem can now easily be found from Proposition 1 by solving a system of cubic equations in $\hat{\omega}_t$ derived from the first order conditions

$$\nabla_{\omega_t} \hat{E}_t[U^4(W_{t+T}; \theta)] \Big|_{\hat{\omega}_t} = \mathbf{0}'.$$

At the optimum $\hat{\omega}_t$ sets the gradient, $\nabla_{\omega_t} \hat{E}_t[U^4(W_{t+T}; \theta)]$ equal to zero and produces a negative definite Hessian matrix, $H_{\omega_t} \hat{E}_t[U^4(W_{t+T}; \theta)]$.

4.1. Empirical Results

As a benchmark, Table 4 reports equity allocations for the single-state model using a short 1-month and a longer 24-month horizon. Our empirical analysis considers returns on five equity portfolios and the world market. To arrive at total portfolio weights we therefore re-allocate the weight assigned to the world market

¹⁸The notation $\kappa_{n,T}$ makes it explicit that the coefficients of the fourth order Taylor expansion depend on the investment horizon through the coefficient v_T , the point around which the approximation is calculated. We follow standard practice and set $v_T = E_t[W_{t+T-1}]$.

¹⁹Based on the evidence in Ang and Bekaert (2002)—who show that the optimal home bias is an increasing function of the coefficient of relative risk aversion—this is also a conservative choice that allows us to examine the effects on the optimal portfolio choice produced by preferences that account for higher order moments.

using the regional market capitalizations as weights.²⁰ Since we are interested in the home bias, we report equity weights as percentages of the total equity portfolio so they sum to unity. The allocation to the risk-free asset (as a percentage of the total portfolio) is also shown for interest rates that vary by up to two standard deviations from the mean. When the T-bill rate is set at its sample mean of 5.9% per annum, at the one-month horizon only 31% of the equity portfolio is invested in US stocks. Slightly less (29%) gets invested in US stocks at the 24-month horizon. Furthermore, the fraction of the equity portfolio allocated to US stocks remains too low in both low and high interest rate environments. These results support earlier findings under mean-variance preferences (e.g. Lewis (1999)) and also show that the home bias puzzle extends to a setting with return predictability from the short T-bill rate.

Turning to the two-state model, Table 4 shows that the allocation to US stocks is much higher in the presence of regimes. This holds both when starting from the steady-state probabilities—i.e. when the investor has very imprecise information about the current state—as well as in the separate bull and bear states. Under steady state probabilities and an average short-term US interest rate the 1-month allocation to US stocks is 70% of the total equity portfolio. This reflects an allocation of 75% in the bear state and a slightly lower allocation of 60% in the bull state. Moreover, this finding is robust to the level of the short US interest rate. Varying this rate predominantly affects the allocation to the risk-free asset versus the overall equity portfolio but has little effect on the regional composition of the equity portfolio.²¹

4.2. *Effect of Higher Moments*

Compared with the benchmark model, our four-moment regime switching model appears capable of significantly increasing the allocation to US stocks but leaves unanswered what accounts for this effect. An economic understanding of the effect of skew and kurtosis on the optimal asset allocation requires studying the co-skew and co-kurtosis properties at the portfolio level. To this end, define the conditional co-skew of the return on stock i with the world market as:

$$S_{i,W}(\mathcal{F}_t, S_t) \equiv \frac{Cov[x_{t+1}^i, (x_{t+1}^W)^2 | \mathcal{F}_t, S_t]}{\{Var[x_{t+1}^i | \mathcal{F}_t, S_t](Var[x_{t+1}^W | \mathcal{F}_t, S_t])^2\}^{1/2}}. \quad (27)$$

The co-skew is normalized by scaling by the appropriate powers of the volatility of the respective portfolios. A security that has negative co-skew with the market portfolio pays low (high) returns when the world market portfolio becomes highly (less) volatile. To a risk averse investor this is an unattractive feature since global market risk rises in periods with low returns. Conversely, positive co-skew is desirable as it means higher expected returns during volatile periods.

Similarly, define the co-kurtosis of the excess return on asset i with the world portfolio as

$$K_{i,W}(\mathcal{F}_t, S_t) \equiv \frac{Cov[x_{t+1}^i, (x_{t+1}^W)^3 | \mathcal{F}_t, S_t]}{\{Var[x_{t+1}^i | \mathcal{F}_t, S_t](Var[x_{t+1}^W | \mathcal{F}_t, S_t])^3\}^{1/2}}. \quad (28)$$

Large positive values are undesirable as they mean that local returns are low (high) when world market returns are largely skewed to the left (right), thus increasing the overall portfolio risk.

²⁰This introduces a very small approximation error as the included stock markets account for only 97% of the world market.

²¹Consistent with the findings in Ang and Bekaert (2002), the allocation to the short-term bond is much higher in the bear state than in the bull state. This happens because equity returns are small and volatile in the bear state, and hence unattractive to risk averse investors.

Table 5 reports estimates of these moments in the bull and bear states as well as under steady state probabilities. The latter gives a measure that is more directly comparable to the full-sample estimates listed in the final column. Short term interest rates are set at the regime-specific unconditional means. As can be seen by comparing the values implied by the two-state model to the full sample estimates, the model generally does a good job at matching the data. Interestingly, with the exception of Japan, in both the bear state and under steady-state probabilities US stocks have the lowest co-kurtosis and co-skew coefficients (essentially zero), explaining why domestic stocks are more attractive to US investors than is revealed by the mean-variance case. Japanese stocks remain unattractive due to their low mean returns over the sample period.

To address the effect of higher order moments on the asset allocation, we next computed the optimal portfolio weights as a function of T and π (the state probability) under mean-variance ($m = 2$) preferences:

$$\hat{E}_t[U^2(W_{t+T}; \theta)] = \kappa_{0,T}(\theta) + \kappa_{1,T}(\theta)E_t[W_{t+T}] + \kappa_{2,T}(\theta)E_t[W_{t+T}^2] \quad (29)$$

where $\kappa_{0,T}(\theta) \equiv v_T^{1-\theta} [(1-\theta)^{-1} - 1 - \frac{1}{2}\theta]$, $\kappa_{1,T}(\theta) \equiv v_T^{-\theta}(1+\theta) > 0$ and $\kappa_{2,T}(\theta) \equiv -\frac{1}{2}\theta v_T^{-(1+\theta)} < 0$. We also consider optimal allocations under three-moment preferences

$$\hat{E}_t[U^3(W_{t+T}; \theta)] = \kappa_{0,T}(\theta) + \kappa_{1,T}(\theta)E_t[W_{t+T}] + \kappa_{2,T}(\theta)E_t[W_{t+T}^2] + \kappa_{3,T}(\theta)E_t[W_{t+T}^3] \quad (30)$$

where now $\kappa_{0,T}(\theta) \equiv v_T^{1-\theta} [(1-\theta)^{-1} - 1 - \frac{1}{2}\theta - \frac{1}{6}\theta(\theta+1)]$, $\kappa_{1,T}(\theta) \equiv v_T^{-\theta} [1 + \theta + \frac{1}{2}\theta(\theta+1)] > 0$, $\kappa_{2,T}(\theta) \equiv -\frac{1}{2}\theta v_T^{-(1+\theta)}(2+\theta) < 0$ and $\kappa_{3,T}(\theta) \equiv \frac{1}{6}\theta(\theta+1)v_T^{-(2+\theta)} > 0$.

Using steady-state probabilities, Table 6 shows that the allocation to US stocks as a portion of the overall equity portfolio remains just above 50% when going from mean-variance to skewness preferences. The introduction of two states on its own thus increases the allocation to US stocks from roughly 30% (as seen in Table 4) to 50%. This allocation rises further to 70% of the equity portfolio when we move to the case with kurtosis aversion. The steady state results conceal large differences in the separate bull and bear states. In the bear state, the large increase in the allocation to US stocks due to introducing higher order moment preferences comes from the skew while the kurtosis plays a similar role in the bull state.

The correlation, co-skew and co-kurtosis between the short interest rate and the stock returns can also affect asset allocations. At the 1-month horizon, the correlation between the risk-free rate and stock returns is zero since the risk-free rate is known. Future short-term spot rates are stochastic, however. This matters to buy-and-hold investors with horizons of $T \geq 2$ months who effectively commit $(1 - \omega'_t \iota_h)$ of their portfolio to roll over investments in T -bills $T-1$ times at unknown future spot rates. We therefore computed the co-skew, $S_{i,R^b}(\mathcal{F}_t, S_t)$, and co-kurtosis, $K_{i,R^b}(\mathcal{F}_t, S_t)$, between the individual stock returns and rolling six-month bond returns assuming steady state probabilities and setting the initial interest rate at its unconditional mean. US stocks were found to generate the second-highest co-skewness coefficient (-0.06) and the second lowest co-kurtosis coefficient (4.44). Only Japanese stocks turn out to be preferable to US stocks, although their conditional mean and variance properties make them undesirable to a US investor. We conclude that the co-moment properties of US stocks against rolling returns on short US T-bills help to explain the high demand for these stocks under three- and four-moment preferences.

5. Interpretation and Robustness of Results

To summarize our results so far, we extended the standard model in two directions: First, by defining preferences over higher moments such as skew and kurtosis and, second, by allowing for the presence of bull and bear regimes tracking periods with very different mean, variances, correlations, skew and kurtosis. In this section we consider the robustness of our results with regard to alternative specifications of investor preferences, estimation errors and dynamic portfolio choice.

5.1. Preference Specification

We first consider the effect of changing the coefficient of relative risk aversion from $\theta = 2$ in the baseline scenario to values of $\theta = 5$ (high) and $\theta = 10$ (very high). Ang and Bekaert (2002) and Das and Uppal (2004) have found that changes in risk aversion have first-order effects on their conclusions on the importance of either regime shifts or systemic (jump) risks. Table 7 shows the effect of such changes. In general there is no monotone relation between θ and the weight on US stocks, although the allocation to US stocks tends to be greater for $\theta = 10$ than for $\theta = 2$. Risk aversion has a first order effect on the choice of T-bills versus stocks but has far less of an effect on the composition of the equity portfolio. Therefore, it does not seem that our conclusions depend on a particular choice of θ .

To make our results comparable to those reported in the literature which assume power utility, we also compare results under four-moment preferences to those under constant relative risk aversion (shown in Table 8). Differences between results computed under power utility and under four-moment preferences appear to be relatively minor.²² In the bear state the allocation to US stocks is 2-4% lower under power utility while conversely the allocation to UK stocks tends to be higher. In the more persistent bull state, results under the four-moment preference specification are similar to those under constant relative risk aversion.

5.2. Precision of Portfolio Weights

Mean-variance portfolio weights are known to be highly sensitive to the underlying estimates of mean returns and covariances. Since such estimates often are imprecisely estimated, this means that the portfolio weights in turn can be poorly determined, see Britten-Jones (1999). As pointed out by Harvey, Liechty, Liechty and Muller (2004), this could potentially be even more of a concern in a model with higher moments due to the difficulty of obtaining precise estimates of moments such as skew and kurtosis.²³ In this situation it becomes important to jointly consider the effect of higher moments and parameter uncertainty.

To address this concern, we computed standard error bands for the portfolio weights under the single state and two-state models using that, in large samples, the distribution of the parameter estimates from a

²²A problem associated with using low-order polynomial utility functionals is the difficulty of imposing restrictions on the derivatives (with respect to the moments of wealth) that apply globally. For example, nonsatiation cannot be imposed by restricting a quadratic polynomial to be monotone increasing and risk aversion cannot be imposed by restricting a cubic polynomial to be globally concave (see Post and Levy (2005) and Post, van Vliet and Levy (2005)). This is why it is important to check through the comparison with power utility that our findings on the optimal portfolio weights are not driven by unreasonable behavior of the utility function.

²³See also the discussion of “Omega” in Cascon, Keating and Shadwick (2003) which is used to capture sample information beyond point estimates through the cumulative density function of returns.

regime switching model is

$$\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \sim N(\mathbf{0}, \mathbf{V}_{\boldsymbol{\theta}}).$$

This allows us to set up the following simulation experiment. In the q th simulation we draw a vector of parameters, $\hat{\boldsymbol{\theta}}^q$, from $N(\hat{\boldsymbol{\theta}}, T^{-1}\hat{\mathbf{V}}_{\boldsymbol{\theta}})$ where $\hat{\mathbf{V}}_{\boldsymbol{\theta}}$ is a consistent estimator of $\mathbf{V}_{\boldsymbol{\theta}}$. Using this draw, $\hat{\boldsymbol{\theta}}^q$, we solve for the associated vector of portfolio weights $\hat{\boldsymbol{\omega}}^q$. We repeat this process Q times. Confidence intervals for the optimal asset allocation $\hat{\omega}_t$ can then be derived from the distribution of $\hat{\boldsymbol{\omega}}^q$, $q = 1, 2, \dots, Q$. This approach is computationally intensive, as (15) must be solved repeatedly, so we set the number of bootstrap trials to $Q = 2,000$.

Results are reported in Table 9. Unsurprisingly, and consistent with the analysis in Britten Jones (1999), the standard error bands are quite wide for the single state model. For example, at the 1-month horizon the 90% confidence band for the weight on the US market in the equity portfolio goes from 2% to 38%—a width of 36%. The width of the confidence bands is roughly similar at the 24-month horizon. For comparison, the width of the US weight in the two-state model under steady state probabilities only extends from 64% to 73%, a width of less than 10%. Even at longer investment horizons, the confidence bands remain quite narrow (e.g. from 50% to 69% under steady state probabilities when $T = 24$ months). In fact, the standard error bands for the portfolio weights are generally narrower under the two-state model than under the single-state model. This suggests that the finding that a large part of the home bias can be explained by the US stock market portfolio's co-skew and co-kurtosis properties in bull and bear states is fairly robust.

There are several reasons for these findings. First, the fact that the portfolio weights do not get less precise even though we account for skew and kurtosis is related to the way we compute these moments from a two-state mixture model. As can be seen from the time-series in figures 2-4, these moments are well behaved without the huge spikes and sampling variations typically observed when such moments are estimated directly from returns data using rolling or expanding data windows. Second, as we saw in Table 3, the two-state model captures many properties of the returns data far better than the single-state model and so reduces one source of noise due to misspecification. Third, and related to this point, the effect of conditioning on states captures more of the return dynamics and means that at least some of the parameters are more precisely estimated compared to the single-state model. Again this reduces the standard error bands on the portfolio weights under the two-state model.

An alternative way to address the effect of parameter estimation error that directly addresses its economic costs is to compute the investor's average (or expected) utility when the estimated parameters as opposed to the true parameters are used to guide the portfolio selection. To this end, Panel A of Table 10 reports the outcome of a Monte Carlo simulation where returns were generated from the two-state model in Table 2. In these simulations, the parameter values were assumed to be unknown to the investor who had to estimate these using a sample of the same length as the actual data before selecting the portfolio weights assuming either a 1-month or a 24-month investment horizon. For comparison, we also report results for alternatives such as using the single state model (7) or adopting the ICAPM weights (i.e. each region is purchased in the proportion that it enters into the global market portfolio).

Even after accounting for the effect of parameter estimation errors, the two-state model produces the highest certainty equivalent return and the highest average wealth at both the 1-month and 24-month horizons. Furthermore, the improvements are meaningful in economic terms, suggesting an increase in the certainty equivalent return of about two percent per annum.

5.3. Out-of-Sample Portfolio Selection

Econometric models fitted to asset returns may produce good in-sample (or historical) fits and imply asset allocations that are quite different from the benchmark ICAPM portfolio. However, this is by no means a guarantee that such models will lead to improvements in ‘real time’ when used on future data. This problem arises, for example, when the proposed model is misspecified. It could also be the result of parameter estimation error as discussed previously.

To address both concerns, we next explored how well the two-state model performs out-of-sample through the following recursive estimation and portfolio selection experiment. We first used data up to 1985:12 to estimate the parameters of the two-state model. Using these estimates, we computed the mean, variance, skew and kurtosis of returns and solved for the optimal portfolio weights at 1-month and 24-month horizons. This exercise was repeated the following month, using data up to 1986:1 to forecast returns and select the portfolio weights. Repeating this until the end of the sample (2005:12) generated a sequence of realized returns from which realized utilities and certainty equivalent returns were computed.²⁴

Results are shown in Panel B in Table 10. Again the two-state model came out ahead of the single-state model and ICAPM specifications in realized utility terms and for both investment horizons.²⁵ For example, at the 1-month horizon, the certainty equivalence return of the two-state model was two percent higher than under the single-state model while it exceeded that of the ICAPM by 80 basis points per annum. Results were very similar at the 24-month horizon. Since this experiment does not assume that the two-state model is the ‘true’ model—realized returns are computed using actual data and not simulated returns—and since the sample (1986-2005) covered several bull and bear markets, this experiment provides an ideal way to test if the two-state model can add value over alternative approaches.²⁶

5.4. Rebalancing

To keep the analysis simple, so far we have ignored the possibility of portfolio rebalancing. We next relax this assumption and allow the investor to rebalance every $\varphi = \frac{T}{B}$ months at B equally spaced points t , $t + \frac{T}{B}$, $t + 2\frac{T}{B}$, ..., $t + (B - 1)\frac{T}{B}$. This requires determining the portfolio weights at the rebalancing times ω_b ($b = 0, 1, \dots, B - 1$). Cumulated wealth can be factored out as a product of interim wealth at the rebalancing points:

$$W_{t+T} = \prod_{b=1}^B \frac{W_{t+\varphi b}(\omega_{b-1})}{W_{t+\varphi(b-1)}(\omega_{b-2})}, \quad (31)$$

where

$$\frac{W_{t+\varphi b}(\omega_{b-1})}{W_{t+\varphi(b-1)}(\omega_{b-2})} = \left\{ (1 - \omega'_{b-1} \iota_h) \exp \left(R_{\varphi(b-1)+1 \rightarrow \varphi b}^b \right) + \omega'_{b-1} \exp \left(\mathbf{R}_{\varphi(b-1)+1 \rightarrow \varphi b}^s \right) \right\},$$

²⁴In this experiment we updated all the parameters once a year while the state probabilities were updated each month using the Hamilton-Kim filter (see Hamilton (1990) for details).

²⁵An investment strategy based on the two-state model fails to produce the highest out-of-sample mean return which is now associated with the ICAPM. However, the ICAPM portfolio weights also generate return volatilities that are 2-3% higher than the portfolio associated with the two-state model. This explains why the two-state portfolio attains higher realized utilities and certainty equivalent returns.

²⁶An analysis of the time-series of recursive portfolio weights showed that the home bias implied by the two-state model has been very persistent over time, oscillating between 65 and 70 percent. This matches the fact that the international exposure US investors achieve through their holdings of domestic stocks has been increasing over time and supports the finding that a considerable home bias is optimal (see Thomas et al. (2004)).

and $\mathbf{R}_{\varphi(b-1)+1 \rightarrow \varphi b}^s \equiv \mathbf{r}_{t+\varphi(b-1)+1}^s + \mathbf{r}_{t+\varphi(b-1)+2}^s + \dots + \mathbf{r}_{t+\varphi b}^s$, $R_{\varphi(b-1)+1 \rightarrow \varphi b}^b \equiv r_{bt+\varphi(b-1)+1} + r_{bt+\varphi(b-1)+2} + \dots + r_{bt+\varphi b}$ are the cumulated risky and riskless returns between periods $\varphi(b-1)+1$ and φb . By the law of iterated expectations, the following decomposition holds:

$$\begin{aligned} M_{t+T}^{(n)} &= E_t[W_{t+T}^n] = E_t \left[\prod_{b=1}^B \left(\frac{W_{t+\varphi b}(\boldsymbol{\omega}_{b-1})}{W_{t+\varphi(b-1)}(\boldsymbol{\omega}_{b-2})} \right)^n \right] \\ &= E_t \left\{ (W_{t+\varphi}(\boldsymbol{\omega}_t))^n E_{t+\varphi} \left[\left(\frac{W_{t+2\varphi}(\boldsymbol{\omega}_{t+\varphi})}{W_{t+\varphi}(\boldsymbol{\omega}_t)} \right)^n E_{t+2\varphi} \left(\left(\frac{W_{t+3\varphi}(\boldsymbol{\omega}_{t+2\varphi})}{W_{t+2\varphi}(\boldsymbol{\omega}_{t+\varphi})} \right)^n \dots \right) \right] \right\} \\ &= M_{0 \rightarrow \varphi}^{(n)}(\boldsymbol{\omega}_0) E_t \left\{ M_{\varphi \rightarrow 2\varphi}^{(n)}(\boldsymbol{\omega}_1) E_{t+\varphi} \left[M_{2\varphi \rightarrow 3\varphi}^{(n)}(\boldsymbol{\omega}_2) E_{t+2\varphi} \left(M_{3\varphi \rightarrow 4\varphi}^{(n)}(\boldsymbol{\omega}_3) \dots \right) \right] \right\} \end{aligned} \quad (32)$$

where $E_{t+\varphi(b-1)}[\cdot]$ is shorthand notation for $E[\cdot | \mathcal{F}_{t+\varphi(b-1)}]$ and $M_{\varphi(b-1) \rightarrow \varphi b}^{(n)}(\boldsymbol{\omega}_{b-1})$ is the n -th (noncentral) moment of the cumulated portfolio returns between $t + \varphi(b-1) + 1$ and $t + \varphi b$, calculated on the basis of time $t + \varphi(b-1)$ information:

$$\begin{aligned} M_{\varphi(b-1) \rightarrow \varphi b}^{(n)}(\boldsymbol{\omega}_{b-1}) &\equiv E_{t+\varphi(b-1)} \left[\left(\frac{W_{t+\varphi b}(\boldsymbol{\omega}_{b-1})}{W_{t+\varphi(b-1)}(\boldsymbol{\omega}_{b-2})} \right)^n \right] \\ &= E_{t+\varphi(b-1)} \left[\left((1 - \boldsymbol{\omega}'_{b-1} \boldsymbol{\iota}_h) \exp \left(R_{\varphi(b-1)+1 \rightarrow \varphi b}^b \right) + \boldsymbol{\omega}'_{b-1} \exp \left(\mathbf{R}_{\varphi(b-1)+1 \rightarrow \varphi b}^s \right) \right)^n \right]. \end{aligned}$$

The decomposition in (32) shows that future moments of wealth depend on future portfolio choices, $\boldsymbol{\omega}_b$.

We use the following recursive strategy to solve the asset allocation problem under m -moment preferences and rebalancing:

1. Solve the time $T - \varphi$ problem

$$\hat{\boldsymbol{\omega}}_{B-1} \equiv \arg \max_{\boldsymbol{\omega}_{B-1}} \sum_{n=0}^m \kappa_n(\boldsymbol{\theta}) \hat{E}_{T-\varphi} \left[M_{T-\varphi \rightarrow T}^{(n)}(\boldsymbol{\omega}_{B-1}) \right].$$

Here $\hat{E}_{T-\varphi}[\cdot]$ is shorthand notation for $E[\cdot | \mathcal{F}_{T-\varphi}]$ calculated on the basis of the filtered state probabilities for time $T - \varphi$.

2. Solve the time $T - 2\varphi$ problem

$$\hat{\boldsymbol{\omega}}_{B-2} \equiv \arg \max_{\boldsymbol{\omega}_{B-2}} \sum_{n=0}^m \hat{E}_{T-2\varphi} \left[\lambda_n^{B-1}(\boldsymbol{\theta}) M_{T-2\varphi \rightarrow T-\varphi}^{(n)}(\boldsymbol{\omega}_{B-2}) \right],$$

where $\lambda_n^{B-1}(\boldsymbol{\theta}) \equiv \kappa_n(\boldsymbol{\theta}) \hat{E}_{T-\varphi} [M_{T-\varphi \rightarrow T}^{(n)}(\hat{\boldsymbol{\omega}}_{B-1})]$ and $\hat{E}_{T-\varphi} [M_{T-\varphi \rightarrow T}^{(n)}(\hat{\boldsymbol{\omega}}_{B-1})]$ is the n -th moment of the optimal wealth process calculated under the solution found in 1.²⁷

3. Solve the problem backward by iterating on steps 1 and 2 up to time $t + \varphi$, to generate a sequence of optimal portfolio choices $\{\hat{\boldsymbol{\omega}}_i\}_{i=1}^{B-1}$. The optimal time t asset allocation, $\boldsymbol{\omega}_0 \equiv \boldsymbol{\omega}_t$, is then found by solving

$$\boldsymbol{\omega}_0 \equiv \arg \max_{\boldsymbol{\omega}_0} \sum_{n=0}^m \hat{E}_t \left[\lambda_n^1(\boldsymbol{\theta}) M_{t \rightarrow t+\varphi}^{(n)}(\boldsymbol{\omega}_0) \right]$$

where

$$\lambda_n^1(\boldsymbol{\theta}) \equiv \kappa_n(\boldsymbol{\theta}) \hat{E}_{t+\varphi} [M_{t+\varphi \rightarrow t+2\varphi}^{(n)}(\boldsymbol{\omega}_b)]. \quad (33)$$

²⁷Maximizing $\hat{E}_{T-2\varphi} [\lambda_n^{B-1}(\boldsymbol{\theta}) M_{T-2\varphi \rightarrow T-\varphi}^{(n)}(\boldsymbol{\omega}_{B-2})]$ implies that the conditional correlation between optimal wealth at time $T - 2\varphi$ and portfolio returns between $T - \varphi$ and T affects portfolio weights.

Table 11 reports optimal portfolio weights for two investment horizons ($T = 6$ and 24 months) and various rebalancing frequencies ($\varphi = 1, 3, 6, 12$ months and $\varphi = T$, the buy-and-hold benchmark of Tables 4 and 6). In these simulations, the US interest rate is set at its mean. As already noted in the literature, rebalancing opportunities give investors incentives to exploit current information more aggressively. This effect is stronger when rebalancing occurs more frequently, i.e. when φ is small. Stock allocations under rebalancing are large and always exceed 60% of current wealth. Starting from the bull state, even at short horizons the allocation under frequent rebalancing ($\varphi = 1$ and 3 months) differs significantly from the buy-and-hold results as the investor attempts to time the market by shifting the portfolio towards Pacific stocks and away from US and UK equities.

However, starting from the bear state or assuming that the initial state is unknown (i.e. adopting steady-state probabilities), very frequent rebalancing ($\varphi = 1$ and 3 months) increases the allocation to US stocks for long horizons ($T = 24$ months), while Japanese stocks also emerge as an attractive investment. For all possible values of φ this implies an even greater allocation to US stocks than under the buy-and-hold strategy. In fact, under frequent rebalancing a US investor with four-moment preferences and a long horizon should hold even more in US securities than under no rebalancing. For example, for $T = 24$, almost 100% of wealth goes into domestic securities, comprising between 60% and 85% in stocks (only 8-12% of total wealth goes into foreign stocks). All told, regime switching combined with preferences that reflect aversion against fat tails and negative skew seem to explain the home bias under a range of assumptions about the rebalancing frequency, especially when investors have little information about the current state (and thus adopt steady state probabilities), which seems to be a plausible assumption.

6. Conclusion

Do regimes or higher moment preferences explain the home bias? The answer seems to be that both play a role. In the absence of regimes, our estimates suggest that a US investor with mean-variance preferences should hold only 30% of the equity portfolio in domestic stocks—less than the US weight in the global market portfolio. Allowing for regimes but maintaining the assumption of mean-variance preferences, the allocation to domestic stocks rises to 50% of the equity portfolio. Introducing both regimes and four-moment preferences, the allocation to US stocks rises further to 70%, a figure that, while not explaining the entire home bias, gets much closer to the actual data.²⁸

Intuition for our finding that US investors should hold a considerably higher proportion of their stocks in domestic equities than under mean-variance preferences comes from the attractive properties that US stocks have for an investor who—besides being risk averse—prefers positively skewed (asymmetric) payoffs and dislikes fat tails (kurtosis). Like Bekaert and Harvey (1995)—who argue that the sources of risk may change when equity markets move from a state of segmentation to a state of integration—we find that risk exposures (covariance, co-skew and co-kurtosis risk), the price of these risks and deviations from the asset pricing restrictions implied by a cubic model for the stochastic discount factor (the alphas) all vary strongly with an underlying state that reflects bull and bear markets and tracks the world business cycle. The correlation of US stock returns with the volatility and skew of the global market portfolio is modest (particularly in the bear state). In addition US stocks have desirable co-movement properties with respect

²⁸For example, Cai and Warnock (2004) estimate US investors' foreign equity holdings at 24% when the foreign exposure of US firms is taken into account.

to future domestic short-term interest rates, thus increasing their weight in the portfolio of a US investor with skew and kurtosis preferences.

An interesting issue that goes beyond the analysis in the current paper is whether our results extend to the home bias observed in investors' equity holdings in other countries. One may conjecture that—because stock and bond markets in the same economy are more likely to be “in phase” than are markets across national borders—the finding that stock returns in one country have attractive co-moment properties with national short-term rates extends beyond our analysis for the US. This would contribute to explain the international evidence of a pervasive home bias in stock holdings.²⁹

Appendix A: Proof of Proposition 1

This appendix derives Proposition 1 and shows how to extend the results to include autoregressive terms in the return process. To derive the n -th moment of the cumulated return on the risky asset holdings in the general case with multiple risky assets (h) and states (k), notice that

$$E_t \left[(\omega'_t \exp(\mathbf{R}_{t+T}^s))^n \right] = E_t \left[\sum_{n_1=1}^n \dots \sum_{n_h=1}^n \lambda(n_1, n_2, \dots, n_h) (\omega_1^{n_1} \times \dots \times \omega_h^{n_h}) \times \exp \left(\sum_{i=1}^T r_{t+i}^1 \right)^{n_1} \dots \times \exp \left(\sum_{i=1}^T r_{t+i}^h \right)^{n_h} \right]. \quad (\text{A1})$$

where the powers $0 \leq n_i \leq n$ ($i = 1, \dots, h$) satisfy the summing-up constraint $\sum_{i=1}^h n_i = n$ and the coefficients λ are given by

$$\lambda(n_1, n_2, \dots, n_h) \equiv \frac{n!}{n_1! n_2! \dots n_h!}.$$

r_{t+i}^i ($i = 1, \dots, h$) represent the one-month return on asset i , $r_{t+i}^i \equiv x_{t+i}^i + r_{t+i}^b$. The sum in (A1) involves $\binom{h+n-1}{n}$ terms and requires solving for moments of the form

$$\begin{aligned} M_{t+T}^{(n)}(n_1, n_2, \dots, n_h) &= E_t \left[\exp \left(\sum_{i=1}^T r_{t+i}^1 \right)^{n_1} \times \dots \times \exp \left(\sum_{i=1}^T r_{t+i}^h \right)^{n_h} \right] \\ &= E_t \left[\exp \left(\sum_{l=1}^h n_l \sum_{i=1}^T r_{t+i}^l \right) \right]. \end{aligned} \quad (\text{A2})$$

(A2) can be decomposed as follows

$$M_{t+T}^{(n)}(n_1, n_2, \dots, n_h) = \sum_{i=1}^k M_{i,t+T}^{(n)}(n_1, n_2, \dots, n_h), \quad (\text{A3})$$

where for $i = 1, \dots, k$,

$$M_{i,t+T}^{(n)}(n_1, n_2, \dots, n_h) = E_t \left[\exp \left(\sum_{l=1}^h n_l \sum_{i=1}^T r_{t+i}^l \right) | S_{t+T} = i \right] \Pr(S_{t+T} = i).$$

²⁹We are grateful to an anonymous referee for pointing our attention in this direction.

Each of these terms satisfies the recursions

$$\begin{aligned}
M_{i,t+T}^{(n)}(n_1, n_2, \dots, n_h) &= \sum_{g=1}^k M_{g,t+T-1}^{(n)}(n_1, n_2, \dots, n_h) E_t \left[\exp \left(\sum_{l=1}^h n_l r_{t+T}^l \right) | S_{t+T} = i, \mathcal{F}_t \right] p_{gi} \\
&= \sum_{g=1}^k p_{gi} M_{g,t+T-1}^{(n)}(n_1, n_2, \dots, n_h) \exp \left(\sum_{l=1}^h n_l \tilde{\mu}_{il} + \sum_{l=1}^h \sum_{u=1}^h n_l n_u \frac{\sigma_{i,lu}}{2} \right), \quad (\text{A4})
\end{aligned}$$

where $\tilde{\mu}_{il}$ is the mean return of asset l in state i (possibly inclusive of risk premia related to covariance, co-skewness and co-kurtosis) and $\sigma_{i,lu} = e_l' \Omega_i e_u$ is the covariance between r_{lt+T} and r_{ut+T} in state $i = 1, 2, \dots, k$. This is a generalization of the result in (19).

Finally, using (A1) and (A2), we get an expression for the n -th moment of the cumulated return:

$$E_t \left[(\omega_t' \exp(\mathbf{R}_{t+T}^s))^n \right] = \sum_{n_1=0}^n \cdots \sum_{n_h=0}^n \lambda(n_1, n_2, \dots, n_h) (\omega_1^{n_1} \times \dots \times \omega_h^{n_h}) M_{t+T}^{(n)}(n_1, \dots, n_h). \quad (\text{A5})$$

Expected utility can now be evaluated in a straightforward generalization of (23):

$$\begin{aligned}
\hat{E}_t[U^m(W_{t+T}; \boldsymbol{\theta})] &= \sum_{n=0}^m \kappa_n \sum_{j=0}^n (-1)^{n-j} v_T^{n-j} {}_n C_j E_t[W_{t+T}^j] \\
&= \sum_{n=0}^m \kappa_n \sum_{j=0}^n (-1)^{n-j} v_T^{n-j} \binom{n}{j} \sum_{i=0}^j \binom{j}{i} E_t[(\omega_t' \exp(\mathbf{R}_{t+T}^s))^i] \left((1 - \omega_t' \boldsymbol{\iota}_h) \exp(T r^f) \right)^{j-i}.
\end{aligned}$$

Inserting (A5) into this expression gives a first order condition that takes the form of an $m - 1$ th order polynomial in the portfolio weights.

Generalizing the results to include autoregressive terms is straightforward. To keep the notation simple, suppose $k = 2$. Using (24) the n -th noncentral moment satisfies the recursions

$$\begin{aligned}
M_{i,t+T}^{(n)} &= M_{i,t+T-1}^{(n)}(n) p_{ii} \exp \left(n \tilde{\mu}_i + n \sum_{j=1}^p b_{j,i} E_t[r_{t+T-j}] + \frac{n^2}{2} \sigma_i^2 \right) + \\
&\quad + M_{-i,t+T-1}^{(n)}(n) (1 - p_{-i-i}) \exp \left(n \tilde{\mu}_i + n \sum_{j=1}^p b_{j,i} E_t[r_{t+T-j}] + \frac{n^2}{2} \sigma_i^2 \right)
\end{aligned}$$

or

$$\begin{aligned}
M_{1,t+1}^{(n)} &= \tilde{\xi}_1^{(n)} M_{1,t}^{(n)} + \tilde{\beta}_1^{(n)} M_{2,t}^{(n)} \\
M_{2,t+1}^{(n)} &= \tilde{\xi}_2^{(n)} M_{1,t}^{(n)} + \tilde{\beta}_2^{(n)} M_{2,t}^{(n)},
\end{aligned}$$

where now

$$\begin{aligned}
\tilde{\xi}_1^{(n)} &= p_{11} \exp \left(n \mu_1 + n \sum_{j=1}^p b_{j,1} E_t[r_{t+T-j}] + \frac{n^2}{2} \sigma_1^2 \right) \\
\tilde{\beta}_1^{(n)} &= (1 - p_{22}) \exp \left(n \mu_1 + n \sum_{j=1}^p b_{j,1} E_t[r_{t+T-j}] + \frac{n^2}{2} \sigma_1^2 \right)
\end{aligned}$$

$$\begin{aligned}\tilde{\xi}_2^{(n)} &= (1 - p_{11}) \exp \left(n\mu_2 + n \sum_{j=1}^p b_{j,2} E_t[r_{t+T-j}] + \frac{n^2}{2} \sigma_2^2 \right) \\ \tilde{\beta}_2^{(n)} &= p_{22} \exp \left(n\mu_2 + n \sum_{j=1}^p b_{j,2} E_t[r_{t+T-j}] + \frac{n^2}{2} \sigma_2^2 \right).\end{aligned}$$

Subject to these changes, the earlier methods can be used with the only difference that terms such as $\exp \left(n\tilde{\mu}_i + \frac{n^2}{2} \sigma_i^2 \right)$ have to be replaced by

$$\exp \left(n\tilde{\mu}_i + n \sum_{j=1}^p b_{j,i} E_t[r_{t+T-j}] + \frac{n^2}{2} \sigma_i^2 \right).$$

The term $\sum_{j=1}^p b_{j,i} E_t[r_{t+T-j}]$ may be decomposed in the following way:

$$\sum_{j=1}^p b_{j,i} E_t[r_{t+T-j}] = I_{\{j>T\}} \sum_{j=1}^p (I_{\{j \geq T\}} b_{j,i} r_{t+T-j} + I_{\{j < T\}} b_{j,i} E_t[r_{t+T-j}]),$$

where $E_t[r_{t+1}], \dots, E_t[r_{t+T-1}]$ can be evaluated recursively, c.f. Doan et al. (1984):

$$\begin{aligned}E_t[r_{t+1}] &= \pi_{1t} \left(\tilde{\mu}_1 + \sum_{j=1}^p b_{j,1} r_{t-j} \right) + (1 - \pi_{1t}) \left(\tilde{\mu}_2 + \sum_{j=1}^p b_{j,2} r_{t-j} \right) \\ E_t[r_{t+2}] &= \pi'_t \mathbf{P} \mathbf{e}_1 \left(\tilde{\mu}_1 + \sum_{j=1}^p b_{j,1} E_t[r_{t+1}] \right) + (1 - \pi'_t \mathbf{P} \mathbf{e}_1) \left(\tilde{\mu}_2 + \sum_{j=1}^p b_{j,2} E_t[r_{t+1}] \right) \\ &\vdots \\ E_t[r_{t+T-1}] &= \pi'_t \mathbf{P}^{T-1} \mathbf{e}_1 \left(\tilde{\mu}_1 + \sum_{j=1}^p b_{j,1} E_t[r_{t+T-2}] \right) + (1 - \pi'_t \mathbf{P}^{T-1} \mathbf{e}_1) \left(\tilde{\mu}_2 + \sum_{j=1}^p b_{j,2} E_t[r_{t+T-2}] \right).\end{aligned}$$

Appendix B: Estimation

This appendix describes the econometric methodology used in estimating the model (8). Defining $\boldsymbol{\eta}_{S_{t+1}}$ as a vector of residuals in state S_{t+1} , the contribution to the log-likelihood function conditional on being in state S_{t+1} at time $t+1$ is given (up to a constant) by:

$$\ln p(\mathbf{y}_{t+1} | \mathcal{F}_t, S_{t+1}; \boldsymbol{\lambda}) \propto -\frac{1}{2} \ln |\boldsymbol{\Omega}_{S_{t+1}}| - \frac{1}{2} \boldsymbol{\eta}'_{S_{t+1}} \boldsymbol{\Omega}_{S_{t+1}}^{-1} \boldsymbol{\eta}_{S_{t+1}},$$

where $\boldsymbol{\lambda} = \{\boldsymbol{\phi}_s, \boldsymbol{\Omega}_s, \mathbf{P}\}_{s=1}^k$ collects the mean ($\boldsymbol{\phi}$), variance ($\boldsymbol{\Omega}$) and transition probability (\mathbf{P}) parameters of the model (8). The expected value of the log-likelihood employed by the EM algorithm is maximized by choosing the parameters $\boldsymbol{\lambda}^{l+1}$ in the $l+1$ iteration to satisfy (see Hamilton (1990, p.51)):

$$\sum_{t=1}^T \sum_{S_{t+1}=1}^k \left. \frac{\partial \ln p(\mathbf{y}_{t+1} | \mathcal{F}_t, S_{t+1}; \boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}} \right|_{\boldsymbol{\lambda}=\boldsymbol{\lambda}^{l+1}} p(S_{t+1} | \mathbf{y}_2, \mathbf{y}_3, \dots, \mathbf{y}_T; \boldsymbol{\lambda}^l) = \mathbf{0}, \quad (\text{B1})$$

where $\{p(S_{t+1}|\mathbf{y}_2, \mathbf{y}_3, \dots, \mathbf{y}_T; \boldsymbol{\lambda}^l)\}_{S_{t+1}=1}^k$ are the smoothed state probabilities for each of the k states. Letting $\mathbf{y} \equiv [\mathbf{y}'_2 \ \mathbf{y}'_3 \ \dots \ \mathbf{y}'_T]'$ and $\boldsymbol{\eta} \equiv [\boldsymbol{\eta}'_1 \ \boldsymbol{\eta}'_2 \ \dots \ \boldsymbol{\eta}'_k]'$, it is useful to re-write the log-likelihood as:

$$\begin{aligned} \ell(\mathbf{y}_1, \dots, \mathbf{y}_T|\boldsymbol{\delta}) &\propto -\frac{1}{2} \sum_{s=1}^k \ln |\boldsymbol{\Omega}_s| \sum_{t=2}^T p(S_{t+1}; \boldsymbol{\lambda}^l) - \frac{1}{2} \sum_{s=1}^k \boldsymbol{\eta}'_s (\tilde{\boldsymbol{\Sigma}}_s \otimes \boldsymbol{\Omega}_s^{-1}) \boldsymbol{\eta}_s \\ &= -\frac{1}{2} \sum_{s=1}^k \ln |\boldsymbol{\Omega}_s| \sum_{t=2}^T p(S_{t+1}; \boldsymbol{\lambda}^l) - \frac{1}{2} \boldsymbol{\eta}' \mathbf{W}^{-1} \boldsymbol{\eta} \end{aligned}$$

where

$$\begin{aligned} \mathbf{Z} &\equiv \begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \\ \vdots \\ \mathbf{Z}_k \end{bmatrix}; \quad \mathbf{Z}_i \equiv \begin{bmatrix} [\mathbf{e}'_i \ \mathbf{e}'_i \otimes \mathbf{y}'_1] \otimes \mathbf{I}_N \\ [\mathbf{e}'_i \ \mathbf{e}'_i \otimes \mathbf{y}'_2] \otimes \mathbf{I}_N \\ \vdots \\ [\mathbf{e}'_i \ \mathbf{e}'_i \otimes \mathbf{y}'_{T-1}] \otimes \mathbf{I}_N \end{bmatrix} \\ \mathbf{W}^{-1} &\equiv \begin{bmatrix} \tilde{\boldsymbol{\Sigma}}_1 \otimes \boldsymbol{\Omega}_1^{-1} & \dots & \mathbf{O} \\ \vdots & \ddots & \vdots \\ \mathbf{O} & \dots & \tilde{\boldsymbol{\Sigma}}_k \otimes \boldsymbol{\Omega}_k^{-1} \end{bmatrix} \\ \tilde{\boldsymbol{\Sigma}}_i &\equiv \text{diag}\{p(s_2 = i; \boldsymbol{\lambda}^l), p(s_3 = i; \boldsymbol{\lambda}^l), \dots, p(s_T = i; \boldsymbol{\lambda}^l)\}. \end{aligned}$$

The EM updating equation for the transition probabilities is based on the smoothed state probabilities and can be found in equation (4.1) of Hamilton (1990, p. 51). Filtered state probabilities are calculated as a by-product. The first order conditions for the mean and variance parameters, $\boldsymbol{\phi}$ and $\boldsymbol{\Omega}$, are:

$$\frac{\partial \ln \ell(\mathbf{y}_t|\boldsymbol{\delta})}{\partial \boldsymbol{\phi}} = -\frac{1}{2} \hat{\boldsymbol{\eta}}' \hat{\mathbf{W}}^{-1} \mathbf{Z} = \mathbf{0} \quad (\text{B2})$$

$$\frac{\partial \ln \ell(\mathbf{y}_t|\boldsymbol{\delta})}{\partial \boldsymbol{\Omega}_s} = -\frac{1}{2} \sum_{t=1}^T p(S_{t+1} = s; \boldsymbol{\lambda}^l) \hat{\boldsymbol{\Omega}}_s^{-1} + \frac{1}{2} \hat{\boldsymbol{\Omega}}_s^{-1} \hat{\boldsymbol{\epsilon}}'_s \tilde{\boldsymbol{\Sigma}}_s \hat{\boldsymbol{\epsilon}}_s \hat{\boldsymbol{\Omega}}_s^{-1} = \mathbf{0} \quad s = 1, 2, \dots, k, \quad (\text{B3})$$

where $\hat{\boldsymbol{\epsilon}}_s \equiv [(\mathbf{y}_2 - \mathbf{Z}_{s_2=i} \hat{\boldsymbol{\phi}})' \ (\mathbf{y}_3 - \mathbf{Z}_{s_3=i} \hat{\boldsymbol{\phi}})' \ \dots \ (\mathbf{y}_T - \mathbf{Z}_{s_T=i} \hat{\boldsymbol{\phi}})']'$ are the residuals in state s and $\hat{\mathbf{W}}^{-1}$ is a function of $\{\hat{\boldsymbol{\Omega}}_s\}_{s=1}^k$. Equation (B2) implies that $\hat{\boldsymbol{\phi}}^{l+1}$ is a GLS estimator once observations are replaced by their smoothed probability-weighted counterparts:

$$\hat{\boldsymbol{\phi}}^{l+1} = (\mathbf{Z}' \hat{\mathbf{W}}^{-1} \mathbf{Z})^{-1} \mathbf{Z}' \hat{\mathbf{W}}^{-1} (\boldsymbol{\iota}_k \otimes \mathbf{y}). \quad (\text{B4})$$

Similarly, (B3) implies a covariance estimator

$$\hat{\boldsymbol{\Omega}}_s = \frac{\hat{\boldsymbol{\epsilon}}'_s \tilde{\boldsymbol{\Sigma}}_s \hat{\boldsymbol{\epsilon}}_s}{\sum_{t=1}^T p(S_{t+1}; \boldsymbol{\lambda}^l)}. \quad (\text{B5})$$

$\hat{\boldsymbol{\phi}}^{l+1}$ and $\{\hat{\boldsymbol{\Omega}}_s^{l+1}\}_{s=1}^k$ must be solved for jointly since $\hat{\boldsymbol{\epsilon}}_s$ enters the expression for the covariance matrix and also depends on $\hat{\boldsymbol{\phi}}^{l+1}$, while the regime-dependent covariance matrices $\{\hat{\boldsymbol{\Omega}}_s^{l+1}\}_{s=1}^k$ enter (B4) via $\hat{\mathbf{W}}^{-1}$. Hence, within each step of the EM algorithm, (B4)-(B5) is iterated upon until convergence of the estimates $\hat{\boldsymbol{\phi}}^{l+1}$ and $\{\hat{\boldsymbol{\Omega}}_s^{l+1}\}_{s=1}^k$.

Finally, notice that (B2) defines $\hat{\boldsymbol{\eta}}$ from

$$\boldsymbol{\eta}_{t+1} \equiv \mathbf{y}_{t+1} - \tilde{\boldsymbol{\mu}}_{S_{t+1}} - \mathbf{B}_{S_{t+1}} \mathbf{y}_t = \mathbf{y}_{t+1} - \boldsymbol{\mu}_{S_{t+1}} - \mathbf{M}_{S_t} \text{vec}(\boldsymbol{\Upsilon}_{S_{t+1}}) - \mathbf{B}_{S_{t+1}} \mathbf{y}_t,$$

so that $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t]$ enters $\mathbf{M}_{S_t} \text{vec}(\mathbf{\Upsilon}_l)$, while $\mathbf{M}_{S_t} \text{vec}(\mathbf{\Upsilon}_l)$ also affects $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t]$, creating a non-linear system of simultaneous equations. For instance, computing $\text{Cov}[\mathbf{x}_{t+1}, x_{t+1}^W|\mathcal{F}_t, S_t]$ requires knowledge of the first h elements of $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t]$. To make estimation possible, within the $(l+1)$ th step of the EM algorithm we use an iterative scheme by which $\mathbf{M}_{S_t} \text{vec}(\mathbf{\Upsilon}_l)$ is first estimated using the values in $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t]$ from the previous optimization step, $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t; \hat{\phi}^l]$. New values of $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t; \hat{\phi}^{l+1}]$ are then computed using estimates of $\mathbf{M}_{S_t} \text{vec}(\mathbf{\Upsilon}_l)$ that employ $E[\mathbf{y}_{t+1}|\mathcal{F}_t, S_t; \hat{\phi}^l]$. We then proceed iteratively until convergence.

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Table 1**Summary Statistics for International Stock Returns**

This table reports sample statistics for six international MSCI portfolios. Returns are denominated in US dollars, include dividends and are in excess of the 1-month US T-bill rate. Returns are monthly and the sample period is 1975:01 – 2005:12. Jarque-Bera is a test for normality based on the skew and kurtosis. Ljung-Box and Ljung-Box squares denote tests for fourth order serial correlation in returns and squared returns, respectively.

Portfolio	Mean	St. Dev.	Skewness	Kurtosis	Jarque-Bera	Ljung-Box	Ljung-Box squares
MSCI United States	0.5415	4.4825	-0.7084	5.9138	162.71**	1.8775	2.4714
MSCI Japan	0.3733	6.4830	0.0700	3.5044	4.2475	6.5087	11.888*
MSCI Pacific ex-Japan	0.3892	7.0538	-2.2723	22.297	5655.6**	2.7472	0.4998
MSCI Europe ex-UK	0.4158	5.0578	-0.5672	4.6124	60.246**	5.9087	12.560*
MSCI United Kingdom	0.7503	6.1898	0.7587	10.316	865.27**	4.1915	19.845**
MSCI World	0.4560	5.1740	-0.8711	6.9133	282.88**	2.3197	1.9827
US 1-month T-bills	0.4906	0.2517	0.8319	3.9949	58.250**	1248.2**	1084.5**

* denotes significance at the 5% level; ** denotes significance at the 1% level.

Table 2

Parameter Estimates for Single State and Two-State Models

Panel A reports parameter estimates for the extended single state ICAPM

$$x_{t+1}^i = \alpha^i + \gamma^i \text{Cov}_t[x_{t+1}^i, x_{t+1}^W] + b^i r_t^{US} + \eta_{t+1}^i \quad i = 1, \dots, 5$$

$$x_{t+1}^W = \alpha^W + \gamma^W \text{Var}_t[x_{t+1}^W] + b^W r_t^{US} + \eta_{t+1}^W$$

$$r_{t+1}^{US} = \alpha^Z + b^Z r_t^{US} + \eta_{t+1}^Z$$

where x_t^i and x_t^W consist of monthly excess returns on the MSCI stock index portfolios (in US dollars). $i = \text{US, Japan, Asia-Pacific (ex-Japan), United Kingdom, and Europe (ex-UK)}$, ' W ' stands for the world market portfolio, and r_t^{US} is the 1-month US T-bill rate. Panel B of the table reports maximum likelihood estimates for the two-state regime switching model:

$$x_{t+1}^i = \alpha_{s_{t+1}}^i + \gamma_{s_{t+1}}^1 \text{Cov}_t[x_{t+1}^i, x_{t+1}^W] + \gamma_{s_{t+1}}^2 \text{Cov}_t[x_{t+1}^i, (x_{t+1}^W)^2] + \gamma_{s_{t+1}}^3 \text{Cov}_t[x_{t+1}^i, (x_{t+1}^W)^3] + b_{s_{t+1}}^1 r_t^{US} + \eta_{t+1}^i \quad i = 1, \dots, 5$$

$$x_{t+1}^W = \alpha_{s_{t+1}}^W + \gamma_{s_{t+1}}^1 \text{Var}_t[x_{t+1}^W] + \gamma_{s_{t+1}}^2 \text{Skew}_t[x_{t+1}^W] + \gamma_{s_{t+1}}^3 K_t[x_{t+1}^W] + b_{s_{t+1}}^W r_t^{US} + \eta_{t+1}^W$$

$$r_{t+1}^{US} = \alpha_{s_{t+1}}^Z + b_{s_{t+1}}^Z r_t^{US} + \eta_{t+1}^Z$$

$\eta_{t+1} \equiv [\eta_{t+1}^{US} \quad \eta_{t+1}^{Jap} \quad \eta_{t+1}^{Pac} \quad \eta_{t+1}^{UK} \quad \eta_{t+1}^{EU}] \sim N(\mathbf{0}, \mathbf{\Omega}_{s_{t+1}})$ is the vector of unpredictable return innovations with regime-specific (heteroskedastic) variances but constant correlations across states. The coefficients b_s^i and b_s^W are set to zero in the first regime. $\text{Var}_t[x_{t+1}^W]$, $\text{Skew}_t[x_{t+1}^W]$ and $K_t[x_{t+1}^W]$ are the conditional variance, skew and kurtosis of excess returns on the world portfolio. Risk premia estimates are reported per unit of covariance, skewness, and kurtosis scaled by the appropriate powers (1, 1.5, and 2) of the variance of excess returns on the world market portfolio. For the covariance matrix we report monthly volatilities on the main diagonal and correlations off the diagonal. HAC standard errors are reported in parenthesis. The sample period is 1975:01 – 2005:12.

	U.S.	Japan	Pacific ex-Japan	Europe ex-UK	United Kingdom	World	U.S. T-bill
Panel A – Single State Gaussian Model							
Cross-sectional risk premia							
Covariance (γ_1)	5.303 (2.574)						
1. Intercepts (α's)	0.463 (0.632)	0.369 (0.691)	1.076 (0.673)	0.726 (0.651)	0.659 (0.652)	0.302 (1.027)	0.022 (0.580)
2. VAR coeffs.							
U.S. T-bill	-0.655 (0.324)	-0.866 (0.523)	-2.500 (1.047)	-1.472 (1.045)	-1.055 (1.059)	-0.843 (0.577)	0.955 (0.105)
3. Volatilities	14.916***	22.112***	23.106***	17.237***	19.903***	14.131***	0.249***
4. Correlations							
U.S.	1						
Japan	0.308**	1					
Pacific ex-Japan	0.540***	0.368**	1				
Europe ex-UK	0.587***	0.476**	0.538***	1			
United Kingdom	0.534***	0.397**	0.555***	0.632***	1		
World	0.845***	0.684***	0.628***	0.790***	0.699***	1	
U.S. T-bill	-0.103	-0.014	-0.131*	-0.046	-0.093	-0.102	1

Table 2 (continued)
Estimates of a Two-State Regime Switching Model

Panel B – Two State Model							
Cross-sectional risk premia							
	Bear State				Bull State		
Covariance ($\gamma_{1,S_{t+1}}$)	9.460 (5.114)				15.874 (5.088)		
Co-skewness ($\gamma_{2,S_{t+1}}$)	-1.077 (1.050)				-3.111 (1.266)		
Co-kurtosis ($\gamma_{3,S_{t+1}}$)	1.669 (2.898)				12.302 (5.048)		
	U.S.	Japan	Pacific ex-Japan	Europe ex-UK	United Kingdom	World	U.S. T-bill
1. Intercepts (α's)							
Bear State	-0.591 (0.323)	-1.756 (0.402)	-0.723 (0.393)	-0.720 (0.339)	-0.776 (0.360)	-0.968 (0.313)	0.0002 (0.0002)
Bull State	1.079 (0.191)	1.621 (0.263)	0.813 (0.274)	0.867 (0.220)	1.218 (0.297)	1.186 (0.236)	0.077 (0.032)
2. VAR coeffs.							
<i>Bear State:</i>							
U.S. T-bill	—	—	—	—	—	—	0.999 (0.064)
<i>Bull State:</i>							
U.S. T-bill	-0.588 (0.039)	-1.118 (0.565)	-1.143 (0.588)	-0.814 (0.475)	-0.527 (0.321)	-0.902 (0.373)	0.864 (0.180)
3. Volatilities							
Bear State	10.883 ^{***}	15.729 ^{***}	17.864 ^{***}	12.242 ^{***}	11.231 ^{***}	10.263 ^{***}	0.439 ^{***}
Bull State	9.165 ^{***}	13.340 ^{***}	13.785 ^{***}	11.155 ^{***}	15.065 ^{***}	8.480 ^{***}	0.546 ^{***}
4. Correlations							
U.S.	1						
Japan	0.276 [*]	1					
Pacific ex-Japan	0.552 ^{***}	0.361 ^{**}	1				
Europe ex-UK	0.619 ^{***}	0.447 ^{**}	0.561 ^{***}	1			
United Kingdom	0.594 ^{***}	0.393 ^{**}	0.645 ^{***}	0.716 ^{***}	1		
World	0.836 ^{***}	0.670 ^{***}	0.639 ^{***}	0.773 ^{***}	0.750 ^{***}	1	
U.S. T-bill	-0.079	-0.122	-0.115	-0.114	-0.097	-0.154	1
4. Transition probabilities							
	Bear State				Bull State		
Bear State	0.899 (0.205)				0.1011		
Bull State	0.0590				0.941 (0.146)		

*denotes significance at the 10% level, ** significance at the 5% level and *** at the 1% level.

Table 3**Model Specification Tests**

This table reports model specification tests based on the principle that under a correct specification, the properly transformed one-step-ahead standardized residuals should follow an independently and identically distributed normal distribution with zero mean and unit variance (Berkowitz (2001)). Significant tests indicated by stars show that the model is misspecified. Jarque-Bera tests whether the normalized residuals have zero skew and excess kurtosis. LR₂ is a test for correct mean and variance (zero and one, respectively); LR₃ tests for first order serial correlation, while LR₆ tests for first and second order serial correlation in the normalized residuals and their squares. This gives the ability to detect the presence of residual ARCH effects.

Model	Number of parameters	Jarque-Bera test	LR₂	LR₃	LR₆
United States					
Linear (VAR(1))	38	118.67**	19.984**	146.93**	193.87**
Two-state model	52	4.779	1.880	5.212	11.939
Japan					
Linear (VAR(1))	38	28.838**	3.644	165.87**	198.349**
Two-state model	52	1.893	3.156	17.396**	19.829**
Asia Pacific (ex-Japan)					
Linear (VAR(1))	38	4,307.4**	11.787**	89.827**	164.57**
Two-state model	52	22.330**	1.565	4.649	8.865
Europe (ex-United Kingdom)					
Linear (VAR(1))	38	38.516**	12.141**	100.60**	139.09**
Two-state model	52	4.063	1.656	7.737	11.803
United Kingdom					
Linear (VAR(1))	38	55.570**	14.647**	98.394**	167.84**
Two-state model	52	4.527	0.574	11.259*	31.734**
World					
Linear (VAR(1))	40	40.903**	0.029	117.74**	167.92**
Two-state model	52	1.952	1.763	7.751	15.188*
1-month T-bill Yield					
Linear (VAR(1))	38	290.58**	35.632**	79.663**	106.82**
Two-state model	54	6.078*	0.423	5.143	21.779**

*denotes significance at the 5% level, ** significance at the 1% level.

Table 4

Optimal Portfolio Weights Under Single State and Two-State Models

Stock holdings are reported as a fraction of the total equity portfolio (and thus sum to 1), while the T-bill holdings are shown as a percentage of the total portfolio. Allocations are computed under interest rates that can deviate by up to two standard deviations from their mean.

		Mean – 2 × SD.	Mean – 1 × SD.	Mean	Mean + 1 × SD.	Mean + 2 × SD.
Panel A - T = 1 month	Single State Benchmark					
	United States	0.228	0.346	0.313	0.156	0.101
	Japan	0.261	0.309	0.375	0.375	0.499
	Pacific (ex-Japan)	0.348	0.222	0.104	0.000	0.000
	United Kingdom	0.130	0.099	0.042	0.000	0.000
	Europe (ex-UK)	0.033	0.025	0.167	0.469	0.400
	1-month US T-bills	0.082	0.191	0.515	0.681	0.800
	Bear State ($\pi = 1$)					
	United States	0.697	0.722	0.746	0.772	0.796
	Japan	0.121	0.093	0.063	0.070	0.093
	Pacific (ex-Japan)	0.061	0.056	0.048	0.035	0.037
	United Kingdom	0.030	0.056	0.048	0.070	0.074
	Europe (ex-UK)	0.091	0.074	0.095	0.053	0.000
	1-month US T-bills	0.668	0.462	0.370	0.431	0.460
	Steady-state probs. ($\pi = 0.33$)					
	United States	0.625	0.696	0.685	0.817	0.851
	Japan	0.125	0.101	0.110	0.070	0.060
	Pacific (ex-Japan)	0.063	0.072	0.082	0.070	0.060
	United Kingdom	0.016	0.000	0.000	0.000	0.000
	Europe (ex-UK)	0.172	0.130	0.123	0.042	0.030
	1-month US T-bills	0.357	0.312	0.269	0.289	0.333
	Bull State ($\pi = 0$)					
	United States	0.535	0.537	0.598	0.656	0.713
	Japan	0.198	0.116	0.098	0.086	0.085
	Pacific (ex-Japan)	0.116	0.074	0.054	0.043	0.032
	United Kingdom	0.023	0.021	0.022	0.011	0.011
	Europe (ex-UK)	0.128	0.253	0.228	0.204	0.160
	1-month US T-bills	0.142	0.053	0.078	0.070	0.062
Panel B - T = 24 months	Single State Benchmark					
	United States	0.310	0.367	0.286	0.000	0.000
	Japan	0.310	0.304	0.381	0.306	0.367
	Pacific (ex-Japan)	0.230	0.177	0.190	0.000	0.000
	United Kingdom	0.149	0.152	0.095	0.056	0.000
	Europe (ex-UK)	0.000	0.000	0.048	0.639	0.633
	1-month US T-bills	0.132	0.208	0.578	0.639	0.698
	Bear State ($\pi = 1$)					
	United States	0.595	0.603	0.623	0.618	0.597
	Japan	0.139	0.141	0.130	0.118	0.125
	Pacific (ex-Japan)	0.089	0.090	0.091	0.079	0.069
	United Kingdom	0.127	0.128	0.117	0.118	0.125
	Europe (ex-UK)	0.051	0.038	0.039	0.066	0.083
	1-month US T-bills	0.208	0.22	0.226	0.240	0.278
	Steady-state probs. ($\pi = 0.33$)					
	United States	0.590	0.593	0.635	0.627	0.622
	Japan	0.108	0.105	0.094	0.108	0.110
	Pacific (ex-Japan)	0.048	0.058	0.059	0.072	0.085
	United Kingdom	0.084	0.081	0.071	0.060	0.061
	Europe (ex-UK)	0.169	0.163	0.141	0.133	0.122
	1-month US T-bills	0.168	0.142	0.149	0.168	0.179
	Bull State ($\pi = 0$)					
	United States	0.565	0.596	0.640	0.655	0.678
	Japan	0.087	0.090	0.093	0.080	0.080
	Pacific (ex-Japan)	0.054	0.056	0.058	0.057	0.046
	United Kingdom	0.054	0.045	0.023	0.023	0.034
	Europe (ex-UK)	0.239	0.213	0.186	0.184	0.161
	1-month US T-bills	0.083	0.109	0.140	0.132	0.131

Table 5

Estimates of Co-Skew and Co-Kurtosis Coefficients with World Market Portfolio

This table reports sample co-skew and co-kurtosis coefficients with the world market portfolio,

$$S_{i,w} \equiv \frac{\text{Cov}[x_t^i (x_t^w)^2 \mid \mathfrak{I}_t]}{\{\text{Var}[x_t^i \mid \mathfrak{I}_t]\}^{1/2} \text{Var}[x_t^w \mid \mathfrak{I}_t]}$$

$$K_{i,w} \equiv \frac{\text{Cov}[x_t^i (x_t^w)^3 \mid \mathfrak{I}_t]}{\{\text{Var}[x_t^i \mid \mathfrak{I}_t]\}^{1/2} \{\text{Var}[x_t^w \mid \mathfrak{I}_t]\}^{3/2}} \quad (i = \text{US, JP, Pac, UK, EU})$$

For comparison we also show the coefficients implied by a two-state regime switching model:

$$x_{t+1}^i = \alpha_{S_{t+1}}^i + \gamma_{S_{t+1}}^1 \text{Cov}_t[x_{t+1}^i, x_{t+1}^w] + \gamma_{S_{t+1}}^2 \text{Cov}_t[x_{t+1}^i, (x_{t+1}^w)^2] + \gamma_{S_{t+1}}^3 \text{Cov}_t[x_{t+1}^i, (x_{t+1}^w)^3] + b_{S_{t+1}}^1 r_t^{\text{US}} + \eta_{t+1}^i \quad i = 1, \dots, 5$$

$$x_{t+1}^w = \alpha_{S_{t+1}}^w + \gamma_{S_{t+1}}^1 \text{Var}_t[x_{t+1}^w] + \gamma_{S_{t+1}}^2 \text{Skew}_t[x_{t+1}^w] + \gamma_{S_{t+1}}^3 K_t[x_{t+1}^w] + b_{S_{t+1}}^w r_t^{\text{US}} + \eta_{t+1}^w$$

$$r_{t+1}^{\text{US}} = \alpha_{S_{t+1}}^Z + b_{S_{t+1}}^Z r_t^{\text{US}} + \eta_{t+1}^Z$$

The coefficients are calculated both conditional on the current state and under steady state probabilities

		Bear state	Bull state	Steady-state probs.	Data
United States	Co-skew	0.151	-0.127	-0.128	-0.052
	Co-kurtosis	3.200	3.434	3.408	3.401
Japan	Co-skew	0.018	-0.001	0.016	0.004
	Co-kurtosis	2.207	2.294	3.303	3.428
Pacific ex-Japan	Co-skew	-0.161	-0.567	-0.677	-0.535
	Co-kurtosis	4.522	5.782	6.561	6.704
United Kingdom	Co-skew	-0.066	-0.252	-0.339	-0.321
	Co-kurtosis	5.297	5.207	5.230	4.910
Europe ex-UK	Co-skew	0.114	-0.167	-0.222	-0.227
	Co-kurtosis	4.192	4.095	4.116	4.113

Table 6

Effects of Preferences (m) on Portfolio Choice

The table reports the optimal allocation to international stocks as a function of the state probability for three choices of the order of the preference polynomial, m : $m=2$ (mean-variance preferences), $m=3$ (three-moment or skew preferences), and $m=4$ (four-moment or skew and kurtosis preferences). T is the investment horizon. Stock holdings are reported as a fraction of the total equity portfolio (and thus sum to 1), while the T-bill holdings are shown as a percentage of the total portfolio.

M		U.S.	Japan	Pacific ex-Japan	UK	EU	US T-bills
Bear State ($\pi = 1$)							
$T=1$	$m=2$	0.661	0.143	0.036	0.036	0.125	0.441
	$m=3$	0.841	0.079	0.016	0.063	0.000	0.373
	$m=4$	0.746	0.063	0.048	0.048	0.095	0.370
$T=6$	$m=2$	0.778	0.032	0.016	0.000	0.175	0.369
	$m=3$	0.721	0.088	0.000	0.162	0.029	0.320
	$m=4$	0.653	0.153	0.056	0.056	0.083	0.282
$T=24$	$m=2$	0.594	0.000	0.058	0.000	0.348	0.309
	$m=3$	0.550	0.163	0.013	0.200	0.075	0.201
	$m=4$	0.623	0.130	0.091	0.117	0.039	0.226
Steady-state state probs. ($\pi = 0.33$)							
$T=1$	$m=2$	0.536	0.014	0.014	0.000	0.435	0.310
	$m=3$	0.521	0.366	0.000	0.113	0.000	0.289
	$m=4$	0.685	0.110	0.082	0.000	0.123	0.269
$T=6$	$m=2$	0.500	0.000	0.056	0.000	0.444	0.282
	$m=3$	0.532	0.286	0.000	0.169	0.013	0.231
	$m=4$	0.646	0.127	0.076	0.051	0.101	0.209
$T=24$	$m=2$	0.525	0.000	0.050	0.013	0.413	0.198
	$m=3$	0.519	0.210	0.012	0.247	0.012	0.190
	$m=4$	0.635	0.094	0.059	0.071	0.141	0.149
Bull State ($\pi = 0$)							
$T=1$	$m=2$	0.262	0.299	0.020	0.181	0.238	0.000
	$m=3$	0.131	0.446	0.002	0.408	0.013	0.000
	$m=4$	0.598	0.098	0.054	0.022	0.228	0.078
$T=6$	$m=2$	0.189	0.232	0.042	0.147	0.389	0.048
	$m=3$	0.215	0.398	0.000	0.366	0.022	0.069
	$m=4$	0.632	0.092	0.057	0.023	0.195	0.125
$T=24$	$m=2$	0.427	0.012	0.049	0.049	0.463	0.180
	$m=3$	0.422	0.241	0.012	0.301	0.024	0.171
	$m=4$	0.640	0.093	0.058	0.023	0.186	0.140

Table 7

Effect of Risk Aversion on Home Bias

The table reports optimal portfolio weights under regime switching when the coefficients of the objective function are evaluated by interpreting the objective as an n -th order Taylor approximation to power utility with constant relative risk aversion θ . The weights are calculated assuming steady state probabilities and a 1-month US T-bill rate that is set at its sample mean of 5.9 percent per annum. The weight on the world market portfolio is re-allocated to the five regional portfolios using their relative market capitalizations. T is the investment horizon. Stock holdings are reported as a fraction of the total equity portfolio (and thus sum to 1), while the T-bill holdings are shown as a percentage of the total portfolio.

	Risk aversion	US	Japan	Pacific ex-Japan	UK	Europe ex-UK	US T-bills
$T = 1$	$\theta = 2$	0.746	0.119	0.090	0.000	0.045	0.329
	$\theta = 5$	0.698	0.111	0.048	0.016	0.127	0.370
	$\theta = 10$	0.727	0.091	0.018	0.055	0.109	0.448
$T = 6$	$\theta = 2$	0.646	0.127	0.076	0.051	0.101	0.213
	$\theta = 5$	0.651	0.143	0.016	0.048	0.143	0.366
	$\theta = 10$	0.737	0.105	0.018	0.053	0.088	0.429
$T = 12$	$\theta = 2$	0.642	0.111	0.062	0.062	0.123	0.190
	$\theta = 5$	0.642	0.119	0.030	0.075	0.134	0.333
	$\theta = 10$	0.721	0.115	0.016	0.066	0.082	0.387
$T = 24$	$\theta = 2$	0.643	0.095	0.060	0.071	0.131	0.158
	$\theta = 5$	0.662	0.099	0.028	0.099	0.113	0.292
	$\theta = 10$	0.710	0.097	0.016	0.129	0.048	0.381

Table 8

Optimal Portfolio Choice under Four-Moment Preferences and Power Utility

This table compares optimal portfolio weights under four-moment preferences with the weights calculated (by simulation, using 60,000 independent draws) under power utility, with θ is set to 2. Returns are generated from a two-state regime-switching model:

$$x_{t+1}^i = \alpha_{S_{t+1}}^i + \gamma_{S_{t+1}}^1 \text{Cov}_t[x_{t+1}^i, x_{t+1}^W] + \gamma_{S_{t+1}}^2 \text{Cov}_t[x_{t+1}^i, (x_{t+1}^W)^2] + \gamma_{S_{t+1}}^3 \text{Cov}_t[x_{t+1}^i, (x_{t+1}^W)^3] + b_{S_{t+1}}^1 r_t^{\text{US}} + \eta_{t+1}^i \quad i=1, \dots, 5$$

$$x_{t+1}^W = \alpha_{S_{t+1}}^W + \gamma_{S_{t+1}}^1 \text{Var}_t[x_{t+1}^W] + \gamma_{S_{t+1}}^2 \text{Skew}_t[x_{t+1}^W] + \gamma_{S_{t+1}}^3 K_t[x_{t+1}^W] + b_{S_{t+1}}^W r_t^{\text{US}} + \eta_{t+1}^W$$

$$r_{t+1}^{\text{US}} = \alpha_{S_{t+1}}^Z + b_{S_{t+1}}^Z r_t^{\text{US}} + \eta_{t+1}^Z$$

where $\eta_{t+1} \equiv [\eta_{t+1}^{\text{US}} \quad \eta_{t+1}^{\text{Jap}} \quad \eta_{t+1}^{\text{Pac}} \quad \eta_{t+1}^{\text{UK}} \quad \eta_{t+1}^{\text{EU}}] \sim I.I.D. \quad N(0, \Omega_{S_{t+1}})$ is the vector of return innovations with regime-specific (heteroskedastic) variances across states. T is the investment horizon. Stock holdings are reported as a fraction of the total equity portfolio (and thus sum to 1), while the T-bill holdings are shown as a percentage of the total portfolio.

	US	Japan	Pacific ex-Japan	UK	Europe ex-UK	US T-bills
Bear state ($\pi = 1$)						
T = 1 – Four moment	0.746	0.063	0.048	0.048	0.095	0.370
T = 1 – CRRA	0.707	0.042	0.067	0.077	0.107	0.368
T = 6 – Four moment	0.653	0.153	0.056	0.056	0.083	0.282
T = 6 – CRRA	0.604	0.143	0.056	0.108	0.089	0.285
T = 24 – Four moment	0.623	0.130	0.091	0.117	0.039	0.226
T = 24 – CRRA	0.602	0.130	0.091	0.167	0.009	0.224
Bull state ($\pi = 0$)						
T = 1 – Four moment	0.598	0.098	0.054	0.022	0.228	0.078
T = 1 – CRRA	0.598	0.098	0.054	0.004	0.246	0.075
T = 12 – Four moment	0.632	0.092	0.057	0.023	0.195	0.125
T = 12 – CRRA	0.632	0.092	0.057	0.006	0.213	0.126
T = 24 – Four moment	0.640	0.093	0.058	0.023	0.186	0.140
T = 24 – CRRA	0.651	0.093	0.058	0.011	0.187	0.141

Table 9

Confidence Bands for Portfolio Weights

The table reports simulated confidence bands for optimal portfolio weights under either a two-state regime switching model or a single-state model. The weights are calculated assuming the 1-month US T-bill rate is set at its mean. The weight on the world market portfolio is re-allocated to the five regional portfolios using their relative market capitalizations as of 2005:12. T is the investment horizon. Stock holdings are reported as a fraction of the total equity portfolio (and thus sum to 1), while the T-bill holdings are shown as a percentage of the total portfolio..

	T = 1 month		T = 6 months		T = 24 months	
	5% Lower Bound	95% Upper Bound	5% Lower Bound	95% Upper Bound	5% Lower Bound	95% Upper Bound
Single-State Model						
United States	0.024	0.379	0.000	0.371	0.000	0.350
Japan	0.088	0.469	0.075	0.451	0.137	0.404
Pacific (ex-Japan)	0.000	0.217	0.000	0.240	0.002	0.259
United Kingdom	0.000	0.314	0.000	0.331	0.000	0.414
Europe (ex-UK)	0.000	0.320	0.000	0.307	0.000	0.272
1-month US T-bills	0.418	0.861	0.423	0.897	0.475	0.937
Two-State Model						
Bear Regime ($\pi = 1$)						
United States	0.586	0.834	0.438	0.964	0.416	0.845
Japan	0.037	0.095	0.045	0.270	0.029	0.196
Pacific (ex-Japan)	0.033	0.065	0.000	0.114	0.006	0.137
United Kingdom	0.020	0.081	0.000	0.150	0.012	0.184
Europe (ex-UK)	0.013	0.220	0.000	0.382	0.000	0.201
1-month US T-bills	0.311	0.381	0.201	0.440	0.138	0.343
Steady-state probs. ($\pi = 0.33$)						
United States	0.636	0.727	0.606	0.665	0.504	0.691
Japan	0.090	0.127	0.107	0.134	0.060	0.108
Pacific (ex-Japan)	0.070	0.093	0.052	0.067	0.038	0.067
United Kingdom	0.004	0.015	0.029	0.059	0.037	0.083
Europe (ex-UK)	0.065	0.225	0.064	0.207	0.114	0.267
1-month US T-bills	0.256	0.289	0.209	0.216	0.047	0.194
Bull Regime ($\pi = 0$)						
United States	0.484	0.744	0.578	0.674	0.518	0.736
Japan	0.015	0.145	0.073	0.106	0.060	0.113
Pacific (ex-Japan)	0.082	0.118	0.079	0.098	0.069	0.109
United Kingdom	0.000	0.047	0.000	0.027	0.000	0.036
Europe (ex-UK)	0.000	0.407	0.000	0.308	0.000	0.317
1-month US T-bills	0.090	0.226	0.124	0.142	0.050	0.230

Table 10

Out-of-Sample Portfolio Performance

The table reports summary statistics for realized utility (using four-moment preferences) and (annualized) portfolio returns based on the portfolio weights associated with the recursive estimates of a two-state regime switching model, a single-state VAR(1) model, and a static ICAPM in which all international portfolios are bought in proportion to their weight in the world market portfolio. Asset allocations across international equity markets are calculated for two investment horizons, $T = 1$ month and $T = 24$ months. The weight on the world market portfolio is re-allocated to the five regional portfolios using their relative market capitalization. SD denotes standard deviations; the CEV is the annualized percentage certainty equivalent of a given mean realized utility. 'Equal weights' is a portfolio that assigns equal weight to all international equity portfolios such that the holdings in 1-month US T-bills matches those from the two-state model. Panel A reports portfolio performance from a simulation experiment in which the data generating process is the two-state regime switching model of Table 2. Panel B uses actual MSCI returns data from the sample period 1986:01 - 2005:12.

Panel A - Simulated Data										
	T=1 month					T=24 months				
	Realized Utility			Annualized Returns		Realized Utility			Annualized Returns	
	Mean	SD	CEV	Mean	SD	Mean	SD	CEV	Mean	SD
Two-state RS	-0.987	0.021	16.42	16.77	7.28	-0.722	0.108	17.69	18.79	12.59
VAR(1)	-0.992	0.017	9.89	11.35	5.89	-0.799	0.070	11.89	12.38	7.99
ICAPM	-0.989	0.011	14.22	14.03	4.16	-0.764	0.094	14.42	15.15	10.11
Equal weights	-0.991	0.015	11.95	12.68	5.54	-0.802	0.066	11.63	12.03	7.50
Panel B - Actual Data										
	T=1 month					T=24 months				
	Realized Utility			Annualized Returns		Realized Utility			Annualized Returns	
	Mean	SD	CEV	Mean	SD	Mean	SD	CEV	Mean	SD
Two-state RS	-0.993	0.029	8.22	8.73	10.05	-0.849	0.158	8.54	10.09	13.08
VAR(1)	-0.995	0.022	6.18	8.72	7.62	-0.872	0.103	7.09	7.89	9.69
ICAPM	-0.994	0.039	7.44	11.35	12.82	-0.850	0.223	7.72	11.45	16.76
Equal weights	-0.994	0.031	7.63	10.03	10.74	-0.849	0.154	7.33	8.72	12.30

Table 11

Effects of Rebalancing

This table reports the optimal allocation to stocks and US T-bills under dynamic rebalancing every ϕ months. The ‘buy-and-hold column’ corresponds to a rebalancing frequency equal to the investment horizon, T. When ϕ exceeds the horizon T, we report a ‘N.A.’ (not available). The weight on the world market portfolio is re-allocated to the five regional portfolios using their relative market capitalization as of 2005:12. Stock holdings are reported as a fraction of the total equity portfolio (and thus sum to 1), while the T-bill holdings are shown as a percentage of the total portfolio.

		$\phi = 1$ month	$\phi = 3$ months	$\phi = 6$ months	$\phi = 12$ months	Buy-and-hold
Panel A - T = 6 month	Bear Regime ($\pi = 1$)					
	United States	0.642	0.666	0.653	N.A.	0.653
	Japan	0.183	0.162	0.153	N.A.	0.153
	Pacific (ex-Japan)	0.095	0.068	0.056	N.A.	0.056
	United Kingdom	0.081	0.045	0.056	N.A.	0.056
	Europe (ex-UK)	0.000	0.059	0.083	N.A.	0.083
	1-month US T-bills	0.383	0.338	0.282	N.A.	0.282
	Steady-state probs. ($\pi = 0.33$)					
	United States	0.596	0.644	0.646	N.A.	0.646
	Japan	0.135	0.131	0.127	N.A.	0.127
	Pacific (ex-Japan)	0.116	0.095	0.076	N.A.	0.076
	United Kingdom	0.032	0.044	0.051	N.A.	0.051
	Europe (ex-UK)	0.120	0.086	0.101	N.A.	0.101
	1-month US T-bills	0.185	0.245	0.209	N.A.	0.209
	Bull Regime ($\pi = 0$)					
	United States	0.000	0.242	0.633	N.A.	0.633
	Japan	0.074	0.088	0.092	N.A.	0.092
	Pacific (ex-Japan)	0.726	0.458	0.057	N.A.	0.057
	United Kingdom	0.015	0.015	0.023	N.A.	0.023
	Europe (ex-UK)	0.186	0.197	0.195	N.A.	0.195
	1-month US T-bills	0.011	0.120	0.126	N.A.	0.126
Panel B - T = 24 months	Bear Regime ($\pi = 1$)					
	United States	0.842	0.830	0.773	0.653	0.623
	Japan	0.146	0.160	0.153	0.137	0.130
	Pacific (ex-Japan)	0.000	0.000	0.015	0.085	0.091
	United Kingdom	0.012	0.010	0.058	0.116	0.117
	Europe (ex-UK)	0.000	0.000	0.000	0.008	0.039
	1-month US T-bills	0.398	0.393	0.355	0.262	0.226
	Steady-state probs. ($\pi = 0.33$)					
	United States	0.837	0.824	0.800	0.684	0.635
	Japan	0.121	0.124	0.118	0.101	0.094
	Pacific (ex-Japan)	0.000	0.000	0.000	0.044	0.059
	United Kingdom	0.000	0.000	0.000	0.057	0.071
	Europe (ex-UK)	0.042	0.052	0.082	0.113	0.141
	1-month US T-bills	0.388	0.369	0.331	0.211	0.149
	Bull Regime ($\pi = 0$)					
	United States	0.000	0.000	0.066	0.451	0.640
	Japan	0.099	0.096	0.097	0.096	0.093
	Pacific (ex-Japan)	0.771	0.774	0.702	0.283	0.058
	United Kingdom	0.000	0.000	0.000	0.011	0.023
	Europe (ex-UK)	0.130	0.130	0.134	0.160	0.186
	1-month US T-bills	0.249	0.198	0.187	0.168	0.140

Figure 1
Bear state probabilities

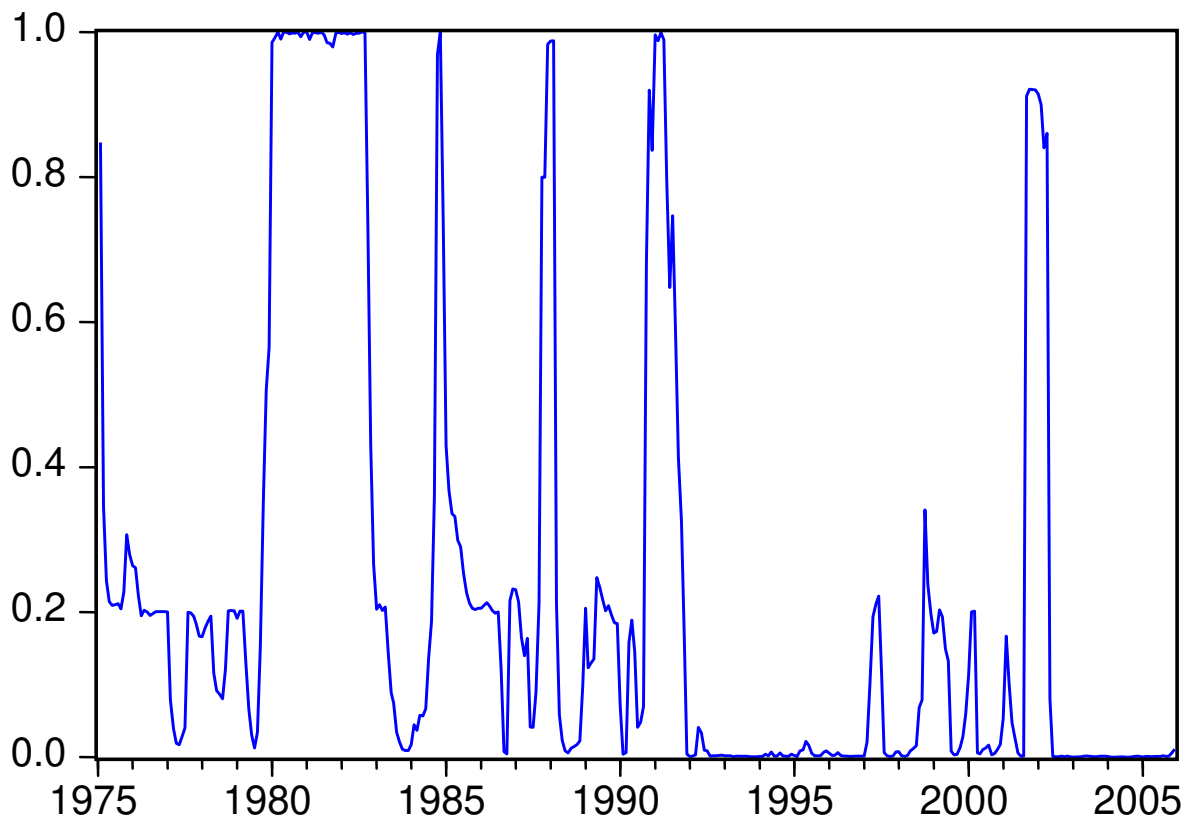
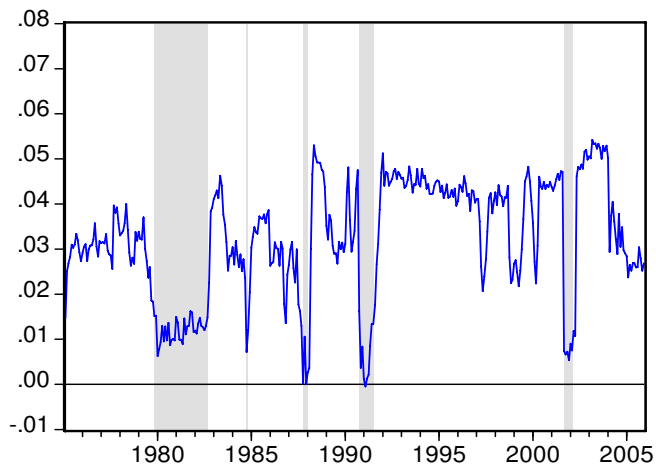
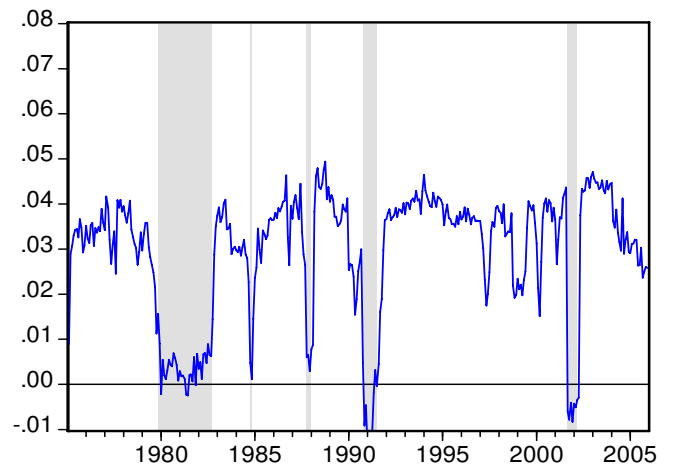


Figure 2

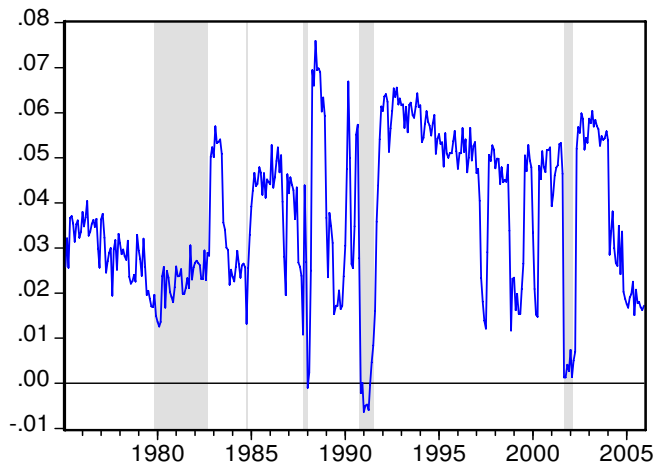
Mean excess returns (per annum) in the two-state model



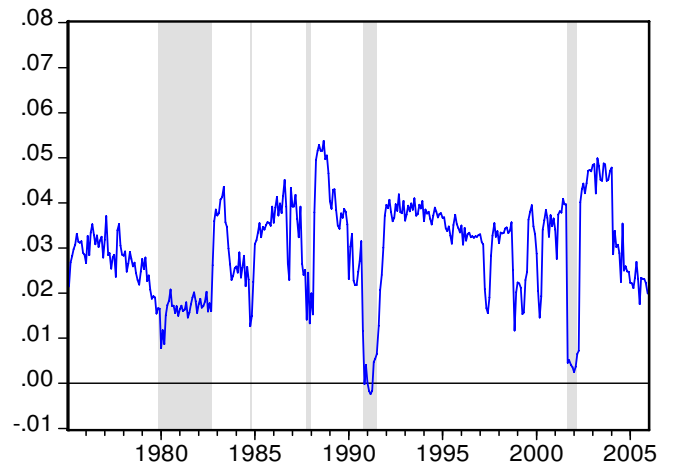
— United States



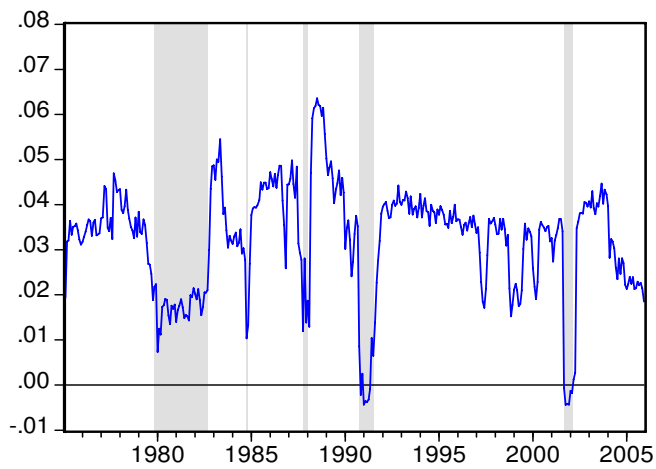
— Japan



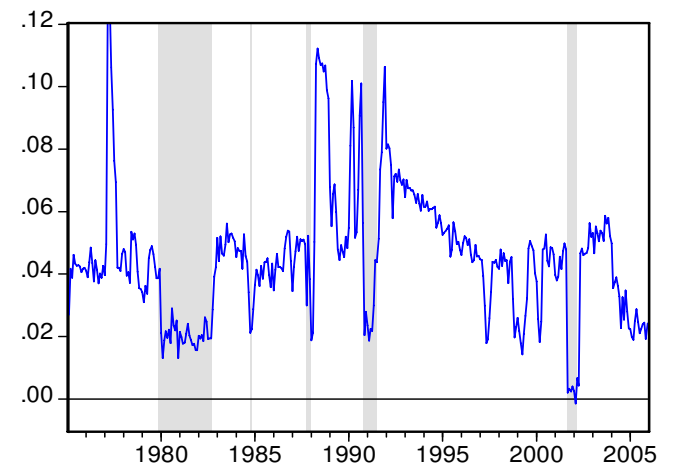
— Pacific (ex-Japan)



— Europe (ex-UK)



— United Kingdom



— World market

Figure 3

Correlations between world market and regional market returns

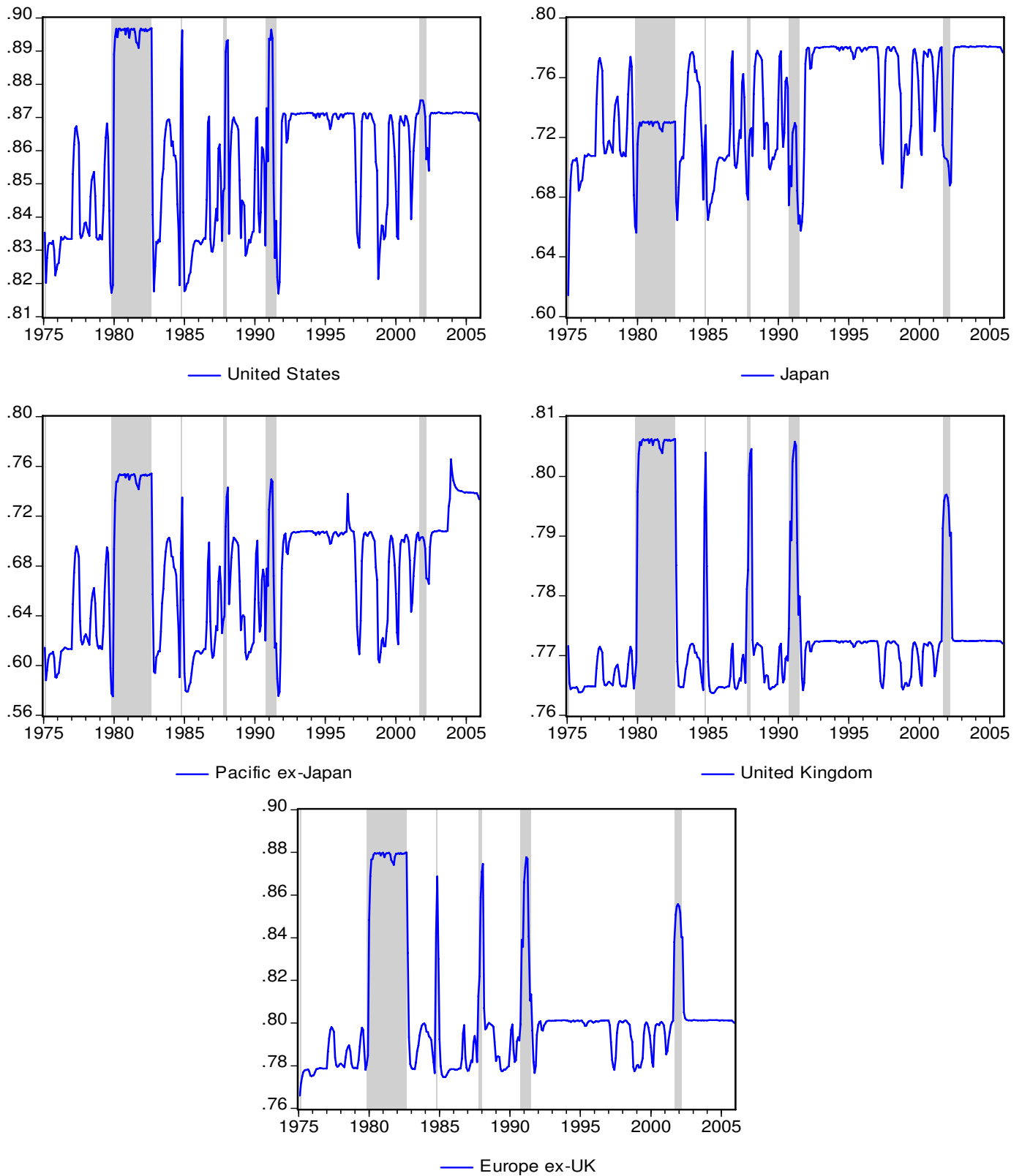


Figure 4

Mean excess returns, volatility, skew and kurtosis of the world market portfolio implied by the two-state model (annualized figures)

