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A Flexible Finite-Horizon Alternative to Long-run Restrictions with an Application to Technology Shocks*

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Abstract

Recent studies using long-run restrictions question the validity of the technology-driven real business cycle hypothesis. We propose an alternative identification that maximizes the contribution of technology shocks to the forecast-error variance of labor productivity at a long, but finite, horizon. In small-sample Monte Carlo experiments, our identification outperforms standard long-run restrictions by significantly reducing the bias in the short-run impulse responses and raising their estimation precision. Unlike its long-run restriction counterpart, when our Max Share identification technique is applied to U.S. data it delivers the robust result that hours worked responds negatively to positive technology shocks.

[JEL: C32, C50, E32]

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1 Introduction

By their nature, long-run restricted structural vector autoregressions are subject to the criticism that restrictions on infinite-order lag polynomials are ill-suited to samples of realistic proportions [see, for example, Sims (1972); Faust (1996); and Faust and Leeper (1997)]. In finite samples, measures of the VAR moving-average parameters at very long horizons are imprecise; when relied on for identification, this parameter uncertainty translates into potentially spurious inference. Using Monte Carlo methods, Erceg, Guerrieri, and Gust (EGG, 2005) and Chari, Kehoe, and McGrattan (CKM, 2008) assesses the extent of these small-sample estimation problems. These papers simulate repeated small samples of data from variations of standard Real Business Cycle (RBC) models and/or New Keynesian sticky price models and apply the long-run (LR) identification to obtain hypothetical small-sample distributions of the impulse responses to technology shocks. Both studies conclude that impulse responses identified by LR restrictions can be substantially biased, either in sign or in magnitude.

Recently, LR restrictions have attracted renewed attention as a means for identifying technology shocks in VARs [see, in particular, Galí (1999) and Francis and Ramey (2005)]. In these papers, identification is based on the assumption that the unit root in labor productivity arises exclusively from technology shocks. Results from some of these studies have led some to question the notion that technological innovation is the preeminent force behind business cycle fluctuations. Positive technology shocks identified using U.S. data yield a decline in hours, apparently contradicting the theoretical predictions of a broad class of RBC models.[1] This result has initiated some controversy, with a number of studies offering conflicting evidence based on alternative specifications of the non-productivity component of Galí's empirical model.[2] This paper focuses instead on the identifying assumption regarding the estimated long-run productivity process.

We offer an alternative approach to identification with the intent of addressing some of the aforementioned shortcomings associated with LR in small-sample estimation. When applied to

[1] Basu, Fernald, and Kimball (2006) and Shea (1999) use different techniques to identify technology and conclude that the hours response is negative.

[2] A number of papers [including Galí and Rabanal (2005) and Christiano, Eichenbaum, and Vigfusson (CEV, 2004)] have focused on the stationarity of hours as the key determinant of the sign of the hours response to a technology shock. A decline in hours is obtained when hours are first-differenced [Galí (1999)], detrended [Fernald (2007)], or demographically-adjusted [Francis and Ramey (2009)]. CEV (2004) argue that *per capita* labor is bounded and cannot have a unit root. If assumed stationary, hours responds to a technology shock positively on impact.

technology shocks, our methodology preserves the association between technology and productivity at frequencies below typical business cycles. Specifically, we identify the technology shock as that associated with the maximum forecast-error variance share (Max Share) in labor productivity at a long, finite horizon.[3]

The Max Share approach has several potential advantages over the conventional LR approach. First, by focusing on a finite horizon, we hope to gain estimation precision over LR, which relies on much longer horizon parameter estimates. Second, in place of the restriction that the unit root in productivity is driven *exclusively* by technology, our approach imposes a weaker restriction that the forecast-error variance in productivity at long horizons is *dominated* by the technology shock. Thus, we essentially allow other shocks to influence labor productivity at all (finite) horizons over which we employ the Max Share algorithm.

The mechanics of this methodology are similar to those introduced by Faust (1998); however, the current application to technology shocks is substantively different in a number of important ways. To our knowledge, we are the first to recognize the suitability of the Max Share approach as a finite-horizon alternative for identification of structural VARs with long-run restrictions. In its original context, Faust identified monetary policy shocks using only the robust predictions of structural VARs identified with short-run restrictions. The two approaches also differ conceptually. Whereas Faust used his objective function as a robustness check of the claim that monetary policy shocks explain only a small portion of output variability, we use our objective function as a necessary condition for identification.[4] Finally, we take advantage of the methodology as a way to obtain better small-sample estimation properties than in long-run structural vector autoregressions (SVARs), something not considered in either Faust or in related work by Uhlig (2004).

The Max Share approach is also similar in spirit to the medium-term driving forces recently proposed by Uhlig (2004) and Comin and Gertler (2006). However, a fundamental difference between these and the Max Share approach is that the latter allows the data to determine the

[3] While our application is to technology shocks, the identification can be applied to any case in which a dominant driving process exists. For example, in work following an earlier draft of this paper, Barsky and Sims (2011) adopt our approach to identify information shocks that are orthogonal to contemporaneous output but have permanent effects on future output. Beaudry, Nam and Wang (2011) identify future changes in total factor productivity and test whether these changes are related to shocks to economic optimism.

[4] Faust makes no presumption—theoretical or otherwise—that the identifying restriction imposed by the optimization criterion necessarily holds in the data-generating process. In contrast, the objective function in our approach serves a fundamental role as a substitute for the restriction that the long-run variance of labor productivity is primarily driven by technology shocks.

relative importance of technology at a predetermined horizon instead of specifying its relative importance at the outset. For instance, Uhlig estimates a model in which technology shocks are identified by a process that explains all of the h-step-ahead forecast revision of labor productivity for some fixed $0 \leq h < \infty$. Our approach, on the other hand, utilizes a maximization routine for horizons up to and including h. We find this more palatable because, in the RBC world, technology explains all of the forecast-error variance, at best, only at $h = \infty$; under the Uhlig assumption the spectrum may be radically shifted in ways that potentially violate the underlying RBC assumption.

Using data simulated from an off-the-shelf RBC model and a standard medium-scale DSGE model with sticky prices, we find that the Max Share approach exhibits less bias (measured by the deviation between the median response and the theoretical response) and less uncertainty (measured by the width of the 68 percent error bands) than the LR approach. These advantages are found to be robust to alternative specifications of the theoretical technology and non-technology shocks. However, relaxing the Galí assumption by allowing non-technology shocks to have nontrivial effects on labor productivity at sufficiently long horizons can qualitatively alter this short-run hours response. Results using the Max Share approach applied to U.S. data are consistent with Galí's original finding that hours decline after a technology shock.

In the next section, we present the Max Share identification approach. We then compare the small-sample performances of the two identification approaches using data simulated from the RBC and sticky price models. In the remainder of the paper, we apply the Max Share approach to postwar U.S. data and examine the robustness of the LR findings to our relaxation of the original identifying assumption. Finally, we incorporate the additional restriction that hours respond positively to a technology shock to examine whether this causes a significant shift in the associated share of the maximum forecast-error variance.

2 Identification

In this section, we provide an overview of the mechanics behind the standard LR and our Max Share identifications. Both approaches isolate the (primary) driver of long-run productivity trend. While the identifying approaches differ in their implementation, the key assumption behind both the Max Share and LR identification is consistent with that imposed by Galí:

Assumption 1: Technology shock is the sole contributor of long-run labor productivity shifts. All other structural innovations having transitory effects on labor productivity.

The assumption identifying technology shocks arises from a broad class of models in which *log* labor productivity, x_t, can be decomposed into two orthogonal components: an unobserved random walk trend component, x_t^T, which we will call technology, and an unobserved cyclical process, x_t^S.[5] The trend-cycle decomposition of productivity is

$$x_t = x_t^T + x_t^S, \tag{1}$$

where

$$x_t^T = x_{t-1}^T + \eta_t, \tag{2}$$

$\eta_t \sim iid\ N(0, \sigma_\eta^2)$, and x_t^S is stationary and ergodic. Since all processes except technology are assumed stationary, the unit root in productivity must arise from x_t^T. Thus, under Assumption 1, technology will necessarily dominate the forecast-error variance of the log-level of productivity at suitably long forecast horizons.[6] Given that x_t^S can be thought of as driven by an amalgamated non-technology shock (composed of fiscal, monetary, and tax shocks), Assumption 1 provides the foundation for both the standard LR identification and our finite-horizon Max Share identification.

2.1 The LR Identification

Assume that the data-generating process can be approximated by the following linear model:

$$A(L)y_t = \varepsilon_t,$$

where $A(L) = \sum_{i=0}^{p} A_i L^i$ is a matrix polynomial in the lag operator, L; ε_t is a structural innovation; $E(\varepsilon_t \varepsilon_t') = I$; and y_t is an $n \times 1$ vector of period-t macroeconomic variables with labor productivity ordered first and entered in *differences*.

[5] Throughout, we will assume that labor productivity is the only $I(1)$ variable.

[6] Recall that the variance of a unit root process, $var(x_t^T) = t\sigma_\eta$, increases with t. When x_t is included in a VAR, the forecast-error variance of x_t grows unbounded as the forecast horizon, h, increases. At sufficiently long horizons, the forecast-error variance is dominated by the non-stationary component [Lütkepohl (1993), p. 377].

To estimate this model, we begin with the reduced-form VAR:

$$B(L)y_t = \mu_t, \tag{3}$$

where $B(L) = \sum_{i=0}^{p} B_i L^i$, $B_0 = I$, and $E(\mu_t \mu_t') = V$. The goal is to find a transformation of the moving-average representation of the VAR:

$$y_t = C(L)A_0^{-1}A_0\mu_t,$$

which identifies the i.i.d. structural shocks of the model:

$$\varepsilon_t = A_0\mu_t,$$

where $C(L) = B(L)^{-1}$ and A_0 is the contemporaneous structural parameter matrix. Conventional long-run identification [e.g., Blanchard and Quah (1989); Shapiro and Watson (1988)] imposes restrictions on the effect of the j-th shock on the i-th variable at an infinite horizon. This is implemented through restrictions on $\left[C(1)A_0^{-1}\right]_{i,j}$, where neutrality implies the restriction $\left[C(1)A_0^{-1}\right]_{i,j} = 0$ for some j. Formally, we have

$$\left[C(1)A_0^{-1}\right]_{i=1,j\neq i} = 0, \tag{4}$$

where $i = 1$ represents labor productivity growth ordered first and $j \neq i$ indicates all non-technology shocks.[7]

2.2 Finite-Horizon Max Share Identification

As in Galí (1999), our objective is to isolate technology shocks by characterizing their effect on productivity at long horizons. However, instead of imposing long-run restrictions, we identify the technology shock by maximizing the forecast-error variance share of productivity at long, finite horizons. This approach is consistent with suggestions in Uhlig (2004) and CEV and is adapted from methods introduced in Faust (1998). We begin by introducing the methodology and then

[7] Equation (4) is isomorphic to the assumption that the zero-frequency spectrum of labor productivity growth is attributable entirely to technology [see also DiCecio and Owyang (2010)].

discuss its practicality.

2.2.1 Methodology

In the Max Share identification, all variables including labor productivity enter the VAR in *log-levels*.[8] The method is operationalized by first expressing the h-step-ahead forecast error for y as a function of realized reduced-form errors:

$$y_{t+h} - \widehat{y}_{t+h} = \sum_{\tau=0}^{h-1} C_\tau \mu_{t+h-\tau}, \tag{5}$$

where $\widehat{y}_{t+h}$ is the h-step-ahead forecast of y conditional on time-t information. Next, we define an orthonormal matrix D, which obtains an alternative linear representation of the reduced-form model:

$$y_{t+h} - \widehat{y}_{t+h} = \sum_{\tau=0}^{h-1} C_\tau D D' \mu_{t+h-\tau}.$$

Then, the h-step-ahead forecast-error variance share for a particular variable i attributable to a particular shock j in this new representation is

$$\omega_{ij}\left(\alpha(h)\right) = \frac{e_i'\left[\sum_{\tau=0}^{h-1} C_\tau \alpha\alpha' C_\tau'\right] e_i}{e_i'\left[\sum_{\tau=0}^{h-1} C_\tau \Omega_\mu C_\tau'\right] e_i}, \tag{6}$$

where e_i is an $n \times 1$ indicator vector that picks out the impulse vector $\alpha = De_j$, the i-th column vector of D.

The technology shock is identified by solving, for a given value of h, the following maximization problem over all possible α:

$$\max_\alpha \omega_{1j}\left(\alpha(h)\right) = \frac{e_i'\left[\sum_{\tau=0}^{h-1} C_\tau \alpha\alpha' C_\tau'\right] e_i}{e_i'\left[\sum_{\tau=0}^{h-1} C_\tau H H' C_\tau'\right] e_i}, \tag{7}$$

where H is obtained by a Cholesky decomposition of Ω_μ making the identified technology shock, $\varepsilon^{tech} = \alpha' H^{-1} \mu_\tau$, orthogonal to other shocks in the system[9] We impose the additional normalization

[8] To execute LR, labor productivity must be entered in differences. In principle, the Max Share identification can be used on systems in which labor productivity enters either in levels or in differences. As an alternative, strong beliefs can be incorporated into a prior.

[9] As described in the appendix in Faust (1998), the maximization problem is solved by obtaining α^*, the eigenvector associated with the maximum eigenvalue of V.

$\alpha'\alpha = 1$, ensuring that the technology shocks have unit variance. The horizon h at which the forecast-error variance for labor productivity is maximized is chosen exogenously. While h is initially fixed at 10 years, we later consider the effect of varying h from 2.5 to 20 years.

2.2.2 Practical Considerations

LR has previously been found to perform poorly in small samples of length comparable to the U.S. postwar sample [see EGG and CKM]. The Max Share approach may demonstrate *less* small-sample bias, in part because it relies on restrictions at a finite horizon.[10] Sims (1972) and, more recently, Faust and Leeper (1997) and CEV (2007) make similar arguments regarding the differences between short- and long-run restrictions. Similar arguments have been made in favor of medium-run restrictions [e.g., Uhlig (2004); Khan and Tsoukalas (2007)].[11]

The econometrician typically uses some criteria to determine the lag order, which may or may not be the appropriate specification. For example, the VAR representation of many theoretical models is a VAR(∞), which necessitates a truncation of the lag order for estimation. If the model is misspecified, the implied responses will be biased, the degree of bias depending on the manner of identification. There may be key differences in the bias that arise from using either LR or Max Share in small samples. LR, for example, places a restriction on $C(1)$, the infinite sum of the transformed/rotated VAR coefficients. The econometrician, however, does not use the true $C(1)$; instead, she uses the estimated $\widehat{C}(1)$ which may bias the identification of the shock.[12] A conjecture is that imposing restrictions on $\sum_{\tau=0}^{h-1} C_\tau \alpha\alpha' C_\tau'$ rather than $C(1)$ can *reduce*—but not eliminate—misspecification bias while preserving the theoretical interpretation of the identification.[13] We investigate this in Monte Carlo experiments below.

The preceding discussion suggests that errors in estimating $C_\tau A_0^{-1}$ may confound the identification, making it possible to attribute too much of the forecast error variance in productivity to

[10] Unlike LR, the Max Share rotation is not preserved as the sample size increases [Mittnik and Zadrozny (1993)] making the formal proof of bias reduction problematic.

[11] Dupor and Kiefer (2008) take a different approach with a similar flavor. They place restrictions on the finite-horizon effects estimated via local projections [Jorda (2005)].

[12] An informal proof of this conjecture can be found in CEV (2007). CEV (2007) contend that using short-run restrictions minimizes the effect of potential misspecification because short-run restrictions do not require $\widehat{C}(1)$ for identification. Moreover, CEV and others argue that identification using very-long-horizon restrictions can be problematic because the spectral mass near the zero frequency can be small.

[13] In addition to bias caused by misspecifying the VAR, the Max Share identification also may be subject to bias caused by choosing h too small. It is not clear *ex ante* which effect will dominate in small samples.

technology. While both LR and Max Share may, to some extent, suffer from this problem, the risks in the LR approach are symmetric around zero. In the Max Share approach, however, the maximization algorithm shrinks the risk of underestimating the forecast error variance share relative to that of overestimating. Given this trade-off, we must determine whether the Max Share identification yields a net advantage in small samples. In the next section, we measure the net effect of employing the Max Share identification by comparing the small-sample performance of the LR and Max Share identifications in Monte Carlo experiments against known data-generating processes.

3 Monte Carlo Experiments

This section outlines the Monte Carlo methods used to assess the ability of the two identification approaches for small samples. We use simulated data from a real business cycle model (RBC) and a New Keynesian (NK) model with nominal rigidities (sticky prices and wages) and real frictions (monopolistic competition, habit formation in consumption, investment adjustment costs, variable capital utilization). For both models, equation (1) holds, allowing us to obtain theoretical impulse responses to a technology shock, but the hours response varies across models. For the RBC model, hours rise in response to a positive technology shock; for the NK model, hours fall. Theoretical impulse responses and simulated data are generated for parameterizations of the two models which vary the persistence of the nontechnology component. Model details and parameterizations appear in the online appendices.

Each Monte Carlo iteration consists of 247 simulated observations of each of the variables, i.e., the same length as the U.S. data sample analyzed below.[14] We estimate a reduced-form VAR and obtain structural impulse responses both by using LR restrictions and by Max Share. The posterior distributions for the impulse responses are simulated from $1,000$ draws utilizing a Normal–Inverse-Wishart prior centered on the OLS estimates [see Sims and Zha (1999)]. To account for potential asymmetries in the impulse responses, we retain the median, 16^{th}, and 84^{th} percentiles for each response. This process is repeated to obtain $1,000$ median estimates and error bands, each corresponding to what an econometrician would estimate given a single set of data. The average

[14] In results not reported here, we found that increasing the sample size reduced the bias between the estimated and theoretical impulse responses for both identification mothods. These results are available upon request.

of these 1,000 median statistics and error bands, which can be interpreted as the expected value of the econometrician's estimates, is reported and compared with their theoretical counterparts. Analysis of the posterior coverage follows. We also analyze the correlation between the theoretical shocks and those estimated by both LR and Max Share.

3.1 Benchmark Results

Both the RBC and NK models are four-variable VAR(4) with the logs of labor productivity, hours, the consumption-output ratio, and the investment-output ratio. Each reduced-form VAR is estimated using OLS. Log productivity enters the VAR in first differences for the LR but enters in levels for the Max Share approach.[15] The benchmark models are parameterized to be consistent with those specified in EGG; the AR(1) coefficient of technology (ρ_z) is set to 1 and non-technology shocks have AR(1) coefficients of 0.98.[16] For Max Share, the technology shock is chosen to be that which maximizes the forecast-error variance share at a horizon of 40 quarters. We consider alternative horizons below.

Figure 1 presents the impulse responses to a 1-standard-deviation technology shock for both the RBC model and the NK model, respectively. The thick solid lines depict the theoretical impulse responses. The averages of the Max Share median responses across Monte Carlo iterations are shown by the thick dotted lines, with the shaded areas representing the accompanying 68 percent coverage bands. The average of the LR median responses and their error bands are shown by dashed lines. In the RBC model, a positive technology shock leads to an immediate increase in both labor productivity and hours. In the NK model, a positive technology shock leads to an increase in labor productivity but a short-run decline in hours.

For the both models, our results corroborate EGG's findings that LR biases the median responses but preserves their shape. The Max Share impulse responses match the theoretical impulse responses qualitatively and display less bias than the LR responses, especially at short horizons. For most variables, the both approaches yield impulse responses biased toward zero. Although

[15] In results not reported here, we considered the effect of entering productivity in differences for the Max Share identification. The results for the RBC model were qualitatively similar to those reported here. Entering productivity in differences for the NK model produced responses essentially identical to those from LR. These results are available upon request.

[16] We also tested the effect of decreasing the persistence of non-technology shocks on both algorithms. Results were qualitatively similar except that LR was able to distinguish the sign of the impact response of hours for the RBC model. These results are reported in the online appendix or available upon request.

the theoretical impulse responses are near the upper tail of the 68 percent probability intervals for both methods, the Max Share responses are considerably closer to the theoretical responses for the first two years following the shock. The probability intervals from the Max Share model are also narrower than their LR counterparts over this horizon. In particular, consider the 68 percent coverage for hours—the series many researchers look to differentiate between models. For the RBC model, the LR coverage interval includes zero, meaning that using LR to infer the true model from the may be problematic. Max Share, on the other hand, clearly differentiates between the two models as both 68 percent coverage intervals exclude zero.

In addition to the responses to the shocks, we can compare the time series of identified shocks to the time series from the generated data. Table 1 presents the correlations between the model-generated and the estimated technology shocks from both identifications. A higher correlation suggests a more accurate identification of the time series of shocks. The median correlation for the Max Share shocks with the true shocks is greater (about 0.81 and 0.81 for the RBC and NK models, respectively) than that for LR shocks (about 0.50 and 0.68 for the RBC and NK models, respectively). Additionally, the median correlation for the LR model lies in the far left tail of the distribution for the Max Share correlations. Taken together with the preceding results, this suggests the technology shock is better identified by Max Share than by LR in small samples similar in length to the U.S. sample.

3.2 Bias and Coverage Analysis

The performance of Max Share depends on choosing h, the identification horizon, large enough to pick out the correct shock but small enough to minimize the misspecification bias. All of the previous results have been obtained with an exogenously chosen Max Share horizon, h, of 40 quarters, Here, we ask how much does changing the identification horizon affect the results? To answer this, we compute the average absolute bias of the responses identified by Max Share while varying the forecast horizon. Figure 2 summarizes the bias properties of the Max Share approach when h varies from 10 to 80 quarters for the benchmark parameterization. The bias is computed over the first four quarters, expressed as a percentage of the true model response. The dotted lines depict the bias computed using the Max Share approach; the dashed line shows a similar measure for LR which is constant by definition. For both models, the bias shown by Max Share is clearly smaller than that of LR for most variables and identification horizons. The Max Share bias is relatively constant over a wide range of h. For example, for the RBC model, the productivity bias for Max Share(80) is approximately 27 percent, well below that for LR (51 percent) but only slightly higher than for Max Share(10) (12 percent). In these model environments, the bias improvement of the Max Share approach is robust to choosing relatively short identification horizons.

These differences in bias also affect the true coverage. In the analysis above, we reported the 68 percent nominal coverage—that is, the means of the interior 68 percent of the posterior distributions of the impulse responses. These nominal coverages may not necessarily contain the true (model) response with 68 percent probability. In fact, the bias in the responses for both Max Share and LR can affect the true coverage leading us to ask how well the 68 percent nominal coverage represents the true coverage. Figure 3 displays the percentage of Monte Carlo iterations in which the 68 percent nominal coverage band contains the true response for the baseline parameterizations of the two models. Because of the bias in both identified responses, the true coverage—especially at short horizons—can be much less than the 68 percent nominal coverage. For the RBC model, Max Share has better coverage for almost all of the variables. The exception is hours, where LR has larger than 68 percent coverage. This suggests that LR produces error bands that are too wide. A similar argument can be made for the NK model. LR more often contains the true response but, in all cases, has greater than 68 percent coverage.

4 Max Share Identification in U.S. Data

Having evaluated the small-sample performance of our identification scheme through Monte Carlo experiments, we turn to U.S. data and estimate a four-variable VAR(4). For the analysis of U.S. data, we use data from the St. Louis Fed's FRED and updated hours data from Francis and Ramey (2009).[17] The sample spans 1948:Q2 throughout 2009:Q4. The VARs include productivity (in log-level or growth rate), log hours, log consumption- and investment-to-output ratios.[18] As in the Monte Carlo section above, the error bands are computed for 68 percent coverage using methods

[17] For a full description of the data, see the online appendix, http://research.stlouisfed.org/fred2/, and http://weber.ucsd.edu/ vramey/research.html.

[18] As some variation exists in the data, we estimated versions of the VAR in which consumption is composed of nondurables, services, and government spending; investment is composed of private investment plus durables; and hours and productivity measures are adjusted for demographic components (Francis and Ramey, 2009). The impulse responses were qualitatively similar across these models.

detailed in Sims and Zha (1999).

4.1 Baseline Results for U.S. Data

The dotted lines in the left column of Figure 4 present the median impulse response to a one-standard-deviation technology shock and the 68 percent probability intervals.[19] The solid lines in these figures represent sign-restricted responses discussed below. In response to a positive technology shock, both consumption and investment increase. If left unrestricted, labor hours fall for the first few quarters and eventually rise above zero.

The right column of Figure 4 displays the impulse responses to a technology shock identified by LR with the associated 68 percent error bands. The median predictions for all of the variables except hours are similar for both the LR and Max Share identifications. The *median* response of hours on impact is positive under LR but not statistically distinguishable from zero. The hours response to the Max Share shock, on the other hand, is negative on impact with zero lying outside the 68 percent coverage interval. In addition, the error bands associated with the Max Share are everywhere narrower.

Figure 5 compares the forecast-error variance shares for output and hours attributable to technology for both Max Share and LR. While the share of output variance is large at most horizons under both identifications, technology explains only a minority share of the variance in hours when identified by Max Share. This result is consistent with CEV (2004) and would suggest that technology is not an important driver of the positive correlation in output and hours at business cycle frequencies.

As we have previously noted, the identifying restriction imposed by Max Share depends on the horizon for which the forecast-error variance share is maximized. To this end, we assess the Max Share impulse responses' sensitivity to the forecast horizon by varying h between 40 and 100 quarters and find them to be qualitatively similar across horizons. As might be expected, the width of the error bands for each response grows as the optimization horizon increases. This result points to the difficulty that longer-horizon restrictions yield more uncertainty in their corresponding short-run predictions.

[19] All of the Max Share results shown are based on $h = 40$ quarters; similar results were obtained for horizons of 20, 60, and 100 quarters. Although the width of the error bands for the Max Share identification increases with h, they are always narrower than those obtained from LR.

4.2 Incorporating Sign Restrictions for U.S. Data

The Max Share approach also allows for exhaustive robustness analysis across a broad class of models. While our finding that hours respond negatively to a positive technology shock appears to corroborate Galí's original result, many papers have questioned the robustness of this prediction. In particular, one can imagine that a small modification to the identifying assumption—perhaps corresponding to a more accommodative monetary policy or a greater influence of non-technology factors—could yield a different qualitative prediction for hours. Taking all such possibilities into account, a more complete robustness test asks: *(1) Is it possible to identify a technology shock that yields a positive response in hours? and (2) If so, what are the features of such a shock?*

To this end, we reestimate the Max Share model with the additional restriction that hours respond positively on impact to a positive technology shock.[20] We then examine whether this has a discernible effect on the FEV share attributable to technology. The left column in Figure 4 also shows the median impulse responses to a technology shock when the additional sign restriction is imposed.[21] The addition of the short-run sign restriction preserves the shape but shifts the point estimates of the responses for each variable. In particular, the median hours response shifts upward above the 68 percent probability interval of the unrestricted hours model for the first five or so quarters. The positive restriction on hours also raises the short-term consumption and investment responses. The net result of restricting technology to raise hours is apparently to amplify technology's effect on short-run output. But how effective is this "restricted" shock at explaining cyclical fluctuations?

Figure 6 displays smoothed kernel densities for the empirical posterior distributions of the maximum forecast-error variance shares estimated at horizons of 40 and 100 quarters. The black line shows hours unrestricted and the gray line shows the share from the model estimated with hours restricted to be positive on impact. The share of labor productivity fluctuations explained by technology declines when hours are restricted to respond positively to a technology shock.

[20] Dedola and Neri (2007) propose an agnostic approach to identify technology shocks using only sign restrictions on the impulse responses. Their results, however, are not directly comparable to ours as they make no restrictions on technology's contribution to productivity's FEV.

[21] As in Faust (1998), sign and shape restrictions on the impulse response of variable i at horizon(s) h can also be incorporated by solving the optimization problem (7) with an additional constraint of the form $e_i' C_t \alpha \geq 0$. The additional restriction on hours is not an overidentifying restriction suitable to a likelihood ratio test. The Max Share identifying assumption (7) is sufficient to identify technology, but the system as a whole is underidentified with or without the additional sign restriction.

Thus, a positive hours response is attainable, but only when the importance of technology shocks significantly diminishes at these horizons.[22] This suggests that a positive hours response may also result when non-technology shocks are influential to labor productivity at an infinite horizon. In other words, the exclusivity assumption in the LR model may be playing an important role in obtaining the negative hours prediction. These results indicate that models identified by restrictions made at long horizons may contain only limited information about short-run movements in hours.

5 Conclusion

We propose an alternative method for identifying shocks in VARs in which long-run restrictions have been ordinarily used. This methodology has the advantage of being robust to relaxing key assumptions about the data-generating process while maintaining the spirit of long-run restrictions. When applied to technology, the shock is identified as that which yields the maximum forecast-error variance share of productivity at some predetermined, yet finite, horizon.

Applied to artificial small samples generated from off-the-shelf RBC and NK models, the Max Share identification outperforms the standard LR identification. In particular, our identification reduces the bias of estimated impulse responses relative to theoretical responses. In addition, the Max Share impulse responses appear to, on average, less biased than those identified using the identifying restrictions proposed by Galí (1999). We also find that the Max Share technology shocks are more highly correlated with the theoretical shocks than those identified by LR. These results reveal a clear improvement over the LR estimates in small samples.

For U.S. postwar data, the Max Share model predicts a negative short-run response in hours, confirming the original LR finding of Galí (1999) and others. However, a positive hours response is attainable if a greater role for non-technology shocks is allowed. Unfortunately, neither model can be rejected based on posterior odds. Nevertheless, our results suggest that the rejection of the RBC framework on the basis of the qualitative response in hours depends critically on the assumption that technology has exclusive influence on long-run productivity.

When we view our model and the infinite-horizon models as a class, our results can be interpreted

[22] This does not necessarily imply that non-technology shocks influence long-run productivity. The odds ratios for the restricted and unrestricted models were indistinguishable, preventing any conclusions based on how well the model fits the data.

as demonstrating the limitations of long- (or infinite-) horizon restrictions in predicting short-run movements in hours. A modest, empirically reasonable adjustment to the assumption regarding the long-run importance of non-technology factors yields different predictions for the direction of the hours response. In light of these findings, we advocate a more flexible identification environment such as the one proposed here.

References

[1] Barsky, Robert B. and Sims, Eric R. "News Shocks and Business Cycles," *Journal of Monetary Economics*, April 2011, *58*(3), pp. 273-89.

[2] Basu, Susanto; Fernald, John G.; and Kimball, Miles S. "Are Technology Improvements Contractionary?" *American Economic Review*, December 2006, *96*(5), pp. 1418-48.

[3] Beaudry, Paul; Nam, Doekwoo; and Wang, Jian. "Do Mood Swings Drive Business Cycles and is it Rational?,", NBER Working Paper No. 17651, December 2011.

[4] Blanchard, Olivier J. and Quah, Danny. "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *American Economic Review,* September 1989, 79(4), pp. 655-73.

[5] Chari, V.V.; Kehoe, Patrick J.; and McGrattan, Ellen R. "Are Structural VARs with Long-Run Restrictions Useful in Developing Business Cycle Theory?," *Journal of Monetary Economics*, November 2008 , *55*(8), pp. 1337–1352.

[6] Christiano, Lawrence J.; Eichenbaum, Martin; and Vigfusson, Robert. "What Happens After a Technology Shock?," Manuscript, August 2004.

[7] Christiano, Lawrence J.; Eichenbaum, Martin; and Vigfusson, Robert. "Assessing Structural VARs," *NBER Macroeconomics Annual 2006*, Volume 21, Eds. Daron Acemoglu, Kenneth Rogoff, and Michael Woodford, 2007, pp. 1-72.

[8] Comin, Diego and Gertler, Mark. "Medium-Term Business Cycles," *American Economic Review*, June 2006, *96*(3), pp. 523-51.

[9] DiCecio, Riccardo and Owyang, Michael. "Identifying Technology Shocks in the Frequency Domain," Federal Reserve Bank of St. Louis Working Paper 2010-025A

[10] Dedola, Luca and Neri, Stafano. "What Does a Technology Shock Do? A VAR Analysis with Model-Based Sign Restrictions," *Journal of Monetary Economics*, March 2007, *54*(2), pp. 512-49.

[11] Dupor, Bill and Kiefer, Leonard. "Executing Long Run Restrictions," Manuscript, October 2008.

[12] Erceg, Christopher J.; Guerrieri, Luca; and Gust, Christopher. "Can Long-Run Restrictions Identify Technology Shocks?," *Journal of the European Economic Association*, December 2005, *3*(6), pp. 1237-78.

[13] Faust, Jon. "Theoretical Confidence Level Problems with Confidence Intervals for the Spectrum of a Time Series," International Finance Discussion Paper No. 575, Board of Governors of the Federal Reserve System, December 1996.

[14] Faust, Jon. "The Robustness of Identified VAR Conclusions about Money," *Carnegie-Rochester Conference Series on Public Policy*, December 1998, *49*(0), pp. 207-44.

[15] Faust, Jon and Leeper, Eric M. "When Do Long-Run Identifying Restrictions Give Reliable Results?," *Journal of Business and Economic Statistics*, July 1997, *15*(3), pp. 345-53.

[16] Fernald, John. "Trend Breaks, Long-Run Restrictions, and Contractionary Technology Improvements," *Journal of Monetary Economics* 54(8), November 2007, 2467-85

[17] Francis, Neville and Ramey, Valerie A. "Is the Technology-Driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited," *Journal of Monetary Economics*, November 2005, *52*(8), pp. 1379-99.

[18] Francis, Neville and Ramey, Valerie A. "Measures of Per Capita Hours and their Implications for the Technology-Hours Debate," *Journal of Money Credit and Banking*, September 2009, Volume 41, Issue 6, pp 1047-263.

[19] Galí, Jordi. "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?," *American Economic Review*, March 1999, *89*(1), pp. 249-71.

[20] Galí, Jordi and Rabanal, Pau. "Technology Shocks and Aggregate Fluctuations: How Well Does the Real Business Cycle Model Fit Postwar U.S. Data?," *NBER Macroeconomics Annual 2004*, Volume 19, Eds. Mark Gertler and Kenneth Rogoff, 2005, pp. 225-88.

[21] Jorda, Oscar. "Estimation and Inference of Impulse Responses by Local Projections," *American Economic Review*, March 2005, *95*(1), pp. 161-82.

[22] Khan, Hashmat and Tsoukalas, John. "Technology Shocks and UK Business Cycles," Working Paper, December 2007.

[23] Lütkepohl, Helmut. *Introduction to Multiple Time Series Analysis*, Second Edition. New York: Springer-Verlag, 1993.

[24] Mittnik, Stefan and Zadrozny, Peter A. "Asymptotic Distributions of Impulse Responses, Step Responses, and Various Decompositions of Estimated Linear Dynamic Models," *Econometrica*, July 1993, *61*(4), pp. 857–70.

[25] Shapiro, Matthew D. and Watson, Mark W. "Sources of Business Cycle Fluctuations," NBER Macroeconomics Annual, 1988, Vol. 3, pp. 111-48.

[26] Shea, John. "What Do Technology Shocks Do?," *NBER Macroeconomics Annual 1998*, Volume 13, Eds. Ben S. Bernanke and Julio J. Rotemberg, 1999, pp. 275-310.

[27] Sims, Christopher A. "The Role of Approximate Prior Restrictions in Distributed Lag Estimation," *Journal of the American Statistical Association*, March 1972, *67*(337), pp. 169-75.

[28] Sims, Christopher A. and Zha, Tao. "Error Bands for Impulse Responses," *Econometrica*, September 1999, *67*(5), pp. 1113-55.

[29] Uhlig, Harald. "Do Technology Shocks Lead to a Fall in Total Hours Worked?," *Journal of the European Economic Association*, April/May 2004, *2*(2/3), pp. 361-71.

Table 1			
Correlation Between Estimated and Model Technology Shocks			
RBC Model			
Identification	$16th$ percentile	median	$84th$ percentile
LR	0.09	0.50	0.75
Max Share	0.67	0.81	0.89
NK Model			
Identification	$16th$ percentile	median	$84th$ percentile
LR	0.39	0.68	0.85
Max Share	0.68	0.81	0.89

Table 1: We generate 1000 draws of artificial data from each model. All technology processes have unit roots. Model parameters are given in the tables in the online appendices. For each parameterization, the implied technology shocks are identified using LR and Max Share. The correlations between the estimated shocks and the artificial shocks are then calculated for each of the 1,000 draws. The median, 16th, and 84th percentiles from the posterior distributions are used for the correlations for each artificial sample as described in Section 3.

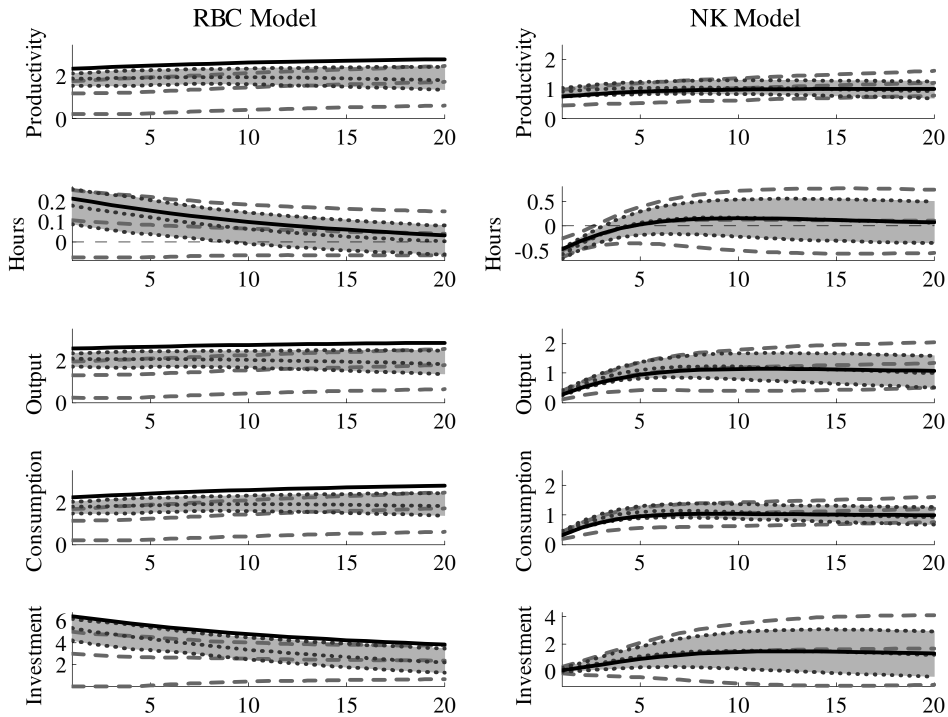

Figure 1: Impulse responses to a technology shock in Monte Carlo experiments (benchmark models).

Theoretical responses [with AR(1) technology coefficient $\rho_z = 1.0$ and nontechnology AR(1) coefficients $\rho = 0.98$] are shown by thick solid lines. Median and 68-percent probability intervals for Max Share are shown with dotted lines and shaded areas, respectively. LR median responses and 68-percent probability intervals are shown by dashed lines. Median estimates and error bands are averages across 1,000 estimates, each representing what an econometrician would estimate based on a sample with 247 observations and 1,000 draws from the posterior distributions for the impulse responses.

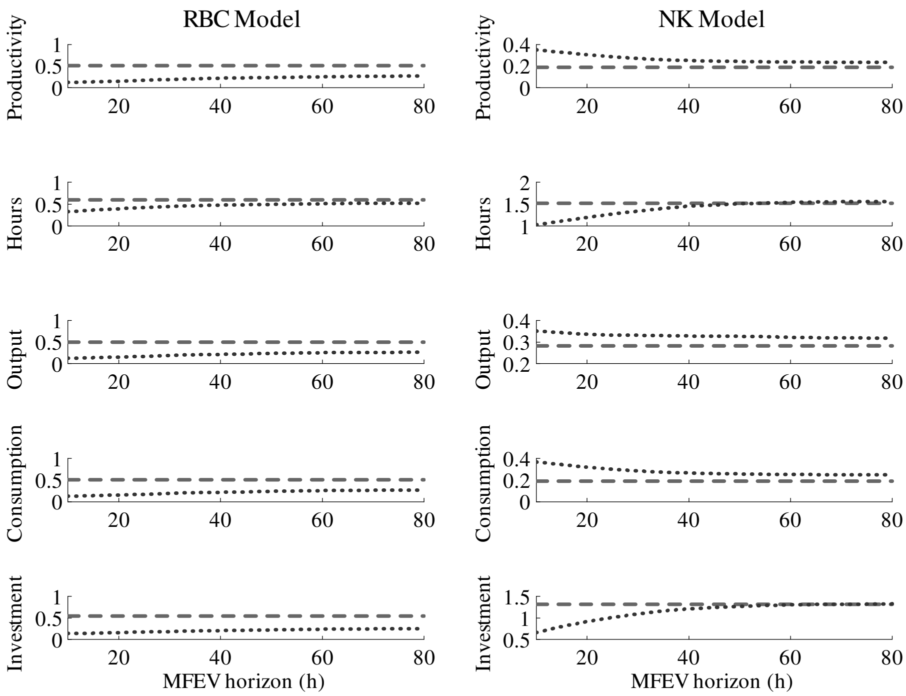

Figure 2: Average absolute bias in LR (dashed) versus Max Share (dotted) across alternative maximization horizons for simulated data.

Bias is measured as the absolute difference between the median Max Share (or LR) and theoretical responses, averaged over the first four quarters. The underlying Max Share and LR responses are averages across 1,000 median estimates, each representing what an econometrician would estimate based on a sample with 247 observations.

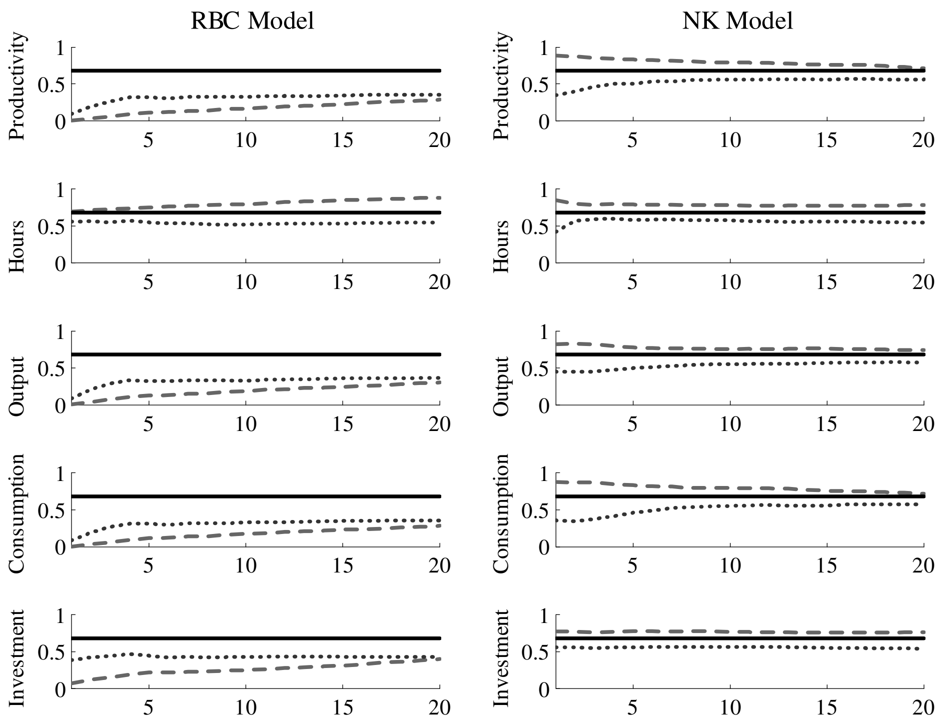

Figure 3: Small sample 68-percent coverage for the model responses.

Percent of MC iterations for which the true response lies within the 68-percent nominal converage interval across 1000 MC iterations. The dashed line is for the LR identification; the dotted line is for Max Share.

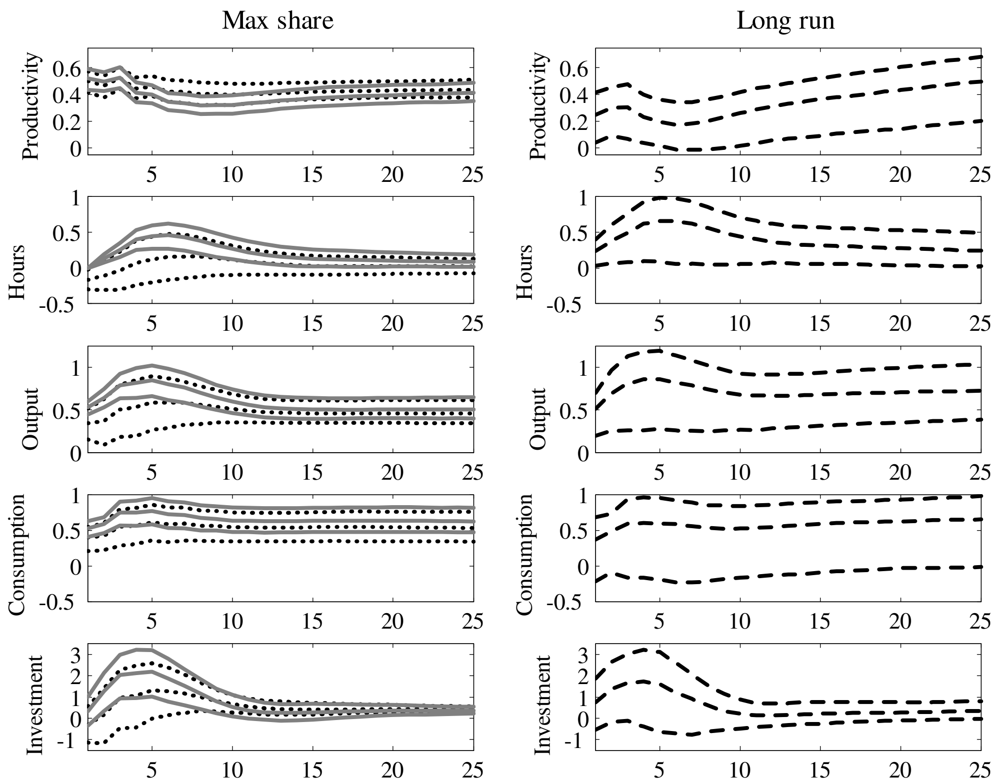

Figure 4: Impulse responses to a technology shock in the Max Share (on left) and the LR models (on right).

Shaded areas are 68 percent Max Share probability intervals, as well as responses when hours are restricted to be positive on impact in the Max Share model (thick solid line on left).

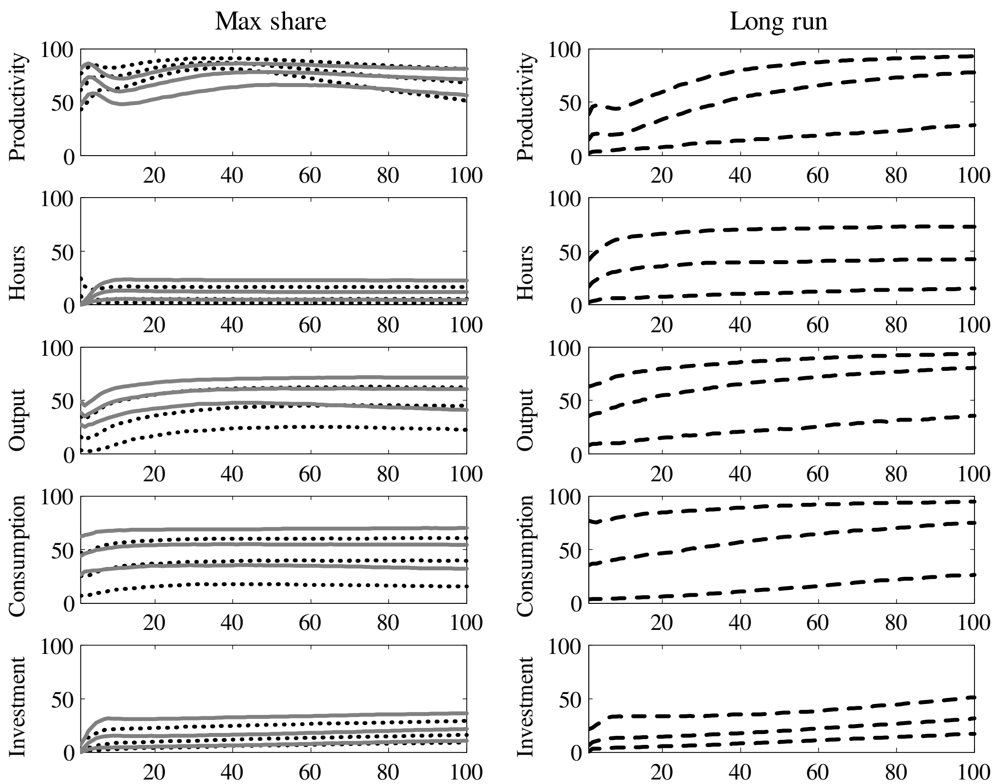

Figure 5: Forecast-error variance share due to a technology shock using Max Share (left) and LR (right) on U.S. data.

Dotted lines on the are computed with hours unrestricted. Solid lines are computed with hours restricted to be positive on impact. Each set of three lines show the median and 68 percent coverage intervals.

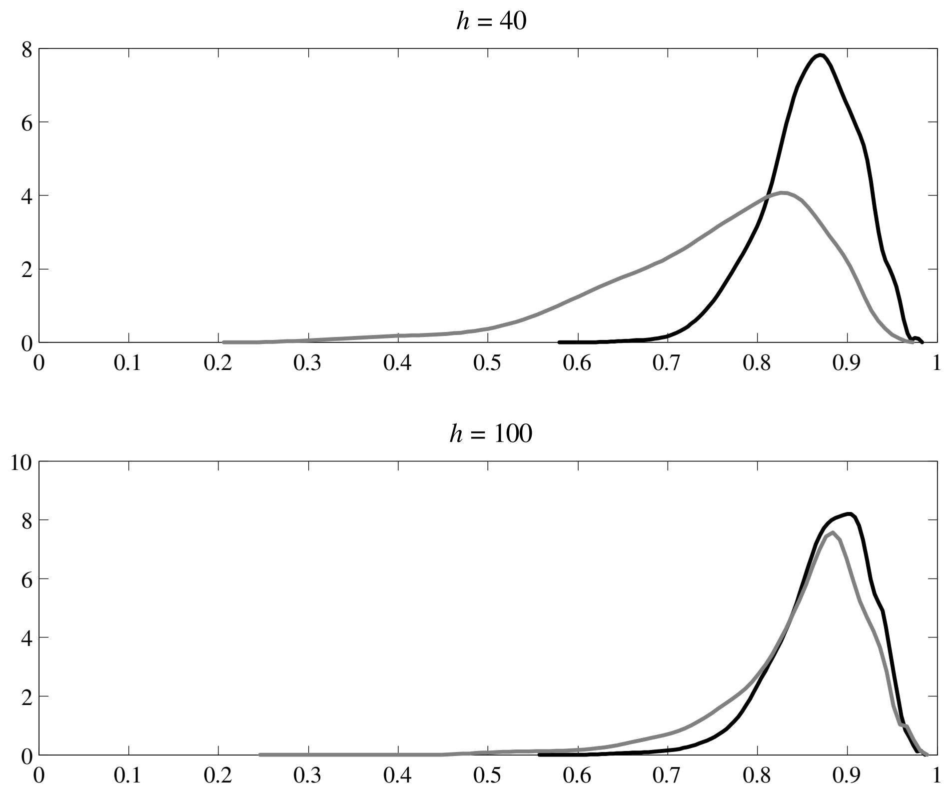

Figure 6: Posterior distributions of the maximized variance share of productivity with hours unrestricted (black) and restricted to be positive (gray).

The two lines show the smoothed kernel densities for the empirical posterior distributions of the maximum forecast-error variance shares estimated at horizons of 40 and 100 quarters using U.S. data.