RESEARCH DIVISION

Working Paper Series

A Specialized Inventory Problem in Banks: Optimizing Retail Sweeps

Richard G. Anderson
and
Suresh K. Nair

Working Paper 2005-023A
https://doi.org/10.20955/wp.2005.023

March 2005

FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
P.O. Box 442
St. Louis, MO 63166

A Specialized Inventory Problem in Banks: Optimizing Retail Sweeps

Suresh K. Nair* • Richard G. Anderson**

School of Business, University of Connecticut, Storrs, CT 06269
Federal Reserve Bank of St. Louis, St. Louis, MO 63102

April 2005

Deposits held at Federal Reserve Banks are an essential input to the business activity of most depository institutions in the United States. Managing these deposits is an important and complex inventory problem, for two reasons. First, Federal Reserve regulations require that depository institutions hold certain amounts of such deposits at the Federal Reserve Banks to satisfy statutory reserve requirements against customers' transaction accounts (demand deposits and other checkable deposits). Second, some inventory of such deposits is essential for banks to operate one of their core lines of business: furnishing payment services to households and firms. including wire transfers, ACH payments, and check clearing settlement. Because the Federal Reserve does not pay interest on such deposits used to satisfy statutory reserve requirements, banks seek to minimize their inventory of such deposits. In 1994, the banking industry introduced a new inventory management tool for such deposits, the *retail deposit sweep program*, which avoids the statutory requirement by reclassifying transaction deposits as savings deposits. In this analysis, we examine two algorithms for operating such sweeps programs within the limits of Federal Reserve regulations.

JEL Codes: D20, G21

Keywords: *Retail Banking; Deposit Sweeps; Regulation D; Required Reserves; Stochastic Dynamic Programming*

*Professor and Ackerman Scholar, Operations and Information Management Department, School of Business, University of Connecticut. Contact: suresh.nair@uconn.edu

**Economist and vice president, Research Division, Federal Reserve Bank of St. Louis. Contact: anderson@stls.frb.org

(This is Federal Reserve Bank of St. Louis Research Division Working Paper 2005-023)

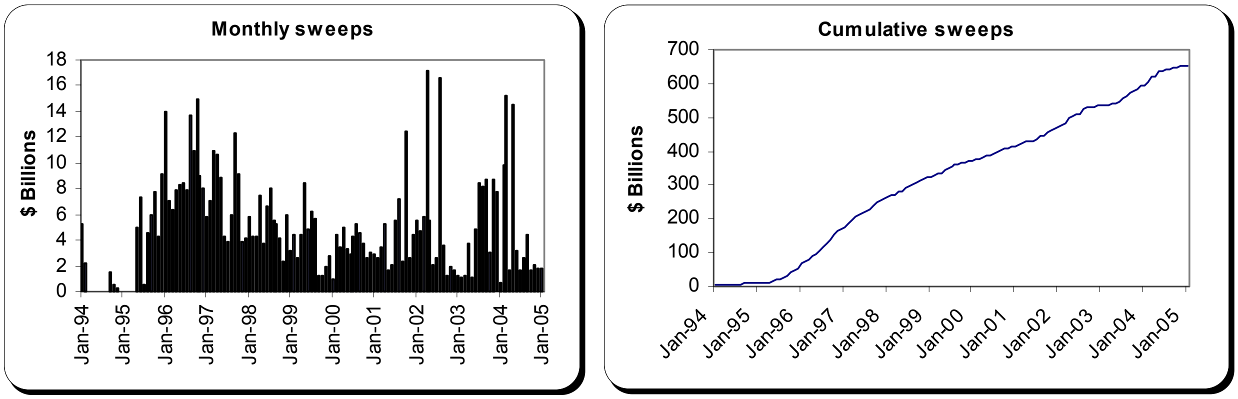
1. Introduction

Deposits held at Federal Reserve Banks are an essential input to the business activity of most depository institutions in the United States. Managing these deposits is an important and complex inventory problem, for two reasons. First, Federal Reserve regulations require that depository institutions hold certain amounts of such deposits at the Federal Reserve Banks to satisfy statutory reserve requirements. Second, an inventory of such transaction deposits is essential for banks to operate one of their core lines of business: furnishing payments services to households and firms, including wire transfers, automated clearing house (ACH) payments, and check clearing settlement.

As competitive firms, banks seek to maximize their profits while complying with applicable laws and regulations. The Monetary Control Act of 1980 authorized the Federal Reserve to require banks and depository institutions to hold statutory reserves against transaction deposits (e.g., checking accounts); since early 1992, a 10 percent reserve-requirement ratio has applied to deposit balances greater than approximately $50 million. Non-transaction deposits (e.g., savings accounts) and most other bank liabilities are subject to a zero percent reserve-requirement ratio. Federal Reserve regulations stipulate that only two bank assets may be used to satisfy statutory reserve requirements: deposits at the Federal Reserve Banks, and vault cash (cash in banks' central vaults, in ATMs, and in transit, etc.). Neither of these assets earns interest for the bank. Penalties are levied if these requirements are violated.

Clever bank managers seek to minimize their holdings of these "sterile reserves." In 1994, for this purpose, a large commercial bank invented the retail deposit sweep program. In such a program, the bank links two companion accounts—a customer's transaction account (demand deposit or similar checkable account), which we hereafter refer as the bank transaction account, or BTA, and a newly created money market deposit account (MMDA, similar to a savings account). Funds are swept regularly from the BTA, where end of day balances are subject to a 10 percent statutory marginal reserve requirement, to the MMDA, which is not subject to any reserve requirement. The MMDA account is invisible to the customer, and is solely a component of the inventory management scheme; transfers between accounts also are invisible to the customer. Unlike numerous earlier reserve-reduction schemes of banks, the Federal Reserve Board has not, as of this writing, objected to banks reducing their

Figure 1: Monthly sweep amounts and cumulative amount since 1994



Source: Board of Governors of the Federal Reserve System
http://research.stlouisfed.org/aggreg/swdata.html

required reserves via such schemes (see Appendix 1 for more details on Federal Reserve history with respect to sweep programs).

In short, the essence of a retail deposit sweep program is the ability of the bank to reclassify, for reserve-requirement purposes, transaction deposits, subject to a 10 percent reserve requirement, as saving deposits (specifically, MMDA), subject to a zero reserve requirement. A retail deposit sweep, essentially, is keeping two sets of books so as to evade the “reserve requirement” tax collector.

Since debits (e.g., check writing) can only be serviced from transaction accounts, it is optimal to always leave some funds in the transaction account and replenish the account from the companion MMDA, as required. To discourage the use of the MMDA (a type of savings account) as if it were a checking account, Federal Reserve regulations (Regulation D) limit to a maximum of 6 the number of withdrawals that can be made from an MMDA account during a calendar month. These limits do not apply to transfers made by personally visiting a branch, or by ATM, or by phone. The consequence of violating the six transfer limit is harsh: the entire amount that has been swept between the BTA and the MMDA becomes subject to the same statutory reserve requirement as if no sweep had been attempted at all. Thus, it is generally optimal to move all remaining balances from the MMDA to the transaction account on the sixth transfer out of the MMDA account.

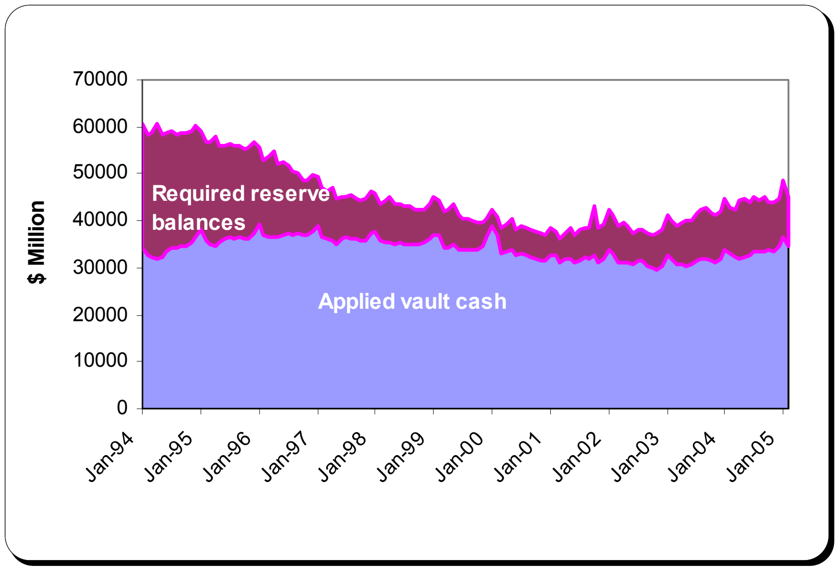
Numerous legal interpretations (e.g., Ireland 1996) from the Federal Reserve emphasize the 6 transfers limit. For a brokerage client with a balance of a million dollars, that would mean a hundred thousand dollars in additional reserves. Therefore, the sweep mechanism would have to ensure that no more than five transfers were necessary from MMDA to BTA every month. This need for judicious sweeping requires sophisticated computer software and algorithms.

Data on the aggregate amount of deposits affected by retail deposit sweep activity is of modest quality, at best. Commercial banks are not required to report data on sweep activity to the Federal Reserve. Hence, in most cases, Federal Reserve staff have imputed the amount being swept from other deposit and reserves data. Sweep activity grew slowly from its inception in January 1994 through mid-1995, and thereafter grew rapidly as banks concluded that the Federal Reserve would not move to prohibit such reserve-requirement reduction schemes. As seen in Figure 1, the cumulative amount of sweep since 1994 reached about $654 billion by January 2005, or more than 50 percent of the total amount of transaction balances at banks and thrift institutions.

Widespread use of sweep programs has significantly reduced the amount of required reserve balances that banks maintain at the Federal Reserve. As a result, even as aggregate transaction deposits (including the amounts in sweeps) has increased, the portion of total require reserves satisfied by vault cash has held almost steady, while the portion satisfied by Federal Reserve Bank deposits (so-called “required reserve balances”) has fallen; see Figure 2. In the past couple of years there has been a slight increase in reserve balances due to low interest rates. There is some concern among economists that this trend can increase volatility in the federal funds rate as the banks try to manage their accounts with very low balances (Bennett and Hilton 1997, Wrase 1998, Anderson and Rasche 2001, Anderson 2002a).

Some economists have expressed concern that aggressive management of deposits of held at Federal Reserve Banks via sweep programs might increase federal funds rate volatility, an outcome regarded by the Federal Open Market Committee as harmful to monetary policy’s effectiveness. According to Furfine (2000), “As recently as a decade ago, relatively high statutory reserve requirements created a fairly predictable demand for bank reserves, making interest rate targeting a relatively easy-to-implement policy choice for the Federal Reserve. Over the last 10 years, however, lower reserve requirements and the widespread adoption of

Figure 2: Components of Reserves and Account Balances at the Fed



Source: Board of Governors of the Federal Reserve System
http://www.federalreserve.gov/releases/H3/hist/h3hist2.txt
http://www.federalreserve.gov/releases/h3/hist/h3hist3.txt

sweep accounts have precipitated a dramatic fall in required reserve balances. These lower reserve balances may increase uncertainty in reserve demand, possibly complicating the Fed's ability to achieve the goals of monetary policy." And Clouse and Dow (2002) state, "Several countries have already moved to monetary systems without reserve requirements so that their demand for reserves is entirely a demand for excess reserves. The United States is also moving in that direction due to the adoption of retail sweep programs by commercial banks. These programs have resulted in significantly lower levels of required reserves, leaving some banks in the position of not needing to hold reserve balances to meet their reserve requirements. Understanding how this demand behaves is becoming an important issue in applied monetary economics."

Some events in recent years have portended an end to retail deposit sweep accounts. But, in our opinion, such events are not likely to reach fruition. In a late 1990s Federal Reserve survey, a sample of sweeping banks responded they would discontinue retail deposit sweep programs if the Federal Reserve paid interest on deposits at the Federal Reserve Banks. Legislation to require the Federal Reserve to start paying interest on sterile reserve balances at the Fed has been introduced but failed to pass the Congress, e.g., the Bank Reserves

Modernization Act of 2000, and the more recent HR 758, The Business Checking Freedom Act of 2003. The latter passed the House of Representatives but was not acted on in the Senate. The Treasury has not endorsed the proposals due to its potential budgetary burden (Abernathy 2003). Certain small community banks are supporting the proposal (Maus 2003) on the grounds that sweep programs are expensive and complicated for small banks to implement, while others (Menzies 2003) like an alternative proposal (HR 974) which would allow 24 transfers from MMDA to BTA each month instead of the present 6.

Although retail deposit sweep programs have been operating for more than a decade — and banks have earned millions of dollars by their use—there are no published papers modeling their implementation, operation, or optimal tuning. These programs have reduced the amount of banks' sterile reserves from $26.3 billion in Jan 1994 to about $10.7 billion in February 2005 (Figure 2). Even assuming a conservative 100 basis points spread on interest rates, the implied earnings are $156 million per year. Clearly this is an interesting optimization problem, with high earnings potential. This paper attempts to begin to bridge this gap in literature.

2. Systems for Sweep Programs

There are a number of algorithms for operating retail deposit sweep programs. Perhaps the earliest algorithm was the simplest: sweep funds from the transaction deposit into the MMDA at the close of business on Friday, and reverse the sweep at the opening of business on Monday; if Monday is a federal banking holiday, postpone the reverse transfer to Tuesday. Because the Federal Reserve's statutory reserve requirements are calculated from end-of-day balances on a 7-day week, this algorithm immediately reduces the amount of transaction deposits subject to reserve requirements. Considerable improvement, however, can be obtained by more sophisticated methods that we discuss next.

In banks, sweep programs are part of the treasury cash management function. While a variety of algorithms undoubtedly exist, anecdotal evidence suggests that two are the most popular. For discussion, we label them the *threshold method* and the *cushion method*. Both these methods try to leverage high-frequency information on customer behavior (daily patterns of net credits and debits) to optimize the division of funds, over the month, between the BTA and MMDA accounts, subject to the constraint of no more than six transfer limit from MMDA to BTA. Note that there is no limit on transfers from the BTA to the MMDA account, although each such transfer may have a small cost for the bank. Recall that if five transfers have already been made for an account, it generally is optimal on the sixth transfer to move the entire MMDA balance to BTA. This can be expensive. If an institutional customer has $10 million in their account and all of it gets transferred to BTA, the reserves will have to be increased by $1 million, for which no interest is earned by the bank. All subsequent activity after the dumping of all MMDA funds to BTA, both debit and credit, is done via the BTA.

For both methods, it is important to keep in mind that the month-by-month six transfer limit imposes a complex time-dependent structure on the functions that define the optimal outcomes. In general, we will show that the thresholds, cushion amounts, and optimal transfers in both directions depend on the position of each specific business day within the calendar month.

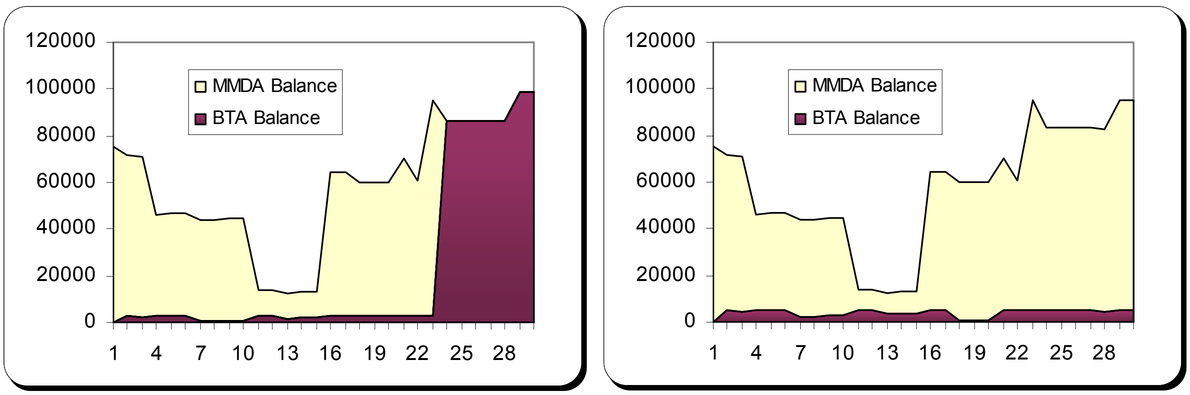
Threshold Method: In this method, for each accountholder, a threshold is set for the BTA account balance. Because there is no restriction on the number of transfers from the BTA to MMDA, and because we assume zero cost of such transfers, optimality always requires that all incoming funds be credited to the BTA. If the account balance in the BTA just prior to the end of the business day exceeds that day's optimal threshold, excess funds are moved from BTA to MMDA before the close of business. If posting incoming debits against the BTA implies a negative balance at the end of business, funds are transferred from the MMDA to the BTA. In our models, we assume that the six transfer rule is a hard constraint. As a result, on the sixth transfer, all funds in the MMDA are moved to the BTA. In fact, this is a stochastic economic decision, and part of the optimal solution. A bank which chooses not to move all funds from the MMDA to the BTA on the sixth transfer is placing a bet that a seventh transfer will not be necessary before the end of the month when a new set of six transfers begins. Determining the correct option price for this bet is beyond the scope of this paper, and part of our current research. In this algorithm, the amount transferred is equal to the threshold plus the amount of negative balance, leaving a balance of the threshold amount in BTA after the transfer. In operation, this method seeks to maintain the BTA balance within an acceptable band. The parameters for this method are the thresholds values. The minimum level here is zero. The method can be generalized to a situation where the minimum level is set to a value greater than zero, in which case the parameters are the minimum and maximum thresholds.

Cushion Method: In this approach, incoming customer funds (deposits) throughout the month are credited to the MMDA so long as the transfer count from MMDA to BTA is less than six. Since debits can only be serviced from the BTA account, the first debit transaction initiates a transfer from MMDA to BTA of the debit amount plus a day-specific cushion amount. The optimal amount generally should be large enough to service a few more debit transactions. When the balance in the BTA is not sufficient to service a subsequent debit transaction, another transfer from MMDA to BTA is initiated. This transfer again includes funds to service the current debit transaction and a cushion amount, which, in general, will differ from the first cushion amount. This pattern is repeated as needed up to five transfers. The parameters in this method are the cushion amount for each transfer.

Although these two methods both can be considered similar to the (s,S) and (r,Q) models in inventory management, the algorithms are not isomorphic, that is, we have been unable to establish (stochastic) circumstances in which the two methods yield the same account balances and profit. In both methods, the item being inventoried is cash, that is, deposits at Federal Reserve Banks. The methods differ primarily in two respects. First, the algorithms differ in where the deposits are made. In the threshold method it is to the BTA, in the cushion method it is to the MMDA. In current work, we are exploring optimal strategies for allocating deposits between the two accounts. Second, they differ in their treatment of the sixth transfer from MMDA to BTA. In further work, we are exploring the optimal handling of the risk incurred by not moving all funds on the sixth transfer. Finally, as noted above, the s and r are set to zero, though this is not necessary and the methodology could benefit from not restricting these to zero.

Discussions with bank treasury managers suggest that most models used in practice to implement sweep programs are very simplistic. In the threshold method, for example, often little variation is permitted by day; in some cases, managers allow the threshold to vary by transfer count ($1000 for first transfer into BTA, $2000 for second transfer, etc.) but not by day of the month. Further, managers may use the same threshold for all accounts, regardless of size and transaction patterns. In addition, the same threshold sometimes is used for transferring balances out of BTA to MMDA and for moving funds back from the MMDA. In the next section, we show that different thresholds for these two types of transfers yield superior earnings relative to one threshold, and that thresholds which vary by day of month and account activity yield further earnings improvements.

Figure 3: The result on MMDA and BTA balances of using different threshold values for the same account ($3000 and $5000 respectively)



Although not discussed further in this analysis, discussions suggest that managers often use similar methods for the cushion approach, with cushion amounts that vary by transfer count (say a cushion of $1000 for the first transfer, $2000 for the second transfer, etc.), but not vary by customer activity or day of month. In the balance of this paper, we examine the threshold method.

An illustration with two different sets of thresholds for the same account is shown in Figure 3. The detailed transaction data for each day is shown in Appendix 2 and 3 for the two cases shown in Figure 3. In the case on the left, all six transfers become necessary, and therefore the sixth transfer dumps the entire MMDA balance into BTA. All subsequent transactions are in BTA. This is an expensive outcome due to a poor choice of thresholds. In the case on the right, because of a better set of thresholds is used, only five transfers are necessary. Therefore, the BTA balance is kept low.

Figure 4: Transition matrix for net transaction activity for an account

Current day		Next Day: Withdrawal Large	Withdrawal Small	No activity	Deposit Small	Deposit Large	Average Amount
Withdrawal	Large	7%	46%	33%	7%	7%	-$3,500
	Small	5%	49%	35%	6%	5%	-$500
No activity		4%	35%	49%	6%	6%	$0
Deposit	Small	4%	44%	41%	6%	5%	$500
	Large	9%	50%	34%	3%	4%	$5,000

3. A Stochastic Dynamic Programming Model

We develop a stochastic dynamic programming model for this problem. For tractability and presentation here, the data in our model is somewhat less rich than the data that a bank treasury manager would have available for use in such a model. Most banks retain customer account activity (debits and credits) for at least several recent months, including daily account balances and net transaction activity. (The net transaction activity is the net of all deposits and withdrawals posted to the account during the day. Positive numbers correspond to net deposits, negative numbers to net withdrawals, and zeros to no activity during the day.) Because the stochastic model could get very unwieldy if actual real-number dollar transaction values are used, we choose to model using discrete *transaction intervals*. Suppose there are *n* transaction intervals in the model. For example, if we choose to model with 8 transaction intervals for net activity, the intervals may be defined to correspond to daily net transaction activity of <-10000, -10,000 to -5001, -5000 to 1, 0, 1 to 5000, 5001 to 7500, 7501 to 15000, >15000. Note that the interval breakpoints do not need to be symmetric, of equal size, or have as many withdrawal intervals and deposit intervals. However, it is important to have one interval for no activity, 0, since on many days there may not be any net activity.

Using the historical transaction data, a (*nxn*) size transition matrix is created. See Figure 4 for an example with 5 intervals. The entries of the transition matrix, p_{ij}, correspond to the probability of having a net transaction in interval *i* on one day, and a transaction in interval *j* the next day, where $\sum_j p_{ij} = 1$, for all *i*.

Figure 5: MMDA and BTA balances in various cases of account activity.
The transfer into BTA threshold is *g* and the transfer out of BTA threshold is *h*.

	Balance at start of day		Net transaction during day, s_i	BTA balance 5 minutes before end of business	Transfer	Balance at end of day		Transfer count
	MMDA	BTA				MMDA	BTA	
Case 1	m	b	Withdrawal	0 <= b+s_i < g	No transfer	m	b+s_i	
Case 2	m	b	Withdrawal	b+s_i < 0, Transfer count < 6	Transfer g-(b+s_i) from MMDA to BTA	m-[g-(b+s_i)]	g	+1
Case 3	m	b	Withdrawal	Transfer count = 6	Transfer entire MMDA balance to BTA	0	m+b+s_i	
Case 4	m	b	No activity	b	No transfer	m	b	
Case 5	m	b	Deposit	b+s_i <= h	No transfer	m	b+s_i	
Case 6	m	b	Deposit	b+s_i > h	Transfer h-(b+s_i) from BTA to MMDA	m+[(b+s_i)-h]	h	

Let s_i ($i \in \{1,\ldots,n\}$) be the average net transaction amount for each interval (last column of Figure 4), where s_i is negative for withdrawal intervals and positive for deposit intervals, *A* denote the MMDA balance at the beginning of the month, l*m* denote the MMDA balance, and *b* denote the BTA balance on day *t*. Suppose the net transaction on day *t* is in transaction interval *i* and *x* transfers from MMDA to BTA have been made to date. Let *X* be the maximum allowable transfers. As per current Federal Reserve regulations, *X* is 6. At the beginning of the month x=0, and we constrain this number to be less than or equal to *X* at the end of the month. Suppose there are *T* working days in the month. Then the state of the system in day *t* can be specified by *(m,b,i,x)*. Let $f_t^T(m,b,i,x)$ be the maximal discounted net present value of being in state *(m,b,i,x)* in period *t* in the *T* period problem when optimal actions are taken in each day from *t* through *T*. The actions pertain to choosing the correct thresholds when transfers become necessary. We will use different thresholds for transfers *into* BTA and *out of* BTA.

Let r_{mb} be the single-day revenue realized from an MMDA balance of *m* and a BTA balance of *b*,

$$r_{mb} = r(m + [1-\delta]b)$$

where r is the interest rate spread that the bank earns on balances (expected marginal asset yield minus its average cost of funds), and δ is the (marginal) fraction of BTA balances that the bank needs to maintain as sterile reserves. Under current regulations, δ is approximately 0.1, or 10%.

Let β be the single-day discount factor. Then the cases that need to be considered depending on the state of the system and the account activity is shown in Figure 5. In what follows, as shorthand, we refer to the hard constraint that all funds in the MMDA must be moved to the BTA on the sixth transfer as a "dump" of the funds back to the BTA.

The following recursive functional equations specify the model:

Withdrawals (s_i is negative):

Case 1 and Case 2 (No dump needed): Suppose the transaction is a withdrawal (that is, $b>s_i$), then we need to select a ***withdrawal threshold***, g, that solves the following recursion.

$$f_t^T(m,b,i,x) = r_{mb} + \sup_{g>=0}\begin{cases} 0 <= b+s_i < g: & \beta[\sum_j p_{ij} f_{t+1}^T(m,b,j,x)] \\ b+s_i < 0: & \beta[\sum_j p_{ij} f_{t+1}^T(m-[g-(b+s_i)],g,j,x+1)] \end{cases} \tag{1}$$

Case 3 (Dump needed): Suppose a transfer from MMDA is necessary and the transfer count $x \geq X$, since all the MMDA balance gets transferred to BTA, we have

$$f_t^T(m,b,i,x) = r_{mb} + \beta[\sum_j p_{ij} f_{t+1}^T(0,m-(b-s_i),j,x+1)] \tag{2}$$

No activity (s_i is zero):

Case 4 (No activity): Suppose the transaction is no activity, and the transfer count $x< X$, then no transfer is made.

$$f_t^T(m,b,i,x) = r_{mb} + \beta[\sum_j p_{ij} f_{t+1}^T(m,b,j,x)] \tag{3}$$

Deposits (s_i is positive):

Case 5 and Case 6 (Deposits): Suppose the transaction is a deposit, we wish to determine a ***deposit threshold***, *h*, that solves the following recursion

$$f_t^T(m,b,i,x) = r_{mb} + \sup_{h>=0} \begin{cases} b + s_i \le h: & \beta[\sum_j p_{ij} f_{t+1}^T(m,b,j,x)] \\ b + s_i > h: & \beta[\sum_j p_{ij} f_{t+1}^T(m + [h-(b+s_i)], h, j, x)] \end{cases} \quad (4)$$

Boundary conditions: The boundary condition for this model is

$$f_T^T(m,b,i,x) = 0 \quad \forall m,b,i,x \quad (5)$$

We solve this model by recursively solving the functional equation $f_0^T(A,0,0,0)$, where *A* is the initial balance in the account.

We have found the above model—a mixed discrete-continuous DP model—difficult to solve. The threshold amounts for withdrawals, *g,* and for deposits, *h,* are continuous real numbers, while the s_i values are discrete. Further, the model has several ridges where derivatives are not continuous due both to zero-bound restrictions such as BTA and MMDA balances cannot be negative, and to the hard constraint of no more than six transfers per month from MMDA to BTA. To solve the model, we simplified the problem so as to use discrete, rather than mixed discrete-continuous, solution methods. Our choices of discrete threshold values are discussed in the next section.

3. 1 The Model with Discrete Thresholds

Suppose $g_1, g_2, \ldots, g_z$ and $h_1, h_2, \ldots, h_y$ are discrete strictly increasing thresholds, where $0<g_1<g_2\ldots<g_z$ and $0<h_1<h_2\ldots<h_z$. Because the threshold needs to be sufficient to cover at least one future withdrawal, g_1 and h_1 should be at least as large as the absolute value of the smallest

average withdrawal, s_i, among the withdrawal intervals. Also, g_z could be the maximum possible threshold that can be transferred without resulting in a negative MMDA balance, that is, $g_z = \max(g : m + b + s_i - g > 0)$. Similarly, $h_y = \max(h : b + (b + s_i) - h > 0)$. We will show later in Figure 7 that these threshold end-points can be selected to make the computations much more efficient. For now, we can modify (1) for discrete thresholds as follows for withdrawals:

$$f_t^T(m,b,i,x) = r_{mb} + \max_{g_1,\cdots g_z} \begin{cases} g_z : & \begin{cases} 0 <= b + s_i < g_z : & \beta[\sum_j p_{ij} f_{t+1}^T(m,b,j,x)] \\ b + s_i < 0 : & \beta[\sum_j p_{ij} f_{t+1}^T(m - [g_z - (b + s_i)], g_z, j, x+1)] \end{cases} \\ \vdots & \\ g_2 : & \begin{cases} 0 <= b + s_i < g_2 : & \beta[\sum_j p_{ij} f_{t+1}^T(m,b,j,x)] \\ b + s_i < 0 : & \beta[\sum_j p_{ij} f_{t+1}^T(m - [g_2 - (b + s_i)], g_2, j, x+1)] \end{cases} \\ g_1 : & \begin{cases} 0 <= b + s_i < g_1 : & \beta[\sum_j p_{ij} f_{t+1}^T(m,b,j,x)] \\ b + s_i < 0 : & \beta[\sum_j p_{ij} f_{t+1}^T(m - [g_1 - (b + s_i)], g_1, j, x+1)] \end{cases} \end{cases} \tag{6}$$

Similarly, we can modify (4) for discrete deposit thresholds as follows:

$$f_t^T(m,b,i,x) = r_{mb} + \max_{h_1,\cdots h_{y_z}} \begin{cases} h_y : & \begin{cases} b + s_i \le h_y : & \beta[\sum_j p_{ij} f_{t+1}^T(m,b,j,x)] \\ b + s_i > h_y : & \beta[\sum_j p_{ij} f_{t+1}^T(m + [h_y - (b + s_i)], h_y, j, x)] \end{cases} \\ \vdots & \\ h_2 : & \begin{cases} b + s_i \le h_2 : & \beta[\sum_j p_{ij} f_{t+1}^T(m,b,j,x)] \\ b + s_i > h_2 : & \beta[\sum_j p_{ij} f_{t+1}^T(m + [h_2 - (b + s_i)], h_2, j, x)] \end{cases} \\ h_1 : & \begin{cases} b + s_i \le h_1 : & \beta[\sum_j p_{ij} f_{t+1}^T(m,b,j,x)] \\ b + s_i > h_1 : & \beta[\sum_j p_{ij} f_{t+1}^T(m + [h_1 - (b + s_i)], h_1, j, x)] \end{cases} \end{cases} \tag{7}$$

4. Structural Results

Our model is a version of the sequential assignment problem (Derman, Lieberman and Ross 1972, Ross 1983) and very similar to the game "So who's counting" (Puterman 1994). In that game, two teams compete to assemble the largest 5-digit number from a set of 0-9 digits randomly and sequentially picked by a spinner in five spins. The teams may place each of the digits that are picked by the spinner in one of the five available slots for digits (the units, tens, hundreds, thousands and ten thousands positions). The team that has a larger number after the five spins wins. In this game, obviously if the first spin results in a 9, one should put it in the first available slot, and if the first spin results in a 0, one should place it in the last available slot. The placement of other digits may not be as obvious, but it turns out that there is an optimal policy that specifies where each digit should be placed given that it is picked in a particular spin. The policy may not result in wins every time, but will be best in the long run if the game is repeatedly played. In this problem there are two dimensions, the spin number and the observed number in a spin. The proof of the optimal policy uses the fact that the value of the solution is monotonic in both these dimensions.

To see that our problem is similar, think of a game where T (the number of working days in the month) spins are allowed, and in each spin, one of 6 transfer counts, $x \in \{0,\ldots,5\}$ may be observed. The analog to the placement of the digits is the threshold value. In our problem, there are three dimensions – t, x, and $(m+b)$. We will show next that the value of the solution, $f_t^T(m,b,i,x)$, is monotonic in each of these dimensions.

Lemma 1:

a) $f_t^T(m,b,i,x)$ is non-increasing in t for all m, b, i, x and T

b) $f_t^T(m,b,i,x)$ is non-decreasing in x for all m, b, i, t and T

c) $f_t^T(m,b,i,x)$ is non-decreasing in $m+b$ for all i, t, x, and T

Proof: We prove these results by induction. For (a), note that the result is true at $t=T-1$ from (5). Suppose the result were true at $t+1$. Then from (1)-(4), (6) and (7), it follows that the result is true at *t*. The result follows by induction. For (b), note that at *x=X*, the result is true because of the dumping of the entire MMDA to BTA in (2). Then supposing that the result were true at *x+1*, it follows from (6) and (7) that the result is true at *x*. The result then follows by induction. (c) follows from the definition of r_{mb}. □

Suppose the optimal thresholds in state *(m,b,i,x)* in period *t* is g^*_{tmbix} and h^*_{tmbix}, where from (6) for withdrawals.

$$g^*_{tmbix} = \arg\max \quad f_t^T(m,b,i,x)$$

And from (7), for deposits

$$h^*_{tmbix} = \arg\max \quad f_t^T(m,b,i,x)$$

Then using the result in Ross (1983), we can present the following theorem.

Theorem 1:

a) g^*_{tmbix} and h^*_{tmbix} are non-increasing in *t* for all *m, b, i, x* and *T*

b) g^*_{tmbix} and h^*_{tmbix} are non-decreasing in *x* for all *m, b, i, t* and *T*

Proof: The proof follows from Ross (1983, page 125, Proposition 7.2) and Lemma 1 above.□

Figure 6: An illustration of optimal thresholds for sweeps, *m*=6500, *b*=0 and data in Figure 4

Large Withdrawal Thresholds, g^*_{tmb1x}					
	Transfer Count, *x*				
Days to EOM, 26-*t*	1	2	3	4	5
25	1000				
24	1000	1000			
23	1000	1000	1500		
22	1000	1000	1500	2500	
21	500	500	1000	2500	3000
20	500	500	1000	2500	3000
:	:	:	:	:	:
5	0	0	0	500	1000
4	0	0	0	0	500
3	0	0	0	0	500
2	0	0	0	0	500
1	0	0	0	0	0

Small Withdrawal Thresholds, g^*_{tmb2x}				
Transfer Count, *x*				
1	2	3	4	5
1000				
1000	1000			
1000	1000	2000		
1000	1000	1500	4500	
1000	1000	1500	4500	6000
1000	1000	1500	4500	6000
:	:	:	:	:
0	0	0	500	1000
0	0	0	0	500
0	0	0	0	500
0	0	0	0	0
0	0	0	0	0

Small Deposit Thresholds, h^*_{tmb4x}					
	Transfer Count, *x*				
Days to EOM, 26-*t*	1	2	3	4	5
25	500				
24	500	500			
23	500	500	500		
22	500	500	500	500	
21	500	500	500	500	500
20	500	500	500	500	500
:	:	:	:	:	:
5	0	0	0	500	500
4	0	0	0	0	500
3	0	0	0	0	500
2	0	0	0	0	0
1	0	0	0	0	0

Large Deposit Thresholds, h^*_{tmb5x}				
Transfer Count, *x*				
1	2	3	4	5
1000				
1000	1500			
1000	1000	1500		
1000	1000	1500	2000	
500	1000	1500	2000	5000
500	1000	1500	2000	5000
:	:	:	:	:
0	0	0	500	1000
0	0	0	0	1000
0	0	0	0	500
0	0	0	0	0
0	0	0	0	0

An illustration of the optimal thresholds for four cases of total balance is shown in Figure 6. Note that in each case the threshold values are non-decreasing from the southwest corner of the table to the northeast corner (which is statements (a) and (b) of Theorem 1). Also, the maximum threshold in each case is the maximum possible. For example, in the top left case of large withdrawal thresholds, note that the MMDA balance is $6500, the BTA balance is $0, and the amount of the transaction from Figure 4 is -$3500. Thus after this transaction is complete, the total MMDA+BTA balance is $3000. The table is asking for all of this to be transferred to BTA as the threshold shown is $3000, if you have already made 5 transfers and have 21 days to go to the end of the month. Of course, as the table indicates, the threshold will be lower if you have fewer transfers done by that date.

The thresholds shown in Figure 6 are in sharp contrast to the simple thresholds used by many banks. For example, a set of thresholds may be 500, 500, 1000, 1500, and 2000 for transfer counts 1 through 5. These thresholds may be the same for all days of the month, and for all transaction types. Further, these may be only marginally adjusted for different customer types.

On the contrary, we are suggesting many more segments (see next section) and as the table indicates, thresholds that vary by day of month and transaction type.

The result of Theorem 1 allows us to limit the search for thresholds and make the model much more efficient. For example, from Figure 6, for the case on the top left (MMDA=6500 and BTA=0), the search for the optimal threshold with 21 days to go, transfer number of 3, can be limited to the range between the number below and the number on the right, that is (1000,2500). We can formalize this using the following result.

Corollary 1: The search for g^*_{tmbix} can be limited to the range $(g^*_{(t+1)mbix}, g^*_{(t+1)mbi(x+1)})$

Proof: Follows immediately from Theorem 1. □

Figure 7: Algorithm for limiting cushion search for *g* (*h* is similar)

Set g^*_{tmbix} to 99 for all *t,m,b,i,x;*
Function *f(t,m,b,i,x)*
Begin

If $g^*_{(t+1)mbix}$ <>99 then $g^*_H = g^*_{(t+1)mbix}$ else $g^*_H = g_z$;

If $g^*_{tmbi(x+1)}$ <>99 then $g^*_L = g^*_{tmbi(x+1)}$ else $g^*_L = 0$;

$$g_t^T(m,b,i,x) = \max_{g=g_L^*,\dots,g_{Hz}^*} \begin{cases} g_H^* : & \begin{cases} 0 <= b+s_i < g_H^* : & \beta[\sum_j p_{ij} f_{t+1}^T(m,b,j,x)] \\ b+s_i < 0 : & \beta[\sum_j p_{ij} f_{t+1}^T(m-[g_H^*-(b+s_i)], g_H^*, j, x+1)] \end{cases} \\ \vdots & \\ g_L^* : & \begin{cases} 0 <= b+s_i < g_L^* : & \beta[\sum_j p_{ij} f_{t+1}^T(m,b,j,x)] \\ b+s_i < 0 : & \beta[\sum_j p_{ij} f_{t+1}^T(m-[g_L^*-(b+s_i)], g_L^*, j, x+1)] \end{cases} \end{cases}$$

$c^*_{tdix} = \text{argmax} \quad h_t^T(d,i,x)$;

End;

In order to use this result, we can implement the simple algorithm shown in Figure 7. This would allow solving for larger problems, with a larger number of transaction intervals, for example. A larger number of intervals will result in a finer granularity of modeling transaction intervals and average transaction amounts.

5. Implementation Issues

The simplifications due to discrete thresholds and the efficiencies gained by application of the above algorithm for reduced search result in the model taking less than a minute on a 1.5GB personal computer for a problem with 7 transaction intervals for each value of initial MMDA balance *A*. However, a few additional steps may be taken to simplify implementation.

Account Segmentation: Notwithstanding the fact that the model is fast and efficient, it may not be practical to run the model for each account holder. Therefore, similar accounts may be grouped into segments and the model run for each segment instead. Appropriate segmentation variables may include the average MMDA and BTA balance for the account, and transaction behavior (such as number of transactions per month, average size of net transactions and the volatility of net transaction amounts).

Balance Segmentation: Even within an account segment it is not prudent to have a threshold table as shown in Figure 6 for each value of balance. That would require numerous tables and will make implementation difficult. Since ranges of balance will have similar threshold amounts, it makes eminent sense to group balance amounts into segments and have a single lookup table of thresholds for each balance segment. Another approach may be to use a regression model to relate threshold values developed at different balance segments to the total balance level in each segment. This regression equation could then be used to return a threshold value for any possible total balance on a continuous scale.

Scaling of dollar amounts: Another simplification would be to scale s_i and the beginning of the month MMDA amount A using some convenient scaling factor. For example, in a 5 transaction interval case, if the s_i are -2100, -150, 0, 300 and 3750 respectively, and A is 1500, then 150 may be a convenient scaling factor (the greatest common divisor of the absolute values of s_i, except for the 0) that coverts s_i into -14, -1,0,2 and 25 respectively, and A to 10. The thresholds should then also be scaled using the same factor. In this example, a threshold g_1 of 150 will get converted to 1. It may be convenient to use $g_2,\ldots,g_z$ that are multiples of g_1, for example, g_2=2, g_3=3, etc. We have found that if the numbers do not scale exactly, approximate scaling works well too.

Modeling customer behavior: An understanding of customer behavior is critical to determination of optimal thresholds. For retail banks with predominantly household accounts, there are some transactions that are more predictable than needs to be modeled by way of a transition matrix. For example, income may be deposited biweekly and is predictable, mortgage payments are predictable, credit card payments are on cycle dates are predictable at least for date, if not for amount. Even the amount can be predicted with some accuracy. Similarly, utility bills are at known dates and fairly stable. Only other sundry and less predictable expenses need to be modeled using the transition matrix. Our stochastic dynamic approach can easily accommodate these refinements to make the model more efficient.

6. Conclusions

We have presented a simple stochastic dynamic programming model for determining the optimal thresholds to reduce sterile reserves in retail banks. We have also presented structural results to make the search for thresholds more efficient, allowing for solving larger problems. In simulations using real data, our model was found to be effective in reducing sterile reserves and is being implemented.

Proposed legislation, if passed by the Congress and signed into law, might make obsolete retail deposit sweep programs by increasing the number of transfers allowed per month or allowing the Federal Reserve to pay interest on all Federal Reserve Bank deposits. Using our model it is straight forward to assess the impact of these changes. Increasing the number of allowed monthly transfers from 6 to 24 unambiguously relaxes a binding constraint, resulting in a reduced level of sterile reserves and increased earnings. The effect of the measure to pay interest on reserve balances with the Fed depends on how much interest is actually paid. Most proposals constrain the Fed to paying significantly less than the overnight federal funds rate. To the extent that the rate is less than the bank's expected interest-rate spread, the need for judicious sweeps would still exist. Both of these measures, if passed, would reduce the earnings realized by banks from our model in relative terms (since threshold amounts will be less critical), but our model would still be valuable in determining effective sweep mechanisms.

References

Abernathy, W. 2003. Testimony of Asst. Secretary for Financial Institutions, US Department of Treasury, on Business Checking Accounts and Reserve Balances before the Subcommittee on Financial Institutions and Consumer Credit of the Committee on Financial Services of the US House of Representatives. March 5. http://www.ustreas.gov/press/releases/js85.htm Downloaded 3/28/2005

Anderson, R. 2002a. Federal Reserve Board Data on OCD Sweep Account Programs. Federal Reserve of St. Louis, http://research.stlouisfed.org/aggreg/swdata.html, Downloaded 3/28/2005.

Anderson, R. 2002b. “Retail Sweep Programs and Money Demand.” Federal Reserve Bank of St. Louis *Monetary Trends,* November. Available at: <http://research/pubs/mt/pdf/200211.pdf>

Anderson, R., Rasche R.H. 2001. Retail Sweep Programs and Bank Reserves. *Federal Reserve Bank of St. Louis Review*, January/February, **83**(1), pp. 51-72. http://research.stlouisfed.org/publications/review/01/0101ra.pdf, Downloaded 3/28/2005

Bennett P., Hilton S. 1997. Falling Reserve Balances and the Federal Funds Rate. *Current Issues in Economics and Finance,* **3** (5), 1-6, Federal Reserve Bank of New York. http://www.newyorkfed.org/rmaghome/curr_iss/ci3-5.pdf. Downloaded 3/28/2005.

Clouse, J. A., J. P. Dow, Jr. 2002. A Computational Model of Banks’ Optimal Reserve Management Policy. *Journal of Economic Dynamics and Control,* **26**, 1787-1814.

Derman C., Lieberman G.J., Ross S.M.1972. A Sequential Stochastic Assignment Problem. *Management Science*, **18**, 7, 349-355.

Furfine, Craig H. 2000. Interbank Payments and the Daily Federal Funds Rate. *Journal of Monetary Economics*, **46**, 535-53.

Ireland, O. 1996. Federal Reserve Board Legal Interpretations from 4/29/96. Federal Reserve Board. http://www.federalreserve.gov/boarddocs/legalint/FederalReserveAct/1996/19960429/ Downloaded 3/28/2005.

Maus, E. 2003. Testimony of America’s Community Bankers on Interest on Business Checking Accounts before the Subcommittee on Financial Institutions and Consumer Credit of the Committee on Financial Services of the US House of Representatives. March 5. http://financialservices.house.gov/media/pdf/030503em.pdf Downloaded 3/28/2005

Menzies, R.M. 2003. Testimony of Independent Community Bankers of America on Interest on Business Checking Accounts before the Subcommittee on Financial Institutions and

Consumer Credit of the Committee on Financial Services of the US House of Representatives. March 5. http://financialservices.house.gov/media/pdf/030503ms.pdf Downloaded 3/28/2005

Puterman, M.L. 1994. Markov Decision Processes, Discrete Stochastic Dynamic Programming, John Wiley and Sons, New York.

Ross, S. 1983. Introduction to Stochastic Dynamic Programming. Academic Press, New York.

Wrase J.M. 1998. Is the Fed Being Swept Out of (Monetary) Control? *Business Review*, Nov-Dec, 1-10. Federal Reserve Bank of Philadelphia. http://www.phil.frb.org/files/br/brnd98jw.pdf . Downloaded 3/28/2005

Appendix 1: The Federal Reserve and Retail Deposit Sweep Programs

Retail deposit sweep programs were introduced by a large U.S. commercial bank in January 1994. In such a program, the bank creates money market deposit accounts that are invisible to its customers and re-classifies customers' transaction deposits as MMDA (that is, savings) deposits so as to avoid the 10 percent marginal statutory reserve requirement ratio on transaction accounts; savings deposits, including MMDA, are currently subject to a zero percent reserve requirement ratio. The 1982 Garn-St. Germain act, which created the money market deposit account as a deposit instrument, prohibited the Federal Reserve Board from classifying such deposits as transaction accounts for reserve-requirement purposes. This prohibition is an essential element of the operation of retail deposit sweep programs. Historically, banks have introduced many inventory management schemes to reduce the amount of deposits at Federal Reserve Banks. But, prior to retail deposit sweep programs, the Federal Reserve Board promptly ended all such schemes. In most cases, it ruled that the scheme was primarily intended to evade statutory requirements and ordered that the affected deposits be subject to the same reserve requirement as other transaction deposits. One of the more clever exploits in sweeps occurred during the early 1990s. In that case, a large bank began sweeping transaction deposits into large-denomination time deposits with seven-day maturities. By placing one-seventh of the swept funds into each of seven different large time deposits maturing on different days of the week, it sought to provide transaction services to its customers while reducing the amount of end-of-day balances subject to statutory reserve requirements. The Federal Reserve Board ruled that this was solely a scheme to evade reserve requirements (as, of course, it was), and re-classified the involved large time as transaction deposits subject to the same reserve requirements as the initial transaction deposits. With its earnings motive destroyed, the bank dropped the sweeping activity.

The six-transfer rule for MMDA is mandated by the Garn-St. Germain act, and promulgated to banks as part of the Federal Reserve's Regulation D. Specifically, the law sets a maximum of six pre-authorized plus third-party payments per month. It does not limit the number of in-person customer withdrawals or transfers. If a customer makes more than 6 such transfers during a month (say, via check), a bank may impose a penalty on the customer but the bank need not report such a violation to the Federal Reserve, which, in turn, is unlikely to impose any penalty on the bank. If a bank regularly or frequently permits more than six such withdrawals by taking no action to require customer compliance with the statutory limit, the MMDA balances may be classified by the Federal Reserve as transaction deposits (rather than MMDA) and hence subject to the same reserve requirements as other transaction deposits. The bank also may be subject to administrative penalties.

We model statutory reserve requirements as a constant 10 percent marginal requirement. Actual statutory requirements on transaction deposits are tiered, with a zero percent ratio applying to approximately the first $5 million, a 3 percent ratio applying to approximately the next $50 million, and, since 1992, a 10 percent ratio applying to the remainder. These break points are indexed to growth in total bank deposits and change annually. In 1980, the highest marginal reserve requirement was 12 percent; it was reduced to 10 percent in 1992. For larger banks, the lower tiers matter not at all.

The role of the Federal Reserve Board, which sets statutory reserve requirements, was an important issue with some analysts and consultants during their early years of retail deposit sweep programs. The 1980 Monetary Control Act set a zero ratio for personal time and savings deposits. In 1990, the ratios for non-personal time and savings deposits, and for Eurocurrency liabilities, were reduced from 3 percent to zero. Strictly speaking, these deposit liabilities remain subject to Federal Reserve statutory reserve requirements but at a zero ratio, and the Federal Reserve Board retains the power to increase these percentages in the future, if desired. Are the invisible MMDA created as part of a retail deposit sweep program personal or non-personal accounts? Some analysts argued that the accounts, although created by banks without their customers' knowledge, are personal savings deposits and hence protected from a non-zero ratio by the Monetary Control Act. Other analysts argued that the accounts were created solely by the bank as a component of the bank's inventory management system for deposits at the Federal Reserve Banks, should be regarded as non-personal deposits which, if the Federal Reserve Board desired, could be made subject to a considerably higher ratio.

Although it is clear that operation of a retail deposit sweep program reduces the amount of sterile reserves held by banks, it is difficult to estimate how much of the related earnings have been retained by banks and how much has been realized by consumers via market competition (even though the underlying inventory technology is invisible to the customer). Recent data suggest that some portion has been realized by consumers; see Anderson (2002b). It has been reported that, more recently, some business customers have become aware of the sweep-into-MMDA scheme, and have requested—and received—a significant share of the resulting earnings. It seems obvious that virtually all households are unaware of the operation of such sweep programs.

There are many additional and complex aspects of bank reserve management that interact with managing deposits at Federal Reserve Banks and retail sweep programs, but these are beyond the scope of this paper. Reserve balances, shown in Figure 2, comprise only about half of the deposits held by banks at the Federal Reserve. The other half are similar deposits but encumbered by clearing balance contracts with the Federal Reserve; under the terms of such contracts, the banks earn interest on the deposits if actual amounts meet or exceed contractual minimums, but incur penalties for shortfalls. The fungibility of deposits makes reserve management complex because deposits held to satisfy clearing balance contracts can, instead, be used to satisfy statutory reserve requirements. In fact, Federal Reserve accounting rules require that available deposits at Federal Reserve Banks first be applied to satisfy statutory reserve requirements and only the excess of such deposits may be applied toward satisfying a clearing balance contract. Modeling capable of handling the dynamics of this stochastic process are beyond the models in this paper.

Appendix 2: Net transactions and MMDA and BTA balances for an account when a threshold of $3000 is used for withdrawals and deposits

	Start of day			5 mins before EOB	Transfer to		End of day		Transfer
Day	MMDA	BTA (5 mins before EOB)	Net transaction	BTA	MMDA	BTA	MMDA	BTA (5 mins before EOB)	Count
1	75000	0	-3000	-3000	0	6000	69000	3000	1
2	69000	3000	-750	2250	0	0	69000	2250	1
3	69000	2250	-25500	-23250	0	26250	42750	3000	2
4	42750	3000	1000	4000	1000	0	43750	3000	2
5	43750	3000	0	3000	0	0	43750	3000	2
6	43750	3000	-2500	500	0	0	43750	500	2
7	43750	500	0	500	0	0	43750	500	2
8	43750	500	500	1000	0	0	43750	1000	2
9	43750	1000	0	1000	0	0	43750	1000	2
10	43750	1000	-30500	-29500	0	32500	11250	3000	3
11	11250	3000	0	3000	0	0	11250	3000	3
12	11250	3000	-1500	1500	0	0	11250	1500	3
13	11250	1500	500	2000	0	0	11250	2000	3
14	11250	2000	0	2000	0	0	11250	2000	3
15	11250	2000	50900	52900	49900	0	61150	3000	3
16	61150	3000	-100	2900	0	0	61150	2900	3
17	61150	2900	-4200	-1300	0	4300	56850	3000	4
18	56850	3000	0	3000	0	0	56850	3000	4
19	56850	3000	0	3000	0	0	56850	3000	4
20	56850	3000	10500	13500	10500	0	67350	3000	4
21	67350	3000	-9500	-6500	0	9500	57850	3000	5
22	57850	3000	34000	37000	34000	0	91850	3000	5
23	91850	3000	-11500	-8500	0	86350	0	86350	6
24	0	86350	-100	86250	0	0	0	86250	6
25	0	86250	0	86250	0	0	0	86250	6
26	0	86250	0	86250	0	0	0	86250	6
27	0	86250	-300	85950	0	0	0	85950	6
28	0	85950	12500	98450	0	0	0	98450	6
29	0	98450	0	98450	0	0	0	98450	6
30	0	98450	-2000	96450	0	0	0	96450	6

Appendix 3: Net transactions and MMDA and BTA balances for an account when a threshold of $5000 is used for withdrawals and deposits

	Start of day			5 mins before EOB	Transfer to		End of day		Transfer
Day	MMDA	BTA (5 mins before EOB)	Net transaction	BTA	MMDA	BTA	MMDA	BTA (5 mins before EOB)	Count
1	75000	0	-3000	-3000	0	8000	67000	5000	1
2	67000	5000	-750	4250	0	0	67000	4250	1
3	67000	4250	-25500	-21250	0	26250	40750	5000	2
4	40750	5000	1000	6000	1000	0	41750	5000	2
5	41750	5000	0	5000	0	0	41750	5000	2
6	41750	5000	-2500	2500	0	0	41750	2500	2
7	41750	2500	0	2500	0	0	41750	2500	2
8	41750	2500	500	3000	0	0	41750	3000	2
9	41750	3000	0	3000	0	0	41750	3000	2
10	41750	3000	-30500	-27500	0	32500	9250	5000	3
11	9250	5000	0	5000	0	0	9250	5000	3
12	9250	5000	-1500	3500	0	0	9250	3500	3
13	9250	3500	500	4000	0	0	9250	4000	3
14	9250	4000	0	4000	0	0	9250	4000	3
15	9250	4000	50900	54900	49900	0	59150	5000	3
16	59150	5000	-100	4900	0	0	59150	4900	3
17	59150	4900	-4200	700	0	0	59150	700	3
18	59150	700	0	700	0	0	59150	700	3
19	59150	700	0	700	0	0	59150	700	3
20	59150	700	10500	11200	6200	0	65350	5000	3
21	65350	5000	-9500	-4500	0	9500	55850	5000	4
22	55850	5000	34000	39000	34000	0	89850	5000	4
23	89850	5000	-11500	-6500	0	11500	78350	5000	5
24	78350	5000	-100	4900	0	0	78350	4900	5
25	78350	4900	0	4900	0	0	78350	4900	5
26	78350	4900	0	4900	0	0	78350	4900	5
27	78350	4900	-300	4600	0	0	78350	4600	5
28	78350	4600	12500	17100	12100	0	90450	5000	5
29	90450	5000	0	5000	0	0	90450	5000	5
30	90450	5000	-2000	3000	0	0	90450	3000	5